

**CHAPTER  
3****Risk and Return  
Analysis****3.1. RETURNS****3.1.1. Concept of Return**

The driving force behind an investment by any investor is, obviously, to have maximum possible level of returns, which poses various limitations including the risks associated with the investment. Return is a form of reward and a motivator, which encourages an investor for making the investment. The significance of returns in any investment decision can be determined from the following factors:

- 1) It helps potential investors in making comparison of returns expected from alternative investment opportunities.
- 2) Measurement of historical (past) returns places the investors in a position to take stock of better performance in the areas of investment.
- 3) Measurement of historical returns also facilitates the assessment of future returns.

**3.1.2. Components of Return**

There are three components of return, composed in an investment:

- 1) **Current Return:** The current return on any investment is the recurring stream of cash inflow (income), in the form of dividend or interest. Current return is measured as the recurring income in relation to the initial investment (beginning price of investment).
- 2) **Capital Return/Gain:** The capital return on an investment could be the change (appreciation or depreciation) in the acquisition cost of the original investment. This is also referred as the 'Capital Gain'. 'Capital Gain' is the predominating constituent in the case of assets like equity shares.

Thus, the total return on any investment (assets) is reflected as follows:

$$\text{Total Return} = \text{Current Return} + \text{Capital Return}$$

An important aspect in respect of the two constituents of return can be stated as, "the current returns can be either zero or positive (it can not be negative), whereas the capital returns can be negative, zero or positive".

- 3) **Total Return:** Total return is the actual rate of return of an investment or a pool of investments over a given evaluation period. Total return includes interest, capital gains, dividends and distributions realized over a given period of time.

Total Return = Yield + Price Change

Or

$$\text{Total Return} = \frac{D_t + [P_t - (P_{t-1})]}{P_{t-1}}$$

where,

TR = Total Return

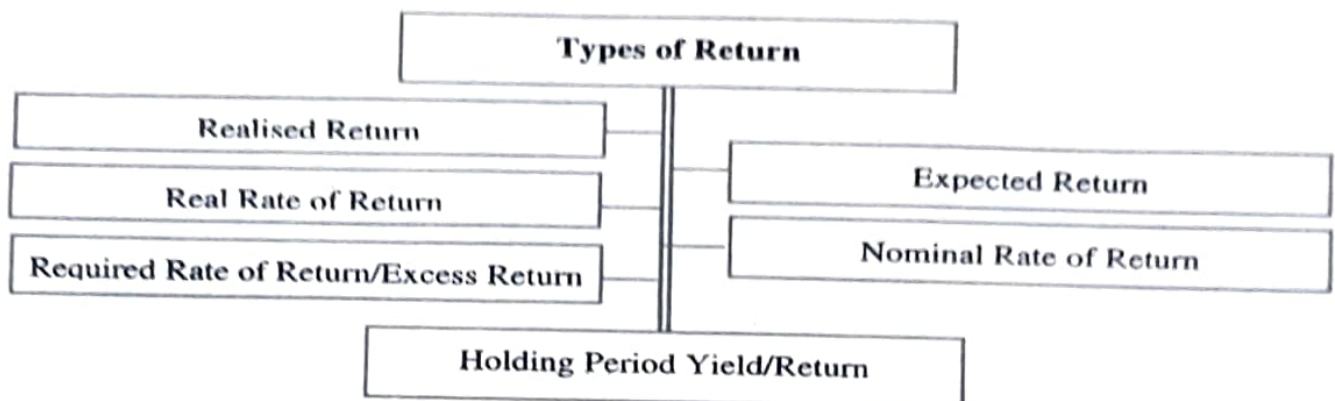
$D_t$  = Cash dividend at the end of the time period  $t$

$P_t$  = Price of stock at time period  $t$

$P_{t-1}$  = Price of stock at time period  $t-1$

### 3.1.3. Types of Return

Two types of return are as follows:



#### 3.1.3.1. Realised Return

Realised return is the return on an investment that has been actually earned. There is an element of certainty and absence of risk in this type of return. An example of bank deposit may be taken to illustrate the above. ₹5,000 deposited in a bank as Fixed Deposit for one year @ 10% would be worth ₹5,500 on completion of one year. Realised return in this case is ₹500.

#### 3.1.3.2. Expected Return

Expected return is the return on an investment, which is anticipated or expected over a period of time in future. There is an element of uncertainty and existence of risk in this type of return, as the expectation may or may not come true. The uncertainty or risk in respect of expected return and its timing get compensated by the substantial high rate of return. The expected rate of return is the weighted average of all the possible returns multiplied by their respective probabilities. In symbolic language, it may be expressed as under:

$$E(R) = \sum_{i=1}^n r_i p_i$$

where,

$E(R)$  = Expected return from the stock,

$r_i$  = Return from stock under state  $i$ ,

$p_i$  = Probability that the state  $i$  occurs,

$n$  = Number of possible state.



**For example,** the table below shows the calculation of expected return of the stock for the given return and probability associated with it:

Return ( $r_i$ )	Probability ( $p_i$ )	Weighted Average Return ( $r_i \times p_i$ )
8	.15	1.20
9	.20	1.80
10	.30	3.00
11	.20	2.20
12	.15	1.80
	1.00	$\Sigma E(r) = 10$

### 3.1.3.3. Real Rate of Return

A real return is the rate of return that investors receive after the impact of inflation. Inflation has a negative impact on investments because money will buy less in the future. To keep real return constant (in other words, to maintain buying power), investor must actually earn more money in nominal terms.

Traditionally, investors have calculated the real return ( $r_r$ ) as simply the nominal return ( $r_n$ ), or the return receive minus the inflation rate ( $\pi$ ). This method is incorrect. It is preferable to always use the following formula:

$$(1 + \text{nominal return } (r_n)) = (1 + \text{real return } (r_r)) \times (1 + \text{inflation } (\pi))$$

To solve for the real return, divide both sides of the equation by  $(1 + \text{inflation } (\pi))$ .

$$\{1 + \text{nominal return } (r_n)\} / \{1 + \text{inflation } (\pi)\} = \{1 + \text{real return } (r_r)\}$$

$$\text{Real return } (r_r) = \{1 + \text{nominal return } (r_n)\} / \{1 + \text{inflation } (\pi)\} - 1$$

A real return is the return on an investment, less the reduction in its value as a result of inflation.

$$\text{Real return} = [(1 + r) \div (1 + i)] - 1 \approx r - i$$

where,

$r$  = Nominal return over a period

$i$  = Inflation over the period

The nominal return is simply the percentage increase measured in currency. The approximation is accurate enough for valuation, except when inflation is very high.

