Module-5
TWO PORT NETWORK PARAMETERS

Objectives:
1. To study the concept of two port networks.
2. To study the concept of z, y, h and transmission parameters.
3. Providing relationship between parameters sets.

7.1 Introduction

A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero. There are several reasons why we should study two-ports and the parameters that describe them. For example, most circuits have two ports. We may apply an input signal in one port and obtain an output signal from the other port. The parameters of a two-port network completely describes its behaviour in terms of the voltage and current at each port. Thus, knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-ports parameters are examined here: impedance, admittance, hybrid, and transmission. We show the usefulness of each set of parameters, demonstrate how they are related to each other.

Fig. 7.1 represents a two-port network. A four terminal network is called a two-port network when the current entering one terminal of a pair exits the other terminal in the pair. For example, $\mathbf{I}_1$ enters terminal a and exits terminal b of the input terminal pair a-b.

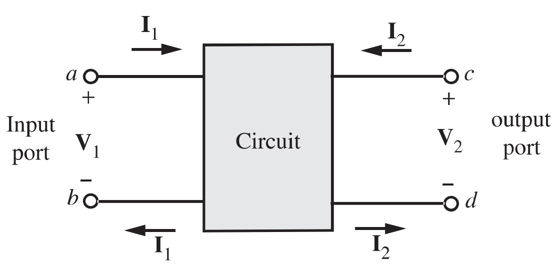

Figure 7.1 A two-port network

We assume that there are no independent sources or nonzero initial conditions within the linear two-port network.

7.2 Admittance parameters

The network shown in Fig. 7.2 is assumed to be linear and contains no independent sources. Hence, principle of superposition can be applied to determine the current $\mathbf{I}_1$, which can be written as the sum of two components, one due to $\mathbf{V}_1$ and the other due to $\mathbf{V}_2$. Using this principle, we can write

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

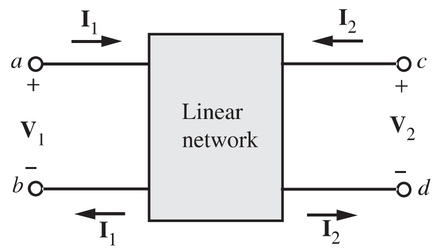

Figure 7.2 A linear two-port network

where $\mathbf{y}_{11}$ and $\mathbf{y}_{12}$ are the constants of proportionality with units of Siemens.

In a similar way, we can write

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

Hence, the two equations that describe the two-port network are

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \tag{7.1}$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \tag{7.2}$$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Here the constants of proportionality $\mathbf{y}_{11}, \mathbf{y}_{12}, \mathbf{y}_{21}$ and $\mathbf{y}_{22}$ are called $\mathbf{y}$ parameters for a network. If these parameters $\mathbf{y}_{11}$, $\mathbf{y}_{12}$, $\mathbf{y}_{21}$ and $\mathbf{y}_{22}$ are known, then the input/output operation of the two-port is completely defined.

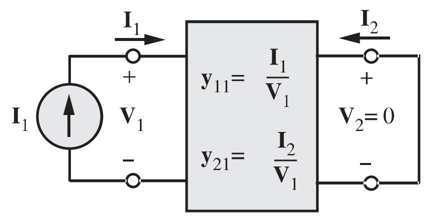

Figure 7.3 Determination of $\mathbf{y}_{11}$ and $\mathbf{y}_{12}$

From equations (7.1) and (7.2), we can determine $\mathbf{y}$ parameters. We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by connecting a current source $\mathbf{I}_1$ to port 1 and short-circuiting port 2 as shown in Fig. 7.3, finding $\mathbf{V}_1$ and $\mathbf{I}_2$, and then calculating,

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

Since $\mathbf{y}_{11}$ is the admittance at the input measured in siemens with the output short-circuited, it is called short-circuit input admittance. Similarly, $\mathbf{y}_{21}$ is called the short-circuit transfer admittance.

Similarly, we obtain $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ by connecting a current source $\mathbf{I}_2$ to port 2 and short-circuiting port 1 as in Fig. 7.4, finding $\mathbf{I}_1$ and $\mathbf{V}_2$, and then calculating,

$$\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$\mathbf{y}_{12}$ is called the *short-circuit transfer admittance* and $\mathbf{y}_{22}$ is called the *short-circuit output admittance*. Collectively the **y** parameters are referred to as short-circuit admittance parameters.

Please note that $\mathbf{y}_{12} = \mathbf{y}_{21}$ only when there are no dependent sources or Op-amps within the two-port network.

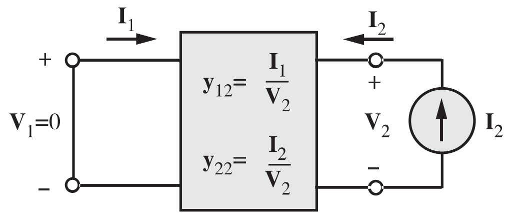

Figure 7.4 Determination of $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$

EXAMPLE 7.1

Determine the admittance parameters of the T network shown in Fig. 7.5.

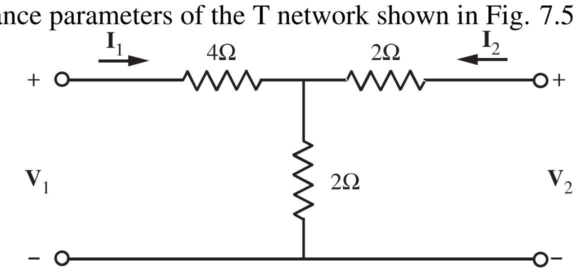

Figure 7.5

SOLUTION

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, we have to short the output terminals and connect a current source $\mathbf{I}_1$ to the input terminals. The circuit so obtained is shown in Fig. 7.6(a).

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{4 + \dfrac{2 \times 2}{2+2}} = \frac{\mathbf{V}_1}{5}$$

Hence, $$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{1}{5}\text{S}$$

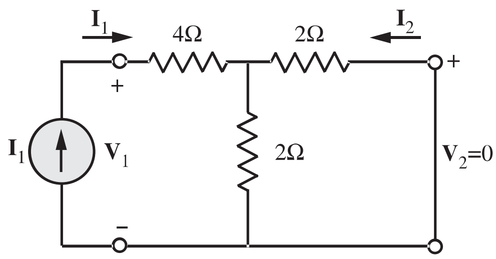

Figure 7.6(a)

Using the principle of current division,

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1 \times 2}{2+2} = \frac{\mathbf{I}_1}{2}$$

$$\Rightarrow \qquad -\mathbf{I}_2 = \frac{1}{2}\left[\frac{\mathbf{V}_1}{5}\right]$$

Hence, $$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{-1}{10}\text{S}$$

To find $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, we have to short-circuit the input terminals and connect a current source $\mathbf{I}_2$ to the output terminals. The circuit so obtained is shown in Fig. 7.6(b).

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2 + \dfrac{4 \times 2}{4+2}}$$

$$= \frac{\mathbf{V}_2}{2 + \dfrac{4}{3}}$$

$$= \frac{3\mathbf{V}_2}{10}$$

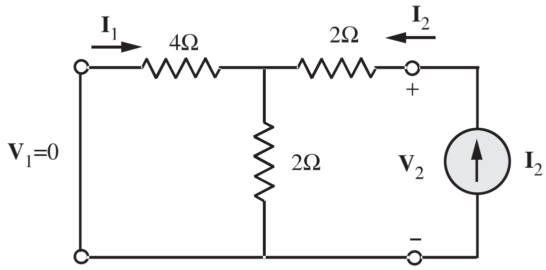

Figure 7.6(b)

Hence, $\mathbf{y}_{22} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \dfrac{3}{10}\text{S}$

Employing the principle of current division, we have

$$-\mathbf{I}_1 = \frac{\mathbf{I}_2 \times 2}{2+4}$$

$$\Rightarrow \quad -\mathbf{I}_1 = \frac{2\mathbf{I}_2}{6}$$

$$\Rightarrow \quad -\mathbf{I}_1 = \frac{1}{3}\left[\frac{3\mathbf{V}_2}{10}\right]$$

Hence, $\mathbf{y}_{12} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \dfrac{-1}{10}\text{S}$

It may be noted that, $\mathbf{y}_{12} = \mathbf{y}_{21}$. Thus, in matrix form we have

$$\mathbf{I} = \mathbf{YV}$$

$$\Rightarrow \quad \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{5} & \dfrac{-1}{10} \\ \dfrac{-1}{10} & \dfrac{3}{10} \end{bmatrix} \begin{bmatrix} \mathbf{V_1} \\ \mathbf{V_2} \end{bmatrix}$$

EXAMPLE 7.2

Find the **y** parameters of the two-port network shown in Fig. 7.7. Then determine the current in a 4Ω load, that is connected to the output port when a 2A source is applied at the input port.

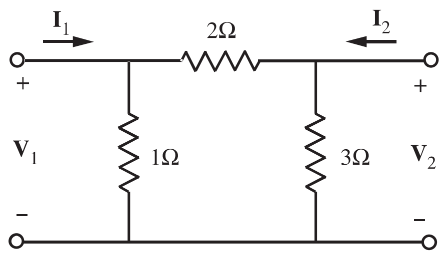

Figure 7.7

SOLUTION

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, short-circuit the output terminals and connect a current source $\mathbf{I}_1$ to the input terminals. The resulting circuit diagram is shown in Fig. 7.8(a).

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1\Omega||2\Omega} = \frac{\mathbf{V}_1}{\dfrac{1\times 2}{1+2}}$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{3}{2}\mathbf{V}_1$$

Hence, $$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{3}{2}\text{S}$$

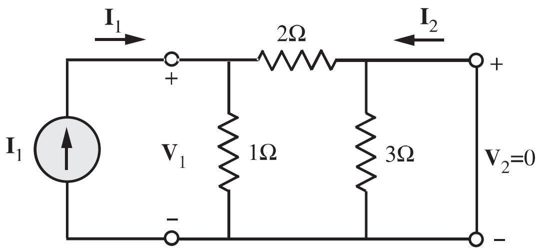

Figure 7.8(a)

Using the principle of current division,

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1\times 1}{1+2}$$

$$\Rightarrow \qquad -\mathbf{I}_2 = \frac{1}{3}\mathbf{I}_1$$

$$\Rightarrow \qquad -\mathbf{I}_2 = \frac{1}{3}\left[\frac{3}{2}\mathbf{V}_1\right]$$

Hence, $$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}\text{S}$$

To find $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, short the input terminals and connect a current source $\mathbf{I}_2$ to the output terminals. The resulting circuit diagram is shown in Fig. 7.8(b).

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2\Omega||3\Omega}$$

$$= \frac{\mathbf{V}_2}{\dfrac{2\times 3}{2+3}} = \frac{5\mathbf{V}_2}{6}$$

$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{5}{6}\text{S}$$

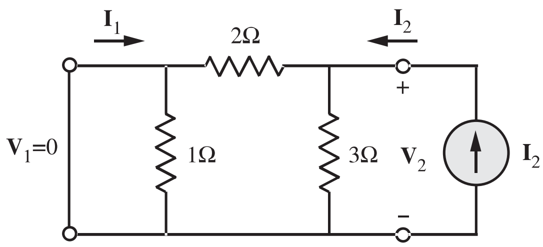

Figure 7.8(b)

Employing the current division principle,

$$-\mathbf{I}_1 = \frac{\mathbf{I}_2\times 3}{2+3}$$

$$\Rightarrow \qquad -\mathbf{I}_1 = \frac{3}{5}\mathbf{I}_2$$

$$\Rightarrow \qquad -\mathbf{I}_1 = \frac{3}{5}\left[\frac{5\mathbf{V}_2}{6}\right]$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{-1}{2}\mathbf{V}_2$$

Hence,

$$\mathbf{y}_{12} = \left.\frac{-\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{-1}{2}\text{S}$$

Therefore, the equations that describe the two-port network are

$$\mathbf{I}_1 = \frac{3}{2}\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 \qquad (7.3)$$

$$\mathbf{I}_2 = -\frac{1}{2}\mathbf{V}_1 + \frac{5}{6}\mathbf{V}_2 \qquad (7.4)$$

Putting the above equations (7.3) and (7.4) in matrix form, we get

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{5}{6} \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

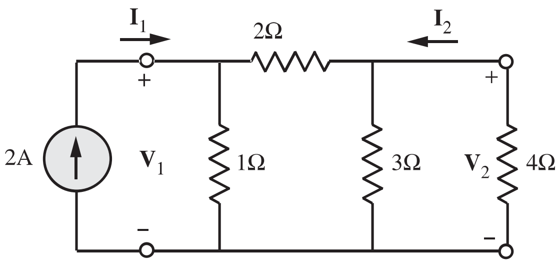

Figure 7.8(c)

Referring to Fig. 7.8(c), we find that $\mathbf{I}_1 = 2\text{A}$ and $\mathbf{V}_2 = -4\mathbf{I}_2$

Substituting $\mathbf{I}_1 = 2\text{A}$ in equation (7.3), we get

$$2 = \frac{3}{4}\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 \qquad (7.5)$$

Multiplying equation (7.4) by -4, we get

$$-4\mathbf{I}_2 = 2\mathbf{V}_1 - \frac{20}{6}\mathbf{V}_2$$

$$\Rightarrow \qquad \mathbf{V}_2 = 2\mathbf{V}_1 - \frac{20}{6}\mathbf{V}_2$$

$$\Rightarrow \qquad 0 = 2\mathbf{V}_1 - \left(\frac{20}{6} + 1\right)\mathbf{V}_2$$

$$\Rightarrow \qquad 0 = \frac{-1}{2}\mathbf{V}_1 + \frac{13}{12}\mathbf{V}_2 \qquad (7.6)$$

Putting equations (7.5) and (7.6) in matrix form, we get

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{13}{12} \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

It may be noted that the above equations are simply the nodal equations for the circuit shown in Fig. 7.8(c). Solving these equations, we get

$$\mathbf{V}_2 = \frac{3}{2}\mathrm{V}$$

and hence, $$\mathbf{I}_2 = \frac{-1}{4}\mathbf{V}_2 = \frac{-3}{8}\mathbf{A}$$

EXAMPLE 7.3

Refer the network shown in the Fig. 7.9 containing a current-controlled current source. For this network, find the **y** parameters.

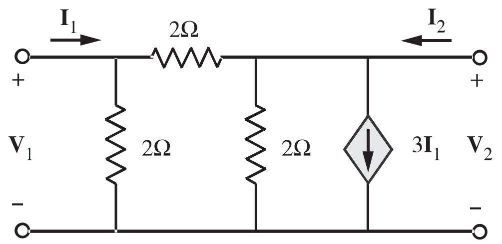

Figure 7.9

SOLUTION

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ short the output terminals and connect a current source $\mathbf{I}_1$ to the input terminals. The resulting circuit diagram is as shown in Fig. 7.10(a) and it is further reduced to Fig. 7.10(b).

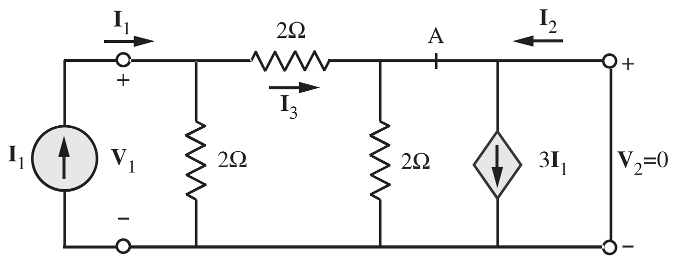

Figure 7.10(a)

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\frac{2\times 2}{2+2}}$$

$$\Rightarrow \qquad \mathbf{I}_1 = \mathbf{V}_1$$

Hence, $$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = 1\mathrm{S}$$

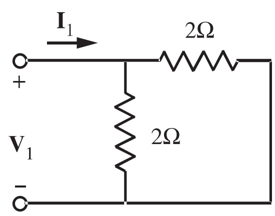

Figure 7.10(b)

Applying KCL at node A gives (Referring to Fig. 7.10(a)).

$$\mathbf{I}_3 + \mathbf{I}_2 = 3\mathbf{I}_1$$

$$\Rightarrow \qquad \frac{\mathbf{V}_1}{2} + \mathbf{I}_2 = 3\mathbf{I}_1$$

$$\Rightarrow \qquad \frac{\mathbf{V}_1}{2} + \mathbf{I}_2 = 3\mathbf{V}_1$$

$$\Rightarrow \qquad \frac{5\mathbf{V}_1}{2} = \mathbf{I}_2$$

Hence, $$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{5}{2}\mathrm{S}$$

To find $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, short the input terminals and connect a current source $\mathbf{I}_2$ at the output terminals. The resulting circuit diagram is shown in Fig. 7.10(c) and further reduced to Fig. 7.10(d).

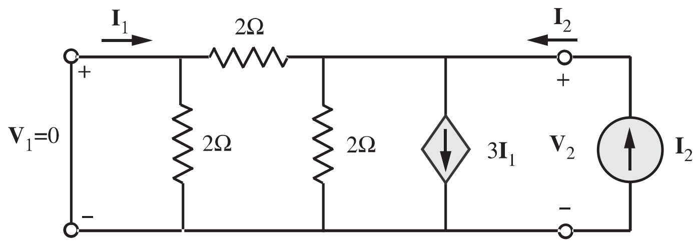

Figure 7.10(c)

$$\mathbf{I}_2 = -\mathbf{I}'_1 = -\frac{\mathbf{V}_2}{2}$$

$$\Rightarrow \qquad -\mathbf{I}_1 = \frac{\mathbf{V}_2}{2}$$

Hence, $$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}\mathrm{S}$$

Applying KCL at node B gives

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2}{2} + 3\mathbf{I}_1$$

But $$\mathbf{I}_1 = \frac{-\mathbf{V}_2}{2}$$

Hence, $$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2}{2} - 3\frac{\mathbf{V}_2}{2}$$

$$\Rightarrow \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = -0.5\mathrm{S}$$

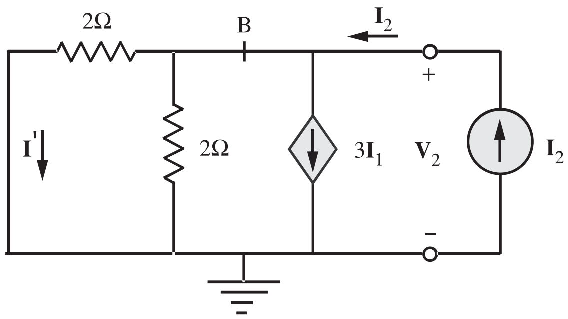

Figure 7.10(d)

Short-cut method:
Referring to Fig. 7.9, we have *KCL at node* $\mathbf{V}_1$:

$$\begin{aligned}\mathbf{I}_1 &= \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} \\ &= \mathbf{V}_1 - 0.5\mathbf{V}_2\end{aligned}$$

Comparing with

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

we get

$$\mathbf{y}_{11} = 1\text{S and } \mathbf{y}_{12} = -0.5\text{S}$$

KCL at node $\mathbf{V}_2$:

$$\begin{aligned}\mathbf{I}_2 &= 3\mathbf{I}_1 + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{2} \\ &= 3\left[\mathbf{V}_1 - 0.5\mathbf{V}_2\right] + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{2} \\ \Rightarrow \qquad \mathbf{I}_2 &= \frac{5}{2}\mathbf{V}_1 - 0.5\mathbf{V}_2\end{aligned}$$

Comparing with $\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$
we get

$$\mathbf{y}_{21} = 2.5\text{S and } \mathbf{y}_{22} = -0.5\text{S}$$

EXAMPLE 7.4

Find the **y** parameters for the two-port network shown in Fig. 7.11.

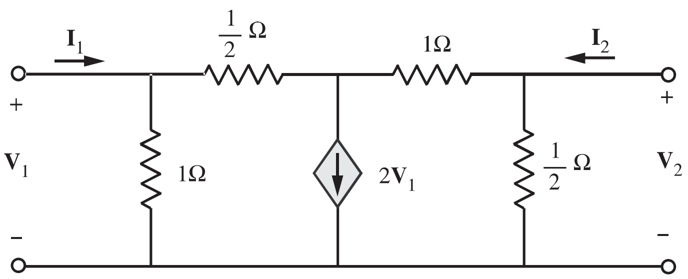

Figure 7.11

SOLUTION

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ short-circuit the output terminals as shown in Fig. 7.12(a). Also connect a current source $\mathbf{I}_1$ to the input terminals.

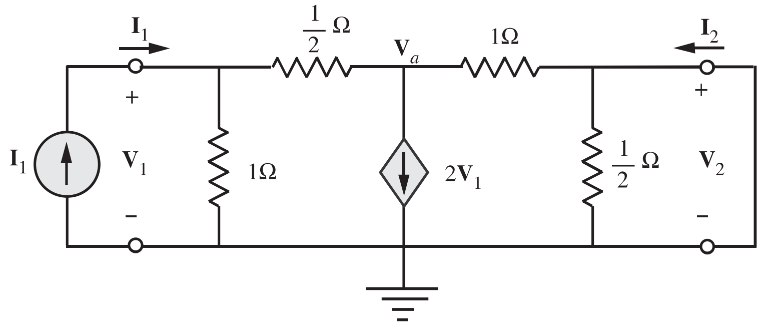

Figure 7.12(a)

KCL at node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_a}{\frac{1}{2}}$$

$$\Rightarrow \qquad 3\mathbf{V}_1 - 2\mathbf{V}_a = \mathbf{I}_1 \tag{7.7}$$

KCL at node $\mathbf{V}_a$:

$$\frac{\mathbf{V}_a - \mathbf{V}_1}{\frac{1}{2}} + \frac{\mathbf{V}_a - 0}{1} + 2\mathbf{V}_1 = 0$$

$$\Rightarrow \qquad 2\mathbf{V}_a - 2\mathbf{V}_1 + \mathbf{V}_a + 2\mathbf{V}_1 = 0$$

$$\Rightarrow \qquad \mathbf{V}_a = 0 \tag{7.8}$$

Making use of equation (7.8) in (7.7), we get

$$3\mathbf{V}_1 = \mathbf{I}_1$$

Hence,
$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = 3\mathrm{S}$$

Since $\mathbf{V}_a = 0, \mathbf{I}_2 = 0$,

$$\Rightarrow \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = 0\mathrm{S}$$

To find $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$ short-circuit the input terminals and connect a current source $\mathbf{I}_2$ to the output terminals. The resulting circuit diagram is shown in Fig. 7.12(b).

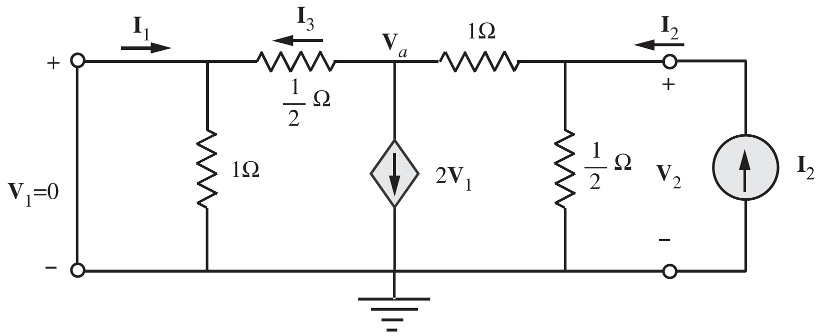

Figure 7.12(b)

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{\frac{1}{2}} + \frac{\mathbf{V}_2 - \mathbf{V}_a}{1} = \mathbf{I}_2$$

$$\Rightarrow \qquad 3\mathbf{V}_2 - \mathbf{V}_a = \mathbf{I}_2 \qquad (7.9)$$

KCL at node $\mathbf{V}_a$:

$$\frac{\mathbf{V}_a - \mathbf{V}_2}{1} + \frac{\mathbf{V}_a - 0}{\frac{1}{2}} + 0 = 0$$

$$\Rightarrow \qquad 3\mathbf{V}_a - \mathbf{V}_2 = 0$$

$$\text{or} \qquad \mathbf{V}_a = \frac{1}{3}\mathbf{V}_2 \qquad (7.10)$$

Substituting equation (7.10) in (7.9), we get

$$3\mathbf{V}_2 - \frac{1}{3}\mathbf{V}_2 = \mathbf{I}_2$$

$$\Rightarrow \qquad \frac{8}{3}\mathbf{V}_2 = \mathbf{I}_2$$

$$\text{Hence,} \qquad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{8}{3}\text{S}$$

$$\text{We have,} \qquad \mathbf{V}_a = \frac{1}{3}\mathbf{V}_2 \qquad (7.11)$$

$$\text{Also,} \qquad \mathbf{I}_1 + \mathbf{I}_3 = 0$$

$$\Rightarrow \qquad \mathbf{I}_1 = -\mathbf{I}_3$$

$$= \frac{-\mathbf{V}_a}{\frac{1}{2}} = -2\mathbf{V}_a \qquad (7.12)$$

Making use of equation (7.12) in (7.11), we get

$$-\frac{\mathbf{I}_1}{2} = \frac{1}{3}\mathbf{V}_2$$

Hence, $$\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{-2}{3}\mathrm{S}$$

EXAMPLE 7.5

Find the **y** parameters for the resistive network shown in Fig. 7.13.

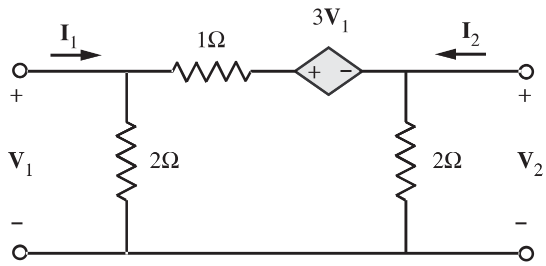

Figure 7.13

SOLUTION

Converting the voltage source into an equivalent current source, we get the circuit diagram shown in Fig. 7.14(a).

