ATME College of Engineering

MACHINE DESIGN BME602

Module-I
Fundamentals of Mechanical Engineering Design

Mr. Rohith S
Assistant Professor,
Dept. of Mechanical Engineering,
ATMECE, Mysuru

Introduction:

- **Definition of design:**
  - Design is a process line to satisfies a particular need by creating some with physical reality.
  - Example: design of a chair
- **Machine:**
  - Machine is a combination of several machine elements arranged to work together as a whole to accomplish specific purpose.
- **Machine Elements:**
  - It is an elementary of a machine or each part of a machine which as motion w.r.t some part is called an machine element.
  - Example: Fastener, Bearings, Gears, Shaft, Keys. Etc.,

- **Machine Design:**
  - It is defined as use of specific principle, technical knowledge and imagination in the description of a machine.
  - Mechanical system to perform specific function with maximum economy and efficiency.

Phases of design – Shigley’s model

- Identification of need
- ↓
- Definition of problem
- ↓
- Synthesis
- ↓
- Analysis and Optimization
- ↓
- Evaluation
- ↓
- Presentation

Engineering Materials

The Engineering materials are mainly classified as,

- Metals and their alloys such as iron, steel, copper, aluminium.
- Non – metals such as Glass, Rubber, Plastic, etc.,

Metals are further classified into :

1. Ferrous metals : Metals which have iron as their main constituent.
   Example: Steel, Cast iron, etc.,
2. Non ferrous metals : Metals which have other than iron as their main constituent.
   Example: Aluminium, Copper, Tin, etc.,

Selection of Materials:

The following are consider while selecting the material

1. Availability of material
2. Cost
3. Suitability of material

Mechanical Properties

1. Strength 2. Elasticity 3.Plasticity 4. Stiffness 5. Resilience

6. Ductility 7. Brittleness 8. Hardness 9. Toughness

Standards:

A set of specification for parts material / process intended to achieve uniformity, efficiency and specified quality.

Types of standards

- Company standard
- National standard – SAE, BS
- International standard – ISO

Codes:

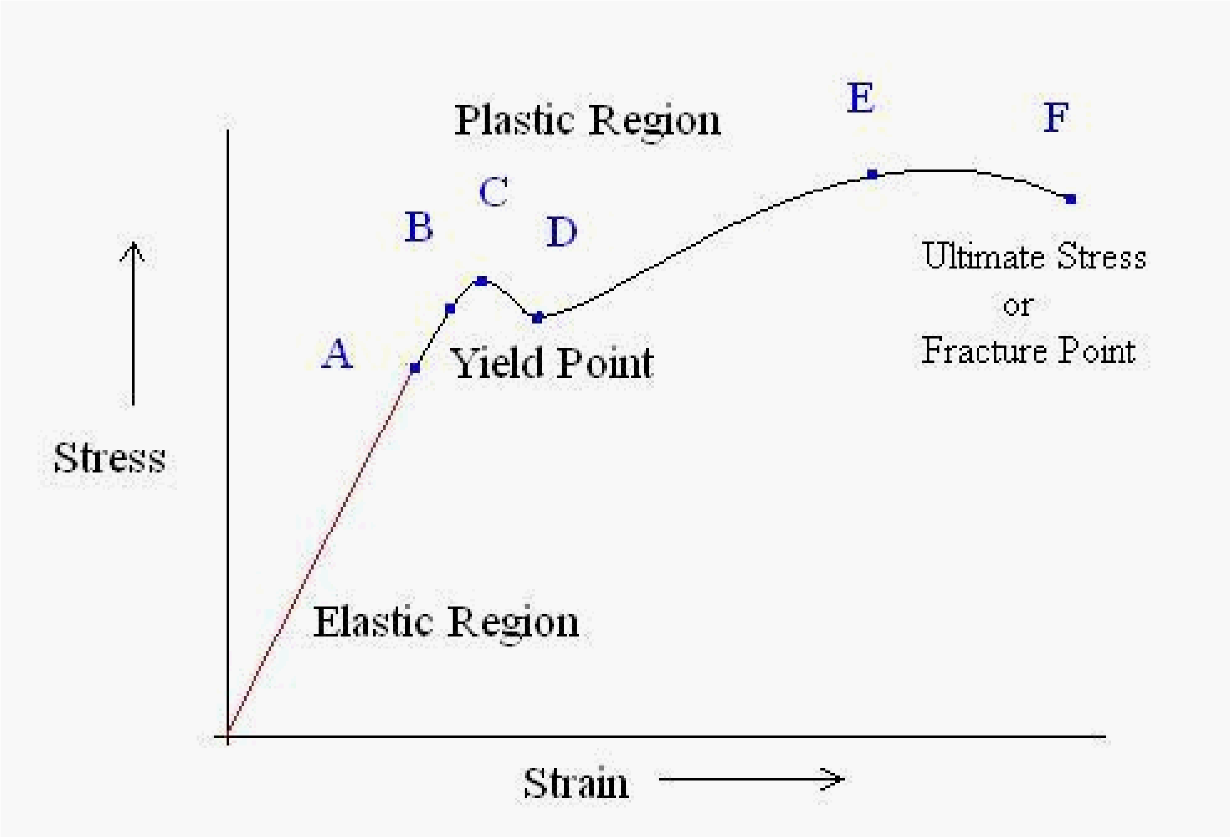
A code is set of specifications for the analysis, design, manufacture and construction.

The purpose of a code is to achieve a specific degree of safety, efficiency and quality.

Codes: ASME, ASM, ASTM, AISI, ISI, BSI

Engineering Stress – Strain diagram

1. A = Proportionality limit
2. B = Elastic limit
3. C = Upper yield point
4. D = Lower yield point
5. E = Ultimate stress
6. F = Fracture / breaking point

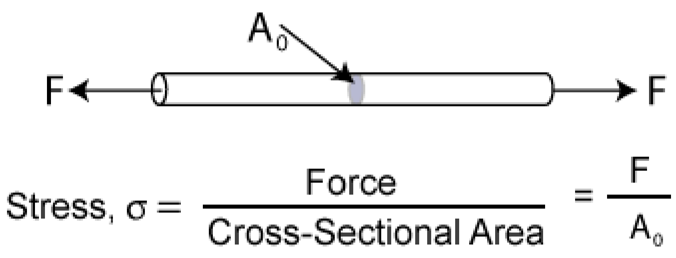


Concepts:

Stress:

The intensity of internally distributed force that tend to resist change in shape of a body is known as stress.

Denoted by “**σ**”



Types of stress:

- Normal stress / direct stress
- Shear stress
- Bending stress

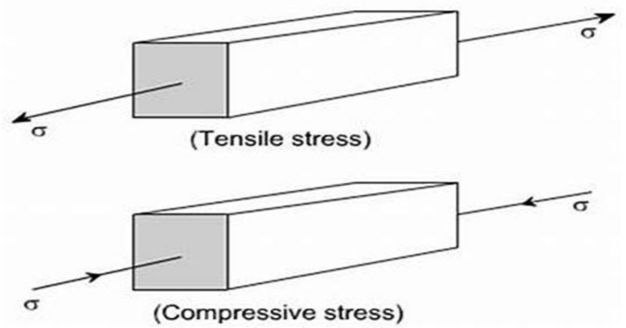
Normal stress / direct stress:

The internal forces and the corresponding stress acting in the direction perpendicular to the surface is known as Normal stress.

Types of Normal stress:

1. Tensile stress:

Two equal and opposite loads acting away from each other which tends to pull apart.



2. Compressive stress:

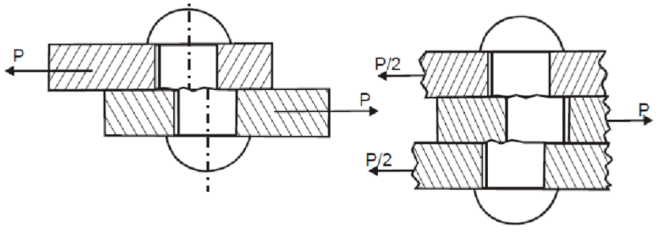
Two equal and opposite loads acting towardseach other which tends to push the particle.

Shear Stress:

Two equal and opposite forces acting tangential on plane is known as shear force.

The stress induced in a body across the resisting section is known as shear stress.

Types of shear stress:



1. Single shear:

   Riveted joints connecting 2 plates, the plate carries tensile load 'F'. The equal and opposite force acting on the rivet at the line of separation causes rivets to shear as shown in figure a.

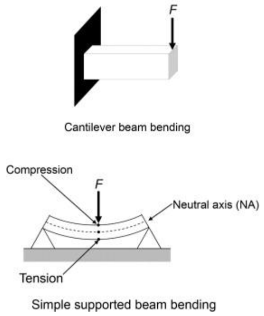
2. Double shear:

   Riveted joints connecting 2 plates with cover plates, the plate carries tensile load 'F'. The force causes the rivets to shear at two places as shown in fig b.

Bending Stress :

The longitudinal stress produced at any section to resist bending is known as bending stress.

$$\frac{M}{I} = \frac{\sigma}{C} = \frac{E}{R}$$ DDHB Equation 1.3a/ page no. 3



Simple stress:

Stress produced by shear, tension and bending are termed as simple stress.

Compound Stress:

Stress produced by torsion and bending are termed as compound stress.

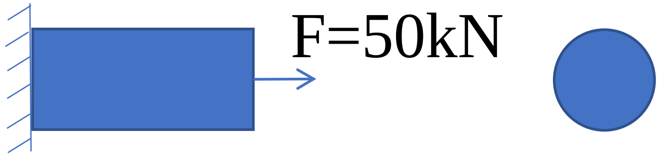
Torsional stress:

The internal stresses which are induced to resist action of twist is termed as Torsional stress.

Problems:

1. Determine the maximum stress induced in a machine element as shown in figure under given loading condition.

a.

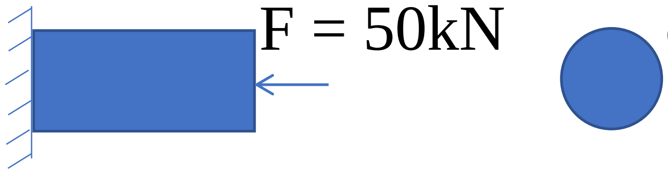


d = 50mm

$$\sigma = \frac{F}{A} = \frac{50*10^3}{\frac{\pi d^2}{4}} = \frac{50*10^3}{\frac{\pi*50^2}{4}}$$

σ = 25.4698 N/mm²

b.

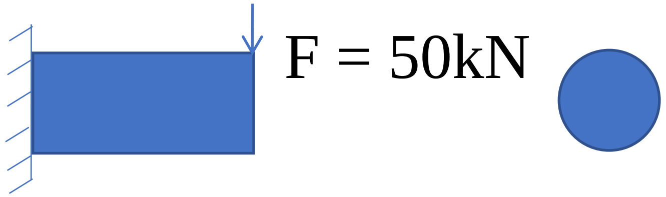


d = 50mm

$$\sigma = \frac{F}{A} = \frac{-50*10^3}{\frac{\pi d^2}{4}} = \frac{-50*10^3}{\frac{\pi*50^2}{4}}$$

σ=-25.4698 N/mm²

c.



d = 50mm

$$\sigma = \frac{MC}{I} = \frac{32*F*L}{\pi*d^3} = \frac{32*50*10^3*500}{\pi*50^3}$$

σ = 2037.1833 N/mm²

2. A round steel rod is subjected to a tensile load of 90kN. Taking the yield stress for the steel as 328.6MPa and FOS as 1.8, determine suitable diameter for the rod.

Given,

Load F = 90kN = 90000N, Yield stress σ_y = 328.6MPa, FOS = 1.8

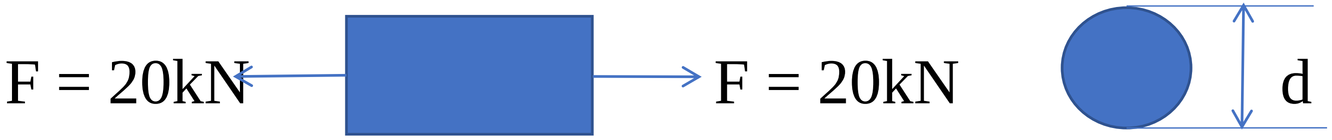
Allowable stress, $\sigma = \sigma_y$ / FOS = 328.6/1.8 = 182.56 MPa

WKT, axial laod $\sigma = \frac{F}{A}$ $\quad 182.56 = \frac{90000}{A}$ $\quad A = \frac{90000}{182.56} = 493mm^2$

For circular rod, $A = \frac{\pi * d^2}{4} = 493$

Diameter d = 25.054mm = **25mm**

3. A tensile bar of SAE 1045 steel annealed. To carry a load of 20kN. Determine the diameter of bar to sustain the load.



Let us assume FOS is n = 2,

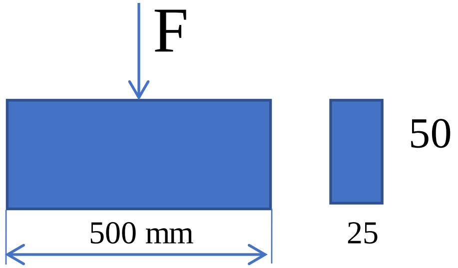
Yield stress is σ_y = 310MPa (DDHB T I.18/pg 473)

$n = \sigma_y/\sigma_{all} \Rightarrow \sigma_{all} = \sigma_y/n = 310/2 = 155\text{MPa}$

$$\sigma = \frac{F}{A} \qquad A = \frac{F}{\sigma} \qquad \frac{\pi * d^2}{4} = \frac{20 * 10^3}{155}$$

d = 12.82 = 13mm

4. A simply supported beam of rectangular c/s of 25*50mm made of steel C40 is loaded as shown in figure and the span of 500mm. Determine the maximum load that could be applied at its centre.



Given,

A = 25*50 1250mm^2 L = 500mm assume FOS n = 3

Yield stress is σ_y = 324MPa (DDHB T I.8/pg 464)

n = σ_y/σ_{all} => σ_{all} = σ_y/n = 324/3 = 108MPa

Since the load is acting at centre bending takes place,

$$\sigma = \frac{MC}{I}$$ C = H/2 = 50/2 = 25mm

$$I = \frac{b * h^3}{12} = \frac{25 * 50^3}{12} = 260.41 * 10^3\ m^4$$

Bending equation

$$\sigma = \frac{MC}{I} \qquad 108 = \frac{M * 25}{260.41 * 10^3}$$

M = 112.49*10^4 N –mm

$$M = \frac{F * l}{4}$$ (DDHB T1.4/pg no. 15)

$$112.49 * 10^4 = \frac{F * 500}{4}$$

F = 9*10^3 N

F = 9kN

5. A steel shaft is used to transmit a torque of 500 N – m. If the diameter of the shaft is 32mm and the material SAE 1045 steel annealed determine FOS.

