Module-4

Controlled Rectifiers: *Introduction, Single phase half wave circuit with RL Load, Single phase half wave circuit with RL Load and Freewheeling Diode, Single phase half wave circuit with RLE Load, Single-Phase Full Converters with RLE Load, Single-Phase Dual Converters, Principle of operation of Three- Phase dual Converters.*
AC Voltage Controllers: *Introduction, Principle of phase control & Integral cycle control, , Single- Phase Full-Wave Controllers with Inductive Loads, Three Phase Full-Wave Controllers.*

Introduction:

Controlled rectifiers are basically AC to DC converters. The power transferred to the load is controlled by controlling triggering angle of the devices. Fig.1 shows this operation.

The triggering angle 'α' of the devices is controlled by the control circuit. The input to the controlled rectifier is normally AC mains. The output of the controlled rectifier is adjustable DC voltage. Hence the power transferred across the load is regulated.

The controlled rectifiers are used in battery chargers, DC drives, DC power supplies etc. The controlled rectifiers can be single-phase or three phase depending upon the load power requirement.

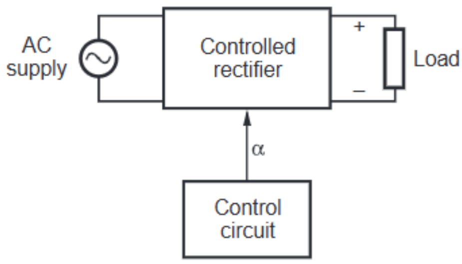


Fig. 1 Principle of operation of a controlled rectifier

Single phase half wave circuit with R Load (Single phase half wave controlled rectifier with R Load):

The principle of phase-controlled operation can be explained with the help of half wave controlled rectifier shown in Fig. 1. secondary of transformer is connected to resistive load through thyristor or SCR Tl. The primary of the transformer is connected to the mains supply. In the positive cycle of the supply, Tl is forward-biased. T1 is triggered at an angle α This is also called as triggering or firing delay angle. Tl conducts and secondary (i.e. supply) voltage is applied to the load. Current io starts flowing through the load. The output current and voltage waveforms are shown in Fig. .2.

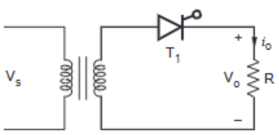


Fig. 1 **Half wave controlled rectifier with R-load**

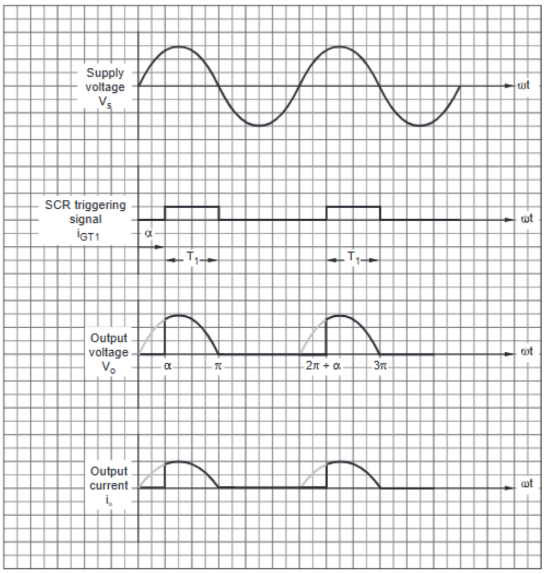


Fig 2 **Phase control principle as applied to half wave controlled rectifier**

Since the load is resistive, output current is given as,

$$i_o = \frac{V_o}{R}$$

Hence the shape of output current waveform is same as output voltage waveform. At π supply voltage drops to zero. Hence current io flowing through Tl becomes zero and it turns off. In the negative half cycle of the supply T 1 is reverse biased and it does not conduct. There is only one pulse of Vo during one cycle of the supply. Hence ripple frequency of the output voltage is,

$$f_{ripple} = 50 \text{ Hz i.e. supply frequency}$$

The average value of output voltage is given as,

$$V_{o(av)} = \frac{1}{T}\int_0^T v_o(\omega t)\, d\omega t$$

The period of one pulse of $v_o(\omega t)$ can be considered as $T = 2\pi$. And $v_o(\omega t) = V_m \sin \omega t$ from α to π. For rest of the period $v_o(\omega t) = 0$. Hence above equation can be written as,

$$V_{o(av)} = \frac{1}{2\pi}\int_\alpha^\pi V_m \sin \omega t\, d\omega t$$

$$= \frac{V_m}{2\pi}\left[-\cos \omega t\right]_\alpha^\pi$$

$$\therefore \quad \boxed{V_{o(av)} = \frac{V_m}{2\pi}(1 + \cos\alpha)}$$

The power transferred to the load will be,

$$P_{o(av)} = \frac{V_{o(av)}^2}{R}$$

Single phase half wave circuit with RL Load (Single phase half wave controlled rectifier with RL Load):

The SCR will be forward biased in the positive half cycle of the supply. Hence SCR is applied with the firing pulses in the positive half cycle. The waveforms are shown in Fig. 1. Fig. 2(a) shows the supply voltage and Fig. 2(b) shows the firing pulses to the SCR.

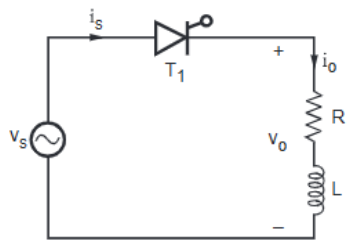
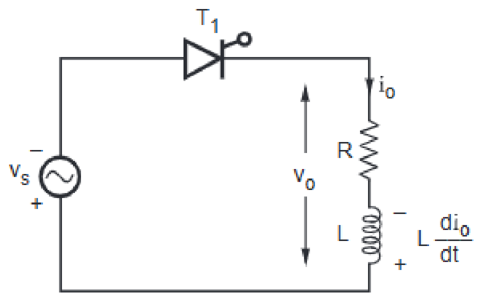


Fig 1 **Half wave controlled rectifier with RL load**

When the SCR is triggered, the supply voltage appears across load. We normally neglect small voltage drop in SCR. Hence Vo = Vs when SCR is conducting. This is shown in Fig.2.(c). Observe that output voltage is same as supply voltage after α. Because of the RL load, output current starts increasing slowly from zero. The shape of io depends upon values of R and L. At π , the supply voltage becomes zero and io is maximum. Due to negative supply voltage after π, SCR tries to turn-off. But energy stored in the load inductance generates the voltage $L\frac{di_0}{dt}$ induced voltage forward biases the SCR and maintains it in conduction. To maintain the flow of io inductance generates the voltage $L\frac{di_0}{dt}$ the polarity as shown in below Fig. This voltage is higher than negative supply voltage. Hence Tl is forward biased and it remains in conduction. The output current and supply current flow in the same loop .Hence io = i_s is all the time. The waveform of io is shown in Fig. 2 (d) and is i_s shown in Fig. 2(e). After π , io (i.e. i_s) flows against the supply. Hence energy is consumed in the supply. io flows due to load inductance energy. In other words, the inductance energy is partially fed to the mains and to the load itself. Therefore energy stored in inductance goes on reducing. Hence io also goes on reducing as shown in Fig. 2 (d).



SCR conducts due to inductance voltage after π

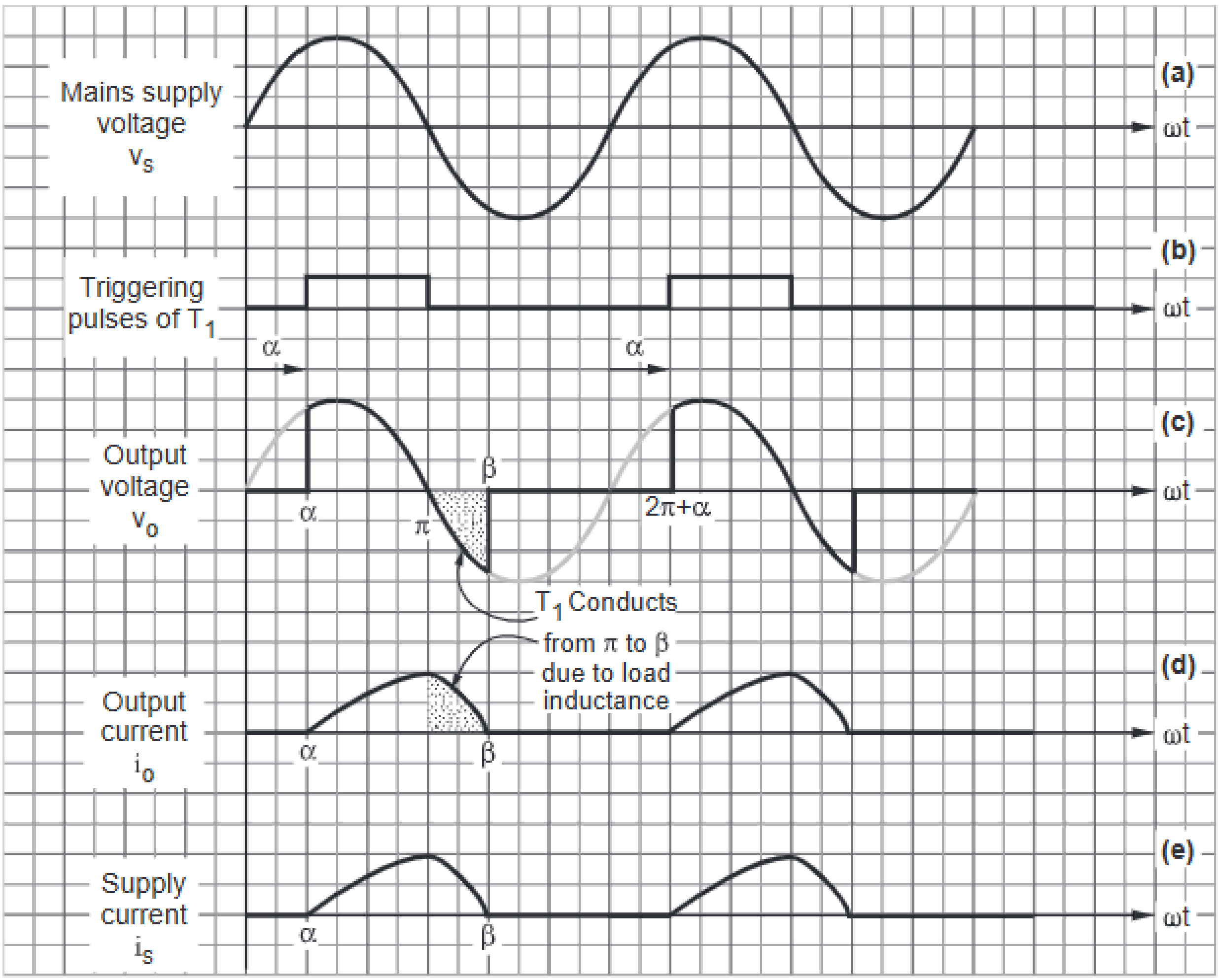


Fig. 2 Waveforms of half wave controlled rectifier for RL load

$$V_{o(av)} = \frac{1}{T}\int_0^T v_o(\omega t)\, d\omega t \qquad \dots (1)$$

In Fig. 4.2.4 observe that,

$$v_o(\omega t) = \begin{cases} v_s = V_m \sin \omega t & \text{from } \alpha \text{ to } \beta \\ 0 & \text{from } 0 \text{ to } \alpha \text{ and } \beta \text{ to } 2\pi \end{cases}$$

Hence equation (4.2.2) can be written as,

$$V_{o(av)} = \frac{1}{2\pi}\int_\alpha^\beta V_m \sin \omega t \; d\omega t$$

$$= \frac{V_m}{2\pi}[-\cos \omega t]_\alpha^\beta$$

$$= \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) \qquad \dots (2)$$

This is an expression for average value of output voltage.