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, the output terminals of Fig. 7.14(a) are shorted and connect a current source $\mathbf{I}_1$ to the input terminals. This results in a circuit diagram as shown in Fig. 7.14(b).

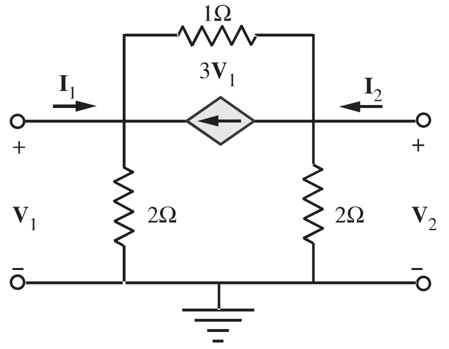

Figure 7.14(a)

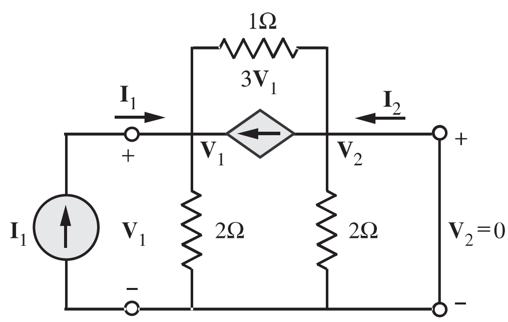

Figure 7.14(b)

KCL at node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} = \mathbf{I}_1 + 3\mathbf{V}_1$$

Since $\mathbf{V}_2 = 0$, we get

$$\frac{\mathbf{V}_1}{2} + \mathbf{V}_1 = \mathbf{I}_1 + 3\mathbf{V}_1$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{-3}{2}\mathbf{V}_1$$

Hence,

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{-3}{2}\mathrm{S}$$

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{2} + 3\mathbf{V}_1 + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1} = \mathbf{I}_2$$

Since $\mathbf{V}_2 = 0$, we get

$$0 + 3\mathbf{V}_1 - \mathbf{V}_1 = \mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{I}_2 = 2\mathbf{V}_1$$

Hence

$$\mathbf{y}_{21} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 2\mathrm{S}$$

To find $\mathbf{y}_{21}$ and $\mathbf{y}_{22}$, the input terminals of Fig. 7.14(a) are shorted and connect a current source $\mathbf{I}_2$ to the output terminals. This results in a circuit diagram as shown in Fig. 7.14(c).

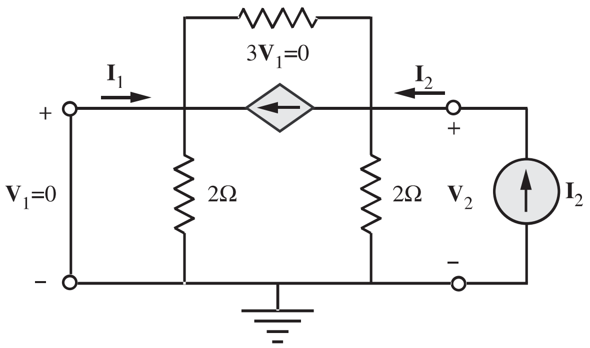

Figure 7.14(c)

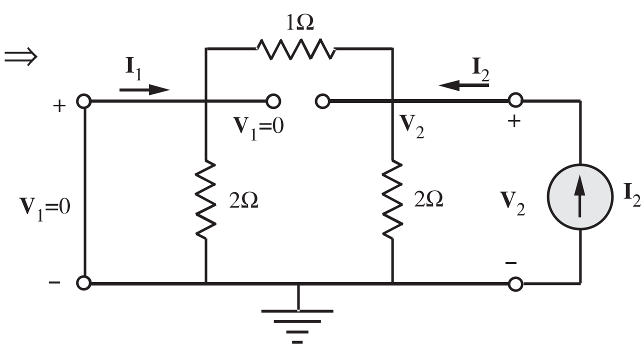

Figure 7.14(d)

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - 0}{1} = \mathbf{I}_2$$

$$\Rightarrow \qquad \frac{3}{2}\mathbf{V}_2 = \mathbf{I}_2$$

Hence, $$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{3}{2}\mathrm{S}$$

KCL at node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} = 0$$

Since $\mathbf{V}_1 = 0$, we get

$$\mathbf{I}_1 = -\mathbf{V}_2$$

Hence, $$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -1\mathrm{S}$$

EXAMPLE 7.6

The network of Fig. 7.15 contains both a dependent current source and a dependent voltage source. Find the **y** parameters.

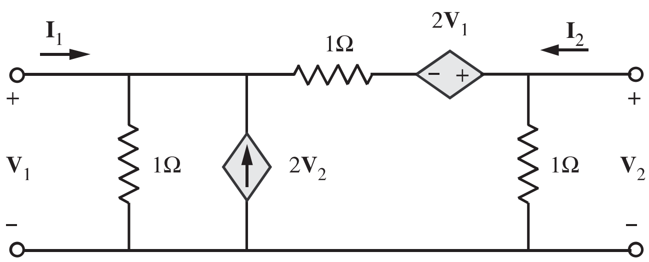

Figure 7.15

SOLUTION

While finding **y** parameters, we make use of *KCL* equations. Hence, it is preferable to have current sources rather than voltage sources. This prompts us to convert the dependent voltage source into an equivalent current source and results in a circuit diagram as shown in Fig. 7.16(a).

To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, refer the circuit diagram as shown in Fig. 7.16(b).

KCL at node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} + 2\mathbf{V}_1 = 2\mathbf{V}_2 + \mathbf{I}_1$$

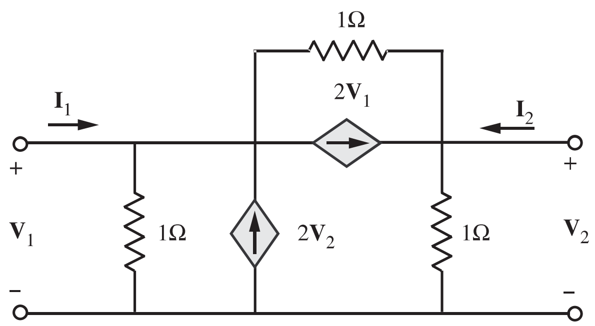

Figure 7.16(a)

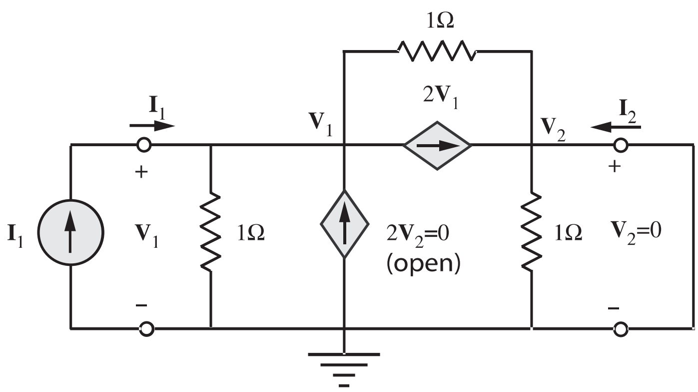

Figure 7.16(b)

Since $\mathbf{V}_2 = 0$, we get

$$\mathbf{V}_1 + \mathbf{V}_1 + 2\mathbf{V}_1 = \mathbf{I}_1$$

$$\Rightarrow \qquad 4\mathbf{V}_1 = \mathbf{I}_1$$

Hence,
$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 4\text{S}$$

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{1} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1} = 2\mathbf{V}_1 + \mathbf{I}_2$$

Since $\mathbf{V}_2 = 0$, we get

$$-\mathbf{V}_1 = 2\mathbf{V}_1 + \mathbf{I}_2$$

$$\Rightarrow \qquad -3\mathbf{V}_1 = \mathbf{I}_2$$

Hence,
$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = -3\text{S}$$

To find $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer the circuit diagram shown in Fig. 7.16(c).

KCL at node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} + 2\mathbf{V}_1 = 2\mathbf{V}_2 + \mathbf{I}_1$$

Since $\mathbf{V}_1 = 0$, we get

$$-\mathbf{V}_2 = 2\mathbf{V}_2 + \mathbf{I}_1$$

$$\Rightarrow \qquad -3\mathbf{V}_2 = \mathbf{I}_1$$

Hence, $$\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = -3\text{S}$$

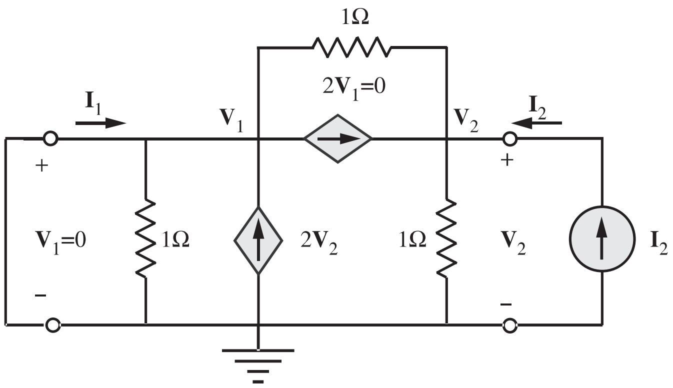

Figure 7.16(c)

KCL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{1} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1} = 2\mathbf{V}_1 + \mathbf{I}_2$$

Since $\mathbf{V}_1 = 0$, we get

$$\mathbf{V}_2 + \mathbf{V}_2 = 0 + \mathbf{I}_2$$

$$\Rightarrow \qquad -2\mathbf{V}_2 = \mathbf{I}_2$$

Hence, $$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = 2\text{S}$$

7.3 Impedance parameters

Let us assume the two port network shown in Fig. 7.17 is a linear network that contains no independent sources. Then using superposition theorem, we can write the input and output voltages as the sum of two components, one due to $\mathbf{I}_1$ and other due to $\mathbf{I}_2$:

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

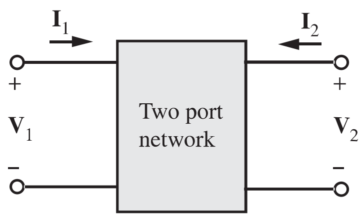

Figure 7.17

Putting the above equations in matrix from, we get

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

The **z** parameters are defined as follows:

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \qquad \mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

In the preceeding equations, letting $\mathbf{I}_1$ or $\mathbf{I}_2 = 0$ is equivalent to open-circuiting the input or output port. Hence, the **z** parameters are called *open-circuit impedance parameters.* $\mathbf{z}_{11}$ is defined as the *open-circuit input impedance,* $\mathbf{z}_{22}$ is called the *open-circuit output impedance,* and $\mathbf{z}_{12}$ and $\mathbf{z}_{21}$ are called the *open-circuit transfer impedances.*

If $\mathbf{z}_{12} = \mathbf{z}_{21}$, the network is said to be **reciprocal network**. Also, if all the **z**-parameter are identical, then it is called a **symmetrical network**.

EXAMPLE 7.7

Refer the circuit shown in Fig. 7.18. Find the **z** parameters of this circuit. Then compute the current in a 4Ω load if a $24\underline{/0^\circ}$ V source is connected at the input port.

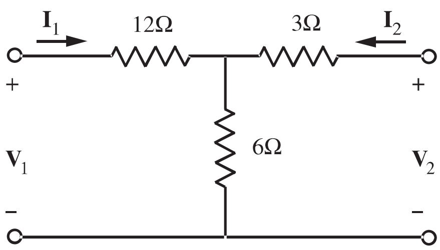

Figure 7.18

SOLUTION

To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, the output terminals are open circuited. Also connect a voltage source $\mathbf{V}_1$ to the input terminals. This gives a circuit diagram as shown in Fig. 7.19(a).

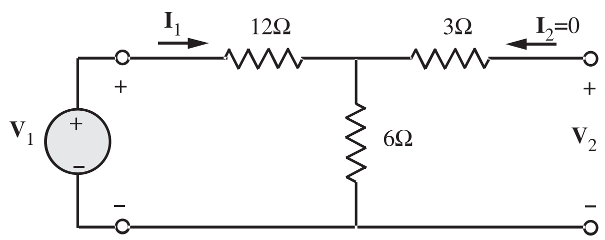

Figure 7.19(a)

Applying KVL to the left-mesh, we get

$$12\mathbf{I}_1 + 6\mathbf{I}_1 = \mathbf{V}_1$$

$$\Rightarrow \qquad \mathbf{V}_1 = 18\mathbf{I}_1$$

Hence, $$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = 18\Omega$$

Applying KVL to the right-mesh, we get

$$-\mathbf{V}_2 + 3 \times 0 + 6\mathbf{I}_1 = 0$$

$$\Rightarrow \qquad \mathbf{V}_2 = 6\mathbf{I}_1$$

Hence, $$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 6\Omega$$

To find $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, the input terminals are open circuited. Also connect a voltage source $\mathbf{V}_2$ to the output terminals. This results in a network as shown in Fig. 7.19(b).

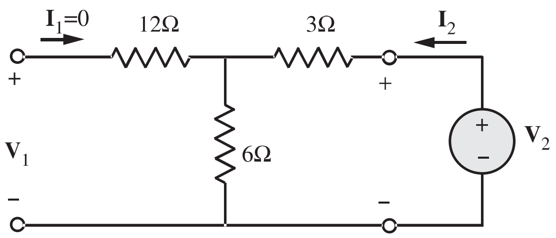

Figure 7.19(b)

Applying KVL to the left-mesh, we get

$$\mathbf{V}_1 = 12 \times 0 + 6\mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{V}_1 = 6\mathbf{I}_2$$

Hence, $$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = 6\Omega$$

Applying KVL to the right-mesh, we get

$$-\mathbf{V}_2 + 3\mathbf{I}_2 + 6\mathbf{I}_2 = 0$$

$$\Rightarrow \qquad \mathbf{V}_2 = 9\mathbf{I}_2$$

Hence, $$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = 9\Omega$$

The equations for the two-port network are, therefore

$$\mathbf{V}_1 = 18\mathbf{I}_1 + 6\mathbf{I}_2 \tag{7.13}$$

$$\mathbf{V}_2 = 6\mathbf{I}_1 + 9\mathbf{I}_2 \tag{7.14}$$

The terminal voltages for the network shown in Fig. 7.19(c) are

$$\mathbf{V}_1 = 24\,\underline{/0^\circ} \tag{7.15}$$

$$\mathbf{V}_2 = -4\mathbf{I}_2 \tag{7.16}$$

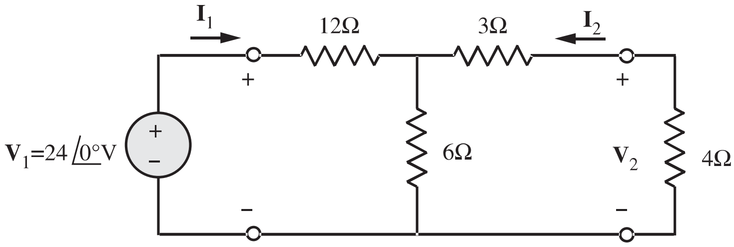

Figure 7.19(c)

Combining equations (7.15) and (7.16) with equations (7.13) and (7.14) yields

$$24\,\underline{/0^\circ} = 18\mathbf{I}_1 + 6\mathbf{I}_2$$

$$0 = 6\mathbf{I}_1 + 13\mathbf{I}_2$$

Solving, we get

$$\mathbf{I}_2 = -0.73\,\underline{/0^\circ}\text{ A}$$

EXAMPLE 7.8

Determine the **z** parameters for the two port network shown in Fig. 7.20.

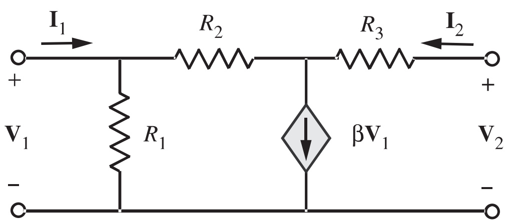

Figure 7.20

SOLUTION

To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, the output terminals are open-circuited and a voltage source is connected to the input terminals. The resulting circuit is shown in Fig. 7.21(a).

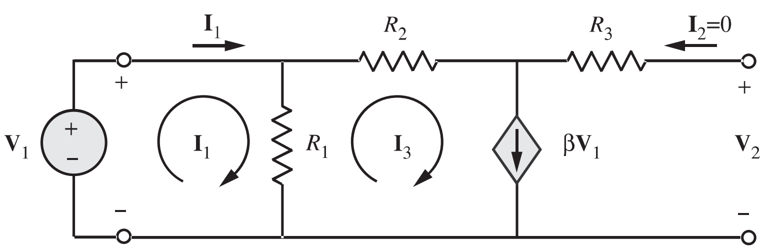

Figure 7.21(a)

By inspection, we find that $\mathbf{I}_3 = \beta \mathbf{V}_1$

Applying KVL to mesh 1, we get

$$\begin{aligned}
& R_1 (\mathbf{I}_1 - \mathbf{I}_3) = \mathbf{V}_1 \\
\Rightarrow \quad & R_1 \mathbf{I}_1 - R_1 \mathbf{I}_3 = \mathbf{V}_1 \\
\Rightarrow \quad & R_1 \mathbf{I}_1 - R_1 \beta \mathbf{V}_1 = \mathbf{V}_1 \\
\Rightarrow \quad & (1 + R_1 \beta) \mathbf{V}_1 = R_1 \mathbf{I}_1 \\
\text{Hence,} \quad & \mathbf{z}_{11} = \left. \frac{\mathbf{V}_1}{\mathbf{I}_1} \right|_{\mathbf{I}_2=0} = \frac{R_1}{1 + \beta R_1}
\end{aligned}$$

Applying KVL to the path $\mathbf{V}_1 \to R_2 \to R_3 \to \mathbf{V}_2$, we get

$$-\mathbf{V}_1 + R_2 \mathbf{I}_3 - R_3 \mathbf{I}_2 + \mathbf{V}_2 = 0$$

Since $\mathbf{I}_2 = 0$ and $\mathbf{I}_3 = \beta \mathbf{V}_1$, we get

$$\begin{aligned}
& -\mathbf{V}_1 + R_2 \beta \mathbf{V}_1 - 0 + \mathbf{V}_2 = 0 \\
\Rightarrow \quad & \mathbf{V}_2 = \mathbf{V}_1 (1 - \beta R_2) \\
& = (1 - \beta R_2) \frac{R_1 \mathbf{I}_1}{1 + \beta R_1} \\
\text{Hence,} \quad & \mathbf{z}_{21} = \left. \frac{\mathbf{V}_2}{\mathbf{I}_1} \right|_{\mathbf{I}_2=0} = \frac{R_1 (1 - \beta R_2)}{1 + \beta R_1}
\end{aligned}$$

The circuit used for finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$ is shown in Fig. 7.21(b).

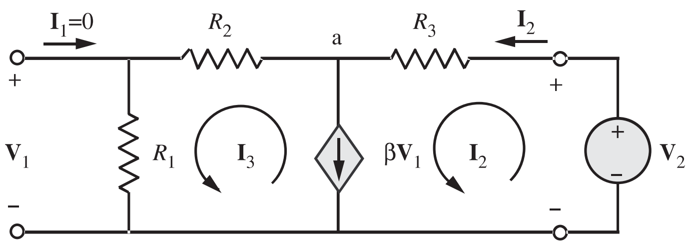

Figure 7.21(b)

By inspection, we find that

$$\mathbf{I}_2 - \mathbf{I}_3 = \beta \mathbf{V}_1 \text{ and } \mathbf{V}_1 = \mathbf{I}_3 R_1$$

$$\begin{aligned}
\Rightarrow \quad & \mathbf{I}_2 - \mathbf{I}_3 = \beta (\mathbf{I}_3 R_1) \\
\Rightarrow \quad & \mathbf{I}_3 (1 + \beta R_1) = \mathbf{I}_2
\end{aligned}$$

Applying KVL to the path $R_3 \rightarrow R_2 \rightarrow R_1 \rightarrow \mathbf{V}_2$, we get

$$R_3\mathbf{I}_2 + (R_2 + R_1)\,\mathbf{I}_3 - \mathbf{V}_2 = 0$$

$$\Rightarrow \quad R_3\mathbf{I}_2 + (R_2 + R_1)\,\frac{\mathbf{I}_2}{1+\beta R_1} = \mathbf{V}_2$$

$$\Rightarrow \quad \mathbf{I}_2\left[R_3 + \frac{R_2 + R_1}{1+\beta R_1}\right] = \mathbf{V}_2$$

Hence,

$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = R_3 + \frac{R_2 + R_1}{1+\beta R_1}\Omega$$

Applying KCL at node a, we get

$$\beta\mathbf{V}_1 + \mathbf{I}_3 = I_2$$

$$\Rightarrow \quad \beta\mathbf{V}_1 + \frac{\mathbf{V}_1}{R_1} = \mathbf{I}_2$$

$$\Rightarrow \quad \mathbf{V}_1\left[\beta + \frac{1}{R_1}\right] = \mathbf{I}_2$$

$$\Rightarrow \quad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{1}{\beta + \dfrac{1}{R_1}} = \frac{R_1}{1+\beta R_1}$$

EXAMPLE 7.9

Construct a circuit that realizes the following **z** parameters.

$$\mathbf{z} = \begin{bmatrix} 12 & 4 \\ 4 & 8 \end{bmatrix}$$

SOLUTION

Comparing **z** with = $\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix}$, we get

$$\mathbf{z}_{11} = 12\Omega, \quad \mathbf{z}_{12} = \mathbf{z}_{21} = 4\Omega, \quad \mathbf{z}_{22} = 8\Omega$$

Let us consider a T network as shown in Fig. 7.22(a). Our objective is to fit in the values of R_1, R_2 and R_3 for the given **z**.

Applying KVL to the input loop, we get

$$\begin{aligned}\mathbf{V}_1 &= R_1\mathbf{I}_1 + R_3\,(\mathbf{I}_1 + \mathbf{I}_2)\\ &= (R_1 + R_3)\,\mathbf{I}_1 + R_3\mathbf{I}_2\end{aligned}$$

Comparing the preceeding equation with

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

we get

$$\mathbf{z}_{11} = R_1 + R_3 = 12\Omega$$
$$\mathbf{z}_{12} = R_3 = 4\Omega$$
$$\Rightarrow \quad R_1 = 12 - R_3 = 8\Omega$$

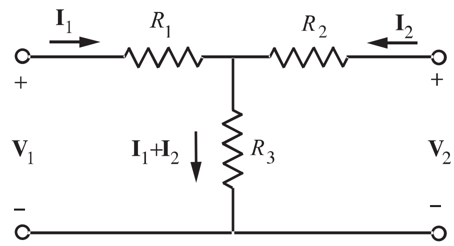

Figure 7.22(a)

Applying *KVL to the output loop*, we get

$$\mathbf{V}_2 = R_2\mathbf{I}_2 + R_3\left(\mathbf{I}_1 + \mathbf{I}_2\right)$$
$$\Rightarrow \quad \mathbf{V}_2 = R_3\mathbf{I}_1 + \left(R_2 + R_3\right)\mathbf{I}_2$$

Comparing the immediate preceeding equation with

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

we get

$$\mathbf{z}_{21} = R_3 = 4\Omega$$
$$\mathbf{z}_{22} = R_2 + R_3 = 8\Omega$$
$$\Rightarrow \quad R_2 = 8 - R_3 = 4\Omega$$

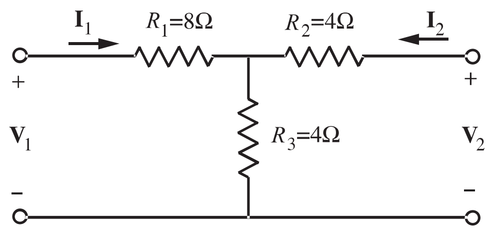

Figure 7.22(b)

Hence, the network to meet the given **z** parameter set is shown in Fig. 7.22(b).

EXAMPLE 7.10

If $\mathbf{z} = \begin{bmatrix} 40 & 10 \\ 20 & 30 \end{bmatrix}\Omega$ for the two-port network, calculate the average power delivered to 50Ω resistor.

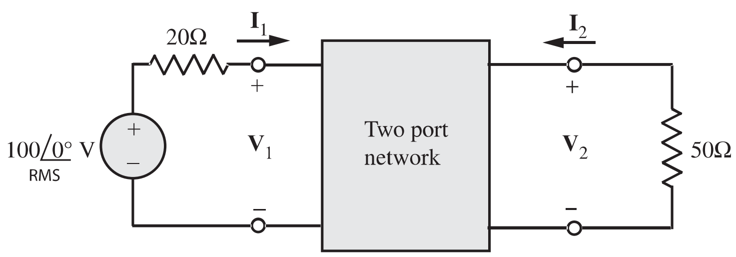

Figure 7.23

SOLUTION

We are given $\mathbf{z}_{11} = 40\Omega \quad \mathbf{z}_{12} = 10\Omega \quad \mathbf{z}_{21} = 20\Omega \quad \mathbf{z}_{22} = 30\Omega$

Since $\mathbf{z}_{12} \neq \mathbf{z}_{21}$, this is not a reciprocal network. Hence, it cannot be represented only by passive elements. We shall draw a network to satisfy the following two *KVL equations*:

$$\mathbf{V}_1 = 40\mathbf{I}_1 + 10\mathbf{I}_2$$
$$\mathbf{V}_2 = 20\mathbf{I}_1 + 30\mathbf{I}_2$$

One possible way of representing a network that is non-reciprocal is as shown in Fig. 7.24(a).

