PSA-2
M2: Load Flow Analysis

Prepared by
Maria Sushma S
Asst Prof.
Dept of EEE
ATMECE

Power Flow Equation or Static Load Flow Eqn in Polar form

- Let us consider a 3ϕ balanced power s/s. The analysis can be carried out on a 1-ϕ. The 1st step in analysis is formulation of suitable eqns for the power flows in the s/s.
- The power s/s is large interconnected s/s, where various buses are connected by TLs.

Power Flow Equation or Static Load Flow Eqn in Polar form cntd.

- At any bus i, the complex power Si (injected) is:

$$S_i = SG_i - SD_i \quad \rightarrow (1)$$

Where:

S_i - *Net complex power injected into bus i*

SG_i - *Complex power injected by the generator @ bus i*

SD_i - *Complex power drawn by the load @ bus i*

- By conservation of complex powers, at any i-th bus, the complex power injected into bus = sum of complex power flows out of bus.

$$\therefore S_i = \Sigma\, S_{ik}\,, \quad i = 1, 2, \ldots, n \quad \rightarrow (2)$$

Where:

S_{ik} - *Sum over all lines connected to the bus*

n - *No. of buses in the system (excluding reference bus)*

- Bus current injected at the i-th bus,

$$I_i = I_{Gi} - I_{Di} \quad ; \quad i = 1, 2, \ldots, n \quad \rightarrow (3)$$

Where:

I_{Gi} - Current injected by the generator at the bus

I_{Di} - Current drawn by the load at the bus

- We know that: $I = Y_{bus} \cdot V \quad \rightarrow (4)$

Where:

$\rightarrow$

$$I = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_n \end{bmatrix}$$

Ybus = Bus admittance matrix

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_n \end{bmatrix}$$

Vector of currents injected at the buses

Vector of complex bus voltages

From Eqn (4) $\Rightarrow$ $\boxed{I_i = \sum_{k=1}^{n} Y_{ik} V_k \quad i = 1, 2, \ldots n}$ $\rightarrow (5)$

In Rectangular form

The complex power S_i is given by

$$S_i = V_i I_i^*$$

$$= V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^*$$

$$= V_i \left(\sum_{k=1}^{n} Y_{ik}^* V_k^* \right) \rightarrow (6)$$

* Let us define V_i: $V_i \angle \delta_i = |V_i| (\cos\delta_i + j \sin\delta_i) \rightarrow (7)$, $\hookrightarrow (7a)$

$\delta_{ik} = \delta_i - \delta_k \rightarrow (7b)$

$Y_{ik} = G_{ik} + j B_{ik} \rightarrow (7c)$ Rectangular form

* Substitute (7) in (6)

$$S_i = |V_i| \cdot \angle\delta_i \times \left[\sum_{k=1}^{n} (G_{ik} + jB_{ik})^* \times V_k^* \right]$$

$$= |V_i| \angle\delta_i \left[\sum_{k=1}^{n} (G_{ik} - j B_{ik}) \cdot |V_k| \angle -\delta_k \right] \quad ; \quad V_k = V_k \angle \delta_k, \; V_k^* = V_k \angle -\delta_k$$

$$= \sum_{k=1}^{n} \left[|V_i| \cdot |V_k| \angle (\delta_i - \delta_k) \cdot (G_{ik} - j B_{ik}) \right]$$

$$= \sum_{k=1}^{n} \left[|V_i| |V_k| \angle \delta_{ik} \cdot (G_{ik} - j B_{ik}) \right]$$

$$= \sum_{k=1}^{n} \left[|V_i| |V_k| \angle \delta_{ik} \cdot (G_{ik} - j B_{ik}) \right]$$

$$S_i = \sum_{k=1}^{n} \left[|V_i| |V_k| (\cos \delta_{ik} + j \sin \delta_{ik}) (G_{ik} - j B_{ik}) \right] \rightarrow (8)$$

* Separate real & imaginary parts in Eqn (8)

$$P_i = \sum_{k=1}^{n} |V_i| |V_k| (G_{ik} \cdot \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \rightarrow (8a)$$

$$Q_i = \sum_{k=1}^{n} |V_i| |V_k| * (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \rightarrow (8b)$$

Power flow Eqn.

