ATME COLLEGE OF ENGINEERING

13th Km Stone, Bannur Road, Mysore - 570028

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

(ACADEMIC YEAR 2025-26)

LABORATORY MANUAL

CONTROL SYSTEM LABORATORY-BEEL606

Prepared By	Verified By	Approved by
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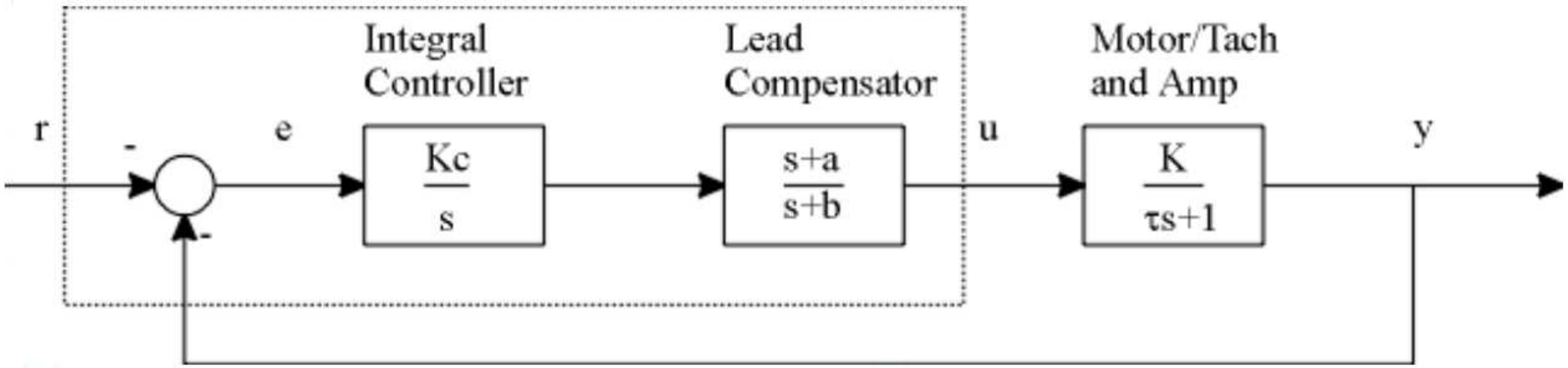


Integral Controller
Lead Compensator
Motor/Tach and Amp
r
e
u
y
-
-
Kc/s
(s+a)/(s+b)
K/(τs+1)

Vision of the Institute

Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

Mission of the Institute

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torch-bearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence

Vision of the Department

To create Electrical and Electronics Engineers who excel to be technically competent and fulfill the cultural and social aspirations of the society.

Mission of the Department

- To provide knowledge to students that builds a strong foundation in the basic principles of electrical engineering, problem solving abilities, analytical skills, soft skills and communication skills for their overall development.
- To offer outcome based technical education.
- To encourage faculty in training & development and to offer consultancy through research & industry interaction.

Program Outcomes (PO's)

PO1: Engineering Knowledge: Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3: Design / Development of Solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4: Conduct Investigations of Complex Problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern Tool Usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6: The Engineer and Society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and Sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9: Individual and Team Work: Function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11: Project Management and Finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12: Life-Long Learning: Recognize the need for and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.

Program Specific Outcomes (PSO's)

PSO1: Apply the concepts of Electrical & Electronics Engineering to evaluate the performance of power systems and also to control industrial drives using power electronics.

PSO2: Demonstrate the concepts of process control for Industrial Automation, design models for environmental and social concerns and also exhibit continuous self- learning.

Program Educational Objectives (PEO's)

PEO1: To produce competent and ethical Electrical and Electronics Engineers who will exhibit the necessary technical and managerial skills to perform their duties in society.

PEO2: To make graduates continuously acquire and enhance their technical and socioeconomic skills**.**

PEO3: To aspire graduates on R&D activities leading to offering solutions and excel in various career paths.

PEO4: To produce quality engineers who have the capability to work in teams and contribute to real time projects.

Control System Laboratory			
Course Code	BEEL606	IA Marks	50
Number of Practical Hours/Week	03	Exam Hours	03
RBT Levels	L1,L2,L3,L4, L5	Exam Marks	50
Credits - 02			

Course objectives:

- To draw the speed torque characteristics of AC and DC servo motor.
- To determine the time and frequency reposes of a given second order system using discrete components.
- To design and analyze Lead, Lag and Lag – Lead compensators for given specifications.
- To study the feedback control system and to study the effect of P, PI, PD and PID controller and Lead compensator on the step response of the system.
- To simulate and write a script files to plot root locus, bode plot, to study the stability of the system

Sl. No	**Experiments**
01	Experiment to draw the speed torque characteristics of (i) AC servo motor (ii) DC servo motor
02	Experiment to draw synchro pair characteristics
03	Experiment to determine frequency response of a second order system
04	A. To design a passive RC lead compensating network for the given specifications, viz, the maximum phase lead and the frequency at which it occurs and to obtain the frequency response. B. To determine experimentally the transfer function of the lead compensating network.
05	A. To design a passive RC lag compensating network for the given specifications, viz, the maximum phase lag and the frequency at which it occurs and to obtain the frequency response B. To determine experimentally the transfer function of the lag compensating network
06	Experiment to draw the frequency response characteristics of the lag – lead compensator network and determination of its transfer function.
Experiments 7 to 11 must be done using MATLAB/SCILAB/OCTAVE only.	
07	A. To simulate a typical second order system and determine step response and evaluate time response specifications. B. To evaluate the effect of additional poles and zeros on time response of second order system. C. To evaluate the effect of pole location on stability. D. To evaluate the effect of loop gain of a negative feedback system on stability.
08	To simulate a second order system and study the effect of (a) P, (b) PI, (c) PD and (d) PID controller onthe step response.
09	A. To simulate a D.C. Position control system and obtain its step response. B. To verify the effect of input waveform, loop gain and system type on steady state errors. C. To perform trade-off study for lead compensator. D. To design PI controller and study its effect on steady state error.
10	A. To examine the relationship between open-loop frequency response and stability, open-loop frequency and closed loop transient response. B. To study the effect of open loop gain on transient response of closed loop system using root locus.
11	A. To study the effect of open loop poles and zeros on root locus contour. B. Comparative study of Bode, Nyquist and root locus with respect to stability

Revised Bloom's Taxonomy Level:	**L_1 – Remembering; L_2 – Understanding; L_3 – Applying;**

Course outcomes: At the end of the course the student will be able to:

1. **Determine** the performance characteristics of AC servomotor, DC servomotors and sychro-transmitter receiver pair. [L3]
2. **Analyse** the effect of P, PI, PD, PID and DC position controllers on the step response of the second order system. [L4]
3. **Analyse** the time response and frequency response of a second order system using software package and discrete components**. [**L4**]**
4. Design and **Analyse** the Lead, Lag and Lag-Lead compensators for the given specifications. [L4]
5. **Evaluate** the stability of the system using root locus, bode plot. And Nyquist plot.[L4]

List of Text Books

1. **Control Systems Engineering,** I. J. Nagarath and M.Gopal, New Age International (P) Limited, 4th Edition – 2005
2. **Modern Control Engineering,** K. Ogata, PHI, 5th Edition, 2010.

List of URLs, Text Books, Notes, Multimedia Content, etc

- http://nptel.ac.in/courses/108101037/
- https://www.youtube.com/watch?v=iyRWW-5OmBA
- https://www.youtube.com/watch?v=FSAfFw_dqgA
- https://www.youtube.com/watch?v=xLhvil5sDcU

Cycle of Experiments

Cycle 1:

Exp. No.	Experiment Title	COs	
1	Experiment to draw the Speed Torque Characteristics of (i) AC servo motor (ii) DC servo motor.	CO1	L3
2	Experiment to draw synchro-pair characteristics	CO1	L3
3	a. To simulate a typical second order system and determine step response and evaluate time response specifications. b. To evaluate the effect of additional poles and zeros on time response of second order system. c. To evaluate the effect of pole location on stability d. To evaluate the effect of loop gain of a negative feedback system on stability.	CO2	L4
4	Experiment to determine frequency response of a second order system	CO2	L4
5	To simulate a second order system and study the effect of (a) P, (b) PI, (c) PD and (d) PID controller on the step response.	CO3	L4

Cycle 2:

Exp. No.	Experiment Title	COs	
6	a. To design a passive RC lead compensating network for the given specifications, viz, the maximum phase lead and the frequency at which it occurs and to obtain the frequency response. b. To determine experimentally the transfer function of the lead compensating network.	CO4	L4
7	a. To design a passive RC lag compensating network for the given specifications, viz, the maximum phase lag and the frequency at which it occurs and to obtain the frequency response. b. To determine experimentally the transfer function of the lag compensating network	CO4	L4
8	Experiment to draw the frequency response characteristics of the lag – lead compensator network and determination of its transfer function.	CO4	L4
9	a. To simulate a D.C. Position control system and obtain its step response. b. To verify the effect of input waveform, loop gain and system type on steady state errors. c. To perform trade-off study for lead compensator. **d.** To design PI controller and study its effect on steady state error.	CO4	L4
10	a. To examine the relationship between open-loop frequency response and stability, open-loop frequency and closed loop transient response. b. To study the effect of open loop gain on transient response of closed loop system using root locus.	CO5	L4
11	a. To study the effect of open loop poles and zeros on root locus contour b. Comparative study of Bode, Nyquist and root locus with respect to stability	CO5	L4

Experiment No: 01 **Date:**

A. Experiment to Draw the Speed Torque Characteristics of AC Servo Motor

Aim: To obtain speed – torque characteristics of AC servo motor

Objective: The students will learn about the characteristics of AC Servo Motor.

Apparatus:

- AC servo motor kit
- Patch cards
- Ammeter

Panel diagram:

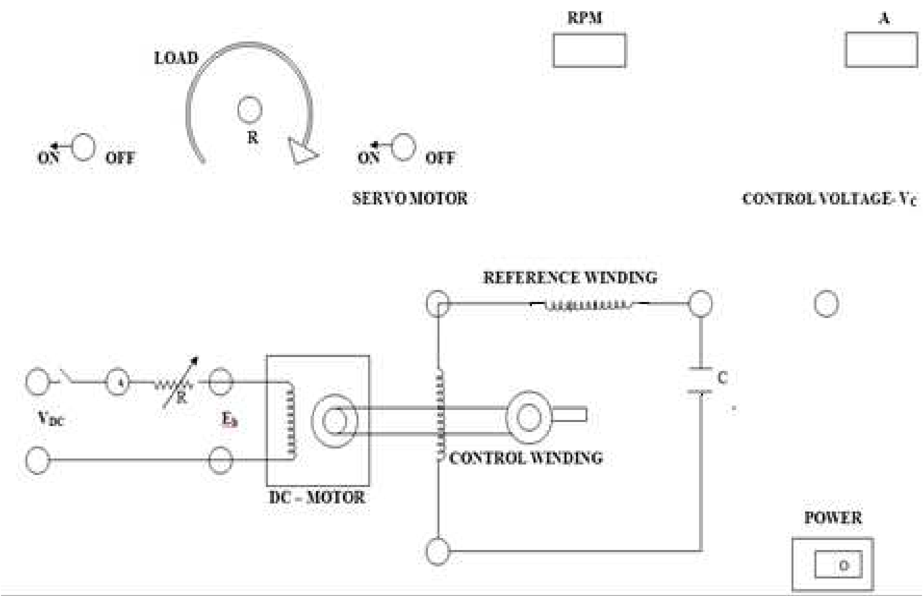


Fig.1.1: AC – Servo Motor speed – torque characteristics study unit

Procedure:

To plot Speed vs. Torque characteristics

1. Study all the controls carefully on the front panel.
2. Initially keep the load control switch at OFF position, keep servomotor supply switch also at OFF position.
3. Keep load potentiometer and control voltage auto transformer at minimum positions.
4. Switch ON main supply to the unit and also AC servomotor supply switch.
5. Set control winding voltage of 180V and reference winding voltage of 240V by varying the autotransformer.
6. With load switch in OFF position, note down the no load speed and back EMF of the DC motor.
7. Switch ON the load switch and vary the load by varying the load potentiometer in steps and note down the corresponding Speed, Back EMF and armature current(I_a) in the tabular column.
8. Repeat the above experiment for control winding Voltage 200V and 180V.
9. Plot the graph of speed Vs torque for the different control winding voltages in a single graph.

To plot Speed vs. Back EMF

1. Initially keep load control switch at OFF position, keep servomotor supply switch also at OFF position.
2. Keep load potentiometer and control voltage auto transformer at minimum positions.
3. Now switch ON main supply to the unit and also AC servomotor supply switch.
4. With load switch in OFF position, vary the speed of the AC servomotor by varying the control voltage and note down back EMF generated by the dc motor in the tabular column.
5. Plot the graph of speed vs. Back EMF.

Tabular Column:

Table 1.1:Speed torque characteristics Readings of AC Servo Motor (Load Test)

Case1:________V					
Sl. No.	**I_a (A)**	**N (rpm)**	**E_b (V)**	**P (W)**	**T (g-cm)**
Case2:________V					
Case3: ________V					

The Torque is given by,

$$T = \frac{P * k * 60}{2\pi N}$$

Where, $P = E_b * I_a$

$k = 1.019*10^4$

Table 1.2: Speed vs. Back EMF Curve (Under No Load Condition):

Sl. No.	**N (rpm)**	**E_b (V)**

Typical Graphs:

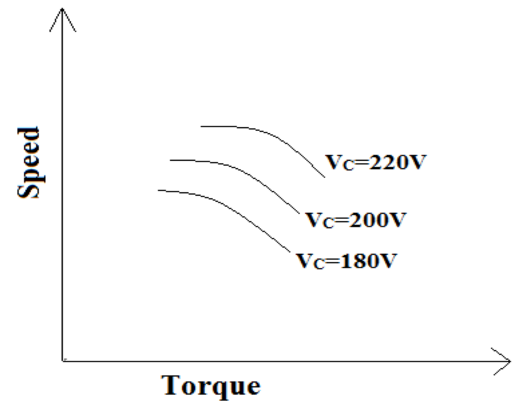


Fig 1.2: Typical graph of Speed vs. Torque Characteristics

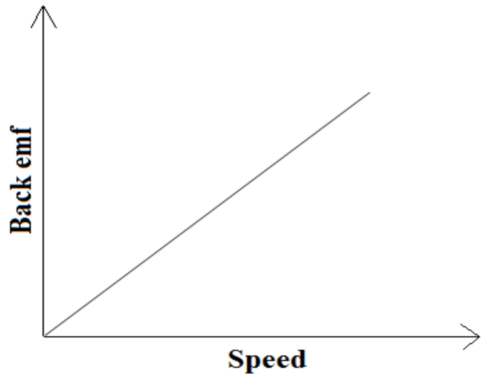


Fig 1.3: Typical graph of Speed vs. Back EMF Characteristics

Free Space for rough work:

Conclusion:

Outcomes: At the end of the experiment

1. The students will acquire the knowledge on the AC servo motor.
2. The students will be able to plot the Speed-Torque Characteristics of AC Servo Motor.

B. Experiment to Draw the Speed Torque Characteristics of DC Servo Motor

Aim: To obtain speed – torque characteristics of DC servo motor.

Objective: The students will learn about the characteristics of DC Servo Motor

Apparatus:

1. DC servo motor Kit
2. Patch cards
3. Ammeter (mA)

DC Servomotor specifications:

1. Type: Permanent magnet type
2. Voltage: 24V D.C
3. Torque: 400 gm-cm
4. Speed: 4400 RPM

Panel Diagram:

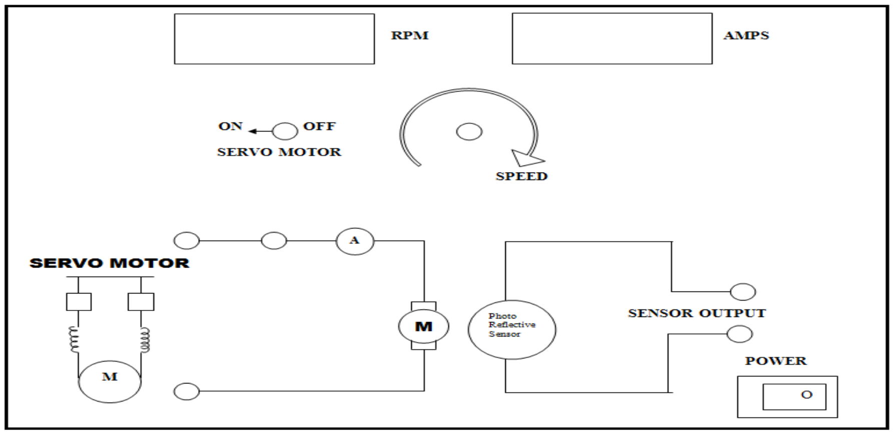
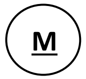


Fig. 1.4: DC– Servo Motor Speed – Torque characteristics study unit

Front panel details:

1. MAINS: Mains ON/OFF switch to the unit with built-in indicator.
2. LCD DISPLAY: LCD display to display the speed in RPM
3. AMMETER(A): Ammeter to measure the DC motor current.
4. SERVOMOTOR ON: AC supply ON/OFF switch to the servomotor.
5. LOAD- ON/OFF: ON/OFF Switch to load the motor.
6. R: Potentiometer to vary the Load-500 Ohms/25 Watts.
7. V_{dc}: 12 V unregulated DC supply to DC motor.
8. 

: PMDC motor terminals to measure the Back EMF.
9. CONTROL WINDING: Control winding terminals of AC Servomotor.
10. REFERENCE WINDING: Reference winding of AC servo motor.
11. CONTROL VOLTAGE: Auto transformer to vary the AC supply to the Control winding.

Procedure:

1. Study all the controls carefully on the front panel.
2. Keep the load at no load condition. Switch On the mains supply to the unit.
3. Switch on the servomotor.
4. Set the rated DC voltage to 24V across the servo motor.
5. Note down the no load current and no load speed.
6. Increase the load in steps till the load current reaches 0.8 Amps(do not exceed 0.8 amps).
7. At each load note down the speed, current and spring balance readings.
8. Calculate the corresponding torque and plot the speed-torque characteristics.
9. Repeat the procedure for 60% and 40% of the rated voltage.

Tabular column:

Table 1.2: Speed torque characteristics Readings of DC Servo Motor (Load Test)

Sl. No.	**I_a (A)**	**N (rpm)**	**Weight-W_1 (g)**	**Weight-W_2 (g)**	**Total Weight W = W_1-W_2 (g)**	**Torque T = W* R (g-cm)**
Set voltage V_1 = _________V						
Set voltage V_1 = _________V						
Set voltage V_1 = _________V						

The Torque is given by,

T = k*W *r (g-cm)

Let k= 1; constant

Where, 'r' is the radius of the pulley in cm and it is obtained from the circumference ($2\pi r$).

Typical graph:

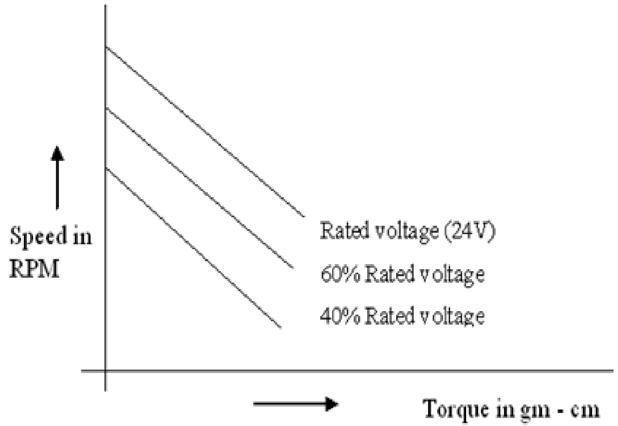


Fig.1.5: Typical Graph of Speed-Torque Characteristics of DC Servo Motor

Conclusion:

Outcomes: At the end of the experiment,

1. The students will acquire the knowledge on the of DC servo motor.
2. The students will be able to plot the Speed-Torque Characteristics of DC Servo Motor.

Viva Questions:

1. Give the comparison between AC Servo Motor and DC Servo Motor.
2. Mention the applications of AC Servo Motor and DC Servo Motor.
3. Define the working principle of Servo Motor.

Preparation	Conduction	Calculation/Result Analysis	Total

Signature of Faculty Member

Free Space for rough work:

Experiment No: 02 **Date:**

Experiment to Draw Synchro Pair Characteristics

Aim: To study the operations of syncro transmitter and receiver pair.

Objective: The students will get knowledge regarding synchro transmitter and receiver and their operations.

Apparatus:

1. Synchro transmitter and receiver set.
2. Patch cards.

Front panel details:

1. POWER:Power ON/OFF switch to the unit with builtin indicator.
2. TRANSMITTER:Synchro Transmitter.
3. SWITCH:Switch for transmitter rotor supply.
4. ROTOR - R1, R2:Synchro Transmitter rotor terminals.
5. S1, S2, S3:Synchro Transmitter stator terminals.
6. RECEIVER:Synchro Receiver
7. SWITCH:ON/OFF Switch for rotor supply.
8. S1', S2', S3':Synchro Receiver stator terminals.
9. ROTOR- R1', R2': Synchro Receiver
10. VOLTMETER:AC Voltmeter to measure stator and rotor voltages.