Single phase half wave circuit with RL Load with free-wheeling diode(Single phase half wave controlled rectifier with RL Load with free-wheeling diode):

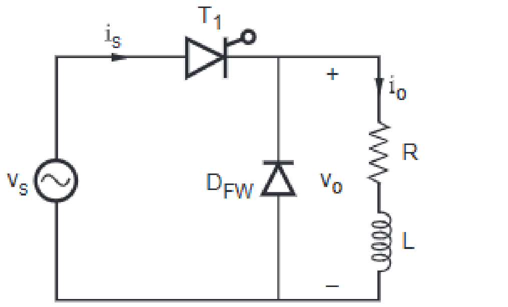
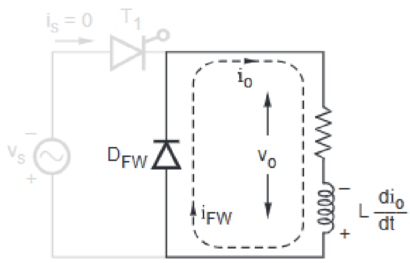


Fig. 1 Freewheeling diode in half wave controlled rectifier



Freewheeling action in half wave controlled rectifier

The circuit diagram shown the half wave controlled rectifier with freewheeling diode across the RL load. The SCR is triggered at firing angle α of in positive half cycle of supply. Hence Vo = Vs. The waveform of shown in Fig.2(c). Observe that from α to π v_o is same as supply voltage v_s. The freewheeling diode (DFW) is reverse biased, hence it does not conduct. The output current io increases from zero as shown in Fig. 2(d).

After π , the supply voltage becomes negative. Hence SCR tries to turn-off. Therefore io tries to go to zero. Observe that io is maximum at π. But the load inductance does not allow io to go to zero.

The induced inductance voltage forward biases freewheeling diode as well as SCR. But freewheeling diode (D_{FW}) is more forward biased. Hence it starts conducting. Therefore T_1 turns-off. The output current now flows through the freewheeling diode. In above figure observe that $i_o = i_{FW}$ when freewheeling diode conducts. Here i_{FW} is freewheeling current. Fig. .2. (d) and (e) shown that $i_o = i_{FW}$ when freewheeling diode conducts. The freewheeling current flows only due to energy stored in the load inductance. The output current flows in the load itself. Thus inductance energy is supplied back to the load itself. This process is called *freewheeling*. If load energy is fedback to the supply (mains), then it is called *feedback*. The energy of inductance goes on decreasing after π . Hence i_o also goes on reducing. At β the inductance energy is finished. Hence i_o becomes zero at β. Thus freewheeling diode conducts from π to β. The output is shorted due to freewheeling diode. Hence v_o = 0 whenever freewheeling diode conducts. This is shown in Fig. .2 (c) also. During freewheeling T_1 is off. Hence no supply current flows. Therefore $i_s = 0$ during freewheeling period. T_1 conducts from α to π . Hence $i_o = i_s$ from α to π as shown in Fig. .2. .

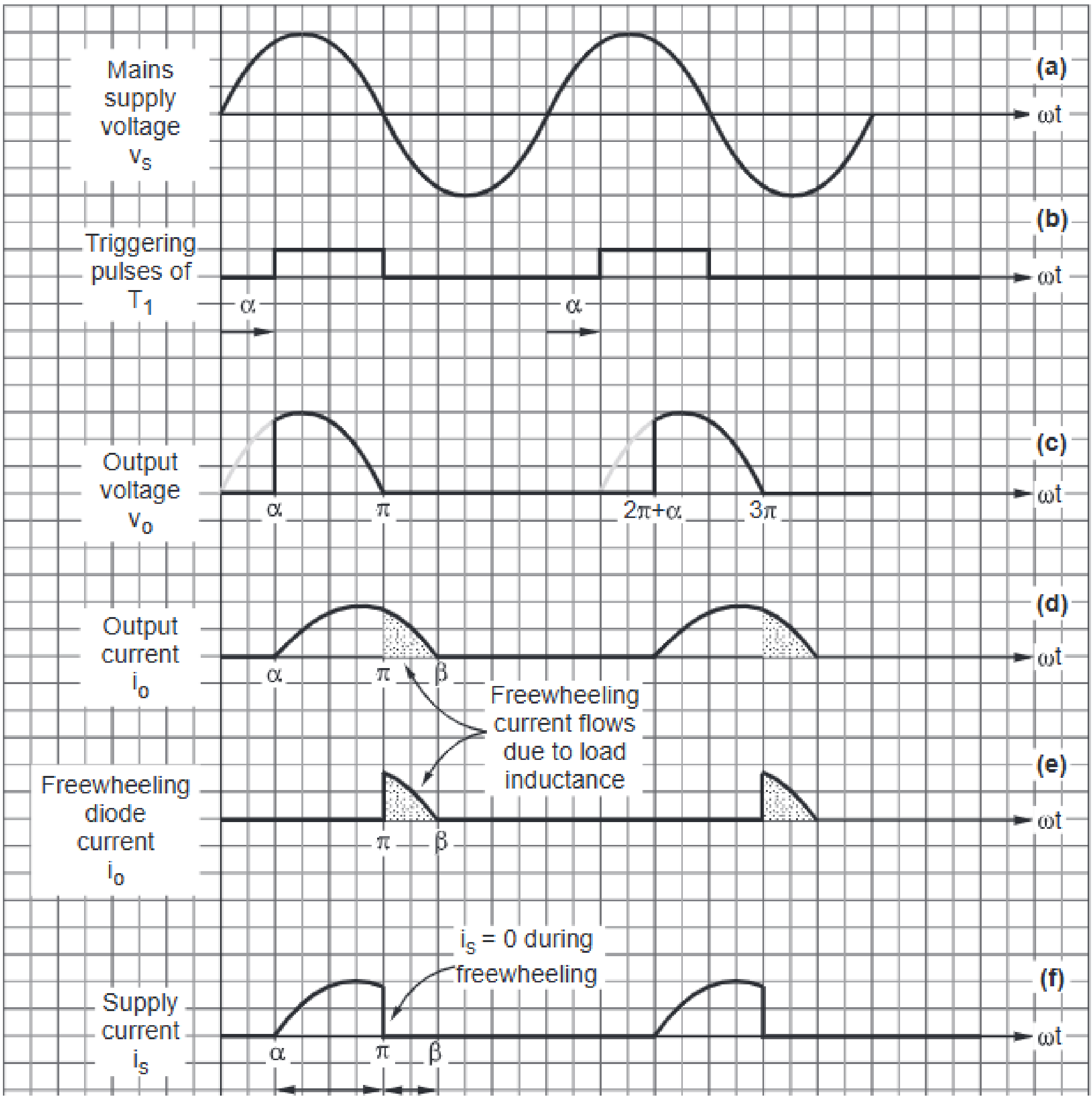


Fig. 2 Waveforms of half wave converter with freewheeling diode

$$v_o = \begin{cases} v_s = V_m \sin \omega t & \text{from } \alpha \text{ to } \pi \\ 0 & \text{from } 0 \text{ } to \text{ } \alpha \text{ and } \pi \text{ to } 2\pi \end{cases}$$

The period of v_o is 2π . The average value is given as,

$$v_{o(av)} = \frac{1}{T}\int_0^T v_o(\omega t)\, d\omega t = \frac{1}{2\pi}\int_\alpha^\pi V_m \sin \omega t \, d\omega t$$

$$= \frac{V_m}{2\pi}[-\cos \omega t]$$

$$= \frac{V_m}{2\pi}[1 + \cos\alpha]$$

The r.m.s. value is given as,

$$V_{o(r.m.s.)} = \left[\frac{1}{T}\int_0^T v_o^2(\omega t)\, d\omega t\right]^{\frac{1}{2}}$$

$$V_{o(r.m.s.)} = \left[\frac{1}{2\pi}\int_\alpha^\pi v_m^2 \sin^2 \omega t \, d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi}\int_\alpha^\pi \frac{1-\cos 2\omega t}{2}\, d\omega t\right]^{\frac{1}{2}}$$

$$= \left\{\frac{V_m^2}{4\pi}\left[\int_\alpha^\pi d\omega t - \int_\alpha^\pi \cos 2\omega t \, d\omega t\right]\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{V_m^2}{4\pi}\left[[\omega t]_\alpha^\pi - \left[\frac{\sin 2\omega t}{2}\right]_\alpha^\pi\right]\right\}^{\frac{1}{2}}$$

$$= \frac{V_m}{2}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{\frac{1}{2}}$$

Advantages of freewheeling diode

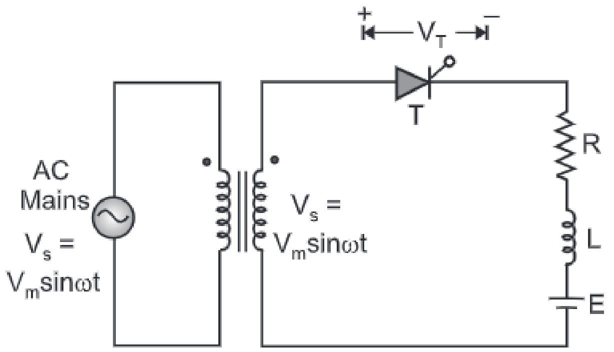
i) It tries to make the load current continuous.

ii) It circulates the energy stored in load inductances in the load itself.