Rectangular form

In Polar form

The complex power S_i is given by

$$S_i = V_i I_i^*$$

$$= V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^*$$

$$= V_i \left(\sum_{k=1}^{n} Y_{ik}^* V_k^* \right) \rightarrow (6)$$

$\boxed{V_i: \quad V_i \angle \delta_i =}$ → (7a)

* An alternate form of P_i & Q_i can be obtained by representing Y_{ik} in polar form. i/e $\boxed{Y_{ik} = |Y_{ik}| \angle \theta_{ik}}$ → (9)

* Substitute (7a) & (9) in (6) Polar form

$$S_i = |V_i| \angle \delta_i \sum_{k=1}^{n} |Y_{ik}| \angle -\theta_{ik} \cdot |V_k| \angle -\delta_k \rightarrow (10)$$

$Y_{ik}^* = |Y_{ik}| \angle -\theta_{ik}$

& $V_k^* = |V_k| \angle -\delta_k$

$$S_i = |V_i| \sum_{k=1}^{n} |Y_{ik}| |V_k| \cdot \angle (\delta_i - \delta_k - \theta_{ik}) \rightarrow (11)$$

WKT : $1 \angle \theta = \cos\theta + j\sin\theta$

$\therefore$ (11) $\Rightarrow S_i = |V_i| \sum_{k=1}^{n} |Y_{ik}| \cdot |V_k| \left[\cos(\delta_i - \delta_k - \theta_{ik}) + j \sin(\delta_i - \delta_k - \theta_{ik}) \right]$

* Extract real & imaginary parts from above Eqn.

$$\boxed{P_i = \sum_{k=1}^{n} |V_i| \cdot |V_k| \cdot |Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k) \quad i = 1, 2, \ldots n} \rightarrow (12)$$

Power flow Eqn

Polar form

$$Q_i = \sum_{k=1}^{n} |V_i| \, |V_k| \cdot |Y_{ik}| \times -\sin(\theta_{ik} - \delta_i + \delta_k)$$

$$\boxed{Q_i = -\sum_{k=1}^{n} |V_i| \cdot |V_k| \cdot |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \quad i = 1, 2, \ldots n} \rightarrow (13)$$

Power flow Eqn

Polar form

$\cos(\delta_i - \delta_k - \theta_{ik})$
$= \cos[-(\theta_{ik} - \delta_i + \delta_k)]$
$= \cos(-\theta) = \cos\theta$
$\Rightarrow \cos(\theta_{ik} - \delta_i + \delta_k)$

$\sin(\delta_i - \delta_k - \theta_{ik})$
$= \sin(-(\theta_{ik} - \delta_i + \delta_k$
$= \sin(-\theta) = -\sin\theta$
$= -\sin(\theta_{ik} - \delta_i + \delta_k)$.

Operating Constraints

In a power system, all equipment (generators, transmission lines, buses) must operate within certain safe limits. These limits are called **operating constraints.**

1. **Voltage Limits**
   - The voltage at each bus must stay between a minimum and maximum value.
   - Too high or too low voltage can damage equipment or affect consumers.

- Voltage at each bus:
  $V_{min} \leq |V| \leq V_{max}$

2. **Active Power Limits**
   - The mechanical input to generators (prime movers like turbines, boilers) decides how much active power (P) can be generated.
   - Active power must remain between a minimum and maximum level.

- Active power generation limit:
  $P_{Gi,min} \leq P_{Gi} \leq P_{Gi,max}$

Operating Constraints cntd.

3. Reactive Power Limits

- The generator's field excitation controls reactive power (Q).
- Reactive power also has limits:

$$Q_{Gi,min} \leq Q_{Gi} \leq Q_{Gi,max}$$

- Reactive power generation limit:

$$Q_{Gi,min} \leq Q_{Gi} \leq Q_{Gi,max}$$

4. Angle Stability Limit

- The power angle (difference between bus voltages) must not exceed a maximum permissible value.
- If angles get too large, the system may lose stability.

- Stability constraint:

$$|\delta_i - \delta_j| \leq (\delta_i - \delta_j)_{max}$$

5. Power Balance

- The total generated power must equal the total demand + losses.
- This applies for both active power and reactive power.

