

Module- 4

Controlled Rectifiers and Ac Voltage Controller

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Module-4

Controlled Rectifiers: Introduction, Single phase half wave circuit with RL Load, Single phase half wave circuit with RL Load and Freewheeling Diode, Single phase half wave circuit with RLE Load, Single-Phase Full Converters with RLE Load, Single-Phase Dual Converters, Principle of operation of Three- Phase dual Converters.

AC Voltage Controllers: Introduction, Principle of phase control & Integral cycle control, Single- Phase Full-Wave Controllers with Inductive Loads, Three Phase Full-Wave Controllers.

Introduction:

Controlled rectifiers are basically AC to DC converters. The power transferred to the load is controlled by controlling triggering angle of the devices. Fig.1 shows this operation.

The triggering angle ' α ' of the devices is controlled by the control circuit. The input to the controlled rectifier is normally AC mains. The output of the controlled rectifier is adjustable DC voltage. Hence the power transferred across the load is regulated.

The controlled rectifiers are used in battery chargers, DC drives, DC power supplies etc. The controlled rectifiers can be single-phase or three phase depending upon the load power requirement.

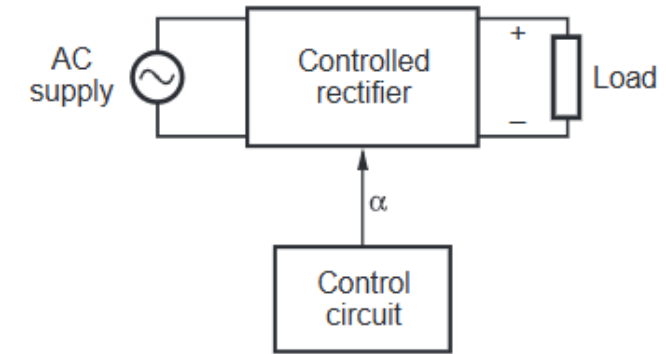


Fig. 1 Principle of operation of a controlled rectifier

Single phase half wave circuit with R Load (Single phase half wave controlled rectifier with R Load):

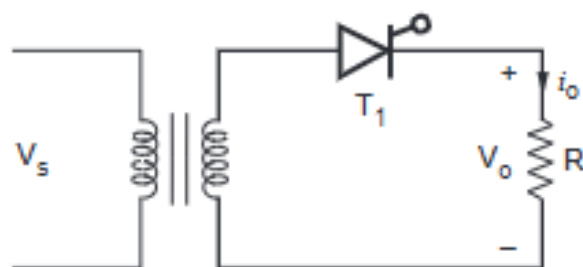
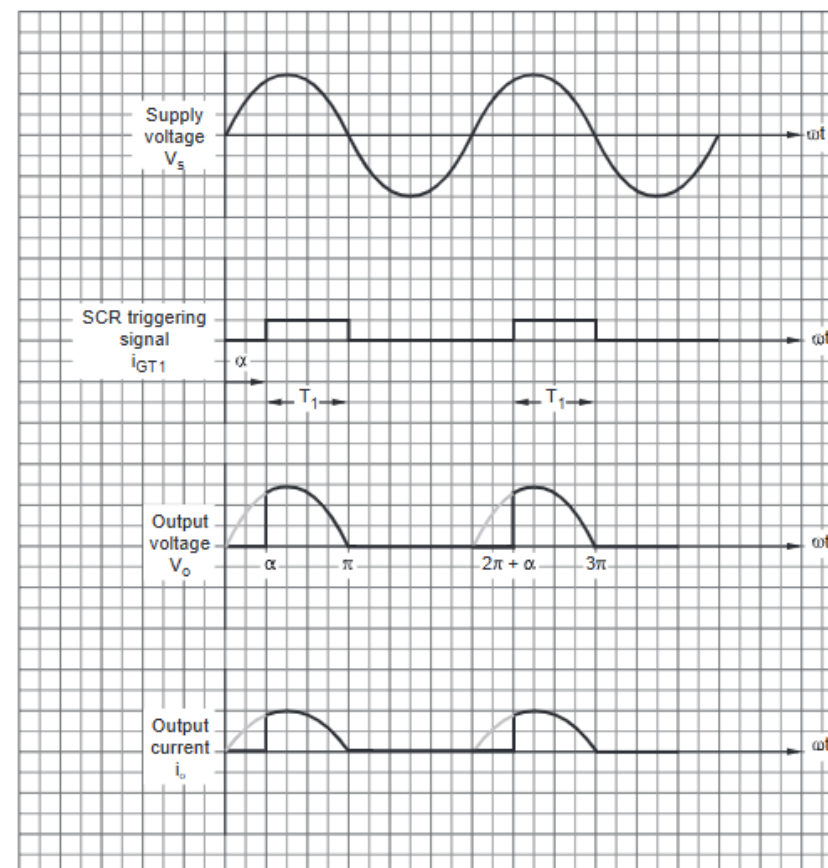


Fig.1 Half wave controlled rectifier with R-load



$$i_o = \frac{V_o}{R}$$

$f_{ripple} = 50$ Hz i.e. supply frequency

The average value of output voltage is given as,

$$V_{o(av)} = \frac{1}{T} \int_0^T v_o(\omega t) d\omega t$$

The period of one pulse of $v_o(\omega t)$ can be considered as $T = 2\pi$. And $v_o(\omega t) = V_m \sin \omega t$ from α to π . For rest of the period $v_o(\omega t) = 0$. Hence above equation can be written as,

$$\begin{aligned} V_{o(av)} &= \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d\omega t \\ &= \frac{V_m}{2\pi} [-\cos \omega t]_{\alpha}^{\pi} \end{aligned}$$

\therefore

$$V_{o(av)} = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

The power transferred to the load will be,

$$P_{o(av)} = \frac{V_{o(av)}^2}{R}$$

Single phase half wave circuit with RL Load (Single phase half wave controlled rectifier with RL Load)

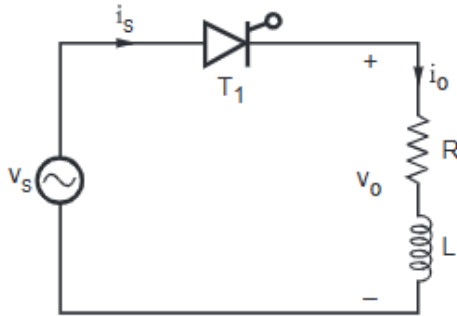
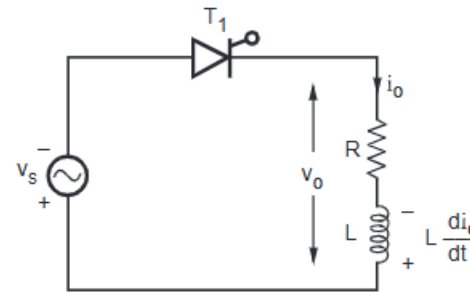


Fig 1 Half wave controlled rectifier with RL load



SCR conducts due to inductance voltage after π

$$V_{o(av)} = \frac{1}{T} \int_0^T v_o(\omega t) d\omega t$$

... (1)

In Fig. 4.2.4 observe that,

$$v_o(\omega t) = \begin{cases} v_s = V_m \sin \omega t & \text{from } \alpha \text{ to } \beta \\ 0 & \text{from } 0 \text{ to } \alpha \text{ and } \beta \text{ to } 2\pi \end{cases}$$

Hence equation (4.2.2) can be written as,

$$\begin{aligned} V_{o(av)} &= \frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin \omega t d\omega t \\ &= \frac{V_m}{2\pi} [-\cos \omega t]_{\alpha}^{\beta} \\ &= \frac{V_m}{2\pi} (\cos \alpha - \cos \beta) \end{aligned}$$

... (2)

This is an expression for average value of output voltage.