Real rates are important as they tell what the actual increase in value was, and how much of a return was just the effect of inflation. Real interest rates are the most widely used type of real return.

### 3.1.3.4. Nominal Rate of Return

Nominal rate of return is a return on investment without any adjustment for inflation. A high nominal return does not guarantee a real profit. **For example,** if the nominal return on an investment is 7% and the inflation rate is 4%, the real rate of return is only 3%. The annual percentage return realized on an investment, which is adjusted for changes in prices due to

inflation or other external effects. This method expresses the nominal rate of return in real terms, which keeps the purchasing power of a given level of capital constant over time. Adjusting the nominal return to compensate for factors such as inflation allows investors to determine how much of their nominal return actually real return is.

**Nominal Return versus Real Return:** Nominal returns are not adjusted for inflation; real returns are. Sometimes, it is useful to know if portfolio is keeping ahead of inflation. Difference between real return and nominal return can be understood by an **example**, if one year return was 2 per cent, and inflation was 3 per cent. Real return is simply investor's return minus inflation. In this example, real return is -1 per cent (2 per cent - 3 per cent) and unadjusted return of 2 per cent is called nominal return. **For example**, If bank pays interest of 5% per year on the funds in savings account. If the inflation rate is currently 3% per year, then the real return on savings today would be 2%. In other words, even though the nominal rate of return on savings is 5%, the real rate of return is only 2%, which means that the real value of savings only increases by 2% during a one-year period.

### 3.1.3.5. Required Rate of Return/Excess Return

Excess return, also known as alpha, is a measure of how much a fund has under or outperformed the benchmark against which it is compared. It can be calculated under the capital asset pricing model (CAPM).

This important financial return metric allows investors to compare sets of funds against each other, in order to see which fund has generated greater excess returns. Of course, there are a number of other measures of performance and so while one investor may favour a fund with high excess returns, others may view the same strategy as too risky. By using excess return (alpha) and volatility risk (as described by beta), investors can evaluate a fund's total performance on a risk-adjusted basis. Excess return can be positive (denoting outperformance relative to the benchmark) or negative (indicating underperformance). It is a measure of the portion of a fund's return which is not explained by overall market returns. As such, an excess return analysis can help determine whether outperformance is the result of a portfolio manager's skill, or simply the result of movements in stock markets. Similarly for alternative indices, excess return can gauge the quality of the index's strategy and underlying stock-selection rules.

#### Calculation of Excess Returns

CAPM, which calculates expected total return, can be reconstructed to show excess return:

$$\text{Excess return} = R_f + \beta[E(R_m) - R_f] - Tr$$

where,

$E(R_j)$	=	expected return on security j
$R_f$	=	risk-free return
$\beta$	=	beta of security j
$E(R_m)$	=	expected return on market portfolio
$Tr$	=	actual or total return from the security



## 3.2. RISK

### 3.2.1. Concept of Risk

'Risk' has been defined in different manners, though the underlying concept remains the same. To have a better understanding of the concept, some of the views are discussed in the following paragraphs:

In simple terms, risk may be defined as the possibility of suffering an injury or loss. It is present in every field or situation. In the context of 'Business or Financial World', it represents the uncertainty associated with an investment. In other words, risk is the possibility that the actual return on an investment may be different from the expected return

According to Fischer and Jordon, risk is the, "The variability of return around the expected average is thus a quantitative description of risk".

### 3.2.2. Upside and Downside Risk

#### 3.2.2.1. Upside Risk

Upside risk measures the extent to which the value of a stock or other investment might go up beyond expected levels. It's when investor stands to get an outcome better than the benchmark against which it's being measured, such as the FTSE 100 index. Risk doesn't just pose threats, it also presents opportunities. Upside risk flies in the face of the stereotype that all risk has the potential for things to get worse. It gives an investor an opportunity to plan what to do, if he get lucky or take positive risk.

An investor can either use technical analysis or fundamental analysis to predict the future price of a stock. A higher upside means the stock has more value than is currently reflected in the price.

Upside beta can be used to measure upside risk. It's calculated using data only from days when an investment's benchmark has gone up. Upside beta is the stock beta measured for periods when the benchmark return is positive. This will allow an investor to understand which stocks have historically generated the highest returns during market upswings. Here, the variance is defined as the variance for periods such that the market return is greater than zero.

Upside risk is the opposite of downside risk, which estimates how much an investor stand to lose.

#### 3.2.2.2. Downside Risk

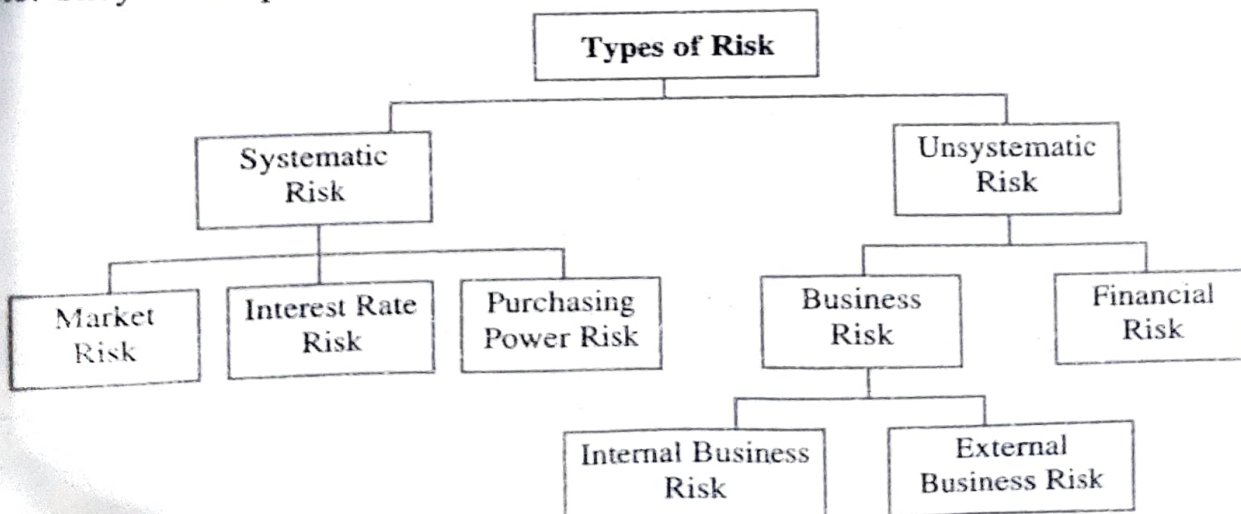
Downside risk refers to the probability that an asset or security will fall in price. It is the potential loss that can result from a fall in the price of an asset as a result of changing market conditions. Downside risk can also be described as a statistical measure that aims to calculate and quantify the worst-case loss that can result from uncertainty in the difference between expected return and realized return in cases where market conditions deteriorate.