- Power balance:

$$\sum P_{Gi} = \sum P_{Di} + P_L$$

$$\sum Q_{Gi} = \sum Q_{Di} + Q_L$$

(Where P_L, Q_L are system losses)

Data for load Flow

7.5.1 System Data

This should give information on

- Number of buses, n.
- Number of PV buses.
- Number of loads.
- Number of transmission lines.
- Number of transformers.
- Number of shunt elements.
- The slack bus number.
- Voltage magnitude of slack bus (angle is generally taken as 0°).
- Tolerance limit.
- Base MVA.
- Maximum permissible number of iterations.

7.5.2 Generator Bus Data

For every PV bus i, the data required is

- Bus number.
- Active power generation P_{Gi}.
- The specified voltage magnitude $|V_{i,sp}|$.
- Minimum reactive power limit $Q_{i,\min}$.
- Maximum reactive power limit $Q_{i,\max}$.

7.5.3 Load Data

For all loads, the data required is

- The bus number.
- Active power demand P_{Di}.
- The reactive power demand Q_{Di}.

7.5.4 Transmission Line Data

For every transmission line connected between buses i and k, data needed is

- Starting bus number i.
- Ending bus number k.
- Resistance of the line.
- Reactance of the line.
- Half line charging admittance.

7.5.5 Transformer Data

For every transformer connected between buses i and k, the data to be given is

- Starting bus number i.
- Ending bus number k.
- Resistance of the transformer.
- Reactance of the transformer.
- Off nominal turns-ratio a.

7.5.6 Shunt Element Data

The data needed for the shunt element is

- Bus number where element is connected.
- Shunt admittance $(G_{sh} + jB_{sh})$.

2.6 Gauss - Seidel Iterative Method : Refer only for Information

2.6.1 Practical Example to understand Iterative method benefits/uses:

Analogy: Finding best route on G maps

Consider a City with 3 buses:

* Bus - 1 : (Slack bus) : A power station fixes Voltage as $1\angle 0^\circ$ pu.
* Bus - 2: (PQ bus): A factory demands 80MW + 40 MVAR
* Bus - 3 (PV bus) : A wind farm generates 50 MW @ fixed Vtg of 1.02 pu.

To find: * The actual Vtgs & angles @ Bus - 2 & Bus - 3.

* Reactive power generator @ Bus - 3.

Understanding how iteration works: (GS or NR method)

1. Start by assuming $V_2 = 1\angle 0°$, $V_3 = 1\angle 0°$.
2. Calculate power mismatch using assumed Vtgs, Calculate the actual Pcalcu, Qcalcu.

   Compare with specified Pspec, Qspec → Find error = Pspec − Pcalc, Qspec − Qcalc.
3. Update Vtgs: Adjust the assumed Vtgs using the mismatch.
   (In GS: directly correct Vtgs
   In NR: Solve Jacobian Eqns to correct)
4. Repeat (Iteration): Keep recalculating until errors are very small < 0.001 pu

2.6.2 Gauss Seidel Method (Analysis)

* The GS method is a popular iterative algorithm for solving non-linear algebraic Equations. An ~~at~~ initial Solution Vector is assumed, @ every subsequent iteration, the Solution is updated till Convergence is reached.

* Initially assume all buses are PQ buses, except slack bus. Here (n-1) complex bus Voltages are to be determined.

* In programming; Slack bus – generally numbered as 1
  PV buses – are numbered in sequence
  PQ buses – Ordered in next sequence.

* Consider: $S_i = V_i \left[\sum_{k=1}^{n} Y_{ik} V_k \right]^* \rightarrow (1)$

Take conjugate on b.s

$S_i^* = V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k \right]^{**}$

$\Rightarrow S_i^* = V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k \right] \rightarrow (2)$

WKT: $S_i^* = P_i - jQ_i \rightarrow (3)$

(3) in (2) $\Rightarrow$ $P_i - jQ_i = V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k \right] \Rightarrow \dfrac{P_i - jQ_i}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k \rightarrow (4)$

Where,

- i = the current bus we are solving for.
- k = other buses that influence i, used inside the summation.

$$P_i - jQ_i = V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k \right] \Rightarrow \frac{P_i - jQ_i}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k \rightarrow (4)$$

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} \cdot V_k \rightarrow (5)$$

By re-arranging the above Eqn,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} \cdot V_k \right] ; \ i = 2, 3, \ldots n$$

$\rightarrow (5)$

By re-arranging the above Eqn,

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik} \cdot V_k\right] \; ; \; i = 2, 3, \ldots n \qquad (5)$$

* In Eqn (5) is an implicit Eqns, since it has unknown Variables on both sides of Eqn. It should be solved by an iterative technique
* In GS method, the Values of @ beginning of the iteration are used for computation of all the buses Voltages in the present iteration. It leads to slow convergence.
* In GS method, the Value of the updated Vtgs are used in computation of subsequent Vtgs in the same iteration. It speeds up convergence.
* Iterations are carried out till the magnitudes of all bus Vtgs do not change more than the tolerance value.