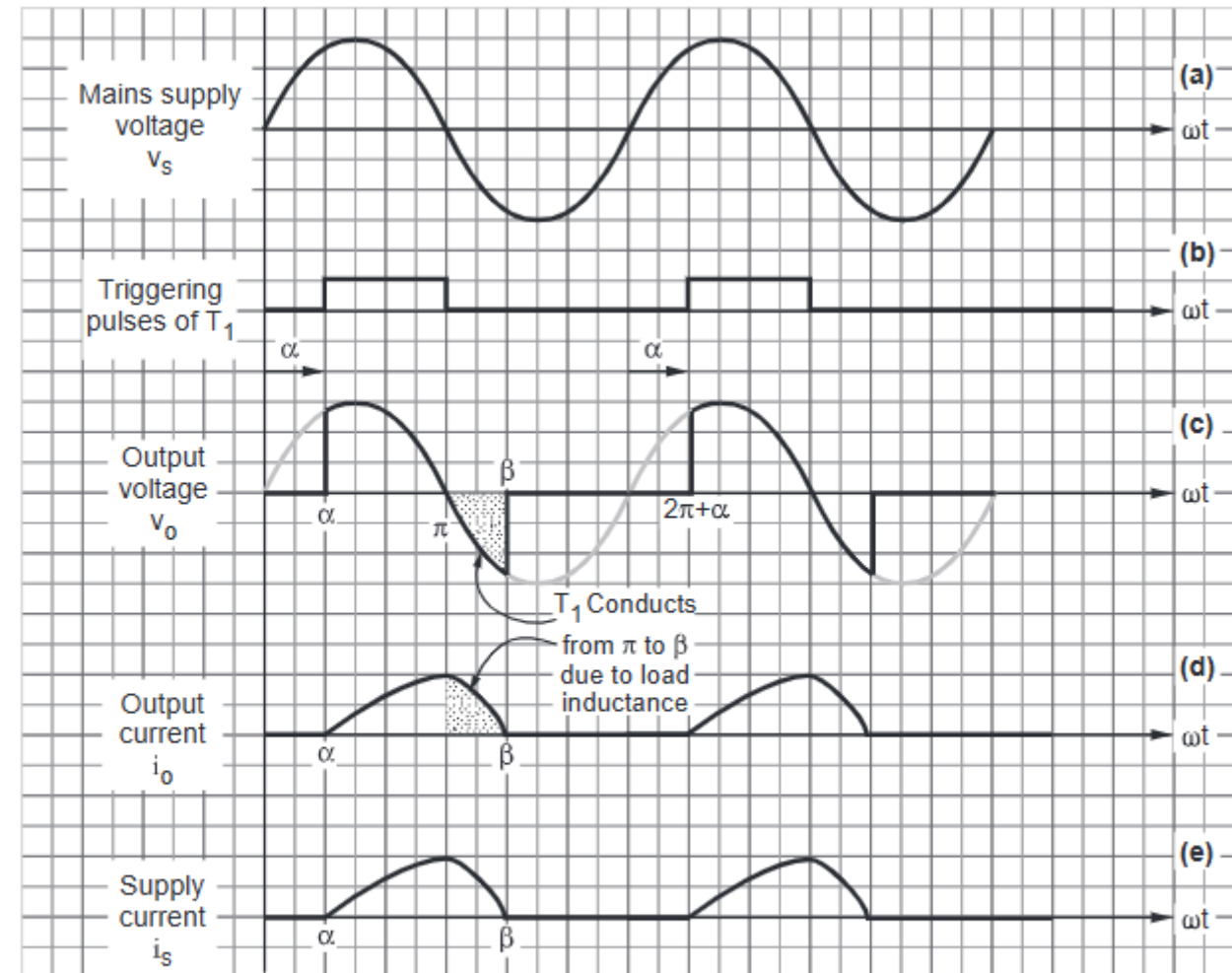


Fig. 4.2 Waveforms of half wave controlled rectifier for RL load

Single phase half wave circuit with RL Load with free-wheeling diode(Single phase half wave controlled rectifier with

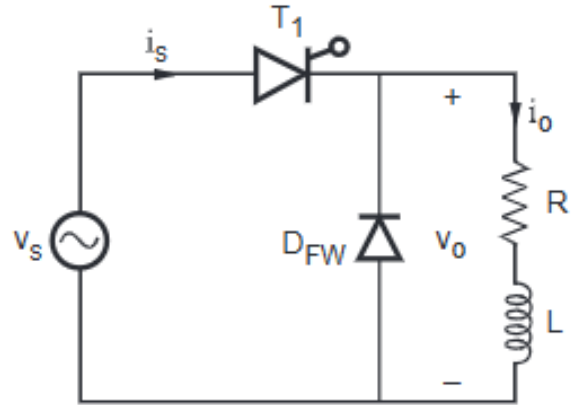
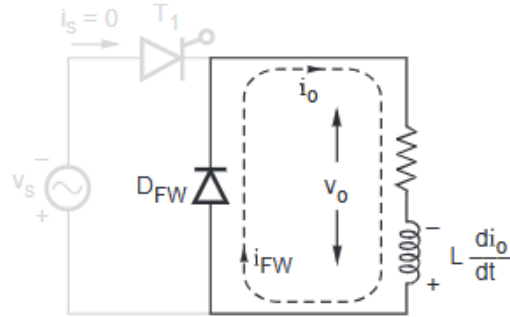
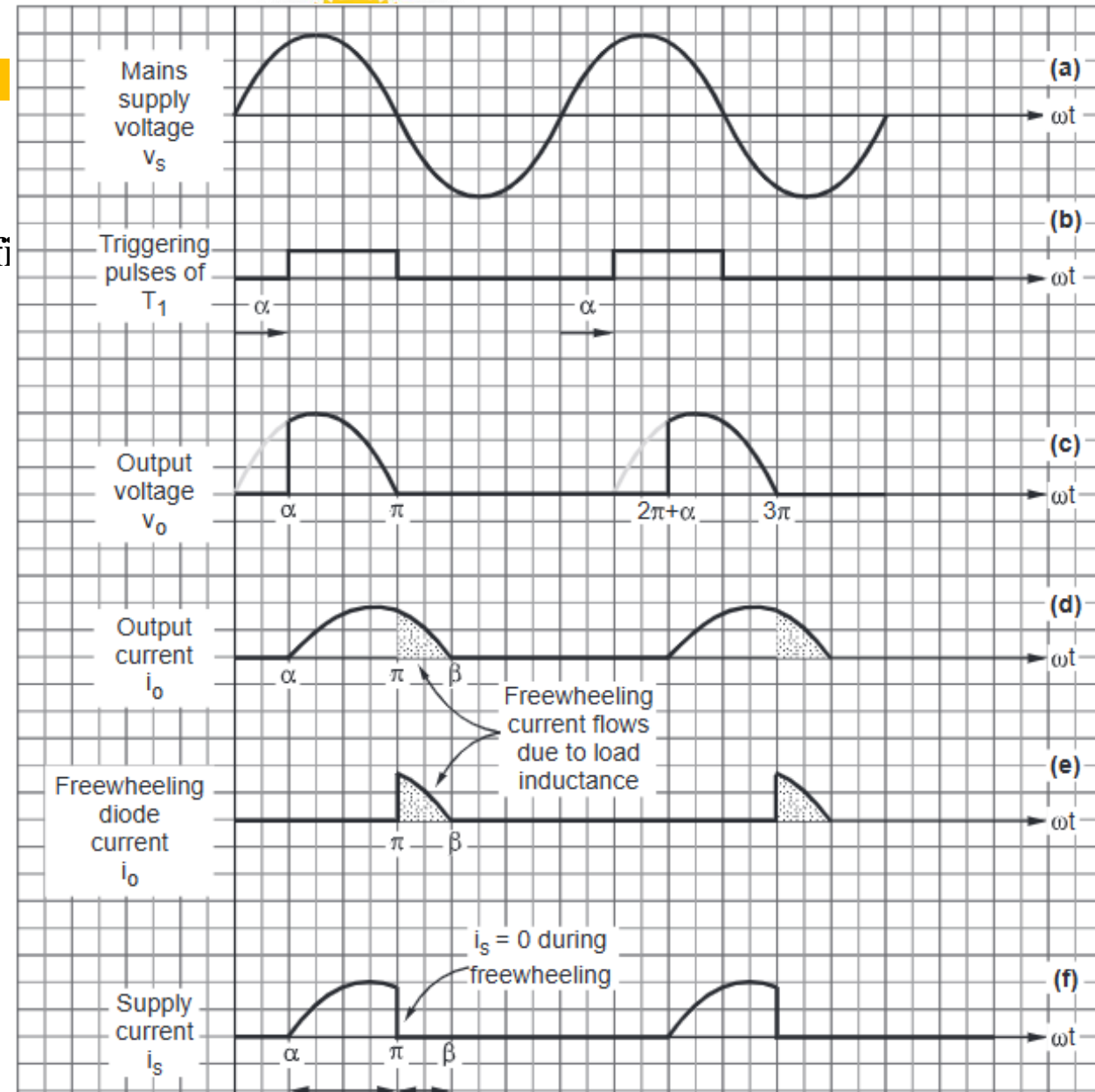


Fig. 1 Freewheeling diode in half wave controlled rectifier



Freewheeling action in half wave controlled rectifier



$$v_o = \begin{cases} v_s = V_m \sin \omega t & \text{from } \alpha \text{ to } \pi \\ 0 & \text{from } 0 \text{ to } \alpha \text{ and } \pi \text{ to } 2\pi \end{cases}$$

The period of v_o is 2π . The average value is given as,

$$\begin{aligned} v_{o(av)} &= \frac{1}{T} \int_0^T v_o(\omega t) d\omega t = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d\omega t \\ &= \frac{V_m}{2\pi} [-\cos \omega t] \\ &= \frac{V_m}{2\pi} [1 + \cos \alpha] \end{aligned}$$

The r.m.s. value is given as,

$$V_{o(r.m.s.)} = \left[\frac{1}{T} \int_0^T v_o^2(\omega t) d\omega t \right]^{\frac{1}{2}}$$

$$\begin{aligned} V_{o(r.m.s.)} &= \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} v_m^2 \sin^2 \omega t d\omega t \right]^{\frac{1}{2}} \\ &= \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} d\omega t \right]^{\frac{1}{2}} \\ &= \left\{ \frac{V_m^2}{4\pi} \left[\int_{\alpha}^{\pi} d\omega t - \int_{\alpha}^{\pi} \cos 2\omega t d\omega t \right] \right\}^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} &= \left\{ \frac{V_m^2}{4\pi} \left[[\omega t]_{\alpha}^{\pi} - \left[\frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} \right] \right\}^{\frac{1}{2}} \\ &= \frac{V_m}{2} \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]^{\frac{1}{2}} \end{aligned}$$