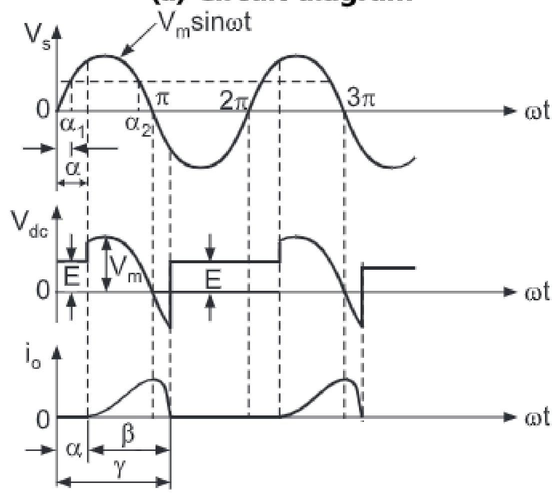
iii) It prevents negative voltage across the load.

iv) It helps in commutation of the SCRs, by imposing reverse voltage across them.

Single phase half wave circuit with RLE Load (Single phase half wave controlled rectifier with RLE Load):



(a) Circuit diagram



(b) Voltage and Current waveforms

Single-phase half-wave rectifier with RLE load

A single-phase half-wave rectifier with RLE or active load (either a battery or a DC motor) is shown in Fig. 2.5 (a). Let us assume that the counter emf is E due to active load. The minimum value of the firing angle is obtained for the conduction when $E = V_m \sin \omega t$. Assuming α_1 to the minimum firing angle, $\alpha_1 = \sin^{-1}\left(\frac{E}{V_m}\right)$.

If the firing angle α of the thyristor is less than α_1, $E > V_s$. This keeps the SCR reverse biased and SCR will not turn on. On the other hand, maximum value of the firing angle is $\alpha_2 = (\pi - \alpha_1)$.

Fig. (b) represents the profile of output voltage and current across the thyristor. It may be noted that during the period of non-conduction, $i_d = 0$ and the load voltage $V_o = E$. When $i_d \neq 0$, the output voltage follows the input. The firing angle being α, β is the conduction angle and γ is the extinction angle. Obviously, $(\gamma - \alpha) = \beta$.

Expression of Average Current and Voltage :

- Since the average voltage across the inductance is zero, thus the average value of load is given by,

$$I_{avg} = \frac{1}{2\pi R}\left[\int_{\alpha}^{\gamma} (V_m \sin \omega t - E)\, d\omega t\right]$$

$$= \frac{1}{2\pi R}[V_m (\cos \alpha - \cos \gamma) - E(\gamma - \alpha)]$$

Conduction angle being β, $\beta = \gamma - \alpha$. Thus $\gamma = \alpha + \beta$,

$$\therefore \quad I_{avg} = \frac{1}{2\pi R}[V_m \{\cos \alpha - \cos (\alpha + \beta)\} - E\beta]$$

or

$$I_{avg} = \frac{1}{2\pi R}\left[2V_m \sin\left(\alpha + \frac{\beta}{2}\right) \sin \frac{\beta}{2} - E\beta\right] \quad \ldots (2.12)$$

$$\left(\text{since } \cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}\right)$$

However, $V_{avg} = E + IdR$

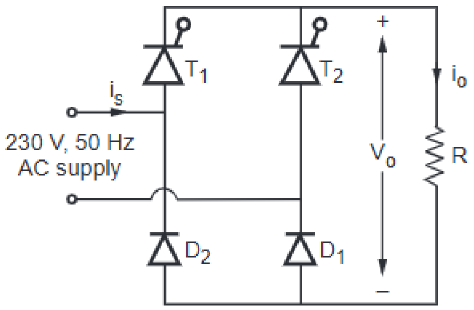
$$= E + \frac{1}{2\pi}\left[2V_m \sin\left(\alpha + \frac{\beta}{2}\right)\sin \frac{\beta}{2} - E\beta\right]$$

or

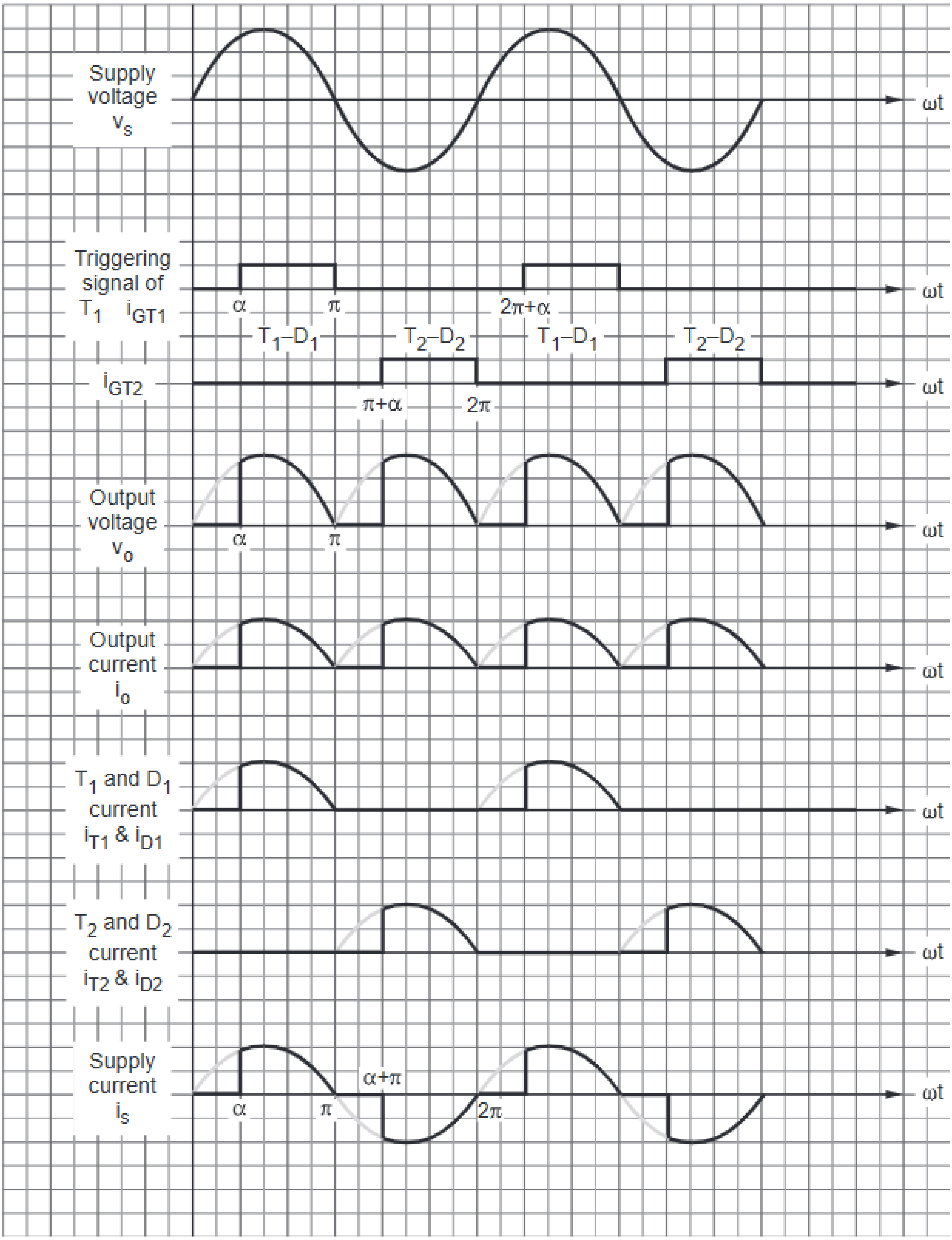
$$V_{avg} = E\left(1 - \frac{\beta}{2\pi}\right) + \frac{V_m}{\pi} \sin\left(\alpha + \frac{\beta}{2}\right) \sin \frac{\beta}{2}$$

Single Phase Semi converters (Half Bridge Converter):

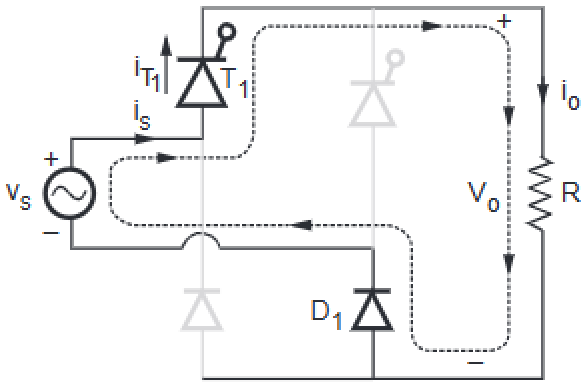
Single Phase Semi converters working with Resistive Load:



Circuit diagram of 1 ϕ semiconverter



Let us consider the working of single phase semi converter having a resistive load. In the positive half cycle of the supply, SCR Tl and diode D2 are forward-biased. SCR Tl is triggered at a firing angle α. Current flows through the load. The equivalent circuit is shown below.



Conduction of Tl and Dl in positive half cycle of the supply. Dotted line shows path of current flow

When Tl —Dl conducts,

$$V_o = V_s \quad \text{(i.e. supply voltage)}$$

and

$$i_o = \frac{V_o}{R} = \frac{V_s}{R}$$

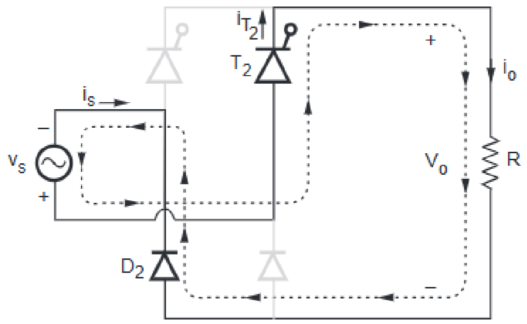
The waveform of Vo is same as supply voltage Vs, when Tl —Dl conducts. Since the load is resistive, the output current waveform is same as voltage waveform. This is because,

$$i_o = \frac{V_o}{R}$$

io, iT1 and is is the same current.

$$i_o = i_s = i_{T1} \quad (\text{when } T_1 - D_1 \text{ conducts})$$

SCR Tl and diode Dl conduct till π . At supply voltage is zero. Hence current through SCR Tl drops to zero. Hence Tl turns off. After ,π the supply voltage is negative and Tl is reverse biased. Hence the output voltage Vo is also zero.



Conduction of T2 - D2 in negative half cycle of the supply. Dotted line shows path Of current flow

At $\pi+\alpha$, SCR T 2 is triggered. It starts conducting, since it is forward biased because of the negative cycle of the supply. The current io flows through load, T2 and 102. Such an equivalent circuit is shown in the above Fig.