- 2) **Competitive Risk:** This risk generally arises as per the behaviour of competitors. A positive behaviour reflects revenue and growth and a negative behaviour of competitors show loss and dropping in product line. Such risks are determined in project management analysis reports. However, actual outcome of behaviour may differ from anticipation. This risk is difficult to be diversified at the firm level as it may impact more than one project. The investors may mitigate such risk by diversifying their personal portfolios. **For example**, if Coca-Cola makes mistakes in understanding risk from its competitors, its shareholders may buy shares of competitor companies to balance their portfolio.
- 3) **Industry Specific Risk:** This source of risk affect the cash inflows and cash outflows of a specific industry. Following are the three main sources of industry specific risk:
  - i) **Technology Risk:** Change in technology may bring about unexpected changes in competition or markets.
  - ii) **Legal Risk:** This category shows the effect of change in regulations and laws.
  - iii) **Commodity Risk:** It shows the effects of price changes in products and services which are used or produced by the industry.
- 4) **International Risk:** This source of risk is faced by the firms which engage in international markets. It may draw revenue or incur costs in foreign markets. Fluctuations in exchange rate or political ups and downs are among various reasons behind such risks. **For example**, Coca-Cola was adversely affected due to the difference in exchange rate in home currency to other currency.
- 5) **Market Risk:** This is the final source of risk and comprises of macro-economic factors which affect all the companies and projects. **For example**, any change in interest rate will affect existing as well as potential projects. Term structure also impacts cashflows and discount rate. This kind of risk cannot be diversified as all risky investments are exposed to such risks.

### 3.2.4. Types of Risk

There are a number of risks a business organisation is exposed to, like 'Market Risk', 'Interest Rate Risk', 'Country Risk', 'Business Risk', 'Financial Risk', etc. They can be put under two broad categories:





### 3.2.4.1. Systematic Risk

The risk, which is inherent to the entire market or system, is termed as '**Systematic Risk**' or '**Uncontrollable Risk**'. It is also referred to as "**Undiversifiable Risk**," as it affects the entire market and not one specific stock or industry. Further, it is associated with economic, social, political and legal aspects of all the securities in the economy. These factors are capable of exerting pressure on all securities in the market in such a manner that all of them would move or change accordingly. During a period of economic upswing, prices of all the securities would move northwards, whereas during a recession period, prices of all the securities would move southwards.

#### Components of Systematic Risk

'Systematic Risk' is a broad category of risk and may be classified into following sub-categories:

- 1) **Market Risk:** Market risk is the risk that the value of an investment will decrease due to movements in market factors. The reason for such uncertainty is market forces represented in two markets, viz. 'Bull Market' and 'Bear Market'. When the economy is booming and other factors and sentiments are positive, the 'Security Index' has a tendency to move upwards and keeps on moving upwards for a considerable period of time. This is a typical 'Bull Market'. The 'Bear Market', on the other hand, is characterised by a tendency of the 'Security Index' to decline from the peak of 'Bull Market' to a lower level. The lowest point of 'Bear Market' is termed as the 'Trough' and it is from this point that economic recovery or 'Bull Market' starts.
- 2) **Interest Rate Risk:** Interest rate risk is the possibility of an unexpected change in interest rates prevailing in the market, which affects the value of an investment adversely. Generally, the value of debt instruments like bonds, debentures, commercial papers, etc. is directly affected by 'Interest Rate Risk'. The movements in market interest rates are prompted by the 'Monetary Policy' of the 'Central Bank' of the country (viz. Reserve Bank of India), which leads to changes in the interest rates of 'Treasury Bills' and 'Government Bonds'.
- 3) **Purchasing Power Risk:** 'Purchasing power risk' is the possible reduction in the purchasing power of the expected returns. Due to a high rate of inflation, there is erosion in the purchasing power of money, which results in decrease in the returns. Increase in the rate of inflation is swifter than the increase in the value of investment.

This results in punishment to the investors in the form of reduced return on their investment. Rising rate of inflation poses a threat to the investor as it is a risk or possible loss for him. Inflation may be 'Demand Pull' (characterised by an increase in aggregate demand and supply cannot keep pace with the demand) or 'Cost Push' (characterised by a reduction in the supply of goods and services, due to increased cost of production).



### 3.2.4.2. Unsystematic Risk

Also known as 'Diversifiable Risk', as it can be managed and controlled, unlike 'Systematic Risk', which is difficult to be managed and controlled, as it is widespread and covers the entire system. 'Unsystematic Risk' may be specific to an industry or company and is caused due to one or more of the following:

- 1) Lack of managerial ability,
- 2) Technological advancement in the process of production,
- 3) Procurement of raw material,
- 4) Lack of human resources, and
- 5) Change in consumer preferences.

Specific causes need to be probed, company/industry-wise separately, to pinpoint and take remedial action.

Changes in consumer preferences matter more for the industries engaged in the production of consumer durable goods like air-conditioners, televisions, washing machines, refrigerators, etc. than the industries engaged in the production of cement, iron, steel, pharmaceuticals, etc. Technological changes directly influence the IT industry. FMCG or consumer durables industries remain rather less affected by the technological changes.

Other factors leading to unsystematic risk may relate to:

- 1) Financial leverage, i.e., Debt-Equity Ratio (DER) of the companies varies from each other,
- 2) Sources of funds and repayment of the loans also involve an element of risk, and
- 3) Sometimes the product may go out of fashion or become obsolete, which is a threat or risk for the company or industry.

### Components of Unsystematic Risk

The unsystematic risk may be categorised/sub-categorised as follows:

- 1) **Business Risk:** Business risk is a part of the unsystematic risk, which basically comes from the operational activities of the business. Due to certain inbuilt deficiencies in the operations of a company, its competitive advantage over the rivals is lost affecting the force of its expansion and/or smooth flow of revenue. This fact is ultimately expressed in the form of poor 'Operating Income' and 'Expected Dividends' of the company.