Internal connection diagram:

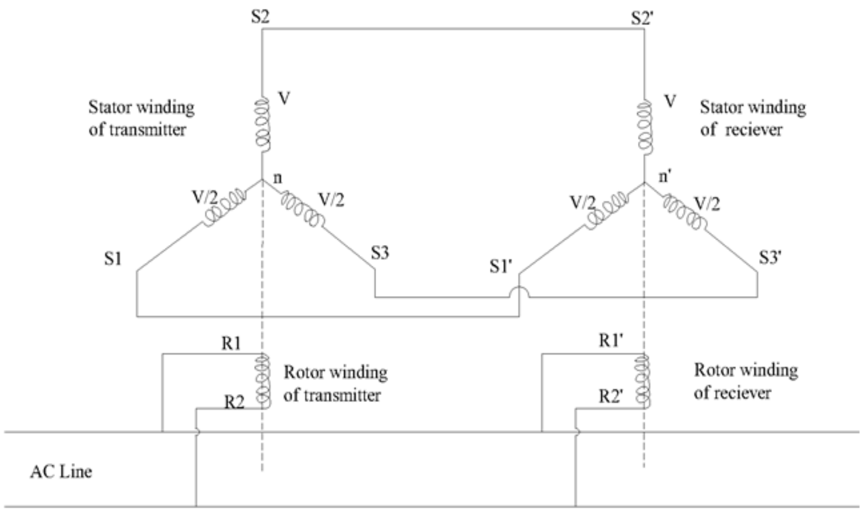


Fig.2.1: Internal connection diagram of Transmitter and Receiver Pair

Procedure:

1. Connect mains supply cable.
2. Connect the stator terminals of transmitter S1, S2 and S3 with stator terminals of receiver S1′, S2′ and S3′ with the help of patch cords respectively.
3. Now at zero angular position of rotor of transmitter, note down that of receiver and tabulate them.
4. Vary the angular positions of rotor of synchro transmitter in steps by 30° and note down the corresponding angular positions of rotor of synchro receiver.
5. It is observed that whenever the rotor of the synchro transmitter is rotated, the rotor of the synchro receiver follows it both directions of rotations and its positions are linear with the initial error.
6. Switch off the mains supply of the kit after bringing back the rotor of the transmitter at 0°.

7. Plot a graph between angular positions of rotor of transmitter and angular positions of rotor of receiver

Tabular Column:

Table.2.1: Synchro Transmitter Rotor Position vs. Stator Voltages for Three Phases (V_{S3S1}, V_{S1S2}, V_{S2S3})

Sl. No.	Rotor position in degrees	V_{S3S1}	V_{S1S2}	V_{S2S3}
1	0			
2	30			
3	60			
4	90			
5	120			
6	150			
7	180			
8	210			
9	240			
10	270			
11	300			
12	330			

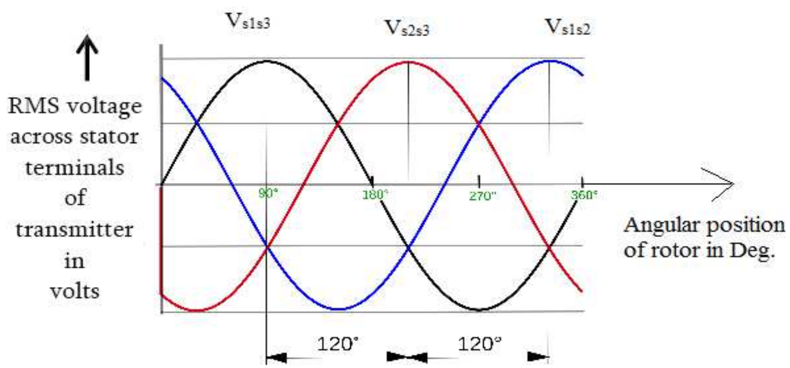


Fig.2.2: Model graph of rotor position vs.stator voltages for three phases

Tabular Column:

Table.2.2: Transmitter angular position and receiver angular position

Sl. No.	Transmitter angular position in degrees	Receiver angular position in degrees
1	0	
2	30	
3	60	
4	90	
5	120	
6	150	
7	180	
8	210	
9	240	
10	270	
11	300	
12	330	

Typical Graph:

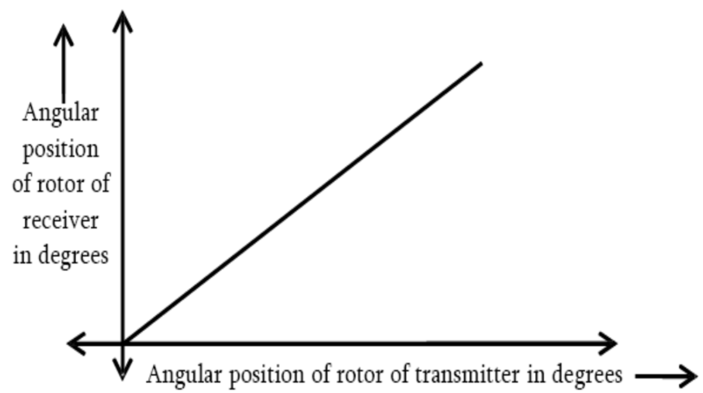


Fig.2.3: Angular position of rotor of receiver vs. angular position of rotor of transmitter

Conclusion:

Outcome: At the end of the experiment,

1. The students will get the knowledge on the synchro transmitter receiver pair

Viva Questions:

1. Define synchro transmitter and synchro receiver.
2. Give the comparison between transmitter and receiver.
3. Mention applications of synchro transmitter and synchro receiver.

Preparation	Conduction	Calculation/Result Analysis	Total

Signature of Faculty Member

Free Space for rough work:

Experiment No: 03 **Date:**

Experiment to Determine Frequency Response of a Second Order System

Aim: To plot the frequency response of second order system and determine frequency domain specifications.

Objective: The students will learn regarding second order system and to determine frequency response for those systems.

Apparatus:

1. DRB, DIB and DCB
2. CRO
3. Patch cards
4. Signal Generator
5. Probes

This Frequency response characteristics unit consists of the following sub units:

1. Sine wave source: 0 to 3.5 Volts, 100Hz to 2000Hz frequency range
2. Digital phase angle / frequency meter
3. Digital voltmeter.
4. RLC Network.

Circuit Diagram:

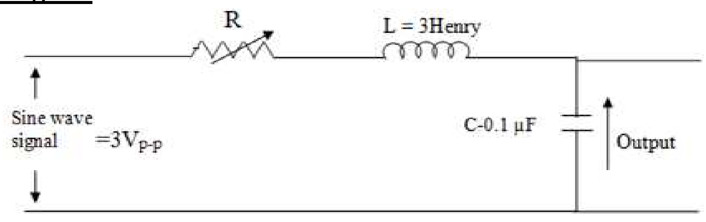


Fig.3.1: Circuit Diagram

Design:

The transfer function of a series RLC circuit with output across the capacitor is:

$$\frac{C(S)}{R(S)} = \frac{1}{\mathrm{LC}s^2 + \mathrm{RCs} + 1}$$

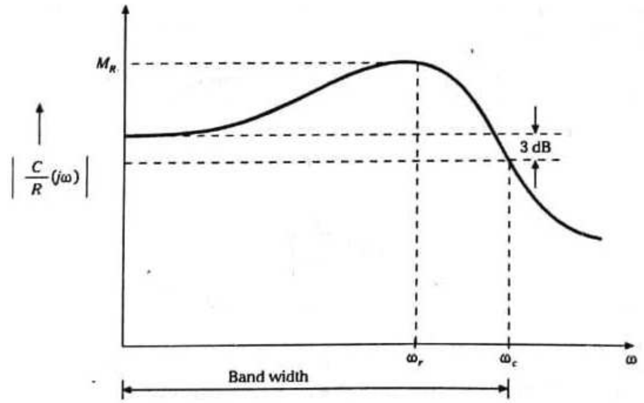
$$\frac{C(S)}{R(S)} = \frac{\frac{1}{\mathrm{LC}}}{s^2 + \left(\frac{\mathrm{R}}{\mathrm{L}}\right)\mathrm{s} + \frac{1}{\mathrm{LC}}}$$

In the frequency domain s=jω

$$\frac{C(S)}{R(S)} = \frac{1}{1 - \omega s^2\mathrm{LC} + \mathrm{j}\omega\mathrm{RC}}$$

From this, the gain and phase response against frequency will be typical of second order system

- Gain: $|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2 LC)^2+(\omega RC)^2}}$
- Phase: $\phi(\omega) = -\tan^{-1} \frac{\omega RC}{1-\omega^2 LC}$



Performance criteria on frequency domain

From the Gain Plots: Maximum gain (Resonant peak Mr), Resonant frequency (ω_r), Cut-off Frequencies ω_b, Bandwidth Δf, can be observed for different ζ

Resonant peak Mr/ Peak Magnitude **Mp**

The maximum Magnitude/gain in dB(resonance peak) of the closed loop frequency response

Resonant Frequency ωr

This is the frequency at which the magnitude of the frequency response is maximum**,** this is the frequency at which the circuit naturally oscillates (maximum voltage across the capacitor in a low-pass filter setup)

$$\omega_r = \omega_n \sqrt{1-2\zeta^2}$$

Cut-off Frequencies ω_c, (also called -3 dB points/-3dB frequency)

The frequencies at which the output power drops to half (or voltage drops to 0.707 of max) where the magnitude falls by 3 dB from peak

Bandwidth Δf:

The range of frequencies over which the output remains significant (gain ≥ 0.707 of max). is inversely proportional to Q (quality factor). A higher R increases damping and widens the bandwidth.

$$\omega_{c1,c2} = \omega_n \quad \sqrt{1-2\zeta^2} \pm \zeta$$

Quality Factor (Q)

Indicates the sharpness of the peak at resonance:

Spec	Formula
Natural frequency	$\omega_n = \frac{1}{\sqrt{LC}}$
Resonant frequency	$\omega_r = \omega_n\sqrt{1-2\zeta^2}$
Peak magnitude	$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$
Bandwidth (rad/s)	$\Delta\omega = 2\zeta\omega_n$
Bandwidth (Hz)	$\Delta f = \frac{\zeta\omega_n}{\pi}$
Quality factor	$Q = \frac{1}{2\zeta}$

Compared with general equation.

$$\frac{C(S)}{R(S)} = \frac{\omega n^2}{S^2 + \mathbf{2\zeta\omega n}s + \omega n^2}$$

Where ωn Natural frequency (rad/s):

The frequency at which the system would oscillate without damping.

$\omega n = \frac{1}{\sqrt{LC}}$, fn = $\frac{\omega}{2\pi}$ (Hz)

Damping Ratio: ζ zeta Describes the level of damping in the system:

$$R = 2\zeta\sqrt{\frac{L}{C}} \quad \Rightarrow \quad \zeta = \frac{R}{2\sqrt{L/C}} = \frac{R}{2}\sqrt{\frac{C}{L}}$$ **(for RLC circuits)**

Let C = 0.1µF and L=1H

$$\omega n = \frac{1}{\sqrt{LC}} = 3162.28\, rad/s$$

$$f = \frac{\omega n}{2\pi} = 504.6\text{Hz}$$

$$R = 2\xi\sqrt{L/C}$$

Calculated R values:

ζ	R (Ω)
0.1	632.5
0.2	1264.9
0.3	1897.4
0.5	3162.3
0.7	4427.2
1.0	6324.5
1.5	9486.8

<u>Procedure</u>:

1. Connect the circuit as shown in the connection diagram.
2. Switch ON the Mains supply to the unit.
3. Connect sine wave output to networks input.
4. Set the amplitude of sine wave to some value 3V peak.
5. Vary the input frequency of the sine wave from 100 Hz to 2000Hz in steps and Measure peak voltage of input and output, calculate gain (magnitude response), Measure phase difference (in degrees or radians) between input and output.
6. Repeat for various values of damping factor ζ = (0.1,0.2,0.3,0.5,0.7, 1, 1.5.).
7. Plot the graph of phase angle vs. frequency and gain v/s frequency for various values of damping factor on a single semi log sheet. Determine the frequency domain specifications from the plots.

<u>Tabular Column</u>:

1. **Under damped. ξ =**

Sl.No.	f(Hz)	Vi (V)	ω in (rad)	Vout (V)	Phase shift (°) φ	Gain (Vo/Vi)	Gain in dB $20\log_{10}$(Vo/Vi)

2. **Critically damped, ξ =**

Sl.No.	f(Hz)	Vi (V)	ω in (rad)	Vout (V)	Phase φ	Gain (Vo/Vi)	Gain in dB 20log(Vo/Vi)

3. Over damped, ξ =

Sl.No.	f(Hz)	Vi (V)	ω in (rad)	Vout (V)	Phase φ	Gain (Vo/Vi)	Gain in dB 20log(Vo/Vi)

Use the formula to calculate the phase difference:

Measure the time difference (Δt) between the two signals using the cursors. This is the phase delay.

Use the formula to calculate the phase difference:

$$\text{Phase Difference (in degrees)} = \frac{\Delta t}{T} \times 360^{\circ}$$

Where T is the period of the input signal.

Typical Frequency response curve:

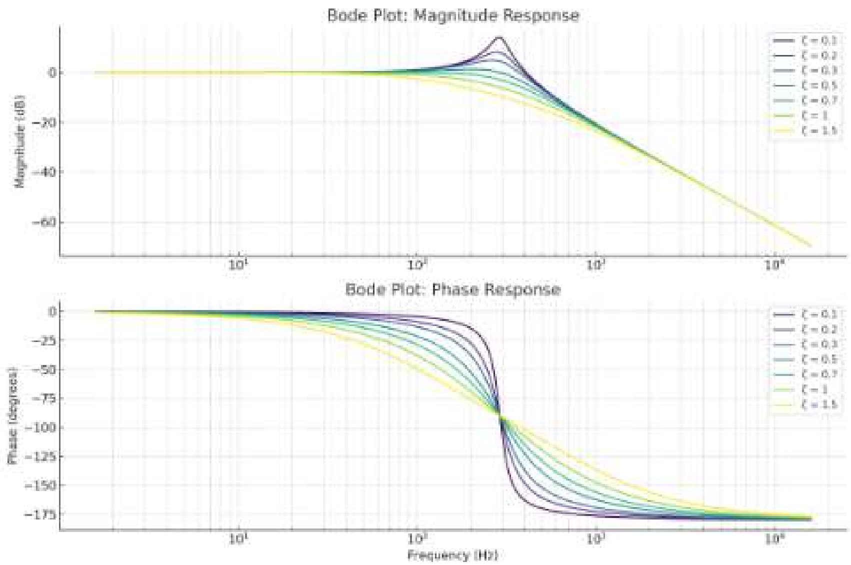


Fig.3.2: Typical frequency response curve

MATLAB Command used to obtain Resonant Peak and resonant frequency are as follows

[mag, phase, w] = bode(num, den, w); or [mag, phase, w] = bode(sys, w);
[Mp, k] = max(mag);
resonant_peak = 20*log10(Mp)
resonant_frequency = w(k)

The bandwidth can be obtained by entering the following lines into the program:

```
n = 1;
while 20*log(mag(n)) >= -3; n = n + 1;
end
bandwidth = w(n)
```

MATLAB/Octave Simulation:

Octave code to calculate the frequency domain specifications Resonant Peak, Resonant Frequency, Cut-off Frequencies and Quality Factor

```
R = 1000;            % Adjust as needed
L = 1;               % Henry
C = 0.1e-6;          % Farad
% Transfer function coefficients
num = [1];
den = [L*C, R*C, 1];
H = tf(num, den);    % creates a transfer function
% Bode plot
bode(H);
[gm, pm, wep, weg] = margin(H);  % Gain margin, Phase margin, and associated frequencies
grid on;
title('Frequency Response of Series RLC Circuit');

% Frequency response at resonant frequency (we = 1/sqrt(LC))
we = 1 / sqrt(L * C);

% Resonant Peak (Maximum gain at resonant frequency)
[mag, phase, freq] = bode(H);
mag = squeeze(mag);  % Remove singleton dimensions
resonant_peak = max(mag);

% Quality Factor (Q)
Q = we * L / R;

% Cut-off Frequencies (for 3 dB down from peak)
cutoff_low = we / sqrt(2);  % Lower cut-off frequency
cutoff_high = we * sqrt(2); % Upper cut-off frequency

% Display results
disp(['Resonant Frequency (we): ', num2str(we), ' rad/s']);
disp(['Resonant Peak: ', num2str(resonant_peak)]);
disp(['Quality Factor (Q): ', num2str(Q)]);
disp(['Lower Cut-off Frequency: ', num2str(cutoff_low), ' rad/s']);
disp(['Upper Cut-off Frequency: ', num2str(cutoff_high), ' rad/s']);
```

Octave code to calculate the frequency domain specifications Resonant Peak, Resonant Frequency, Cut-off Frequencies and Quality Factor

- **Simulates a 3V sine wave input from 100 Hz to 1000 Hz**
  Calculates Gain & Phase shift for each damping factor
  Saves results to an Excel-readable .csv **f**

```
% Given values
L = 1;              % Inductance (Henry)
C = 0.1e-6;         % Capacitance (Farad)
wn = 1 / sqrt(L*C); % Natural frequency (rad/s)
fn = wn / (2*pi);   % Natural frequency (Hz)
Vin_peak = 3;       % Peak input voltage (Volts)

% Frequency sweep range
freqs = linspace(100, 1000, 10);  % Sweep from 100 Hz to 1000 Hz (10
points)

% List of damping factors (Corrected)
zeta_values = [0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5];

% Time settings
t = linspace(0, 0.05, 5000);   % Simulate for 50 ms

% Initialize table to collect data
data = {};

% Header for CSV file
header = {'Zeta', 'Resistance (Ohm)', 'Input Frequency (Hz)', 'Gain', 'Phase
Shift (deg)'};

% Loop over each damping factor
for i = 1:length(zeta_values)
   zeta = zeta_values(i);

   % Resistance Calculation
   R = 2 * zeta * sqrt(L/C);

   % Transfer function H(s)
   num = [1];
   den = [L*C, R*C, 1];
   H = tf(num, den);

   printf('\n--- Damping factor ζ = %.1f (R = %.2f Ω) ---\n', zeta, R);

   % Loop over each input frequency
   for j = 1:length(freqs)
      f_in = freqs(j);            % Input frequency (Hz)
      omega_in = 2*pi*f_in;       % Angular frequency (rad/s)

      % Create sine wave input
      Vin = Vin_peak * sin(omega_in * t);

      % Simulate system output
      Vout = lsim(H, Vin, t);

      % Calculate Gain (output peak/input peak)
      Vout_peak = max(abs(Vout));
      Gain = Vout_peak / Vin_peak;

      % Calculate Phase shift
      [~, idx_in] = max(Vin);
      [~, idx_out] = max(Vout);
```

```
        phase_delay_time = t(idx_out) - t(idx_in);  % Time difference
        phase_shift_deg = -phase_delay_time * omega_in * (180/pi);  %
Negative for lag

     % Display Results
        printf('Input Freq = %4d Hz | Gain = %.3f | Phase shift = %.2f deg\n',
...
               f_in, Gain, phase_shift_deg);

        % Store in table
        data(end+1,:) = {zeta, R, f_in, Gain, phase_shift_deg};
    end
end

% Now save collected data into a CSV file
cell2csv('frequency_response_data.csv', [header; data]);

printf('\n✅Data saved to "frequency_response_data.csv"\n');
```

Conclusion:

Outcomes: At the end of the experiment,

1. The students will be able to design the RLC circuits for various damping ratios, discuss the impact of the resistor (damping)
2. Compare experimental gain and phase to the theoretical Bode plot.

3. Identify resonant frequency, cutoff frequency and bandwidth.

Viva Questions:

1. Define damping ratio.
2. Give the comparison between under damped, critically damped and over damped.
3. Mention the advantages of second order system.
4. Mention the differences between first order system and second order system.

Preparation	Conduction	Calculation/Result Analysis	Total

Free Space for rough work:

Signature of Faculty Member

COMPENSATION

A well designed control system should have the following properties:

1. It should be stable.
2. The controlled output should follow the changes in the reference input without unduly oscillations or overshoots. That is, it should have suitable damping.
3. It should be less sensitive to system parameter changes.
4. It should be able to reduce the effect of undesirable disturbances.
5. It should have as little error as possible

A feedback control system that provides an optimum performance without any necessary adjustments is rare. Usually it is necessary to compromise among the many conflicting and demanding specifications.

Some adjustments in system parameters are to be made to provide suitable and acceptable performance when it is not possible to obtain all the desired optimum specifications. The alteration or adjustment of a control system in order to provide a suitable performance is called compensation. That is, compensation is the adjustment of a system in order to make up for deficiencies or inadequacies.

An additional component or circuit called compensator is inserted into the system to compensate for the deficient performance.