Given,

Material = SAE 1045 steel annealed, T = 500N-m, d = 32mm, n = ?

Since torque is given, consider tortional stress,

$\frac{T}{J} = \frac{\tau}{r} = \frac{G * \theta}{l}$ (DDHB Eq no. 1.3b/pg no. 3)

from DDHB T1.3b/pg 14

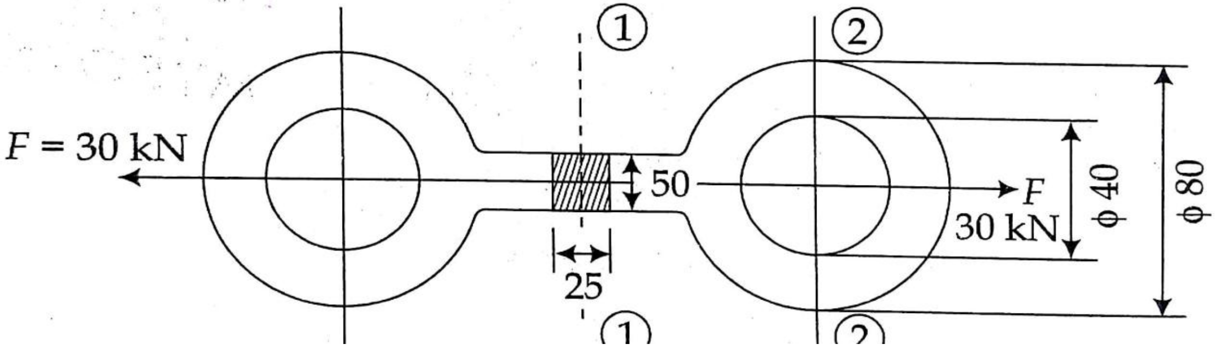
$\frac{T}{J} = \frac{\tau}{r}$ $J = \frac{\pi * d^4}{32}$ $r = \frac{d}{2}$ $\tau = \frac{16T}{\pi * d^3}$ $\frac{16 * 500 * 10^{-3}}{\pi * (32 * 10^{-3})^3}$ τ = 77.71 Mpa

FOS = τ_y / τ_{all}

Yield stress is τ_y = 180MPa (DDHB T I.1.18/pg 473)

FOS = 180/77.71 **= 2.31**

6. Determine the maximum stress induced in a link loaded as shown in figure



Given, Load F = 30kN = 30000N

Section 1-1 is a rectangular c/s, $A_1 = b*h$

Area = $A_1 = 25*50 = 1250mm^2$

Tensile stress, $\sigma 1 = \frac{F}{A} = \frac{30000}{1250}$ = **24MPa**

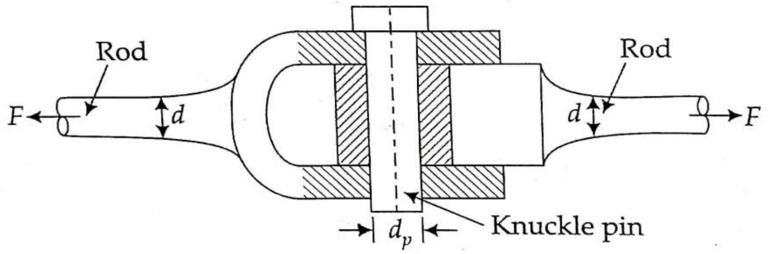
Section 2-2 is hollow

Area = $A_2 = 25(80-40) = 1000mm^2$

Tensile stress, $\sigma 2 = \frac{F}{A} = \frac{30000}{1000}$ = **30MPa**

Maximum tensile stress = **30MPa** at section 2-2

7. Figure shows a knuckle joint carrying a tensile load of 90kN. Find suitable diameter of rods and knuckle pin taking allowable shear and tensile stresses as 72 Mpa and 120MPa respectively.



Given,

Load F = 90kN, Tensile stress σ = 120MPa , shear stress τ = 72MPa

Rods are subjected to tensile stress and the knuckle pin subjected to double shear

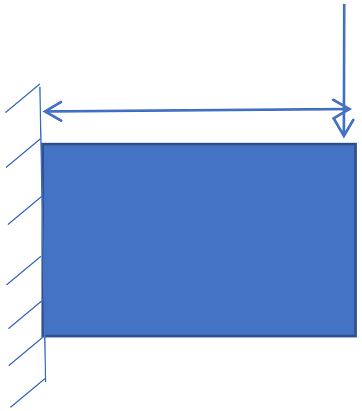
For rods, $\sigma = \frac{F}{A}$ $A = \frac{F}{\sigma}$ $\frac{\pi * d^2}{4} = \frac{90 * 10^3}{120}$

d = 30.901 = 31mm

For knuckle pin, $\tau = \frac{F}{2A}$ $2A = \frac{F}{\tau}$ $\frac{2\pi * d^2}{4} = \frac{90 * 10^3}{72}$

d = 28.2mm = 29mm

8. A beam of uniform c/s is fixed at one end and carries transverse load of 1.8kN at a distance of 0.9m from the fixed end. The material used is C30 steel (σ_y = 294.2MPa) and FOS is 2.5. Find the width and depth of c/s if the depth is twice the width.



Given,

Load F = 1.8kN = 1800N, Length L = 0.9m = 900mm,

yield stress = σ_y = 294.2MPa, FOS = 2.5, depth of c/s, h =2b

Allowable stress, $\sigma_{all} = \sigma_y$ / FOS = 294.2/3 = 117.68MPa

The beam is subjected to bending

WKT, bending equation = $\frac{M}{I} = \frac{\sigma}{C} = \frac{E}{R}$

$$\sigma = \frac{MC}{I}$$

M = bending moment = F * L = 1800*900 = $1.62*10^6$ N-mm

I = moment of inertia = $I = \frac{b*h^3}{12} = \frac{b*(2b)^3}{12} = \frac{8b^4}{12} = 0.667b^4$ (T1.3a/pg 12)

C = distance from NA = h/2 = 2*b/2 = b

σ = allowable stress = 117.68 Mpa

Substituting $\sigma = \frac{MC}{I}$ $117.68 = \frac{1.62*10^6*b}{0.667*b^4}$ $117.68 = \frac{2.428*10^6}{b^3}$

Width, b = 27.42mm = 28mm

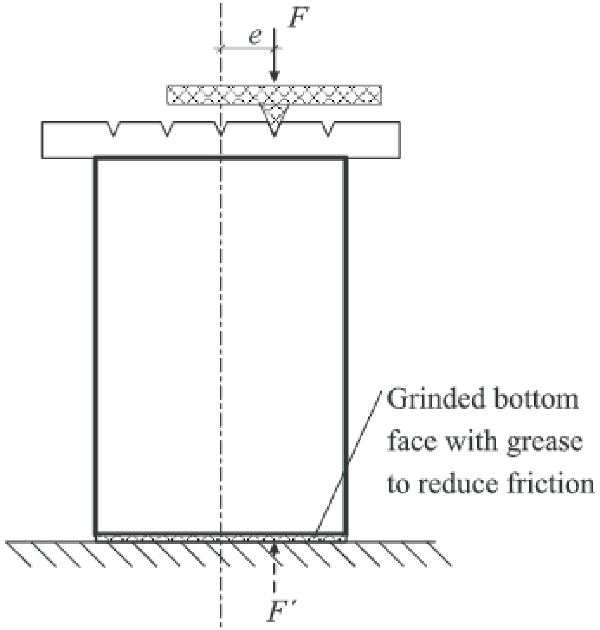
Depth of cross section = h = 2*b = 2*28

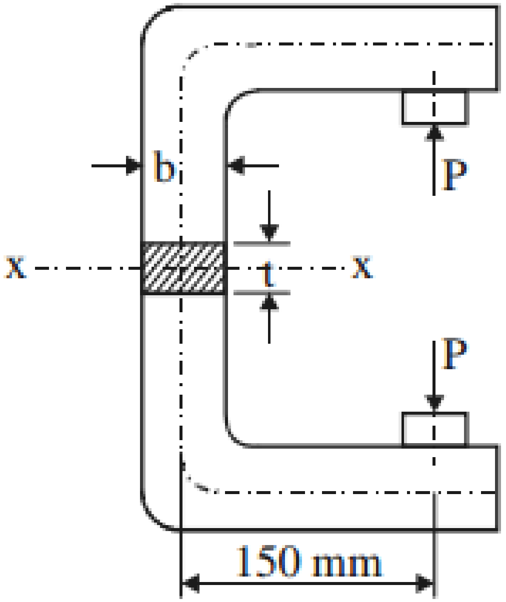
h = 56 mm

Eccentric loading

An external load whose line of action is parallel but **does not coincide** with centroidal axis of a machine component is known as eccentric loading.

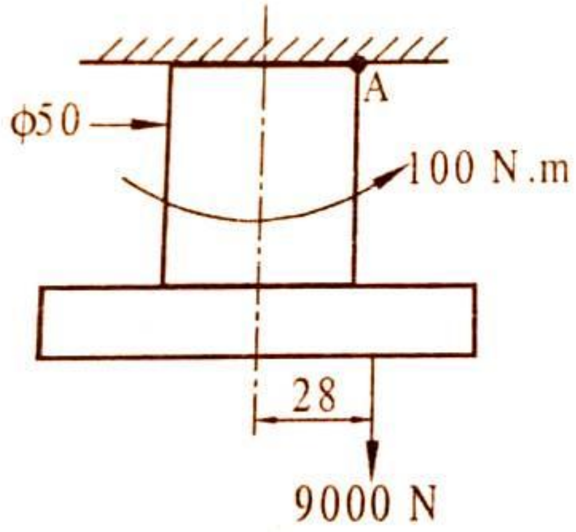
Eccentric load is replaced by parallel force acting through the centroidal axis with couple. Resultant of those two force will be $\boldsymbol{\sigma = \sigma_d + \sigma_b}$





Problems on eccentric loading

1. A 50mm diameter steel rod supports 9kN load and in addition it is subjected to a torsion moment of 100N-m as shown in figure. Determine maximum tensile and maximum shear stress



Given, F = 9kN, T = 100 N-m

Since the load is subjected to eccentric load the rod is subjected to

a. Direct stress, b. Bending stress.

Combined stress at point A

$$\sigma = \sigma_d + \sigma_b = \frac{F}{A} + \frac{MC}{I}$$

$$\sigma = \frac{9*10^3}{\frac{\pi*50^2}{4}} + \frac{9*10^3*28*25}{\frac{\pi*50^2}{64}}$$

σ = 25.11MPa

Shear stress τ:

$$\tau = \frac{16T}{\pi * d^3} = \frac{16 * 100 * 10^3}{\pi * 50^3}$$

τ = 4.074MPa

Maximum normal stress at A

$$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{2} + \tau^2}$$ (DDHB eq no. 1.5a/ pg 3)

$$\sigma_{max} = \frac{25.11}{2} + \sqrt{(\frac{25.11}{2})^2 + 4.074^2}$$

σ_{max} = 25.75MPa

Maximum shear stress at A

$$\tau_{max} = \sqrt{(\frac{\sigma}{2})^2 + \tau^2}$$ (DDHB eq no. 1.5b/ pg 3)

$$\tau_{max} = \sqrt{(\frac{25.11}{2})^2 + 4.074^2}$$

τ_{max} = 13.19MPa

2. A mild steel bracket shown in figure is subjected to a pull of 10kN. The bracket has a rectangular c/s whose depth is twice the width. If the allowable stress for the material is 80MPa, determine c/s of the bracket

Given,

F = 10kN, σ = 80MPa, h = 2b

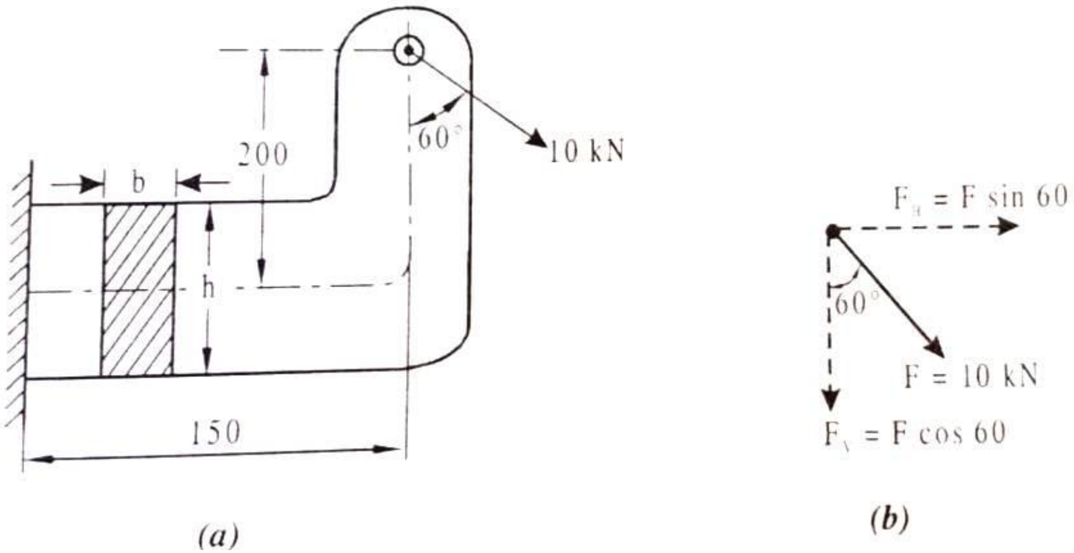
On resolving,

Vertical component.,

F_v = Fcos60° = 10000*cos60° = 5000N

Horizontal component.,

F_H = Fsin60° = 10000*sin60° = 8660.254N = 8.66*10^3N



a. Consider the vertical component,

Bending moment due to vertical = $M_{bv} = F_v*150 = 5000*150 = 75*10^4$ N-mm

Bending stress due to BM = M_{bv} = $\sigma_1 = \frac{M_{bv}C}{I}$

Where, $I = \frac{b*h^3}{12} = \frac{b*(2b)^3}{12} = \frac{2b^4}{3}$ c = h/2 = 2b/2 = b

$$\sigma_1 = \frac{75*10^4*b}{\frac{2*b^4}{3}} = \frac{112.5*10^4}{b^3}$$

b. Consider horizontal component

$\sigma = \sigma_d + \sigma_b = \frac{F}{A} + \frac{MC}{I}$

i. Direct stress = $\frac{F}{A} = \frac{8.66*10^3}{2*b^2} = \frac{4.38*10^3}{b^2}$

ii. Bending stress

Bending moment due to horizontal = $8.66*10^3*200=1.732*10^6$

$$\frac{M_{bH}C}{I} = \frac{1.732*10^6*\frac{2*b}{2}}{\frac{2*(2b)^3}{12}} = \frac{2.598*10^6}{b^3}$$