Business risk may arise due to internal causes or external causes and accordingly it is categorised under two heads:

- i) **Internal Business Risk:** Internal business risk is related to the operational effectiveness of a company. The operational effectiveness of a company is measured in terms of the level of its targeted achievements and keeping the promises made to its investors. Operational effectiveness varies from company to company, and is influenced by the following factors:



- a) **Sales Variation:** Maintenance of sales figures are of utmost importance for a company in order to keep the operational income at an appropriate level. An ongoing monitoring is required in this regard, so as to ensure that the company's competitive edge is not lost to the rivals and customers remain loyal to the company, which calls for a good customer service.
- b) **Research and Development (R&D):** For any business organisation, Research and Development (R&D) plays an important role in ensuring that the style and form of products keep pace with the changing trend, remain competitive, and obsolescence is avoided. Companies, that lack far-sightedness, sometimes, fail to give proper attention to R&D aspect of the business, and as a result their operational capabilities are hit.
- c) **Personnel Management:** The Personnel Management of a company is a crucial contributory factor to its operational competency. Repeated labour unrests or lockouts, etc. in a company have adverse impact on its productivity and lead to revenue and capital loss. Human resource policy of a company needs to be drawn and implemented very carefully. This kind of risk is faced by almost all the companies.
- d) **Fixed Cost:** Fixed cost of production, as the name itself suggests, is fixed and cannot be reduced. If it happens to be at a higher level in the overall cost component of a company, an internal risk may creep in. In case of an emergency, the company would not be able to reduce its fixed cost and reduction in the variable cost would not have a substantial impact on the overall cost. Hence, the high fixed cost is a potential risk for the company.
- e) **Single Product:** A company engaged in the production of a single product reflects lack of diversification on its part and it has higher level of risk. If, due to any unforeseen reason, the demand of that particular product declines, the company has no alternative other than facing the disaster.

On the other hand, if a company has diversified products, impact of decline in the demand of one or two products would not be so disastrous, as the remaining products would take care of the situation and the company will be able to survive and grow in future. Thus, the diversification of products is necessary for a company to sustain in the market.

- ii) **External Business Risk:** External business risks are the risks caused by the circumstances, which are external to a company's business. The company has no control over these circumstances or factors, which are:

- a) **Social and Regulatory Factors:** Regulatory framework and laws relating to different issues like environment, pollution, etc. may act as impediments to the profitability of companies. Various controls



imposed upon by the Government like price control, volume control, import/export control, drug control, etc. are not favourable while conducting business. In general, they tend to reduce profitability of companies. Such risk is more prominent in the public utility sectors, like banking, transport, telecommunication, etc. Telecommunication is a glaring example of impact of control (Government's tariff policy) on the earnings of the players in the sector. Similarly, the profitability of banks is affected by some of the regulatory directions issued on the lending policies.

- b) **Political Risk:** Frequent changes in the Government and its policies have a negative impact on the business environment. The policies framed by one ruling party's Government are co-terminus with the Government of that party. This risk exists especially in the case of foreign investments.
- c) **Business Cycle:** The variations of 'Business Cycle' are prevalent in almost all the industries, which result in the variation in the company's earnings. Slowdown in the economy may result in a reduced output in most of the industries. Impact of the business cycle (alternate 'Recession' and 'Boom') is more pronounced in Cloth Industry and Consumer Durable Goods. During the Boom cycle, their demand is at a high level, whereas during Recession cycle their demand is at a low level.

2) **Financial Risk:** Financial risk of a company is closely related to the manner in which the funds have been raised to design its capital structure. Financial strength of a company depends upon the level and stability of its earnings. An inappropriate 'Capital Structure' may not be able to ensure stable earnings and as a result of such variation in earnings, the company would be exposed to financial risk. A high level of debt component in a company's capital structure indicates the presence of a low cost of capital, which provides financial leverage for the shareholders. The low cost of capital ensures that:

- i) Earnings of the company remain higher than the cost of capital, and
- ii) EPS shows an increasing trend.

Financial risk may be classified into following sub-divisions:

- i) **Credit Risk:** Credit risk arises from the possibility of a borrower defaulting in timely payments, as and when it falls due or makes payment with delay. Failure on the part of a borrower in repayment is a credit risk, which directly affects the profitability of the lender. Even a delay in repayment of borrowed funds carries a cost element with it.
- ii) **Currency Risk:** The possibility of a loss due to unfavourable movements of exchange rate in foreign exchange market, either short-term or long-term, is referred to as the 'Currency Risk'. A company engaged in the business of export, import, forex dealing or has foreign subsidiaries may be a victim of 'Currency Risk'. It may also affect a company which has a competition with a foreign entity in the domestic market.



iii) **Country Risk:** 'Country Risk' is a collection of risks associated with doing business in a foreign country. It varies from one country to another. Some countries have risks, high enough to discourage foreign investment. These risks include political risk, regulatory risk, and economic risk.

- a) **Political Risk:** Political risk arises from the possibility of downfall in the financial health of a country as a result of (i) unstable Government, or (ii) change of Government or (iii) distress regarding Government's actions. Political risk is highest in the countries with political instability. Sudden change in a Government creates an environment of doubt and unpredictability with regard to the new Government's policies about exchange control, treatment with foreign business houses, etc.
- b) **Regulatory Risk:** The risk arising out of the tension that new regulations would be introduced or the existing regulations would be implemented in a more rigorous manner is termed as 'Regulatory Risk'. Possibility of a country's Central Bank to increase the CRR (Cash Reserve Ratio) or SLR (Statutory Liquidity Ratio) requirements for banks operating in that country is an example of the 'Regulatory Risk'.
- c) **Economic Risk:** The possibility of a country's economic conditions becoming negative and having a damaging impact on its financial health, especially on inflation, interest rates, and forex rates, is termed as 'Economic Risk'.
- iv) **Liquidity Risk:** 'Liquidity Risk' is the probability of a security market (bond or stock) being non-marketable, making it difficult for the holders of such security to sell them at a fair price.

### 3.2.4.3. Difference between Systematic and Unsystematic Risk

Basis of Difference	Systematic Risk	Unsystematic Risk
1) <b>Diversification of Risk</b>	The systematic risk cannot be diversified away through investment in domestic securities	The unsystematic risk can be diversified away in domestic securities.
2) <b>Performance</b>	Systematic risk is a macro-economic risk. It is inherent in the performance of the economy as a whole.	The unsystematic risk is firm-specific and so simultaneous investment in other securities may lower it.
3) <b>Measurement</b>	Systematic risk can be measured by Beta.	Unsystematic risk is measured by standard deviation.
4) <b>Reduction of Risk</b>	Systematic risk too can be reduced through international investment as the macro-economic fundamentals vary in different countries.	It cannot be reduced through diversification in the domestic market.



if the large portion of the risky investment does not provide any return. The analysis of this data would result in deriving the qualitative assessment of the risk aversion and then the appropriate asset class of the investor can be known. Contrary to this, an indirect measurement is done by asking question regarding the individuals' attitude in selecting the asset class at a given age, income and income to be invested by the investor.

The analysis of this data gives helps in arriving at the results regarding the individuals' feeling about the investment in the risky securities. In order to determine that which family needs this management tool for their retirement we have to begin the process from the indirect measurement of risk aversion by keeping in view the direct measurements of risk aversion and end it up by comparisons and implications.