From the above equivalent circuit observe that positive of Vs is connected to positive of Vo. Hence Vo remains positive even if supply polarity (i.e. negative cycle) is reversed. Hence we can write,

$$V_o = -V_s$$

and $$i_o = \frac{V_o}{R} = -\frac{V_s}{R}$$

$$i_s = -i_o$$

i) Average output voltage

The average output voltage is given as,

$$V_{o(av)} = \frac{1}{T}\int_0^T V_o(\omega t)\, d\omega t$$

$$V_{o(av)} = \frac{1}{\pi}\int_\alpha^\pi V_m \sin \omega t\, d\omega t$$

In the above equation $V_o(\omega t) = V_m \sin \omega t$ from α to π. Solving the above integration we get,

$$V_{o(av)} = \frac{V_m}{\pi}(1+\cos\alpha)$$

ii) R.M.S. output voltage

R.M.S. output voltage is given as,

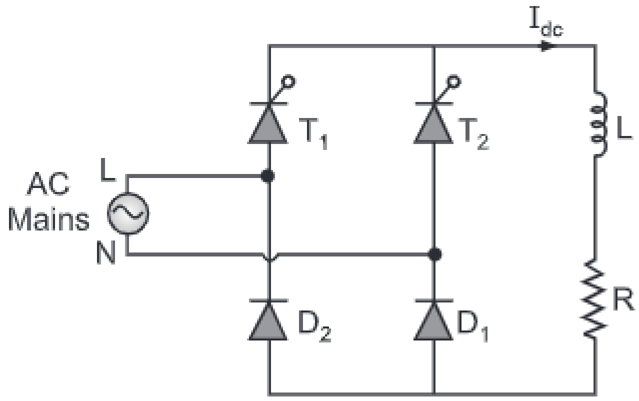
$$V_{o(r.m.s.)} = \left[\frac{1}{T}\int_0^T V_o^2(\omega t)\, d\omega t\right]^{1/2}$$

Putting the values in above equation,

$$V_{o(r.m.s.)} = \left[\frac{1}{\pi}\int_\alpha^\pi V_m^2 \sin^2 \omega t\, d\omega t\right]^{\frac{1}{2}} = \left\{\frac{V_m^2}{\pi}\int_\alpha^\pi \left[\frac{1-\cos(2\omega t)}{2}\right] d\omega t\right\}^{\frac{1}{2}}$$

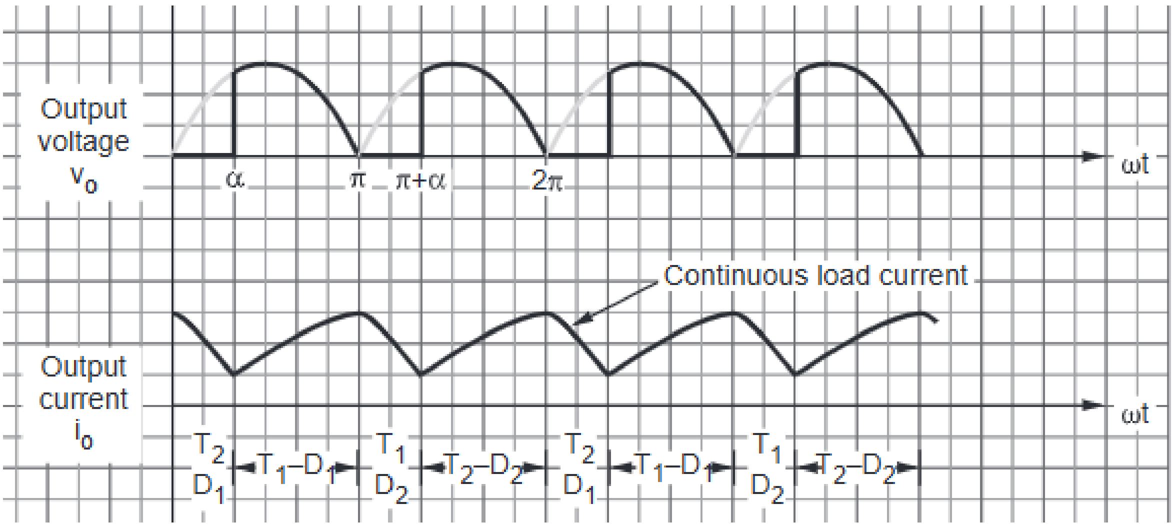
$$= \left\{\frac{V_m^2}{2\pi}\left[\pi - \alpha + \frac{1}{2}\sin 2\alpha\right]\right\}^{\frac{1}{2}}$$

Single Phase Semi converters working with RL Load:



(a) Circuit diagram

Continuous current mode :



In this mode, the current flows continuously in the load because of inductive effect. The waveforms of load current and load voltage are shown in Fig. In these waveforms observe that SCR T_1 and diode D_1 conducts from α to π. Since the load is inductive current keeps on increasing (saturating) and it is maximum at π. At π, even though the supply voltage is zero, current does not go to zero. This is because load inductance opposes this sudden change of current. The load inductance generates a large voltage so as to maintain load current. This current flows through T_1 and D_2. The equivalent circuit of this operation is shown in Fig. The SCR T_1 conducts even after π, since it is forward biased due to voltage induced in the load inductance i.e. $L\frac{di}{dt}$. Diode D_2 is also forward biased due to this voltage. Hence current does not flow through supply i.e. i_s when freewheeling action takes place. Thus the energy stored in the load inductance is fedback to load itself in freewheeling action.

SCR T_2 is triggered at $\pi+\alpha$ and the output current starts increasing. Since the current i_o is continuous, it is called continuous current mode of semiconverter. The similar operation takes place when T_2 and D_2 conducts in negative half cycle of the supply. Fig. shows supply current (i_s), freewheeling current and other waveforms for inductive load. Note that the output voltage waveform remains same. If there is freewheeling diode in semiconverter, then freewheeling current flows through this diode.

$$V_{o(av)} = \frac{V_m}{\pi}(1+\cos\alpha)$$

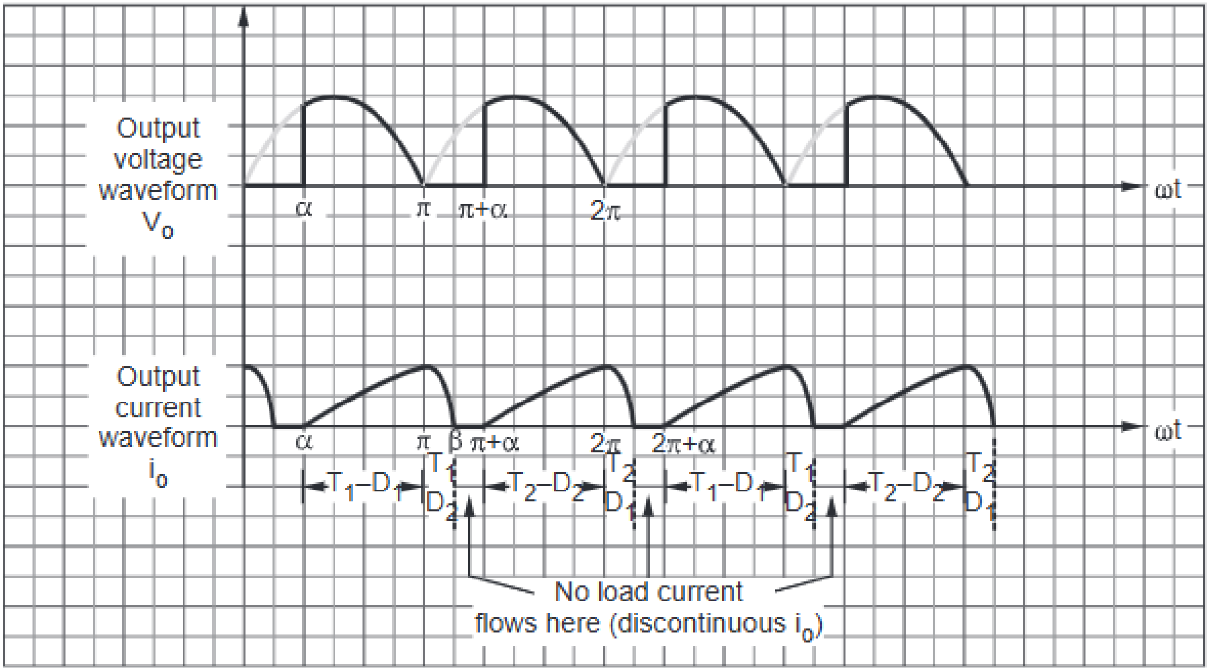
$$V_{o(rms)} = \left\{\frac{V_m^2}{2\pi}\left[\pi-\alpha+\frac{1}{2}\sin 2\alpha\right]\right\}^{\frac{1}{2}}$$

ii) Discontinuous current mode

In this mode, the current through the load becomes zero for some duration. Hence it is called discontinuous current mode. Fig. shows the waveforms of discontinuous current mode of semiconverter.

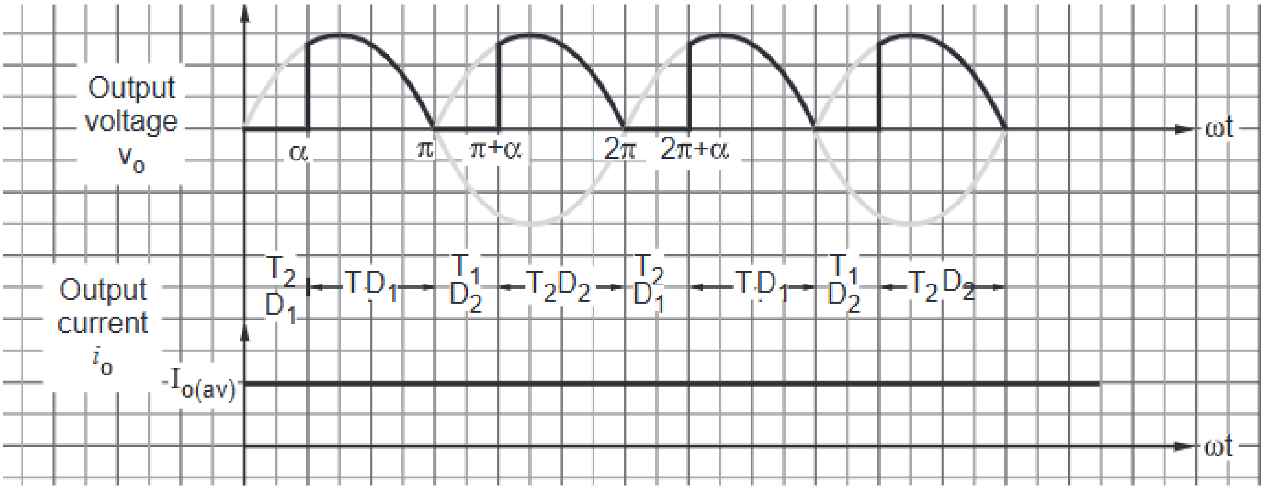
As shown in waveforms, $T_1 - D_1$ conducts from α to π and the load current i_o goes on increasing. At π supply voltage is zero. But because of inductance, i_0 does not go to zero. The load inductance induces a large voltage $L\frac{di_o}{dt}$ to maintain current in the same direction. Hence i_o continuous to flow and it goes to zero at β. Since next SCR T_2 is triggered at $\pi+\alpha$ output current is discontinuous. Freewheeling takes place from π to β. The freewheeling current flows through T_1 and D_2. Similar operation repeats in next half cycle.