$\sigma = \sigma_d + \sigma_b$

$$\sigma_2 = \frac{4.38 * 10^3}{b^2} + \frac{2.598 * 10^6}{b^3}$$

Total stress

$\sigma = \sigma_1 + \sigma_2$

$$80 = \frac{1.125 * 10^6}{b^3} + \frac{4.38 * 10^3}{b^2} + \frac{2.598 * 10^6}{b^3}$$

b = 36.4710mm

Now, h = 2b

h = 2* 36.4710

h = 72.94mm

3. A steel member is loaded as shown in figure. Find the magnitude of maximum normal stress and minimum normal stress.

Given, F = 9kN = 9000N

Tensile stress, $\sigma = \sigma_d + \sigma_b = \frac{F}{A} + \frac{MC}{I}$

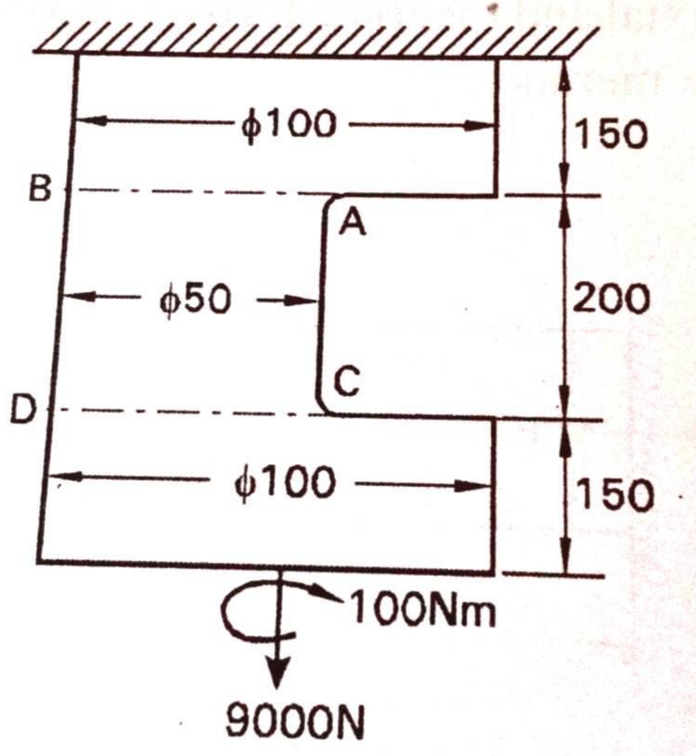
$$\frac{F}{A} = \frac{9 * 10^3}{\frac{\pi * 50^2}{4}} = 4.583MPa \qquad \frac{MC}{I} = \frac{9000 * 25 * 25}{\frac{\pi * 50^4}{64}} = 18.33MPa$$

σ = 4.583+18.33 = 22.91MPa

Shear stress,

$$\tau = \frac{16T}{\pi * d^3} = \frac{16 * 100 * 10^3}{\pi * 50^3}$$

τ= 4.07MPa



Consider the point A,

Maximum normal stress at A

$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{2} + \tau^2}$ (DDHB Eq no. 1.5a/Pg 3) $\sigma_{max} = \frac{22.91}{2} + \sqrt{(\frac{22.91}{2})^2 + 4.07^2}$

σ_{max} = 23.61MPa (Tensile)

Minimum normal stress at A

$\sigma_{min} = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{2} + \tau^2}$ $\sigma_{min} = \frac{22.91}{2} - \sqrt{(\frac{22.91}{2})^2 + 4.07^2}$

σ_{min} = -0.7MPa (Compressive)

Maximum shear stress at A

$\tau_{max} = \sqrt{(\frac{\sigma}{2})^2 + \tau^2}$ (DDHB Eq no. 1.5b/Pg 3) $\tau_{max} = \sqrt{(\frac{22.91}{2})^2 + 4.07^2}$

τ_{max} = 12.16MPa

Consider the point B,

$\sigma = \sigma_d - \sigma_b = 4.583 – 18.33 = -13.75MPa, \tau = 4.07MPa$

Maximum normal stress at B

$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{2} + \tau^2}$ (DDHB Eq no. 1.5a/Pg 3) $\sigma_{max} = \frac{-13.75}{2} + \sqrt{(\frac{-13.75}{2})^2 + 4.07^2}$

σ_{max} = 1.116 MPa (Tensile)

Minimum normal stress at B

$\sigma_{min} = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{2} + \tau^2}$ $\sigma_{min} = \frac{-13.75}{2} - \sqrt{(\frac{-13.75}{2})^2 + 4.07^2}$

σ_{min} = -14.86 MPa (Compressive)

Maximum shear stress at B

$\tau_{max} = \sqrt{(\frac{\sigma}{2})^2 + \tau^2}$ (DDHB Eq no. 15b/Pg 3) $\tau_{max} = \sqrt{(\frac{22.91}{2})^2 + 4.07^2}$

τ_{max} = 7.99 MPa

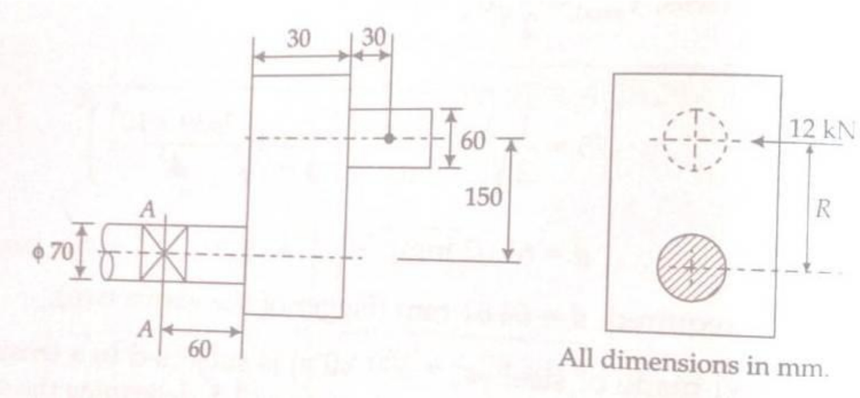
Therefore,

Maximum normal stress $\boldsymbol{\sigma_{max}}$ **= 23.61MPa (Tensile)**

Minimum normal stress $\boldsymbol{\sigma_{min}}$ **= -14.86 Mpa (Compressive)**

Maximum shear stress $\boldsymbol{\tau_{max}}$ **= 12.16 Mpa**

4. Determine normal and shear stresses induced at section AA, when a load of 12kN is applied at the centre of crank pin as shown in figure.



Given, Load F = 12kN = 12000N, diameter d = 70mm,

The given data acting on the crank pin as

two effects at section A-A

a. Bending stress due to cantilever subjected to BM

BM = F*L, where L = Distance from section AA to load

L = 60+30+30=120mm

M= 12000*120=$1.44*10^6$ N-mm

Bending stress = $\sigma = \frac{MC}{I} = \frac{1.44 * 10^6 * 35}{\frac{\pi * 70^4}{64}} = 42.76MPa$

b. Shear stress due to torque = T = F*r = 12000*150=$1.8*10^6$ N-mm

Shear stress $\tau = \frac{16T}{\pi * d^3} = \frac{16 * 1.8 * 10^6}{\pi * 70^3}$

τ = 26.72Mpa

Now, $\sigma = \sigma_d + \sigma_{b,}$ where load is acting only bending so direct stress is $\sigma_d = 0$

$\sigma = \sigma_b$ = 42.76 Mpa and τ = 26.72Mpa

Therefore,

Combined stress,

$$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{2} + \tau^2} \quad \sigma_{max} = \frac{42.76}{2} + \sqrt{(\frac{42.76}{2})^2 + 26.72^2}$$

σ_{max} = 55.608MPa

Shear stress,

$$\tau_{max} = \sqrt{(\frac{\sigma}{2})^2 + \tau^2} \quad \tau_{max} = \sqrt{(\frac{42.76}{2})^2 + 26.72^2}$$

τ_{max} = 34.22MPa

Stress concentration :

It is defined as the concentration of stress in a machine member either due to change in c/s or due to irregularity present in the component.

Stress concentration factor:

It is defined as the ratio of actual maximum stress induced to the theoretical or nominal stress.

$$k_{\sigma t} = \frac{\sigma_{max}}{\sigma_{nom}} \qquad k_{\tau t} = \frac{\tau_{max}}{\tau_{nom}}$$

K_σ = is used for axial and bending stresses.

K_τ = is used for shear stresses.

σ_{max} or τ_{max} is maximum stress induced

σ_{nom} or τ_{nom} is nominal or theoretical stress

Determination of stress concentration factor:

The theoretical stress concentration factor is determined by two methods,

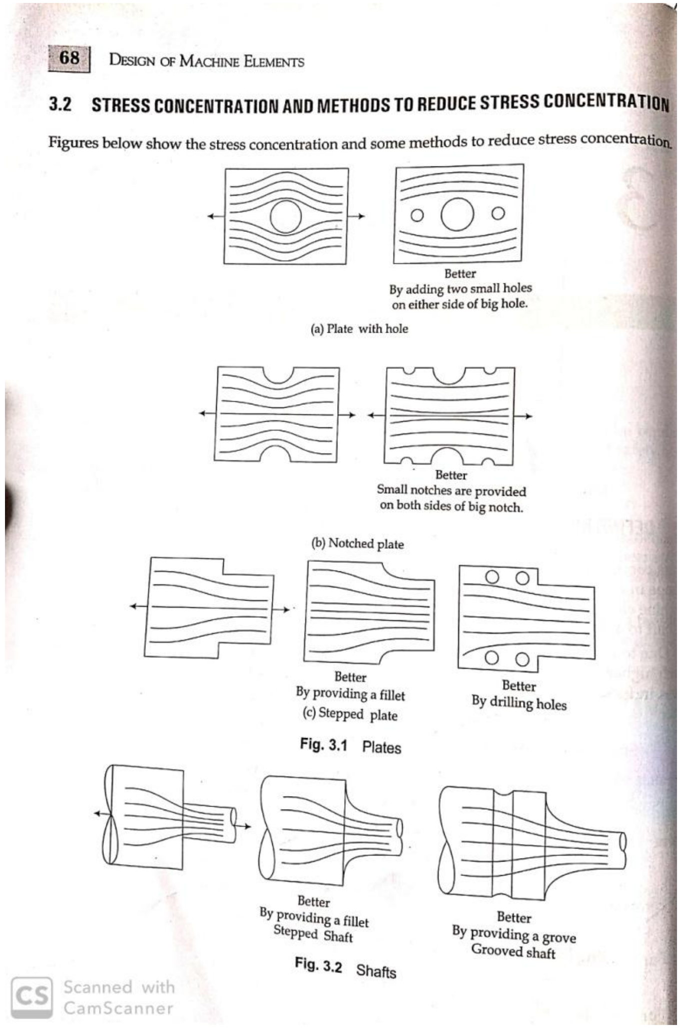
a. Experimental technique:

    1. Photo elasticity
    2. brittle coating method
    3. electrical strain gauge method

b. Numerical technique:

    1. Finite element method
    2. Theory of elasticity approach

Methods of reducing stress concentration:



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3.2 STRESS CONCENTRATION AND METHODS TO REDUCE STRESS CONCENTRATION

Figures below show the stress concentration and some methods to reduce stress concentration.

Better
By adding two small holes on either side of big hole.

(a) Plate with hole

Better
Small notches are provided on both sides of big notch.

(b) Notched plate

Better
By providing a fillet

(c) Stepped plate

Better
By drilling holes

Fig. 3.1 Plates

Better
By providing a fillet
Stepped Shaft

Better
By providing a grove
Grooved shaft

Fig. 3.2 Shafts

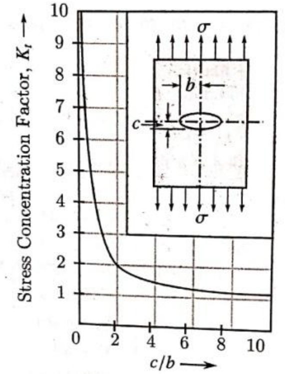


Fig. 2.11: ***Stress Concentration at the Edge of an Elliptical Hole in a Plate***

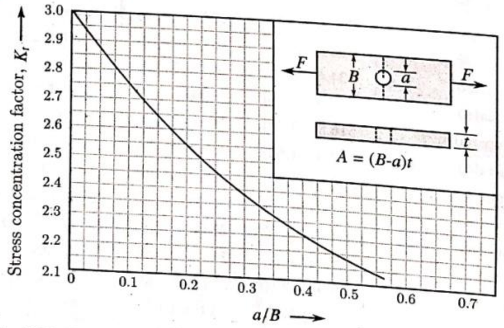


Fig. 2.12: ***Stress Concentration Factor K_t for a plate with hole in tension***

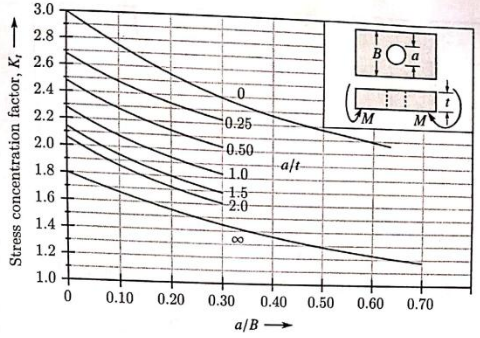


Fig. 2.13: *Stress concentration factor for a flat bar with a transverse hole in bending*