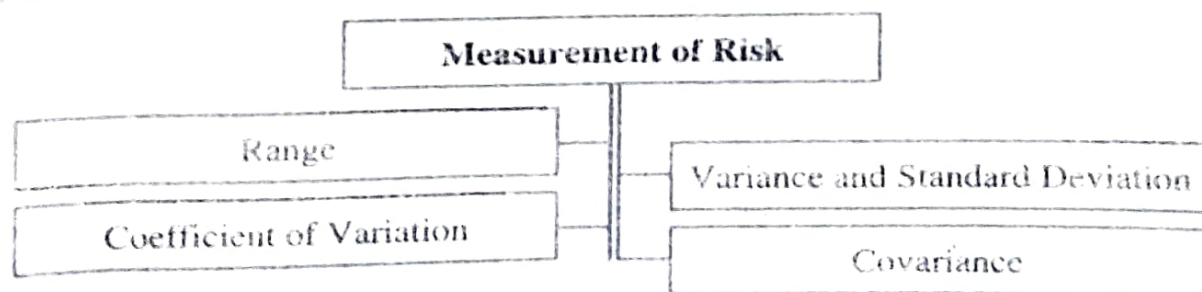
Table		
Age	Wealth (₹)	Equity
25	15,000	60%
35	1,50,000	50%
45	4,00,000	50%
55	7,00,000	20%
65	10,00,000	10%
75	8,00,000	5%

In the above table the age of the investor is given at which he is supposed to have his own income to the retiring age. The wealth column represents the income available for investment. From the table it can be seen that at the age of 25 years the attitude of the family is to save the money. With the passage of time the investment grows and this investment grows even more if both the spouses are earning. This makes the largest wealth level at the age of family retirement but it decreases during the retirement years.

The investor at the young age is a risk-lover and prefers in investing the securities which have more risks. So he invests in the equity shares in order to earn high rate of return rather than the securities which offer fixed income. And after the age of 45 the investors tend to become risk avoider and invest less percentage in the equity shares.

#### 4.2.6. Measurement of Risk

Various statistical techniques are applied to measure the approximate level of risk, which are as follows:





### 3.2.6.1. Range

The difference between two extreme ends of returns, viz. maximum and minimum is taken as 'Range', which reflects the quantum of risk. It may be expressed in mathematical terms as under:

$$\text{Range} = \text{Maximum Value of Return} - \text{Minimum Value of Return}$$

This can be illustrated by an example of two shares 'A' and 'B': Suppose the current market price of share 'A' is ₹100. Forecasting of its future price is ₹200 under favourable market conditions and ₹50 under depressed market conditions. For another share 'B', current market price is same at ₹100. Future maximum and minimum prices are forecasted at ₹300 and ₹25 respectively. The 'Range' of shares 'A' and 'B' comes to be ₹100 (₹200 – ₹100) and ₹275 (₹300 – ₹25) respectively. On the basis of the 'Range', share 'B' would be considered as more risky. This technique (Range), however, has limitations, as it does not take into account the probability of extra-ordinary or extreme events taking place in the future. In order to have a better assessment of risk, the range of values are not enough to be considered, the probabilities of different values also needs to be considered.

### 3.2.6.2. Variance and Standard Deviation

This technique involves measurement of the values of the variables around its mean. In other words, it is the 'square root of the sum of the squared deviations from the mean' divided by the 'number of observation'. Two companies may have the same arithmetic mean of the returns, but there may be wide variations in returns. The variance and standard deviation of a historical return series may

$$\text{be expressed as follows: } \sigma^2 = \left[ \frac{\sum_{i=1}^n (R_i - \bar{R})^2}{n - 1} \right] \quad \sigma = \sqrt{\sigma^2}$$

Where,  $\sigma^2$  = Variance of return

$\sigma$  = Standard deviation of return

$R_i$  = Return from the stock in period  $i$  ( $i = 1, \dots, n$ )

$\bar{R}$  = Arithmetic mean of return

$n$  = Number of periods

**Example 3:** Find out variance and standard deviation for company A and B.

#### Stocks

Company A			Company B		
Return	Probability	Weighted Average Return	Return	Probability	Weighted Average Return
( $r_i$ )	( $p_i$ )	( $r_i \times p_i$ )	( $r_i$ )	( $p_i$ )	( $r_i \times p_i$ )
8	0.15	1.20	9	0.30	2.70
9	0.20	1.80	10	0.40	4.00
10	0.30	3.00	11	0.30	3.30
11	0.20	2.20			
12	0.15	1.80			
	1.00	$\Sigma E(r) = 10$		1.00	$\Sigma E(r) = 10$



Stocks of Company 'A' and Company 'B' have identical expected average returns. But the spread is different. The range in Company 'A' is from 8 to 12 and for Company 'B' it ranges between 9 to 11 only. The range does not imply greater risk. The spread or dispersion can be measured by standard deviation.

**Solution:** To find out the variation, standard deviation technique is applied.

$$\sigma = \sqrt{\sum_{i=1}^N p_i [r_i - E(r)]^2}$$

Company A					Company B				
$r_i$	$p_i$	$r_i - E(r)$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$	$r_i$	$p_i$	$r_i - E(r)$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$
8	0.15	-2	4	0.6	9	0.30	-1	1	0.30
9	0.20	-1	1	0.20	10	0.40	0	0	0
10	0.30	0	0	0	11	0.30	1	1	0.30
11	0.20	1	1	0.20					
12	0.15	2	4	0.6					
				1.6					0.60

#### Company A:

Variance  $\sigma^2 = 1.6$

Standard Deviation  $\sigma = \sqrt{1.6} = 1.26$

#### Company B:

Variance  $\sigma^2 = 0.60$

Standard Deviation  $\sigma = \sqrt{0.60} = 0.77$

**Example 4:** Techno Ltd is considering an investment in one of the two securities. Given the information that follows, which investment is better, based on risk (as measured by the standard deviation) and return?