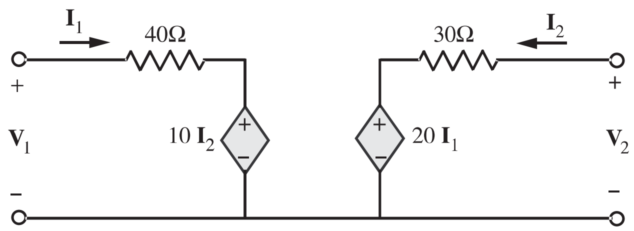

Figure 7.24(a)

Now connecting the source and the load to the two-port network, we get the network as shown in Fig. 7.24(b).

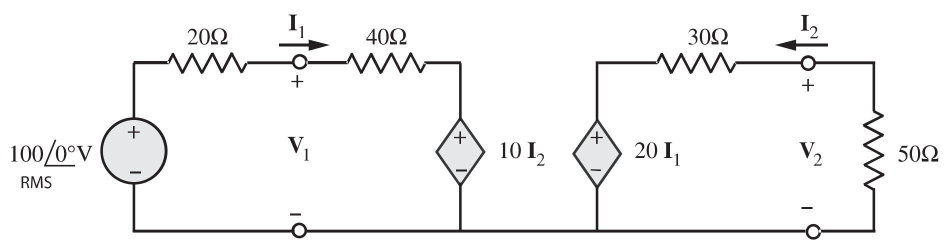

Figure 7.24(b)

KVL for mesh 1:

$$60\mathbf{I}_1 + 10\mathbf{I}_2 = 100$$
$$\Rightarrow \qquad 6\mathbf{I}_1 + \mathbf{I}_2 = 10$$

KVL for mesh 2:

$$80\mathbf{I}_2 + 20\mathbf{I}_1 = 0$$
$$\Rightarrow \qquad 4\mathbf{I}_2 + \mathbf{I}_1 = 0$$
$$\Rightarrow \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Solving the above mesh equations, we get

$$-24\mathbf{I}_2 + \mathbf{I}_2 = 10$$

$$\Rightarrow \quad -23\mathbf{I}_2 = 10$$

$$\Rightarrow \quad \mathbf{I}_2 = \frac{-10}{23}$$

Power supplied to the load $= |\mathbf{I}_2|^2 R_L$

$$= \frac{100}{(23)^2} \times 50$$

$$= \mathbf{9.45\ W}$$

EXAMPLE 7.11

Refer the network shown in Fig. 7.25. Find the **z** parameters for the network. Take $\alpha = \frac{4}{3}$

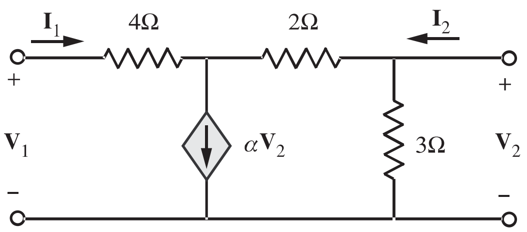

Figure 7.25

SOLUTION

To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, open-circuit the output terminals as shown in Fig. 7.26(a). Also connect a voltage source $\mathbf{V}_1$ to the input terminals.

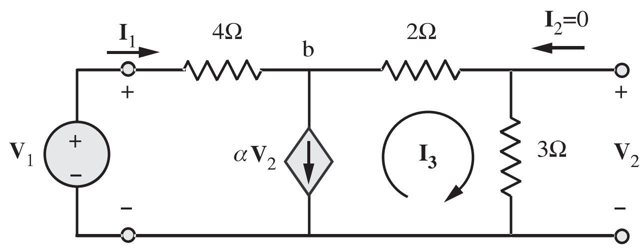

Figure 7.26(a)

Applying KVL around the path $\mathbf{V}_1 \rightarrow 4\Omega \rightarrow 2\Omega \rightarrow 3\Omega$, we get

$$4\mathbf{I}_1 + 5\mathbf{I}_3 = \mathbf{V}_1 \tag{7.17}$$

Also,

$$\mathbf{V}_2 = 3\mathbf{I}_3, \text{ so } \mathbf{I}_3 = \frac{\mathbf{V}_2}{3} \tag{7.18}$$

KCL at node b leads to

$$\mathbf{I}_1 - \alpha\mathbf{V}_2 - \mathbf{I}_3 = 0 \tag{7.19}$$

Substituting equation (7.18) in (7.19), we get

$$\mathbf{I}_1 = \alpha\mathbf{V}_2 + \frac{\mathbf{V}_2}{3} = \left[\alpha + \frac{1}{3}\right]\mathbf{V}_2$$

$$= \left[\frac{4}{3} + \frac{1}{3}\right]\mathbf{V}_2$$

Hence, $$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \frac{3}{5}\Omega$$

Substituting equation (7.18) in (7.17), we get

$$\mathbf{V}_1 = 4\mathbf{I}_1 + 5\frac{\mathbf{V_2}}{\mathbf{3}}$$

$$= 4\mathbf{I_1} + \frac{5}{3}\left(\mathbf{I}_1 \times \frac{3}{5}\right) \qquad \left(\text{Since } \frac{\mathbf{V_2}}{\mathbf{I_1}} = \frac{3}{5}\right)$$

Therefore, $$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 5\Omega$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, open-circuit the input terminals as shown in Fig. 7.26(b). Also, connect a voltage source $\mathbf{V}_2$ to the output terminals.

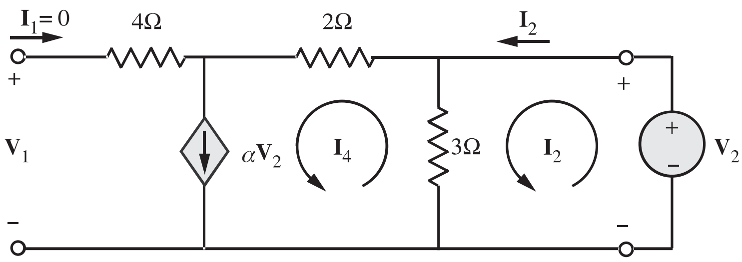

Figure 7.26(b)

KVL for the mesh on the left:

$$\mathbf{V}_1 + 5\mathbf{I}_4 - 3\mathbf{I}_2 = 0 \tag{7.20}$$

KVL for the mesh on the right:

$$\mathbf{V}_2 + 3\mathbf{I}_4 - 3\mathbf{I}_2 = 0 \tag{7.21}$$

Also, $$\mathbf{I}_4 = \alpha\mathbf{V}_2 \tag{7.22}$$

Substituting equation (7.22) in (7.21), we get

$$\mathbf{V}_2 + 3\alpha\mathbf{V}_2 - 3\mathbf{I}_2 = 0$$

$$\Rightarrow \qquad \mathbf{V}_2(1+3\alpha) = 3\mathbf{I}_2$$

Hence,

$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{3}{1+3\alpha}$$

$$= \frac{3}{1+3\left(\frac{4}{3}\right)} = \frac{3}{5}\Omega$$

Substituting equation (7.22) in (7.20), we get

$$\mathbf{V}_1 + 5\alpha\mathbf{V}_2 = 3\mathbf{I}_2$$

Substituting $\mathbf{V}_2 = \frac{3}{5}\mathbf{I}_2$, we get

$$\mathbf{V}_1 + 5\alpha\left(\frac{3}{5}\times\mathbf{I}_2\right) = 3\mathbf{I}_2$$

Hence,

$$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$= 3 - 3\alpha$$

$$= 3 - 3\frac{4}{3} = -1\Omega$$

Finally, in the matrix form, we can write

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ \frac{5}{3} & \frac{3}{5} \end{bmatrix}$$

Please note that $\mathbf{z}_{12} \neq \mathbf{z}_{21}$, since a dependent source is present in the circuit.

EXAMPLE 7.12

Find the Thevenin equivalent circuit with respect to port 2 of the circuit in Fig. 7.27 in terms of **z** parameters.

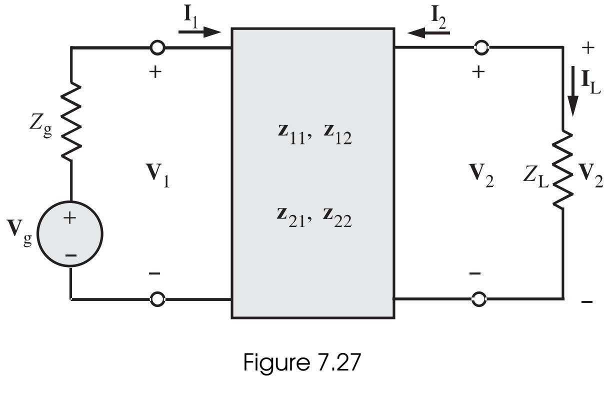

Figure 7.27

SOLUTION

The two port network is defined by

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2;$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2;$$

here, $\mathbf{V}_1 = \mathbf{V}_g - Z_g\mathbf{I}_1$
and $\mathbf{V}_2 = \mathbf{I}_L Z_L = -\mathbf{I}_2 Z_L$

To find Thevenin equivalent circuit as seen from the output terminals, we have to remove the load resistance R_L. The resulting circuit diagram is shown in Fig. 7.28(a).

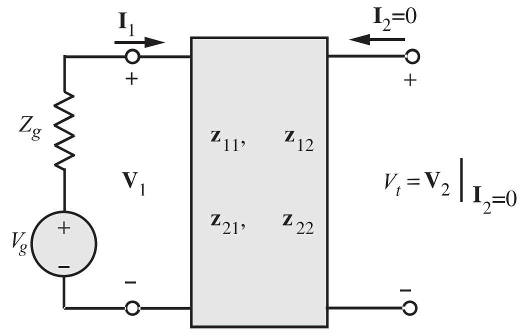

Figure 7.28(a)

$$\begin{aligned} V_t &= \mathbf{V}_2\big|_{\mathbf{I}_2=0} \\ &= \mathbf{z}_{21}\mathbf{I}_1 \end{aligned} \tag{7.23}$$

With $\mathbf{I}_2 = 0$, we get

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{z}_{11}} = \frac{V_g - \mathbf{I}_1 Z_g}{\mathbf{z}_{11}}$$

Solving for $\mathbf{I}_1$, we get

$$\mathbf{I}_1 = \frac{V_g}{\mathbf{z}_{11} + Z_g} \tag{7.24}$$

Substituting equation (7.24) into equation (7.23), we get

$$V_t = \frac{\mathbf{z}_{21}V_g}{\mathbf{z}_{11} + Z_g}$$

To find Z_t, let us deactivate all the independent sources and then connect a voltage source $\mathbf{V}_2$ across the output terminals as shown in Fig. 7.28(b).

$$Z_t = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_g=0}; \text{ where } \quad \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

We know that $\qquad \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$

Substituting, $\mathbf{V}_1 = -\mathbf{I}_1 Z_g$ in the preceeding equation, we get

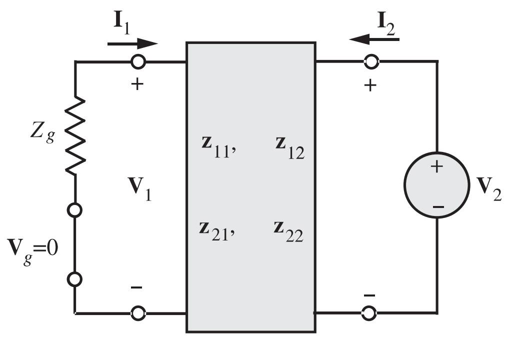

Figure 7.28(b)

$$-\mathbf{I}_1 Z_g = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\text{Solving,} \qquad \mathbf{I}_1 = \frac{-\mathbf{z}_{12}\mathbf{I}_2}{\mathbf{z}_{11} + Z_g}$$

We know that,

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2$$
$$= \mathbf{z}_{21}\left[\frac{-\mathbf{z}_{12}\mathbf{I}_2}{\mathbf{z}_{11} + Z_g}\right] + \mathbf{z}_{22}\mathbf{I}_2$$

Thus,
$$Z_t = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \mathbf{z}_{22} - \frac{\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11} + Z_g}$$

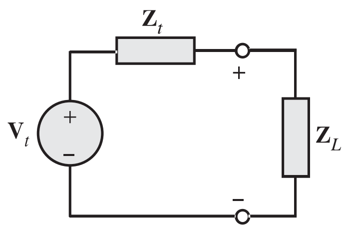

Figure 7.28(c)

The Thevenin equivalent circuit with respect to the output terminals along with load impedance Z_L is as shown in Fig. 7.28(c).

EXAMPLE 7.13

(a) Find the **z** parameters for the two-port network shown in Fig. 7.29.

(b) Find $\mathbf{V}_2(t)$ for $t > 0$ where $v_g(t) = 50u(t)\mathbf{V}$.

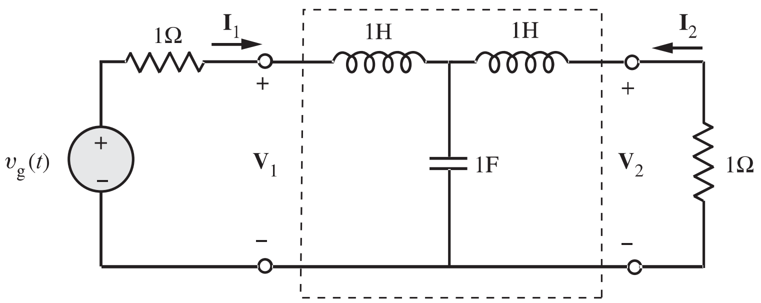

Figure 7.29

SOLUTION

The Laplace transformed network with all initial conditions set to zero is as shown in Fig. 7.30(a).

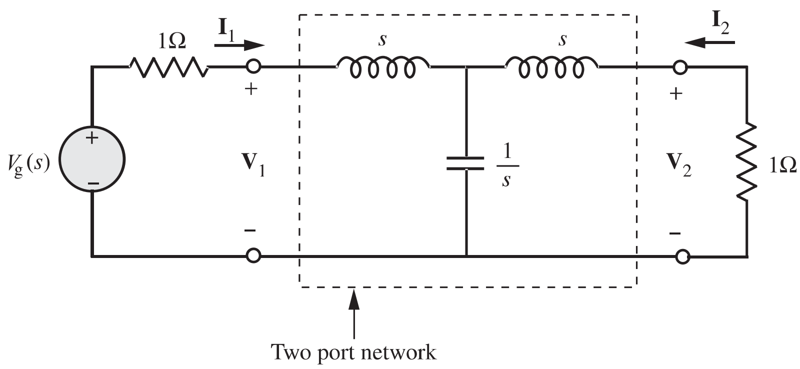

Figure 7.30(a)

(a) To find $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, open-circuit the output terminals and then connect a voltage source $\mathbf{V}_1$ across the input terminals as shown in Fig. 7.30(b).

Applying KVL to the left mesh, we get

$$\mathbf{V}_1 = \left(s + \frac{1}{s}\right)\mathbf{I}_1$$

Hence,
$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}$$
$$= s + \frac{1}{s} = \frac{s^2+1}{s}$$

Also,
$$\mathbf{V}_2 = \mathbf{I}_1\frac{1}{s}$$

Hence,
$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \frac{1}{s}$$

To find $\mathbf{z}_{21}$ and $\mathbf{z}_{22}$, open-circuit the input terminals and then connect a voltage source $\mathbf{V}_2$ across the output terminals as shown in Fig. 7.30(c).

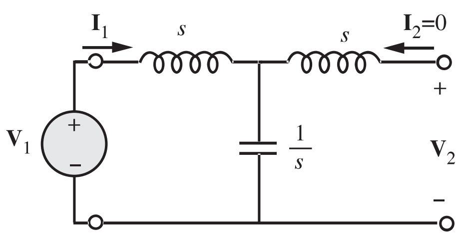

Figure 7.30(b)

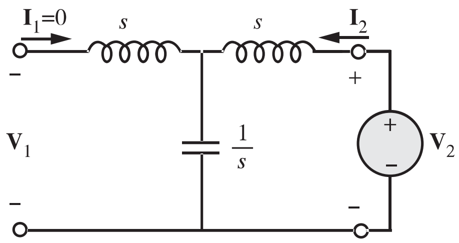

Figure 7.30(c)

Applying KVL to the right mesh, we get

$$\mathbf{V}_2 = \left[s + \frac{1}{s}\right]\mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{s^2+1}{s}$$

Also,
$$\mathbf{V}_1 = \frac{1}{s}\mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{1}{s}$$

Summarizing,

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+1}{s} & \dfrac{1}{s} \\ \dfrac{1}{s} & \dfrac{s^2+1}{s} \end{bmatrix}$$

(b)

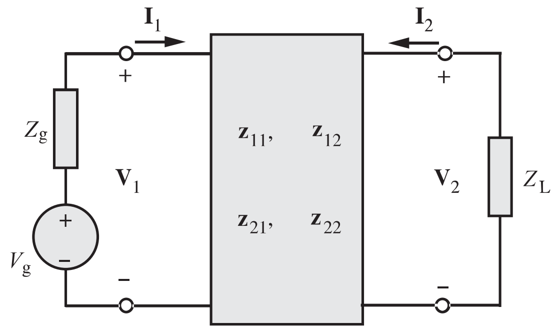

Figure 7.30(d)

Refer the two port network shown in Fig. 7.30(d).

$$\mathbf{V}_1 = V_g - \mathbf{I}_1 Z_g = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\Rightarrow \qquad V_g = (\mathbf{z}_{11} + Z_g)\,\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\Rightarrow \qquad V_g = (\mathbf{z}_{11} + Z_g)\,\mathbf{I}_1 + \mathbf{z}_{12}\left[\frac{-\mathbf{V}_2}{Z_L}\right] \tag{7.25}$$

and

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 - \mathbf{z}_{22}\frac{\mathbf{V}_2}{Z_L}$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{1}{\mathbf{z}_{21}}\left[1 + \frac{\mathbf{z}_{22}}{Z_L}\right]\mathbf{V}_2 \tag{7.26}$$

Substituting equation (7.26) in equation (7.25) and simplifying, we get

$$\frac{\mathbf{V}_2}{V_g} = \frac{\mathbf{z}_{21}\mathbf{z}_L}{(Z_L + \mathbf{z}_{22})\,(\mathbf{z}_{11} + Z_g) - \mathbf{z}_{12}\mathbf{z}_{21}} \tag{7.27}$$

Substituting for Z_L, Z_g and **z**-parameters, we get

$$\frac{\mathbf{V}_2(s)}{V_g(s)} = \frac{\frac{1}{s}}{\left(\frac{s^2+1}{s}+1\right)\left(\frac{s^2+1}{s}+1\right) - \frac{1}{s^2}}$$

$$= \frac{s}{\left(s^2+s+1\right)^2 - 1}$$

$$\Rightarrow \qquad \frac{\mathbf{V}_2(s)}{V_g(s)} = \frac{1}{s^3 + 2s^2 + 3s + 2}$$

$$= \frac{1}{(s+1)\,(s^2+s+2)}$$

Hence,

$$\mathbf{V}_2(s) = \frac{\mathbf{V}_g(s)}{(s+1)\,(s^2+s+2)} \tag{7.28}$$

The equation $s^2 + s + 2 = 0$ gives

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

This means that, $$\mathbf{V}_2(s) = \frac{V_g(s)}{(s+1)\left(s+\frac{1}{2}-j\frac{\sqrt{7}}{2}\right)\left(s+\frac{1}{2}+j\frac{\sqrt{7}}{2}\right)}$$

Given $$v_g(t) = 50u(t)$$

$$\Rightarrow \quad V_g(s) = \frac{50}{s}$$

Hence, $$\mathbf{V}_2(s) = \frac{50}{s(s+1)\left(s+\frac{1}{2}-j\frac{\sqrt{7}}{2}\right)\left(s+\frac{1}{2}+j\frac{\sqrt{7}}{2}\right)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

By performing partial fraction expansion, we get

$$K_1 = 25, \quad K_2 = -25, \quad K_3 = 9.45\,\underline{/90^\circ}$$

Hence, $$\mathbf{V}_2(s) = \frac{25}{s} - \frac{25}{s+1} + \frac{9.45\,\underline{/90^\circ}}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{9.45\,\underline{/-90^\circ}}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

Taking inverse Laplace transform of the above equation, we get

$$\mathbf{V}_2(t) = \left[25 - 25e^{-t} + 18.9e^{-0.5t}\cos(1.32t + 90^\circ)\right]u(t)\text{V}$$

Verification:

$$\mathbf{V}_2(0) = 25 - 25 + 18.9\cos 90 = 0$$
$$\mathbf{V}_2(\infty) = 25 + 0 + 0 = 25\text{V}$$

Please note that at $t = \infty$, the circuit diagram of Fig. (7.29) looks as shown in Fig. 7.30(e).

$$\mathbf{I}(\infty) = \frac{50}{2} = 25\mathbf{A}$$

Hence, $\mathbf{V}_2(\infty) = V_C(\infty) = 25\text{V}$

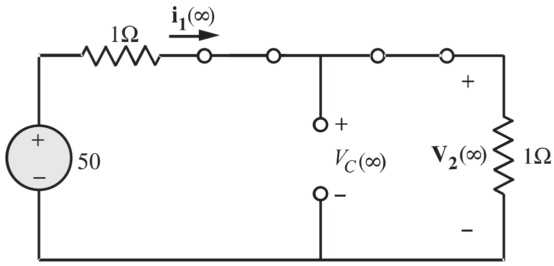

Figure 7.30(e)

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EXAMPLE 7.14

The following measurements were made on a resistive two-port network:

Measurement 1: With port 2 open and 100V applied to port 1, the port 1 current is 1.125A and port 2 voltage is 104V.

Measurement 2: With port 1 open and 50V applied to port 2, the port 2 current is 0.3A, and the port 1 voltage is 30 V.

Find the maximum power that can be delivered by this two-port network to a resistive load at port 2 when port 1 is driven by an ideal voltage source of 100 Vdc.

SOLUTION

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \frac{100}{1.125} = 88.89\Omega$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \frac{104}{1.125} = 92.44\Omega$$

$$\mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{30}{0.3} = 100\Omega$$

$$\mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \frac{50}{0.3} = 166.67\Omega$$

We know from the previous example 7.12 that,

$$\begin{aligned} Z_t &= \mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{11} + Z_g} \\ &= 166.67 - \frac{92.44 \times 100}{88.89 + 0} \\ &= 166.67 - 103.99 \\ &= 62.68\Omega \end{aligned}$$

For maximum power transfer, $Z_L = Z_t$

$$= 62.68\Omega \text{ (For resistive load)}$$

$$\begin{aligned} \mathbf{V}_t &= \frac{\mathbf{z}_{21}V_g}{\mathbf{z}_{11} + Z_g} \\ &= \frac{92.44 \times 100}{88.89 + 0} \\ &= 104\ \mathbf{V} \end{aligned}$$

The Thevenin equivalent circuit with respect to the output terminals with load resistance is as shown in Fig. 7.31.

$$P_{\max} = \mathbf{I}_t^2 R_L$$

$$= \left[\frac{104}{62.68 \times 2}\right]^2 \times 62.68$$

$$= \mathbf{43.14\ W}$$

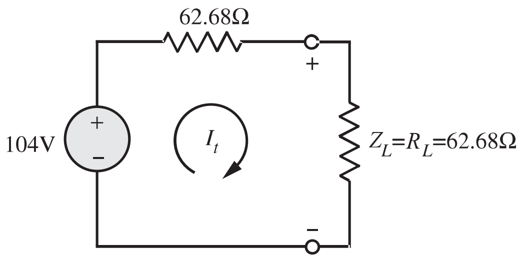


Figure 7.31

EXAMPLE 7.15

Refer the network shown in Fig. 7.32(a). Find the impedance parameters of the network.