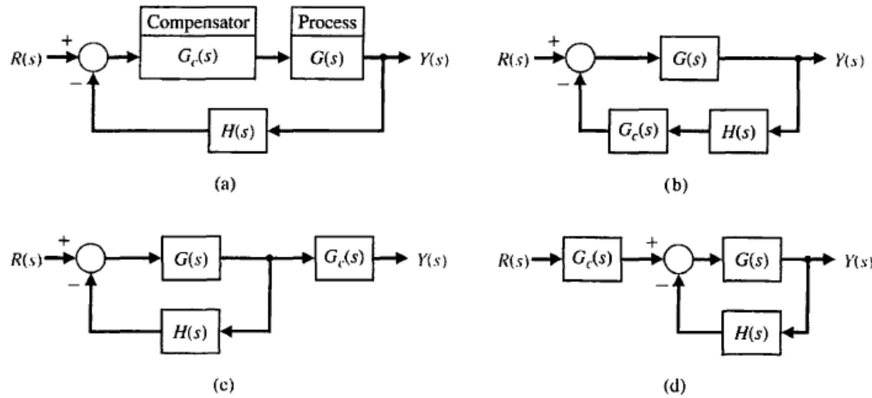
The compensating device may be electric, mechanical, hydraulic, pneumatic, or some other type of device or network. Electric networks are often used as compensators in many control systems

The performance of a control system is specified by a set of specifications in time domain and/or in frequency domain such as damping ratio, peak overshoot, rise time, settling time, gain margin, phase margin, bandwidth, steady-state error, etc.

The term compensation is used for the modification of the performance characteristics of a system so that the required characteristics are obtained. A compensator is thus an additional component which is added into a control system to modify the closed-loop performance and compensate for a deficient performance. A compensator placed in the forward path is called a cascade or series compensator (Figure A).

Types of compensation.

(a) Cascade compensation.
(b) Feedback compensation.
(c) Output, or load, compensation.
(d) Input compensation



The transfer function of a compensator is designated as $G_c(s) = E_0(s)/E_{in}(s)$, and the compensator can be placed in a suitable location within the structure of the system. The selection of the compensation scheme depends upon a consideration of the specifications, the power levels at various signal nodes in the system, and the networks available for use

SELECTION OF A COMPENSATOR

Selection of the compensation scheme depends upon the specifications, the power levels at various signals nodes in the system, available components, economic ions and the designer's experience.

In general, there are two situations in which compensation is required. In the first case, the system is absolutely unstable and the compensation is required to stabilize it and also to achieve a specified performance. In the second case, the system is stable, but the compensation is required to obtain the desired performance.

APPROACHES TO SYSTEM DESIGN

The performance of a control system can be described in terms of the time-domain performance measures or the frequency-domain performance measures. The performance of a system can be specified by time-domain Parameters requiring a certain peak time Tp, maximum overshoot, and settling-time for a step input. Furthermore, it is usually necessary to specify the maximum allowable steady-state error for several test signal inputs and disturbance inputs. These performance specifications can be defined in terms of the desirable location of the poles and zeros of the closed-loop system transfer function,

When the specification is given in time-domain, the root locus approach is normally used for designing the compensator. The location of the closed-loop poles determines the time-domain response and a pair of dominant complex conjugate poles dominates the transient response.

The locus of the roots of the closed-loop system can be readily obtained for the variation of one system parameter. However, when the locus of roots does not result in a suitable root configuration, we must add a compensating network (Figure A) to alter the locus of the roots as the parameter is varied

Therefore, we can use the root locus method and determine a suitable compensator network transfer function so that the resultant root locus yields the desired closed loop root configuration

Alternatively, we can describe the performance of a feedback control system in terms of frequency performance measures. Then a system can be described in terms of the resonant peak of the closed-loop frequency response Mr, the resonant frequency(ωr), the bandwidth, and the phase margin of the system. We can add a suitable compensation network, if necessary, in order to satisfy the system specifications.

The root locus approach to design is very effective when the specifications are given in terms of time-domain quantities such as the damping ratio and the undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time and settling time.

The design of the compensating network, represented by Gc(s), is developed in terms of the frequency response as portrayed on the polar plane, the Bode diagram, or the Nichols chart. Because a cascade transfer function is readily accounted for on a Bode plot by adding the frequency response of the network, we usually prefer to approach the frequency response methods by utilizing the Bode diagram

Thus, the design of a system is concerned with the alteration of the frequency response or the root locus of the system in order to obtain a suitable system performance.

For frequency response methods, we are concerned with altering the system so that the frequency response of the compensated system will satisfy the system specifications. Hence, in the frequency response approach, we use compensation networks to alter and reshape the system characteristics represented on the Bode diagram and Nichols chart

Alternatively, the design of a control system can be accomplished in the s-plane by root locus methods. For the case of the s-plane, the designer wishes to alter and reshape the root locus so that the roots of the system will lie in the desired position in the s-plane

The compensation network function Gc(s) is cascaded with the specified process G(s) in order to provide a suitable loop transfer function L(s) = Gc(s)G(s)H(s). The compensator Gc(s) can be chosen to alter either the shape of the root locus or the frequency response. In either case, the network may be chosen to have a transfer function

$$G_c(s) = \frac{K\prod_{i=1}^{M}(s + z_i)}{\prod_{j=1}^{n}(s + p_j)}.$$

Then the problem reduces to the judicious selection of the poles and zeros of the compensator.

To illustrate the properties of the compensation network, we will consider a first-order compensator. The compensation approach developed on the basis of a first-order compensator can then be extended to higher-order compensators, for example, by cascading several first-order compensators.

A compensator *Gc(s)* is used with a process *G(s)* so that the overall loop gain can be set to satisfy the steady-state error requirement, and then *Gc(s)* is used to adjust the system dynamics favorably without affecting the steady-state error

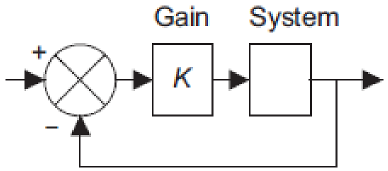
Consider the first-order compensator with the transfer function

$$G_c(s) = \frac{K(s + z)}{s + p}.$$

The design problem then becomes the selection of z, p and K in order to provide a suitable performance. When \z\ < \p\, the network is called a **phase-lead network**

Changing the Gain

Consider the effects of adjusting the performance of a control system by changing the gain in the forward path (Figure 2). The effect of increasing the gain is to shift upwards the gain-frequency Bode plot by an equal



(FIGURE 2) Adjusting the performance of a system by introducing gain.

amount over all the frequencies; this is because we are adding a constant gain element to the Bode plot. Figure 3 illustrates this. There is no effect on the phase-frequency Bode plot. Increasing the gain thus shifts the 0 dB crossing point of the gain plot to the right and so to a higher frequency. This decreases the stability of the system since it decreases the gain margin and the phase margin. Hence, if an increase in stability is required then the gain needs to be reduced.

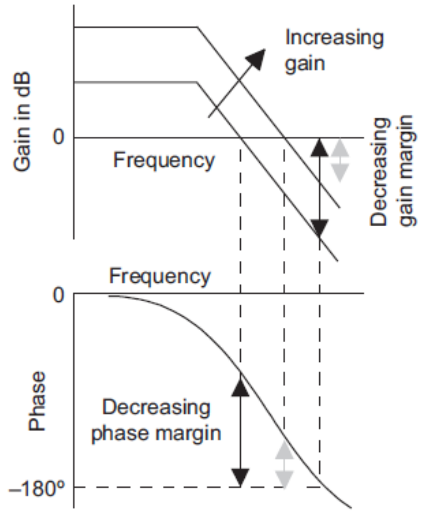


FIGURE 3: The effect of increasing the gain.

Phase-Lead Compensation

The phase-lead compensation transfer function can be obtained with the network shown in Figure 1, in this network sinusoidal output leads the sinusoidal input. The angle of lead is a function of the frequency

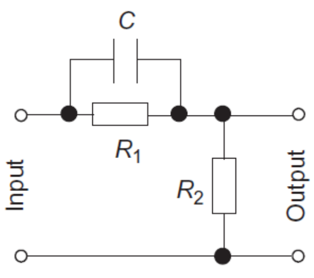


Fig. 1 Phase-lead network.

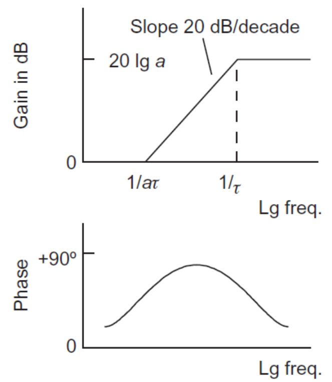


Fig. 2 Bode diagram of the phase-lead network.

The transfer function of a phase-lead compensator is of the form:

$$Gc(s) = \frac{Eo(s)}{Ei(s)} = \frac{1}{a}\left[\frac{(1+a\tau s)}{(1+\tau s)}\right] = \frac{K}{a}\left[\frac{(1+a\tau s)}{(1+\tau s)}\right]$$

with a > 1, $a = \left[\frac{(R1+R2)}{R2}\right]$ and $\tau = \left[\frac{(R1\times R2\times C)}{(R1+R2)}\right]$

$\tau = \frac{1}{P}$ and $a = \frac{P}{Z} \rangle 1$

Figure 2. shows the Bode plot for a phase-lead compensator (it can be obtained by adding the Bode plots for the numerator term and the denominator term).

The term phase-lead is used because the compensator has a positive phase and so is used to add phase to an uncompensated system. The maximum value of the phase lead occurs at a frequency ωm which is midway between the pole and zero frequencies of P = 1/τ and z =1/aτ on the logarithmic scale and so:

$$\log_{10}\omega_m = \frac{1}{2}\left[\log_{10}\frac{1}{a\tau} + \log_{10}\frac{1}{\tau}\right]$$

and hence, Maximum phase lead occurs at a frequency

$$\omega_m = \sqrt{ZP} = \frac{1}{\tau\sqrt{a}}$$

Lead Phase angle is given by $\phi = \tan^{-1}(a\omega\tau) - \tan^{-1}(\omega\tau)$

The value of this maximum phase angle φ_m at a frequency ωm is given by:

$\sin\phi_m = \frac{a-1}{a+1}$, we can calculate the value of ''a'' $a = \frac{1+\sin\phi_m}{1-\sin\phi_m}$

With a cascade phase-lead compensator we add its Bode plot to that of the system being compensated in order to obtain the required specification. As a consequence, we can increase the phase margin. The cascade phase-lead compensator is thus used to provide a satisfactory phase margin for a system

Effect/Advantages of Phase Lead Compensation

1. **The damping of the closed-loop system is increased**, reducing Maximum overshoot and oscillations.
2. The velocity constant K_v increases, **improving the system's ability to track inputs accurately**
3. The slope of the magnitude curve of the bode plot is reduced at the gain crossover frequency, therefore, the **relative stability improved**
4. **Phase margin of the closed-loop system is increased**. making the system more stable and less prone to oscillations.
5. The bandwidth of the system is increased, this corresponds to **faster time response** because it shifts gain crossover frequency to a higher value (faster rise times, settling times, and overall transient response speed)
6. **Increased Bandwidth:** The lead network enhances the system's ability to handle high-frequency inputs, contributing to a faster response.
7. Simplified Design: Compared to lag-lead compensators, phase-lead compensators can be simpler to design and implement.

Disadvantages/ Limitations of Phase Lead Compensation

1.**No Steady-State Error Improvement:**

Lead compensation primarily focuses on transient response and doesn't significantly improve steady-state error.

2.**Increased Sensitivity to Noise:**

Increased bandwidth can also amplify noise level at system output, potentially impacting the system's accuracy.

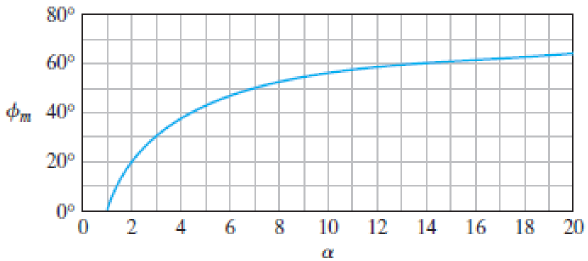
3.**Reduced Input Gain:**

While bandwidth increases, the overall gain of the system might be slightly reduced, depending on the specific design of the lead compensator.

For getting large phase margin we required large value of 'a' which increases the bandwidth, this may increase the transmission of noise. In practice the value of 'a' should not be greater than 15.

The phase angle readily obtainable from this network is not much greater than 70°, there are practical limitations on the maximum value of *a* that we should attempt to obtain. Therefore, if we required a maximum angle greater than 70°, **If the large phase lead is required two or more cascade compensation should be used**

If the phase shift decreases rapidly near the gain crossover frequency, phase lead compensation becomes ineffective because the additional phase lead at new gain crossover frequency is added to a much smaller phase angle than that at the old gain cross over frequency. The desired phase margin can be achieved only with large value of 'a' which is not desirable.



Maximum phase angle ϕm versus 'a' for a phase-lead network.

6.3.1. Phase Lead Network

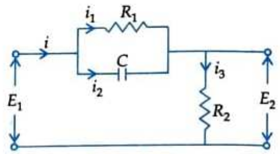


Fig. 6.3.

From circuit diag.

$$i = i_1 + i_2$$

$$i_1 = \frac{E_1 - E_2}{R_1}$$

$$i_2 = C\frac{d}{dt}(E_1 - E_2)$$

$$\therefore \quad i = \frac{E_1 - E_2}{R_1} + C\frac{d}{dt}(E_1 - E_2) \quad ...(6.3)$$

$$i_3 = i = \frac{E_2}{R_2} \quad ...(6.4)$$

Put the value of i from equation 6.4 to (6.3)

$$\frac{E_2}{R_2} = \frac{E_1 - E_2}{R_1} + C\frac{d}{dt}(E_1 - E_2) \quad ...(6.5)$$

Laplace transform of equation (6.5)

$$\frac{1}{R_2}E_2(s) = \frac{1}{R_1}[E_1(s) - E_2(s)] + sC[E_1(s) - E_2(s)]$$

$$\therefore \quad \frac{E_2(s)}{E_1(s)} = \frac{R_2 + R_1R_2Cs}{R_1 + R_2 + R_1R_2Cs} \quad ...(6.6)$$

Equation (6.6) can be written as

$$\frac{E_2(s)}{E_1(s)} = \frac{R_2}{R_1 + R_2}\left[\frac{1 + R_1Cs}{1 + \frac{R_1R_2Cs}{R_1 + R_2}}\right] \quad ...(6.7)$$

Put

$$a = \frac{R_1 + R_2}{R_2} \qquad a > 1$$

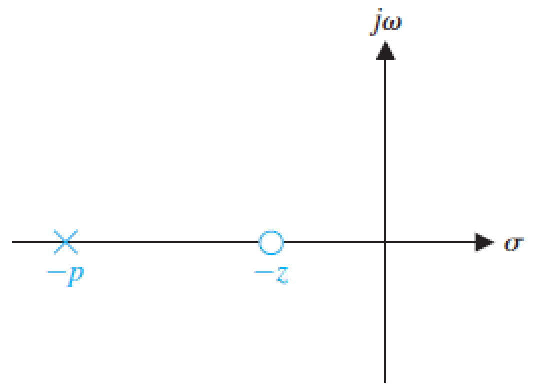
$$T = \frac{R_1R_2}{R_1 + R_2}\cdot C$$

From equation (6.7)

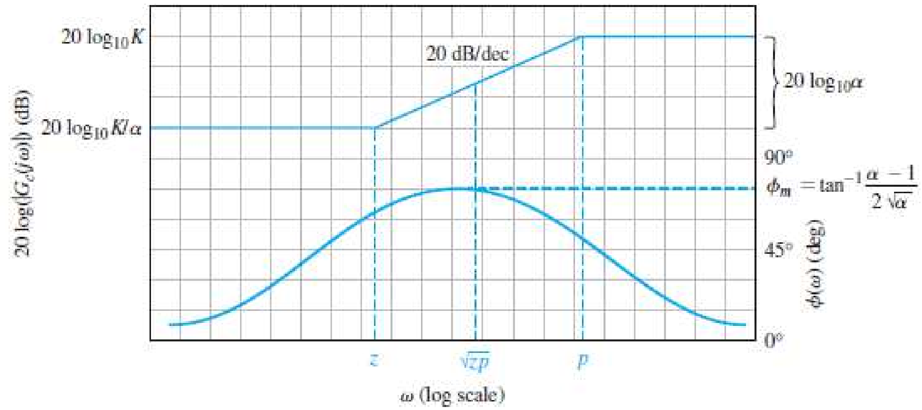
$$\frac{E_2(s)}{E_1(s)} = \frac{R_2}{R_1 + R_2}\left[\frac{1 + \left(\frac{R_1 + R_2}{R_2}\right)\left(\frac{R_1R_2C}{R_1 + R_2}\right)s}{1 + \frac{R_1R_2Cs}{R_1 + R_2}}\right]$$

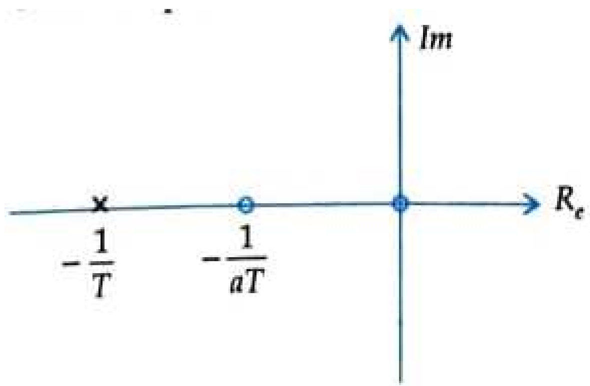
$$\therefore \quad \frac{E_2(s)}{E_1(s)} = \frac{1}{a}\left[\frac{1 + aTs}{1 + Ts}\right] \quad ...(6.8)$$

Pole-Zero plot of transfer function of a phase-lead compensator

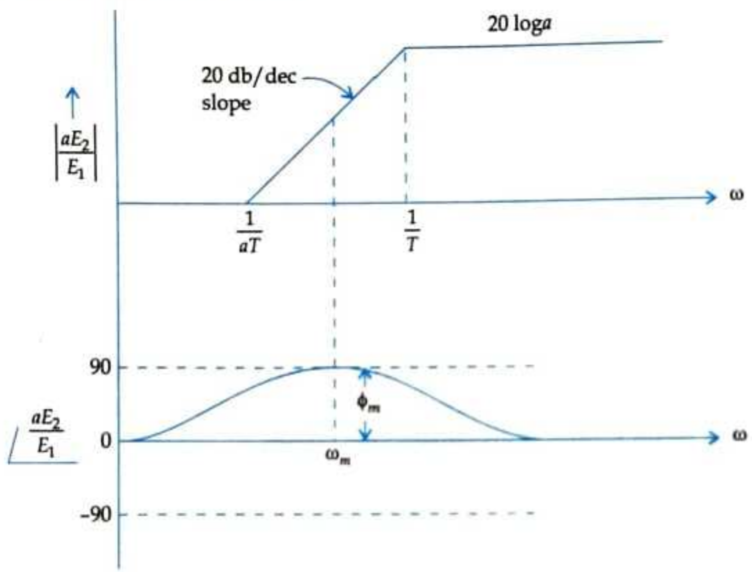


Bode diagram of the phase-lead network





Bode Plot for Phase Lead Network



Phase-lag compensation

In this electrical network sinusoidal output lags the sinusoidal input by an angle which is a function of the frequency

The phase-lag compensator has a negative phase angle and so is used to subtract phase from an uncompensated system.
A phase-lag compensator has a transfer function of the form

$$Gc(s) = \frac{Eo(s)}{Ei(s)} = Ka \times \left[\frac{(1+\tau s)}{(1+a\tau s)} \right]$$ where a is greater than 1.

$a = \left[\frac{R1+R2}{R2} \right]$ and $\tau = [R2 \times C]$ where z =1/τ and P = z/a = 1/aτ

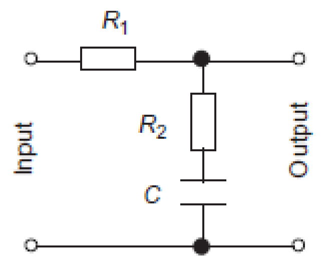


Figure 1. Phase-lag compensator

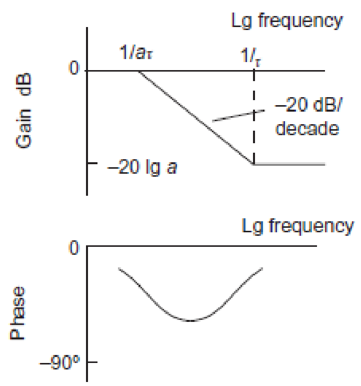


Figure 2. shows the Bode plot

Maximum phase lag occurs at a frequency $\omega_m = \sqrt{ZP} = \frac{1}{\tau\sqrt{a}}$,

Lag Phase angle is given $\phi = \tan^{-1}\omega\tau - \tan^{-1}a\omega\tau$

Because the phase-lag compensator adds a negative phase angle to a system, the phase lag is not a useful effect of the compensation and does not provide a direct means of improving the phase margin.