Observe that the voltage waveform remains same in discontinuous mode also. Hence $V_{o(av)}$ and $V_{o(RMS)}$ are same as that of resistive load.



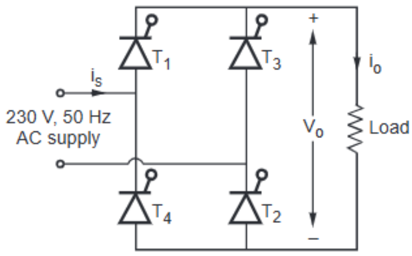
iii) Continuous and ripple free current for large inductive load

As the load inductance increases, the ripple in i_o reduces. When the load inductance is very large, the ripple in i_o will be negligible. And i_o can be treated as continuous and ripple free. Fig. shows the waveforms of 1ϕ semiconverter for large inductive load. The load current is continuous and ripple free. Observe that the output voltage waveform is same as resistive load. But the current waveforms are different.



The output current is constant DC of amplitude $I_{o(av)}$. The SCRs conduct for π radians. Hence SCR current is square wave. The supply current has the amplitudes of $\pm I_{o(av)}$. The supply current is zero whenever freewheeling action takes place.

Single Phase Full Converters with R load: Fig. shows the block diagram of 1 phase full bridge converter. It contains four SCRs Tl , T2 , T 3 and T4. The conduction of all these SCRs is controlled. Hence this is called full converter. The input to this converter is AC supply. The output is controllable DC. The full bridge converter is mainly used for speed control of DC motors.



Circuit diagram of 1ϕ full converter

In the positive half cycle of the supply SCRs Tl and T2 are triggered at firing angle a. Hence current starts flowing through the load.

when Tl and T2 conducts,

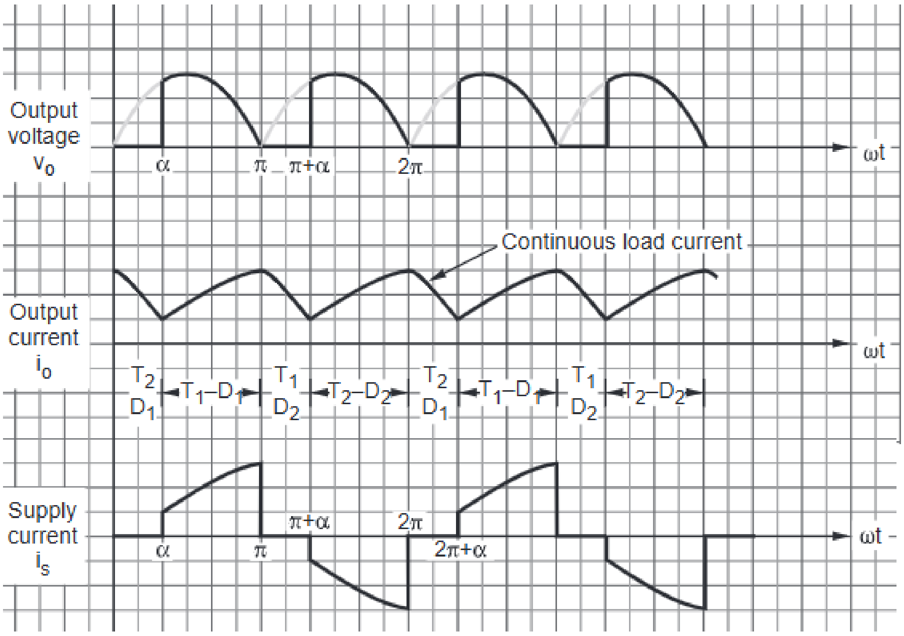
$$V_o = V_s \quad \text{(i.e. supply voltage)}$$

$$i_o = \frac{V_o}{R} = \frac{V_s}{R}$$

The load voltage is same as supply voltage from α to π . Since the load is resistive, waveforms of Vo and io are same. The supply current is and io are in the same direction hence is=io. Tl and T2 turn off when supply voltage becomes zero at . In the negative half cycle T3 and T4 are triggered at $\pi + \alpha$.

The supply current is and load current io flow through the same loop. But directions of is and io are opposite hence

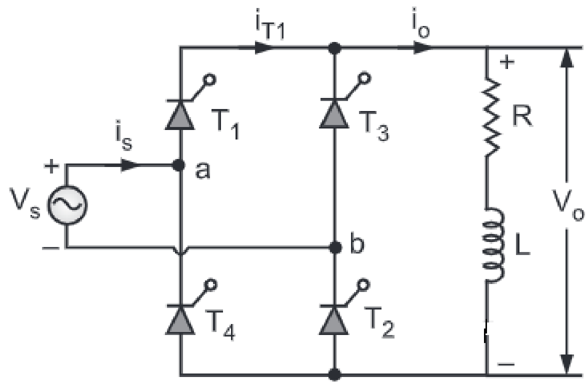
$$i_s = -i_o$$



$$V_{o(av)} = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$V_{o(rms)} = \frac{V_m^2}{2\pi}\left[\pi-\alpha+\frac{1}{2}\sin 2\alpha\right]^{\frac{1}{2}}$$

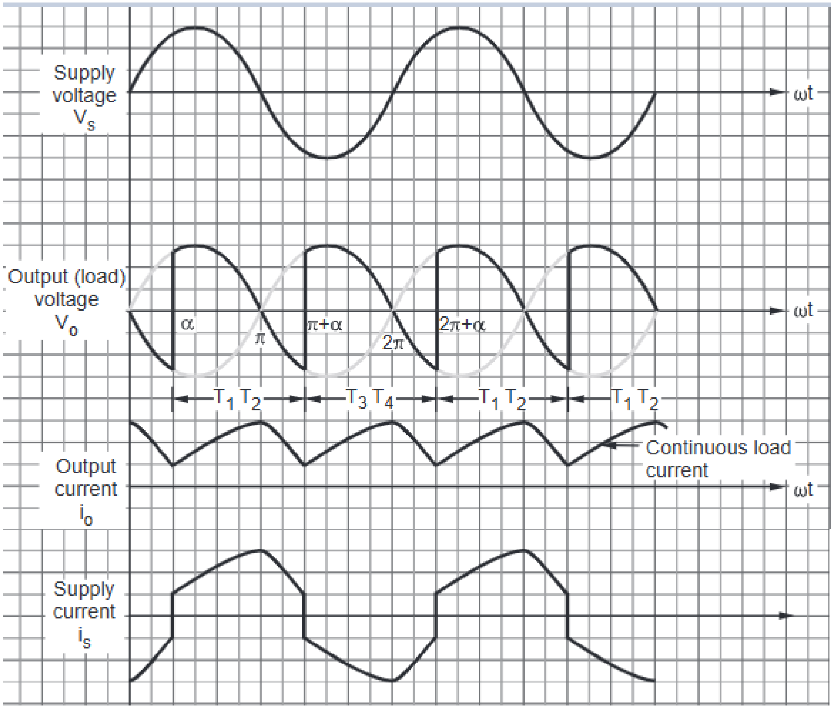
Single Phase Full Converters with RL load:



Continuous load current:

In the continuous load current, the load or output current io flows continuously.

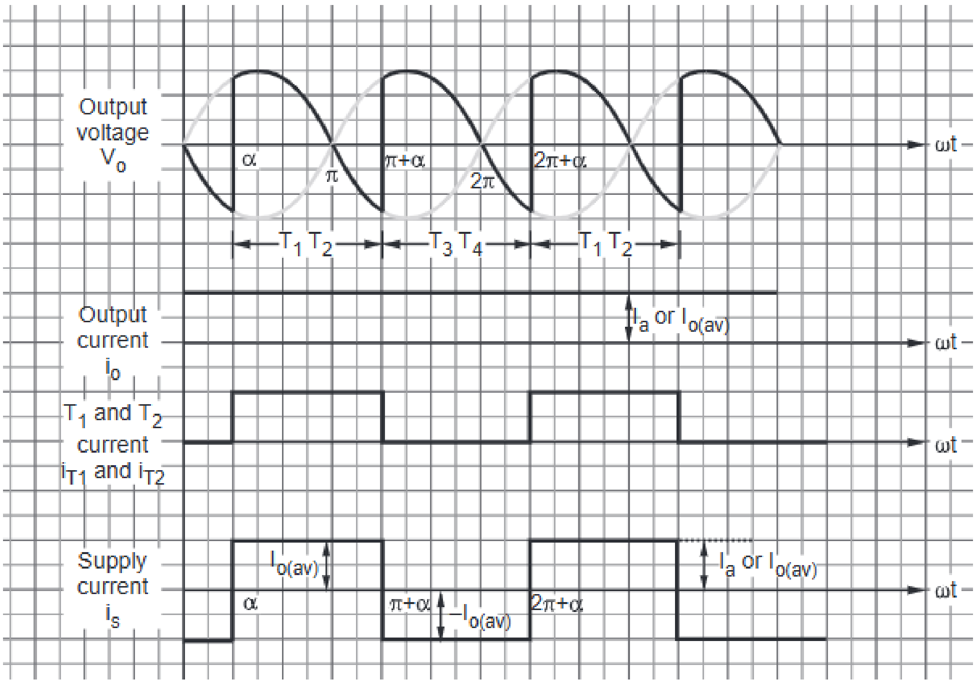
As shown in the waveforms of Fig. · · , T_1 and T_2 conduct from α to π . The nature of the load current depends upon values of R and L in the inductive load. Because of the inductance, i_0 keeps on increasing and becomes maximum at π . At π , the supply voltage reverses but SCRs T_1 and T_2 does not turn off. This is because, the load inductance does not allow the current i_0 to go to zero instantly. The load inductance generates a large voltage $L\,\frac{di_0}{dt}$.



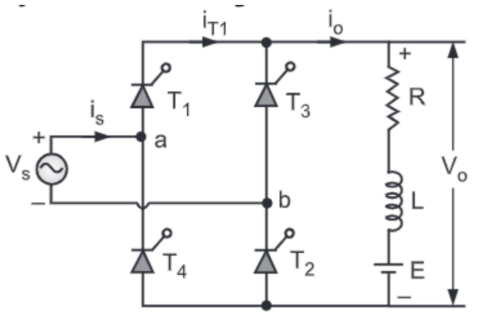
This voltage forward biases Tl and T 2. the load current flows against the supply voltage. The energy stored in the load inductance is supplied partially to the mains supply and to the load itself. Hence this is also called as feedback operation. The output voltage is negative from π to $\pi + \alpha$ since supply voltage is negative. But the load current keeps on reducing.

At $\pi + \alpha$, SCRs T 3 and T4 are triggered. The load current starts increasing. The load current remains continuous in the load. The similar operation repeats. The ripple in the load current reduces as the load inductance is increased.