Single phase half wave circuit with RLE Load (Single phase half wave controlled rectifier with RLE Load):

Expression of Average Current and Voltage :

- Since the average voltage across the inductance is zero, thus the average value of load is given by,

$$I_{avg} = \frac{1}{2\pi R} \left[\int_{\alpha}^{\gamma} (V_m \sin \omega t - E) d\omega t \right]$$

$$= \frac{1}{2\pi R} [V_m (\cos \alpha - \cos \gamma) - E(\gamma - \alpha)]$$

Conduction angle being β , $\beta = \gamma - \alpha$. Thus $\gamma = \alpha + \beta$,

$$\therefore I_{avg} = \frac{1}{2\pi R} [V_m \{ \cos \alpha - \cos (\alpha + \beta) \} - E\beta]$$

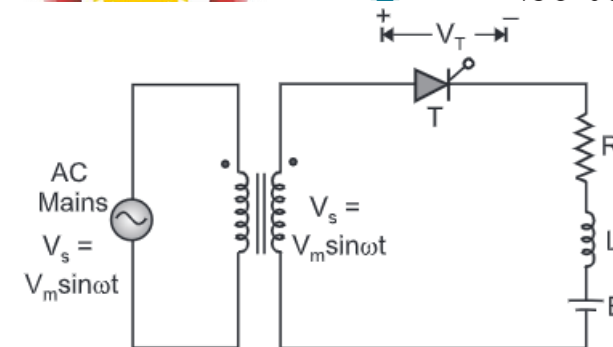
$$\text{or } I_{avg} = \frac{1}{2\pi R} \left[2V_m \sin \left(\alpha + \frac{\beta}{2} \right) \sin \frac{\beta}{2} - E\beta \right] \quad \dots (2.12)$$

$$\left(\text{since } \cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{B-A}{2} \right)$$

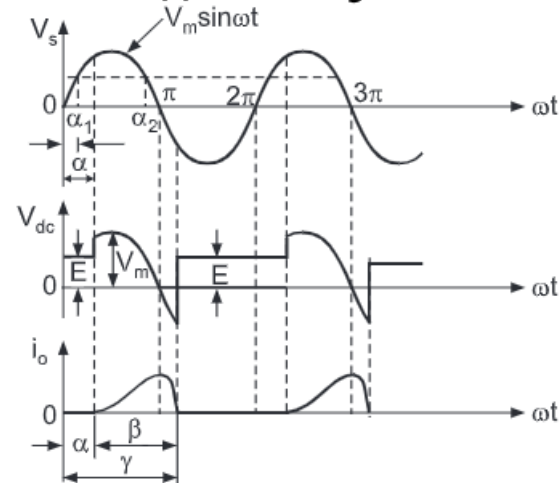
However, $V_{avg} = E + I_{avg}R$

$$= E + \frac{1}{2\pi} \left[2V_m \sin \left(\alpha + \frac{\beta}{2} \right) \sin \frac{\beta}{2} - E\beta \right]$$

$$\text{or } V_{avg} = E \left(1 - \frac{\beta}{2\pi} \right) + \frac{V_m}{\pi} \sin \left(\alpha + \frac{\beta}{2} \right) \sin \frac{\beta}{2}$$



(a) Circuit diagram

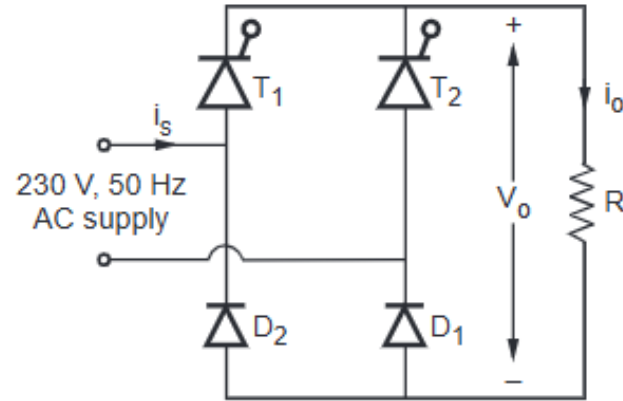


(b) Voltage and Current waveforms

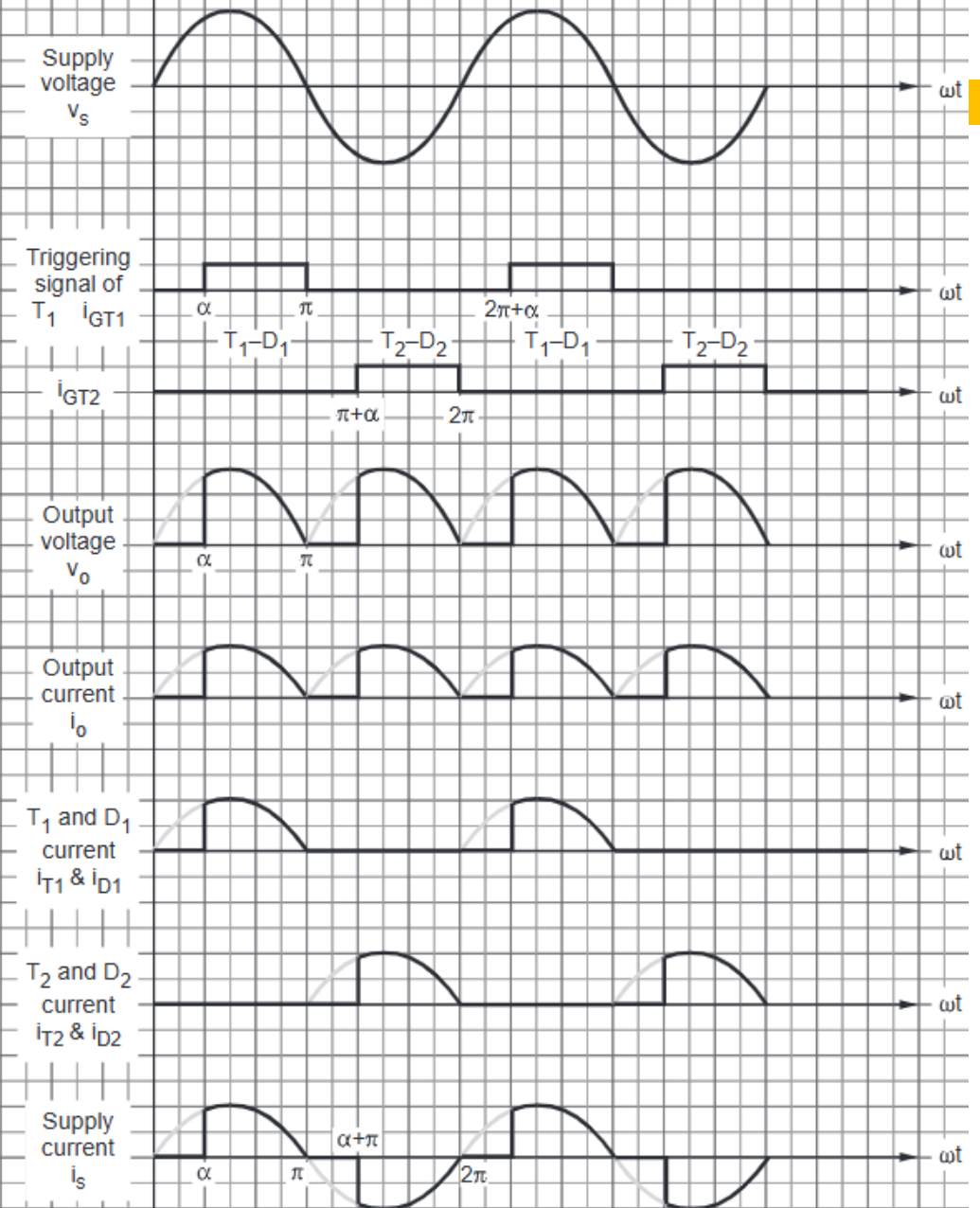
Single-phase half-wave rectifier with RLE load

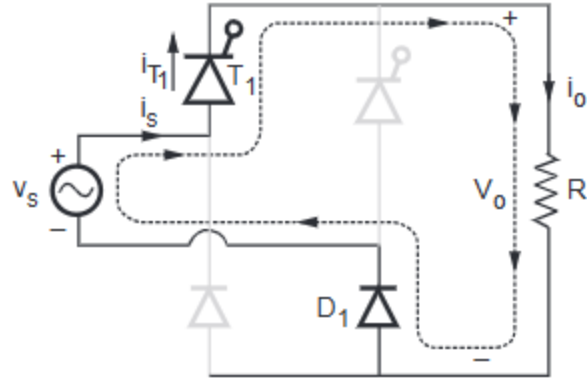
Single Phase Semi converters (Half Bridge Converter):

Single Phase Semi converters working with Resistive Load:



Circuit diagram of 1 ϕ semiconverter





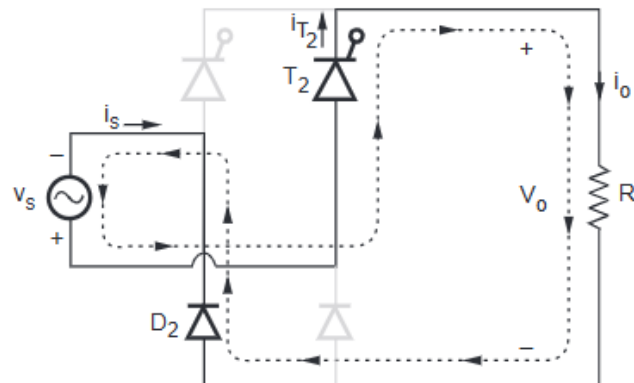
and

$$V_o = V_s \quad (\text{i.e. supply voltage})$$

$$i_o = \frac{V_o}{R} = \frac{V_s}{R}$$

$$i_o = \frac{V_o}{R}$$

$$i_o = i_s = i_{T1} \quad (\text{when } T_1 - D_1 \text{ conducts})$$



i) Average output voltage

The average output voltage is given as,

$$V_o(av) = \frac{1}{T} \int_0^T V_o(\omega t) d\omega t$$

$$V_o(av) = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d\omega t$$

In the above equation $V_o(\omega t) = V_m \sin \omega t$ from α to π . Solving the above integration we get,

$$V_o(av) = \frac{V_m}{\pi} (1 + \cos \alpha)$$

ii) R.M.S. output voltage

R.M.S. output voltage is given as,

$$V_o(rms) = \left[\frac{1}{T} \int_0^T V_o^2(\omega t) d\omega t \right]^{1/2}$$

Putting the values in above equation,

$$\begin{aligned} V_o(rms) &= \left[\frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t d\omega t \right]^{1/2} = \left\{ \frac{V_m^2}{\pi} \int_{\alpha}^{\pi} \left[\frac{1 - \cos(2\omega t)}{2} \right] d\omega t \right\}^{1/2} \\ &= \left\{ \frac{V_m^2}{2\pi} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right] \right\}^{1/2} \end{aligned}$$

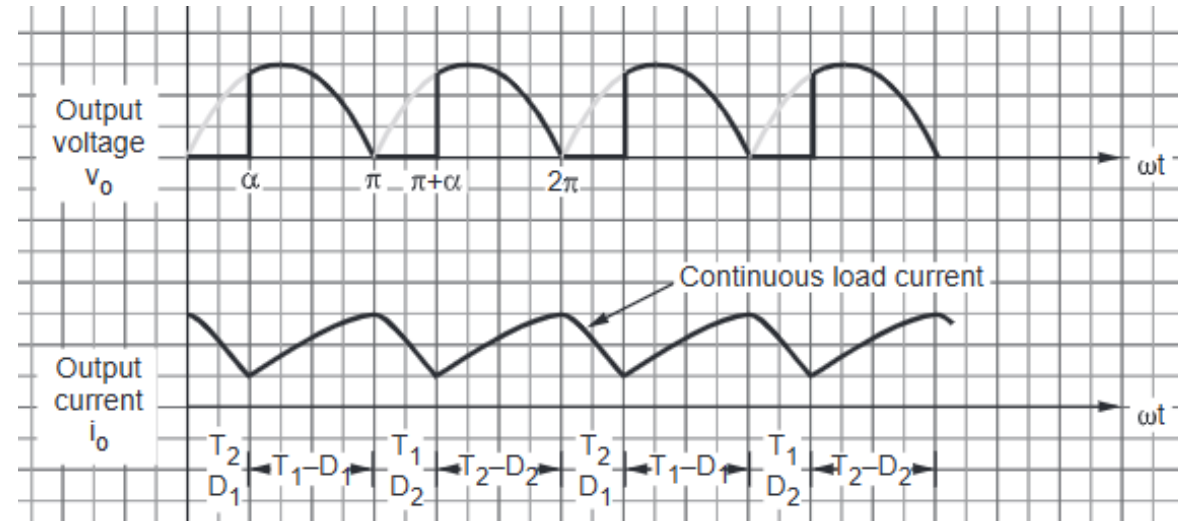
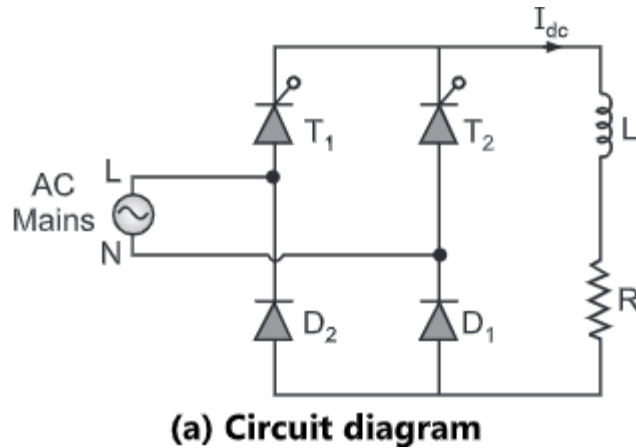
and

$$V_o = -V_s$$

$$i_o = \frac{V_o}{R} = -\frac{V_s}{R}$$

Single Phase Semi converters working with RL Load:

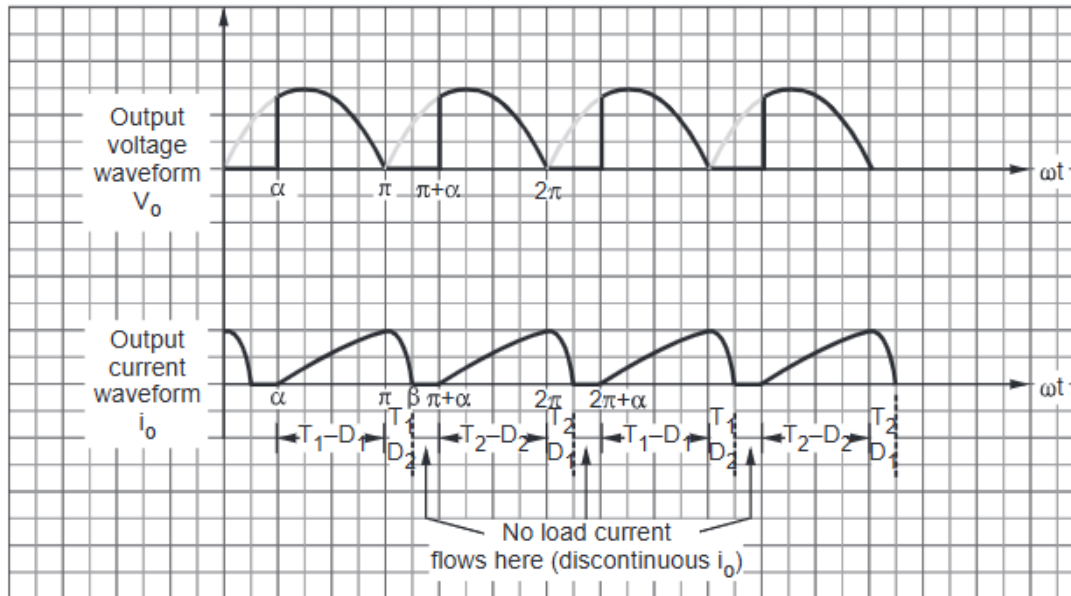
Continuous current mode :



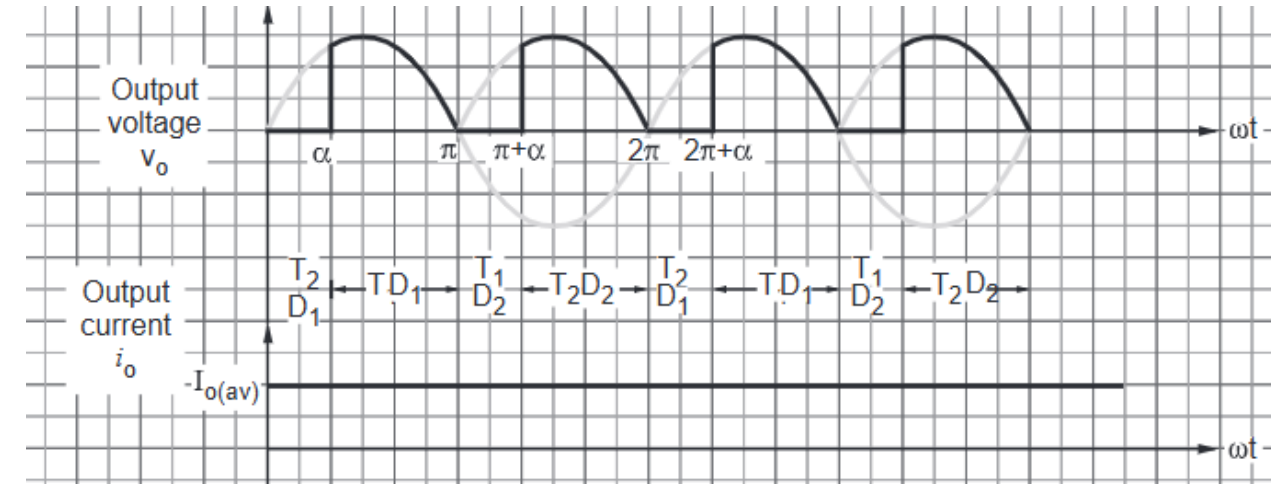
$$V_o(av) = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$V_o(rms) = \left\{ \frac{V_m^2}{2\pi} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right] \right\}^{\frac{1}{2}}$$