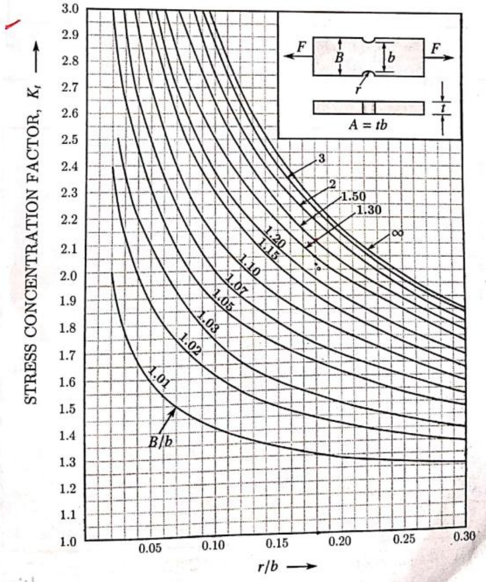
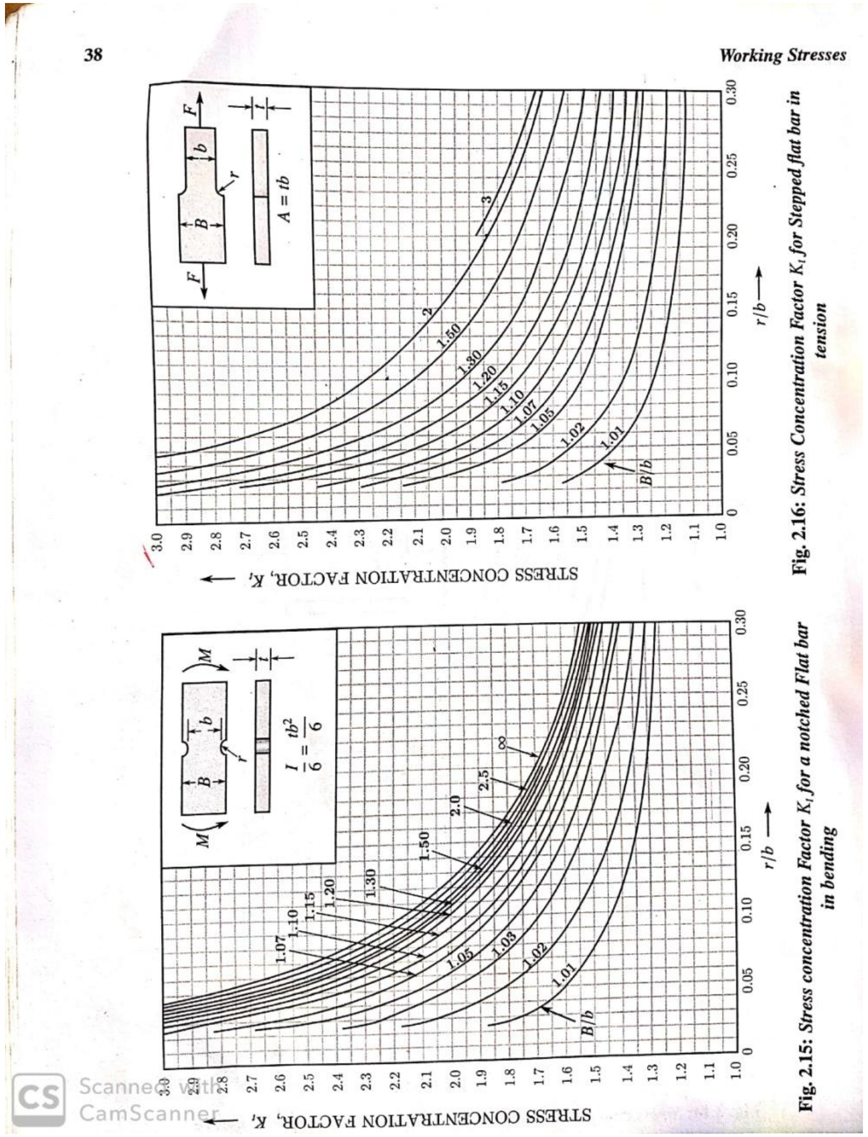


Fig. 2.14: *Stress concentration Factor K_t for a notched Flat bar in tension*


38
Working Stresses
Fig. 2.16: Stress Concentration Factor K_t for Stepped flat bar in tension
Fig. 2.15: Stress concentration Factor K_t for a notched Flat bar in bending
STRESS CONCENTRATION FACTOR, K_t
r/b
$A = tb$
$\frac{I}{6} = \frac{tb^2}{6}$
B/b
Scanned with CamScanner

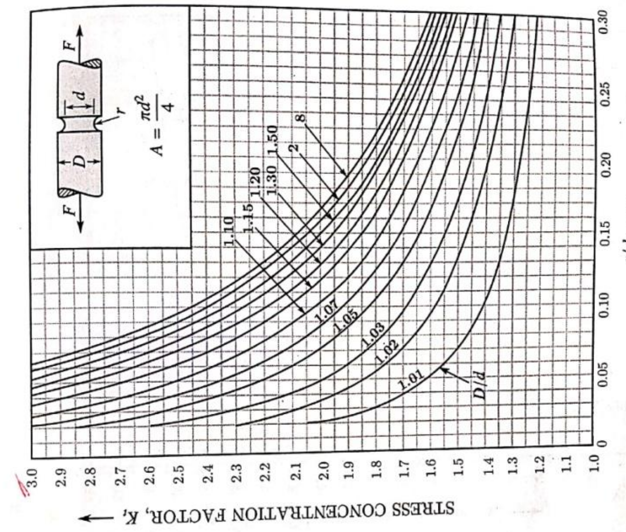


Fig. 2.18: Stress Concentration Factor K_t for a grooved shaft in tension

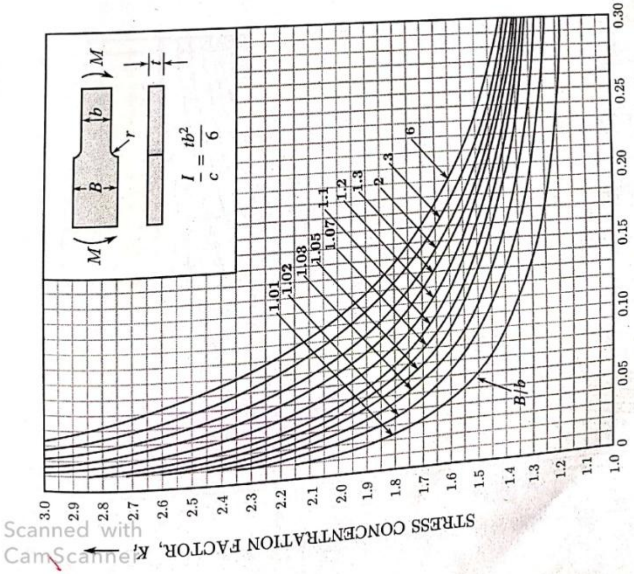


Fig. 2.17: Stress Concentration Factor K_t for Stepped flat bar in Bending

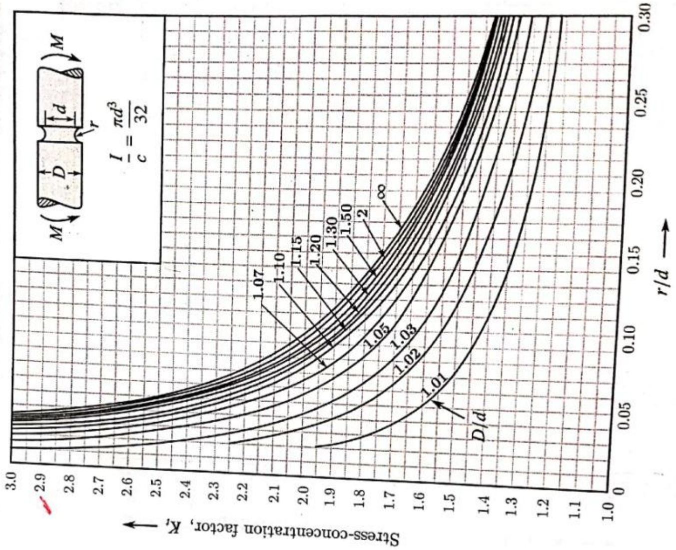


Fig. 2.20: Stress Concentration Factor K_t for a grooved shaft in bending

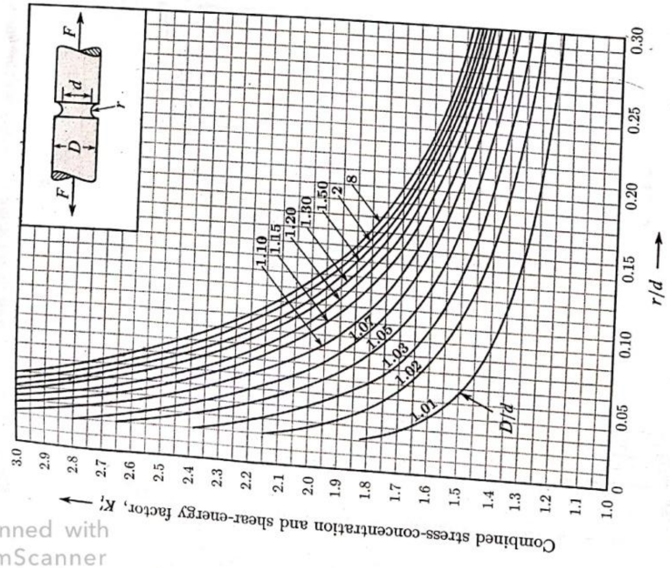


Fig. 2.19: Combined factor K_t' for a grooved shaft in tension

Problems on Stress concentration factor:

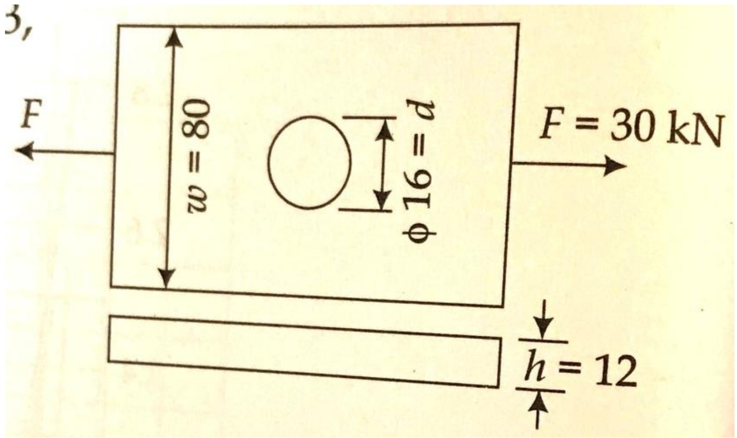
1. Determine the maximum stress induced in the following stress concentration into account, A rectangular plate 80mm wide, 12mm thick with a central hole of dia 16mm subjected to a tensile load of 30kN.

Given,

a = 16mm, B = 80mm, t = 12mm, F = 30kN = 30000N

A = (B-a)t = (80-16)12 = 768mm^2 (DDHB fig 2.12/ pg 36)

$$\sigma_{nom} = \frac{F}{A} = \frac{30000}{768} = 39.06MPa$$



To find K_t, From fig 2.12/pg 36

a/B = 16/80 = 0.2, k_t = 2.55

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad 2.55 = \frac{\sigma_{max}}{39.06}$$

σ_{max} = 99.603MPa

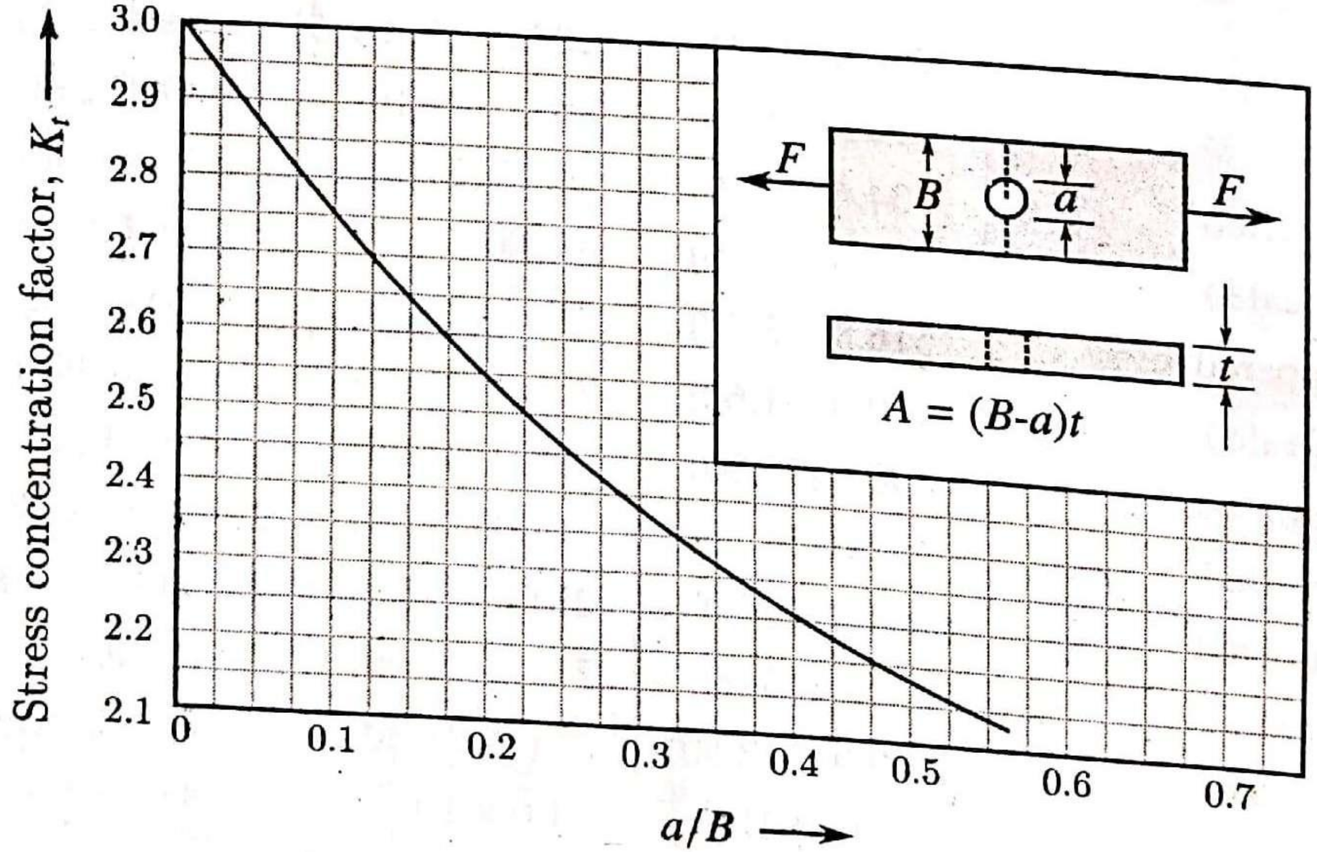


Fig. 2.12: ***Stress Concentration Factor K_t for a plate with hole in tension***

2. Determine the maximum stress induced in the following stress concentration into account, A stepped shaft stepped down from 60mm diameter to 40mm diameter with a fillet radius of 8mm subjected to a twisting moment of 120kNm.