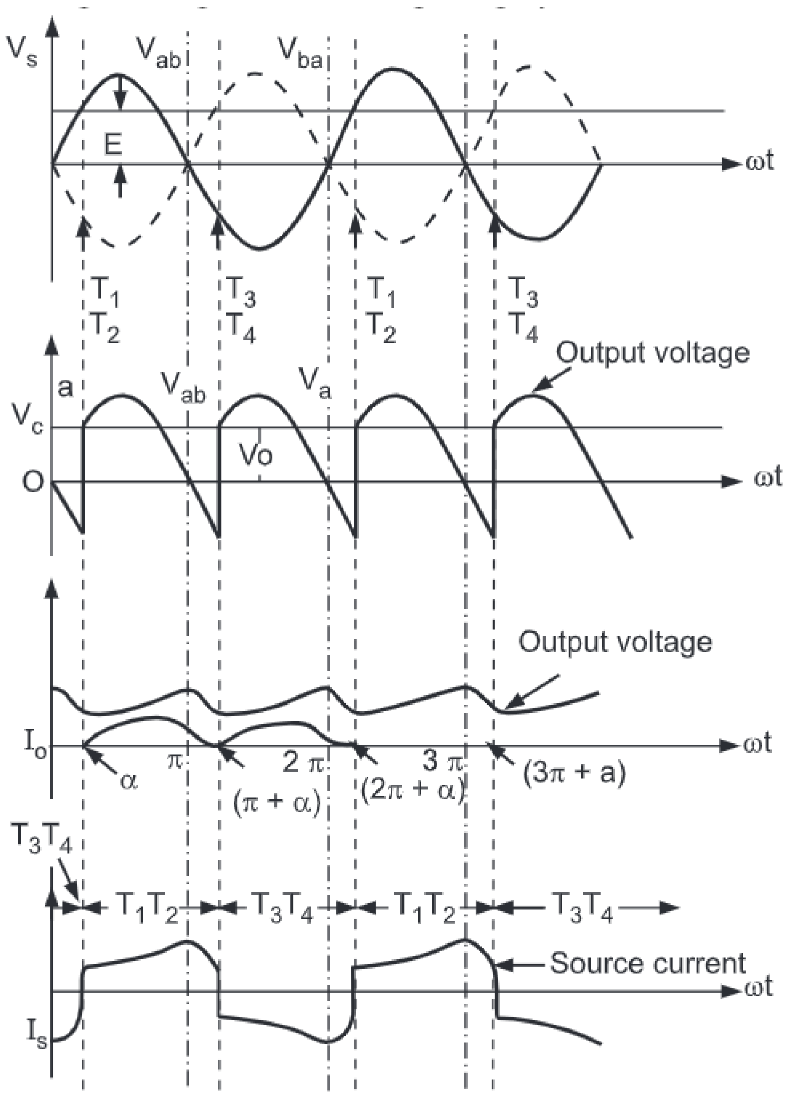
Continuous and ripple free current for large inductive load: Let us consider the case when there is large inductance in the load. Because of the large inductance, the ripple in the load current is very small and it can be neglected. Hence load current will be totally DC.



Single Fully Controlled Bridge Rectifier with RLE Load:



Circuit diagram of single phase fully controlled rectifier



Single phase fully controlled rectifier waveforms

Mode 1 : (α to π) :

During positive half cycle of AC input voltage, leave point L is positive with respect to cathode, therefore thyristors T_1T_2 are fired at $\omega t = \alpha$. Thus, the average output voltage is equal to the instantaneous supply voltage.

In this mode of operation, the shape of load voltage is identical to that of supply voltage. The load voltage is positive and constant. The load current is also positive as that of the supply current 'i_s'. Both load voltage and load current are positive, the inductive load will store energy.

Mode 2 : (π to $\pi + \alpha$) :

In this step of operation, at instant $\omega t = \pi$, the supply goes through zero and after π radians supply reverses its polarities and it becomes negative. Therefore, the conducting thyristors T_1 and T_2 will try to turn-off due to natural reversal of supply voltage (i.e. natural commutation or line commutation). But due to stored energy in inductive load, it will oppose any change in the current flow through load. So thyristors T_1 and T_2 will continue to conduct in negative half for some period. In this mode of operation, the load voltage becomes negative and load current is always positive, continuous and constant. Both load voltage and load currents are opposite in polarities. So the stored energy in inductive load will return back to the supply again.

Mode 3 : ($\pi + \alpha$ to 2π) :

At instant $\omega t = \pi + \alpha$, the conducting thyristors T_1 and T_2 are turned off due to natural or line commutation, at the same time other pair of SCRs T_3 and T_4 are fired at $\omega t = \pi + \alpha$. Therefore, the average output voltage is equal to the instantaneous supply voltage. The load current is instantaneously transferred from one pair of SCRs (T_1, T_2) to other pair of SCR (T_3, T_4).

In this mode of operation, both load voltage and load currents are positive, the inductive load will again store energy.

Mode 4 : (2π to $2\pi + \alpha$) :

At instant $\omega t = 2\pi$ radians, the input voltage goes through zero after 2π it becomes positive. i.e. during positive half cycle of AC input, the conducting thyristors T_3, T_4 try to turn off, the inductive load will oppose any change in current through it, in order to maintain the load current constant and in some direction, a self-induced voltage appears across the load. This maintains conducting thyristors T_3 and T_4 forward biased, inspite of the change in the polarity of supply voltage. The load voltage becomes negative and equal to the supply voltage whereas the load current continues positive. Therefore, load acts as a source and the stored energy in inductive load will be returned back to supply again.

Mathematical Analysis :

1. Expression for the Average Output Voltage (V_{avg}) :

$$V_{avg} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \cdot d\omega t$$

$$= \frac{1}{\pi}\left[-V_m \cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi}\left[\cos\alpha - \cos(\pi+\alpha)\right]$$

$$= \frac{V_m}{\pi}\left[\cos\alpha - \cos\pi\cdot\cos\alpha + \sin\pi\cdot\sin\alpha\right]$$

$$V_{avg} = \frac{2V_m}{\pi}\cos\alpha$$

Expression for the Average Load Current (I_{avg}) :

$$I_{avg} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}\cos\alpha$$

Expression for the rms Load Voltage (V_{rms}) :

$$V_{rms} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t \cdot d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} (1 - \cos 2\omega t) \cdot d\omega t\right]^{1/2}$$

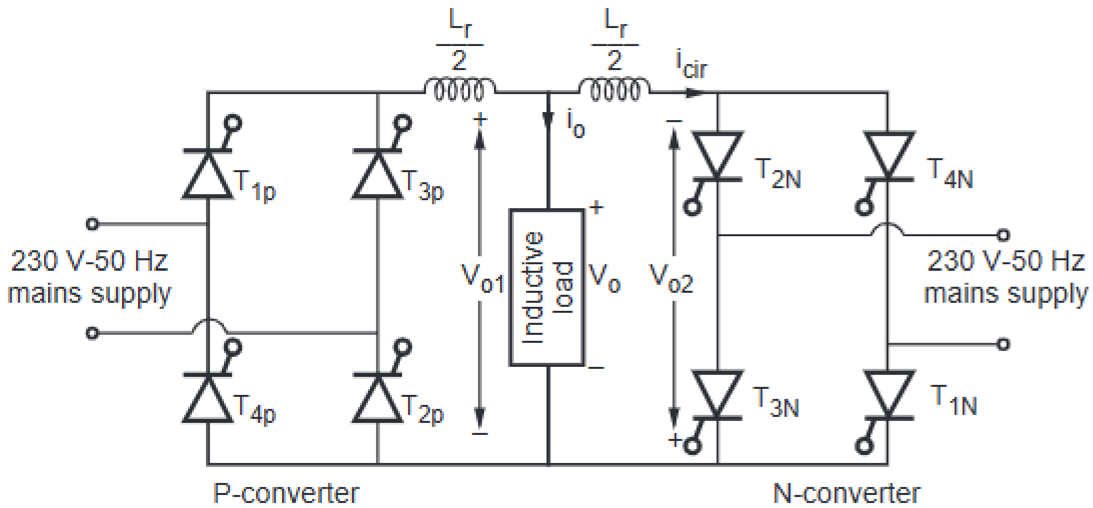
$$= \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi+\alpha}\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi + \alpha - \frac{\sin(2\pi + 2\alpha)}{2} - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Single Phase Dual Converters:



- Dual converters are suitable for high power applications but not for low power applications.
- The below circuit explains sigle phase dual converters with circulating current.

In circulating current mode

Load current is continuous and it is fast process

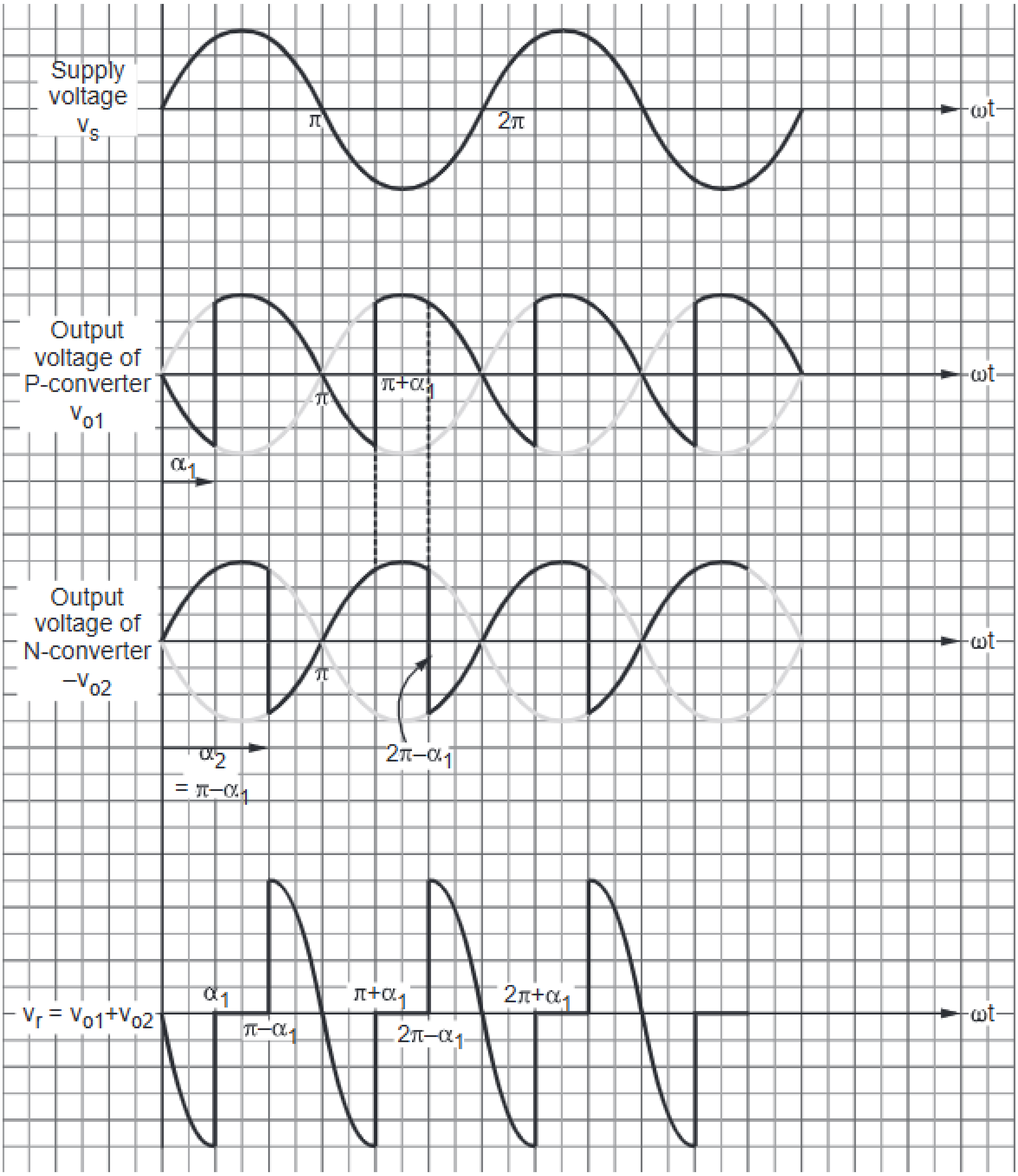
- Two converters are simultaneously operated
- The circuit will protect with current limiting reactors
- Average output load will be more than the load.
- The dual converters consists of two full converters one with positive and another with negative output voltages.