By re-arranging the above Eqn,

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}\cdot V_k\right]; \quad i=2,3,\dots n \rightarrow (5)$$

2.6.3 Algorithm for GS Method :

1. Prepare the data for load flow.
2. Formulate the bus admittance matrix Y_{bus}. Usually done by Inspection method
3. Assume initial voltages for all buses. Complex bus voltages @ all $(n-1)$ buses (except slack bus) are taken as $1.0\angle 0^\circ$.

4 (*) Update the voltages. In any $(r+1)^{th}$ iteration, from Eqn (5), the Vtgs are

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik}(V_k)^{(r)}\right] \rightarrow (6)$$

$$i = 2, 3, \dots n$$

Where,

- i = the current bus we are solving for.
- k = other buses that influence i, used inside the summation.

4 ⓧ Update the voltages. In any $(r+1)^{th}$ iteration, from Eqn ⑤, the Vtgs are

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} (V_k)^{(r)}\right] \rightarrow ⑥$$

$i = 2, 3, \ldots n$

🔷 Case 1: Buses before i (k < i)

- Those buses (like 2, 3, ..., i–1) have **already been updated** in this same iteration.
- So, their **new values** $V_k^{(r+1)}$ are available.
- We immediately use them → this makes Gauss–Seidel converge faster.

🔷 Case 2: Buses after i (k > i)

- Those buses (like i+1, ..., n) have **not yet been updated** in this iteration.
- Only the **old values** $V_k^{(r)}$ from the previous iteration are available.
- So, we must use the older iteration values.

- Before i → use new values $(r+1)$ because they're already updated.
- After i → use old values (r) because they're not yet updated.

4 (x) Update the voltages. In any $(r+1)^{th}$ iteration, from Eqn (5), the Vtgs are

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} (V_k)^{(r)}\right] \rightarrow (6)$$

$i = 2, 3, \dots n$

(x) When computation is carried out for i^{th} bus, updated values are already available for buses 2, 3, ... $(i-1)$ in the current $(r+1)^{th}$ iteration so these values are used. For buses $(i+1), \dots n$, values from previous r^{th} iteration are used.

(x) Since P_i & Q_i are constants @ PQ buses, let

$$L_i = \frac{P_i - jQ_i}{Y_{ii}} \quad \& \quad M_{ik} = \frac{Y_{ik}}{Y_{ii}}$$ be computed initially.

(x) Eqn (6) is written as:

$$V_i^{(r+1)} = \frac{L_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} M_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} M_{ik} V_k^{(r)} \rightarrow (7)$$

5. Continue iterations till:

$$\left|\Delta V_i^{(r+1)}\right| = \left|V_i^{(r+1)} - V_i^{(r)}\right| < \varepsilon \quad ; \; i = 2, 3, \ldots n \rightarrow (8)$$

ε — tolerance value $= 0.0001$ pu

$$\rightarrow S_i^* = V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k\right] \rightarrow (2)$$

6. Compute slack bus power after voltages are converged using Eqn (2) (Assume Bus 1 as slack bus)

$$S_1^* = P_1 - jQ_1 = V_1^* \left[\sum_{k=1}^{n} Y_{1k} V_k\right] \rightarrow (9)$$

7. (x) Compute all lines flows. Consider a line connected b/w buses i & k as shown.