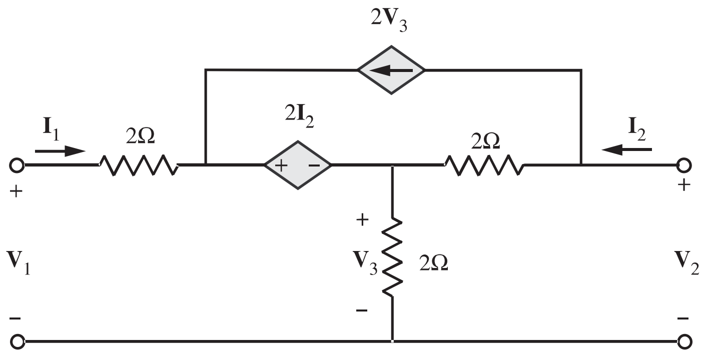


Figure 7.32(a)

SOLUTION

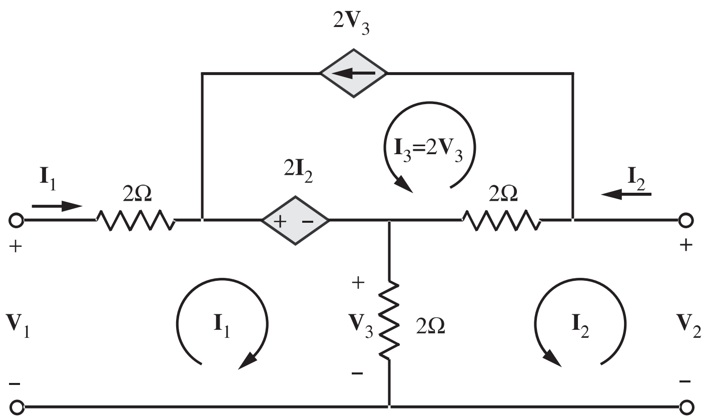


Figure 7.32(b)

Referring to Fig. 7.32(b), we can write

$$\mathbf{V}_3 = 2\left(\mathbf{I}_1 + \mathbf{I}_2\right)$$

KVL for mesh 1:

$$\begin{aligned} & 2\mathbf{I}_1 + 2\mathbf{I}_2 + 2\left(\mathbf{I}_1 + \mathbf{I}_2\right) = \mathbf{V}_1 \\ \Rightarrow \quad & 4\mathbf{I}_1 + 4\mathbf{I}_2 = \mathbf{V}_1 \end{aligned}$$

KVL for mesh 2:

$$\begin{aligned} & 2\left(\mathbf{I}_2 - 2\mathbf{V}_3\right) + 2\left(\mathbf{I}_1 + \mathbf{I}_2\right) = \mathbf{V}_2 \\ \Rightarrow \quad & 2\mathbf{I}_2 - 4 \times 2\left(\mathbf{I}_1 + \mathbf{I}_2\right) + 2\left(\mathbf{I}_1 + \mathbf{I}_2\right) = \mathbf{V}_2 \\ \Rightarrow \quad & 2\mathbf{I}_2 - 6\left(\mathbf{I}_1 + \mathbf{I}_2\right) = \mathbf{V}_2 \\ \Rightarrow \quad & -6\mathbf{I}_1 - 4\mathbf{I}_2 = \mathbf{V}_2 \end{aligned}$$

$$\begin{aligned} \mathbf{z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \left.\frac{4\mathbf{I}_1 + 4\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = 4\Omega \\ \mathbf{z}_{21} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \left.\frac{-6\mathbf{I}_1 - 4\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = -6\Omega \\ \mathbf{z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \left.\frac{4\mathbf{I}_1 + 4\mathbf{I}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = 4\Omega \\ \mathbf{z}_{22} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = \left.\frac{-6\mathbf{I}_1 - 4\mathbf{I}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} = -4\Omega \end{aligned}$$

EXAMPLE 7.16

Is it possible to find **z** parameters for any two port network ? Explain.

SOLUTION

It should be noted that for some two-port networks, the **z** parameters do not exist because they cannot be described by the equations:

$$\left.\begin{aligned} \mathbf{V}_1 &= \mathbf{I}_1\mathbf{z}_{11} + \mathbf{I}_2\mathbf{z}_{12} \\ \mathbf{V}_2 &= \mathbf{I}_1\mathbf{z}_{21} + \mathbf{I}_2\mathbf{z}_{22} \end{aligned}\right\} \qquad (7.29)$$

As an example, let us consider an ideal transformer as shown in Fig. 7.33.

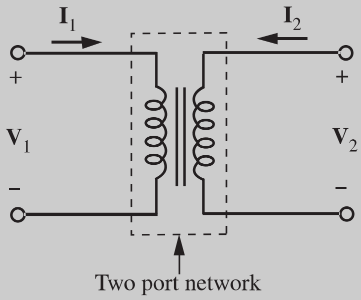


Figure 7.33

The defining equations for the two-port network shown in Fig. 7.33 are:

$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2 \quad \mathbf{I}_1 = -n\mathbf{I}_2$$

It is not possible to express the voltages in terms of the currents, and viceversa. Thus, the ideal transformer has no **z** parameters and no **y** parameters.

7.4 z and y parameters by matrix partitioning

For **z** parameters, the mesh equations are

$$\begin{aligned}
\mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 + \cdots + \mathbf{z}_{1n}\mathbf{I}_n \\
\mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 + \cdots + \mathbf{z}_{2n}\mathbf{I}_n \\
0 &= \cdots\cdots \\
0 &= \mathbf{z}_{n1}\mathbf{I}_1 + \mathbf{z}_{n2}\mathbf{I}_2 + \cdots + \mathbf{z}_{nn}\mathbf{I}_n
\end{aligned}$$

By matrix partitioning, the above equations can be written as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hline 0 \\ - \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cc} \mathbf{z}_{11} & \mathbf{z}_{12} & - & \mathbf{z}_{1n} \\ \mathbf{z}_{21} & \mathbf{z}_{22} & - & \mathbf{z}_{2n} \\ \hline \mathbf{z}_{31} & \mathbf{z}_{32} & - & \mathbf{z}_{3n} \\ - & - & - & - \\ \mathbf{z}_{n1} & \mathbf{z}_{n2} & - & \mathbf{z}_{nn} \end{array}\right] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \hline \mathbf{I}_3 \\ - \\ \mathbf{I}_n \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hline 0 \\ - \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{M} & \mathbf{N} \\ \hline \mathbf{P} & \mathbf{Q} \end{array}\right] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \hline \mathbf{I}_3 \\ - \\ \mathbf{I}_n \end{bmatrix}$$

The above equation can be simplified as (exact analysis not required)

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{M} - \mathbf{N}\ \mathbf{Q}^{-1}\ \mathbf{P} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$\mathbf{M} - \mathbf{N}\mathbf{Q}^{-1}\mathbf{P}$ gives **z** parameters.
Similarly for **y** parameters,

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ 0 \\ - \\ 0 \end{bmatrix} = \left[\begin{array}{cc:cc} \mathbf{y}_{11} & \mathbf{y}_{12} & - & \mathbf{y}_{1n} \\ \mathbf{y}_{21} & \mathbf{y}_{22} & - & \mathbf{y}_{2n} \\ \hdashline \mathbf{y}_{31} & \mathbf{y}_{32} & - & \mathbf{y}_{3n} \\ - & - & - & - \\ \mathbf{y}_{n1} & \mathbf{y}_{n2} & - & \mathbf{y}_{nn} \end{array}\right] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hdashline \mathbf{V}_3 \\ - \\ \mathbf{V}_n \end{bmatrix}$$

$$\left[\begin{array}{c} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \hdashline 0 \\ - \\ 0 \end{array}\right] = \begin{bmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{P} & \mathbf{Q} \end{bmatrix} \left[\begin{array}{c} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hdashline \mathbf{V}_3 \\ - \\ \mathbf{V}_n \end{array}\right]$$

$$\Rightarrow \qquad \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{M} - \mathbf{N}\ \mathbf{Q}^{-1}\ \mathbf{P} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$\mathbf{M} - \mathbf{N}\mathbf{Q}^{-1}\mathbf{P}$ gives **y** parameters.

EXAMPLE 7.17

Find **y** and **z** parameters for the resistive network shown in Fig. 7.34(a). Verify the result by using $\mathrm{Y} - \Delta$ transformation.

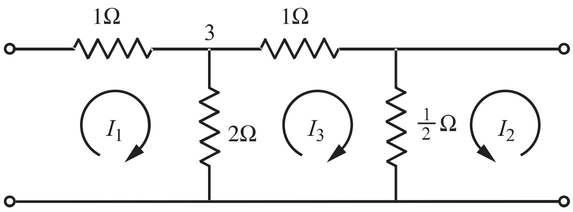


Figure 7.34(a)

SOLUTION

For the loops indicated, the equations in matrix form,

$$\left[\begin{array}{c} \mathbf{V}_2 \\ \mathbf{V}_2 \\ \hdashline 0 \end{array}\right] = \left[\begin{array}{cc:c} 3 & 0 & -2 \\ 0 & 0.5 & 0.5 \\ \hdashline -2 & 0.5 & 3.5 \end{array}\right] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix}$$

Then, $$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \left\{ \begin{bmatrix} 3 & 0 \\ 0 & 0.5 \end{bmatrix} - \frac{1}{3.5} \begin{bmatrix} -2 \\ 0.5 \end{bmatrix} \begin{bmatrix} -2 & 0.5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \right\}$$

$$\Rightarrow \qquad \begin{bmatrix} 1.8571 & 0.2857 \\ 0.2857 & 0.4285 \end{bmatrix} = [\mathbf{z}]$$

$$\mathbf{y} = \mathbf{z}^{-1} = \begin{bmatrix} 0.6 & -0.4 \\ -0.4 & 2.5 \end{bmatrix}$$

Verification

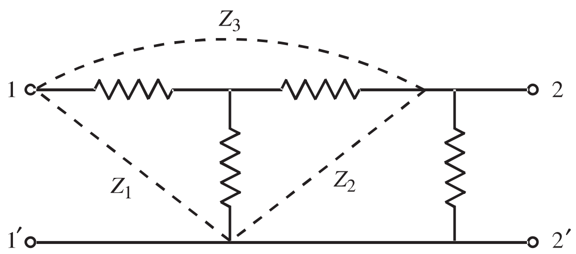


Figure 7.34(b)

Refer Fig 7.34(b), converting T of 1, 1', 2 into equation,

$$Z_1 = \frac{1 \times 1 + 1 \times 2 + 1 \times 2}{1} = 5$$

$$Z_2 = 5$$

$$Z_3 = \frac{5}{2}$$

Therefore, $$Z_2^{'} = \frac{5 \times \frac{1}{2}}{5.5} = \frac{5}{11}$$

$$\mathbf{y}_{11} = \frac{3}{5}; \quad \mathbf{y}_{12} = \mathbf{y}_{21} = -\frac{2}{5}; \quad \mathbf{y}_{22} = \frac{13}{5}$$

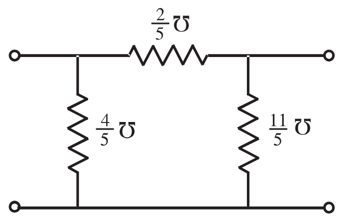


Figure 7.34(c)

The values with transformed circuit is shown in Fig 7.34(c).

EXAMPLE 7.18

Find **y** and **z** parameters for the network shown in Fig.7.35 which contains a current controlled source.

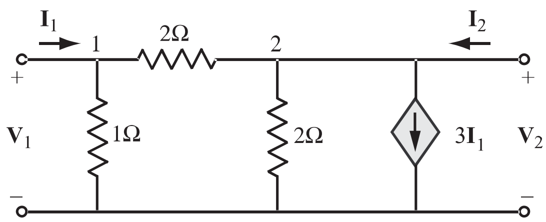


Figure 7.35

SOLUTION

At node 1,

$$1.5\mathbf{V}_1 - 0.5\mathbf{V}_2 = \mathbf{I}_1$$

At node 2,

$$-0.5\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{I}_2 - 3\mathbf{I}_1$$

In matrix form,

$$\begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\text{Therefore,} \qquad [\mathbf{z}] = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{z}]^{-1} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$$

7.5 Hybrid parameters

The **z** and **y** parameters of a two-port network do not always exist. Hence, we define a third set of parameters known as hybrid parameters. In the pair of equations that define these parameters, $\mathbf{V}_1$ and $\mathbf{I}_2$ are the dependent variables. Hence, the two-port equations in terms of the hybrid parameters are

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \tag{7.30}$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \tag{7.31}$$

or in matrix form,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

These parameters are particularly important in transistor circuit analysis. These parameters are obtained via the following equations:

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \qquad \mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

The parameters $\mathbf{h}_{11}, \mathbf{h}_{12}, \mathbf{h}_{21}$ and $\mathbf{h}_{22}$ represent the *short-circuit input impedance*, the *open-circuit reverse voltage gain,* the *short-circuit forward current gain,* and the open-circuit *output admittance* respectively. Because of this mix of parameters, they are called **hybrid parameters.**

EXAMPLE 7.19

Refer the network shown in Fig. 7.36(a). For this network, determine the **h** parameters.

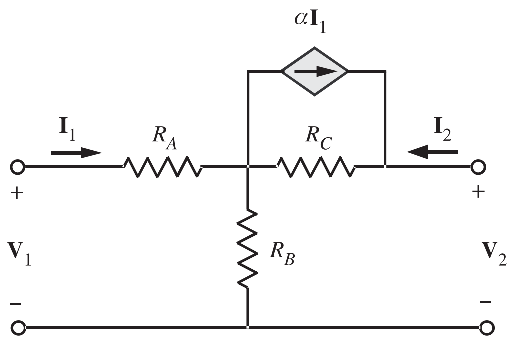


Figure 7.36(a)

SOLUTION

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ short-circuit the output terminals so that $\mathbf{V}_2 = 0$. Also connect a current source $\mathbf{I}_1$ to the input port as in Fig. 7.36(b).

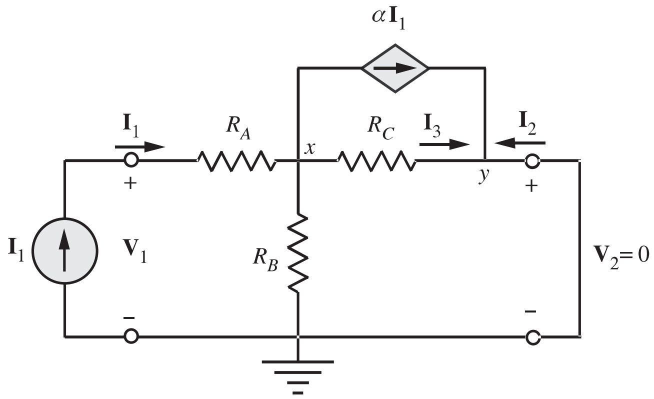


Figure 7.36(b)

Applying KCL at node x:

$$-\mathbf{I}_1 + \frac{V_x}{R_B} + \frac{V_x - 0}{R_C} + \alpha\mathbf{I}_1 = 0$$

$$\Rightarrow \qquad \mathbf{I}_1\left[\alpha - 1\right] = -V_x\left[\frac{1}{R_B} + \frac{1}{R_C}\right]$$

$$\Rightarrow \qquad V_x = \frac{(1-\alpha)\mathbf{I}_1 R_B R_C}{R_B + R_C}$$

Hence,

$$\begin{aligned}\mathbf{h}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \\ &= \left.\frac{V_x + \mathbf{I}_1 R_A}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \\ &= \frac{(1-\alpha)\mathbf{I}_1 R_B R_C}{(R_B + R_C)\,\mathbf{I}_1} + R_A \mathbf{I}_1 \\ &= \frac{(1-\alpha) R_B R_C}{R_B + R_C} + R_A\end{aligned}$$

KCL at node y:

$$\alpha\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 0$$

$$\Rightarrow \qquad \alpha\mathbf{I}_1 + \mathbf{I}_2 + \frac{\mathbf{V}_x - 0}{R_C} = 0$$

$$\Rightarrow \qquad \alpha\mathbf{I}_1 + \mathbf{I}_2 + \frac{1}{R_C}\left[\frac{(1-\alpha)\mathbf{I}_1 R_B R_C}{R_B + R_C}\right] = 0$$

Hence,

$$\begin{aligned}\mathbf{h}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \\ &= -\alpha - \frac{(1-\alpha) R_B}{R_B + R_C} \\ &= \frac{-(\alpha R_C + R_B)}{R_B + R_C}\end{aligned}$$

To find $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$ open-circuit the input port so that $\mathbf{I}_1 = 0$. Also, connect a voltage source $\mathbf{V}_2$ between the output terminals as shown in Fig. 7.36(c).

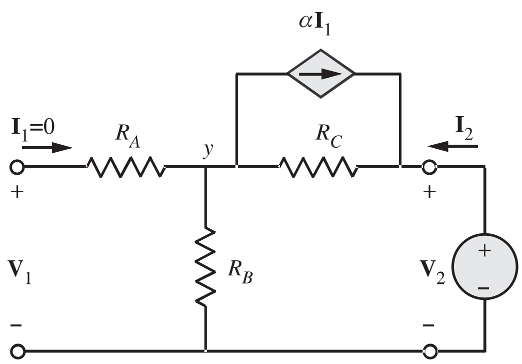


Figure 7.36(c)

KCL at node y:

$$\frac{\mathbf{V}_1}{R_B} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_C} + \alpha\mathbf{I}_1 = 0$$

Since $\mathbf{I}_1 = 0$, we get

$$\frac{\mathbf{V}_1}{R_B} + \frac{\mathbf{V}_1}{R_C} - \frac{\mathbf{V}_2}{R_C} = 0$$

$$\mathbf{V}_1 \left[\frac{1}{R_B} + \frac{1}{R_C}\right] = \frac{\mathbf{V}_2}{R_C}$$

$$\Rightarrow \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \frac{R_B}{R_B + R_C}$$

Applying KVL to the output mesh, we get

$$-\mathbf{V}_2 + R_C\,(\alpha\mathbf{I}_1 + \mathbf{I}_2) + R_B\mathbf{I}_2 = 0$$

Since $\mathbf{I}_1 = 0$, we get

$$R_C\mathbf{I}_2 + R_B\mathbf{I}_2 = \mathbf{V}_2$$

$$\Rightarrow \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \frac{1}{R_C + R_B}$$

EXAMPLE 7.20

Find the hybrid parameters for the two-port network shown in Fig. 7.37(a).

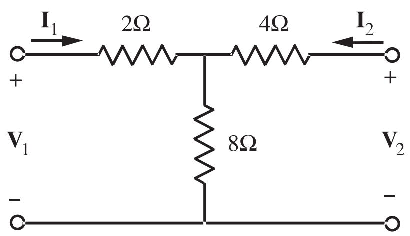


Figure 7.37(a)

SOLUTION

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as shown in Fig. 7.37(b).

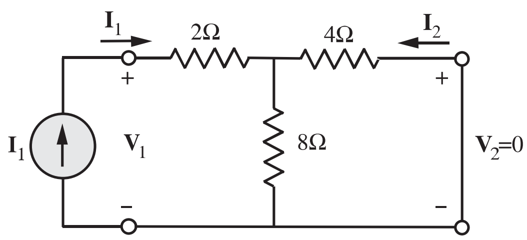


Figure 7.37(b)

Referring to Fig. 7.37(b), we find that

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{I}_1\,[2\Omega + (8\Omega||4\Omega)] \\ &= \mathbf{I}_1 \times 4.67\end{aligned}$$

Hence,
$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = 4.67\Omega$$

By using the principle of current division, we find that

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1 \times 8}{8+4} = \frac{2}{3}\mathbf{I}_1$$

Hence,
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \frac{-2}{3}$$

To obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, open-circuit the input port and connect a voltage source $\mathbf{V}_2$ to the output port as in Fig. 7.37(c).

Using the principle of voltage division,

$$\mathbf{V}_1 = \frac{8}{8+4}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

Hence,
$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

Also,
$$\begin{aligned}\mathbf{V}_2 &= (8+4)\mathbf{I}_2 \\ &= 12\mathbf{I}_2\end{aligned}$$

$$\Rightarrow \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \frac{1}{12}\text{S}$$

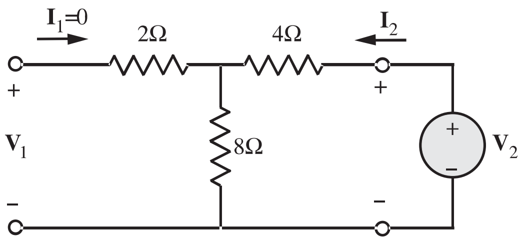


Figure 7.37(c)

EXAMPLE 7.21

Determine the **h** parameters of the circuit shown in Fig. 7.38(a).

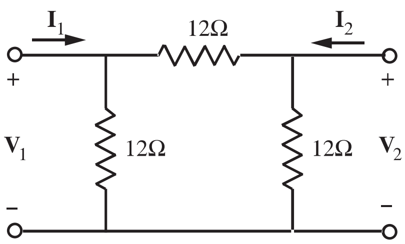


Figure 7.38(a)

SOLUTION

Performing Δ to Y transformation, the network shown in Fig. 7.38(a) takes the form as shown in Fig. 7.38(b). Please note that since all the resistors are of same value, $R_Y = \frac{1}{3}R_\Delta$.

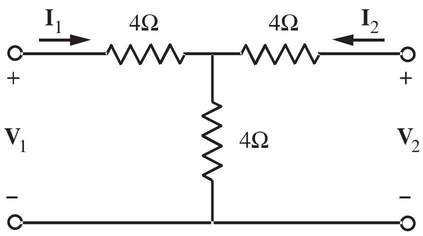


Figure 7.38(b)

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as in Fig. 7.38(c).

$$\mathbf{V}_1 = \mathbf{I}_1\,[4\Omega + (4\Omega||4\Omega)]$$
$$= 6\mathbf{I}_1$$

Hence, $$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = 6\Omega$$

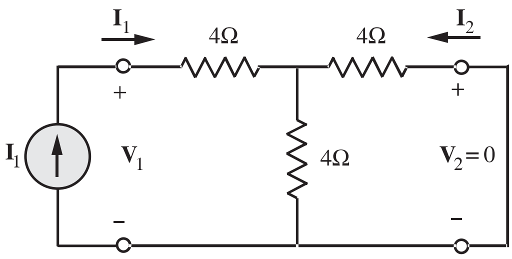


Figure 7.38(c)

Using the principle of current division,

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1}{4+4} \times 4$$

$\Rightarrow$ $$-\mathbf{I}_2 = \frac{\mathbf{I}_1}{2}$$

Hence, $$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \frac{-1}{2}$$

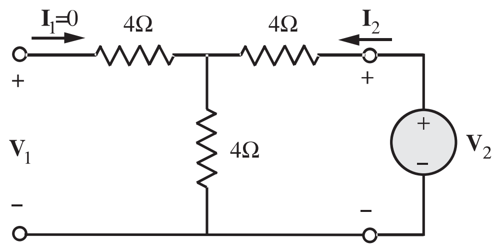


Figure 7.38(d)

To find $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, open-circuit the input port and connect a voltage source $\mathbf{V}_2$ to the output port as shown in Fig. 7.38(d).

Using the principle of voltage division, we get

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{4+4} \times 4$$

$\Rightarrow$ $$\mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \frac{1}{2}$$

Also, $$\mathbf{V}_2 = [4+4] \times \mathbf{I}_2 = 8\mathbf{I}_2$$

$\Rightarrow$ $$\mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \frac{1}{8}\ \text{S}$$

EXAMPLE 7.22

Determine the Thevenin equivalent circuit at the output of the circuit in Fig. 7.39(a).

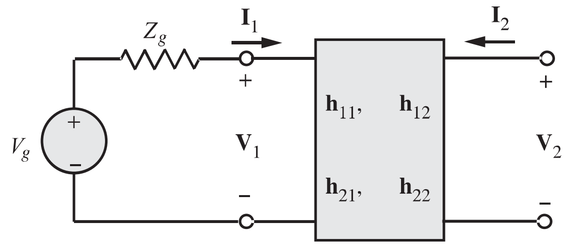


Figure 7.39(a)

SOLUTION

To find Z_t, deactivate the voltage source V_g and apply a 1 V voltage source at the output port, as shown in Fig. 7.39(b).

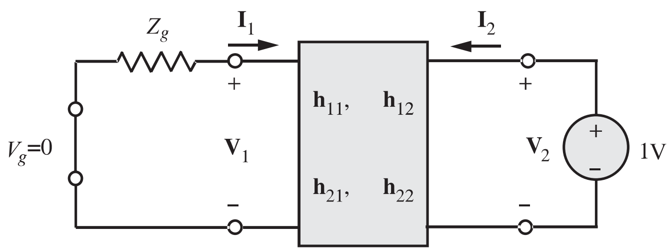


Figure 7.39(b)

The two-port circuit is described using **h** parameters by the following equations:

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \tag{7.32}$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \tag{7.33}$$

But $\mathbf{V}_2$ = 1 V and $\mathbf{V}_1 = -\mathbf{I}_1 Z_g$

Substituting these in equations (7.32) and (7.33), we get

$$-\mathbf{I}_1 Z_g = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}$$

$$\Rightarrow \quad \mathbf{I}_1 = \frac{-\mathbf{h}_{12}}{Z_g + \mathbf{h}_{11}} \tag{7.34}$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22} \tag{7.35}$$

Substituting equation (7.34) into equation (7.35), we get

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{h}_{22} - \frac{\mathbf{h}_{21}\mathbf{h}_{12}}{\mathbf{h}_{11}+Z_g}\\ &= \frac{\mathbf{h}_{11}\mathbf{h}_{22}-\mathbf{h}_{21}\mathbf{h}_{12}+\mathbf{h}_{22}Z_g}{\mathbf{h}_{11}+Z_g}\end{aligned}$$

Therefore,
$$Z_t = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \frac{\mathbf{h}_{11}+Z_g}{\mathbf{h}_{11}\mathbf{h}_{22}-\mathbf{h}_{12}\mathbf{h}_{21}+\mathbf{h}_{22}Z_g}$$

To get V_t, we find open circuit voltage $\mathbf{V}_2$ with $\mathbf{I}_2 = 0$. To find V_t, refer the Fig. 7.39(c).

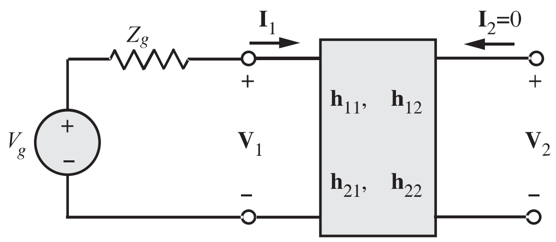


Figure 7.39(c)

At the input port, we can write

$$\begin{aligned}-V_g + \mathbf{I}_1 Z_g + \mathbf{V}_1 &= 0\\ \Rightarrow \qquad \mathbf{V}_1 &= V_g - \mathbf{I}_1 Z_g \qquad (7.36)\end{aligned}$$

Substituting equation (7.36) into equation (7.32), we get

$$\begin{aligned}V_g - \mathbf{I}_1 Z_g &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2\\ \Rightarrow \qquad V_g &= (\mathbf{h}_{11}+Z_g)\,\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \qquad (7.37)\end{aligned}$$

and substituting $\mathbf{I}_2 = 0$ in equation (7.33), we get

$$\begin{aligned}0 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2\\ \Rightarrow \qquad \mathbf{I}_1 &= \frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2 \qquad (7.38)\end{aligned}$$

Finally substituting equation (7.38) in (7.37), we get

$$V_g = (\mathbf{h}_{11}+Z_g)\left(\frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2\right) + \mathbf{h}_{12}\mathbf{V}_2$$

$$\Rightarrow \qquad \mathbf{V}_2 = V_t = \frac{V_g\mathbf{h}_{21}}{\mathbf{h}_{12}\mathbf{h}_{21}-\mathbf{h}_{11}\mathbf{h}_{22}-Z_g\mathbf{h}_{22}}$$

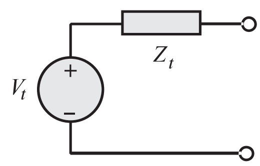


Figure 7.39(d)

Hence, the Thevenin equivalent circuit as seen from the output terminals is as shown in Fig. 7.39(d).