- For highly inductive load the dual converters will operate in four quadrants with continuous current mode.

When the circuit is fed with the supply the thyristor TI and T3 conducts positive half cycle at converter I and T5, T6 conducts positive half cycle at converter 2

For negative half cycle the thyristors T3, T4, T 7, T8 are takes the active position at converter 1 and converter 2.

- For positive half cycle the current direction is given by
  For converter 1: P – T1 – Lr1/2 – LOAD – T2 – N
  For converter 2: N – T6 – Lr2/2 – LOAD – T5 – P
- For negative half cycle the current direction is given by
  Converter 1: N – T3 – Lr1/2 – LOAD – T4 – P
  Converter 2: P – T8 – Lr2/2 – LOAD – T7 – N



Three Phase Dual Converters:

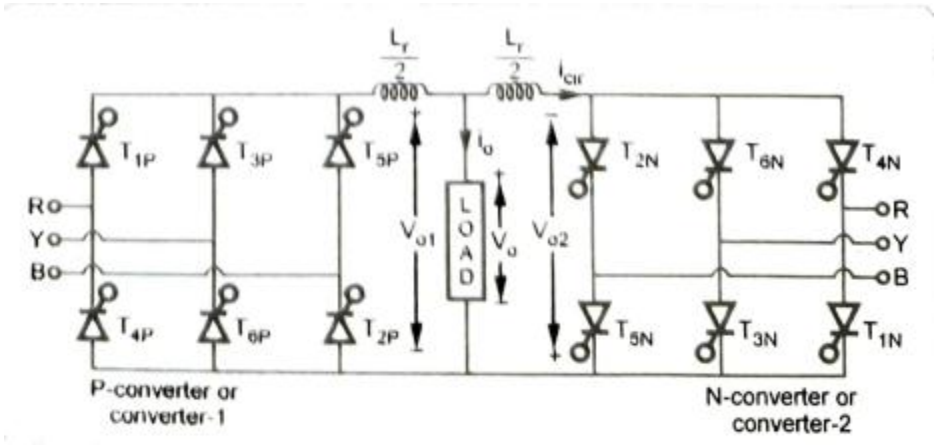


Fig. shows circuit diagram of 3 phase dual converter. It uses two 3 phase full converters.

- We know that the average output voltage of 3 ϕ full converter is given for highly inductive as,

$$V_{o(av)} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha \qquad \ldots(1)$$

Hence the outputs of the two converters will be,

$$V_{o1\,(av)} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha_1$$

and

$$V_{o2\,(av)} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha_2$$

From Fig. 4.9.1 it is clear that,

$$V_{o\,(av)} = V_{o1\,(av)} = -V_{o2\,(av)} \qquad \ldots 2)$$

From equation (4.9.1) and above equation we have,

$$\frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha_1 = -\frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha_2$$

$$\cos\alpha_1 = -\cos\alpha_2$$

or

$$\alpha_2 = \pi - \alpha_1 \qquad \ldots 3)$$

This is the relationship between triggering angles of the two converters. The converter having $\alpha < 90^o$ operates in rectifying mode. And the converter having $\alpha > 90^o$ operates in inverting mode. Fig. shows the waveforms of 3ϕ dual converter. Fig. (c) shows the output voltage waveform of converter 1 for $\alpha_1 = 60^o$. It operates in rectification mode. Fig. (d) shows the output voltage waveform of converter-2 (inverted output i.e. $-v_{o2}$) for $\alpha_2 = 120^o$. It operates in the inverting mode.

The two outputs v_{o1} and v_{o2} are same in magnitude. But their instantaneous values are different. Hence small amount of circulating current flows from one converter to other converter. This current can be limited by the circulating current reactor L_r connected between the two converters.

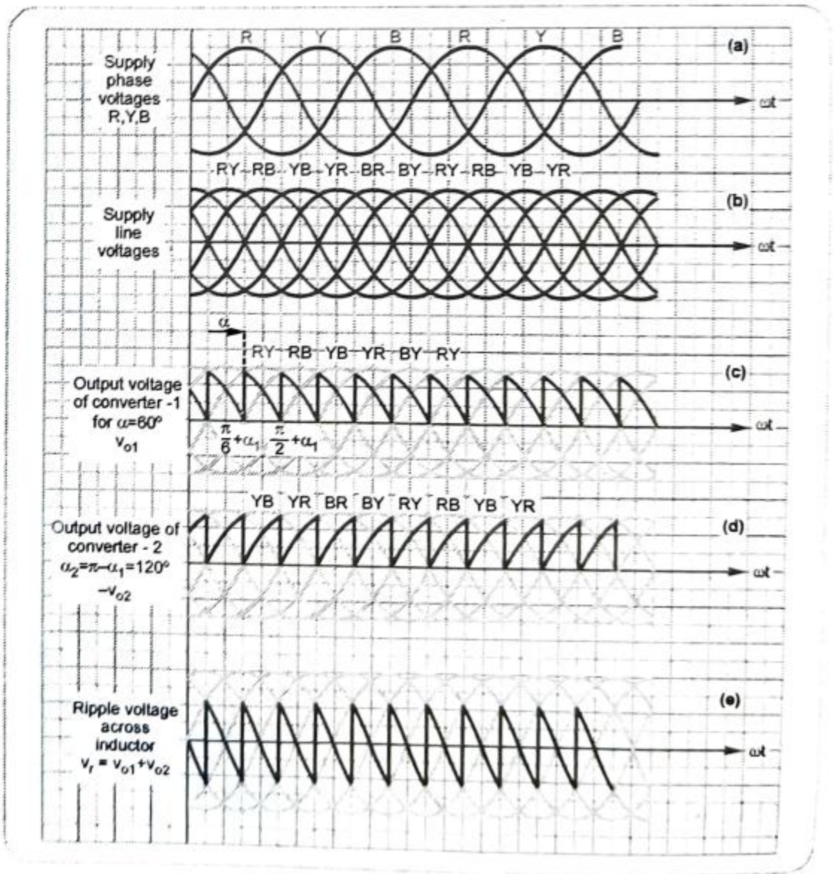


Fig. **Waveforms of 3 φ dual converter**

Derive an expression for circulating current in 3 φ dual converter.

Solution : Consider the interval from $\left(\frac{\pi}{6}+\alpha_1\right)$ to $\left(\frac{\pi}{2}+\alpha_1\right)$ in Fig. 2.9.2. The voltage $v_{o1} = V_{RY}$ and $v_{o2} = V_{YB}$ in this interval.

$$v_{o1} = v_{RY} = \sqrt{3}\,V_m \sin\left(\omega t+\frac{\pi}{6}\right)$$

and
$$v_{o2} = v_{RB} = \sqrt{3}\,V_m \sin\left(\omega t-\frac{\pi}{2}\right)$$

Hence the ripple voltage across the inductor during the interval $\left(\frac{\pi}{6}+\alpha_1\right) \le \omega t \le \left(\frac{\pi}{2}+\alpha_1\right)$ will be,

$$v_r = v_{o1}-v_{o2} = v_{RY}-v_{YB} = \sqrt{3}\,V_m \sin\left(\omega t+\frac{\pi}{6}\right)-\sqrt{3}\,V_m \sin\left(\omega t-\frac{\pi}{2}\right)$$

$$= 3V_m \cos\left(\omega t-\frac{\pi}{6}\right) \quad \ldots$$

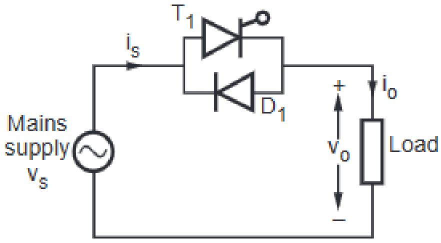
The waveform of this voltage is shown in Fig. 4.9.2 (e). Now the circulating current can be obtained as,

$$i_{cir}(t) = \frac{1}{\omega L_r}\int_{\frac{\pi}{6}+\alpha_1}^{\omega t} v_r(\omega t)\,d\omega t = \frac{1}{\omega L_r}\int_{\frac{\pi}{6}+\alpha_1}^{\omega t} 3\,V_m \cos\left(\omega t-\frac{\pi}{6}\right) d\omega t$$

$$= \frac{3V_m}{\omega L_r}\left[\sin\left(\omega t-\frac{\pi}{6}\right)-\sin\alpha_1\right]$$

Explain the principle of on-off control. Obtain an expression for rms voltage, rms current and power factor.