Given, d_1 = 60mm, d_2 = 40mm, r = 8mm, T = 120kN-m

$$k_{\tau t} = \frac{\tau_{max}}{\tau_{nom}} \qquad \tau_{max} = \tau_{nom} * k_{\tau t}$$

$$\tau = \frac{16T}{\pi * d^3} = \frac{16 * 120 * 10^3}{\pi * 40^3} = 9.54\text{Mpa}$$

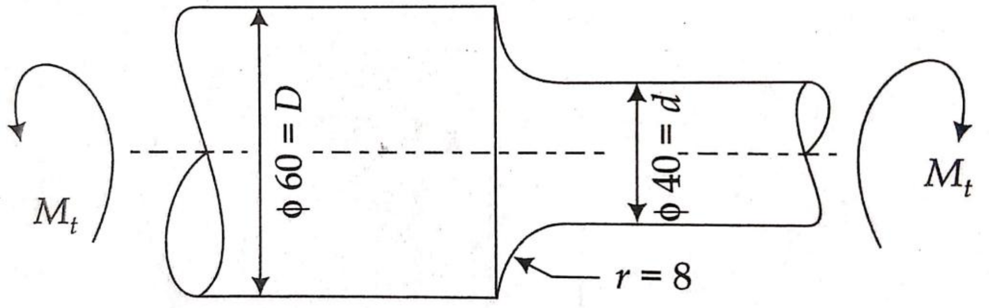


Fig. 3.16

To find $k_{\tau t}$

From fig 2.27/pg 44

r/d = 8/40 = 0.2, D/d = 60/40 = 1.5

$k_{\tau t}$ = 1.24

Therefore, τ_{max} = 9.54*1.24 = 11.84Mpa

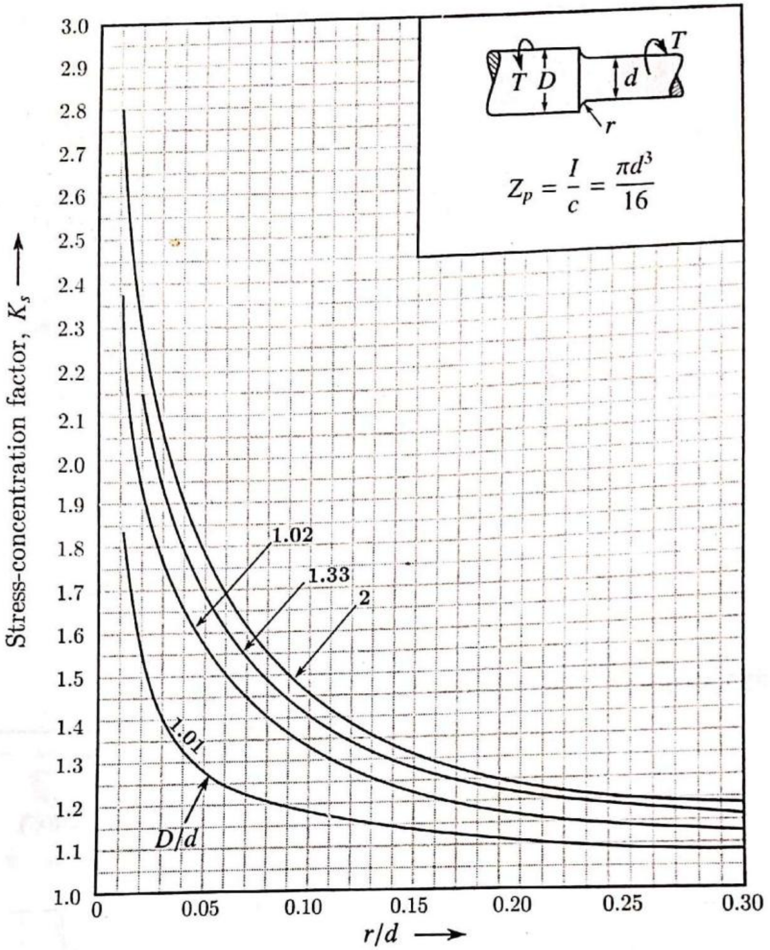


Fig. 2.27: ***Stress-concentration factor K_{ts} for a stepped shaft in torsion***

3. Determine the thickness of the shaft plate loaded as shown fig, the permissible shear stress is 100Mpa for the flat plate

Given, F = 250kN = 250*10^3 N τ_{all} = 100Mpa

Consider section B-B,

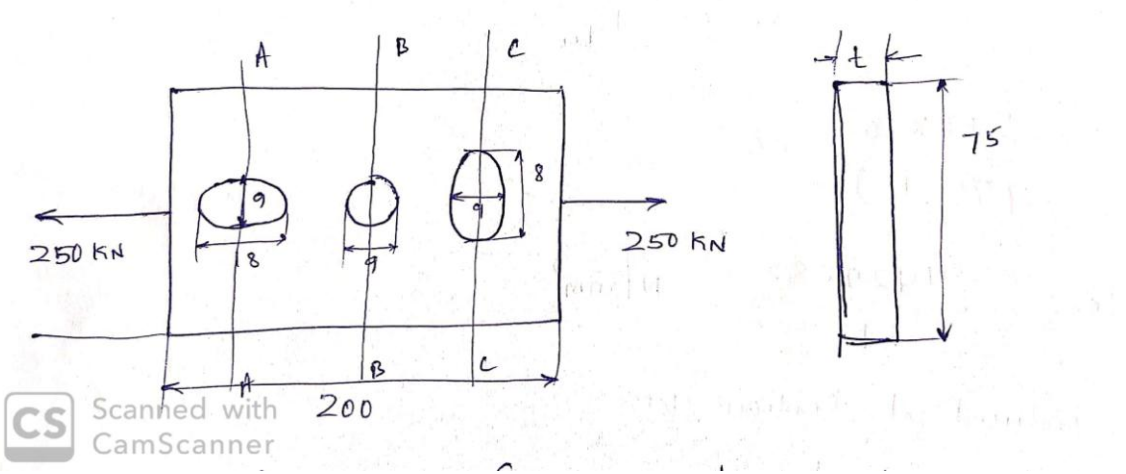
From fig 2.12/ pg 36, a/B = 9/75 = 0.12 k_t = 2.7

$$\sigma_{nom} = \frac{F}{A} = \frac{250 * 10^3}{(75-9)t} = \frac{250 * 10^3}{66t}$$

Wkt,

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad \sigma_{max} = \sigma_{nom} * k_t = 2.7 * \frac{250 * 10^3}{66t}$$

$$\sigma_{max} = \frac{10.22 * 10^3}{t} \text{ --------- } 1$$



Consider section A-A

From DDHB T2.1/pg 30

An elliptical hole in a plate, the major axis along the load $k_t = 1 + \frac{2c}{b}$

From fig 2.11/ pg 36, b = semi major axis and c = semi minor axis

$k_t = 1 + \frac{2\frac{9}{2}}{\frac{18}{2}}$ $k_t = 2$

$$\sigma_{nom} = \frac{F}{A} = \frac{250 * 10^3}{(75 - 9)t} = \frac{250 * 10^3}{66t}$$

$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$ $\sigma_{max} = k_t * \sigma_{nom} = 2 * \frac{250 * 10^3}{66t}$

$\sigma_{max} = \frac{7.55 * 10^3}{t}$ -------- 2

Consider section C-C

From DDHB T2.1/pg 30

An elliptical hole in a plate, the major axis normal to the load $k_t = 1 + \frac{2b}{c}$

From fig 2.11/ pg 36, b = semi major axis and c = semi minor axis

$k_t = 1 + \frac{2\frac{18}{2}}{\frac{9}{2}}$ $k_t = 5$

$$\sigma_{nom} = \frac{F}{A} = \frac{250 * 10^3}{(75 - 18)t} = \frac{250 * 10^3}{57t}$$

$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$ $\sigma_{max} = k_t * \sigma_{nom} = 5 * \frac{250 * 10^3}{57t}$

$\sigma_{max} = \frac{22.72 * 10^3}{t}$ -------- 3

Wkt,

$\tau_{all} = \sigma_{all}/2$

$\sigma_{all} = 2 * \tau_{all}$

$\sigma_{all} = 2 * 100 = 200$Mpa

Since the maximum stress is at section C-C, the thickness of the plate is calculated

i.e., $\sigma_{max} = \dfrac{22.72 * 10^3}{t}$

$200 = \dfrac{22.72 * 10^3}{t}$

Therefore,

t = 113.6mm

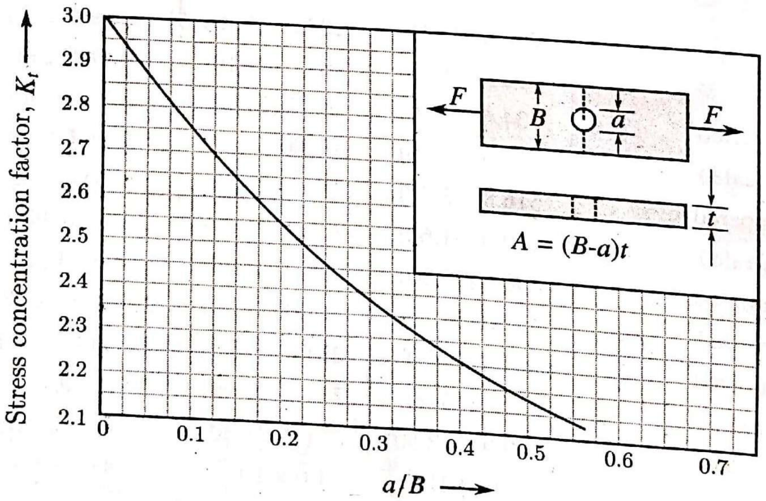


Fig. 2.12: *Stress Concentration Factor K_t for a plate with hole in tension*

36 *Working Str*

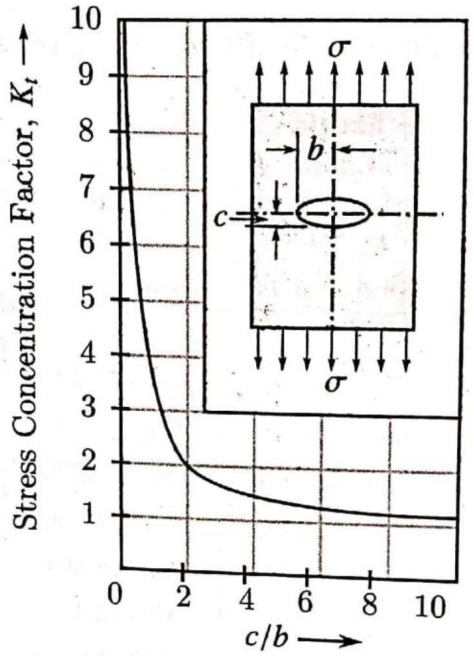


Fig. 2.11: *Stress Concentration at the Edge of an Elliptical Hole in a Plate*

4. Find the maximum stress induced in machine element as shown in fig,

Given,

F = 10kN = 10000 N, r = 12mm, B = 70mm, b =?

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad \sigma_{nom} = \frac{F}{A} = \frac{F}{t*b}\frac{10*10^3}{10*46} = 21.73MPa$$

From DDHB fig 2.14 / pg 37

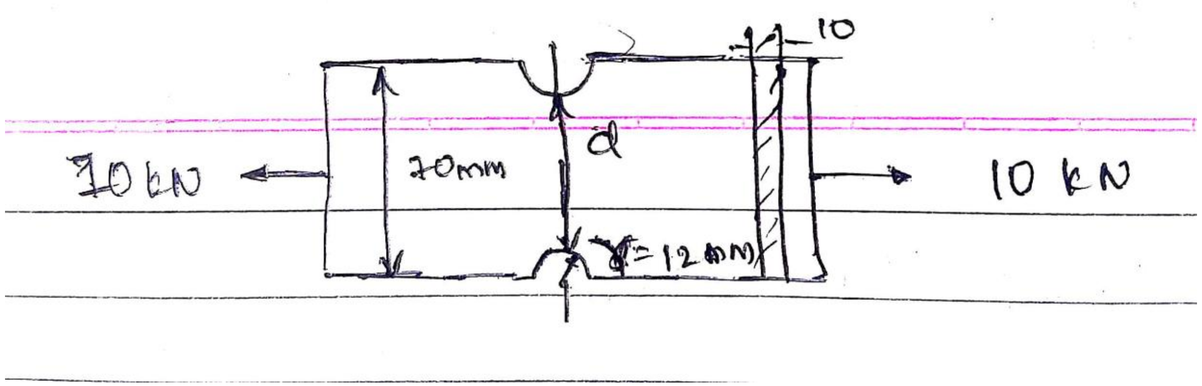
r/b = 12/46 = 0.26, B/b = 70/46 = 1.52

$K_t = 1.87$

$\sigma_{max} = \sigma_{nom} * k_t$

$\sigma_{max} = 21.73*1.87$

σ_{max} = 40.63Mpa



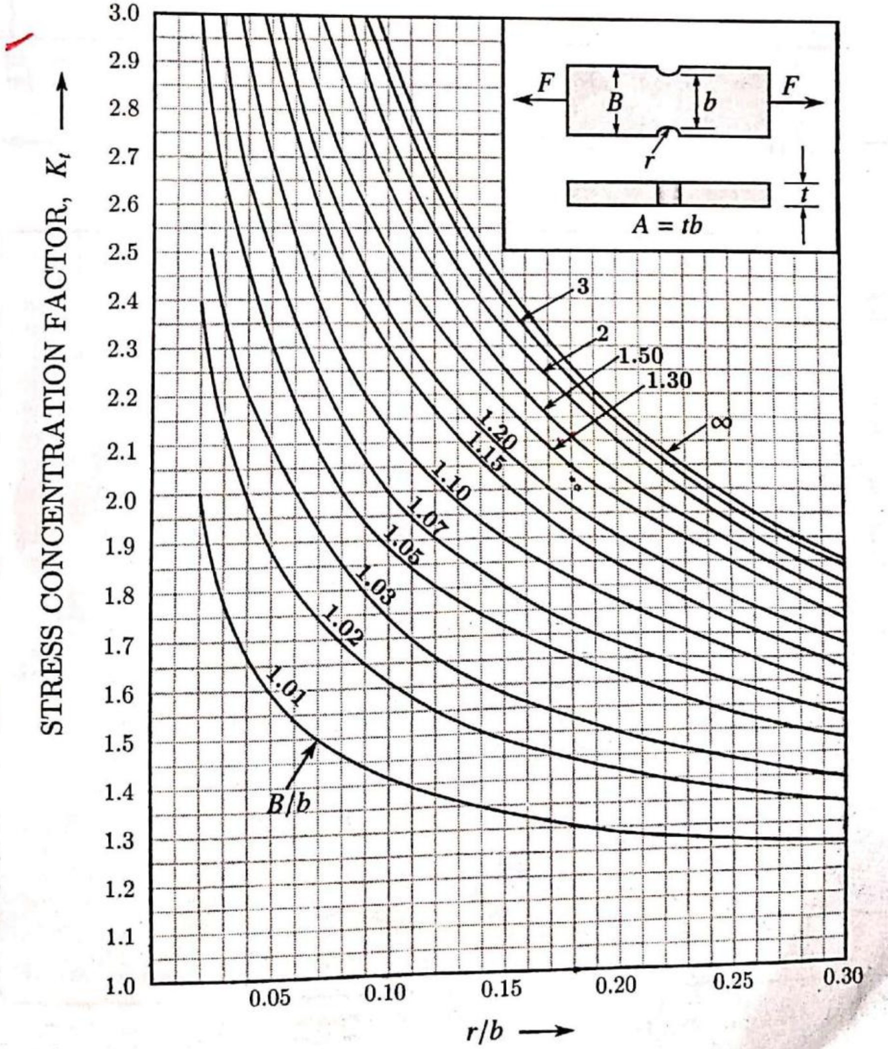


Fig. 2.14: ***Stress concentration Factor K_t for a notched Flat bar in tension***

5. A notched flat plate shown in fig, is subjected to bending moment of 10N-m. Determine the maximum stress induced in the member by taking the stress concentration into account.