EXAMPLE 7.23

Find the input impedance of the network shown in Fig. 7.40.

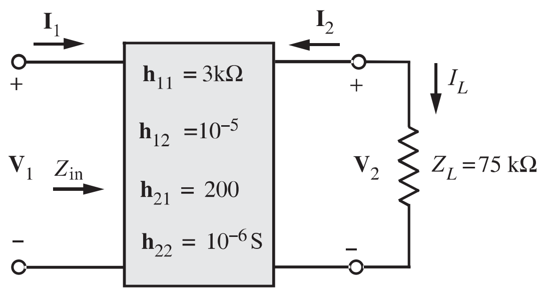


Figure 7.40

SOLUTION

For the two-port network, we can write

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \tag{7.39}$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \tag{7.40}$$

But $\mathbf{V}_2 = I_L Z_L = -\mathbf{I}_2 Z_L$ (7.41)

where $Z_L = 75\ \text{k}\Omega$

Substituting the value of $\mathbf{V}_2$ in equation (7.40), we get

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 - \mathbf{h}_{22}\mathbf{I}_2 Z_L$$

$$\Rightarrow \qquad \mathbf{I}_2 = \frac{\mathbf{h}_{21}\mathbf{I}_1}{1 + Z_L\mathbf{h}_{22}} \tag{7.42}$$

Substituting equation (7.42) in equation (7.41), we get

$$\mathbf{V}_2 = \frac{-Z_L\mathbf{h}_{21}\mathbf{I}_1}{1 + Z_L\mathbf{h}_{22}} \tag{7.43}$$

Substituting equation (7.43) in equation (7.39), we get

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 - \frac{\mathbf{h}_{12}Z_L\mathbf{h}_{21}\mathbf{I}_1}{1 + Z_L\mathbf{h}_{22}}$$

Hence

$$Z_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

$$= \mathbf{h}_{11} - \frac{Z_L\mathbf{h}_{12}\mathbf{h}_{21}}{1 + Z_L\mathbf{h}_{22}}$$

$$= 3 \times 10^3 - \frac{75 \times 10^3 \times 10^{-5} \times 200}{1 + 75 \times 10^3 \times 10^{-6}}$$

$$= \mathbf{2.86k\Omega}$$

EXAMPLE 7.24

Find the voltage gain, $\dfrac{\mathbf{V}_2}{V_g}$ for the network shown in Fig. 7.41.

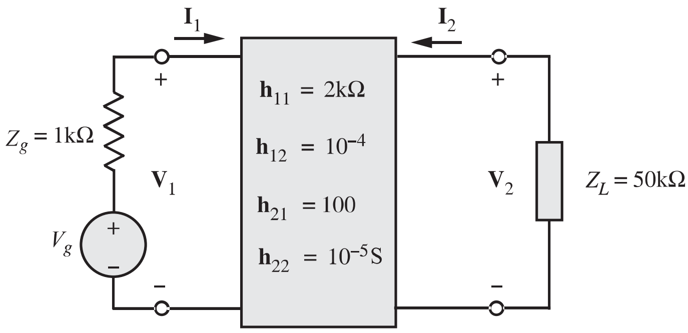


Figure 7.41

SOLUTION

For the two-port network we can write,

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2, \quad \text{here} \quad \mathbf{V}_1 = V_g - Z_g\mathbf{I}_1$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2, \quad \text{here} \quad \mathbf{V}_2 = -Z_L\mathbf{I}_2$$

Hence,
$$V_g - Z_g\mathrm{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

$$\Rightarrow \quad V_g = (\mathbf{h}_{11} + Z_g)\,\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

$$\Rightarrow \quad \mathbf{I}_1 = \frac{V_g - \mathbf{h}_{12}\mathbf{V}_2}{\mathbf{h}_{11} + Z_g}$$

Also,
$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{Z_L} = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\Rightarrow \quad \frac{-\mathbf{V}_2}{Z_L} = \mathbf{h}_{21}\left[\frac{V_g - \mathbf{h}_{12}\mathbf{V}_2}{\mathbf{h}_{11} + Z_g}\right] + \mathbf{h}_{22}\mathbf{V}_2$$

From the above equation, we find that

$$\begin{aligned}
\frac{\mathbf{V}_2}{V_g} &= \frac{-\mathbf{h}_{21}Z_L}{(\mathbf{h}_{11}Z_g)\,(1 + \mathbf{h}_{22}Z_L) - \mathbf{h}_{12}\mathbf{h}_{21}Z_L} \\
&= \frac{-100 \times 50 \times 10^3}{(2 \times 10^3 + 1 \times 10^3)\,(1 + 10^{-5} \times 50 \times 10^3) - (10^{-4} \times 100 \times 50 \times 10^3)} \\
&= \mathbf{-1250}
\end{aligned}$$

EXAMPLE 7.25

The following dc measurements were done on the resistive network shown in Fig. 7.42(a).

Measurement 1	Measurement 2
$\mathbf{V}_1 = 20$ V	$\mathbf{V}_1 = 35$ V
$\mathbf{I}_1 = 0.8$ A	$\mathbf{I}_1 = 1$ A
$\mathbf{V}_2 = 0$ V	$\mathbf{V}_2 = 15$ V
$\mathbf{I}_2 = -0.4$ A	$\mathbf{I}_2 = 0$ A

Find the value of R_o for maximum power transfer.

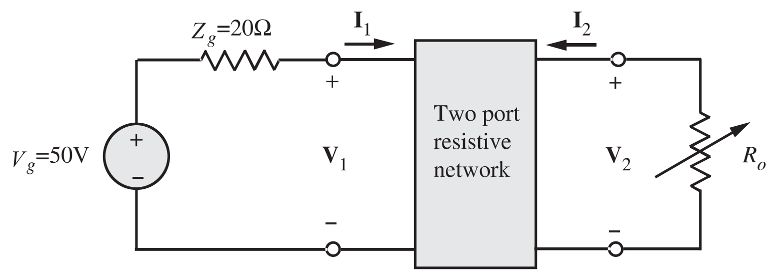


Figure 7.42(a)

SOLUTION

For the two-port network shown in Fig. 7.41, we can write:

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

From measurement 1:

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \frac{20}{0.8} = 25\Omega$$
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \frac{-0.4}{0.8} = -0.5$$

From measurement 2:

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\Rightarrow \quad 35 = 25 \times 1 + \mathbf{h}_{12} \times 15$$
$$\Rightarrow \quad \mathbf{h}_{12} = \frac{10}{15} = 0.67$$

Then,

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$
$$\Rightarrow \quad 0 = \mathbf{h}_{21} \times 1 + \mathbf{h}_{22} \times 15$$
$$\mathbf{h}_{22} = \frac{-\mathbf{h}_{21}}{15} = \frac{0.5}{15} = 0.033 \text{ S}$$

For example (7.22),

$$\mathbf{V}_t = \frac{V_g \mathbf{h}_{21}}{\mathbf{h}_{12}\mathbf{h}_{21} - \mathbf{h}_{11}\mathbf{h}_{22} - Z_g \mathbf{h}_{22}}$$

$$= \frac{50 \times (-0.5)}{0.67 \times (-0.5) - 25 \times 0.033 - 20 \times 0.033}$$

$$= \frac{-25}{-1.82} = 13.74 \text{ Volts}$$

$$Z_t = \frac{\mathbf{h}_{11} + Z_g}{\mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21} + \mathbf{h}_{22} Z_g}$$

$$= \frac{25 + 20}{25 \times 0.033 - 0.67 \times (-0.5) + 0.033 \times 20}$$

$$= \frac{45}{1.82} = 24.72\ \Omega$$

For maximum power transfer, $Z_L = Z_t = 24.72\ \Omega$ (Please note that, Z_L is purely resistive).

The Thevenin equivalent circuit as seen from the output terminals along with Z_L is shown in Fig. 7.42(b).

$$P_{\max} = I_t^2 \times 24.72$$

$$= \left[\frac{13.74}{24.72 + 24.72}\right]^2 \times 24.72$$

$$= \frac{(13.74)^2}{4 \times 24.72} = \mathbf{1.9\ Watts}$$

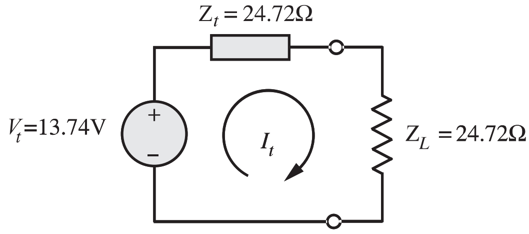


Figure 7.42(b)

EXAMPLE 7.26

Determine the hybird parameters for the network shown in Fig. 7.43.

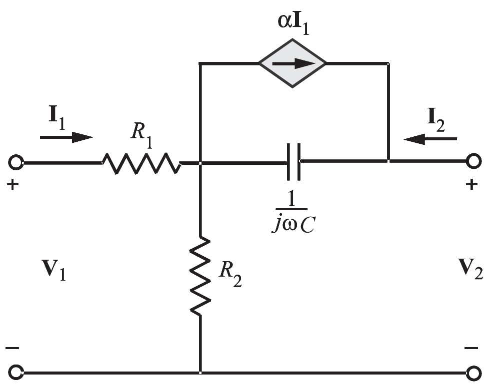


Figure 7.43

SOLUTION

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, short-circuit the output terminals so that $\mathbf{V}_2 = 0$. Also connect a current source $\mathbf{I}_1$ to the input port as shown in Fig. 7.44(a).

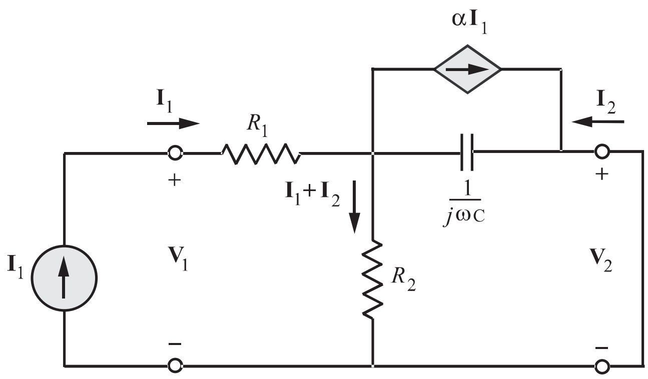


Figure 7.44(a)

Applying *KVL to the mesh on the right side*, we get

$$R_2\left[\mathbf{I}_1 + \mathbf{I}_2\right] + \frac{1}{j\omega C}\left[\alpha\mathbf{I}_1 + \mathbf{I}_2\right] = 0$$

$$\Rightarrow \qquad \left[R_2 + \frac{\alpha}{j\omega C}\right]\mathbf{I}_1 + \left[R_2 + \frac{1}{j\omega C}\right]\mathbf{I}_2 = 0$$

$$\Rightarrow \qquad \left[\alpha + j\omega R_2 C\right]\mathbf{I}_1 = -\left[1 + j\omega C R_2\right]\mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{I}_2 = \frac{-\left[\alpha + j\omega R_2 C\right]}{1 + j\omega R_2 C}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = -\left[\frac{\alpha + j\omega C R_2}{1 + j\omega R_2 C}\right]$$

Applying *KVL to the mesh on the left side*, we get

$$\begin{aligned}\mathbf{V}_1 &= R_1\mathbf{I}_1 + R_2\left[\mathbf{I}_1 + \mathbf{I}_2\right] \\ &= \left[R_1 + R_2\right]\mathbf{I}_1 + R_2\mathbf{I}_2 \\ &= \left[R_1 + R_2 - \frac{R_2\left(\alpha + j\omega C R_2\right)}{1 + j\omega R_2 C}\right]\mathbf{I}_1\end{aligned}$$

Hence,

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}$$

$$= R_1 + R_2 - \frac{R_2\,(\alpha + j\omega R_2 C)}{1 + j\omega R_2 C}$$
$$= \frac{R_1 + R_2\,(1-\alpha) + j\omega R_1 R_2 C}{1 + j\omega R_2 C}$$

To find $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, open-circuit the input terminals so that $\mathbf{I}_1 = 0$. Also connect a voltage source $\mathbf{V}_2$ to the output port as shown in Fig. 7.44(b). The dependent current source is open, because $\mathbf{I}_1 = 0$.

$$\mathbf{V}_1 = \mathbf{I}_2\mathbf{R}_2$$
$$= \frac{\mathbf{V}_2}{R_2 + \dfrac{1}{j\omega C}} R_2$$

Hence, $$\mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$
$$= \frac{j\omega C R_2}{1 + j\omega C R_2}$$
$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{R_2 + \dfrac{1}{j\omega C}} = \frac{j\omega C \mathbf{V}_2}{1 + j\omega C R_2}$$
$$\Rightarrow \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} = \frac{j\omega C}{1 + j\omega C R_2}$$

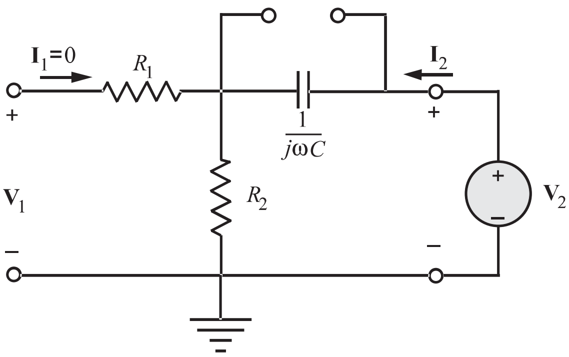


Figure 7.44(b)

7.6 Transmission parameters

The transmission parameters are defined by the equations:

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

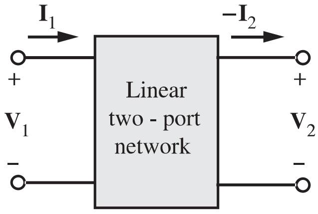


Figure 7.45 Terminal variables used to define the ABCD Parameters

Putting the above equations in matrix form we get

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

Please note that in computing the transmission parameters, $-\mathbf{I}_2$ is used rather than $\mathbf{I}_2$, because the current is considered to be leaving the network as shown in Fig. 7.45.

These parameters are very useful in the analysis of circuits in cascade like transmission lines and cables. For this reason they are called Transmission Parameters. They are also known as **ABCD** parameters. The parameters are determined via the following equations:

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \qquad \mathbf{B} = \left.\frac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \qquad \mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \qquad \mathbf{D} = \left.\frac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

A, **B**, **C** and **D** represent the *open-circuit voltage ratio*, the *negative short-circuit transfer impedance*, the *open-circuit transfer admittance*, and the *negative short-circuit current ratio*, respectively. When the two-port network does not contain dependent sources, the following relation holds good.

$$\mathbf{AD} - \mathbf{BC} = 1$$

EXAMPLE 7.27

Determine the transmission parameters in the s domain for the network shown in Fig. 7.46.

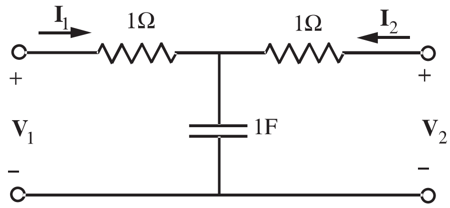


Figure 7.46

SOLUTION

The s domain equivalent circuit with the assumption that all the initial conditions are zero is shown in Fig. 7.47(a).

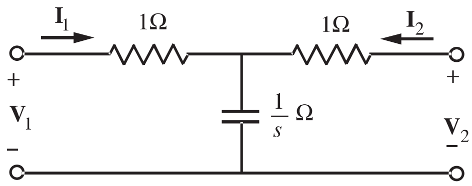
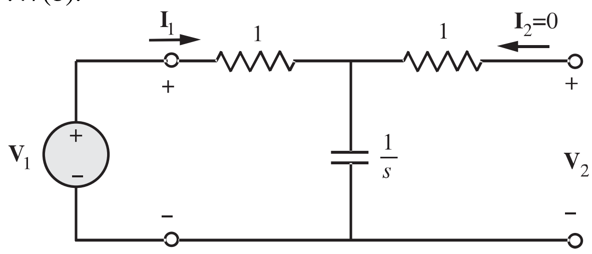


Figure 7.47(a)

To find the parameters **A** and **C**, open-circuit the output port and connect a voltage source $\mathbf{V}_1$ at the input port. The same is shown in Fig. 7.47(b).



$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1+\frac{1}{s}} = \frac{s\mathbf{V}_1}{s+1}$$

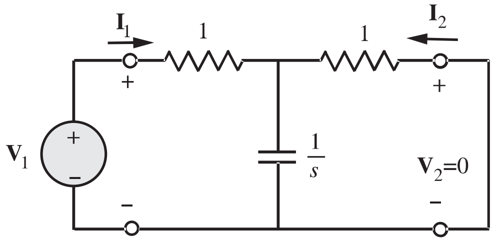
Then $$\mathbf{V}_2 = \frac{1}{s}\mathbf{I}_1$$

$$\Rightarrow \quad \mathbf{V}_2 = \frac{1}{s}\frac{s\mathbf{V}_1}{s+1} = \frac{\mathbf{V}_1}{s+1}$$

$$\Rightarrow \quad \mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = s+1$$

Also, $$\mathbf{V}_2 = \frac{1}{s}\mathbf{I}_1$$

$$\Rightarrow \quad \mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = s$$



To find the parameters **B** and **D**, short-circuit the output port and connect a voltage source $\mathbf{V}_1$ to the input port as shown in Fig. 7.47(c).

The total impedance as seen by the source $\mathbf{V}_1$ is

$$\mathbf{Z} = 1 + \frac{\frac{1}{s}\times 1}{\frac{1}{s}+1}$$

$$= 1 + \frac{1}{s+1} = \frac{s+2}{s+1}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}} = \frac{\mathbf{V}_1(s+1)}{(s+2)} \tag{7.44}$$

Using the principle of current division, we have

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1\left(\frac{1}{s}\right)}{\frac{1}{s}+1} = \frac{\mathbf{I}_1}{s+1} \tag{7.45}$$

Hence, $$\mathbf{D} = \left.\frac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = s+1$$

From equation (7.44) and (7.45), we can write

$$-\mathbf{I}_2(s+1) = \frac{\mathbf{V}_1(s+1)}{(s+2)}$$

Hence, $$\mathbf{B} = \left.\frac{-\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = s+2$$

Verification

We know that for a two port network without any dependent sources,

$$\mathbf{AD} - \mathbf{BC} = 1$$
$$(s+1)(s+1) - s(s+2) = 1$$

EXAMPLE 7.28

Determine the **ABCD** parameters for the two port network shown in Fig. 7.48.

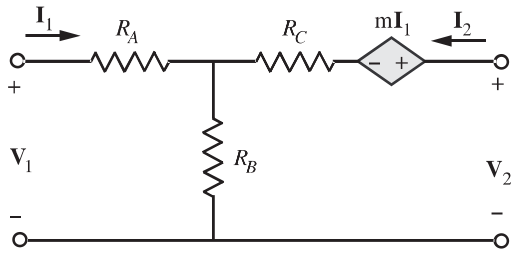


Figure 7.48

SOLUTION

To find the parameters **A** and **C**, open-circuit the output port as shown in Fig. 7.49(a) and connect a voltage source $\mathbf{V}_1$ to the input port.

Applying KVL to the output mesh, we get

$$-\mathbf{V}_2 + \mathrm{m}\mathbf{I}_1 + 0 \times R_C + \mathbf{I}_1 R_A = 0$$

$$\Rightarrow \quad \mathbf{V}_2 = \mathbf{I}_1 (\mathrm{m} + R_A)$$

Hence, $\mathbf{C} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \dfrac{1}{\mathrm{m} + R_A}$

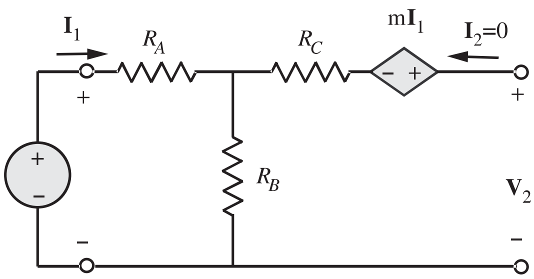


Figure 7.49(a)

Applying KVL to the input mesh, we get

$$\mathbf{V}_1 = \mathbf{I}_1 (R_A + R_B)$$

Hence, $$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \frac{R_A + R_B}{\mathrm{m} + R_A}$$

To find the parameters **B** and **D**, short-ciruit the output port and connect a voltage source $\mathbf{V}_1$ to the input port as shown in Fig. 7.49(b).

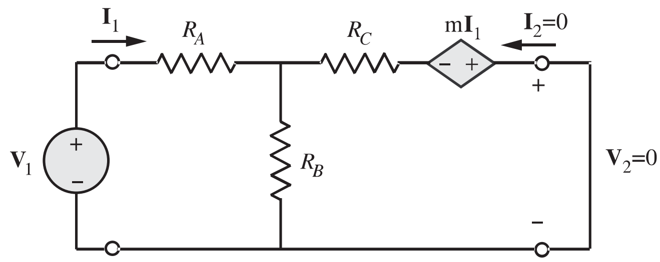


Figure 7.49(b)

Applying KVL to the right-mesh, we get

$$\mathrm{m}\mathbf{I}_1 + R_C\mathbf{I}_2 + R_B(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$\Rightarrow \quad (\mathrm{m} + R_B)\,\mathbf{I}_1 = -(R_C + R_B)\,\mathbf{I}_2$$

$$\Rightarrow \quad \mathbf{I}_1 = \frac{-(R_C + R_B)}{(\mathrm{m} + R_B)}\mathbf{I}_2$$

Hence,
$$\mathbf{D} = \left.\frac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \frac{(R_C + R_B)}{(\mathrm{m} + R_B)}$$

Applying KVL to the left-mesh, we get

$$-\mathbf{V}_1 + R_A\mathbf{I}_1 + R_B(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$\Rightarrow \quad \mathbf{V}_1 = (R_A + R_B)\,\mathbf{I}_1 + R_B\mathbf{I}_2$$

$$= (R_A + R_B)\left[\frac{-(R_C + R_B)}{(\mathrm{m} + R_B)}\mathbf{I}_2\right] + R_B\mathbf{I}_2$$

$$= -\left[\frac{R_CR_A + R_CR_B + R_BR_A - \mathrm{m}R_B}{\mathrm{m} + R_B}\right]\mathbf{I}_2$$

Hence,
$$\mathbf{B} = \left.\frac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$= \frac{R_CR_A + R_CR_B + R_BR_A - \mathrm{m}R_B}{\mathrm{m} + R_B}$$

EXAMPLE 7.29

The following direct-current measurements were done on a two port network:

Port 1 open	**Port 1 Short-circuited**
$\mathbf{V}_1 = 1$ mV	$\mathbf{I}_1 = -0.5\ \mu$A
$\mathbf{V}_2 = 10$ V	$\mathbf{I}_2 = 80\ \mu$A
$\mathbf{I}_2 = 200\ \mu$A	$\mathbf{V}_2 = 5$ V

Calculate the transmission parameters for the two port network.

SOLUTION

For the two port network, we can write

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

From $\mathbf{I}_1 = 0$ (port 1 open): $\quad 1 \times 10^{-3} = \mathbf{A} \times 10 - \mathbf{B} \times 200 \times 10^{-6}$

From $\mathbf{V}_1 = 0$ (Port 1 short): $\quad 0 = \mathbf{A} \times 5 - \mathbf{B} \times 80 \times 10^{-6}$

Solving simultaneously yields,

$$\mathbf{A} = -4 \times 10^{-4}, \mathbf{B} = -25\Omega$$

From $\mathbf{I}_1 = 0$: $\quad 0 = \mathbf{C} \times 10 - \mathbf{D} \times \left(200 \times 10^{-6}\right)$

From $\mathbf{V}_1 = 0$: $\quad -0.5 \times 10^{-6} = \mathbf{C} \times 5 - \mathbf{D} \times 80 \times 10^{-6}$

Solving simultaneously yields,

$$\mathbf{C} = -5 \times 10^{-7}\text{S}, \quad \mathbf{D} = -0.025$$

In summary,

$$\mathbf{A} = -4 \times 10^{-4}$$
$$\mathbf{B} = -25\Omega$$
$$\mathbf{C} = -5 \times 10^{-7}\ \text{S}$$
$$\mathbf{D} = -0.025$$

EXAMPLE 7.30

Find the transmission parameters for the network shown in Fig. 7.50.

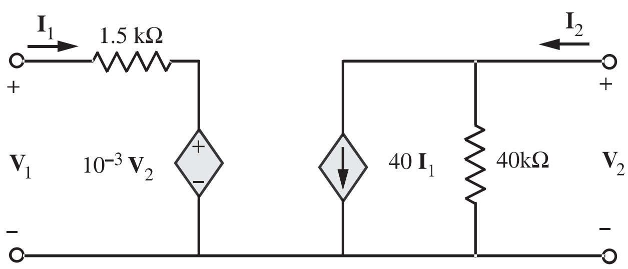


Figure 7.50

SOLUTION

To find the parameters **A** and **C**, open the output port and connect a voltage source $\mathbf{V}_1$ to the input port as shown in Fig. 7.51(a).