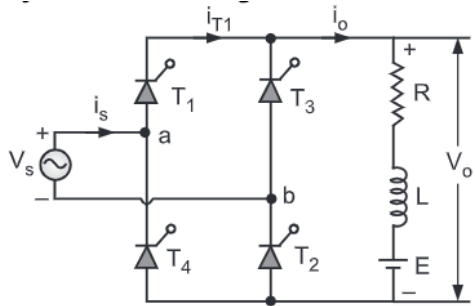
Discontinuous current mode :



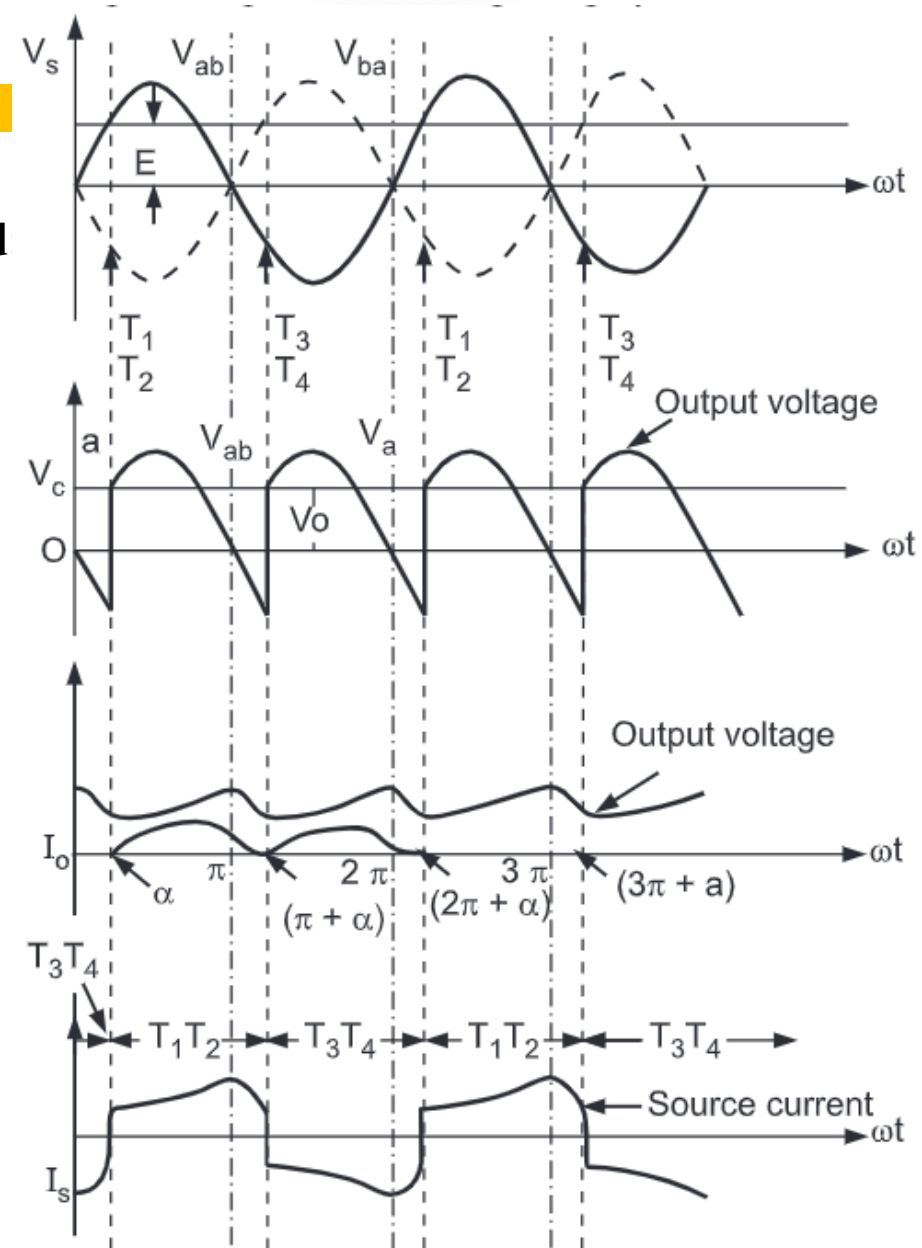
Continuous and ripple free current for large inductive load



Single Fully Controlled Bridge Rectifier with RLE Load



Circuit diagram of single phase fully controlled rectifier



Single phase fully controlled rectifier waveforms

Mathematical Analysis :

1. Expression for the Average Output Voltage (V_{avg}) :

$$\begin{aligned} V_{avg} &= \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} V_m \sin \omega t \cdot d\omega t \\ &= \frac{1}{\pi} [-V_m \cos \omega t]_{\alpha}^{\pi + \alpha} \\ &= \frac{V_m}{\pi} [\cos \alpha - \cos (\pi + \alpha)] \\ &= \frac{V_m}{\pi} [\cos \alpha - \cos \pi \cdot \cos \alpha + \sin \pi \cdot \sin \alpha] \\ V_{avg} &= \frac{2V_m}{\pi} \cos \alpha \end{aligned}$$

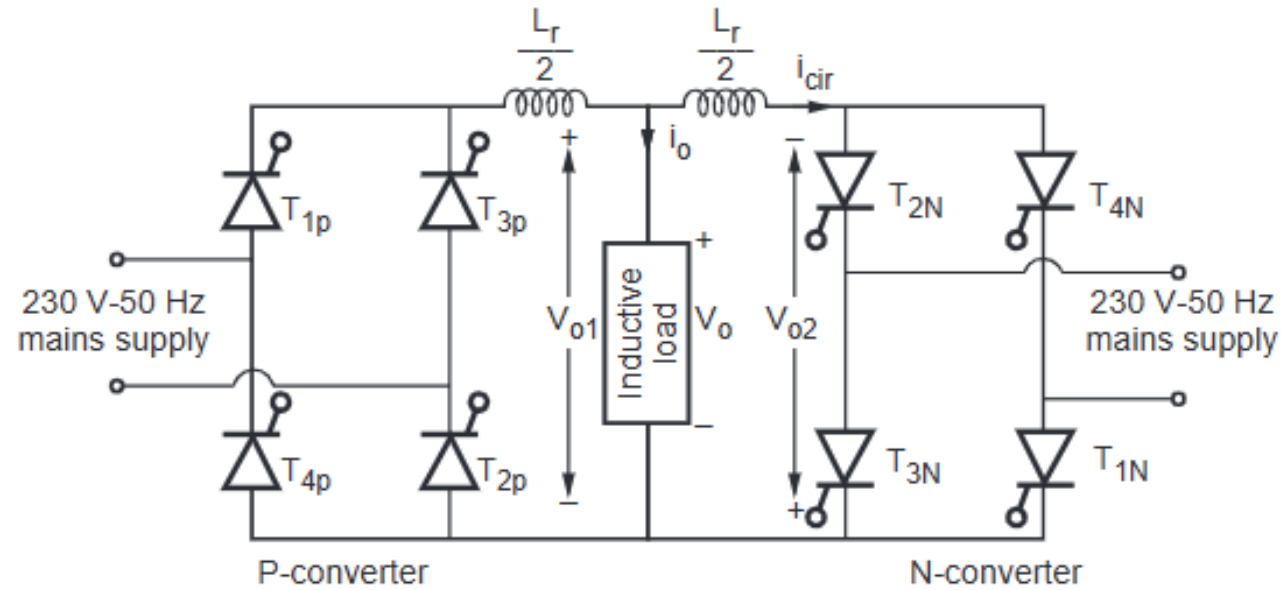
Expression for the Average Load Current (I_{avg}) :