The phase-lag compensator does, however, reduce the gain and so can be used to lower the crossover frequency. A consequence of this is that, as usually the phase margin of the system is higher at the lower frequency, the stability can be improved. The phase-lag compensator is used, not to provide a phase-lag angle, which is normally a destabilizing influence, but rather to provide an attenuation and to increase the steady-state error constant

Effects of Lag Compensation

1. For a given relative stability, the value of error (**Velocity) constant is increased.**
2. **There is decrease in the gain crossover frequency decreasing the bandwidth of the system**
3. High gain at low frequencies improves steady-state performance and low gain at high frequencies improves phase margin. i.e. **Phase margin increases**
4. Response becomes slower

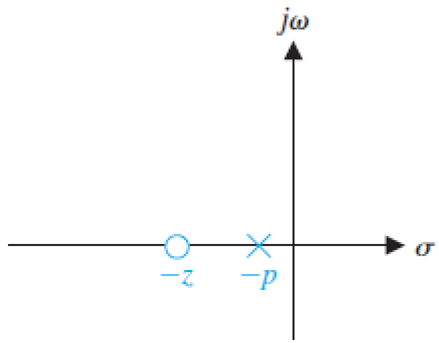
Advantages of Phase Lag Compensation

1. Lag compensator has a high gain at low frequencies and low gain at higher frequencies. Therefore, a **lag compensator acts as a low-pass filter**.
2. It attenuates high frequencies, so **it acts as a high-frequency noise filter**, Phase lag network allows low frequencies

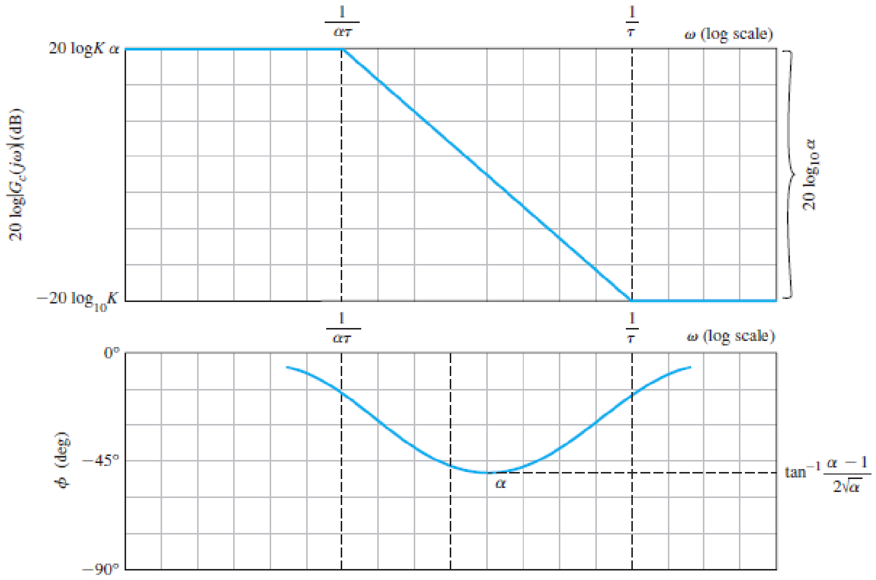
Disadvantages of Phase Lag Compensation

1. Lag compensator increases the order of the system by one. Therefore, the transient response becomes sluggish. i.e the speed of the system decreases.
2. The phase lag characteristic of a lag compensator is of no use for compensation purposes.
3. Since the attenuation due to lag compensation decreases, the gain crossover frequency decrease, thus the bandwidth of the system is reduced. The rise time and settling time are increased (becomes large) and the settling time becomes large

Pole–zero diagram of the phase-lag compensator.



Bode plot of the phase-lag compensator



The transfer function of the phase-lag compensator is of the form

$$G_c(s) = K\frac{s+z}{s+p} = K\alpha\frac{1+\tau s}{1+\alpha\tau s},$$

where

$$z = \frac{1}{\tau} \quad \text{and} \quad p = z/\alpha.$$

A Summary of the Characteristics of Phase-Lead and Phase-Lag Compensators

	Compensation	
	Phase-Lead	Phase-Lag
Approach	Addition of phase-lead angle near cross-over frequency on Bode plot. Add lead compensator to yield desired dominant roots in s-plane.	Addition of phase-lag to yield an increased error constant while maintaining desired dominant roots in s-plane or phase margin on Bode plot
Results	1. Increases system bandwidth 2. Increases gain at higher frequencies	1. Decreases system bandwidth
Advantages	1. Yields desired response 2. Improves dynamic response	1. Suppresses high-frequency noise 2. Reduces steady-state error
Disadvantages	1. Requires additional amplifier gain 2. Increases bandwidth and thus susceptibility to noise	1. Slows down transient response
Applications	1. When fast transient response is desired	1. When error constants are specified
Situations not applicable	1. When phase decreases rapidly near crossover frequency	1. When no low-frequency range exists where phase is equal to desired phase margin

Design steps via root locus

Lag Compensation Design

1. Determine the factor by which the steady state error constant must be increased for satisfactory steady state error.
2. Place the compensator zero at 10% of the distance between the origin and the first pole or zero on the LHP from the origin.
3. Pick system gain such that the dominant closed loop pole pair before compensation is changed minimally.
4. Check transient response and adjust accordingly.

The main motivation of the lag compensator is to improve the steady state error without affecting the shape of the root locus of the uncompensated system.

Lead compensator Design

Add the compensator so that the closed loop transfer function has a pair of complex poles with the desired damping ratio and natural frequency.

The complex poles dominate the system response

1. Determine the damping ratio for desired overshoot and resonant frequency for desired closed loop response time. Use this to identify desired pole.
2. Determine the phase angle of the plant at the desired pole and place the pole such that is it 180 degrees from the plant.
3. Place the zero such that the root locus has all the poles on either far left hand plane or near the zeros.
4. Determine the gain to satisfy steady state error requirement.

   In the design process it is key to go through step 3 several times to ensure proper overshoot. If required overshoot is not achieved after several trails, the damping ratio and resonant frequency would need adjustment.

LAG COMPENSATOR	LEAD COMPENSATOR
1. Magnitude compensator	1. Phase compensator
2. Improves the gain characteristic at the expense of phase characteristic	2. Improves the phase characteristic at the expense of gain characteristic
3. Improves steady state response but slows transient response	3. Speeds up transient response but hurts steady state response
4. Acts as low pass filter	4. Acts as a high pass filter
5. Decreases bandwidth	5. Increases bandwidth
6. System more tolerant to high frequency noise	6. System more susceptible to high frequency noise

LAG-LEAD COMPENSATOR

Lag-lead compensators are used when both transient as well as steady-state responses are to be improved. Only single compensator, lag or lead cannot meet both the requirements simultaneously. Lead compensators are employed to improve transient performance. To meet the steady-state requirements, lag compensators are used.

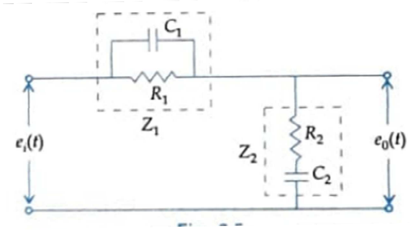
Both phase lag and lead occur in the output in different frequency regions. Phase lag occurs in the low-frequency region and phase lead occurs in the high-frequency region. In this way, the phase angle varies from lag to lead when the frequency is increased from zero to infinity.

The characteristics of Phase lead compensators are such that the gain crossover frequency shifts to a higher value and these increase the bandwidth and speed up the response and decrease the maximum overshoot for a step input but steady state error does not show much improvement.

In Phase Lag compensators, the gain crossover frequency shifts to a lower value, increase the low frequency gain and reduce the steady-state error but response speed slows down due to reduced bandwidth.

The speed of response and steady state error can be simultaneously improved if both phase lead and Phase lag compensation network are used

Figure 12.11 shows a single lag-lead compensator, which combines the characteristics of the lag and lead compensators. This is called a lag-lead network because the phase of the sinusoidal response Eo, compared with the sinusoidal input Ei, varies from a lag to a lead angle as the reference frequency is increased from zero to infinity. For frequencies from zero to value ω_x, the output voltage lags the input voltage. For frequencies above ω_x, the output voltage leads the sinusoidal input voltage.



A Lag-Lead compensator has a transfer function of the form

$$G(s) = \frac{\left(\frac{1+aT_1s}{1+T_1s}\right)\left(\frac{1+bT_2s}{1+T_2s}\right)}{lead\ lag} \quad \textbf{Eq. 1}$$

$$Z_1 = \frac{R_1}{1+R_1C_1s} \quad \text{and} \quad Z_2 = \frac{1+R_2C_2s}{C_2s}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} = \frac{(1+R_2C_2s)/C_2s}{\frac{1+R_2C_2s}{C_2s}+\frac{R_1}{1+R_1C_1s}}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{(1+R_1C_1s)(1+R_2C_2s)}{1+(R_1C_1+R_2C_2+R_1C_2)s+R_1R_2C_1C_2s^2} \quad \textbf{Eq.2}$$

Comparing Equation 1 with Equation 2

$$aT_1 = R_1C_1 \quad \textbf{Eq.3}$$
$$bT_2 = R_2C_2 \quad \textbf{Eq.4}$$
$$T_1T_2 = R_1R_2C_1C_2 \quad \textbf{Eq.5}$$

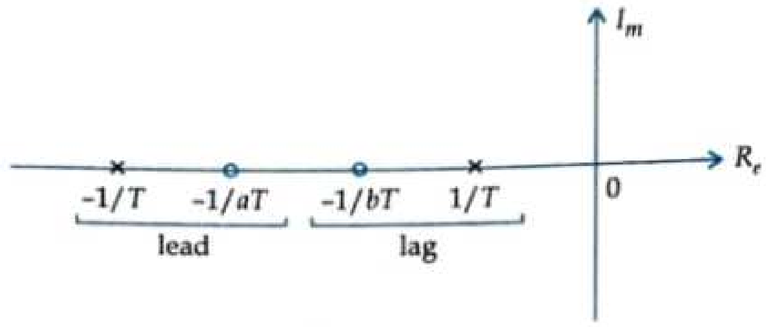
Multiplying the equations 3 and 4

$$abT_1T_2 = R_1R_2C_1C_2 \quad \textbf{Eq.6}$$

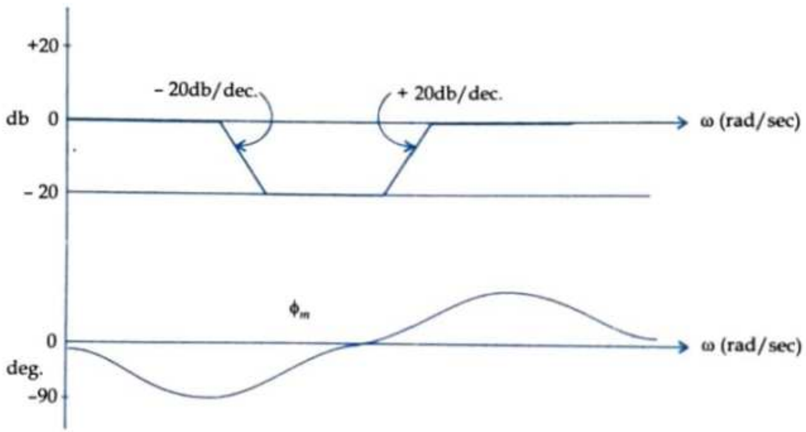
Comparing Equation 5 with Equation 6

ab =1 , where a > 1(Lead composition) and b < 1(Lag composition)

Pole–zero diagram of the Lag-Lead compensator.



Bode plot of the Lag-Lead compensator



Advantages of Phase Lag Lead Compensation

1. Due to the presence of phase lag-lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lag-lead network accuracy is improved
3. This compensator improves the steady state, bandwidth of the system and make the system response fast.

Experiment No: 04 **Date:**

A. **To design a passive RC Lead compensating network for the given specifications, viz., the maximum phase lead and the frequency at which it occurs and to obtain the frequency response.**
B. **To determine experimentally the transfer function of the lead compensating network.**

Aim:

- To design a passive RC lead compensating network for the given specifications,
- To obtain its frequency response curve.
- To determine experimentally the transfer function of the given lead compensating network.

Objectives:

1. The students will get knowledge regarding lead compensating network and its design.
2. The students will learn about frequency response curve and determination of transfer function for the lead network.

Apparatus required:

1. Function generator
2. DRB and DCB
3. Phase angle detector
4. voltmeter

Circuit diagram:

Basic lead Compensator:

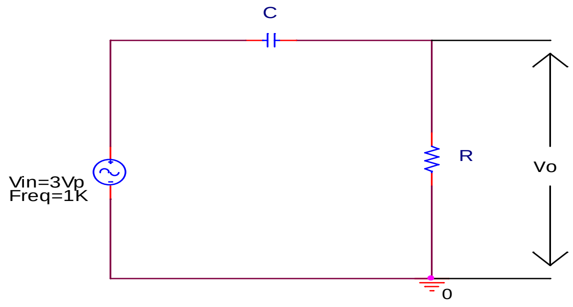


Fig.4.1: Basic Lead Compensator Circuit

Limited lead Compensator:

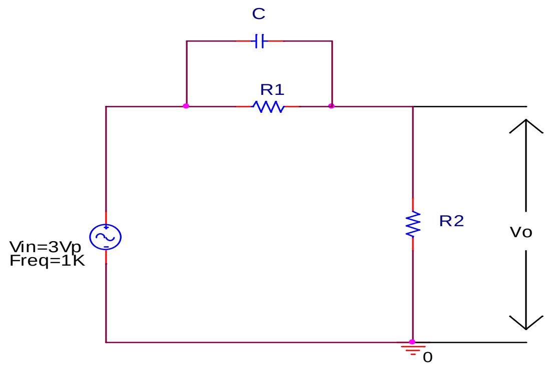


Fig.4.2: Limited Lead Compensator Circuit

Design:

Basic Lead Compensator:

Design lead compensator for a phase angle of 58° at 1 kHz

Phase angle φ of a basic lead circuit shown in Fig.4.1 is given by

$$\varphi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Assuming the value of C, Value of R is calculated.

Limited Lead Compensator:

Design a limited lead compensator for a phase angle of 40° at 500 Hz

Transfer function of limited lag compensator shown in Fig.4.2 is given by,

$$G(s) = \frac{\alpha(1+sT)}{1+sT\alpha}, \quad \alpha < 1 \quad \text{.................1}$$

Where, $\alpha = \frac{R_1}{R_1 + R_2}$,.............................2

T = R_1C...3

Maximum possible Phase angle of Lead network depends on the value of α and is given by $\sin\phi_m = \frac{1-\alpha}{1+\alpha}$

$$\alpha = \frac{1-\sin\varphi_m}{1+\sin\varphi_m}, \text{.................4}$$

Lower the value of α, we get higher lead angle.

Frequency at which max phase angle occurs is given by,

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \text{..............................5}$$

Lead angle is given by,

$$\varphi = \tan^{-1}(\omega T) - \tan^{-1}(\alpha\omega T) \text{..........6}$$

Simplifying above equation, we get

$$\tan(\varphi) = \frac{\omega T(1-\alpha)}{1+\alpha(\omega T)^2} \text{....................7}$$

- For the given phase angle φ, value of α is correctly calculated using the equation-4 and value of T is found using equation-5.
- By assuming the value of C, value of R_1 is found using equation-3.
- Value of R_2 is found using equation-2.

Procedure:

To Plot Frequency Response Curve

1. A passive RC lead compensating network is designed for the given specifications.
2. Connections are made as per the circuit diagram.
3. The output voltage of sine generator is set to 3 V (peak) and is supplied as input to the RC lead compensator.
4. The input frequency of the circuit is varied in steps 100Hz to 1MHz and the corresponding output voltage, frequency and Phase angle between input and output signal is tabulated.
5. The plots of gain in dB Vs frequency and phase angle vs. frequency are plotted in semi log sheet.

To Determine the Transfer function of the compensator experimentally

1. From the plot of phase angle vs. frequency, obtain the maximum phase lead φ_m in degrees and the frequency at which this maximum phase lead occurs f_m in Hz.
2. Using the formula $\omega m = 2 * \pi * f_m$, compute the maximum frequency in rad/s.
3. Using the formula, $\alpha = \frac{1-\sin\varphi_m}{1+\sin\varphi_m}$, compute the constant factor α.
4. Using the formula, $\omega_m = \frac{1}{T\sqrt{\alpha}}$, compute the time constant T in sec.
5. Transfer function of Lead Compensation is determined using the formula, $G(s) = \frac{\alpha(1+sT)}{1+sT\alpha}$
6. Verify the same, as follows.

Verification:

1. From the given RC phase lead compensator network, collect the values of the components R_1, R_2 and C.
2. With the help of the formula $\alpha = \frac{R_1}{R_1 + R_2}$ Compute the value of α.

3. With the help of the formula, $T = R_1 * C$ obtain the value of time constant T in sec.
4. Theoretical transfer function of Lag Compensation is determined using the formula $G(s) = \frac{\alpha(1+sT)}{1+sT\alpha}$.

Tabular Column:

Input Voltage, V_i = 3V

Sl.No.	ω = 2πf in red/sec	V_O (V)	Phase angle Φ in degrees	Gain = (V_O/V_i)	Gain in dB $20\log_{10}(V_o/V_i)$

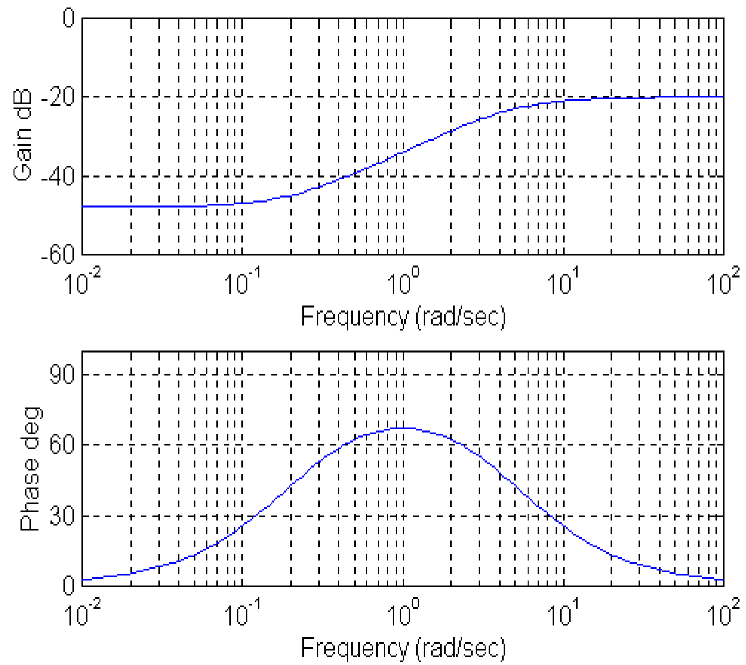


Fig.4.3: Typical Frequency Response Curve of Lead Compensator

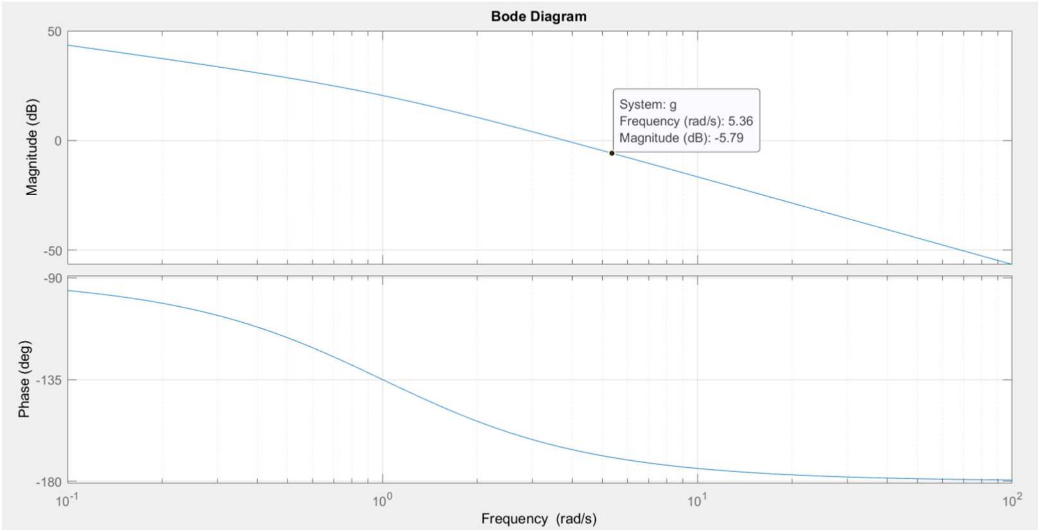
Typical Graph:

Program:

MATLAB program for lead compensator for a unity feedback system with the open loop transfer function $G(s)=k/s(1+s)$. Design a suitable lead compensator so that phase margin is $\geq 45^{o}$ and $e_{ss} \leq 1/15$.

```
clc;
clear all;
s=tf('s');
ess=1/15;
kv=1/ess;
k=kv;
g=k/(s*(1+s))
w=logspace(-1,2,200);
bode(g,w)
grid on
[gm,pm,wcp,wcg]=margin(g);
disp('Uncompensated Phase margin is')
disp(pm)
byem=45-pm+5
byem1=byem*(pi/180);
alpha=(1-sin(byem1))/(1+sin(byem1))
dbtowm=-20*log10(1/sqrt(alpha))
wm=5.36;
T=1/(wm*sqrt(alpha));
zc=1/T;
pc=1/(alpha*T);
gc=(s+zc)/(s+pc)
go=(1/alpha)*gc*g;
[gm,pm,wcp,wcg]=margin(go);
disp('compensated Phase margin is')
disp(pm)
```

Expected output: Response of Uncompensated system



Transfer function:
15
----
s^2 + s

Uncompensated Phase margin is 14.7105
byem = 35.2895

alpha = 0.2677

dbtowm = -5.7241

Transfer function:
s + 2.773
----
s + 10.36

compensated Phase margin is 45.9165

Conclusion:

Outcomes: At the end of the experiment,

1. The students will be able to design lead compensator circuit for a given phase angle.
2. The students will acquire the knowledge on the applications of lead compensator circuit.