Investment in Security ABC			Investment in Security XYZ		
Return ( $r_i$ ) (%)	Probability ( $p_i$ )	Expected Return (in %) ( $r_i \times p_i$ )	Return ( $r_i$ ) (%)	Probability ( $p_i$ )	Expected Return (in %) ( $r_i \times p_i$ )
19	0.30	5.7	22	0.20	4.4
15	0.40	6.0	6	0.30	1.8
11	0.30	3.3	14	0.30	4.2
			-5	0.20	-1.0
<b>E(r) = 15.0</b>			<b>E(r) = 9.4</b>		

**Solution:** To find out the variation, standard deviation technique is applied.

$$\sigma = \sqrt{\sum_{i=1}^N p_i [r_i - E(r)]^2}$$



Investment in Security ABC					Investment in Security XYZ				
$r_i$	$p_i$	$r_i - E(r)$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$	$r_i$	$p_i$	$r_i - E(r)$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$
19	0.30	4	16	4.8	22	0.20	12.6	158.76	31.75
15	0.40	0	0	0	6	0.30	-3.4	11.56	3.47
11	0.30	-4	16	4.8	14	0.30	4.6	21.16	6.35
					-5	0.20	-14.4	207.36	41.47
				<b>9.6</b>					<b>83.04</b>

**Investment in Security ABC:**

Variance  $\sigma^2 = 9.6$

Standard Deviation  $\sigma = \sqrt{9.6} = 3.09$

**Investment in Security XYZ:**

Variance  $\sigma^2 = 83.04$

Standard Deviation  $\sigma = \sqrt{83.04} = 9.11$

**Example 5:** The return on two securities X and Z are given below; select the security according to risk and return.

Return on Security X (%)	Return on Security Y (%)	Probability
10	40	0.3
14	16	0.5
20	20	0.2

**Solution:** To find out the variation, standard deviation technique is applied.

$$\sigma = \sqrt{\sum_{i=1}^N p_i [r_i - E(r)]^2}$$

Investment in Security X						Investment in Security Y					
$r_i$	$p_i$	$r_i \times p_i$	$r_i - E(r)$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$	$r_i$	$p_i$	$r_i \times p_i$	$r_i - E(r)$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$
10	0.3	3	-4	16	4.8	40	0.3	12	16	256	76.8
14	0.5	7	0	0	0	16	0.5	8	-8	64	32
20	0.2	4	6	36	7.2	20	0.2	4	-4	16	3.2
	<b>1.0</b>	<b><math>\Sigma E(r)</math> =14</b>			<b>12</b>		<b>1.0</b>	<b><math>\Sigma E(r)</math> =24</b>			<b>112</b>

**Security X:**

Variance  $\sigma^2 = 12$

Standard Deviation  $\sigma = \sqrt{12} = 3.46$

**Security Y:**

Variance  $\sigma^2 = 112$

Standard Deviation  $\sigma = \sqrt{112} = 10.58$

respectively. Now, in absolute terms both 'A' and 'B' shares carry the same level of risk, but in relative terms the risk level of share 'B' would be lower, due to its higher expected returns. The relationship between 'Standard Deviation' and 'Expected Returns' may be expressed as 'Coefficient of Variation'. It is defined as the ratio of 'Standard Deviation' to the 'Expected Value'.

Mathematically it can be exhibited as following:

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}}{\text{Expected Value}} = \frac{\sigma}{E(R)}$$

Extreme caution needs to be taken while using the 'Coefficient of Variation' as a measure of risk. 'Coefficient of Variation' for an expected value of zero would be a value, viz. infinity, which is difficult to explain. Further, its tendency is to exaggerate the risk for lower values of mean, and minimize the risk for a higher expected value.

### 3.2.6.4. Covariance

Covariance is used in statistics to describe the linear relationship between two variables. Covariance provides a measure of the strength of the correlation between two or more sets of random variables. The covariance for two random variables X and Y, each with sample size N, is defined by

$$\text{Cov}(X, Y) = \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{N} \text{ or } \frac{\sum_{i=1}^N x_i y_i}{N}$$

For uncorrelated variables the covariance is zero. However, if the variables are correlated in some way, then their covariance will be non zero. In fact, if  $\text{Cov}(X, Y) > 0$ , then Y tends to increase as X increases, and if  $\text{Cov}(X, Y) < 0$ , then Y tends to decrease as X increases.

**Example 7:** Find covariance of the following series:

X	8	13	15	17	20	22	24	25
Y	25	30	32	30	37	40	42	45

**Solution:**

(X)	(Y)	(X - $\bar{X}$ )	(Y - $\bar{Y}$ )	(X <sub>i</sub> - $\bar{X}$ )(Y <sub>i</sub> - $\bar{Y}$ )
8	25	-10	-10	100
13	30	-5	-5	25
15	32	-3	-3	9
17	30	-1	-5	5
20	37	2	2	4
22	40	4	5	20
24	42	6	7	42
25	45	7	10	70
$\Sigma X = 144$	$\Sigma Y = 281$			$\Sigma (X_i - \bar{X})(Y_i - \bar{Y}) = 275$



$$\bar{X} = \frac{\sum X}{N} = \frac{144}{8} = 18; \bar{Y} = \frac{\sum Y}{N} = \frac{281}{8} = 35,$$

$$\text{Cov}(X, Y) = \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{N} = \frac{275}{8} = 34.4$$

**Example 8:** Refer to the following data for computing betas for:

- 1) Security X,
- 2) Security Y,
- 3) For an equally weighted portfolio of securities X and Y.

Security i	Correlation Coefficients i with Market	Standard Deviation of i
X	0.5	0.25
Y	0.3	0.30

$$E(R_m) = 0.12, T = 0.05, \sigma_m^2 = 0.01$$

**Solution:**  $\text{Cov}_{xy} = r_{xy} \sigma_x \sigma_y$

$$\beta_{im} = \frac{\text{CoV}(R_i, R_m)}{\sigma_m^2} = \frac{\frac{\sum x_m - \bar{x} \cdot \bar{m}}{n}}{\frac{\sum m^2}{n} - (\bar{m})^2}$$

$$1) \text{Cov}_x = r_x \sigma_x \sigma_y = (0.5) (0.25) (0.30) = 0.0375$$

$$\beta_x = \frac{0.0375}{0.010} = 3.75$$

$$2) \text{Cov}_y = r_y \sigma_x \sigma_y = (0.3) (0.30) (0.25) = 0.0225$$

$$\beta_y = \frac{0.0225}{0.01} = 2.25$$

$$3) \beta_p = \sum_{i=1}^n \omega_i \beta_{i,p} = 0.5 (3.75) + 0.5 (2.25) = 3.00$$

The average product of deviations of individual values in a given series is known as covariance (Cov).

$$\text{Cov} = \frac{1}{n} \sum_{j=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

#### Calculation of Correlation between M and W

Year	Returns on M (x)	Returns on W (y)	(x - $\bar{x}$ )	(y - $\bar{y}$ )	(x - $\bar{x}$ ) (y - $\bar{y}$ )
2007	40%	-10%	25%	-25%	-625
2008	-10%	40%	-25%	25%	-625
2009	35%	5%	20%	-20%	-400
2010	-5%	35%	-20%	20%	400
2011	15%	15%	0%	0%	0
<b>Average</b>	<b>15%</b>	<b>15%</b>			<b><math>\Sigma =</math> -</b>
<b>Standard Deviation</b>	<b>22.6</b>	<b>22.6</b>			<b>2,050</b>

$$\text{Cov}_{(xy)} = \frac{\sum (x - \bar{x})(y - \bar{y})}{n} = \frac{-2,050}{5} = -410.0$$

$$\text{Correlation (r)} = \text{Cov}_{(xy)} / \sigma_x \sigma_y = -410.0 / (22.6 \times 22.6) = -802$$

With the help of this correlation values, the Portfolio Risk can be estimated by solving the formula of  $(a+b)^2$ .