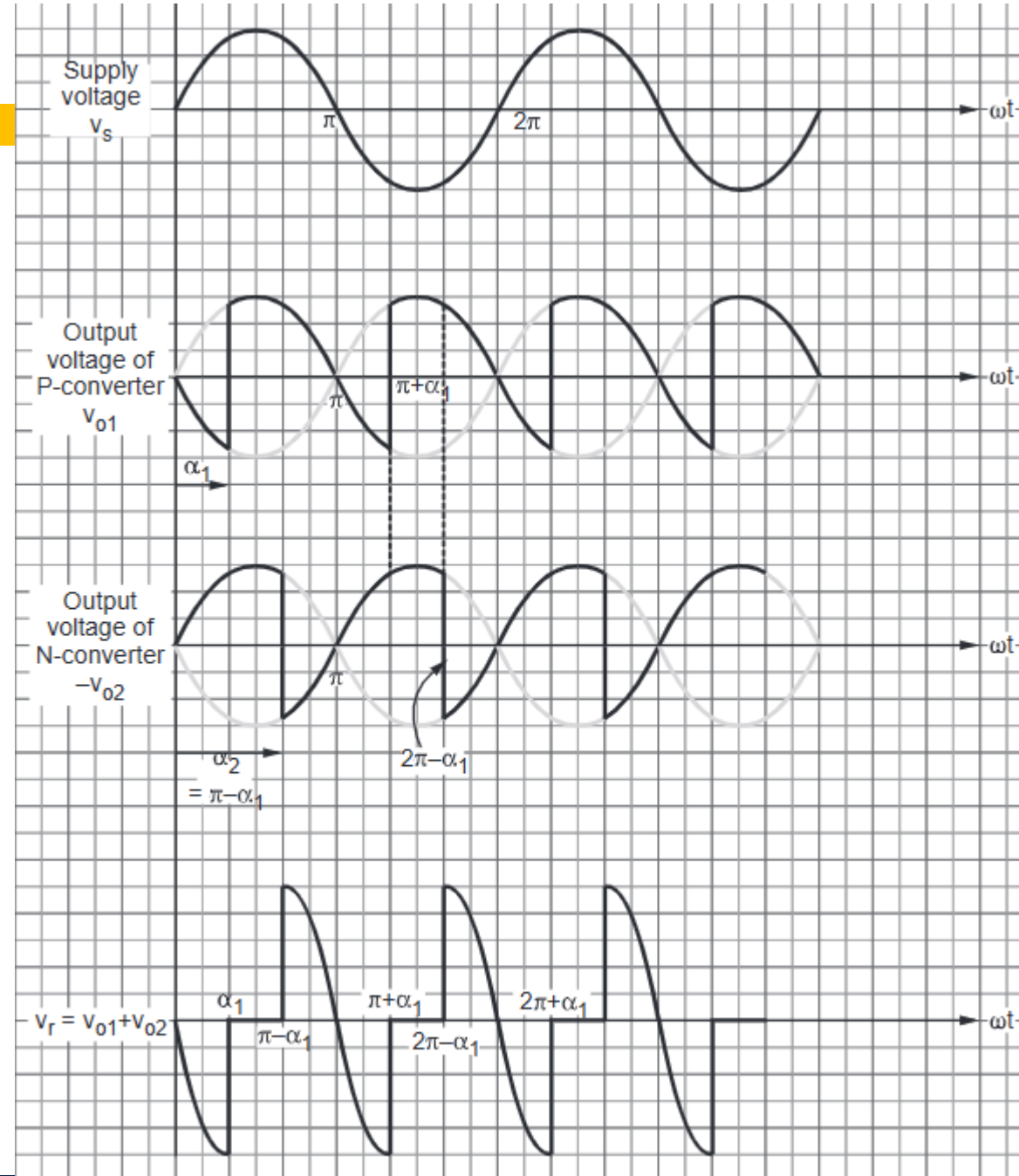
$$I_{avg} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R} \cos \alpha$$

Expression for the rms Load Voltage (V_{rms}) :

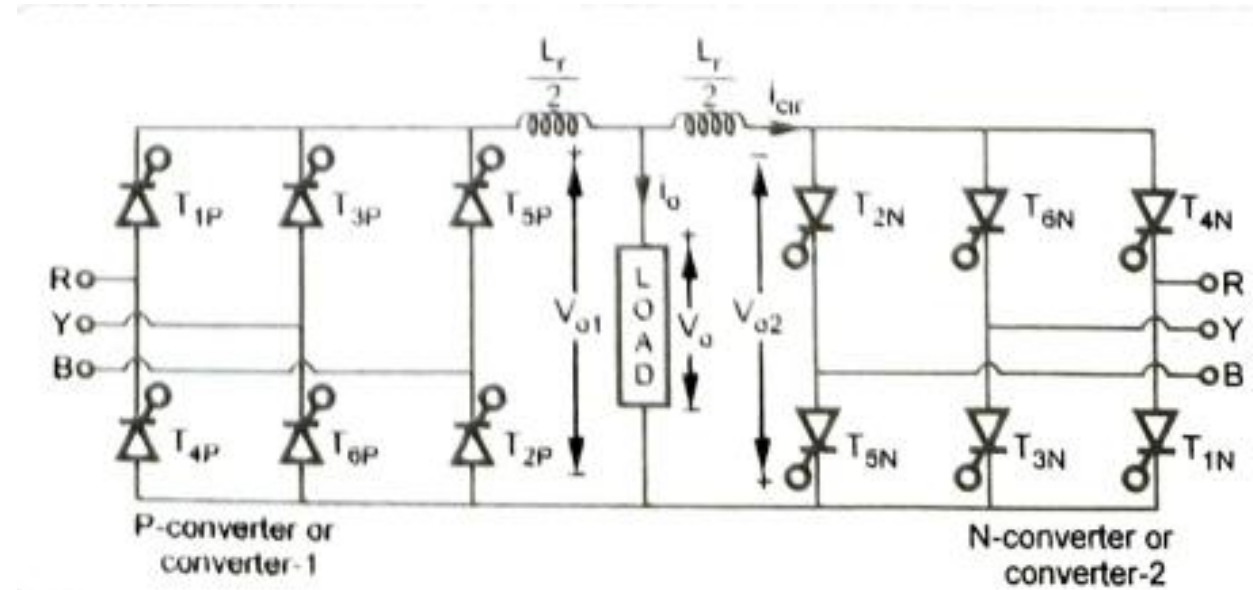
$$\begin{aligned} V_{rms} &= \left[\frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} V_m^2 \sin^2 \omega t \cdot d\omega t \right]^{1/2} \\ &= \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \cdot d\omega t \right]^{1/2} \\ &= \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\omega t - \frac{\sin 2\omega t}{2} \right)_{\alpha}^{\pi + \alpha} \right]^{1/2} \\ &= \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi + \alpha - \frac{\sin (2\pi + 2\alpha)}{2} - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \\ &= \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \\ V_{rms} &= \frac{V_m}{\sqrt{2}} \end{aligned}$$

Single Phase Dual Converters:





Three Phase Dual Converters:



- We know that the average output voltage of 3 ϕ full converter is given for highly inductive as,

$$V_{o(av)} = \frac{3\sqrt{3} V_m}{\pi} \cos \alpha \quad \dots (1)$$

Hence the outputs of the two converters will be,

$$V_{o1(av)} = \frac{3\sqrt{3} V_m}{\pi} \cos \alpha_1$$

and

$$V_{o2(av)} = \frac{3\sqrt{3} V_m}{\pi} \cos \alpha_2$$

From Fig. 4.9.1 it is clear that,

$$V_o(av) = V_{o1(av)} = -V_{o2(av)} \quad \dots (2)$$

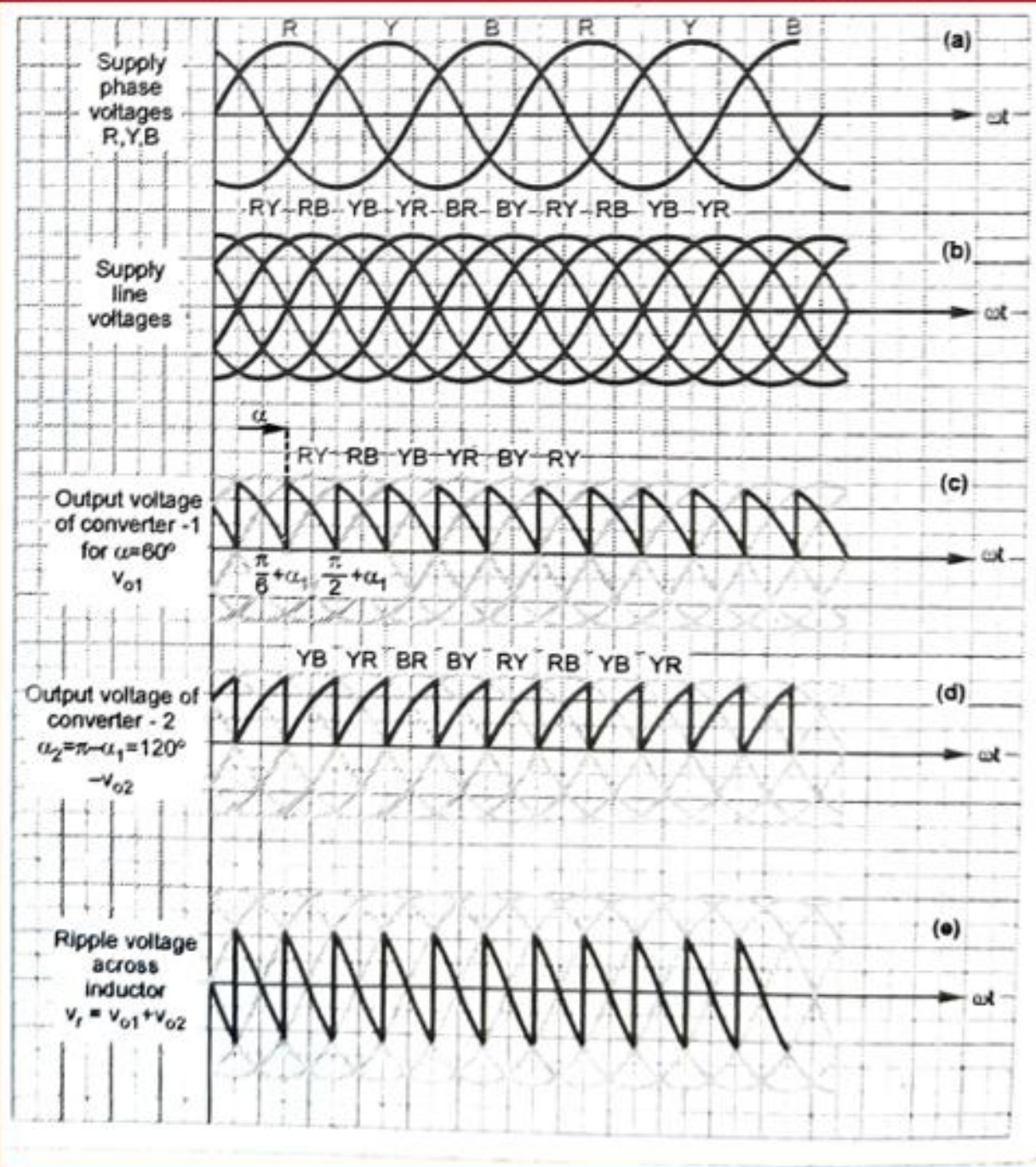
From equation (4.9.1) and above equation we have,

$$\frac{3\sqrt{3} V_m}{\pi} \cos \alpha_1 = -\frac{3\sqrt{3} V_m}{\pi} \cos \alpha_2$$