Viva Questions:

1. Mention the advantages of lead compensator.
2. Give the differences between lead compensator and lag compensator.
3. Mention the effects of phase lead compensation.

Preparation	Conduction	Calculation/Result Analysis	Total

Signature of Faculty Member

Free Space for rough work:

Experiment No: 05 **Date:**

A. To design a passive RC Lag compensating network for the given specifications, viz., the maximum phase lag and the frequency at which it occurs and to obtain the frequency response.

B. To determine experimentally the transfer function of the lag compensating network.

Aim:

- To design a passive RC phase lag compensating network for the given specifications,
- To obtain its frequency response curve.
- To determine experimentally the transfer function of the given lag compensating network.

Objectives:

1. The students will get knowledge regarding phase lag compensating network and its design.
2. The students will learn about frequency response curve and determination of transfer function for the lead network

Apparatus required:

- Function generator
- DRB and DCB
- Phase angle detector
- voltmeter

Circuit diagram:

Basic Lag Compensator:

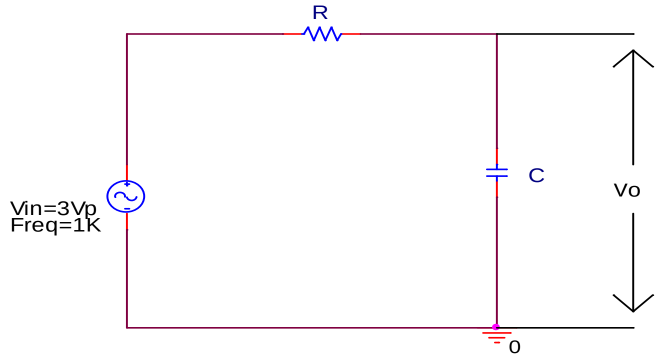


Fig.5.1: Basic Lag Compensator Circuit

Limited Lag Compensator:

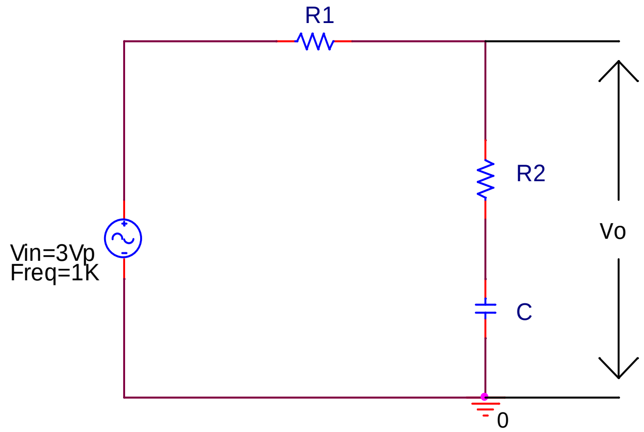


Fig.5.2: Limited Lag Compensator Circuit

Design:

Basic Lag Compensator:

Design a lag compensator for a phase angle of 81° at 1 kHz

Phase angle φ of a basic lag circuit shown in Fig.5.1 is given by,

$$\varphi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Assuming the value of C, Value of R is calculated.

Limited Lag Compensator:

Design a limited phase lag compensator for a maximum phase Lag angle of 50° at 1 kHz Transfer function of limited lag compensator shown in Fig. 2 is given by,

$$G(s) = \frac{1+sT}{1+sT\beta}, \quad \beta > 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots..1$$

$$\text{Where, } \beta = \frac{R_1 + R_2}{R_2} \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.2$$

$$T = R_2C\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3$$

Maximum possible Phase angle of lag network depends on the value of β and is given by

$$\sin\phi_m = \frac{\beta-1}{\beta+1}, \quad \beta = \frac{1+\sin\varphi_m}{1-\sin\varphi_m}, \ldots\ldots\ldots\ldots\ldots\ldots4$$

As the value of β increases, we get higher Phase Lag angle.

Frequency at which max phase angle occurs is given by,

$$\omega_m = \frac{1}{T\sqrt{\beta}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.5$$

Lag angle is given by,

$$\varphi = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)\ldots\ldots.6$$

Simplifying above equation, we get

$$\tan(\varphi) = \frac{\omega T(1-\beta)}{1+\beta(\omega T)^2}\ldots\ldots\ldots\ldots\ldots\ldots7$$

- For the given maximum phase lag angle φ, value of β is correctly chosen using the equation-4 and value of T is found using equation-5.
- By assuming the value of C, value of R_2 is found using equation-3.
- Value of R_1 is found using equation-2.

Procedure:

To Plot Frequency Response Curve

1. A passive RC lag compensating network is designed and for the given specifications.
2. Connections are made as per the circuit diagram.
3. The output voltage of sine generator is set to 3V (peak) and is supplied as input to the RC lag compensator.
4. The input frequency of the circuit is varied in steps 100Hz to 1MHz and the corresponding output voltage, frequency and Phase angle between input and output signal is tabulated.
5. The plots of gain in dB Vs frequency and phase angle vs. frequency are plotted in semi log sheet.

To Determine the Transfer function of the compensator experimentally

1. From the plot of phase angle vs. frequency, obtain the maximum phase lag φ_m in degrees and the frequency at which this maximum phase lag occurs f_m in Hz
2. Using the formula $\omega_m = 2*\pi*f_m$, compute the maximum frequency in rad/s.
3. Using the formula, $\beta = \frac{1+\sin\varphi_m}{1-\sin\varphi_m}$ compute the constant factor β.
4. Using the formula $\omega_m = \frac{1}{T\sqrt{\beta}}$, compute the time constant T in sec.
5. Transfer function of Lag Compensation is determined using the formula $G(s) = \frac{1+sT}{1+sT\beta}$
6. Step 6: Verify the same, as follows.

Verification:

1. From the given RC phase lag compensator network, collect the values of the components R_3, R_2 and C.
2. With the help of the formula, $\beta = \frac{R_1 + R_2}{R_2}$ compute the value of β.
3. With the help of the formula, T = R2∗C, obtain the value of time constant T in sec.
4. Theoretical transfer function of Phase Lag Compensation is determined using the formula $G(s) = \frac{1+sT}{1+sT\beta}$.

Tabular Column:

Input Voltage V_i= 3V

Sl.No.	ω = 2πf in red/sec	V_O (V)	Phase angle Φ in degrees	Gain = (V_O/V_i)	Gain in dB 20log(V_o/V_i)

Typical Graphs:

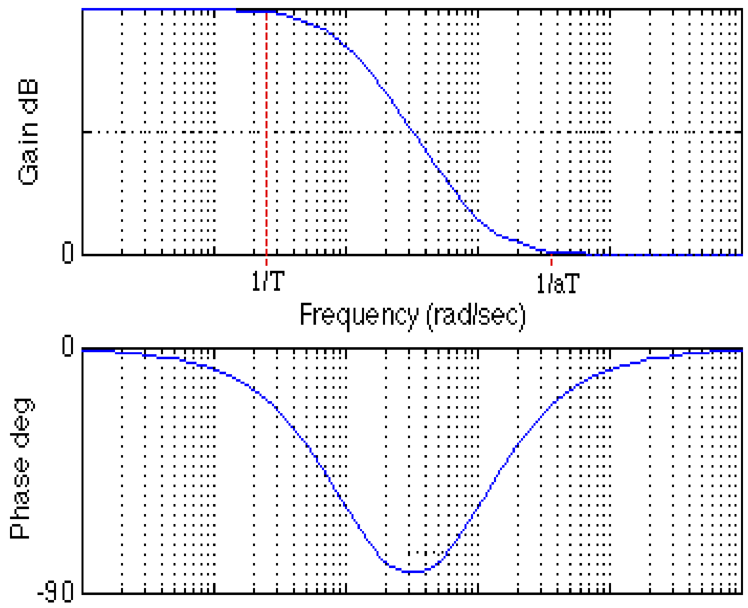


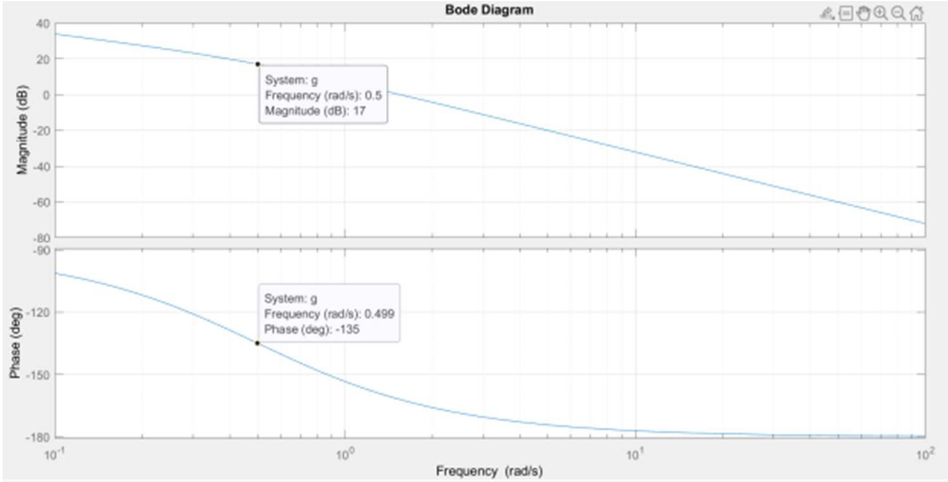
Fig.5.3: Model Frequency Response Curve of Lag Compensator

Program:

MATLAB program for lag compensator for a unity feedback system with the open loop transfer function $G(s)=k/s(1+2s)$. Design a suitable lag compensator so that phase margin is 40º and $ess \leq 0.2$.

```
clc;
clear all;
s=tf('s');
ess=0.2;
kv=1/ess;
k=kv;
g=k/(s*(1+2*s))
w=logspace(-1,2,200);
bode(g,w)
grid on
[gm,pm,wcp,wcg]=margin(g);
disp('Uncompensated Phase margin is')
disp(pm)
pmnew=40+5;
byegcn=pmnew-180
wgcn=0.499;
Agcn=17;
beta=10^(Agcn/20);
T=10/wgcn;
zc=1/T;
pc=1/(beta*T);
gc=(s+zc)/(s+pc)
go=(1/beta)*(gc)*(g)
[gm,pm,wcp,wcg]=margin(go);
disp('compensated Phase margin is')
disp(pm)
```

Expected output: Response of Uncompensated system




```
Transfer function:
5
---------
2 s^2 + s

Uncompensated Phase margin is 17.9642
byegcn =  -135

Transfer function:
s + 0.0499
------------
s + 0.007049

Transfer function:
0.7063 s + 0.03524
------------------------------
2 s^3 + 1.014 s^2 + 0.007049 s

compensated Phase margin is  40.0513
```

Conclusion:

Outcomes: At the end of the experiment,

1. The students will be able to design lag compensator circuit for a given phase angle.
2. The students will acquire the knowledge on the applications of lead compensator circuit.

Viva Questions:

1. Mention the applications of lag compensator.
2. Give the advantages and disadvantages of lag compensator.
3. Mention the effects of phase lag control.

Preparation	Conduction	Calculation/Result Analysis	Total

Signature of Faculty Member

Free Space for rough work:

Experiment No: 06 **Date:**

Experiment to draw the frequency response characteristics of the Lag –lead compensator network and determination of its Transfer function

Aim: Experiment to draw the frequency response of a given lead-lag compensating network.

Objective: The students will learn regarding frequency response characteristics of the lag – lead compensator network and also to determine the transfer function.

Apparatus Required:

1. Resistors – 10k – 2 Nos
2. Capacitors – 0.1μF – 2 Nos
3. Wires
4. Multimeter
5. Phase- frequency meter.

Circuit Diagram:

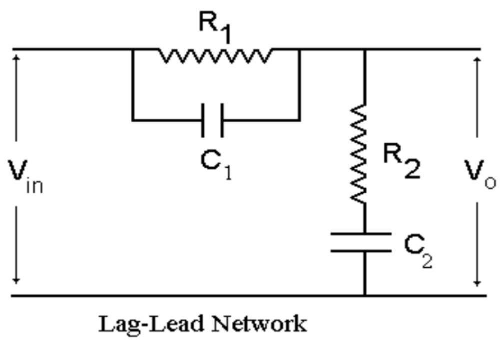


Fig.6.1: Circuit diagram Lag-Lead Compensator Network

Procedure:

1. Derive the transfer function for the lag lead network given above.
2. Connections are made as per the Lag lead circuit diagram by the selecting the proper values.
3. Switch ON the mains supply and apply sinusoidal wave by selecting suitable amplitude.
4. The frequency of the signal is varied in steps and at each step note down the corresponding magnitude of output and phase angle.
5. Draw the frequency response plot and hence find the transfer function & compare it with the design.

Derivation of transfer function:

Write the above circuit in Laplace form.

$Vi(s) = (Z_1+Z_2)*I(s)$, Where I(s) is the current in the circuit.

$Z_1 = (R_1//C_1S)$ and $Z_2 = (R_2 + 1/C_2S)$

$V_O(s) = Z_2*I(s)$

$Vo(s)/Vi(s) = Z_2 /(Z_1+Z_2)$

After simplification, $GC(S) = \dfrac{(S+ 1/ T_1)(S+ 1/ T_2)}{(S+ 1/ \beta T2)(S+ \beta/ T1)}$

Where, $T_1 = R_1C_1$, $T_2 = R_2C_2$, $R_1C_1+ R_2C_2 + R_1C_2 = 1/ \beta T_2 + \beta/ T_1$

Tabular Column:

V_i =………… V			
Sl. No.	**V_O, RMS (V)**	**Φ in degrees**	**Gain in dB = 20 log (V_O/V_i)**

Typical Graph:

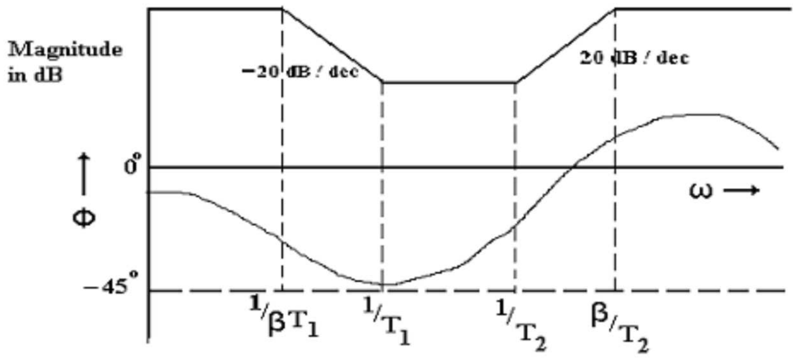


Fig.6.2: Typical response of Lag-lead compensator

Conclusion:

Outcome: At the end of the experiment,

1. The students will acquire the knowledge of the effect of lag-lead circuit on any system.

Viva Questions:

1. Mention the advantages and disadvantages of lag-lead compensator.
2. Give the differences between lag-lead and lead compensator.
3. Mention the applications of lag-lead compensator.

Preparation	Conduction	Calculation/Result Analysis	Total

Signature of Faculty Member

Free Space for rough work:

Experiment No: 07 **Date:**

- **A. Simulation of a typical second order system using MATLAB and determination of step response and evaluation of time domain specifications.**
- **B. Evaluation of effect of additional poles and zeros on time response of second order system.**
- **C. Evaluation of effect of pole location on Stability.**
- **D. Effect of loop gain of a negative feedback system on stability.**

Aim: To determine the step response of second order system and study the effect of adding poles and zeros on the response of the system.

Objective: The students will learn about program to determine step response of second order system and evaluate the time domain specifications.

Second-order systems are commonly encountered in practice, and are the simplest type of dynamic system to exhibit oscillations. In fact many real higher order systems are modelled as second-order to facilitate analysis. Typical examples are the mass-spring-damper systems and RLC circuits

Procedure:

Case I: To simulate a typical second order system and determine step response and evaluate time response specifications.

Standard form of transfer function of a second order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} ------(1)$$

The dynamic behavior of a second order system can be described in terms of ξ the damping ratio and ω_n natural frequency

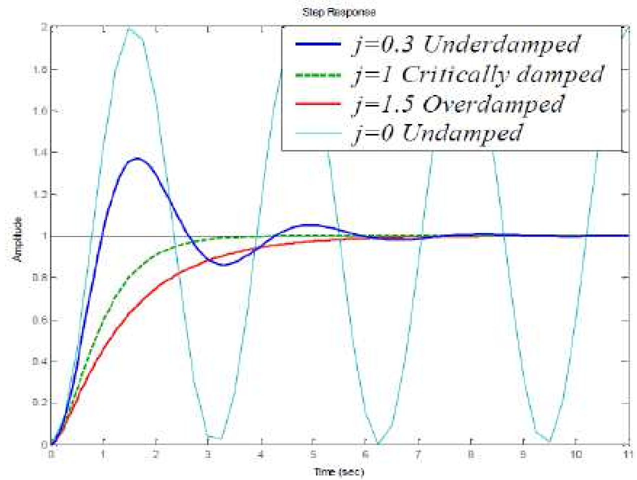
If, $0 < \xi < 1$, the closed loop poles are complex conjugate and lie in the left half of the s-plane. The system is then under damped and the transient response is oscillatory.

If, $\xi = 1$, the system is called critically damped.

If, $\xi > 1$, the system is called over damped.

If, $\xi = 0$, the system is undamped and the transient response does not die out; but oscillation continues indefinitely

$\zeta = 0$ *Undamping*	The system has two imaginary poles. There is no damping
$0 < \zeta < 1$ *Under-damping*	The system has stable imaginary poles. Quickly tends to equilibrium, but with oscillation
$\zeta > 1$ *Over-damping*	The system has stable real poles. Tends slower to equilibrium, but when it reach, remains in balance
$\zeta = 1$ *Critical damping*	The system has a double real stable pole. Tends to balance the maximum possible time without oscillation.



The qualitative analysis of first and second order systems regards the characteristics of the transient response and the steady state errors. The analysis of these characteristics helps determine the quality of the response regarding specific design requirements.

For example, when controlling the flow of a water tank, we want to stabilize the water capacity to a steady level before the tank overflows. This in control theory terms is the minimization of the overshoot. If in addition we desire this transition to a specific water lever to be done in as little time as possible, then we must study the rise and settling time of the system.

As another example, when we design a remote controlled navigation system, it is our aim to input specific coordinates for the system to follow. So in this case we want to minimize the difference between input and output, i.e. the steady state error.

Overall, the basic characteristics that we study are the following:

Overshoot: Describe the difference in the response of the system between the transitional and permanent state, when the system is stimulated by the unit step input. We are interested in the maximum elevation as well as the time it happens

Rise time: The time required to switch the system's response (in step input) from10% to 90% of it's final value

Settling time: The time needed to switch the system's response (in step input) and remain within a certain range of the final price (typically ±2%).

Peak time: The time required to reach the first, or maximum peak

<u>Step response of second order system and command window output</u>

```
wn= input('frequency')
zeta= input('damping factor')
k= input('constant')
num= [k*wn^2]
den= [1 2*zeta*wn wn^2]
g= tf(num, den)
t= feedback(g,1)
step(t, 'r')
data = stepinfo(t)
```

```
wn= input('frequency')
zeta= input('damping factor')
k= input('constant')
num= [k*wn^2]
deno= [1 2*zeta*wn wn^2]
g= tf(num, deno)
t= feedback(g,1)
step(g)
hold on
step(t, 'r')
grid
legend('Open-loop system’ ' closed-loop system')
data=stepinfo(g)
data=stepinfo(t)
```

GNU Octave code for plotting the unit step response of a second-order system for various values of damping ratio ξ and natural frequency ω_n= 1 rad/sec

```
clc;
t = 0:0.2:12;                          % Time vector
zeta = [0 0.2 0.4 0.6 0.8 1 1.5];      % Damping ratios
figure;
hold on; grid on;
for i = 1:length(zeta)
   num = [0 0 1];                      % Numerator: ωn^2 = 1
   den = [1 2*zeta(i) 1];              % Denominator for each zeta
   sys = tf(num, den);                 % Transfer function
   step(sys, t);                       % Plot step response
   data=stepinfo(sys)
end
xlabel('Time, t (sec)');
ylabel('Unit Step Response, c(t)');
legend('\zeta = 0','0.2','0.4','0.6','0.8','1.0','1.5');
title('Unit Step Responses for Various Damping Ratios');
```

MATLAB program to plot the unit step response of a second-order system for various values of natural frequency of oscillation ω_n, with fixed damping ratio ξ = 0.4

```
clc;
t = 0:0.01:10;
zeta = 0.4;
omega_n_values = [0.5, 1, 2, 4, 8];
figure;
hold on; grid on;
for i = 1:length(omega_n_values)
    wn = omega_n_values(i);
    num = [wn^2];
    den = [1, 2*zeta*wn, wn^2];
    sys = tf(num, den);
    step(sys, t);
    data=stepinfo(sys)
end
xlabel('Time (s)');
ylabel('Unit Step Response');
legend('\omega_n = 0.5','1','2','4','8');
title('Step Response for Varying \omega_n (Damping Ratio \zeta = 0.4)');
```

Time Response Characteristics:

Rise Time (T_R): Rise time is defined as the time required for the response to rise from 10% to 90% of the final value for an under damped system.

$$t_r = \frac{\pi - tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_n\sqrt{1-\xi^2}} ------(2)$$

Settling Time (T_S): Settling time is defined as the time required for the response to decrease and stay within the specified percentage (2% to 5%) of its final value.

$$t_s(5\%) = 3\tau = \frac{3}{\xi\omega_n} ---------(3)$$

$$t_s(2\%) = 4\tau = \frac{4}{\xi\omega_n} ---------(4)$$

Peak Overshoot (M_P): Peak overshoot is defined as the maximum response of the system measured from the final value.

$$M_P = e^{-\pi\xi\sqrt{1-\xi^2}} ---------- (5)$$

Peak Time (T_P): The time required to reach the first peak overshoot is defined as peak time.