#### 4.1.4.2. Portfolio Risk in One Asset

The formula for calculating the expected rate of return of one security is as follows:

$$\bar{R} = \sum_{i=1}^n R_i P_i$$

where,

- $\bar{R}$  = Expected return on investment in a security or asset
- $R_i$  = Outcome of i
- $P_i$  = Probability of occurrence of i
- $n$  = Total number of outcomes

**Example 3:** An investor holds a security for one year. The probability distribution of possible returns of the investment is given here. For this particular investment, determine the expected rate of return.

Probability of Occurrence	Possible Rate of Return (%)
0.04	0.10
0.05	0.02
0.10	0.08
0.25	0.12
0.30	0.15
0.26	0.20



**Solution:** The expected return ( $\bar{R}$ ) on the investment can be computed as follows:

$$\begin{aligned}\bar{R} &= \sum_{i=1}^n R_i P_i \\ &= (0.10 \times 0.04) + (0.02 \times 0.05) + (0.08 \times 0.10) + (0.12 \times 0.25) + (0.15 \times 0.30) + (0.20 \times 0.26) \\ &= 0.14 \text{ or } 14 \text{ per cent}\end{aligned}$$

The standard deviation of the returns ( $\sigma$ ), a measure of risk of a security, can be calculated as follows:

$$\sigma = \sqrt{\sum_{i=1}^n (R_i - \bar{R})^2 P_i}$$

Using the data given in example above, the standard deviation can be calculated as:

$$\begin{aligned}\sigma &= \sqrt{(0.10 - 0.14)^2 0.04 + (0.02 - 0.14)^2 0.05 + (0.08 - 0.14)^2 0.10 \\ &\quad + (0.12 - 0.14)^2 0.25 + (0.15 - 0.14)^2 0.30 + (0.20 - 0.14)^2 0.26} \\ &= \sqrt{(0.0016 \times 0.04) + (0.0144 \times 0.05) + (0.0036 \times 0.10) \\ &\quad + (0.0004 \times 0.25) + (0.0001 \times 0.30) + (0.0036 \times 0.26)} \\ &= \sqrt{0.000064 + 0.00072 + 0.00036 + 0.0001 + 0.00003 + 0.000936} \\ &= \sqrt{0.00221} = 0.047\end{aligned}$$

#### 4.1.4.3. Portfolio Risk in Two Asset Model

The formula of Two-Asset case are as follows:

$$\sigma_P = \sqrt{W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 \cdot W_A \cdot W_B \cdot r_{AB} \cdot \sigma_A \cdot \sigma_B}$$

where,

- $W_A$  = Weightage/proportion of Asset A in the Total portfolio,
- $W_B$  = Proportion of investment in Asset B,
- $\sigma_A, \sigma_B$  = Standard deviations of Stock A and Stock B,
- $r_{AB}$  = Correlation coefficient between the returns of Two Stocks.

**Example 4:** Calculate the portfolio standard deviation for a two-asset portfolio comprised of the following two assets if the correlation of their returns is 0.5.

Particulars	Asset A	Asset B
Expected return	10%	20%
Standard deviation of expected returns	5%	20%
Amount invested	₹20,000	₹30,000

**Solution:**  $\sigma_P = \sqrt{W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 \cdot W_A \cdot W_B \cdot r_{AB} \cdot \sigma_A \cdot \sigma_B}$

$$\begin{aligned}\sigma_P &= \sqrt{(0.4^2)(0.05^2) + (0.6^2)(0.2^2) + 2(0.4)(0.6)(0.5)(0.05)(0.2)} \\ &= \sqrt{[(0.16)(0.0025)] + [(0.36) + (0.04)] + 2(0.0012)} \\ &= \sqrt{0.0004 + 0.0144 + 0.0024} \\ &= \sqrt{0.0172} = 0.131148 \text{ or } 13\%\end{aligned}$$

**Note:** Weight of Stock =  $\frac{\text{Stock's Value}}{\text{Total Portfolio Value}} \times 100$

$$W_A = \frac{20,000}{20,000 + 30,000} \times 100 = \frac{20,000}{50,000} \times 100 = 40\% \text{ or } 0.4$$

$$W_B = \frac{30,000}{20,000 + 30,000} \times 100 = \frac{30,000}{50,000} \times 100 = 60\% \text{ or } 0.6$$

**Example 5:** Determine portfolio risk involve with the help of following information.

S. No.	Scrip Name	Weight of Scrip (%)	Standard Deviation (%)	Correlation between I and J
1	I	30	45	-0.90
2	J	70	10	

**Solution:** Portfolio Risk ( $\sigma_P$ ) =  $\sqrt{W_I^2 \sigma_I^2 + W_J^2 \sigma_J^2 + 2 \cdot W_I \cdot W_J \cdot r_{IJ} \cdot \sigma_I \cdot \sigma_J}$

where,

$W_I$  = Weightage/proportion of Asset I in the Total portfolio,

$W_J$  = Proportion of investment in Asset J,

$\sigma_I, \sigma_J$  = Standard deviations of Stock I and Stock J,

$r_{IJ}$  = Correlation coefficient between the returns of Two Stocks.

$$\begin{aligned}\sigma_P^2 &= \sqrt{(0.3)^2 (0.45)^2 + (0.7)^2 (0.1)^2 + 2 \times (0.3) \times (0.7) \times (-0.90 \times 0.45 \times 0.1)} \\ &= \sqrt{(0.09 \times 0.2025) + (0.49 \times 0.01) + (0.42 \times -0.0405)} \\ &= \sqrt{0.018225 + 0.0049 + (-0.01701)} \\ &= \sqrt{0.006115} \\ &= 0.078\end{aligned}$$



**Example 6:** Stock TATA and BIRLA display the following returns over the past three years:

Year	Return	
	TATA	BIRLA
1994	14	12
1995	16	18
1996	20	15

Answer following:

- 1) What is the expected return on portfolio made up of 40 per cent of TATA and 60 per cent of BIRLA?
- 2) What is the standard deviation of each stock?
- 3) Determine the correlation coefficient of stock TATA and BIRLA.
- 4) What is the portfolio risk of a portfolio made up of 40 per cent TATA and 60 per cent BIRLA?