Given, M = 10N-m = $10*10^3$ N-mm, t = 10mm, B = 60mm, r = 10mm, b =?

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad \sigma = \frac{MC}{I}$$

C = d/2 = 40/2 = 20

$$I = \frac{b*h^3}{12} = \frac{t*(b)^3}{12} = \frac{10*40^3}{12} = 53.33*10^3$$

$$\sigma = \frac{MC}{I} = \frac{10*10^3*20}{53.33*10^3} = 3.75MPa$$ σ_{nom} = 3.75Mpa

From DDHB fig no. 2.15/pg 38

r/b = 10/40 = 0.25, B/b = 60/40 = 1.5

K_t = 1.56

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$$ $\sigma_{max} = \sigma_{nom} * k_t$ = 3.75 * 1.56 = **5.85Mpa**

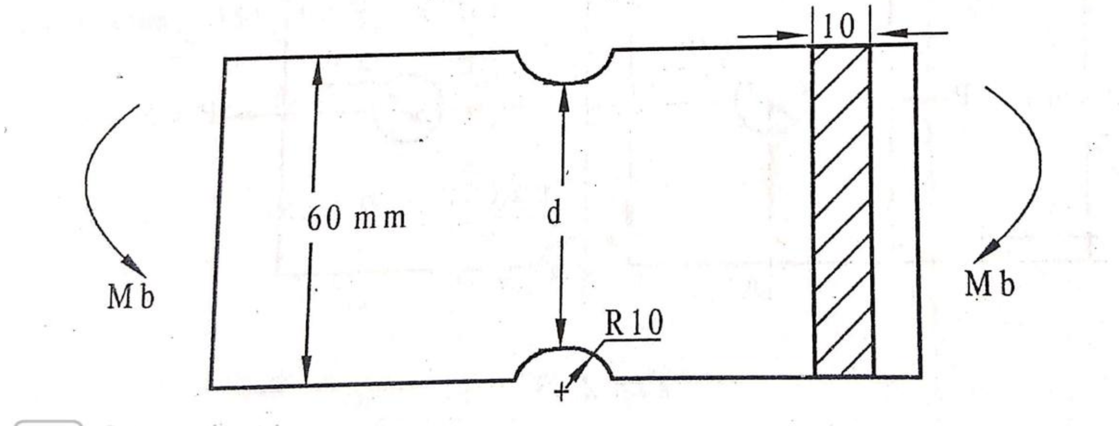


Fig. 2.42

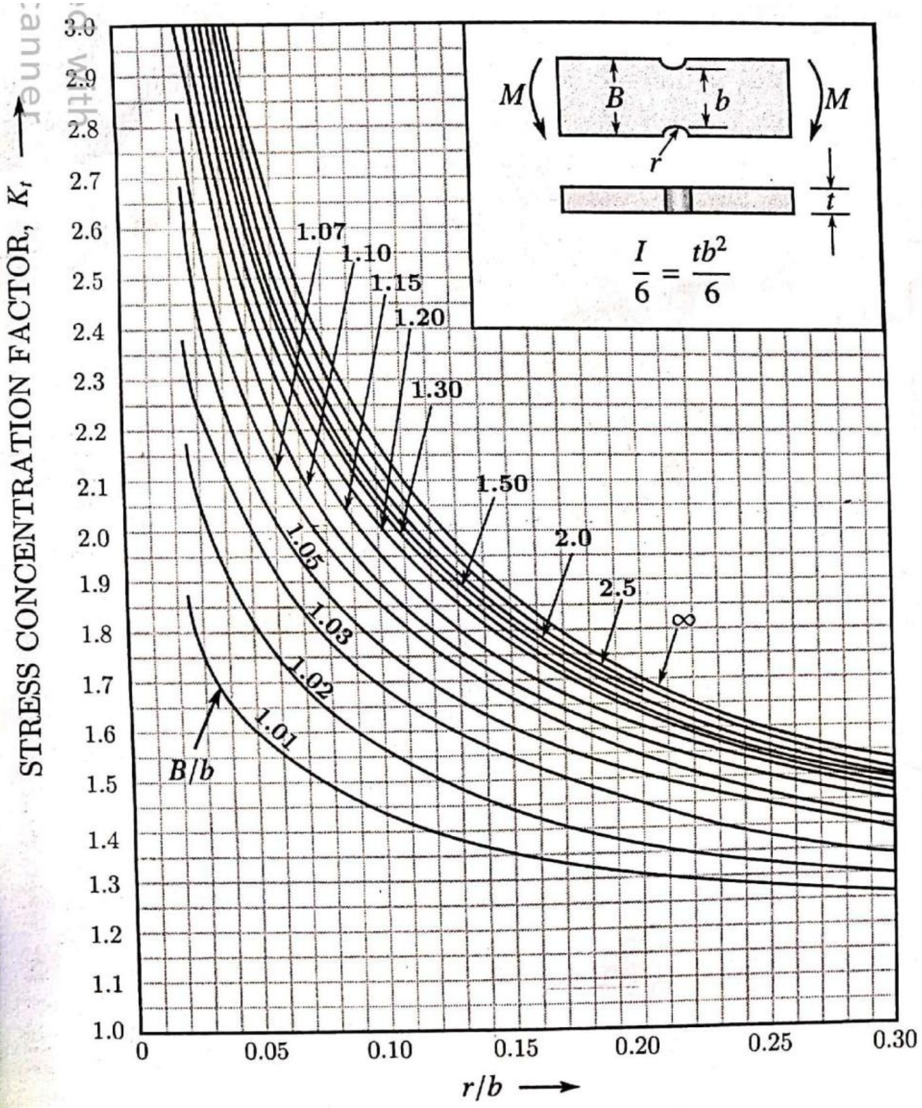


Fig. 2.15: ***Stress concentration Factor K_t for a notched Flat bar in bending***

6. Find the value of maximum stress induced on the fillet if the stress concentration factor for the filleted flat bar shown in fig having a B/b ratio as 1.2 also determine FOS. If the flat box is made up of steel having yield stress of 640Mpa thickness of the box is 25mm.

Given, F = 120kN, B/b = 1.2, t = 25mm, σ_y = 640Mpa, r = 10mm, σ_{max} = ? FOS = ?

B/b = 1.2 B = 1.2*b

From fig, B = b+2r => 1.2b = b + 2r => 1.2b = b+2*10 => b = 100mm

Therefore, B/b = 1.2 => B/100=1.2 => B =120mm

From fig 2.16/pg 38

r/b = 10/100 = 0.1, B/b = 1.2

Therefore,

K_t = 1.8

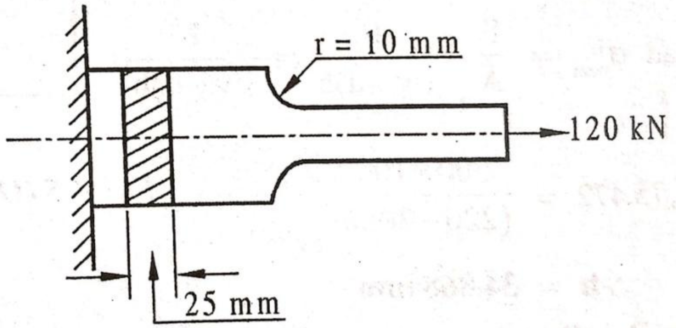


Fig. 2.44

Now,

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad \sigma_{nom} = \frac{F}{A} = \frac{F}{tb} = \frac{120 * 10^3}{25 * 100} = 48Mpa$$

$$1.8 = \frac{\sigma_{max}}{48}$$

σ_{max} = 86.4Mpa

To find FOS,

$$FOS = \frac{\sigma_y}{\sigma_{all}} = \frac{640}{86.4}$$

FOS = 7.4

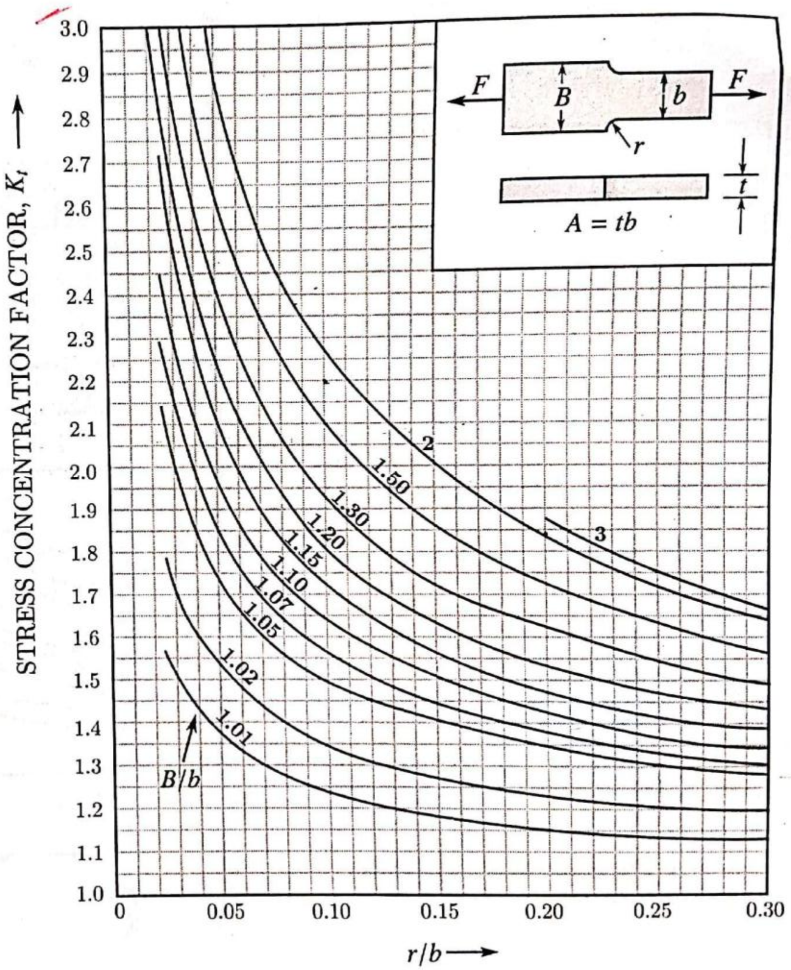


Fig. 2.16: ***Stress Concentration Factor K_t for Stepped flat bar in tension***

7. Find the diameter of the hole for the following fig if the stress concentration factor at the hole is same as at fillet.

i. Consider across fillet

From fig 2.16/pg 38

B = 160mm, b = 80mm, r = 8mm

r/b = 8/80 = 0.1, B/b = 160/80 = 2

Therefore, $K_t = 2.26$

ii. Consider across hole

From fig 2.12/pg 36

Wkt, $K_t = 2.26$

So a/B = 0.39

a = 0.39*160 => **a = 62.4mm**

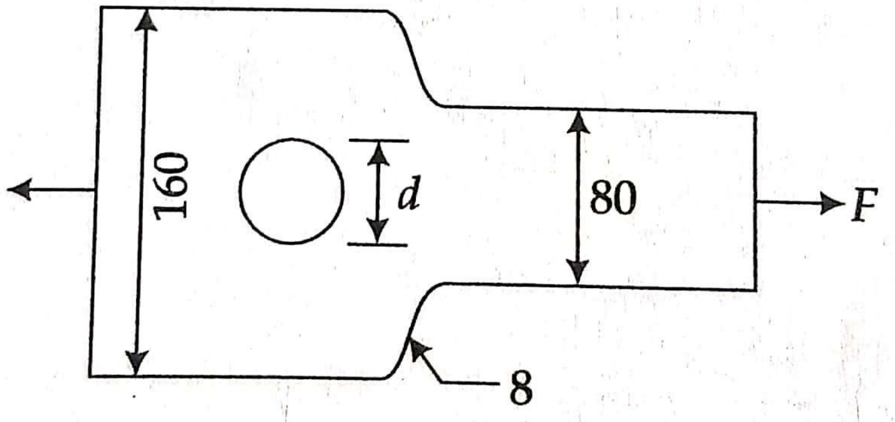


Fig. 3.22

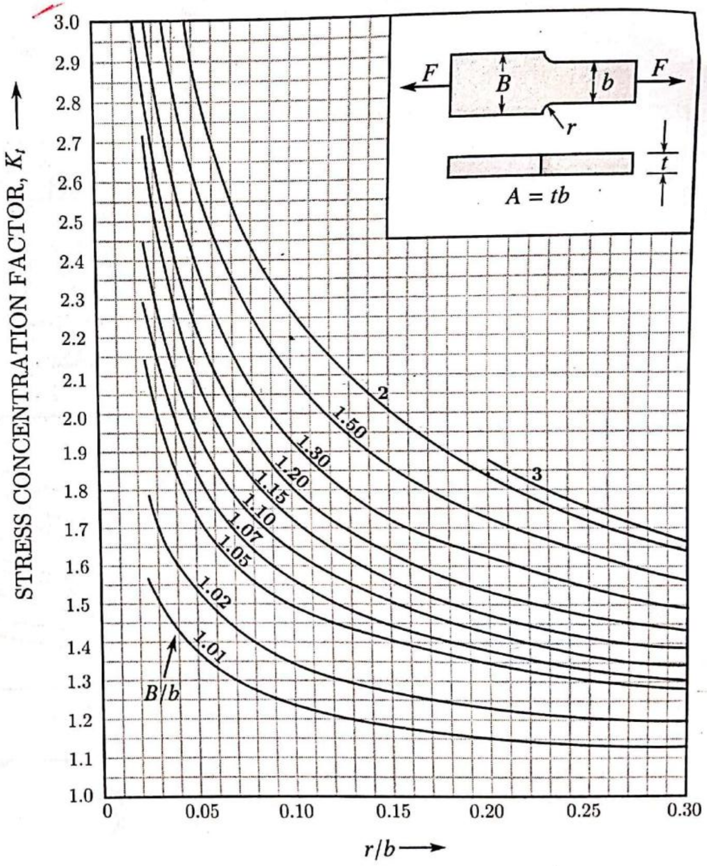


Fig. 2.16: ***Stress Concentration Factor K_t for Stepped flat bar in tension***

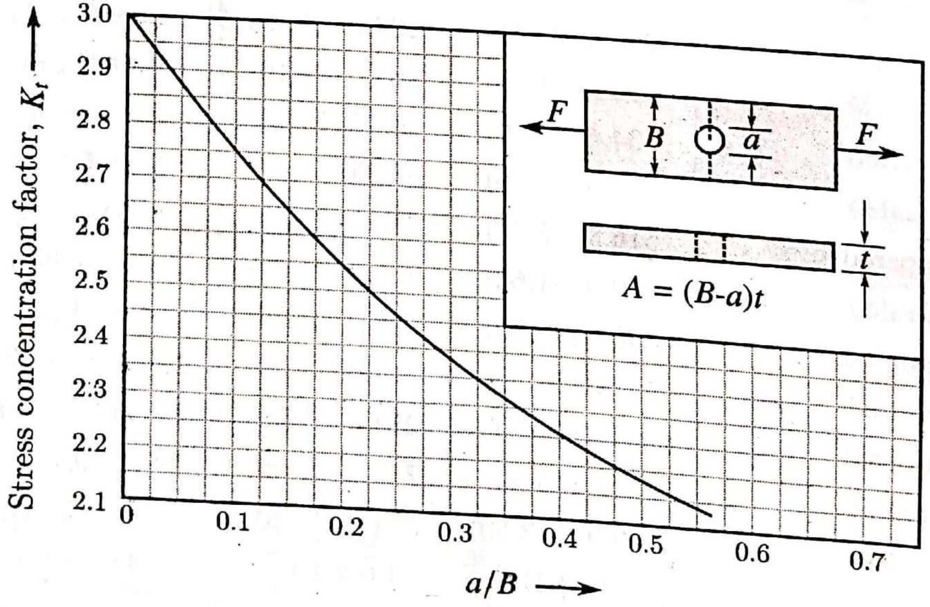


Fig. 2.12: ***Stress Concentration Factor K_t for a plate with hole in tension***

8. A grooved shaft shown in fig is to transmit 5kW at 120rpm. Determine the diameter of the shaft at the groove if it is made of C15 steel (σ_y = 235.4Mpa). FOS is 2

Given, Material C15 = σ_y = 235.4Mpa, FOS = 2, P = 5kW, N = 120rpm.