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} ---------- (6)$$

1. Consider an example for the second order system with an open loop transfer function as $G(S) = \frac{5}{S(S+1)}$ and unity feedback system H(s) = 1.
2. The closed loop transfer function is determined by using the equation, $TF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} -------- (7)$
3. By substituting the values of G(s) and H(s) in above equation we get, $TF = \frac{C(s)}{R(s)} = \frac{5}{s^2+s+5} ----------(8)$
4. Comparing Equation (1) and (8), we getω_n= 2.236 and ξ= 0.224. Since ξ < 1, the given system is under damped.
5. Write the programusing GNU OCTAVE to simulate step response of the givensystem and save the file and load the control package.
6. Evaluate time domain specifications :
   a. Run the program. Step response appears on the screen.
   b. By using pointer read the time domain specifications.

Program:

Program to obtain step response of second order system

```
clc
% prg c(t)
num = [5];
den = [1 1 5];
G = tf(num, den)
kp=dcgain(G)
ess=1/(1+kp)
w = sqrt (den(3))
zeta = den(2) / (2*w)
TD=(1+0.7*zeta)/w
TS = 4/ (zeta*w)
TP = pi/ (w*sqrt(1-zeta^2))
TR=(pi-atan((sqrt(1-zeta^2))/zeta))/(w*sqrt(1-zeta^2))
Percentovershoot= exp(-zeta*pi/ sqrt(1-zeta^2))*100
step(G)
title('Step response of c(t)')
figure
pzmap(G)
title('pole zero map of c(t)')
figure
```

Output Response:

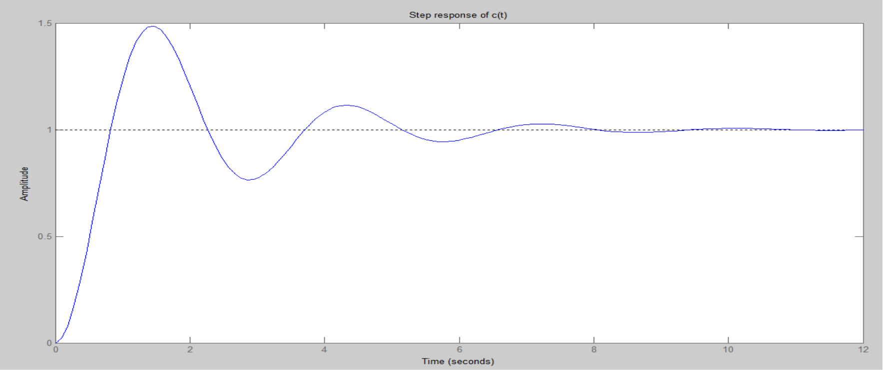


Fig.7.1: Step Response Curve

Results:

Particulars	Theoretical Value	Simulated Value
Delay Time		
Rise Time		
Peak Time		
Settling Time		
Peak Overshoot		

Case – II: Evaluation of the effect of additional poles and zeroes on time response of second order system.

Effect of adding pole to the transfer function:

1. If pole is added at -2 for the open loop transfer function.

$$G(S) = \frac{5}{S(S+1)}.$$

The transfer function gets modified as,

$$G(s) = \frac{5}{s(s+1)(s+2)}$$

2. The closed loop transfer function for the given system is obtained as,

$$TF = \frac{5}{s^3 + 3s^2 + 2s + 5}$$

3. Effect of adding pole to transfer function results in
   a. Increases the order of the system.
   b. Increases the overshoot.
   c. Reduces the stability.
   d. Increases the Rise time of the step response.

Program:

```
clc
% prg c(t)
num = [5];
den = [1 32 5];
G = tf(num, den)
step(G)
```

```
title('Step response of c(t)')
figure
pzmap(G)
title('pole zero map of c(t)')
figure
```

Output Response:

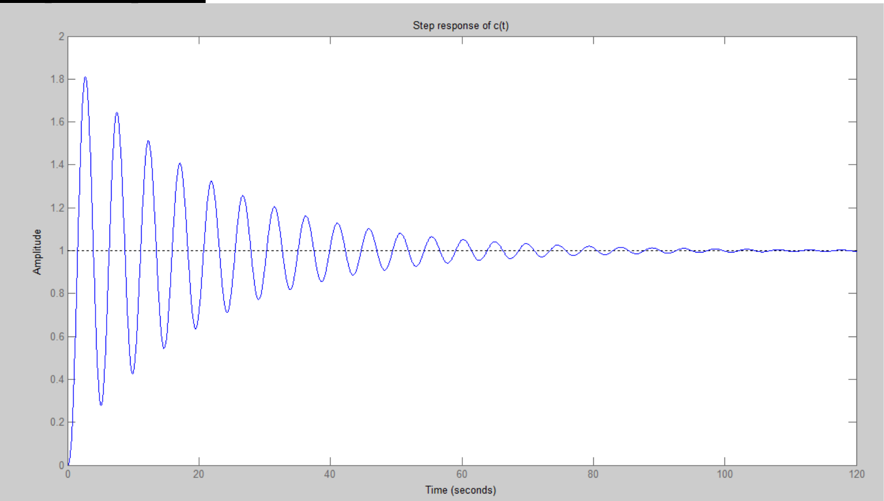


Fig.7.2: Step response after adding pole to the transfer function

Results:

Particulars	Simulated Value
Delay Time	
Rise Time	
Peak Time	
Settling Time	
Peak Overshoot	

Effect of adding zero to the transfer function:

1. If zero is added at -2 for the open loop transfer function.
   $G(S) = \frac{5}{S(S+1)}$.
   The transfer function gets modified as,
   $$G(s) = \frac{5(s+2)}{s(s+1)}$$
2. The closed loop transfer function for the given system is obtained as,
   $$TF = \frac{5s+10}{s^2+6s+10}$$
3. Comparison of location for zeroes with respect to overshoot and damping parameters.

Time response parameters	Location of Zeros		
	Far away from the imaginary axis	Zero moving towards right	Zero moves closer to origin
Overshoot	Large	Reduced	Almost nil
Damping	Very poor	Improved	Improves

Program:

```
clc
% prg c(t)
num = [5 10];
den = [1 6 10];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Output Response:

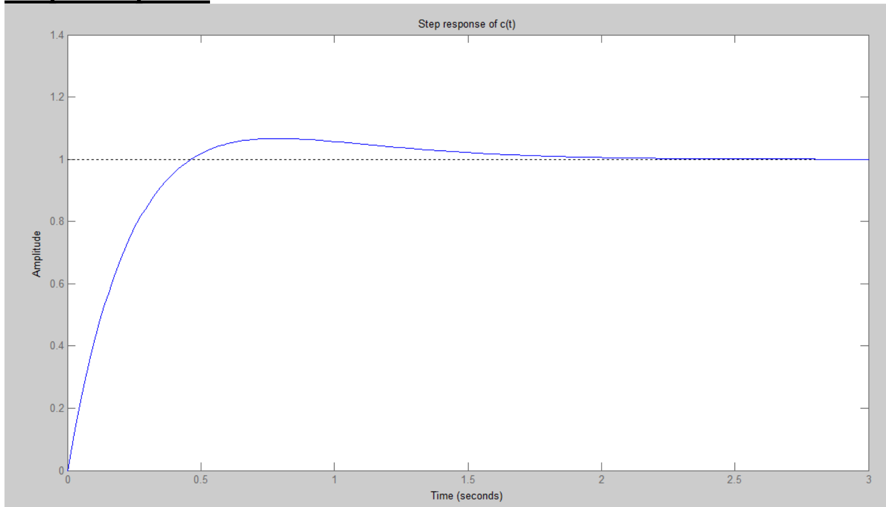


Fig.7.3: Step response after adding zeros to the transfer function

Results:

Particulars	Simulated Value
Delay Time	
Rise Time	
Peak Time	
Settling Time	
Peak Overshoot	

Case – III: Evaluation of effect of pole location on Stability:

1. The stability of a feedback system is directly related to the location of the roots of the characteristic equation of the system transfer function.
2. "A linear system will be stable if and only if all the poles of the transfer function are located on the left half of the 'S' plane".
3. Consider the below examples to study the effect of location of pole on stability of the system.

Example 1(a): $\frac{C(S)}{R(S)} = \frac{1}{S+5}$ i.e. Pole at s = -5

Program:

```
clc
% prg c(t)
num = [1];
den = [1 5];
G = tf(num, den)
impulse(G)
title('Step response of c(t)')
figure
```

Output Response:

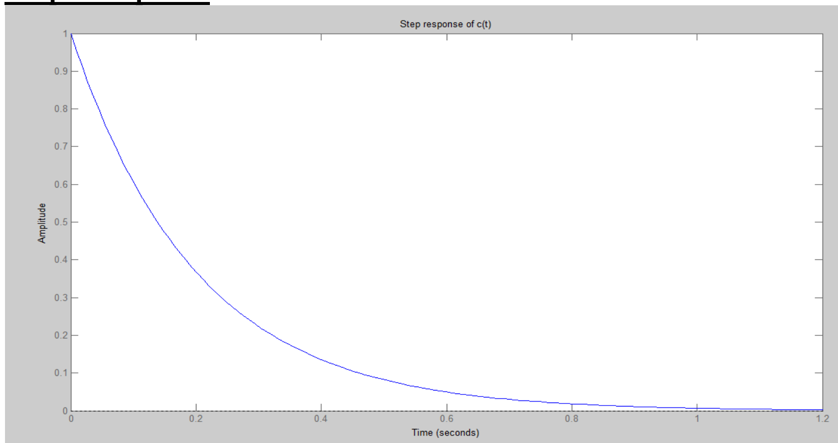


Fig.7.4: Time response for $\frac{C(S)}{R(S)} = \frac{1}{S+5}$

Example 1(b): $\frac{C(S)}{R(S)} = \frac{1}{S-5}$ i.e. Pole at s = 5

Program:

```
clc
% prg c(t)
num = [1];
den = [1 -5];
G = tf(num, den)
impulse(G)
title('Step response of c(t)')
figure
```

Output Response:

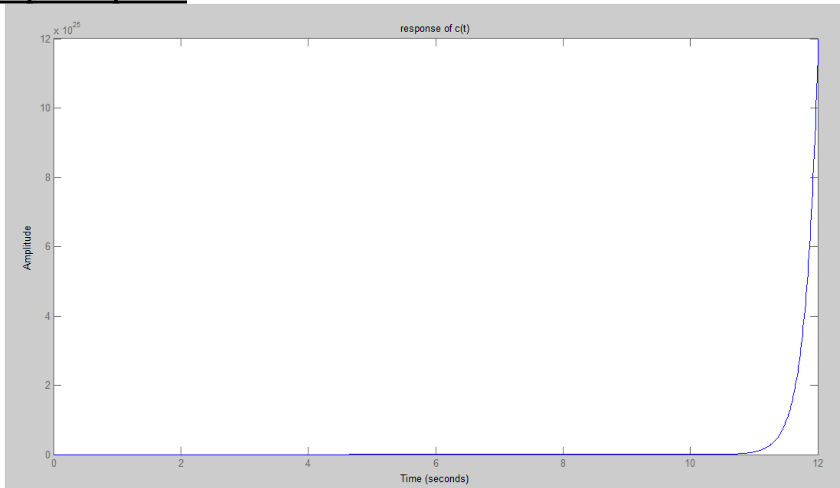


Fig.7.5: Time response for $\frac{C(S)}{R(S)} = \frac{1}{S-5}$

Example 2(a): $\frac{C(S)}{R(S)} = \frac{1}{S^2+5S+29}$ i.e. Pole at s = -2.5$\pm j4.77$

Program:

```
clc
% prg c(t)
num = [1];
den = [1 5 29];
G = tf(num, den)
impulse(G)
title('Step response of c(t)')
figure
```

Output Response:

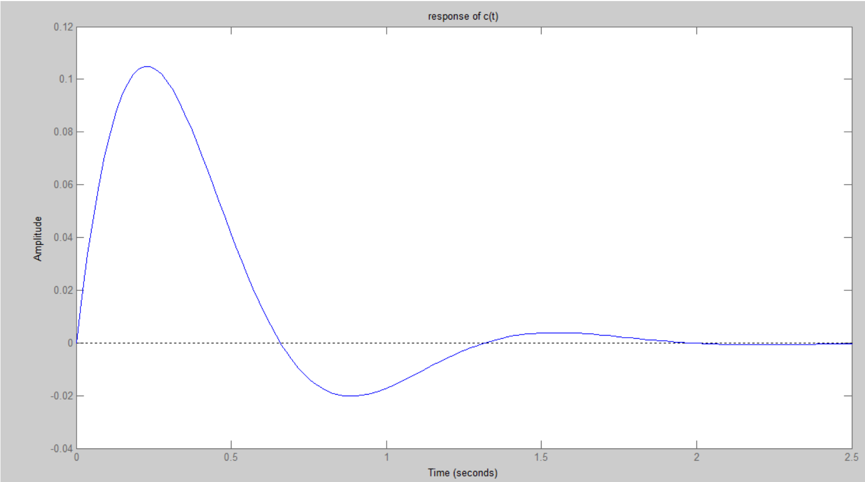


Fig.7.6: Time response for $\frac{C(S)}{R(S)} = \frac{1}{S^2+5S+29}$

Example 2(b): $\frac{C(S)}{R(S)} = \frac{1}{S^2-5S+29}$ i.e. Pole at s = 2.5$\pm j4.77$

Program:

```
clc
% prg c(t)
num = [1];
den = [1 -5 29];
G = tf(num, den)
impulse(G)
title('Step response of c(t)')
figure
```

Output Response:

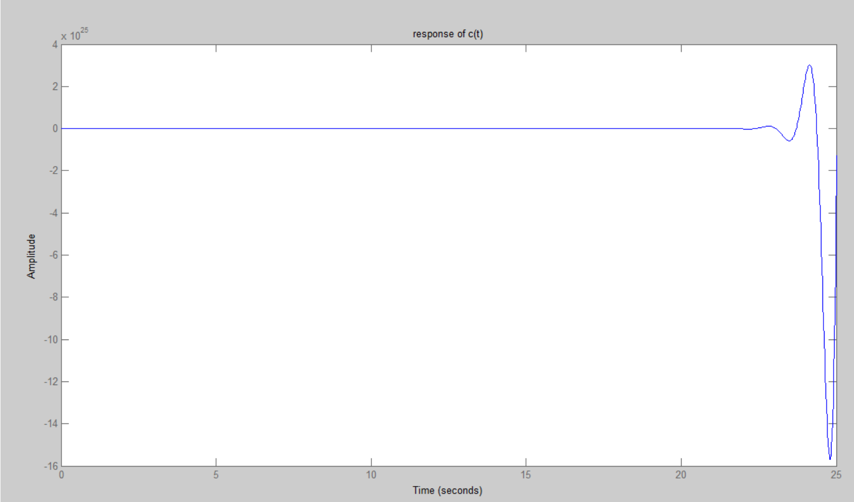


Fig.7.7: Time response for $\frac{C(S)}{R(S)} = \frac{1}{S^2-5S+29}$

Case – IV: Effect of loop gain of a negative feedback system on stability

1. Effect of loop gain of a negative feedback system on stability of a system can be identified by knowing characteristic equation of the system and applying "Routh's Stability Criterion".
2. Consider an open loop transfer function of a system as,

$$G(s)H(s) = \frac{K}{s(s^2+4s+20)(s+4)}$$

3. Determine the value of 'K' loop gain for which the system is stable. Consider the system with unity feedback H(s) = 1.
4. The closed loop transfer function is obtained as,

$$TF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s^4+8s^3+36s^2+80s+K}$$

5. From closed loop TF, Determine characteristic equation

i.e. $S^4 + 8S^3 + 36S^2 + 80S + K = 0$

6. Form Routh's stability array,

S^4	1	36	K
S^3	8	80	$\div$ *by* 8
S^3	1	10	
S^2	26	K	
S^1	(260-K)/26	0	
S^0	K		

For a system to be stable, it is necessary and sufficient that each term of first column of Routh array of its characteristic equation be positive. If this condition is not met, the system is unstable and number of sign changes of the terms of the first column of the Routh array corresponds to the number of roots of the characteristic equation in the right half of the s-plane.

7. For stability all the coefficients in the first column of the array must be positive, therefore

$$K > 0 \text{ and } (260-K)/26 > 0$$

By rearranging the terms the range of K for which the system is stable is $0 < K < 260$. When K =260, the system gives sustained oscillations.

Program:

```
clc
% prg c(t)
num = [1];
den = [1 8 36 80 0];
G = tf(num, den)
[r,k]=rlocus(G,[240:10:280])
title('response of c(t)')
figure
```

Results:

r = (Pole location)				
-3.9061+ 3.1037i	-3.9542 + 3.13i	-4.0000 + 3.1623i	-4.0436 + 3.1900i	-4.0853+3.2169i
-3.9061 - 3.1037i	-3.9542 - 3.135i	-4.0000 - 3.1623i	-4.0436 - 3.1900i	-4.0853 - 3.2169i
-0.0939 + 3.103i	-0.0458 + 3.13i	0.0000 + 3.1623i	0.0436 + 3.1900i	0.0853 + 3.2169i
-0.0939 - 3.1037i	-0.0458 - 3.133i	0.0000 - 3.1623i	0.0436 - 3.1900i	0.0853 - 3.2169i
k = (value of loop gain)				
240	250	260	270	280

By seeing the above table, we can tell at

a. K < 260 system is stable because all the 4 poles are in left half of s-plane
b. K = 260 system is marginally stable because 2 poles are on the imaginary axis
c. K > 260 system is unstable because 2 poles are located on right half of s-plane.

Conclusion:

Outcomes: At the end of the experiment,

1. Students will be able to find Delay Time, Rise Time, Peak Time, Settling Time and Peak Overshoot from OCTAVE simulation.
2. The students will know about the effect of adding poles and zeros to the second order system.
3. Effect pole location and loop gain on stability of the system.

Viva Questions:

1. Mention the time response specifications and define them.
2. Define steady state error and mention error constants.
3. Define static position error constant and static velocity error constant.

Preparation	Conduction	Calculation/Result Analysis	Total

Free Space for rough work:

Signature of Faculty Member

Experiment No: 08 **Date:**

To simulate a second order system and study the effect of (a) P, (b) PI, (c) PD and (d) PID controller on the step response

Aim: To study the effect of different controllers on the step response of second order system.

Objective: The students will learn regarding different controllers and their effects on step response of second order system.

Procedure:

1. Consider an open loop transfer function $G(S) = \frac{1}{s^2+10s+20}$ with unity feedback system as shown in block diagram.
2. Obtain the closed loop transfer function and write the program using GNU OCTAVE to simulate step response of the given system and save the file and load the control package.
3. Evaluate time domain specifications :
   e. Run the program. Step response appears on the screen.
   f. By using pointer read the time domain specifications and also determine the steady state error.

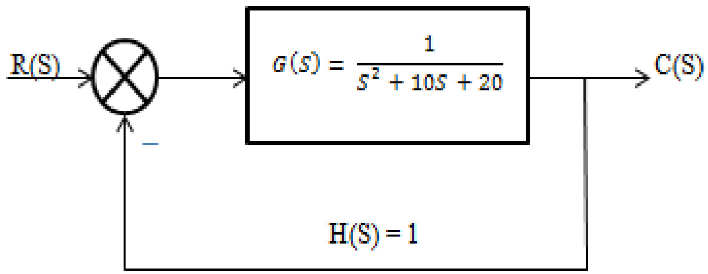


Fig.8.1: Closed loop system

Closed Loop Transfer Function $\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$

$$\frac{C(S)}{R(S)} = \frac{1}{S^2 + 10S + 21}$$

Program:

```
clc
% prg c(t)
num = [1];
den = [1 10 21];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Tabular Column:

Time domain specification parameter	Simulated values
Delay Time	
Rise Time	
Peak Time	
Settling Time	
Peak Overshoot	
Steady State Error	

Proportional (P) Controller:

1. In Proportional Controller the actuating signal is proportional to the error signal. The error signal is the difference between reference input signal and feedback signal obtained from output.
2. Analyse the above system by including proportional controller as shown in block diagram.
3. Obtain the closed loop transfer function and write the program using GNU OCTAVE to simulate step response of the given system and save the file and load the control package
4. The proportional controller reduces the rise times and steady state error and increases the peak overshoot. For different values of K_P obtain the time specifications and steady state error.