**Solution:**

$$1) \left( \text{Expected Rate of Return} = \frac{\sum R}{N} \right)$$

$\sum R$  is the total of Returns,  $N$  is the number of observations

$$\text{Returns of Stock TATA} = \frac{14 + 16 + 20}{3} = \frac{50}{3} = 16.67$$

$$\text{Returns of Stock BIRLA} = \frac{12 + 18 + 15}{3} = \frac{45}{3} = 15$$

$$\text{Portfolio Return} = \sum_{i=1}^N X_i R_i$$

$X_i$  being the proportion held by each security which includes 40% of TATA and 60% of BIRLA

$$= (0.4 \times 16.67) + (0.6 \times 15) = 6.67 + 9 = 15.67$$

2) **Standard Deviation of Each Stock**

$$\sigma = \sqrt{\frac{\sum (R_i - \bar{R}_i)^2}{N}}$$

$$\sigma_{\text{TATA}} = \sqrt{\frac{(14 - 16.67)^2 + (16 - 16.67)^2 + (20 - 16.67)^2}{3}}$$

$$= \sqrt{\frac{(-2.67)^2 + (-0.67)^2 + (3.33)^2}{3}}$$

$$= \sqrt{\frac{7.13 + 0.45 + 11.09}{3}} = \sqrt{\frac{18.67}{3}} = \sqrt{6.22}$$

$$\sigma_{TATA} = 2.49$$

$$\begin{aligned}\sigma_{BIRLA} &= \sqrt{\frac{(12-15)^2 + (18-15)^2 + (15-15)^2}{3}} \\ &= \sqrt{\frac{(-3)^2 + (3)^2 + (0)^2}{3}} \\ &= \sqrt{\frac{9+9+0}{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}\end{aligned}$$

$$\sigma_{BIRLA} = 2.45$$

### 3) Correlation Coefficient of Stock TATA and BIRLA

$$\begin{aligned}\text{COV}_{TATABIRLA} &= \frac{\sum(R_1 - \bar{R}_1)(R_2 - \bar{R}_2)}{N} \\ &= \frac{(14-16.67)(16-16.67)(20-16.67) + (12-15)(18-15)(15-15)}{3} \\ &= \frac{(-2.67 \times -0.67 \times 3.33) + (-3 \times 3 \times 0)}{3} = \frac{5.96 + 0}{3} = 1.98\end{aligned}$$

$$\begin{aligned}r &= \frac{\text{Covariance TATABIRLA}}{\sigma_x \sigma_y} \\ &= \frac{1.98}{2.49 \times 2.45} = \frac{1.98}{6} = 0.33\end{aligned}$$

### 4) Portfolio Risk

$$\text{Portfolio Risk } (\sigma_P) = \sqrt{(x_1)^2(\sigma_1)^2 + (x_2)^2(\sigma_2)^2 + 2X_1X_2(r_{12}\sigma_1\sigma_2)}$$

$$\begin{aligned}R_P &= \sqrt{\sigma_P^2} \\ &= \sqrt{(0.4)^2(2.49)^2 + (0.6)^2(2.45)^2 + 2 \times (0.4) \times (0.6) \times (0.33 \times 2.49 \times 2.45)} \\ &= \sqrt{(0.16)(6.1009) + (0.36)(6.0025) + 2 \times (0.24) \times (2.0131)} \\ &= \sqrt{0.98 + 2.16 + 0.97} \\ &= \sqrt{4.11} \\ &= 2.03\end{aligned}$$

#### 4.1.4.4. Portfolio Risk in n-Asset Model

The portfolio risk in n-asset model is used when the number of assets are more than two in a portfolio. It needs one to start with the calculation of covariance-variance matrix between all assets under consideration. The presence of various computer software packages has helped the matrix calculation easier for a sample which is not more than 30 assets. Once the correlation matrix is calculated so that the portfolio risk is found by the square rooting the weighted



products of all the cells present in the matrix. The covariance-variance matrix in case of 3 assets is as follows:

	$W_A$ A	$W_B$ B	$W_C$ C
A	$\sigma_A^2$	$Cov_{AB}$	$Cov_{AC}$
B	$Cov_{AB}$	$\sigma_B^2$	$Cov_{BC}$
C	$Cov_{AC}$	$Cov_{BC}$	$\sigma_C^2$

Portfolio Risk =

$$\sigma_P = \sqrt{(W_A \times W_A \times \sigma_A^2) + (W_A \times W_B \times Cov_{AB}) + (W_A \times W_C \times Cov_{AC}) + (W_B \times W_A \times Cov_{AB}) + (W_B \times W_B \times \sigma_B^2) + (W_B \times W_C \times Cov_{BC}) + (W_C \times W_A \times Cov_{AC}) + (W_C \times W_B \times Cov_{BC}) + (W_C \times W_C \times \sigma_C^2)}$$

**Example 7:** The correlation between Security A and B is  $-0.35$  and the correlation between security B and C are  $0.20$  and the correlation coefficient between security A and C is  $-0.60$ .

Security	Risk	Return
A	15%	12%
B	20%	18%
C	25%	22%

Calculate the portfolio return and risk characteristic of an equally weighted portfolio of three securities.

**Solution:** Portfolio Return =  $\sum_{i=1}^N X_i R_i$

$$= (0.33 \times 12) + (0.33 \times 18) + (0.33 \times 22)$$

$$= 3.96 + 5.94 + 7.26$$

$$= 17.16\%$$

Portfolio Risk ( $\sigma_P$ ) =  $\sqrt{(x_1)^2(\sigma_1)^2 + (x_2)^2(\sigma_2)^2 + 2X_1X_2(r_{12}\sigma_1\sigma_2)}$

$$\sigma_P = \sqrt{(0.33^2 \times 15^2) + (0.33^2 \times 20^2) + (0.33^2 \times 25^2) + [2(0.33^2 \times -0.35 \times 15 \times 20)] + [2(0.33^2 \times 0.20 \times 20 \times 25)] + [2(0.33^2 \times -0.60 \times 15 \times 25)]}$$

$$= \sqrt{(0.1089 \times 225) + (0.1089 \times 400) + (0.1089 \times 625) + (2 \times 0.1089 \times -105) + (2 \times 0.1089 \times 100) + (2 \times 0.1089 \times -225)}$$

$$= \sqrt{24.50 + 43.56 + 68.06 + (-22.87) + 21.78 + (-49)}$$

$$= \sqrt{136.12 - 22.87 + 21.78 - 49}$$

$$= \sqrt{86.03}$$

$$= 9.275$$