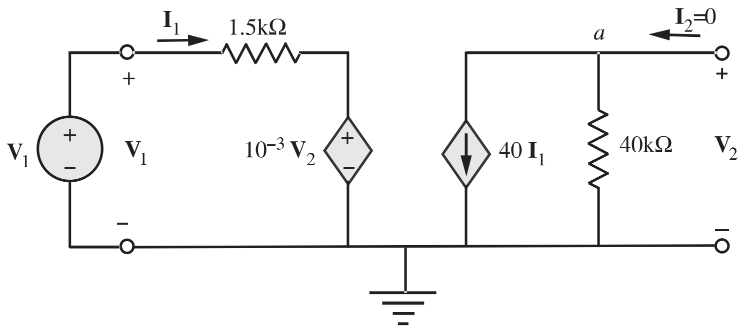


Figure 7.51(a)

Applying KVL to the input loop, we get

$$\mathbf{V}_1 = 1.5 \times 10^3 \mathbf{I}_1 + 10^{-3}\mathbf{V}_2$$

Also *KCL at node a* gives

$$40\mathbf{I}_1 + \frac{\mathbf{V}_2}{40 \times 10^3} = 0$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{-\mathbf{V}_2}{160 \times 10^3} = -6.25 \times 10^{-6}\mathbf{V}_2$$

Substituting the value of $\mathbf{I}_1$ in the preceeding loop equation, we get

$$\mathbf{V}_1 = 1.5 \times 10^3 \left(-6.25 \times 10^{-6}\mathbf{V}_2\right) + 10^{-3}\mathbf{V}_2$$

$$\Rightarrow \qquad \mathbf{V}_1 = -9.375 \times 10^{-3}\mathbf{V}_2 + 10^{-3}\mathbf{V}_2$$

$$= -8.375 \times 10^{-3}\mathbf{V}_2$$

Hence, $$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \mathbf{-8.375 \times 10^{-3}}$$

Also, $$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \mathbf{-6.25 \times 10^{-6}}$$

To find the parameters **B** and **D**, refer the circuit shown in Fig. 7.51(b).

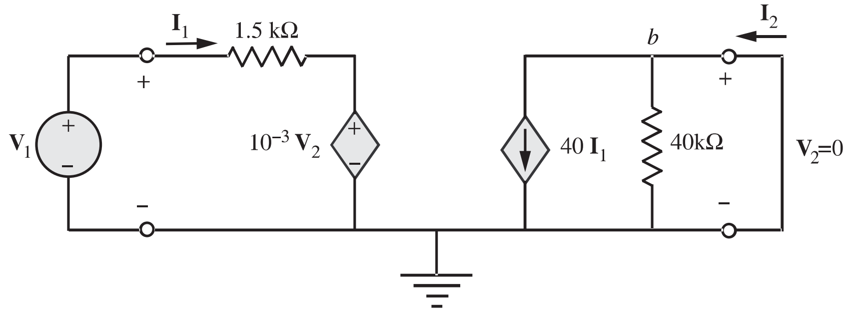


Figure 7.51(b)

Applying KCL at node b, we find

$$40\mathbf{I}_1 + 0 = \mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{I}_2 = 40\mathbf{I}_1$$

Hence, $$\mathbf{D} = \left.\frac{-\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \frac{\mathbf{-1}}{\mathbf{40}}$$

Applying KVL to the input loop, we get

$$\mathbf{V}_1 = 1.5 \times 10^3\mathbf{I}_1$$

$$\Rightarrow \qquad \mathbf{V}_1 = 1.5 \times 10^3 \times \frac{\mathbf{I}_2}{40}$$

Hence, $$\mathbf{B} = \left.\frac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$= \frac{-1.5}{40} \times 10^3$$

$$= \mathbf{-37.5\Omega}$$

EXAMPLE 7.31

Find the Thevenin equivalent circuit as seen from the output port using the transmission parameters for the network shown in Fig. 7.52.

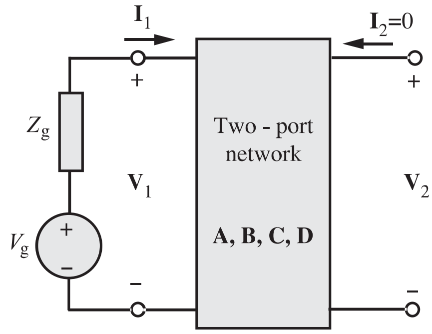


Figure 7.52

SOLUTION

For the two-port network, we can write

$$\mathbf{V}_1 = \mathbf{AV}_2 - \mathbf{BI}_2 \tag{7.46}$$

$$\mathbf{I}_1 = \mathbf{CV}_2 - \mathbf{DI}_2 \tag{7.47}$$

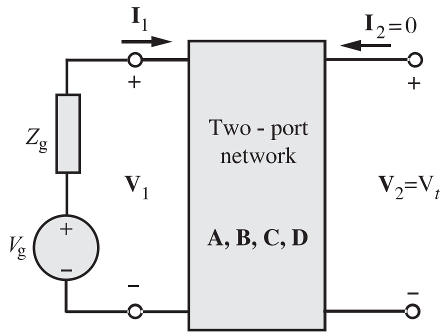


Figure 7.53(a)

Refer the network shown in Fig. 7.53(a) to find V_t.
At the input port,

$$V_g - \mathbf{I}_1 Z_g = \mathbf{V}_1 \tag{7.48}$$

Also,

$$\mathbf{I}_2 = 0 \tag{7.49}$$

Making use of equations (7.48) and (7.49) in equations (7.46) and (7.47) we get,

$$V_g - \mathbf{I}_1 Z_g = \mathbf{A}\mathbf{V}_2 \tag{7.50}$$

and

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 \tag{7.51}$$

Making use of equation (7.51) in (7.50), we get

$$V_g - \mathbf{C}\mathbf{V}_2 Z_g = \mathbf{A}\mathbf{V}_2$$

$$\Rightarrow \qquad \mathbf{V}_2 = V_t = \frac{V_g}{\mathrm{A} + \mathbf{C}Z_g}$$

To find R_t, deactivate the voltage source V_g and then connect a voltage source $\mathbf{V}_2 = 1$ V at the output port. The resulting circuit diagram is shown in Fig. 7.53(b).

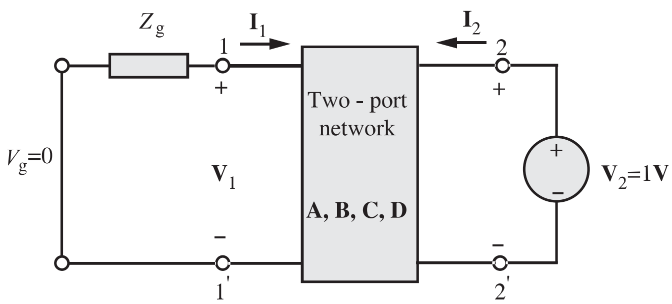


Figure 7.53(b)

Referring Fig. 7.53(b), we can write

$$\mathbf{V}_1 = -\mathbf{I}_1 Z_g$$

Substituting the value of $\mathbf{V}_1$ in equation (7.46), we get

$$-\mathbf{I}_1 Z_g = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\Rightarrow \qquad \mathbf{I}_1 = \frac{-\mathbf{A}}{Z_g}\mathbf{V}_2 + \frac{\mathbf{B}}{Z_g}\mathbf{I}_2 \tag{7.52}$$

Equating equations (7.47) and (7.52) results

$$\mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 = \frac{-\mathbf{A}}{Z_g}\mathbf{V}_2 + \frac{\mathbf{B}}{Z_g}\mathbf{I}_2$$

$$\mathbf{V}_2\left[\mathbf{C} + \frac{-\mathbf{A}}{Z_g}\right] = \left[\mathbf{D} + \frac{\mathbf{B}}{Z_g}\right]\mathbf{I}_2$$

Hence

$$Z_t = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{D} + \dfrac{\mathbf{B}}{Z_g}}{\mathbf{C} + \dfrac{\mathbf{A}}{Z_g}}$$

$$= \frac{\mathbf{B} + \mathbf{D}Z_g}{\mathbf{A} + \mathbf{C}Z_g}$$

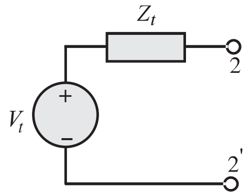


Figure 7.54

Hence, the Thevenin equivalent circuit as seen from the output port is as shown in Fig. 7.54.

EXAMPLE 7.32

For the network shown in Fig. 7.55(a), find R_L for maximum power transfer and the maximum power transferred.

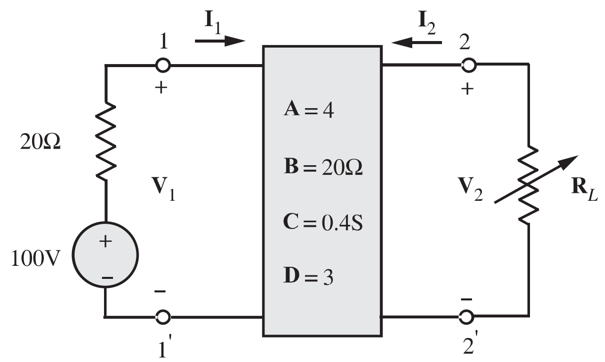


Figure 7.55(a)

SOLUTION

From the previous example 7.31,

$$Z_t = \frac{\mathbf{B}+\mathbf{D}Z_g}{\mathbf{A}+\mathbf{C}Z_g} = \frac{20+3\times 20}{4+0.4\times 20} = \frac{20+60}{4+8} = \mathbf{6.67\Omega}$$

$$V_t = \frac{V_g}{\mathbf{A}+\mathbf{C}Z_g} = \frac{100}{4+0.4\times 20} = \frac{100}{12} = \mathbf{8.33V}$$

For maximum power transfer,

$R_L = Z_t = 6.67\Omega$ (purely resistive)

Hence, the Thevenin equivalent circuit as seen from output terminals along with R_L is as shown in Fig. 7.55(b).

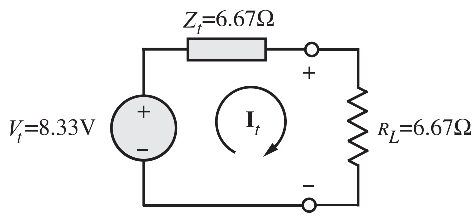


Figure 7.55(b)

$$I_t = \frac{8.33}{6.67+6.67} = 0.624\text{A}$$

$$\begin{aligned}(P_L)_{\max} &= I_t^2 \times 6.67\\ &= (0.624)^2 \times 6.67\\ &= \mathbf{2.6Watts}\end{aligned}$$

EXAMPLE 7.33

Refer the bridge circuit shown in Fig. 7.56. Find the transmission parameters.

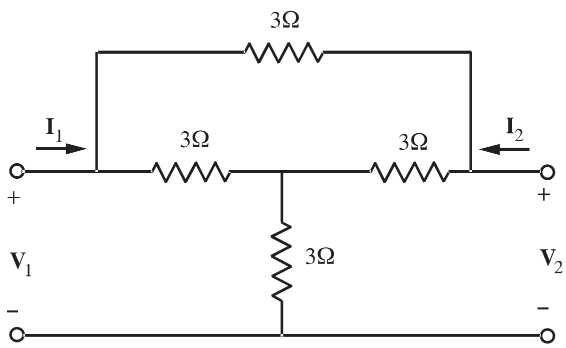


Figure 7.56

SOLUTION

Performing **Δ** to Y transformation, as shown in Fig. 7.57(a) the network reduces to the form as shown in Fig. 7.57(b). Please note that, when all resistors are of equal value,

$$R_Y = \frac{1}{3}R_\Delta$$

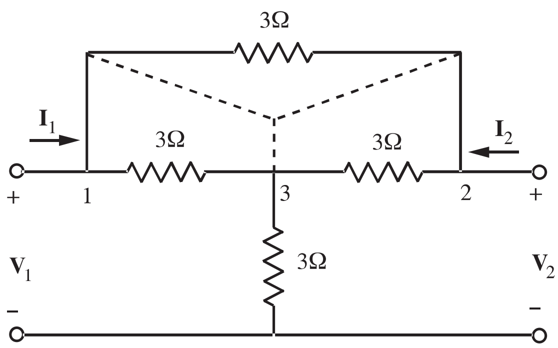


Figure 7.57(a)

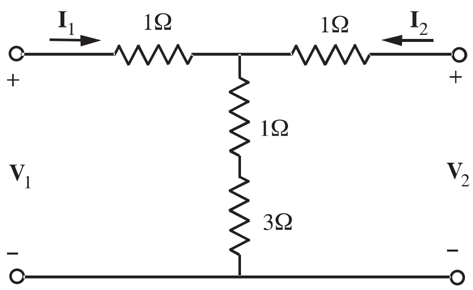


Figure 7.57(b)

To find the parameters **A** and **D**, open the output port and connect a voltage source $\mathbf{V}_1$ at the input port as shown is Fig. 7.57(c).

Applying KVL to the input loop we get

$\mathbf{I}_1 + 4\mathbf{I}_1 = \mathbf{V}_1$

$$\Rightarrow \quad \mathbf{V}_1 = 5\mathbf{I}_1$$

Also, $$\mathbf{I}_1 = \frac{\mathbf{V}_2}{4} \;\Rightarrow\; \mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \frac{1}{4}\mathrm{S}$$

Also, $$\Rightarrow \quad \mathbf{V}_1 = 5\mathbf{I}_1 = \frac{5}{4}\mathbf{V}_2$$

Hence, $$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \frac{5}{4}$$

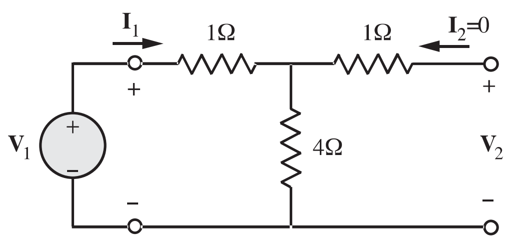


Figure 7.57(c)

To find the parameters **B** and **D**, refer the circuit shown in Fig. 7.57(d).

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1 \times 4}{4+1} = \frac{4}{5}\mathbf{I}_1$$

Hence $\quad \mathbf{D} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{v}_2=0} = \frac{5}{4}$

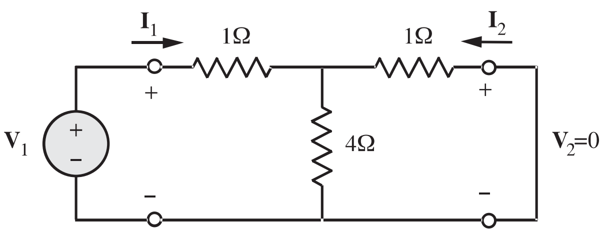


Figure 7.57(d)

Applying KVL to the input loop, we get

$$-\mathbf{V}_1 + 1 \times \mathbf{I}_1 + 4 \times (\mathbf{I}_1 + \mathbf{I}_2) = 0$$

Substituting $\mathbf{I}_1 = -\frac{5}{4}\mathbf{I}_2$ in the preceeding equation, we get

$$-\mathbf{V}_1 - \frac{5}{4}\mathbf{I}_2 + 4\left(-\frac{5}{4}\mathbf{I}_2 + \mathbf{I}_2\right) = 0$$

$$\Rightarrow \quad -\mathbf{V}_1 - \frac{5}{4}\mathbf{I}_2 - 5\mathbf{I}_2 + 4\mathbf{I}_2 = 0$$

$$\Rightarrow \quad 4\mathbf{V}_1 = -9\mathbf{I}_2$$

Hence, $\quad \mathbf{B} = \left.\frac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{v}_2=0} = \frac{9}{4}\Omega$

Verification:
For a two port network which does not contain any dependent sources, we have

$$\mathbf{AD} - \mathbf{BC} = 1$$

$$\frac{5}{4} \times \frac{5}{4} - \frac{1}{4} \times \frac{9}{4} = \frac{25}{16} - \frac{9}{16} = 1$$

7.7 Relations between two-port parameters

If all the two-port parameters for a network exist, it is possible to relate one set of parameters to another, since these parameters interrelate the variables $\mathbf{V}_1, \mathbf{I}_1, \mathbf{V}_2$ and $\mathbf{I}_2$. To begin with let us first derive the relation between the **z** parameters and **y** parameters.

The matrix equation for the **z** parameters is

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Rightarrow \quad \mathbf{V} = \mathbf{zI} \tag{7.53}$$

Similarly, the equation for **y** parameters is

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Rightarrow \quad \mathbf{I} = \mathbf{yV} \tag{7.54}$$

Substituting equation (7.54) into equation (7.53), we get

$$\mathbf{V} = \mathbf{zyV}$$

Hence,
$$\mathbf{z} = \mathbf{y}^{-1} = \frac{\mathrm{adj}(\mathbf{y})}{\mathbf{\Delta y}}$$

where
$$\mathbf{\Delta y} = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{21}\mathbf{y}_{12}$$

This means that we can obtain **z** matrix by inverting **y** matrix. It is quite possible that a two-port network has a **y** matrix or a **z** matrix, but not both.

Next let us proceed to find **z** parameters in terms of **ABCD** parameters.

The **ABCD** parameters of a two-port network are defined by

$$\mathbf{V}_1 = \mathbf{AV}_2 - \mathbf{BI}_2$$

$$\mathbf{I}_1 = \mathbf{CV}_2 - \mathbf{DI}_2$$

$$\Rightarrow \qquad \mathbf{V}_2 = \frac{1}{\mathbf{C}}\left(\mathbf{I}_1 + \mathbf{DI}_2\right)$$

$$\Rightarrow \qquad \mathbf{V}_2 = \frac{1}{\mathbf{C}}\mathbf{I}_1 + \frac{\mathbf{D}}{\mathbf{C}}\mathbf{I}_2 \tag{7.55}$$

$$\mathbf{V}_1 = \mathbf{A}\left(\frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{DI}_2}{\mathbf{C}}\right) - \mathbf{BI}_2$$

$$= \frac{\mathbf{AI}_1}{\mathbf{C}} + \left(\frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B}\right)\mathbf{I}_2 \tag{7.56}$$

Comparing equations (7.56) and (7.55) with

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

respectively, we find that

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} \qquad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} \qquad \mathbf{z}_{21} = \frac{1}{\mathbf{C}} \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Next, let us derive the relation between hybrid parameters and **z** parameters.

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \tag{7.57}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \tag{7.58}$$

From equation (7.58), we can write

$$\mathbf{I}_2 = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{\mathbf{V}_2}{\mathbf{z}_{22}} \tag{7.59a}$$

Substituting this value of $\mathbf{I}_2$ in equation (7.57), we get

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\left[\frac{-\mathbf{z}_{21}\mathbf{I}_1}{\mathbf{z}_{22}} + \frac{\mathbf{V}_2}{\mathbf{z}_{22}}\right]$$

$$= \left[\frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22}}\right]\mathbf{I}_1 + \frac{\mathbf{z}_{12}\mathbf{V}_2}{\mathbf{z}_{22}} \tag{7.59b}$$

Comparing equations (7.59b) and (7.59a) with

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

we get,

$$\mathbf{h}_{11} = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22}} = \frac{\Delta\mathbf{z}}{\mathbf{Z}_{22}}$$

$$\mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \qquad \mathbf{h}_{21} = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}}$$

where

$$\Delta\mathbf{z} = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}$$

Finally, let us derive the relationship between **y** parameters and **ABCD** parameters.

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \tag{7.60}$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \tag{7.61}$$

From equation (7.61), we can write

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2$$

$$= \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}}\mathbf{I}_2 \tag{7.62}$$

Substituting equation (7.62) in equation (7.60), we get

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11}\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \mathbf{y}_{12}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2$$

$$= \frac{-\Delta\mathbf{y}}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2 \tag{7.63}$$

Comparing equations (7.62) and (7.63) with the following equations,

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

we get

$$\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}} \qquad \mathbf{B} = \frac{-1}{\mathbf{y}_{21}} \qquad \mathbf{C} = \frac{-\Delta\mathbf{y}}{\mathbf{y}_{21}} \quad \mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}$$

where

$$\Delta\mathbf{y} = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$$

Table 7.1 lists all the conversion formulae that relate one set of two-port parameters to another. Please note that $\Delta\mathbf{z}$, $\Delta\mathbf{y}$, $\Delta\mathbf{h}$, and $\Delta\mathbf{T}$, refer to the determinants of the matrices for **z**, **y**, **hybrid**, and **ABCD** parameters respectively.

Table 7.1 Parameter relationships

	$\mathbf{z}$	$\mathbf{y}$	$\mathbf{T}$	$\mathbf{h}$
$\mathbf{z}$	$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{y}_{22}}{\Delta\mathbf{y}} & \frac{-\mathbf{y}_{12}}{\Delta\mathbf{y}} \\ \frac{-\mathbf{y}_{21}}{\Delta\mathbf{y}} & \frac{\mathbf{y}_{11}}{\Delta\mathbf{y}} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta\mathbf{T}}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta\mathbf{h}}{\mathbf{h}_{22}} & \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \frac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \frac{1}{\mathbf{h}_{22}} \end{bmatrix}$
$\mathbf{y}$	$\begin{bmatrix} \frac{\mathbf{z}_{22}}{\Delta\mathbf{z}} & \frac{-\mathbf{z}_{12}}{\Delta\mathbf{z}} \\ \frac{-\mathbf{z}_{21}}{\Delta\mathbf{z}} & \frac{\mathbf{z}_{11}}{\Delta\mathbf{z}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{D}}{\mathbf{B}} & \frac{-\Delta\mathbf{T}}{\mathbf{B}} \\ \frac{-1}{\mathbf{B}} & \frac{\mathbf{A}}{\mathbf{B}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\mathbf{h}_{11}} & \frac{-\mathbf{h}_{22}}{\mathbf{h}_{11}} \\ \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \frac{\Delta\mathbf{h}}{\mathbf{h}_{11}} \end{bmatrix}$
$\mathbf{T}$	$\begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta\mathbf{z}}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}} & \frac{-1}{\mathbf{y}_{21}} \\ \frac{-\Delta\mathbf{y}}{\mathbf{y}_{21}} & \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta\mathbf{h}}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \frac{-1}{\mathbf{h}_{21}} \end{bmatrix}$
$\mathbf{h}$	$\begin{bmatrix} \frac{\Delta\mathbf{z}}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{\mathbf{y}_{11}} & \frac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \frac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \frac{\Delta\mathbf{y}}{\mathbf{y}_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\mathbf{B}}{\mathbf{D}} & \frac{\Delta\mathbf{T}}{\mathbf{D}} \\ -\frac{1}{\mathbf{D}} & \frac{\mathbf{C}}{\mathbf{D}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix}$

$$\Delta\mathbf{z} = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}, \Delta\mathbf{y} = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}, \Delta\mathbf{h} = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21}, \Delta\mathbf{T} = \mathbf{AD} - \mathbf{BC}$$

EXAMPLE 7.34

Determine the **y** parameters for a two-port network if the **z** parameters are

$$\mathbf{z} = \begin{bmatrix} 10 & 5 \\ 5 & 9 \end{bmatrix}$$

SOLUTION

$$\Delta\mathbf{z} = 10 \times 9 - 5 \times 5 = 65$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta\mathbf{z}} = \frac{9}{65}\mathrm{S}$$

$$\mathbf{y}_{12} = \frac{-\mathbf{z}_{12}}{\Delta\mathbf{z}} = \frac{-5}{65}\mathrm{S}$$

$$\mathbf{y}_{21} = \frac{-\mathbf{z}_{21}}{\Delta\mathbf{z}} = \frac{-5}{65}\mathrm{S}$$

$$\mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta\mathbf{z}} = \frac{10}{65}\mathrm{S}$$

EXAMPLE 7.35

Following are the hybrid parameters for a network:

$$\begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

Determine the **y** parameters for the network.

SOLUTION

$$\Delta\mathbf{h} = 5 \times 6 - 3 \times 2 = 24$$

$$\mathbf{y}_{11} = \frac{1}{\mathbf{h}_{11}} = \frac{1}{5}\text{S}$$

$$\mathbf{y}_{12} = \frac{-\mathbf{h}_{22}}{\mathbf{h}_{11}} = \frac{-6}{5}\text{S}$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{3}{5}\text{S}$$

$$\mathbf{y}_{22} = \frac{\Delta\mathbf{h}}{\mathbf{h}_{11}} = \frac{24}{5}\text{S}$$

Reinforcement problems

R.P 7.1

The network of Fig. R.P. 7.1 contains both a dependent current source and a dependent voltage source. Determine **y** and **z** parameters.