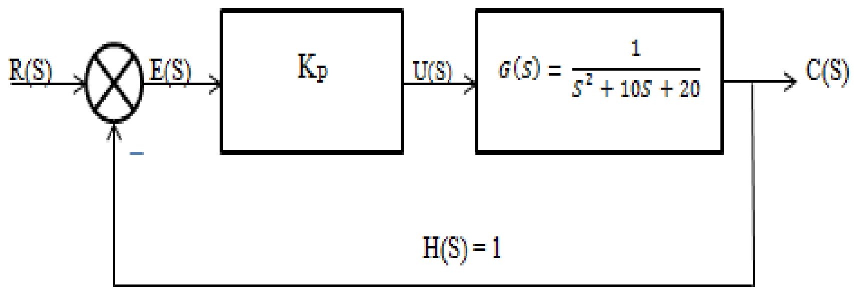


Fig.8.2: Block diagram of Second order system with Proportional Controller

Closed Loop Transfer Function

$$\frac{C(S)}{R(S)} = \frac{K_P}{S^2 + 10S + 20 + K_P}$$

Program:

```
clc
% prg c(t)
num = [10];
den = [1 10 30];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Tabular Column:

Time domain specification parameter	**Simulated values**			
K_P	**10**	**100**	**300**	**500**
Delay Time				
Rise Time				
Peak Time				
Settling Time				
Peak Overshoot				
Steady State Error				

Proportional Integral (PI) Controller:

1. In PI controller the actuating signal consists of proportional-error signal along with the integral of the error signal.
2. Analyse the given second order system by including proportional Integral (PI) controller as shown in block diagram.
3. Obtain the closed loop transfer function and write the program using GNU OCTAVE to simulate step response of the given system and save the file and load the control package.
4. The proportional Integral controller reduces the steady state error and improves the transient response but it also increases the system settling

time. For different values of K_I keeping K_P constant obtain the time specifications and steady state error.

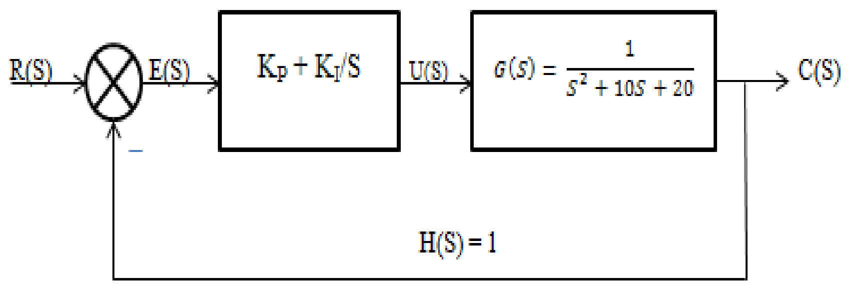


Fig.8.3: Block diagram of Second order system with Proportional Integral Controller

Closed Loop Transfer Function

$$\frac{C(S)}{R(S)} = \frac{SK_P + K_I}{S^3 + 10S^2 + (20 + K_P)S + K_I}$$

Program:

```
clc
% prg c(t)
num = [30 50];
den = [1 10 50 50];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Tabular Column:

Time domain specification parameter	Simulated values		
	$K_P = 30$, $K_I = 50$	$K_P = 30$, $K_I = 70$	$K_P = 30$, $K_I = 100$
Delay Time			
Rise Time			
Peak Time			
Settling Time			
Peak Overshoot			
Steady State Error			

Proportional Derivative (PD) Controller:

1. In PD controller the actuating signal consists of proportional error signal and also the derivative of error signal.
2. Analyse the given second order system by including proportional Derivative (PD) controller as shown in block diagram.
3. Obtain the closed loop transfer function and write the program using GNU OCTAVE to simulate step response of the given system and save the file and load the control package
4. The derivative controller reduces both the overshoot and the settling time and had a small effect on rise time and steady state error.
5. To control the steady state error the derivative gain K_D must be high.
6. The PD controller reduces the response times of the system and can make it susceptible to noise.

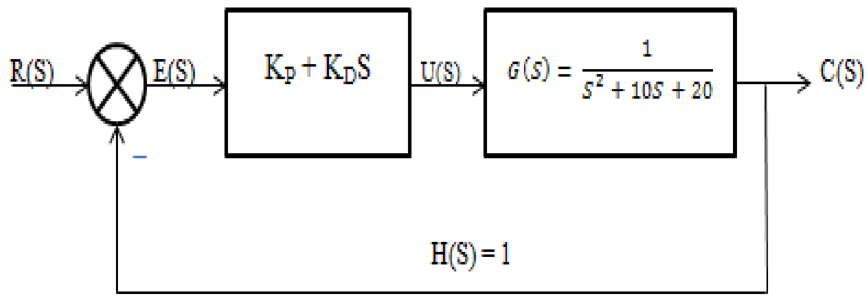


Fig.8.4: Block diagram of Second order system with Proportional Derivative Controller

Closed Loop Transfer Function

$$\frac{C(S)}{R(S)} = \frac{K_P + K_D S}{S^2 + (10 + K_D)S + 20 + K_P}$$

Program:

```
clc
% prg c(t)
num = [10 500];
den = [1 20 520];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Tabular Column:

Time domain specification parameter	Simulated values	
	K_P = 500, K_D = 10	K_P = 500, K_D = 50
Delay Time		
Rise Time		
Peak Time		
Settling Time		
Peak Overshoot		
Steady State Error		

Proportional Integral Derivative Controller (PID) Controller:

1. For PID controller, the actuating signal consists of proportional error signal and also the derivative and integral of error signal.
2. Analyse the given second order system by including proportional Integral Derivative (PD) controller as shown in block diagram.
3. Obtain the closed loop transfer function and write the program using GNU OCTAVE to simulate step response of the given system and save the file and load the control package.
4. The PID controller removes the steady state error and reduces the settling time while maintaining reasonable transient response.

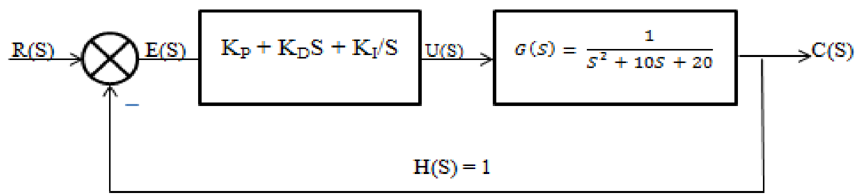


Fig.8.5: Block diagram of Second order system with PID Controller

Closed Loop Transfer Function

$$\frac{C(S)}{R(S)} = \frac{S^2K_D + SK_P + K_I}{S^3 + (10 + K_D)S^2 + (20 + K_P)S + K_I}$$

Program:

```
clc
% prg c(t)
num = [10 500];
den = [1 20 520];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Tabular Column:

Time domain specification parameter	Simulated values
	$K_P = 500, K_D = 50, K_I = 400$
Delay Time	
Rise Time	
Peak Time	
Settling Time	
Peak Overshoot	
Steady State Error	

Conclusion:

Outcomes: At the end of the experiment,

1. The students have knowledge on PI, PD and PID controller.
2. The students will be able to design PI, PD and PID controller.

PID Controller: $G_c(s) = K_P + K_D s + \frac{K_I}{s} = \frac{K_D(s^2 + as + b)}{s}$

$a = K_P/K_D$ and $b = K_I/K_D$

PI Controller: $G_c(s) = K_P + \frac{K_I}{s} = \frac{K_P(s + c)}{s}$

$c = K_I/K_P$

PD Controller: $G_c(s) = K_P + K_D s = K_D(s + a)$

$a = K_P/K_D$

Effect of Increasing the PID Gains K_P, K_D, and K_I on the Step Response

PID Gain	Percent Overshoot	Settling Time	Steady-State Error
Increasing K_P	Increases	Minimal impact	Decreases
Increasing K_I	Increases	Increases	Zero steady-state error
Increasing K_D	Decreases	Decreases	No impact

Viva Questions:

1. Define PI, PD and PID Controllers
2. Give the comparison between PI and PD Controller.
3. Why differential control is not used alone?
4. Mention the applications of PID Controller.

Preparation	Conduction	Calculation/Result Analysis	Total

Free Space for rough work:

Signature of Faculty Member

Experiment No: 09 **Date:**

A. To simulate a DC Position control system and obtain its step response.
B. To verify the effect of the input wave form, system type on steady state errors.
C. To perform a trade-off study for lead compensation.

Case A:

Aim: To study the DC position control system of DC motor and obtain its step response.

Objective: The students will learn regarding DC position control system of DC motor.

Circuit Diagram:

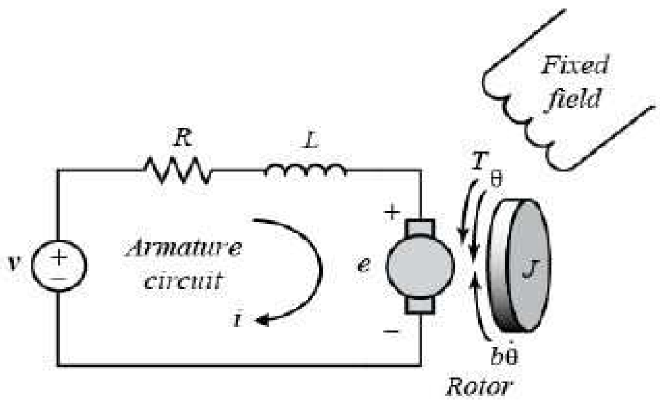


Fig.9.1: Armature controlled DC motor

Design:

The closed loop transfer function of the system is given by

$$\frac{\theta(S)}{V(S)} = K_T/s[(Js+b)(Ls+R)+K_TK_B]$$

Where,

J = Moment of inertia of the rotor = $3.228 * 10^{-6}$ kgm^2

B = Motor viscous friction constant = $3.5077 * 10^{-6}$ Nms

K_B = Electromotive force constant = 0.0274 V/rad/sec

K_T = Motor torque constant = 0.0274 Nm/A

R = Electric Resistance = 4Ω

L = Electric Inductance = $2.75 * 10^{-6}$ H

Program:

```
clc
% prg c(t)
num = [];
den = [];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Procedure:

1. Obtain the closed loop transfer function for the system given by substituting the given values.
2. Write the program in GNU Octave after entering the coefficients of numerator and denominator.
3. Obtain the step response of the given system and determine different time specifications and steady state error.

4. Tabulate the results.

Tabular Column:

Time domain specification parameter	Simulated Values
Delay Time	
Rise Time	
Peak Time	
Settling Time	
Peak Overshoot	
Steady State Error	

Case B: To verify the effect of the input wave form, system type on steady state errors.

The open loop transfer function of a unity feedback system can be written in pole zero form as given below.

$$G(s) = \frac{K(s+Z_1)(s+Z_2)\ldots\ldots}{s^n(s+P_1)(s+P_2)\ldots\ldots}$$

The above equation involves S^n in denominator which corresponds to number of integrations in the system. As S tends to zero, this term dominates in determining steady state error. Control systems are therefore classified in accordance with number of integrations in open loop transfer function G(S) as Type 0, Type 1 and Type 2 systems for n=0,1,2 respectively.

Example-1: Consider a type 0 system whose transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{(s+1)}{(s^2+2s+6)}$$

Step response of the system can be simulated as given in the program below.

Program:

```
clc
% prg c(t)
num = [1 1];
den = [1 2 6];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Output Response:

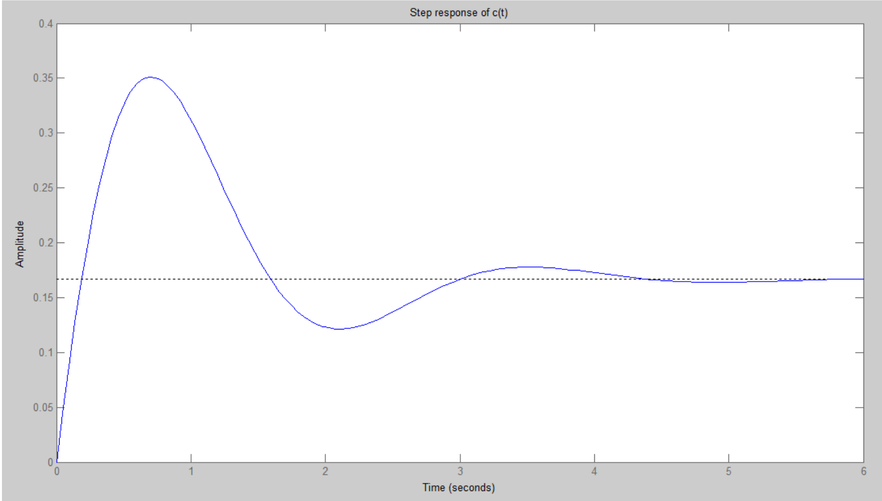


Fig 9.2: Step Response of Type 0 System

Example-2: Consider a type 1 system given by transfer function

$$\frac{C(s)}{R(s)} = \frac{(s+1)}{s(s^2+2s+6)}$$

Step response of the system can be simulated as given in the program below.

Program:

```
clc
% prg c(t)
num = [1 1];
den = [1 2 6 0];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Output Response:

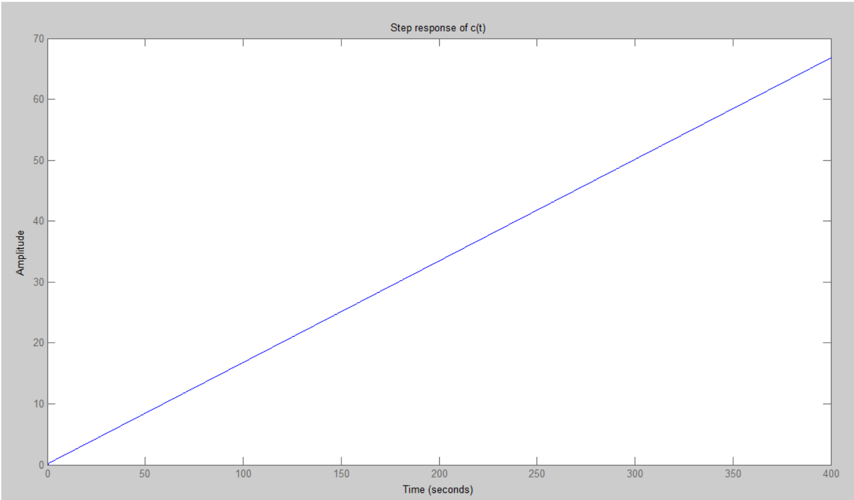


Fig 9.3: Step Response of Type 1 System

Example-3: Consider a type 2 system given by transfer function

$$\frac{C(s)}{R(s)} = \frac{(s+1)}{s^2(s^2+2s+6)}$$

Step response of the system can be simulated as given in the program below.

Program:

```
clc
% prg c(t)
num = [1 1];
den = [1 2 6 0 0];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Output Response:

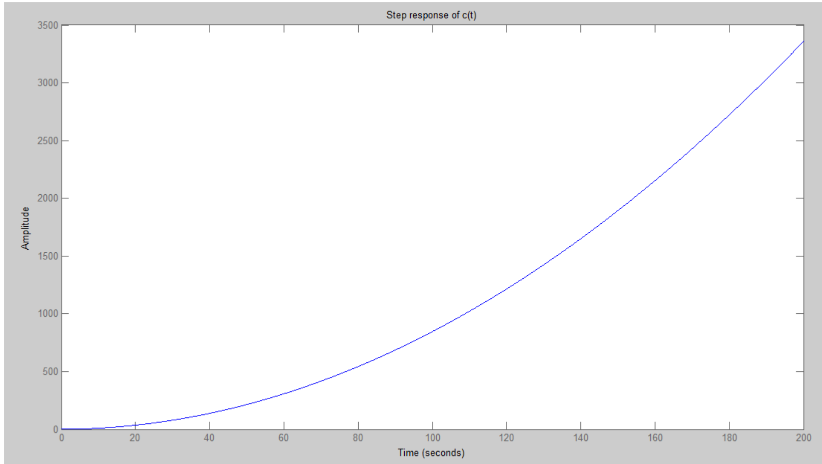


Fig 9.4: Step Response of Type 2 System

Observation:

System Type	Steady state error for unit step input
Type 0	$1/1+K_p$
Type 1	∞
Type 2	∞

Example-4: Consider the type 0, type 1 and type 2 systems given in examples 1, 2 and 3. The effect on steady state errors for these systems for a ramp input can be simulated by the program given below.

Program:

```
sys1=tf([1 1],[1 2 6 ]);
sys2=tf([1 1],[1 2 6 0]);
sys3=tf([1 1],[1 2 6 0 0]);
t=0:0.1:10;
u1=t;
lsim(sys1,'r', sys2,'m', sys3,'g', u1,t)
```

Output Response:

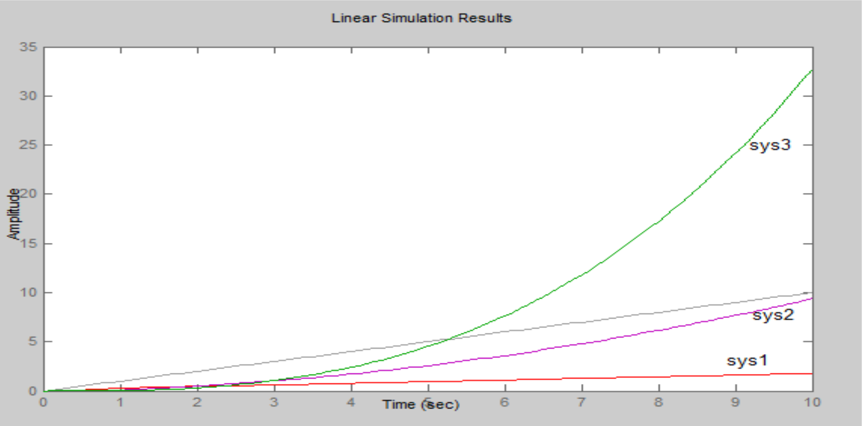


Fig.9.5: Effect of ramp input on steady state errors of type 0(sys1), type 1(sys2) and type 2(sys3) systems

Observation:

System Type	Steady state error for ramp input
Type 0	∞
Type 1	$1/K_v$
Type 2	0

Example-5: Consider the type 0, type 1 and type 2 systems given in examples 1, 2 and 3. The effect on steady state errors for these systems for a parabolic input can be simulated by the program given below.

Program:

```
sys1=tf([1 1],[1 2 6 ]);
sys2=tf([1 1],[1 2 6 0]);
sys3=tf([1 1],[1 2 6 0 0]);
t=0:0.1:10;
u1=t.*t;
lsim(sys1,'r', sys2,'m', sys3,'g', u1,t)
```

Output Response:

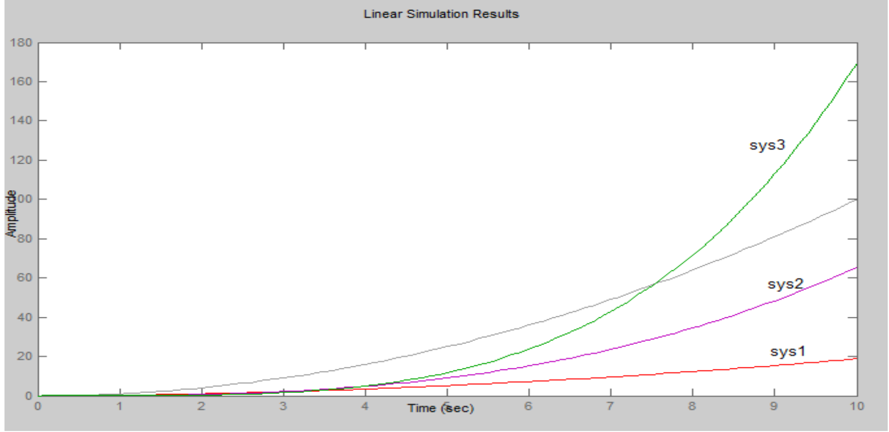


Fig 9.6: Effect of parabolic input on steady state errors of type 0(sys1), type 1(sys2) and type 2(sys3) systems

Observation:

System Type	Steady state error for Parabolic input
Type 0	0
Type 1	0
Type 2	$1/K_a$

Case C: To perform a trade-off study for lead compensation

The lead compensation on bode plots proceeds by adjusting the system error constant to a desired value.

The phase margin of the uncompensated system is then checked. Iffound satisfactory, the lead compensation is designed to meet the specified phase margin.With lead compensation introduced in the system, following observations are made.

- Phase margin is increased.
- Bandwidth increases.
- Peak resonance is reduced.
- Resonance frequency is increased.

Thus in general, the effect of lead compensator increases the margin of stability and speed of response.

Example: Consider a type 1 unity feedback system with an open loop transfer function.

$$G_f(s) = \frac{12}{S(S+1)}.$$

The bode plot of the given system is obtained by the program given below.