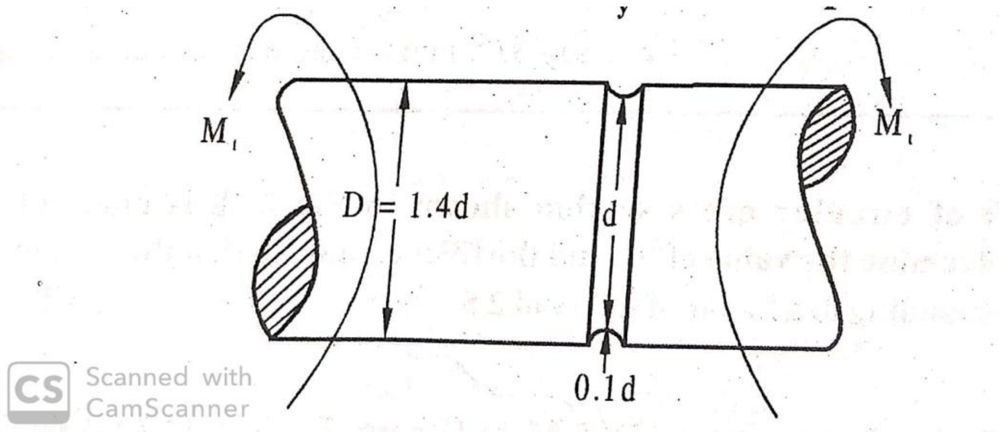
$$FOS = \frac{\sigma_y}{\sigma_{all}} \qquad \sigma_{all} = \frac{\sigma_y}{FOS} = \frac{235.4}{2} = 117.7Mpa \qquad \tau_{max} = \frac{\sigma_{all}}{2} = \frac{117.7}{2} = 58.85Mpa$$

$$k_{\tau t} = \frac{\tau_{max}}{\tau_{nom}}$$

From fig 2.22/pg 41

r/d = 0.1d/d = 0.1, D/d = 1.4d/d = 1.4

K_t = 1.57



$$\tau_{nom} = \frac{\tau_{max}}{k_t} = \frac{58.85}{1.57}$$

τ_{nom} = 37.48Mpa

$$\tau_{nom} = \frac{T * R}{J}$$

$$P = \frac{2 * \pi * N * T}{60} \qquad 5 * 10^3 = \frac{2 * \pi * 120 * T}{60}$$

T = 397.88N-mm

$$\tau_{nom} = \frac{T * R}{J} = \frac{16T}{\pi * d^3}$$

$$37.48 = \frac{16 * 397.88}{\pi * d^3}$$

d = 3.78mm

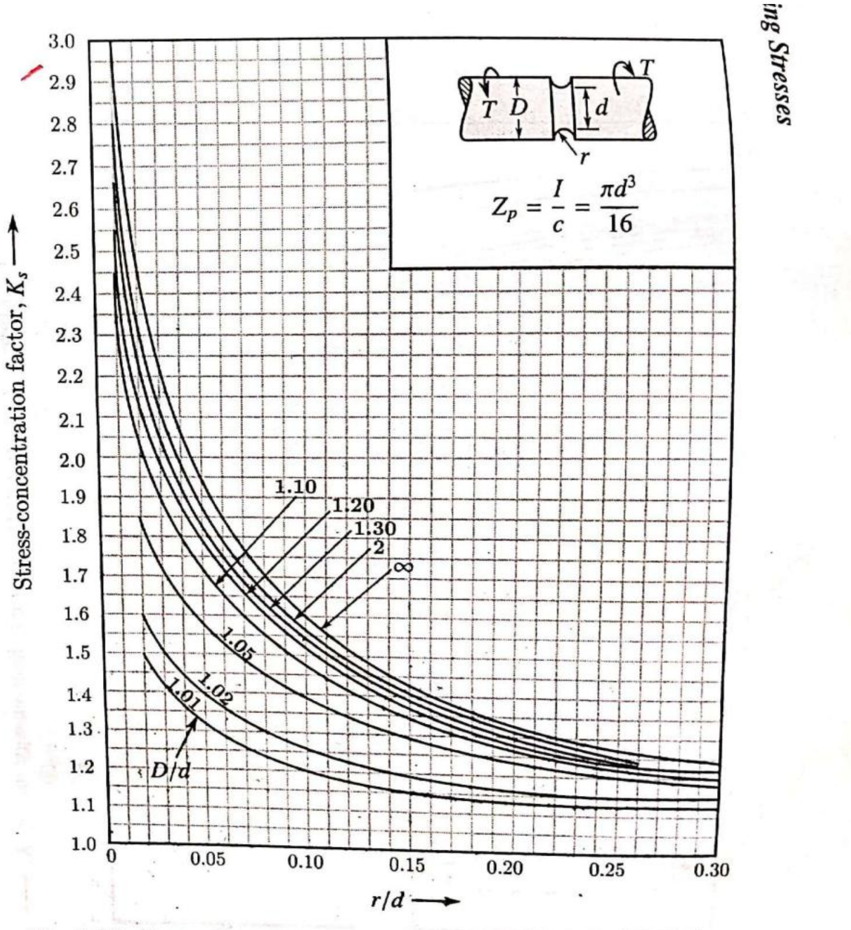


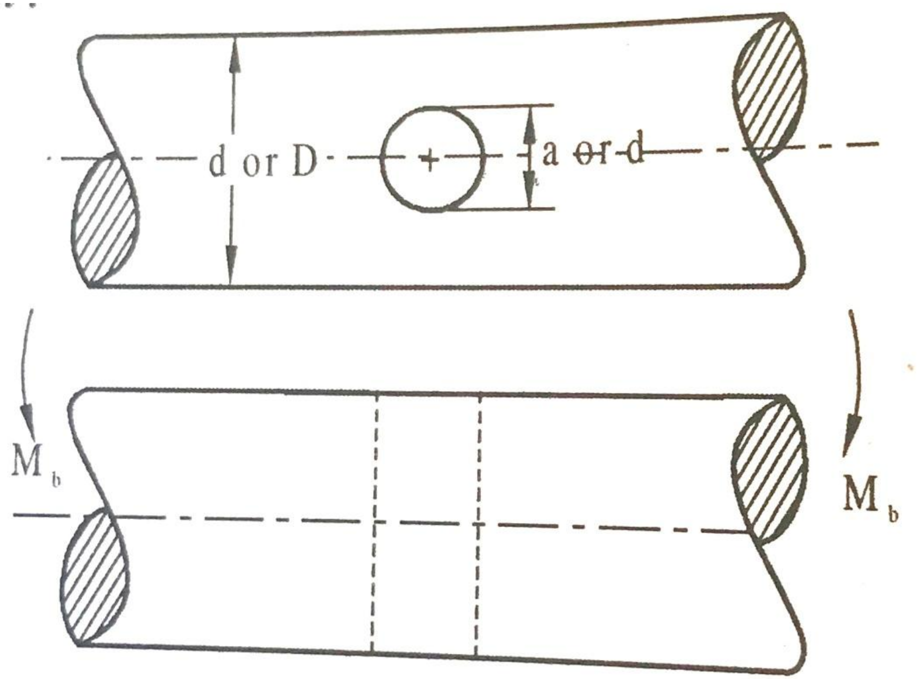
Fig. 2.22: *Stress Concentration Factor K_s for a grooved shaft in torsion*

9. A round shaft 50mm diameter is subjected to a bending moment 100 Nm. If there is a transverse circular hole of 10mm diameter is drilled on the shaft, determine the maximum stress induced on the shaft.

Given, D = 50mm, BM = 100 Nm = 100*10^3 N-mm, a = 10mm

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad \sigma_{nom} = \frac{MC}{I} = \frac{100 * 10^3 * \frac{50}{2}}{\frac{\pi * 50^4}{64}}$$

σ_{nom} = 8.14Mpa



From Fig 2.28/ pg 44

a/D = 10/50 = 0.2, k_t = 2.02

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \qquad 2.02 = \frac{\sigma_{max}}{8.14}$$

σ_{max} = 16.44Mpa

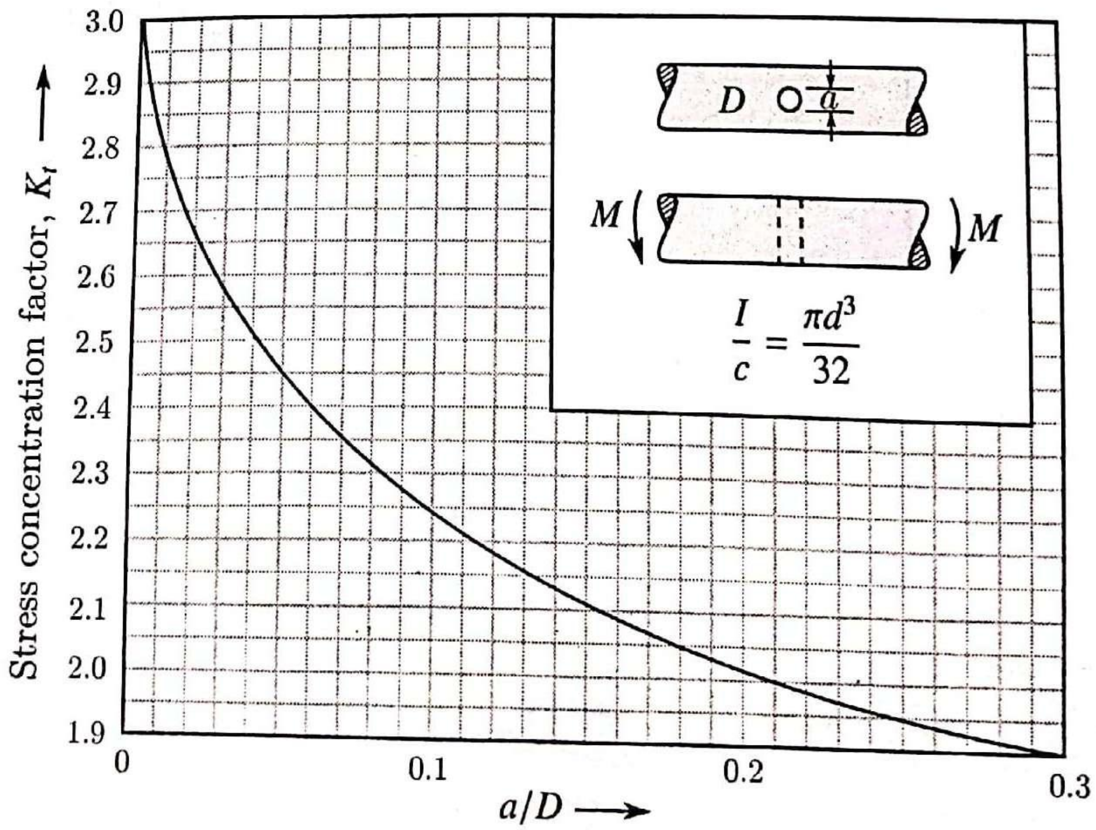


Fig. 2.28: *Stress-concentration factor K_t for a shaft with transverse hole*

ed with

10. A stepped shaft is subjected to transfer load of 8kN as shown in fig, a shaft is made of steel with ultimate tensile strength of 400Mpa. Determine the shaft diameter based on FOS 2.