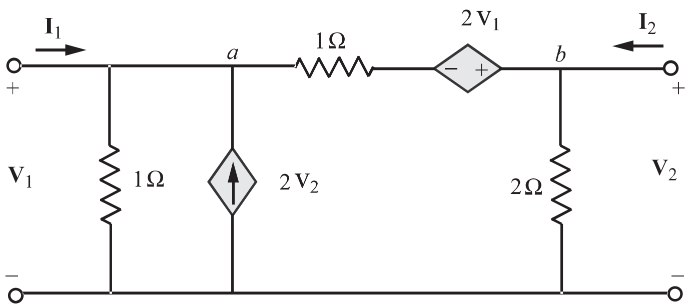


Figure R.P. 7.1

SOLUTION

From the figure, the node equations are

$$\mathbf{I}_{ab} = -\left(\mathbf{I}_2 - \frac{\mathbf{V}_2}{2}\right)$$

At node a,

$$\mathbf{I}_1 = \mathbf{V}_1 - 2\mathbf{V}_2 - \left(\mathbf{I}_2 - \frac{\mathbf{V}_2}{2}\right)$$

At node b,

$$\mathbf{V}_1 = \mathbf{V}_2 - 2\mathbf{V}_1 - \left(\mathbf{I}_2 - \frac{\mathbf{V}_2}{2}\right)$$

Simplifying, the nodal equations, we get

$$\mathbf{I}_1 + \mathbf{I}_2 = \mathbf{V}_1 - \frac{3}{2}\mathbf{V}_2$$

$$\mathbf{I}_2 = -3\mathbf{V}_1 + \frac{3}{2}\mathbf{V}_2$$

In matrix form,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{3}{2} \\ -3 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -\frac{3}{2} \\ -3 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Therefore,

$$\mathbf{y} = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix}$$

and

$$\mathbf{Z} = -\frac{1}{3} \begin{bmatrix} \frac{3}{2} & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ 1 & -\frac{4}{3} \end{bmatrix}$$

R.P 7.2

Compute **y** parameters for the network shown in Fig. R.P. 7.2.

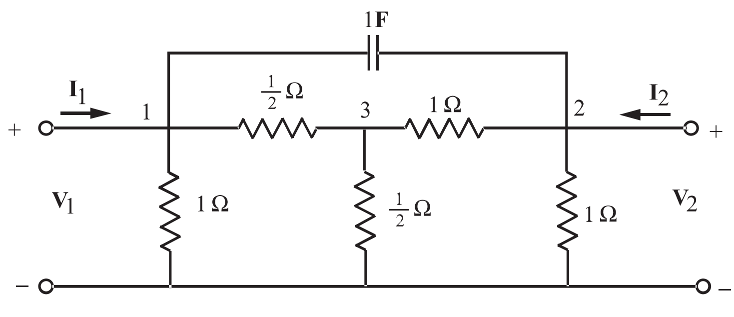


Figure R.P. 7.2

SOLUTION

The circuit shall be transformed into s-domain and then we shall use the matrix partitioning method to solve the problem. From Fig 7.2, Node equations in matrix form,

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cc|c} s+3 & -s & -2 \\ -s & s+2 & -1 \\ \hline -2 & -1 & 5 \end{array}\right] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hline \mathbf{V}_3 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{P} & \mathbf{Q} \\ - & - \\ \hline \mathbf{M} & \mathbf{N} \end{array}\right] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hline \mathbf{V}_3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{P} - \mathbf{Q}\ \mathbf{N}^{-1}\ \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} s+3 & -s \\ -s & s+2 \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} \right\} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} s+3 & -s \\ -s & s+2 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} s+2.2 & -(s+0.4) \\ -(s+0.4) & s+1.8 \end{bmatrix}$$

R.P 7.3

Determine for the circuit shown in Fig. R.P. 7.3(a): (a) Y_1, Y_2, Y_3 and g_m in terms of **y** parameters. (b) Repeat the problem if the current source is connected across Y_3 with the arrow pointing to the left.

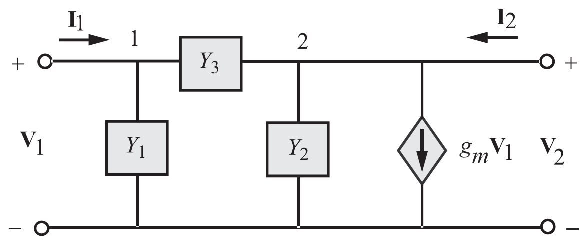


Figure R.P. 7.3(a)

SOLUTION

(a) Refering Fig. R.P. 7.3(a), the node equations are:

At node 1

$$\begin{aligned} \mathbf{I}_1 &= Y_1\mathbf{V}_1 + (\mathbf{V}_1 - \mathbf{V}_2)Y_3 \\ &= \mathbf{V}_1(Y_1 + Y_3) - Y_3Y_2 \end{aligned} \tag{7.64}$$

At node 2

$$\begin{aligned} \mathbf{I}_2 &= g_m\mathbf{V}_1 + \mathbf{V}_2Y_2 + (\mathbf{V}_2 - \mathbf{V}_1)Y_3 \\ &= (g_m - Y_3)\mathbf{V}_1 + (Y_2 + Y_3)\mathbf{V}_2 \end{aligned} \tag{7.65}$$

Then from equations (7.64) and (7.65),

$$\mathbf{y}_{11} = Y_1 + Y_3; \quad \mathbf{y}_{12} = -Y_3$$
$$\mathbf{y}_{21} = g_m - Y_3; \quad \mathbf{y}_{22} = Y_2 + Y_3$$

Solving,

$$Y_3 = -\mathbf{y}_{12}; \quad Y_1 = \mathbf{y}_{11} + \mathbf{y}_{12}$$
$$Y_2 = \mathbf{y}_{22} + \mathbf{y}_{12}; \quad g_m = \mathbf{y}_{21} - \mathbf{y}_{12}$$

(b) Making the changes as given in the problem, we get the circuit shown in Fig R.P. 7.3(b).

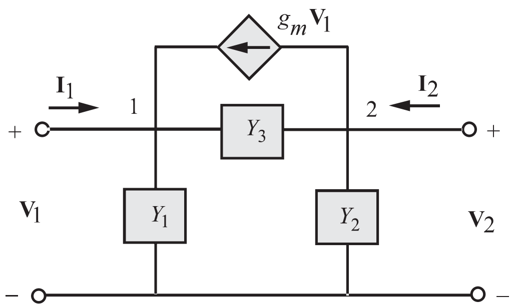


Figure R.P. 7.3(b)

Node equations : At node 1

$$\begin{aligned}\mathbf{I}_1 &= Y_1\mathbf{V}_1 + (\mathbf{V}_1 - \mathbf{V}_2)Y_3 - g_m\mathbf{V}_1 \\ &= (Y_1 + Y_3 - g_m)\mathbf{V}_1 - Y_3\mathbf{V}_2 \qquad (7.66)\end{aligned}$$

At node 2,

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{V}_2Y_2 + (\mathbf{V}_2 - \mathbf{V}_1)Y_3 + g_m\mathbf{V}_1 \\ &= (g_m - Y_3)\mathbf{V}_1 + \mathbf{V}_2(Y_2 + Y_3) \qquad (7.67)\end{aligned}$$

From equations (7.66) and (7.67),

$$\mathbf{y}_{11} = Y_1 + Y_3 - g_m; \quad \mathbf{y}_{12} = -Y_3$$
$$\mathbf{y}_{21} = g_m - Y_3; \quad \mathbf{y}_{22} = Y_2 + Y_3$$

Solving,

$$Y_3 = -\mathbf{y}_{12}; \quad Y_2 = \mathbf{y}_{22} - \mathbf{y}_{12}$$
$$g_m = \mathbf{y}_{21} - \mathbf{y}_{12}$$
$$\begin{aligned}Y_1 &= \mathbf{y}_{11} - Y_3 + g_m \\ &= \mathbf{y}_{11} + \mathbf{y}_{12} + \mathbf{y}_{21} - \mathbf{y}_{12} = \mathbf{y}_{11} - \mathbf{y}_{21}\end{aligned}$$

R.P 7.4

Complete the table given as part of Fig. R.P. 7.4. Also find the values for **y** parameters.

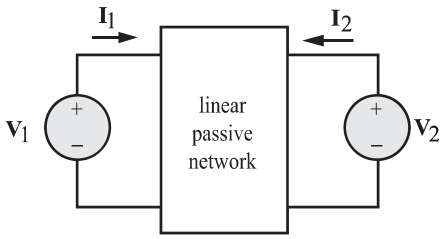


Figure R.P. 7.4

Table

Sl.no	V_1	V_2	I_1	I_2
1	50	100	-1	27
2	100	50	7	24
3	200	0	–	–
4	–	–	20	0
5	–	–	10	30

SOLUTION

From article 7.2,

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Substituting the values from rows 1 and 2,

$$\begin{bmatrix} -1 & 7 \\ 27 & 24 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} 50 & 100 \\ 100 & 50 \end{bmatrix}$$

Post multiplying by $[\mathbf{V}]^{-1}$,

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 27 & 24 \end{bmatrix} \begin{bmatrix} 50 & 100 \\ 100 & 50 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix}$$

For row 3:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \end{bmatrix} = \begin{bmatrix} 20 \\ 28 \end{bmatrix}$$

For row 4:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 140.84 \\ -98.59 \end{bmatrix}$$

For row 5:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.06 \\ 0.14 & 0.2 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 30 \end{bmatrix} = \begin{bmatrix} 133.8 \\ 56.338 \end{bmatrix}$$

R.P 7.5

Find the condition on a and b for reciprocity for the network shown in Fig. R.P. 7.5.

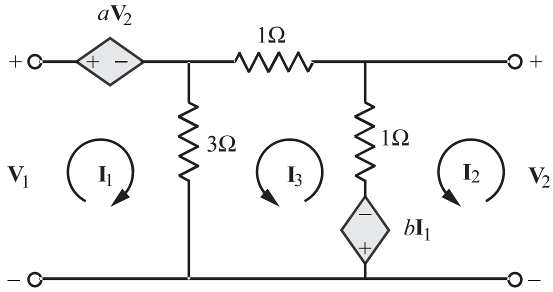


Figure R.P. 7.5

SOLUTION

The loop equations are

$$\mathbf{V}_1 - a\mathbf{V}_2 = 3(\mathbf{I}_1 + \mathbf{I}_3) \tag{7.68}$$

$$\mathbf{V}_2 = (\mathbf{I}_2 - \mathbf{I}_3) - b\mathbf{I}_1 \tag{7.69}$$

$$\begin{aligned}\mathbf{I}_3 &= \mathbf{V}_2 - (\mathbf{V}_1 - a\mathbf{V}_2)\\ &= (1+a)\mathbf{V}_2 - \mathbf{V}_1\end{aligned} \tag{7.70}$$

Substituting equation (7.70) in equations (7.68) and (7.69),

$$\mathbf{V}_1 - a\mathbf{V}_2 = 3\mathbf{I}_1 + 3(1+a)\mathbf{V}_2 - 3\mathbf{V}_1$$

$$\Rightarrow \quad 4\mathbf{V}_1 = (3+4a)\mathbf{V}_2 + 3\mathbf{I}_1 \tag{7.71}$$

$$\mathbf{V}_2 = \mathbf{I}_2 - [(1+a)\,\mathbf{V}_2 - \mathbf{V}_1] - b\mathbf{I}_1$$

$$\Rightarrow \quad -\mathbf{V}_1 + (2+a)\,\mathbf{V}_2 = -b\mathbf{I}_1 + \mathbf{I}_2 \tag{7.72}$$

Putting equations (7.71) and (7.72) in matrix form and solving

$$\begin{aligned}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} &= \begin{bmatrix}4 & -(3+4a)\\ -1 & 2+a\end{bmatrix}^{-1}\begin{bmatrix}3 & 0\\ -b & 1\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}\\ &= \frac{1}{\Delta}\begin{bmatrix}2+a & 3+4a\\ 1 & 4\end{bmatrix}\begin{bmatrix}3 & 0\\ -b & 1\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}\\ &= \frac{1}{\Delta}\begin{bmatrix}M & 3+4a\\ 3-4b & N\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}\end{aligned}$$

For reciprocity,

$$3 + 4a = 3 - 4b$$

Therefore, $$a = -b$$

R.P 7.6

For what value of a is the circuit reciprocal? Also find **h** parameters.

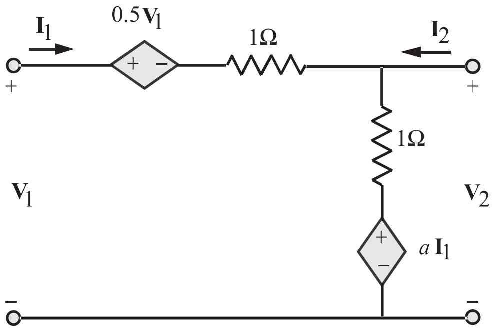


Figure R.P. 7.6

SOLUTION

The node equations are

$$\mathbf{V}_1 - 0.5\mathbf{V}_1 - \mathbf{I}_1 = \mathbf{V}_2$$
$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)2 + a\mathbf{I}_1$$
$$\mathbf{h} = \begin{bmatrix} 0.5 & 0 \\ 0 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 2+a & -1 \end{bmatrix}$$
$$= \frac{1}{\Delta} \begin{bmatrix} -2 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2+a & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 2 \\ \dfrac{2+a}{2} & 0.5 \end{bmatrix} \qquad (\Delta = -1)$$

For reciprocity,

$$\mathbf{h}_{12} = -\mathbf{h}_{21}$$
$$\Rightarrow \qquad 2 = \frac{2+a}{2}$$
$$\Rightarrow \qquad -4 = 2 + a; \quad a = 2$$

Therefore $$\mathbf{h} = \begin{bmatrix} 2 & 2 \\ 2 & 0.5 \end{bmatrix}$$

R.P 7.7

Find $\mathbf{y}_{12}$ and $\mathbf{y}_{21}$ for the network shown in Fig. R.P. 7.7 for n = 10. What is the value of n for the network to be reciprocal?

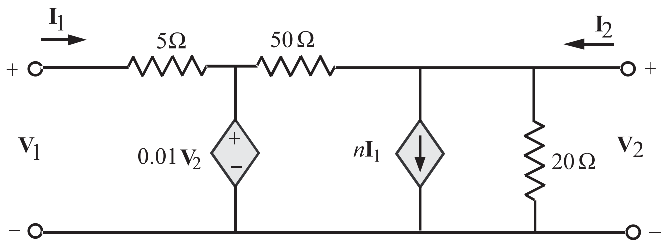


Figure R.P. 7.7

SOLUTION

Equations for $\mathbf{I}_1$ and $\mathbf{I}_2$ are

$$\begin{aligned}\mathbf{I}_1 &= \frac{\mathbf{V}_1 - 0.01\mathbf{V}_2}{5} \\ &= 0.2\mathbf{V}_1 - 0.002\mathbf{V}_2 \qquad (7.73)\\ \mathbf{I}_2 &= \frac{\mathbf{V}_2}{20} + n\mathbf{I}_1 + \frac{\mathbf{V}_2 - 0.01\mathbf{V}_2}{50} \qquad (7.74)\end{aligned}$$

Substituting the value of $\mathbf{I}_1$ from equation (7.73) in equation (7.74), we get

$$\mathbf{I}_2 = n(0.2\mathbf{V}_1 - 0.002\mathbf{V}_2) + \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_2 - 0.01\mathbf{V}_2}{20} \qquad (7.75)$$

Simplifying the above equation with n = 10,

$$\mathbf{I}_2 = 2\mathbf{V}_1 + 0.0498\mathbf{V}_2 \qquad (7.76)$$

From equation (7.73), $\mathbf{y}_{12} = -0.002$

and from equation (7.75), $\mathbf{y}_{21} = 0.2n$

For reciprocity $\mathbf{y}_{12} = \mathbf{y}_{21}$

$\Rightarrow$ $-0.002 = 0.2n$

Hence, $n = -0.01$

R.P 7.8

Find **T** parameters (ABCD) for the two-port network shown in Fig. R.P. 7.8.

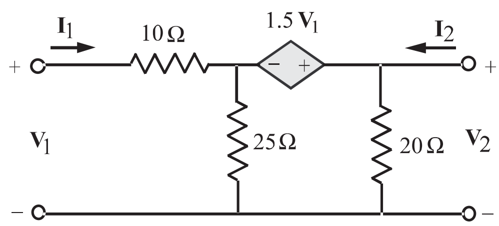


Figure R.P. 7.8

SOLUTION

Network equations are

$$\mathbf{V}_1 - 10\mathbf{I}_1 = \mathbf{V}_2 - 1.5\mathbf{V}_1 \tag{7.77}$$

$$\mathbf{I}_1 - \frac{\mathbf{V}_2 - 1.5\mathbf{V}_1}{25} + \mathbf{I}_2 - \frac{\mathbf{V}_2}{20} = 0 \tag{7.78}$$

Simplifying,

$$2.5\mathbf{V}_1 - 10\mathbf{I}_1 = \mathbf{V}_2$$

$$0.06\mathbf{V}_1 + \mathbf{I}_1 = 0.09\mathbf{V}_2 - \mathbf{I}_2$$

In matrix form,

$$\begin{bmatrix} 2.5 & -10 \\ 0.06 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.09 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

Therefore

$$\mathbf{T} = \begin{bmatrix} 2.5 & -10 \\ 0.06 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0.09 & 1 \end{bmatrix} = \begin{bmatrix} 0.613 & 3.23 \\ 0.053 & 0.806 \end{bmatrix}$$

R.P 7.9

(a) Find **T** parameters for the active two port network shown in Fig. R.P. 7.9.

(b) Find new **T** parameters if a 20 Ω resistor is connected across the output.

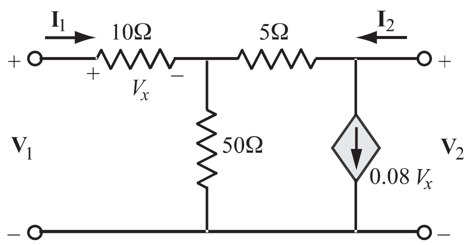


Figure R.P. 7.9

SOLUTION

(a) With $V_x = 10\mathbf{I}_1$,

$$0.08V_x = 0.8\mathbf{I}_1$$

Therefore,

$$\begin{aligned} \mathbf{V}_1 - 10\mathbf{I}_1 &= \mathbf{V}_2 - 5(\mathbf{I}_2 - 0.08V_x) \\ &= \mathbf{V}_2 - 5\mathbf{I}_2 + 4\mathbf{I}_1 \end{aligned} \tag{7.79}$$

$$\mathbf{I}_1 + \mathbf{I}_2 - 0.8\mathbf{I}_1 = \frac{\mathbf{V}_1 - 10\mathbf{I}_1}{50} \tag{7.80}$$

Simplying the equations (7.79) and (7.80),we get

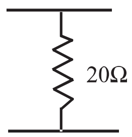
$$\mathbf{V}_1 - 14\mathbf{I}_1 = \mathbf{V}_2 - 5\mathbf{I}_2$$

and
$$-\mathbf{V}_1 + 20\mathbf{I}_1 = -50\mathbf{I}_2$$

Therefore,

$$\mathbf{T} = \begin{bmatrix} 1 & -14 \\ -1 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 5 \\ 0 & 50 \end{bmatrix} = \begin{bmatrix} 3.33 & 133.33 \\ 0.167 & 9.17 \end{bmatrix}$$

(b) Treating 20 Ω across the output as a second T network for which



$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ \frac{1}{20} & 1 \end{bmatrix}$$

Then new **T**-parameters,

$$\mathbf{T} = \begin{bmatrix} 3.33 & 133.33 \\ 0.167 & 9.17 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{20} & 1 \end{bmatrix} = \begin{bmatrix} 10 & 133.33 \\ 0.625 & 9.17 \end{bmatrix}$$

R.P 7.10

Obtain **z** parameters for the network shown in Fig. R.P. 7.10.

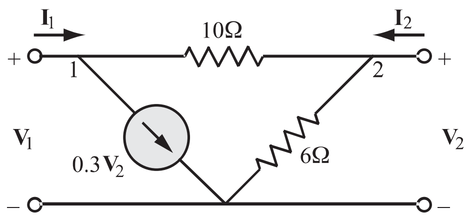


Figure R.P. 7.10

SOLUTION

At node 1,

$$\begin{aligned} \mathbf{V}_1 &= (\mathbf{I}_1 - 0.3\mathbf{V}_2)10 + \mathbf{V}_2 \\ &= 10\mathbf{I}_1 - 2\mathbf{V}_2 \qquad (7.81) \end{aligned}$$

At node 2,

$$\mathbf{V}_2 = \left(\mathbf{I}_2 - \frac{\mathbf{V}_2}{6}\right)10 + \mathbf{V}_1 = 10\mathbf{I}_2 + \mathbf{V}_1 - \frac{5}{3}\mathbf{V}_2$$

$$\Rightarrow \quad \frac{8}{3}\mathbf{V}_2 = \mathbf{V}_1 + 10\mathbf{I}_2 \qquad (7.82)$$

Putting in matrix form,

$$\begin{bmatrix} 1 & 2 \\ 1 & \dfrac{-8}{3} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & -10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Therefore,

$$\mathbf{z} = \begin{bmatrix} 1 & 2 \\ 1 & \dfrac{-8}{3} \end{bmatrix}^{-1} \begin{bmatrix} 10 & 0 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} 5.71 & -4.286 \\ 2.143 & 2.143 \end{bmatrix}$$

R.P 7.11

Obtain **z** and **y** parameters for the network shown in Fig. R.P. 7.11.

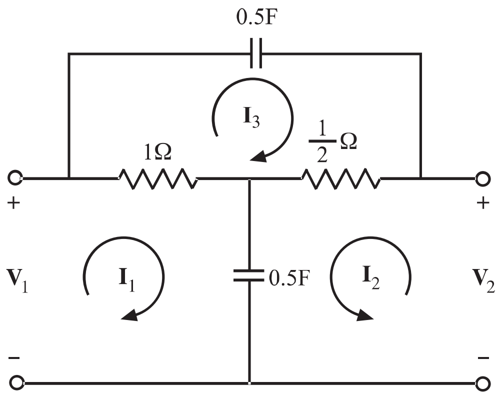


Figure R.P. 7.11

SOLUTION

For the meshes indicated, the equations in matrix form is

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cc:c} 1+\dfrac{2}{s} & \dfrac{2}{s} & -1 \\ \dfrac{2}{s} & \dfrac{1}{2}+\dfrac{2}{s} & \dfrac{1}{2} \\ \hdashline -1 & \dfrac{1}{2} & \dfrac{3}{2}+\dfrac{2}{s} \end{array}\right] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \hline \mathbf{I}_3 \end{bmatrix}$$

By matrix partitioning,

$$\mathbf{z} = \begin{bmatrix} \frac{s+2}{s} & \frac{2}{s} \\ \frac{2}{s} & \frac{s+4}{2s} \end{bmatrix} - \left[\frac{2s}{3s+4}\right] \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+2}{s} & \frac{2}{s} \\ \frac{2}{s} & \frac{s+4}{2s} \end{bmatrix} - \left[\frac{2s}{3s+4}\right] \begin{bmatrix} 1 & \frac{-1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+2}{s} & \frac{2}{s} \\ \frac{2}{s} & \frac{s+4}{2s} \end{bmatrix} - \begin{bmatrix} \frac{2s}{3s+4} & \frac{-s}{3s+4} \\ \frac{-s}{3s+4} & \frac{-8s}{3s+4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s^2+10s+8}{s(3s+4)} & \frac{s^2+6s+8}{s(3s+4)} \\ \frac{s^2+6s+8}{s(3s+4)} & \frac{s^2+8s+8}{s(3s+4)} \end{bmatrix}$$

R.P 7.12

Find **z** and **y** parameters at $\omega = 10^8$ rad/sec for the transistor high frequency equivalent circuit shown in Fig. R.P. 7.12.

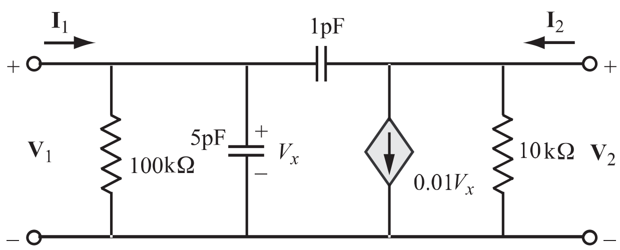


Figure R.P. 7.12

SOLUTION

In the circuit, $V_x = \mathbf{V}_1$. Therefore the node equations are

$$\mathbf{I}_1 = (10^{-5} + j6 \times 10^{-4})\mathbf{V}_1 - j10^{-4}\mathbf{V}_2$$
$$\mathbf{I}_2 = -j10^{-4}\mathbf{V}_1 + 0.01\mathbf{V}_1 + 10^{-4}(1+j)\mathbf{V}_2$$

Simplifying the above equations,

$$\mathbf{I}_1 = 10^{-4}[(0.1 + j6)\mathbf{V}_1 - j1\mathbf{V}_2]$$
$$\mathbf{I}_2 = 10^{-4}[(100 + j1)\mathbf{V}_1 + (1 + j)\mathbf{V}_2]$$

Therefore, $$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 0.1 + j6 & -j1 \\ 100 - j1 & 1 + j1 \end{bmatrix} \times 10^{-4} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\omega C_1 = 10^8 \times 5 \times 10^{-12} = 5 \times 10^{-4}$$

$$\omega C_2 = 10^8 \times 10^{-12} = 10^{-4}$$

$$\Delta = 10^{-8}[(0.1 + j6)(1 + j) + (100 - j1)(j1)]$$
$$= 10^{-8} \times 106.213 \angle 92.64^\circ$$

Therefore, $$\mathbf{y} = \begin{bmatrix} 6 \angle 89^\circ & -j1 \\ 100 \angle -0.6^\circ & \sqrt{2} \angle 45^\circ \end{bmatrix} \times 10^{-4}$$

Then, $$\mathbf{z} = \mathbf{y}^{-1} = \begin{bmatrix} \sqrt{2} \angle 45^\circ & j1 \\ 100 \angle -180.6^\circ & 6 \angle 89^\circ \end{bmatrix} \times 10^{-4} \div \Delta$$

$$= \begin{bmatrix} \sqrt{2} \angle 45^\circ & j1 \\ 100 \angle -180.6^\circ & 6 \angle 89^\circ \end{bmatrix} \times \frac{10^{-4}}{10^{-8} \times 106.213 \angle 92.64^\circ}$$

$$= \begin{bmatrix} 133.15 \angle -47.64^\circ & 94.16 \angle -2.64^\circ \\ 94.16 \angle 86.8^\circ & 565 \angle -31.6^\circ \end{bmatrix}$$

R.P 7.13

Obtain $\mathbf{T}_A$, $\mathbf{T}_B$, $\mathbf{T}_C$ for the network shown in Fig. R.P. 7.13 and obtain overall $\mathbf{T}$.