Fig represents π Model of TL:

(x) $I_{ik} = (V_i - V_k) y_{ik} + V_i y_{ik0} \rightarrow$ (10a)

$I_{ki} = (V_k - V_i) y_{ik} + V_k y_{ik0} \rightarrow$ (10b)

$S_{ik} = V_i I_{ik}^* \rightarrow$ (10c)

$S_{ki} = V_k . I_{ki}^* \rightarrow$ (10d)

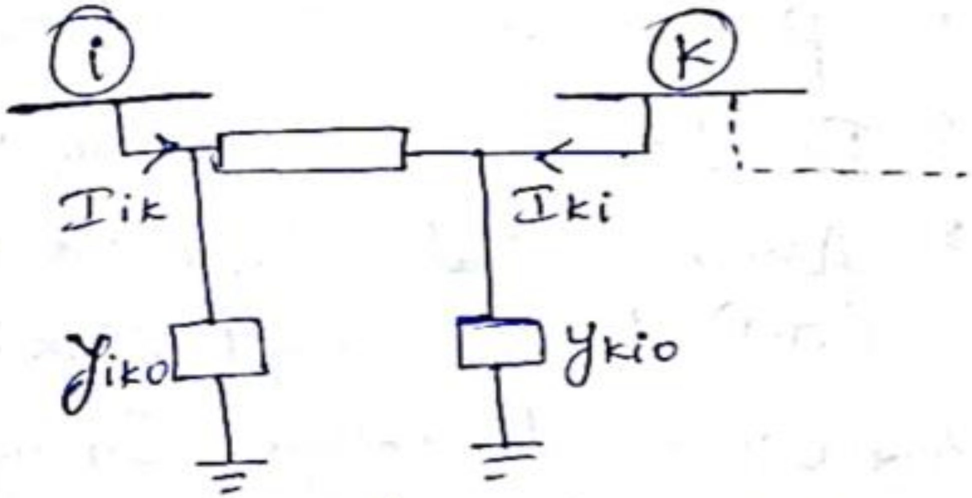

fig: Computation of line flow

(x). Complex power loss in the line is given by $S_{ik} + S_{ki}$. The total loss in the s/m is calculated by summing the loss over all the lines.

2.6.4 Modification of Algorithm to include PV Buses:

* At the PV buses, V is specified, hence it is required to calculate Q_i to be used in Eqn (6)

From Eqn (2), we have

$$Q_i = -\underset{\substack{\downarrow \\ \text{Imaginary Part}}}{Im}\left[V_i^* \sum_{k=1}^{n} Y_{ik} V_k\right] \longrightarrow (11)$$

* At any $(r+1)^{th}$ iteration, at the i^{th} PV bus,

$$Q_i^{(r+1)} = -\mathrm{Im}\left[\left(V_i^{(r)}\right)^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + \left(V_i^{(r)}\right)^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)} \right] \rightarrow (12)$$

* The steps for i^{th} PV bus are as follows:

(i) Compute $Q_i^{(r+1)}$ using Eqn (12)

(ii) Calculate V_i using Eqn (6) with $Q_i = Q_i^{(r+1)}$

2.6.5: Q-Limit Violations

* In previous algorithm if the Q limit @ the voltage controlled bus i.e $Q_i^{(r+1)}$ computed again using Eqn (12) is violated during any iteration is either less than $Q_{i,min}$ or greater than $Q_{i,max}$.
* It means that the Vtg cannot be maintained @ the specified value due to lack of reactive power support.
* This bus is treated as a PQ bus in the $(r+1)^{th}$ iteration & the Vtg is calculated with the value of Q_i set as:

If, $Q_i < Q_{i,min}$ then $Q_i = Q_{i,min}$

If, $Q_i > Q_{i,max}$, then $Q_i = Q_{i,max}$

* If in the subsequent iteration, Q_i falls within the limits, the bus can be switched back to PV bus.

2.6.6: Acceleration of Convergence:

* In GS method of load flow, the no of iterations increase with increase in the size of the s/s.
* The no. iterations reqd can be reduced if the correction in the vtg @ each bus is accelerated, by multiplying with a constant α, called the acceleration factor.
* In the $(r+1)^{th}$ iteration, $V_i^{(r+1)}(\text{accelerated}) = V_i^{(r)} + \alpha\left(V_i^{(r+1)} - V_i^{(r)}\right)$

→ where α - real no. , when $\alpha = 1$, the value of $V_i^{(r+1)}$ is the computed value

→ If $1 < \alpha < 2$, → then the value computed is extrapolated.

→ Generally α is taken b/w 1.6 to 2.0.

* At PQ buses, if the magnitude of vtg violates the limit, it indicates that the specified reactive power demand cannot be supplied, with the vtg maintained within acceptable limits