Program:

```
num = [12];
den = [1 1 0];
sys = tf (num , den)
bode (sys)
margin(sys)
```

Output Response:

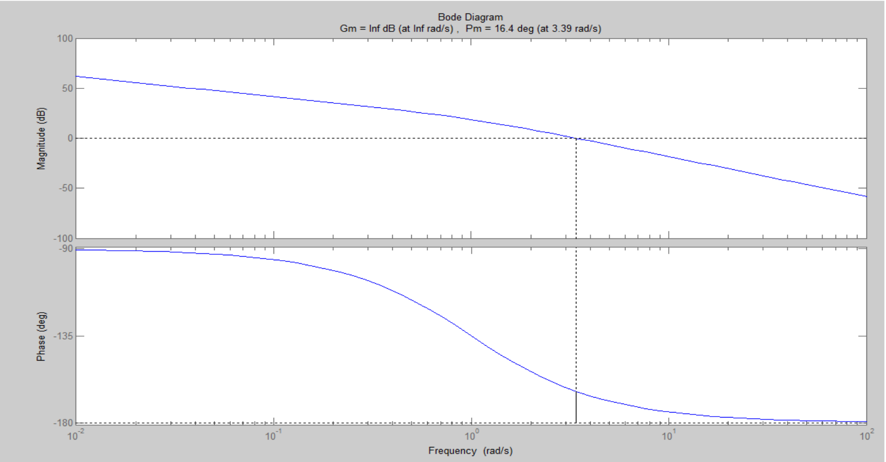


Fig 9.7 Bode plot of a type 1 system without compensation

Consider a lead compensator whose transfer function is given by;

$$G_c(s) = \frac{0.377s+1}{0.128s+1}$$

The open loop transfer function of the compensated system is now given by;

$$G_c(s) = \frac{12(0.377s+1)}{S(S+1)(0.128s+1)}$$

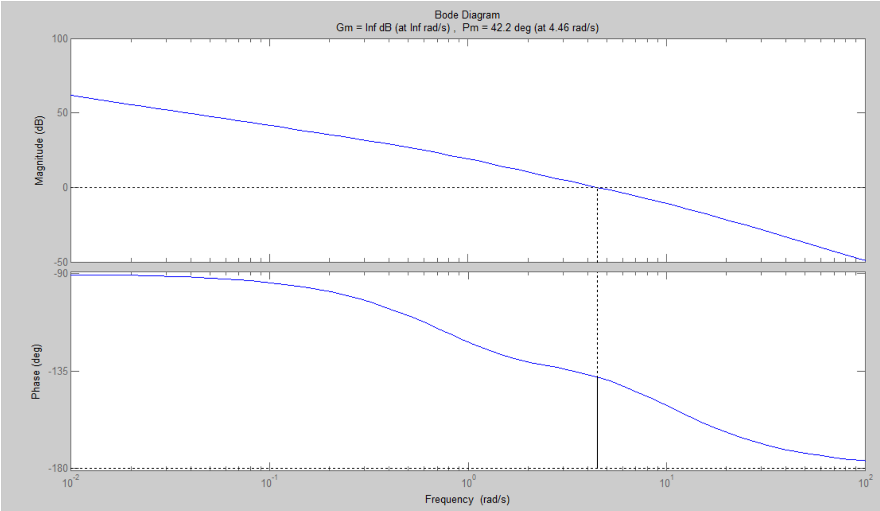


Fig 9.8: Bode plot of a compensated system

Observation:

System	Phase margin
Uncompensated system	16.4 degrees at 3.3 rad/sec
Compensated system	42.2 degrees at 4.46 rad/sec

Conclusion:

Outcome: At the end of experiment,

The students will have knowledge of DC position control of DC motor and also effect of type of system and input waveform on steady state error.

Viva Questions:

1. Define DC position of a system.
2. Differentiate between armature controlled and field controlled DC motor.
3. Define steady state error and mention different error coefficients.

Preparation	Conduction	Calculation/Result Analysis	Total

Free Space for rough work:

Signature of Faculty Member

Experiment No: 10 **Date:**

To examine the relationships between open-loop frequency response and stability, open loop frequency and closed loop transient response

Aim: To study the relationship between open loop frequency response and stability, open loop frequency and closed loop transient response.

Objective: The students will learn regarding open loop frequency response, stability and closed loop transient response.

Stability of a system can be accessed by determining the gain margin and phase margin. These gain and phase margins can be found by drawing the frequency response plots. **Polar plots, Bode plots, Nyquist plots** are the examples of frequency response plots.

Gain margin (GM): This is the factor by which the magnitude of G(jω)H(jω) at the phase cross over frequency is to be multiplied to make it unity. If the phase cross over frequency is ω_p, this is the frequency at which the phase plot of G(jω)H(jω) is crossing the -180° line (phase angle = -180°).

Therefore $|G(j\ \omega p)H(j\omega p)|GM = 1$ or

20log $|G(j\ \omega p)H(j\omega p)|$ + 20log (GM) = 0

Hence gain margin in db is the value by which the log magnitude plot is shifted to make 0db at the phase cross over frequency.

To determine the gain margin, find the phase cross over frequency from the phase plot and then find the gain at this frequency.

Therefore GM = 0 – gain in db at the phase cross over frequency.

Phase margin (PM): The phase margin is the amount of phase to be added to the phase angle at the gain cross over frequency to make the phase angle = -180°. If ω_g is the gain cross over frequency. This is the frequency at which the magnitude of G(jω)H(jω) is unity or 0db.

To determine the phase margin, find the gain cross over frequency from the magnitude plot and then find the phase angle at this frequency.

PM = phase angle at the gain cross over frequency + 180°

Condition for stability through PM and GM: For a system to be stable both gain margin and phase margins must be positive. Also $\omega_{p}>\omega_g$.

Construct the Bode plot and find gain margin and phase margin and comment on stability for the given transfer function.

$$G(s) = \frac{50}{s(1+0.5s)\,(1+0.05s)}$$

Program:

```
num = 50;
den =[0.025 0.55 1 0];
sys = tf(num,den);
bode(sys);
margin(sys);
```

From the plot note down the gain margin (GM), phase margin (PM) and the corresponding cross over frequencies. For unstable systems, GM and PM will not be displayed correctly but can be obtained by clicking on the plot at suitable points.

a. Click on the phase angle curve and find the frequency at which the curve crosses the 180^0 line. This gives the phase cross over frequency ω_p.
b. Click on the magnitude curve and find the magnitude at $\omega=\omega_p$. The gain margin is calculated as, GM = 0 - magnitude at phase cross over frequency.
c. Click on the magnitude curve and find the frequency at which the curve crosses 0dB line. This will give the gain cross over frequency ω_g.
d. Click on the phase angle curve and find the phase angle at $\omega=\omega_g$. The phase margin is calculated as PM = phase angle at the gain cross over frequency + 180° at $\omega=\omega_g$.

The GM, PM and the cross over frequencies can also be obtained using the following function.

[GM PM WCF WCG] = margin (sys);

Where GM = gain margin in abs unit (20logGM is the GM in db)

PM = Phase margin
WCF = phase cross over frequency
WCG = Gain cross over frequency.
These parameters will be displayed in the command window.

Check for the stability of the system.For a system to be stable both gain margin and phase margins must be positive. Also $\omega_{p>}\omega_{g}$ .If this condition is satisfied then the system is stable or otherwise the system is unstable.

Output Response:

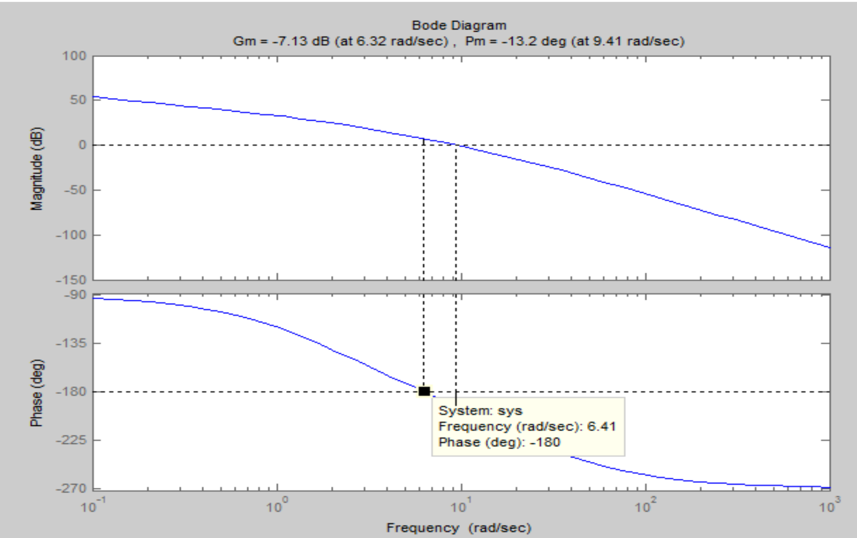


Fig.10.1: Frequency response of the system

Conclusion:

Outcome: At the end of experiment,

The students will have knowledge of obtaining the stability analysis of the system by using the Bode plot and the relationship between the open loop frequency and the stability.

Viva Questions:

1. Define phase margin?
2. Define gain margin?
3. Define phase cross over frequency?
4. Define gain cross over frequency?

Preparation	Conduction	Calculation/Result Analysis	Total

Free Space for rough work:

Signature of Faculty Member

Experiment No: 11 **Date:**

To study the effect of open loop poles and zeros on root locus contour and Comparative study of bode, Nyquist and root locus with respect to stability.

Aim: To obtain Root locus of a given T. F. and hence finding breakaway point, intersection point on imaginary axis and analysis complementary information of linear control system stability by applying graphical methods: Bode plot, Nyquist Plot and Root locus method for common transfer function.

Objectives:

1. The students will learn about procedure for drawing root locus diagram and also obtaining the same by using simulation.
2. The students will knowledge about the breakaway point and intersection point.
3. The students will compare complementary information of linear control system stability by applying different graphical methods Bode plot, Nyquist Plot and Root locus and how each method complements the other.

Procedure:

1. Open the GNU OCTAVE command window.
2. Click on file/new/M file to open the editor window. In editor window enter theprogram
3. Save the program as .M file.
4. Execute the program by selecting run.
5. Copy the plot obtained, note down the breaking point, intersection point.

Program for root locus:

Given $G(s)H(s)=\dfrac{K}{s(s+1)(s+2)(s+3)}$

Program:

```
num=[1 ];
den=[1 6  11 6 0];
sys=tf(num,den);
printsys(num,den);
[r,k]=rlocus(sys)
rlocus(sys)
```

Typical Graph:

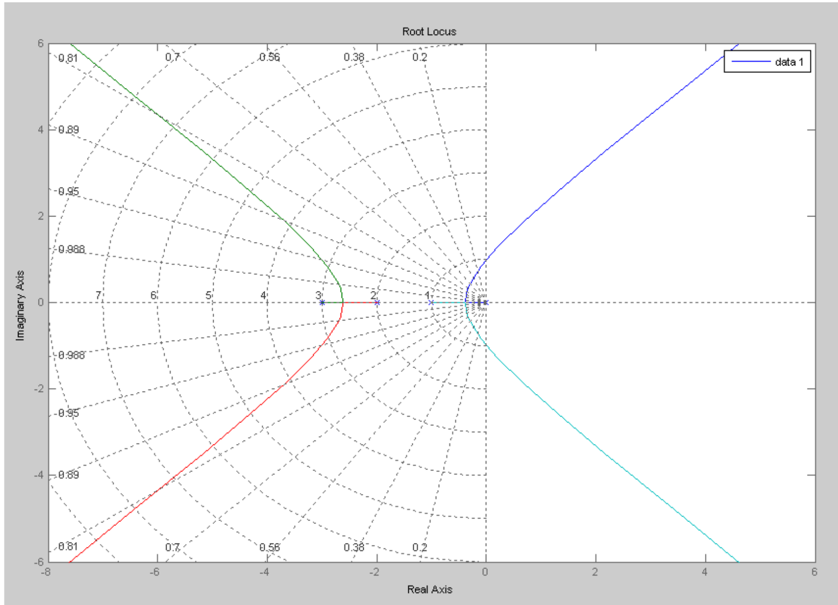


Fig.11.1: Root locus curve

Tabular Column:

	Theoretical Values	Simulated Values
Poles		
Breakaway Point		
Gain		
Imaginary Axis Crossover		

Effect of adding a zero on root locus plot

1. The root locus shifts towards left half of S-plane.
2. Relative stability of system is improved.

Consider a zero to be added to the given transfer function as given by

$$G(s)H(s) = \frac{K(s+1)}{s(s+1)(s+2)(s+3)}$$

The program is modified as

```
num=[1 1 ];
den=[1 6  11 6 0];
sys=tf(num,den);
printsys(num,den);
[r,k]=rlocus(sys)
rlocus(sys)
```

Typical Graph:

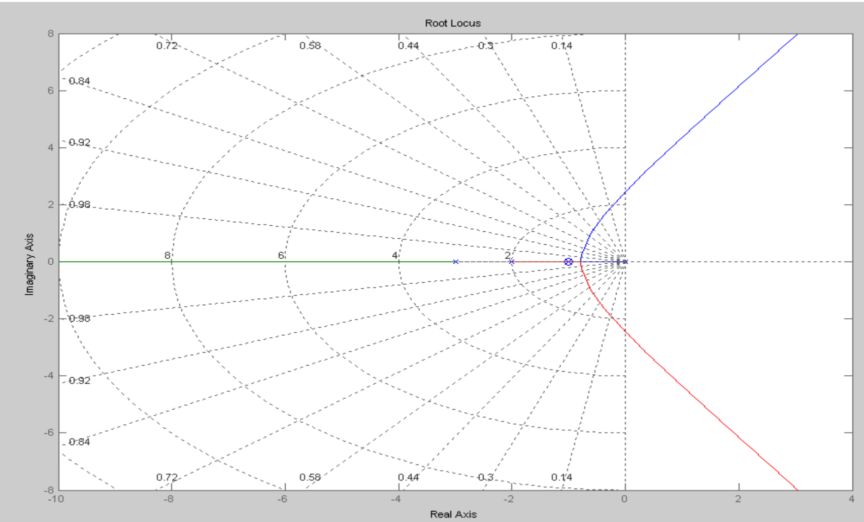


Fig.11.2: Root locus curve

Effect of adding a pole on root locus plot

a. Root locus shifts towards the right half of S-plane.
b. Angles of asymptotes reduce.
c. Relative stability of the system is decreased.

Consider a pole to be added to the given transfer function as given by

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)(s+3)(s+4)}$$

The program is modified as

```
num=[1 ];
den=[1 10 35 50 24  0];
sys=tf(num,den);
printsys(num,den);
[r,k]=rlocus(sys)
rlocus(sys)
```

Typical Graph:

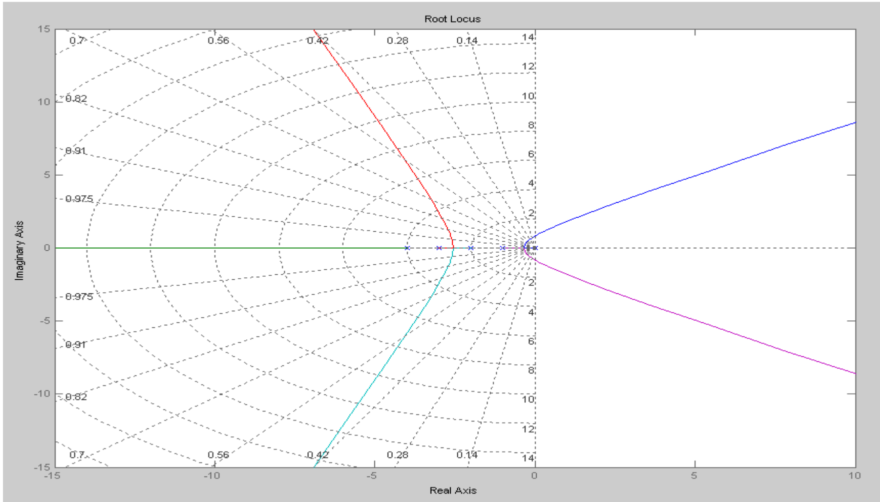


Fig.11.3: Root locus curve

B. Comparative study of bode, Nyquist and root locus with respect to stability

Example 1

Given $G(s)H(s) = \frac{80}{s(s+2)(s+20)}$

$$\boldsymbol{G(s) = \frac{80}{s(s+2)(s+20)} = \frac{80}{s^3 + 22s^2 + 40s}}$$

Program for root locus:

Program:

```
num = [80];
den = [1 22 40 0];
sys=tf(num,den);
printsys(num,den);
[r,k]=rlocus(sys)
rlocus(sys)
```

Program for Bode Plots:

Given $G(s)H(s) = \frac{80}{s(s+2)(s+20)}$

Program:

```
num = [80];
den = [1 22 40 0];
G = tf(num, den);
[gm, pm, wep, weg] = margin(G)
bode(G), grid
```

```
num = [80];
den = [1 22 40 0];
G = tf(num, den);
margin(G)
%grid
```

Program for Nyquist Plots:

$G(s)H(s) = \frac{80}{s(s+2)(s+20)}$

Program:

```
clc
num = [80];
den = [1 22 40 0];
sys=tf(num, den);
nyquist(sys)
[re, im, w] =nyquist(sys)
```

Example 2

Given $G(s)H(s) = \frac{K}{s(s+4)(s+5)}$

Program for root locus:

Program:

```
num = [1];
den = [1 5 4 20 0];
sys=tf(num,den);
printsys(num,den);
[r,k]=rlocus(sys)
 rlocus(sys)
```

Program for Bode Plots:

Given $G(s)H(s) = \frac{K}{s(s+4)(s+5)}$

Program:

```
num = [1];
den = [1 5 4 20 0];
G = tf(num, den);
[gm, pm, wep, weg] = margin(G)
bode(G), grid
```

```
num = [1];
den = [1 5 4 20 0];
G = tf(num, den);
margin(G)
%grid
```

Program for Nyquist Plots:

Given $G(s)H(s) = \frac{K}{s(s+4)(s+5)}$

Program:

```
clc
num = [1];
den = [1 5 4 20 0];
sys=tf(num, den);
nyquist(sys)
[re, im, w] =nyquist(sys)
```

Conclusion:

Outcomes: At the end of the experiment,

1. The students will be able to plot root loci using root locus.
2. The students will be able to Bode plot and determine its specifications.
3. The students will be able to Nyquist plot and determine its specifications.
4. The students will able to determine stability of a system.
5. The students will be able to Evaluate complementary information of linear control system stability by applying graphical methods method for a common transfer function and how each method complements the other.

Viva Questions:

1. What is the significance of root locus method?
2. What are the rules of construction of root locus?
3. Mention the advantages and disadvantages of root locus method.
4. What is meant by Asymptotes?
5. Define phase margin?
6. Define gain margin?
7. Define phase cross over frequency?
8. Define gain cross over frequency?
9. Define Nyquist criterion.

Preparation	Conduction	Calculation/Result Analysis	Total

Signature of Faculty Member

Free Space for rough work:

DESIGN STEPS:

To design the value of R1, R2 and C for specified values of Maximum Phase Lead angle **ϕm=45.6°** at frequency **f=fm=780Hz** and calculate experimentally the performance of the network in terms of frequency response.

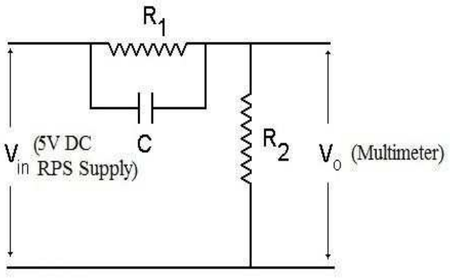


Fig (a) Lead Network

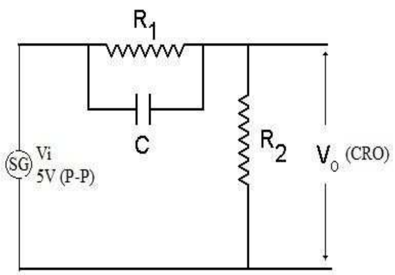


Fig (b) Lead Network

Since The value of this maximum phase angle φm at a frequency ωm is given by: $\sin\phi_m = \dfrac{a-1}{a+1}$, we can calculate the value of ‘‘a”

$$a = \frac{1+\sin\phi_m}{1-\sin\phi_m}$$

a = 6

$$a = \left[\frac{(R1+R2)}{R2}\right]$$

Let R_2 =2 KΩ; Therefore, R1=10KΩ

Maximum phase lead occurs at a frequency $\omega_m = \sqrt{ZP} = \dfrac{1}{\tau\sqrt{a}}$

$\omega_m = 2\Pi f_m$ rad/sec = 4898 rad/sec

Therefore $\tau = \dfrac{1}{\omega_m\sqrt{a}}$ $= 8.4\times10^{-5}$

and $\tau = \left[\dfrac{(R1\times R2\times C)}{(R1+R2)}\right]$

Therefore C = $0.05\mu\ F$

PROCEDURE:

Procedure to find Transfer function of Lead Network:

1. Connections are made as shown in the figure(a)
2. Select the components R1=10kΩ, R2=2kΩ, & C=0.05µF
3. Apply 5V DC using RPS and measure output voltage using digital voltmeter.
4. Calculate the Gain i.e. |V0 / Vin| = 6 = ‘a’ ensures phase lead network.
5. Replace RPS unit with Signal Generator as shown in figure (b)
6. Adjust the amplitude of input to 5V
7. Measure the Output voltage at a frequency of 1 kHz and calculate the Gain. The value of gain at this frequency (1KHz) gives |G(jω)| i.e. |G(jω)| = V0 / Vin at 1KHz.
8. Knowing the value of ‘a’ and |G(jω)|, the value of T and hence Transfer Function is evaluated.

The transfer function of a phase-lead compensator is of the form:

$$Gc(s) = \frac{Eo(s)}{Ei(s)} = \frac{1}{a}\left[\frac{(1+a\tau s)}{(1+\tau s)}\right] = \frac{K}{a}\left[\frac{(1+a\tau s)}{(1+\tau s)}\right]$$

with a > 1, $a = \left[\frac{(R1 + R2)}{R2} \right]$ and $\tau = \left[\frac{(R1 \times R2 \times C)}{(R1 + R2)} \right]$

$\tau = \frac{1}{P}$ and $a = \frac{P}{Z} \rangle 1$