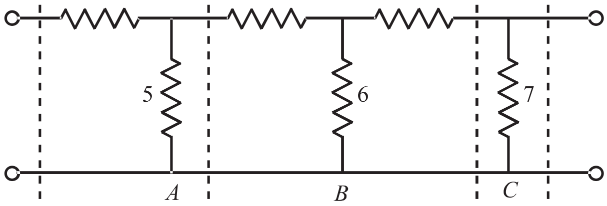
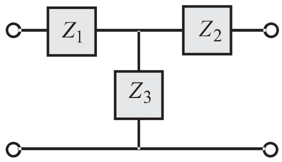


Figure R.P. 7.13

SOLUTION

Using the equation for **T**-parameters for a T-network

$$\mathbf{A} = \frac{Z_1 + Z_3}{Z_3}; \quad \mathbf{B} = \frac{\sum Z_1 Z_3}{Z_3}; \quad \mathbf{C} = \frac{1}{Z_3}; \quad \mathbf{D} = \frac{Z_2 + Z_3}{Z_3}$$



We have for A $\qquad \mathbf{T}_A = \begin{bmatrix} \frac{7}{5} & 2 \\ \frac{1}{5} & 1 \end{bmatrix}$

For B, $\qquad \mathbf{T}_B = \begin{bmatrix} \frac{9}{6} & \frac{54}{6} \\ \frac{1}{6} & \frac{10}{6} \end{bmatrix}$

For C, $\qquad \mathbf{T}_C = \begin{bmatrix} 1 & 0 \\ \frac{1}{7} & 1 \end{bmatrix}$

Overall **T** : $\qquad \mathbf{T} = [\mathbf{T}_A][\mathbf{T}_B][\mathbf{T}_C] = \begin{bmatrix} 4.709 & 15.93 \\ 0.962 & 3.46 \end{bmatrix}$

This dervation is left as an exercise to the reader.

Verification:

Using T $-$ Δ transformation, that is changing T (3, 4, 6) of Fig. R.P. 7.13, in to Δ,

$$Z_{xz} = 13.5$$
$$Z_{xy} = 9$$
$$Z_{yz} = 18$$

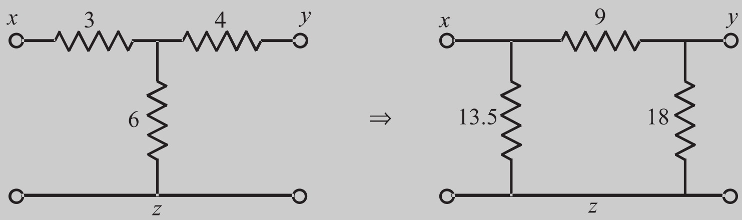


Figure R.P.7.13(a)

Putting the values in the circuit of Fig. R.P. 7.13, we get

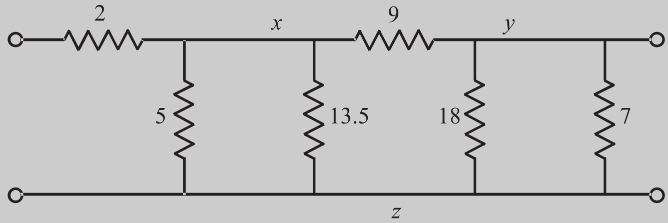


Figure R.P.7.13(b)

Reducing, the above circuit, we get the circuit shown in Fig. R.P. 7.13c.

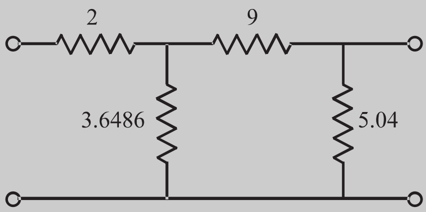


Figure R.P. 7.13(c)

Converting the circuit into T, we get the circuit shown in Fig. R.P. 7.13(d).
Now from Fig. R.P. 7.13(d),

$$\mathbf{A} = \frac{3.8564 + 1.0396}{1.0396} = 4.709$$

$$\mathbf{B} = \frac{1.0396(3.8564 + 2.5644) + 3.8564 \times 2.5644}{1.0396}$$

$$= 15.93\ \Omega$$

$$\mathbf{C} = \frac{1}{Z_p} = \frac{1}{1.0396} = 0.962$$

$$\mathbf{D} = \frac{2.5644 + 1.0396}{1.0396} = 3.46$$

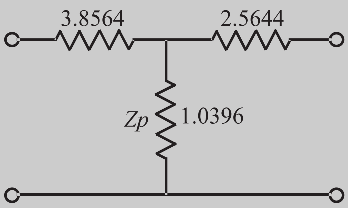


Figure R.P. 7.13(d)

Exercise Problems

E.P 7.1

Find the **y** parameters for the network shown in Fig. E.P. 7.1.

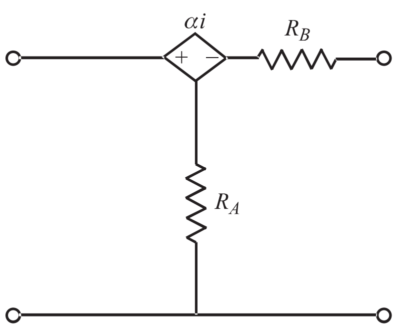


Figure E.P. 7.1

Ans: $\mathbf{y_{11}} = \dfrac{\boldsymbol{\alpha + R_A + R_B}}{\boldsymbol{R_A R_B}}, \mathbf{y_{12}} = \dfrac{\mathbf{-1}}{\boldsymbol{R_B}}, \mathbf{y_{21}} = \dfrac{\boldsymbol{-(\alpha + R_A)}}{\boldsymbol{R_A R_B}}, \mathbf{y_{22}} = \dfrac{\mathbf{1}}{\boldsymbol{R_B}}.$

E.P 7.2

Find the **z** parameters for the network shown in Fig. E.P. 7.2.

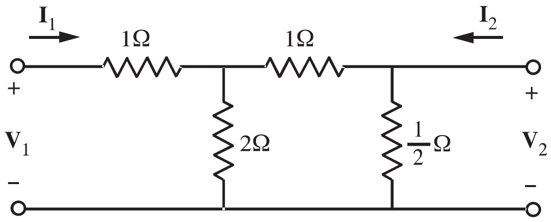


Figure E.P. 7.2

Ans: $\mathbf{z_{11}} = \dfrac{\mathbf{13}}{\mathbf{7}}\mathbf{\Omega}$, $\mathbf{z_{12}} = \dfrac{\mathbf{2}}{\mathbf{7}}\mathbf{\Omega}$, $\mathbf{z_{21}} = \dfrac{\mathbf{2}}{\mathbf{7}}\mathbf{\Omega}$, $\mathbf{z_{22}} = \dfrac{\mathbf{3}}{\mathbf{7}}\mathbf{\Omega}$.

E.P 7.3

Find the **h** parameters for the network shown in Fig. E.P. 7.3.

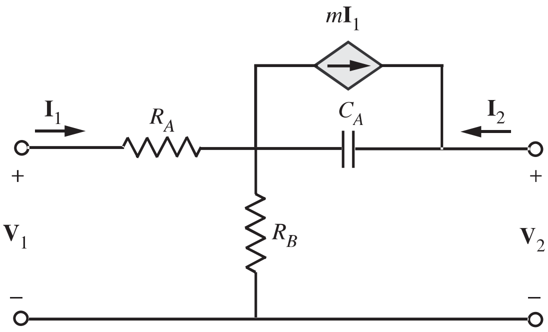


Figure E.P. 7.3

Ans: $\mathbf{h_{11}} = \dfrac{sC_AR_AR_B + R_A + (1-m)R_B}{sC_AR_B + 1}$, $\mathbf{h_{21}} = \dfrac{sC_AR_B + m}{sC_AR_B + 1}$.

$\mathbf{h_{12}} = \dfrac{sC_AR_B}{sC_AR_B + 1}$, $\mathbf{h_{22}} = \dfrac{sC_A}{sC_AR_B + 1}$.

E.P 7.4

Find the **y** parameters for the network shown in Fig. E.P. 7.4.

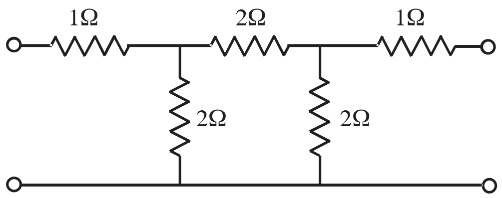


Figure E.P. 7.4

Ans: $\mathbf{y_{11}} = \mathbf{y_{22}} = \dfrac{\mathbf{7}}{\mathbf{15}}\mathbf{S}$, $\mathbf{y_{12}} = \mathbf{y_{21}} = \dfrac{\mathbf{-2}}{\mathbf{15}}\mathbf{S}$.

E.P 7.5

Find the **y** parameters for the network shown in Fig. E.P. 7.5. Give the result in s domain.

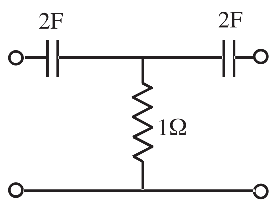


Figure E.P. 7.5

Ans: $\mathbf{y_{11} = y_{22} = \dfrac{2s(2s+1)}{4s+1}}, \qquad \mathbf{y_{12} = y_{21} = \dfrac{-4s^2}{4s+1}}.$

E.P 7.6

Obtain the **y** parameters for the network shown in Fig. E.P. 7.6.

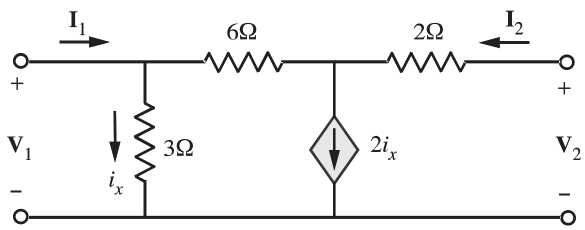


Figure E.P. 7.6

Ans: $\mathbf{y_{11} = 0.625\ S}, \qquad \mathbf{y_{12} = -0.125\ S}, \qquad \mathbf{y_{21} = 0.375\ S}, \qquad \mathbf{y_{22} = 0.125\ S}.$

E.P 7.7

Find the **z** parameters for the two-port network shown in Fig. E.P. 7.7. Keep the result in s domain.

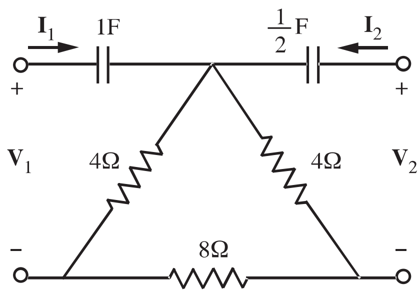


Figure E.P. 7.7

Ans: $\mathbf{z_{11} = \dfrac{2s+1}{s}}, \qquad \mathbf{z_{12} = z_{21} = 2}, \qquad \mathbf{z_{22} = \dfrac{2s+2}{s}}.$

E.P 7.8

Find the **h** parameters for the network shown in Fig. E.P. 7.8. Keep the result in s domain.

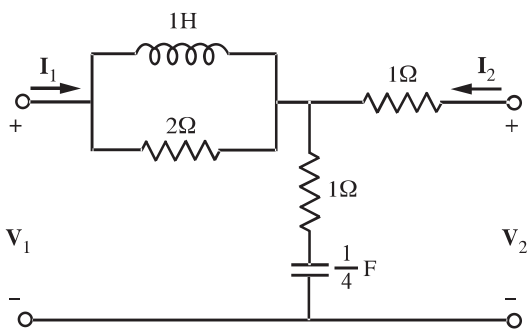


Figure E.P. 7.8

Ans: $\mathbf{h_{11}} = \dfrac{\mathbf{5s+4}}{\mathbf{2(s+2)}}, \quad \mathbf{h_{12}} = \dfrac{\mathbf{s+4}}{\mathbf{2(s+2)}}, \quad \mathbf{h_{21}} = \dfrac{\mathbf{-(s+4)}}{\mathbf{2(s+2)}}, \quad \mathbf{h_{22}} = \dfrac{\mathbf{s}}{\mathbf{2(s+2)}}.$

E.P 7.9

Find the transmission parameters for the network shown in Fig. E.P. 7.9. Keep the result in s domain.

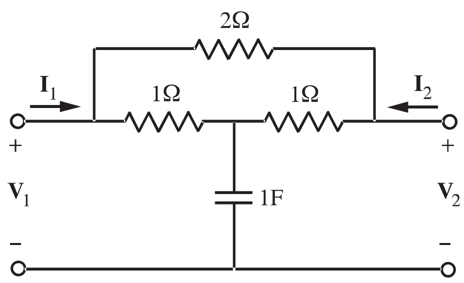


Figure E.P. 7.9

Ans: $\mathbf{A} = \dfrac{\mathbf{3s+4}}{\mathbf{s+4}}, \quad \mathbf{B} = \dfrac{\mathbf{2s+4}}{\mathbf{s+4}}, \quad \mathbf{C} = \dfrac{\mathbf{4s}}{\mathbf{s+4}}, \quad \mathbf{D} = \dfrac{\mathbf{3s+4}}{\mathbf{s+4}}.$

E.P 7.10

For the same network described in Fig. E.P. 7.9, find the **h** parameters using the defining equations. Then verify the result obtained using conversion formulas.

Ans: $\mathbf{h_{11}} = \dfrac{\mathbf{2s+4}}{\mathbf{3s+4}}, \quad \mathbf{h_{12}} = \dfrac{\mathbf{s+4}}{\mathbf{3s+4}}, \quad \mathbf{h_{21}} = \dfrac{\mathbf{-(s+4)}}{\mathbf{3s+4}}, \quad \mathbf{h_{22}} = \dfrac{\mathbf{4s}}{\mathbf{3s+4}}.$

E.P 7.11

Select the values of R_A, R_B, and R_C in the circuit shown in Fig. E.P. 7.11 so that **A** =1, **B** = 34 Ω, **C** = 20 mS and **D** = 1.4.

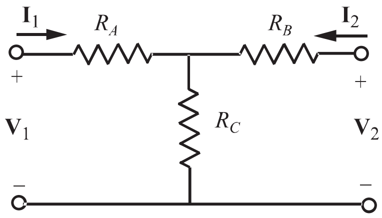


Figure E.P. 7.11

Ans: $\boldsymbol{R_A = 10\Omega,} \qquad \boldsymbol{R_B = 20\Omega,} \qquad \boldsymbol{R_C = 50\Omega.}$

E.P 7.12

Find the s domain expression for the **h** parameters of the circuit in E.P. 7.12.

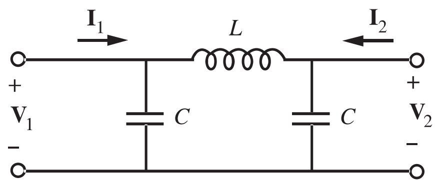


Figure E.P. 7.12

Ans: $$\mathbf{h_{11}} = \frac{\frac{1}{C}s}{s^2 + \frac{1}{LC}}, \qquad \mathbf{h_{12} = h_{21}} = \frac{-\frac{1}{LC}}{s^2 + \frac{1}{LC}}, \qquad \mathbf{h_{22}} = \frac{Cs\left(s^2 + \frac{2}{LC}\right)}{s^2 + \frac{1}{LC}}.$$

E.P 7.13

Find the **y** parameters for the network shown in Fig. E.P. 7.13.

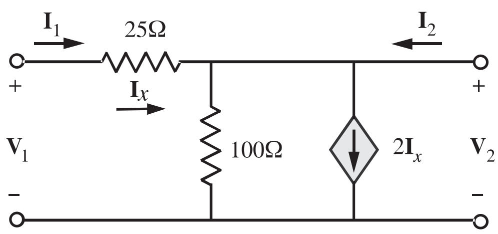


Figure E.P. 7.13

Ans: $\mathbf{y_{11} = 0.04S,} \qquad \mathbf{y_{12} = -0.04S,} \qquad \mathbf{y_{21} = 0.04S,} \qquad \mathbf{y_{22} = -0.03S.}$

E.P 7.14

Find the two-port parameters $\mathbf{h}_{12}$ and $\mathbf{y}_{12}$ for the network shown in Fig. E.P. 7.14.

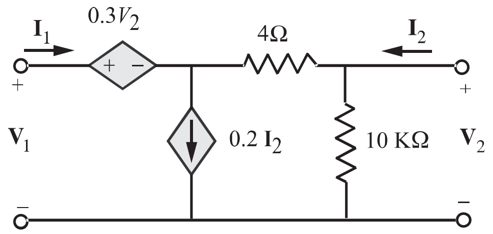


Figure E.P. 7.14

Ans: $\mathbf{h_{12} = 1.2}$, $\mathbf{y_{12} = 0.24S}$.

E.P 7.15

Find the **ABCD** parameters for the 4Ω resistor of Fig. E.P. 7.15. Also show that the **ABCD** parameters for a single 16Ω resistor can be obtained by **(ABCD)**^{4}.

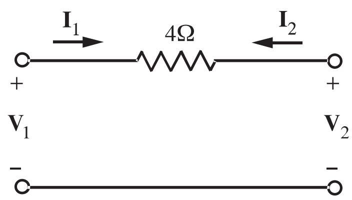


Figure E.P. 7.15

Ans: Verify your answer using the relation between the parameters.

E.P 7.16

For the T-network shown in Fig. E.P. 7.16. show that **AD** − **BC** =1.

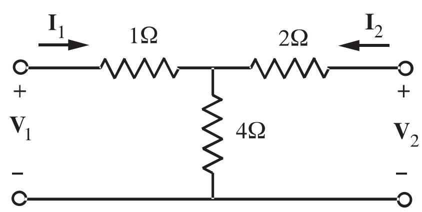


Figure E.P. 7.16

E.P 7.17

Find $\mathbf{y}_{21}$ for the network shown in Fig. E.P. 7.17.

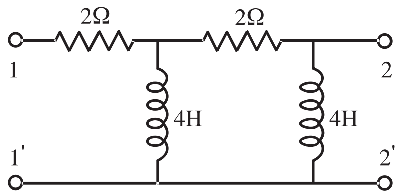


Figure E.P. 7.17

Ans: $\mathbf{y_{21} = \dfrac{-s}{4s+1}}$.

E.P 7.18

Determine the **y**-parameters for the network shown in Fig. E.P. 7.18.

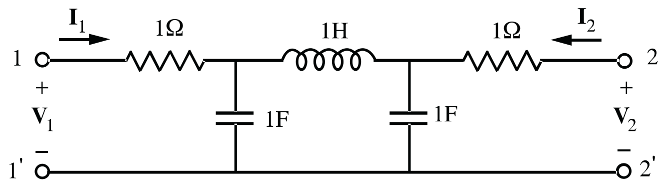


Figure E.P. 7.18

Ans: $\mathbf{y_{11} = \dfrac{s^3+s^2+2s+1}{s(s^2+2)}}$, $\mathbf{y_{12} = y_{21} = \dfrac{-1}{s(s^2+2)}}$, $\mathbf{y_{22} = \dfrac{s^3+s^2+2s+1}{s(s^2+2)}}$.

E.P 7.19

Obtain the **h**-parameters for the network shown in Fig. 7.19.

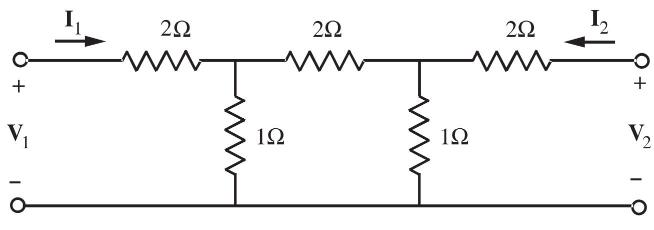


Figure E.P. 7.19

Ans: $\mathbf{h_{11} = \dfrac{30}{11}\Omega}$, $\mathbf{h_{21} = \dfrac{-1}{11}}$, $\mathbf{h_{12} = \dfrac{1}{11}}$, $\mathbf{h_{22} = \dfrac{4}{11}\mho}$

E.P 7.20

The following equations are written for a two-port network. Find the transmission parameters for the network. (Hint: use relation between **y** and **T** parameters).

$$\mathbf{I_1 = 0.05V_1 - 0.4V_2 \qquad I_2 = -0.4V_1 + 0.1V_2}$$

E.P 7.21

Find the network shown in figure, determine the **z** and **y** parameters.

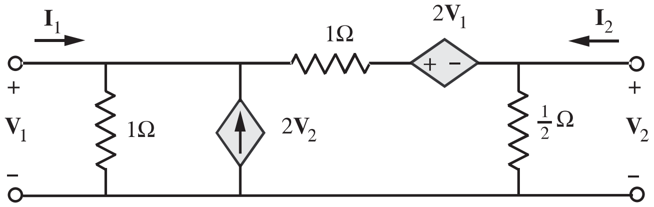


Figure E.P. 7.21

Ans: $\mathbf{y_{11} = 4\mho^*}$, $\mathbf{y_{22} = 3\mho}$, $\mathbf{y_{12} = y_{21} = -3\mho}$,

$\mathbf{z_{11} = 1\Omega}$, $\mathbf{z_{22} = \frac{4}{3}\Omega}$, $\mathbf{z_{12} = z_{21} = 1\Omega}$.

E.P 7.22

Determine the **z**, **y** and Transmission parameters of the network shown in Fig. 7.22.

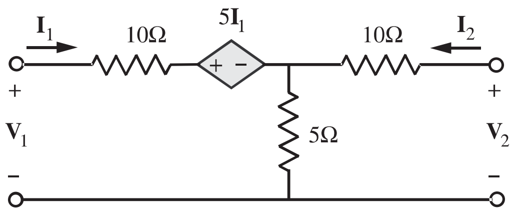


Figure E.P. 7.22

Ans: $\mathbf{y_{11} = \frac{3}{55}\mho}$, $\mathbf{y_{12} = y_{21} = \frac{1}{55}\mho}$, $\mathbf{y_{22} = \frac{4}{55}\mho}$,

$\mathbf{z_{11} = 20\Omega}$, $\mathbf{z_{12} = z_{21} = 5\Omega}$, $\mathbf{z_{22} = 15\Omega}$

$\mathbf{A = 55\Omega}$, $\mathbf{B = 55\Omega}$, $\mathbf{C = 0.2\mho}$, $\mathbf{D = 3}$.

E.P 7.23

For the network shown in Fig. E.P. 7.23 determine **z** parameters.

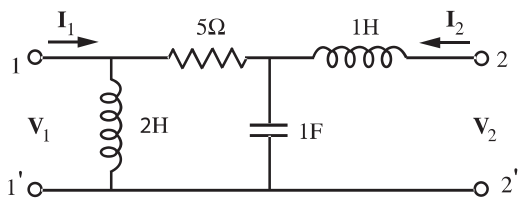


Figure E.P. 7.23

Ans: $\mathbf{z_{11} = \frac{2s(5s+1)}{2s^2+5s+1}}$, $\mathbf{z_{12} = z_{21} = \frac{2s}{2s^2+5s+1}}$, $\mathbf{z_{22} = \frac{2s^3+5s^2+3s+5}{2s^2+5s+1}}$

* The unit $\mho$ and S are same

E.P 7.24

Determine the **y** parameters of the two-port network shown in Fig. E.P. 7.24.

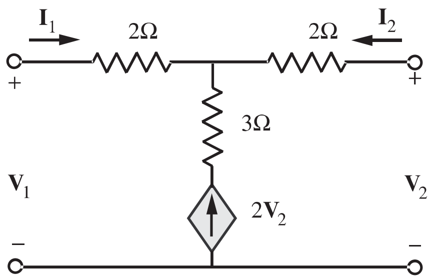


Figure E.P. 7.24

Ans: $\mathbf{y_{11}} = \dfrac{\mathbf{1}}{\mathbf{4}}\mho, \quad \mathbf{y_{21}} = \dfrac{\mathbf{-1}}{\mathbf{4}}\mho, \quad \mathbf{y_{12}} = \dfrac{\mathbf{-5}}{\mathbf{4}}\mho, \quad \mathbf{y_{22}} = \dfrac{\mathbf{-4}}{\mathbf{3}}\mho.$

Outcomes:

1. To analyze circuit systems using concept of two port networks.
2. Apply the concept of z, y, h and transmission parameters cascade and cascode networks.
3. Be able to calculate y-parameters, z-parameters, hybrid parameters, and transmission parameters of two-port networks, and perform transformations between the various representations.
4. Ability to analyze two-port networks finding the different parameters to model two-port networks.
5. Application of network parameters in the analysis of transistor circuits and the synthesis of ladder networks.

Resources:

1. http://en.wikipedia.org/wiki/Two-port_network
2. http://nptel.ac.in/courses/108102042/
3. http://jcatsc.com/media/ee541/LectureSupplements/01-LinearTwoPortNetworks.pdf
4. http://freevideolectures.com/Course/2336/Circuit-Theory/26