Given, F=8kN = 8000N, σu = 400Mpa, FOS = 2, r =d/10

$R_a + R_b = 8kN$

BM @ a,

$8000*(250+100) – R_b*500 = 0$

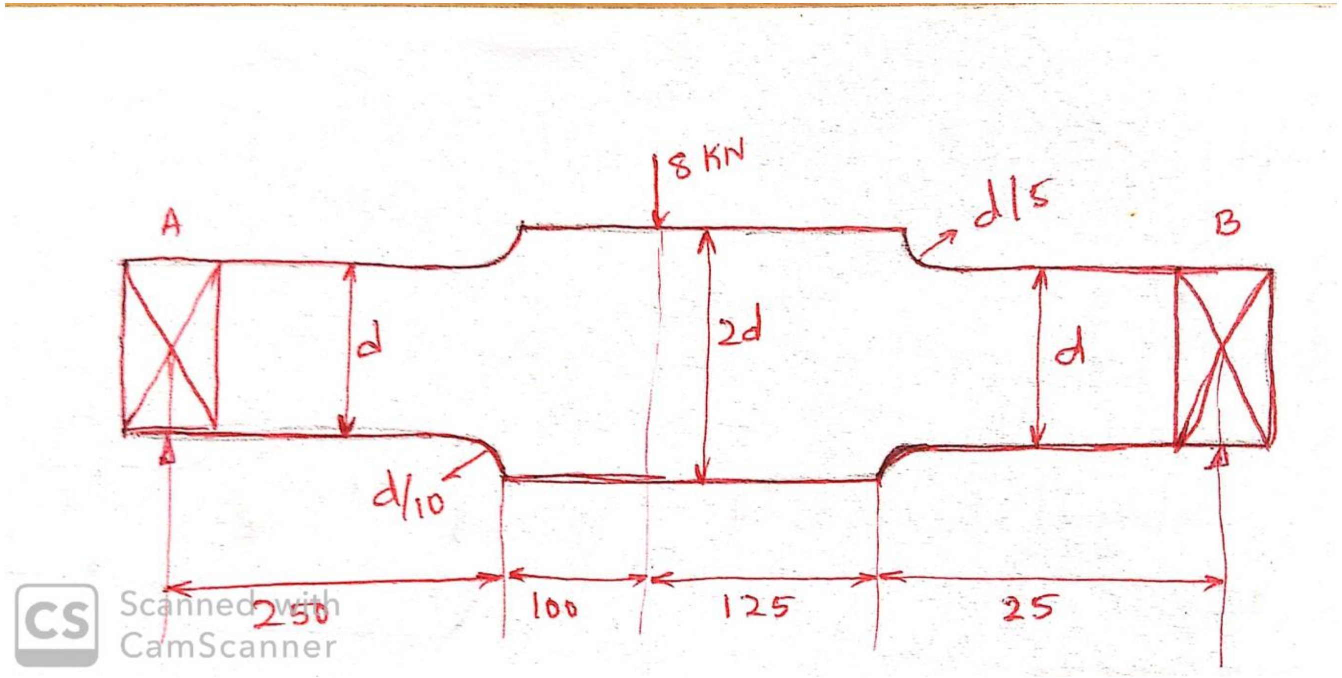
$R_b = 5600N$

$R_a = 2400N$

Consider section 1-1,

From fig 2.25 / pg 43

$r/d = 0.1$, $D/d = 2$ $\therefore k_t = 1.73$



Wkt, FOS = σ_u/σ_{all}, $\sigma_{all} = \sigma_u/FOS = 400/2 = 200$Mpa

$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$ $\sigma_{nom} = \frac{\sigma_{max}}{k_t} = \frac{200}{1.73}$ σ_{nom} = 115.60Mpa

$\sigma_{nom} = \frac{MC}{I}$ $115.60 = \frac{2400 * 250 * \frac{d}{2}}{\frac{\pi * d^4}{64}}$

d = 37.53mm ---------- 1

Consider section 2-2

From fig 2.25 / pg 43

r/d = 0.2, D/d = 2 $k_t = 1.43$

Wkt, FOS = σ_u/σ_{all}, $\sigma_{all} = \sigma_u/FOS = 400/2 = 200$Mpa

$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$ $\sigma_{nom} = \frac{\sigma_{max}}{k_t} = \frac{200}{1.43}$ σ_{nom} = 139.86Mpa

$\sigma_{nom} = \frac{MC}{I}$ $139.86 = \frac{5600 * 25 * \frac{d}{2}}{\frac{\pi * d^4}{64}}$

d = 21.68mm ---------- 2

By considering both sections

maximum diameter was in section 1-1

i.e., **d = 37.53mm**

Therefore,

D = 2*d = 2*37.53

D = 74.86mm

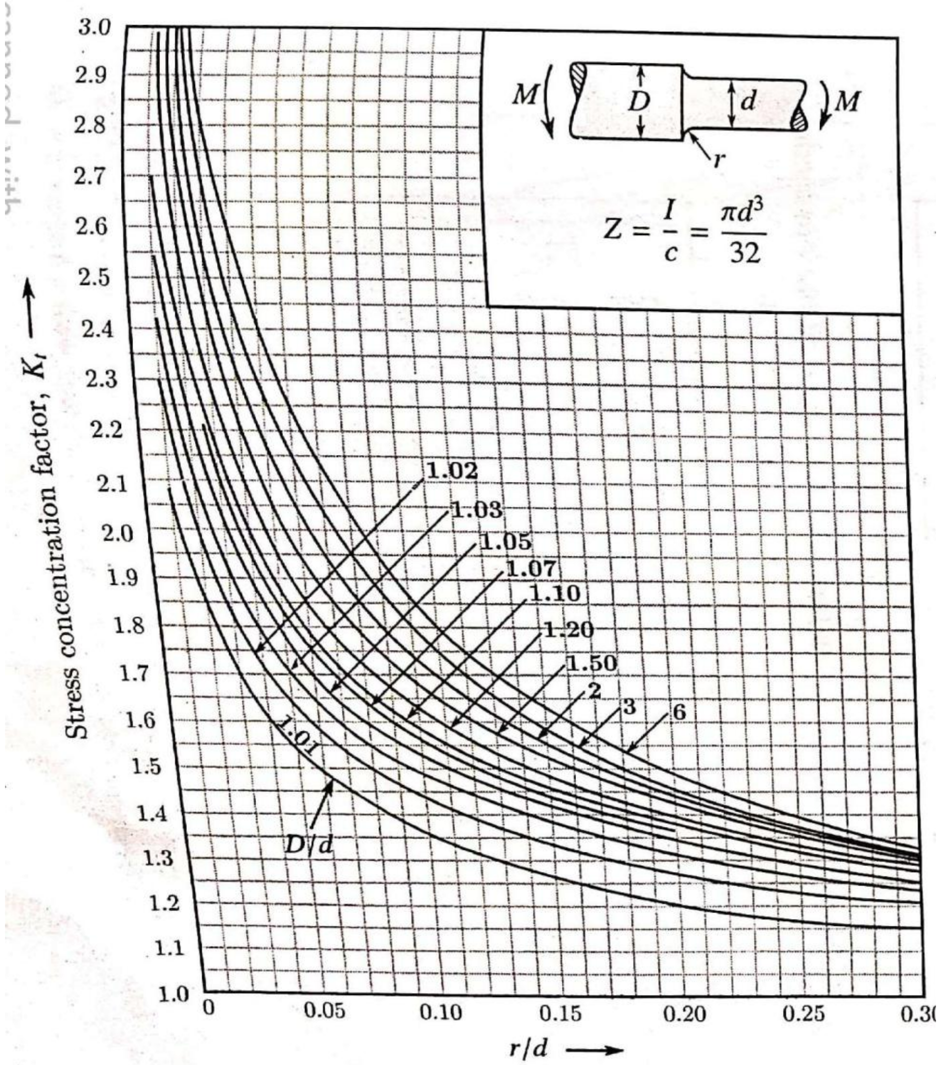


Fig. 2.25: *Stress-concentration factor K_t for a stepped shaft in bending*

Failure of brittle materials:

- Failure of brittle materials takes place by fracture
- The maximum principal stresses theory will be used for the design of brittle material
- Separate design equation should be used in tension and compression
- Since the yield point is not well defined in the brittle material, the ultimate strength is used as the basis for determining the allowable or design stress
- Design eqaution for brittle material is $\sigma_{max} \leq \frac{\sigma_u}{n}$

Failure of Ductile materials:

- Ductile material fail by yielding
- Since yield point is well defined for ductile material, the working or allowable stress is based on yield point
- Maximum shear stress theory is easy and quick to use
- Design equation for ductile material is $\tau_{max} \leq \frac{\sigma_y}{2n}$

Theories of failure

1. Rankine theory or maximum normal stress theory:

This theory states that the failure of the mechanical component subjected to bi axial or tri axial stresses occurs when the maximum normal stress reaches the yield or ultimate strength of the material.

According to this theory stress will be equivalent

$$\sigma_e = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(DDHB Eq no. 2.8a/pg 21), where σ_x and σ_y are principle stresses

For safe design then, $\sigma_e <= \sigma_{all}$

2. Guest theory or maximum shear stress theory:

This theory states that the failure of a mechanical component subjected to bi axial or triaxial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the simple tension test, when yielding starts,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(DDHB Eq no. 1.8f/pg 5)

For safe design, $\tau_{max} <= \tau_y$

For 2D stress, τ_{max} is the largest among these three, $\frac{\sigma_1}{2}$, $\frac{\sigma_2}{2}$, $\frac{\sigma_1 - \sigma_2}{2}$

If principle stresses σ_1 and σ_2 are opposite in direction then $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

If principle stresses σ_1 and σ_2 are same in direction then $\tau_{max} = \frac{\sigma_1}{2}$ or $\frac{\sigma_2}{2}$

For 3D stress, τ_{max} is the largest among these three $\frac{\sigma_1 - \sigma_2}{2}$, $\frac{\sigma_2 - \sigma_3}{2}$, $\frac{\sigma_3 - \sigma_1}{2}$

Where, σ_1, σ_2, σ_3 are principle stresses in 3 planes

3. Hencky – von misses theory / Distortion energy theory / Shear energy theory:

According to this theory, the member fails if the equivalent stress exceeds normal yield stress.

$$\sigma = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$ DDHB Eq no. 2.8c/ Pg 22

4. Saint vanant's theory / Maximum strain theory:

This theory states that the failure of the mechanical component subjected to bi axial or triaxial stresses occurs when the strain energy of distortion per unit volume.

This theory is good for brittle material

For biaxial, $\sigma = \sigma_1 - \mu\, \sigma_2$

For triaxial,

$$\sigma = (1-\mu)\left(\frac{\sigma_x + \sigma_y}{2}\right) + (1+\mu)\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ Eq no. 2.8b/Pg no. 22

Where, μ = poisson's ratio

Problems on Theories of failure:

1. A machine element is subjected to following stresses σ_x = 60Mpa, σ_y = 45Mpa and τ_{xy} = 30Mpa, find the FOS if it is made of C45 steel having yield stress at 353Mpa, using following theories
   1. Maximum normal stress theory
   2. Maximum shear stress theory
   3. Shear energy theory
   4. Maximum strain theory, taking poison's ratio μ = 0.3

Given, σ_x = 60Mpa, σ_y = 45Mpa, τ_{xy} = 30Mpa σ_{yp} = 353MPa μ = 0.3

1. Maximum normal stress theory:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \sigma = \frac{60+45}{2} + \sqrt{\left(\frac{60-45}{2}\right)^2 + 30^2}$$

σ = 83.42Mpa

FOS = σ_{yp}/σ_{all} = 353/83.42 **FOS = 4.23**

2. Maximum shear stress theory:

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \tau_{max} = \pm\sqrt{\left(\frac{60 - 45}{2}\right)^2 + 30^2}$$

τ_{max} = 30.92Mpa

$$\tau_{max} \leq \frac{\sigma_y}{2n}$$

30.92 = 353/2*n = > **FOS = 5.708**

3. Shear energy theory:

$$\sigma = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} \qquad \sigma = \sqrt{60^2 - 60 * 45 + 45^2 + 3 * 30^2}$$

σ = 75Mpa

FOS = σ_{yp}/σ_{all} = 353/75

FOS = 4.7

4. Maximum strain theory:

$$\sigma = (1-\mu)\left(\frac{\sigma_x+\sigma_y}{2}\right) + (1+\mu)\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma = (1-0.3)\left(\frac{60+45}{2}\right) + (1+0.3)\sqrt{\left(\frac{60-45}{2}\right)^2 + 30^2}$$

σ = 76.25Mpa

FOS = σ_{yp}/σ_{all} = 353/76.25

FOS = 4.58

2. A material has a maximum yield strength in tension and compression of σ_y = 100Mpa. Compute the FOS for the following theories of failure using the following stresses. i) Maximum normal stress theory, ii) Maximum shear stress theory iii) Distortion energy theory.

a) σ_1 = 70Mpa, σ_2 = 70Mpa, σ_3 = 0

b) σ_1 = 70Mpa, σ_2 = 30Mpa, σ_3 = 0

c) σ_1 = 0, σ_2 = -30Mpa, σ_3 = -70Mpa

i. Maximum normal stress theory:

a. σ_1 = 70Mpa, σ_2 = 70Mpa, σ_3 = 0
$\sigma_{max} = \sigma_{all}$ = 70Mpa
Wkt, FOS = σ_y/σ_{all}
$\sigma_{all} = \sigma_y$/FOS => 100/70
FOS = 1.43

b. σ_1 = 70Mpa, σ_2 = 30Mpa, σ_3 = 0
$\sigma_{max} = \sigma_{all}$ = 70Mpa
Wkt, FOS = σ_y/σ_{all}
$\sigma_{all} = \sigma_y$/FOS => 100/70
FOS = 1.43

c. σ_1 =0, σ_2 = -30Mpa, σ_3 = -70Mpa
$\sigma_{max} = \sigma_{all}$ = -70Mpa
Wkt, FOS = σ_y/σ_{all}
$\sigma_{all} = \sigma_y$/FOS => 100/-70
FOS = 1.43

ii. Maximum Shear stress theory:

For tri axial stress state τmax is the largest among the three values of $\frac{\sigma_1-\sigma_2}{2}$ $\frac{\sigma_2-\sigma_3}{2}$ $\frac{\sigma_3-\sigma_1}{2}$

a. $\sigma_1 = 70$Mpa, $\sigma_2 = 70$Mpa, $\sigma_3 = 0$

$\tau_{all} = \frac{\sigma_2-\sigma_3}{2} = \frac{70-0}{2} = 35$Mpa

Wkt, $\tau_{max} \leq \frac{\sigma_y}{2n}$

$\tau_{max} = 100/2*n \Rightarrow 35 = 100/2*n$

FOS = 1.43

b. $\sigma_1 = 70$Mpa, $\sigma_2 = 30$Mpa, $\sigma_3 = 0$

$\tau_{all} = \frac{\sigma_1-\sigma_3}{2} = \frac{70-0}{2} = 35$Mpa

Wkt, $\tau_{max} \leq \frac{\sigma_y}{2n}$

$\tau_{max} = 100/2*n \Rightarrow 35 = 100/2*n$

FOS = 1.43

c. $\sigma_1 = 0$Mpa, $\sigma_2 = -30$Mpa, $\sigma_3 = -70$Mpa

$\tau_{all} = \frac{\sigma_1-\sigma_3}{2} = \frac{0-(-70)}{2} = 35$Mpa

Wkt, $\tau_{max} \leq \frac{\sigma_y}{2n}$

$\tau_{max} = 100/2*n \Rightarrow 35 = 100/2*n$

FOS = 1.43

iii. Distortion energy theory:

According to this theory the design equation for tri axial stress state is,

a. $\sigma_1 = 70\text{Mpa}, \sigma_2 = 70\text{Mpa}, \sigma_3 = 0$

$$\left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3 \qquad \left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_1\sigma_2$$

$$\left(\frac{100}{n}\right)^2 = 70^2 + 70^2 + 70 * 70$$

FOS = 1.43

b. $\sigma_1 = 70\text{Mpa}, \sigma_2 = 30\text{Mpa}, \sigma_3 = 0$

$$\left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3 \qquad \left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_1\sigma_2$$

$$\left(\frac{100}{n}\right)^2 = 70^2 + 30^2 - 70 * 30$$

FOS = 1.644

c. $\sigma_1 = 0, \sigma_2 = -30\text{Mpa}, \sigma_3 = -70\text{Mpa}$

$$\left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3 \qquad \left(\frac{\sigma_y}{n}\right)^2 = \sigma_2^2 + \sigma_3^2 + \sigma_2\sigma_3$$

$$\left(\frac{100}{n}\right)^2 = -30^2 + (-70)^2 + (-30 * -70)$$

FOS = 1.644

THANK YOU