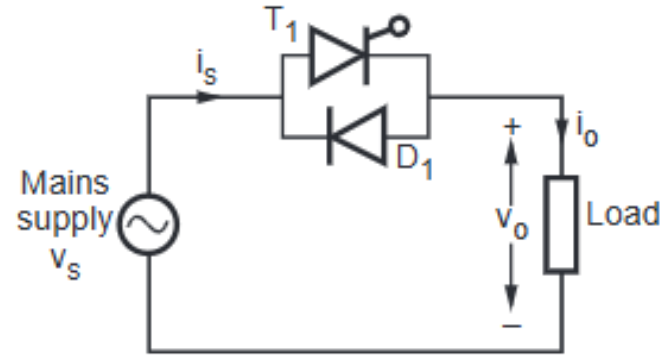
$$\cos \alpha_1 = -\cos \alpha_2$$

or

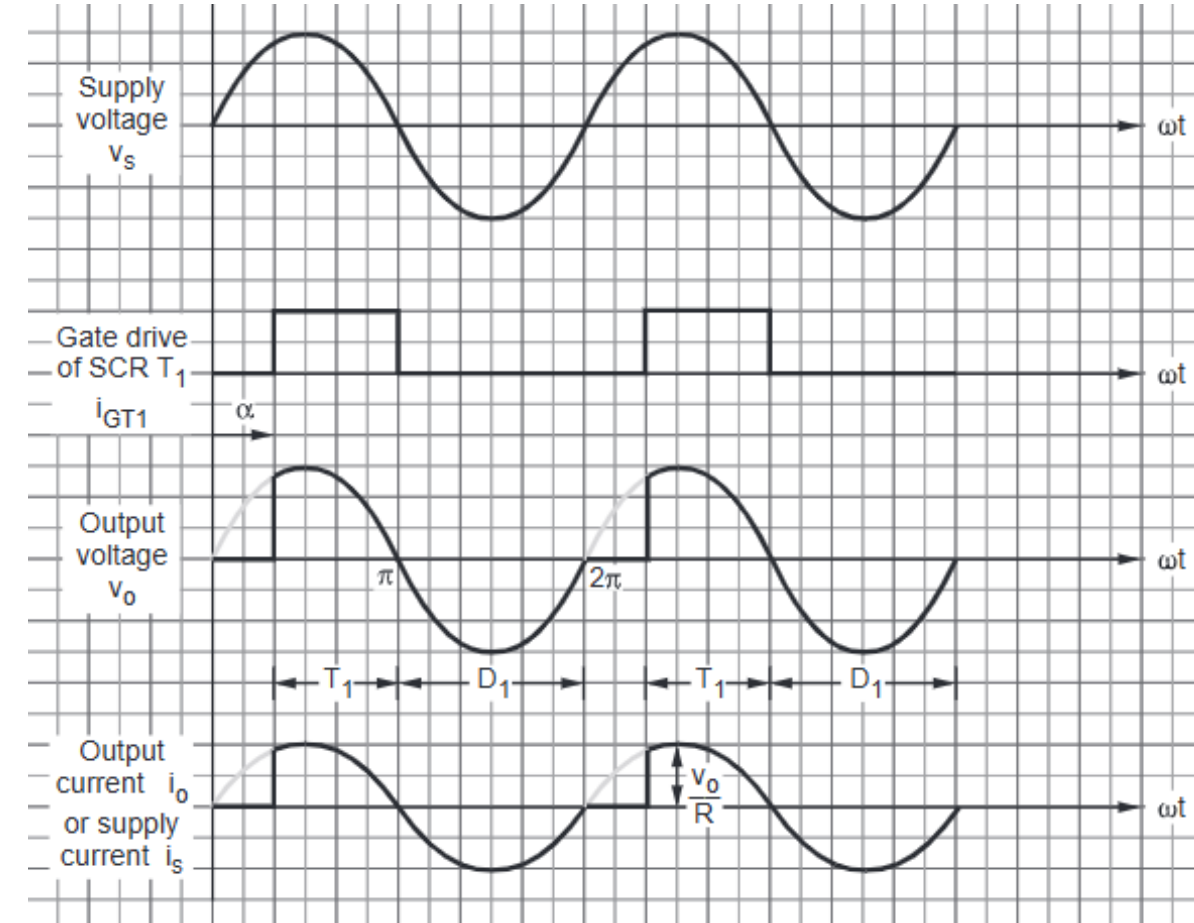
$$\alpha_2 = \pi - \alpha_1 \quad \dots (3)$$



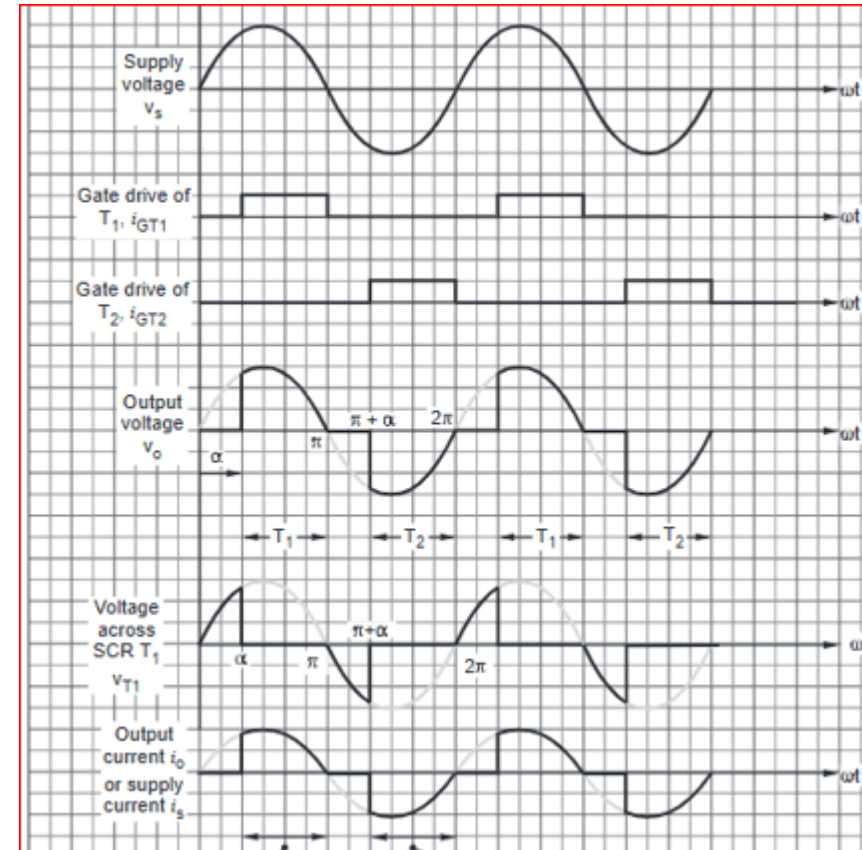
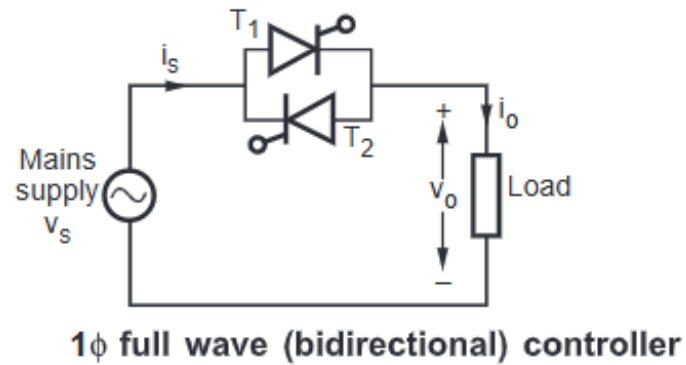
Principle of Phase Control



**1 ϕ half wave controller
(unidirectional controller)**



Single Phase Controllers with Resistive Loads (Bidirectional Controllers):



Single Phase Controllers with Inductive Loads:

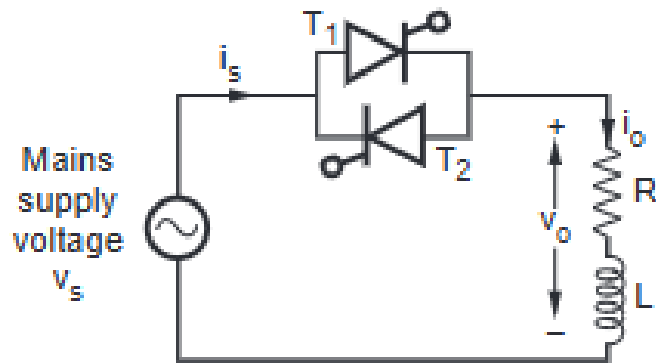
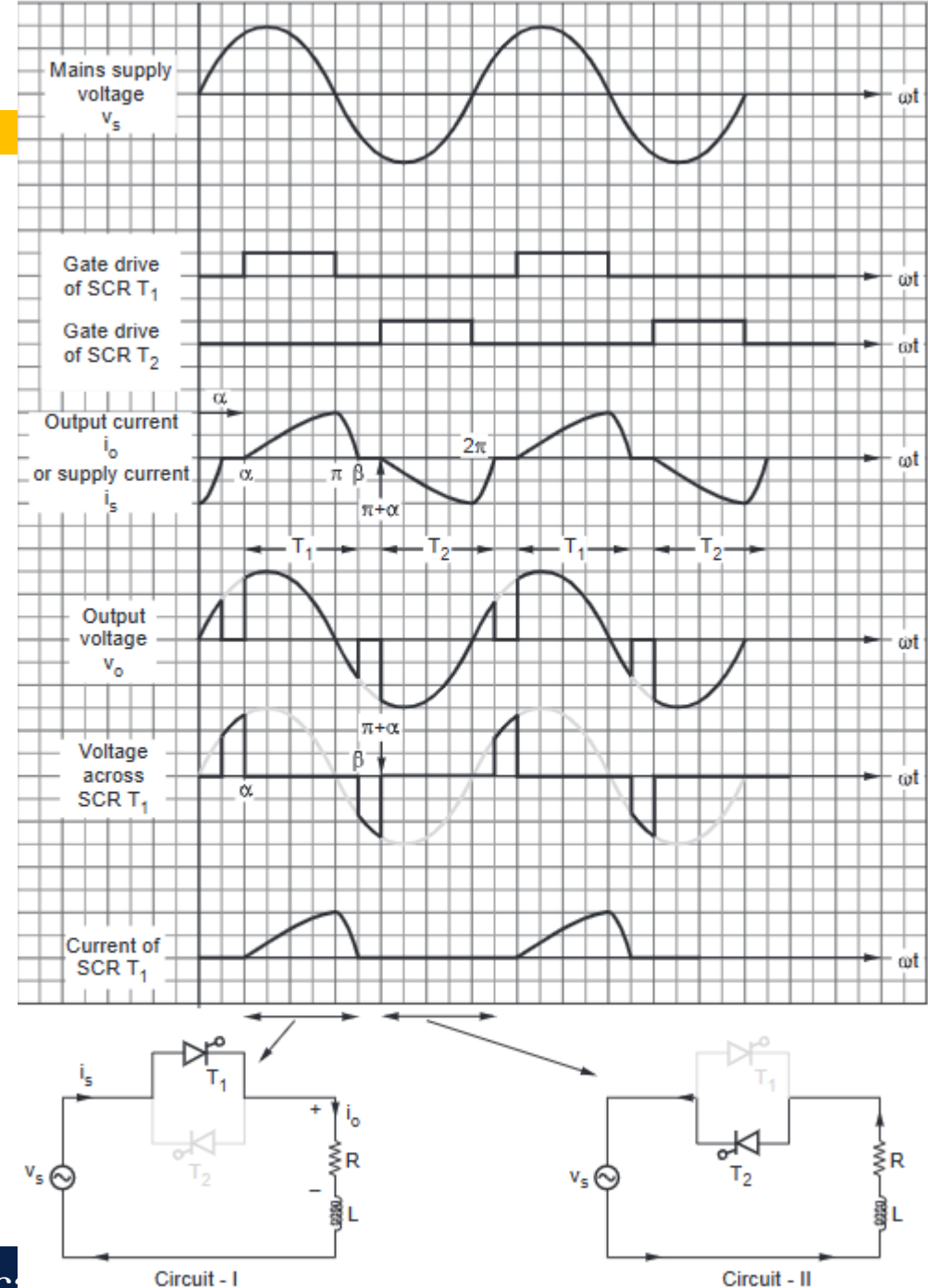
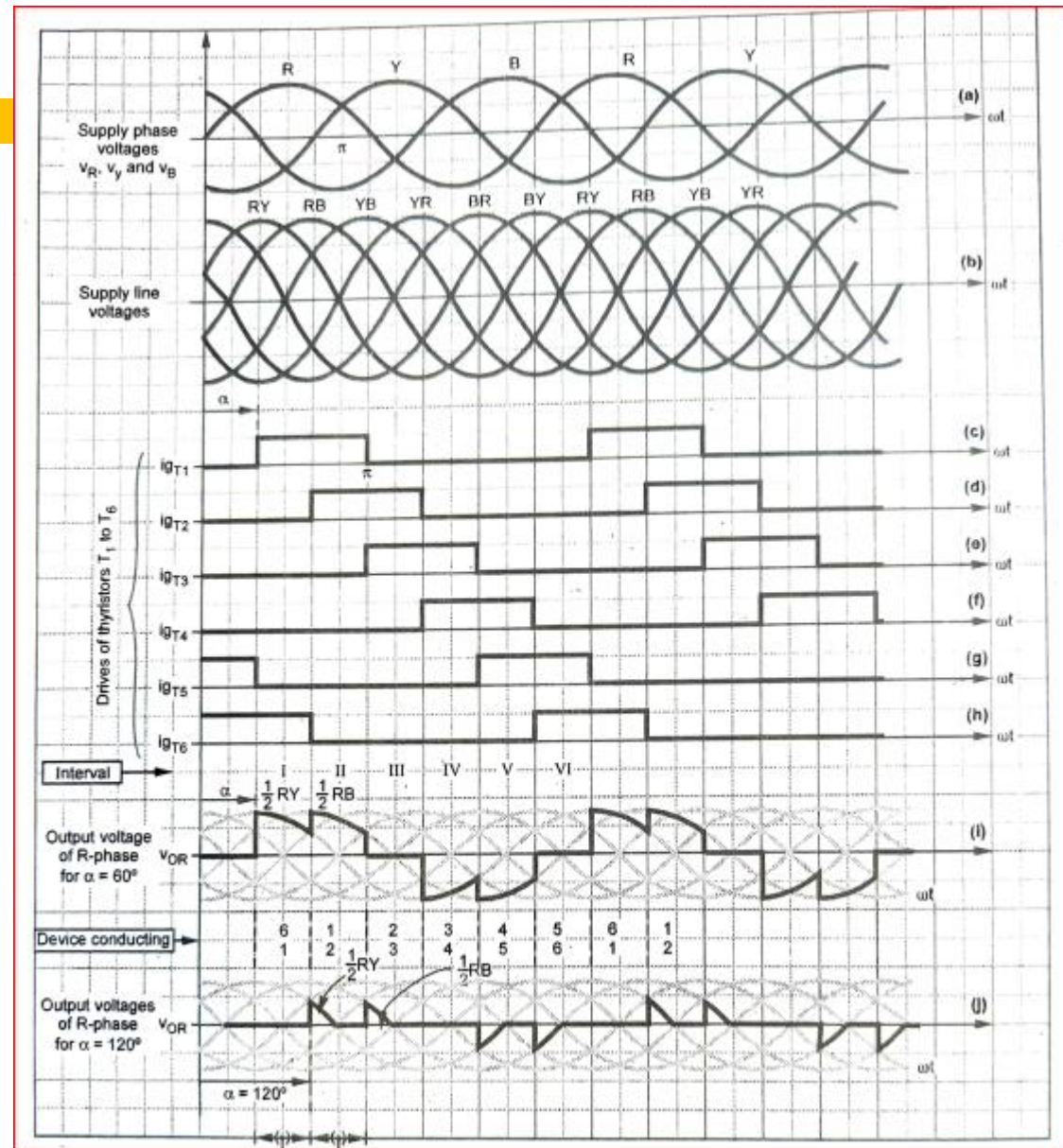
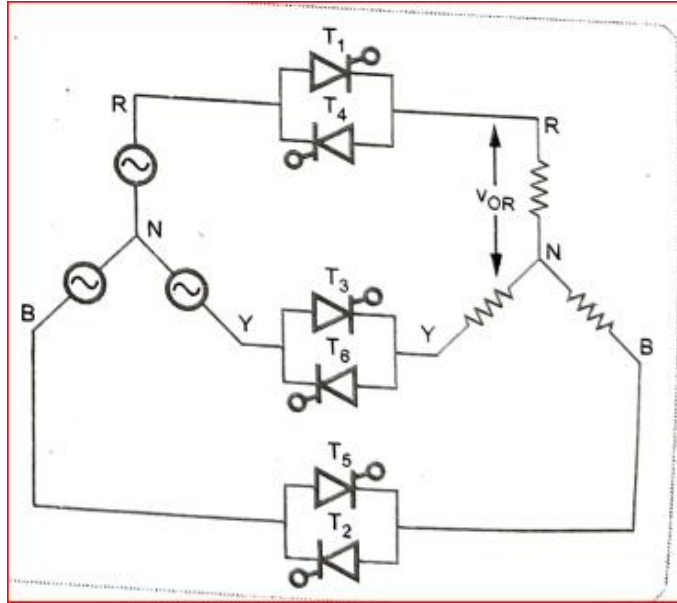


Fig 1 1 ϕ full wave controller with inductive load



3 phase Full Wave AC Voltage Controllers:





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