Principle of Phase Control:



1 ϕ half wave controller (unidirectional controller)

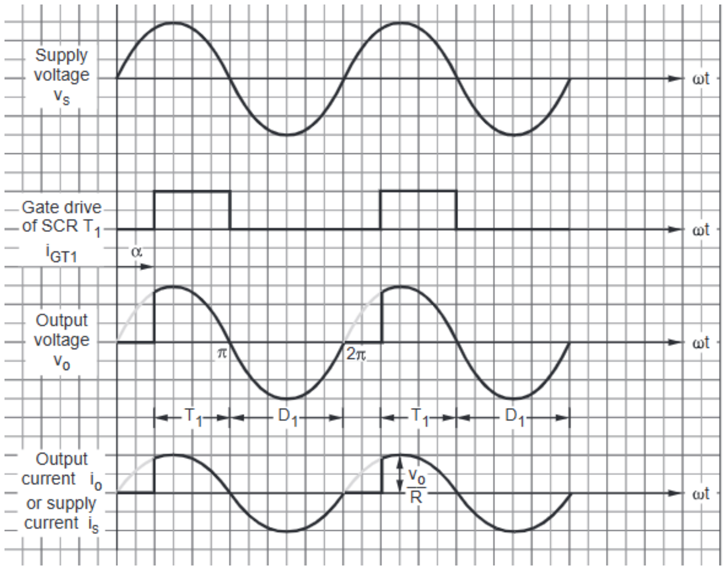


Fig. shows the circuit diagram of single phase half wave controller employing phase angle control. Observe that there is one SCR Tl and antiparallel diode Dl. Hence only positive half cycle of the supply is controlled. The negative half cycle is not controlled since diode Dl conducts fully. Fig shows the waveforms of this circuit for resistive load. Observe that the output voltage waveform is not symmetric. Hence it will have a d.c. component. This may saturate the loads like induction motors, pumps etc. Observe that the output current waveform is not symmetric. The supply current waveform is same as output current waveform. Hence the supply current is is not symmetric. The supply current also contains d.c. component. If there is input transformer, then it will be saturated due to d.c. component of supply current.

The supply voltage is,

$$v_s(\omega t) = V_m \sin \omega t$$

The r.m.s. value is given as,

$$V_{o(r.m.s.)} = \left[\frac{1}{T}\int_0^T v_o^2(\omega t)\, d\,\omega t\right]^{\frac{1}{2}}$$

From output voltage waveform of Fig. . we can write above equation as,

$$V_{o(r.m.s.)} = \left\{\frac{1}{2\pi}\left[\int_\alpha^\pi v_o^2(\omega t)\, d\omega t + \int_\pi^{2\pi} v_o^2(\omega t)\, d\omega t\right]\right\}^{\frac{1}{2}}$$

In the waveform observe that whenever SCR or diode conducts, the supply voltage v_s is applied to the load. Hence $v_{o(\omega t)} = v_s(\omega t)$. Hence above equation can be written as,

$$V_{o(r.m.s.)} = \left\{ \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \, d\omega t + \int_{\pi}^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \right] \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{V_m^2}{2\pi} \left[\int_{\alpha}^{\pi} \sin^2 \omega t \, d\omega t + \int_{\pi}^{2\pi} \sin^2 \omega t \, d\omega t \right] \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{V_m^2}{2\pi} \left[\int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} \, d\omega t + \int_{\pi}^{2\pi} \frac{1 - \cos 2\omega t}{2} \, d\omega t \right] \right\}^{\frac{1}{2}}$$

Solving the above integration,

$$V_{o(r.m.s.)} = \frac{V_m}{2} \sqrt{\frac{2\pi - \alpha + \frac{\sin 2\alpha}{2}}{\pi}}$$

And r.m.s. value of current will be,

$$I_{o(r.m.s.)} = \frac{V_{o(r.m.s.)}}{R}$$

We know that the average value is given as,

$$V_{o(av)} = \frac{1}{T} \int_0^T v_o(\omega t) \, d\omega t$$

We know that $v_o(\omega t) = v_s(\omega t)$ whenever SCR or diode conducts. Hence above equation can be written as,

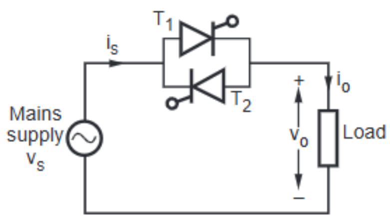
$$V_{o(av)} = \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} V_m \sin \omega t \, d\omega t + \int_{\pi}^{2\pi} V_m \sin \omega t \, d\omega t \right]$$

$$= \frac{V_m}{2\pi} \left[\int_{\alpha}^{\pi} \sin \omega t \, d\omega t + \int_{\pi}^{2\pi} \sin \omega t \, d\omega t \right]$$

Solving the above integration,

$$V_{o(av)} = \frac{V_m}{2\pi} (\cos \alpha - 1)$$

Single Phase Controllers with Resistive Loads (Bidirectional Controllers):



1ϕ full wave (bidirectional) controller

Fig 1

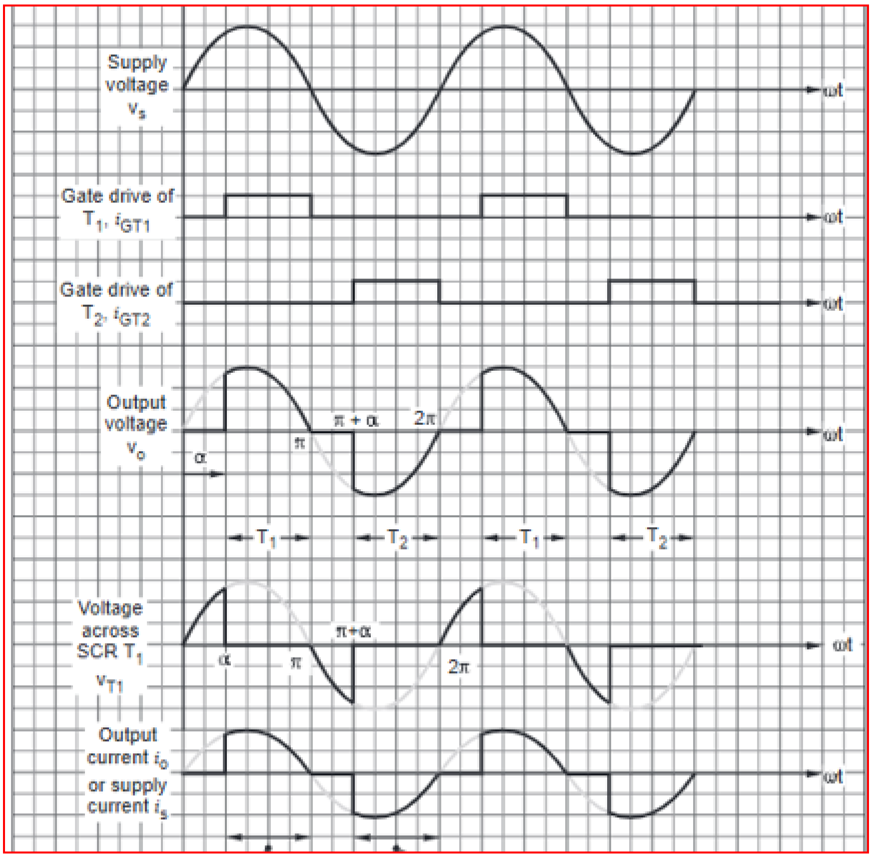


Fig. 1 shows the circuit diagram of 1 phase full wave controller. It has two SCRs, Tl and T2. In the positive half cycle of the supply Tl controls the power flow to the load. And in the negative half cycle of the supply T2 controls the power flow to the load. The waveforms of this circuit are shown in Fig. 2 for resistive load.

The output current waveform is shown for resistive load. It is similar to the voltage waveform. The output current and the supply current flow in the same loop. Hence $i_v = i_s$. Observe that the voltage and current waveforms are symmetric. Hence there is no dc component in v_o , i_o and i_s, Also, it is possible to control the output fully from zero to maximum value. The output is controlled in positive as well as negative half cycles due to two SCRs.

We know that the rms value is given as,

$$V_{o(r.m.s.)} = \left[\frac{1}{T}\int_0^T v_o^2 \, (\omega t) \, d\,\omega t\right]^{\frac{1}{2}}$$

Whenever the SCRs conduct, the output voltage is same as supply voltage.

The supply voltage is given as

$$v_s(\omega t) = V_m \sin \omega t$$

From the output voltage waveform of Fig. 8.4.2 we can write equation 8.4.1 as,

$$V_{o(r.m.s.)} = \left\{ \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} v_o^2(\omega t)\, d\omega t + \int_{\pi+\alpha}^{2\pi} v_o^2(\omega t)\, d\omega t \right] \right\}^{\frac{1}{2}}$$

When SCRs conduct, $v_o(\omega t) = v_s(\omega t)$. Hence above equation will be,

$$V_{o(r.m.s.)} = \left\{ \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t\, d\omega t + \int_{\pi+\alpha}^{2\pi} V_m^2 \sin^2 \omega t\, d\omega t \right] \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{V_m^2}{2\pi} \left[\int_{\alpha}^{\pi} \frac{1-\cos 2\omega t}{2}\, d\omega t + \int_{\pi+\alpha}^{2\pi} \frac{1-\cos 2\omega t}{2}\, d\omega t \right] \right\}^{\frac{1}{2}}$$

On simplifying the above integration, we get,

$$V_{o(r.m.s.)} = V_m \sqrt{\frac{\pi - \alpha + \frac{\sin 2\alpha}{2}}{2\pi}}$$

In the above equation,

when $\alpha = 0$ $\quad V_{o(r.m.s.)} = \frac{V_m}{\sqrt{2}} = V_{s(r.m.s.)}$

when $\alpha = \pi$ $\quad V_{o(r.m.s.)} = 0$

Thus the output can be controlled from zero to $V_{s(r.m.s.)}$ by varying firing angle from π to zero. Since the output current and voltage as well as supply current waveforms are symmetric, their d.c./average values are zero. Hence transformer saturation problems are absent.

Single Phase Controllers with Inductive Loads:

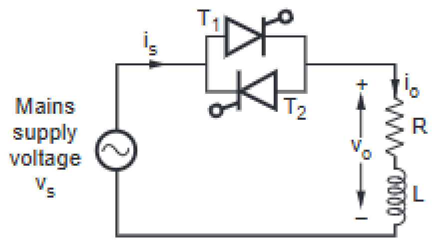


Fig 1 1ϕ full wave controller with inductive load

Most of the times the load on the controllers is inductive. Such loads are induction motors, water pumps, fans, blowers etc. The output current waveform is different in case of inductive load. Fig.1 shows the circuit diagram of 1 phase full wave controller having inductive load.

The SCR Tl is triggered in positive half cycle with delay angle of α. The output current starts increasing from zero. The waveforms are shown in Fig. 2. At π, the supply voltage is zero, but output current is not The load inductance to maintain the current in the same direction. Hence SCR T'; keeps on conducting. The SCR T 1 conducts from $\pi\ to\ \beta$ to due to energy Stored in the load inductance. At ß the output current becomes zero. Hence Tl turns off by natural commutation.

In the waveforms of Fig. 2 observe that T2 is triggered at $\pi + \alpha$. The output current starts increasing in the negative direction from zero. And negative supply voltage appears across the load. Thus negative cycle of and io starts. The supply current is same as output current. These currents are nonsinusoidal due to inductive load.

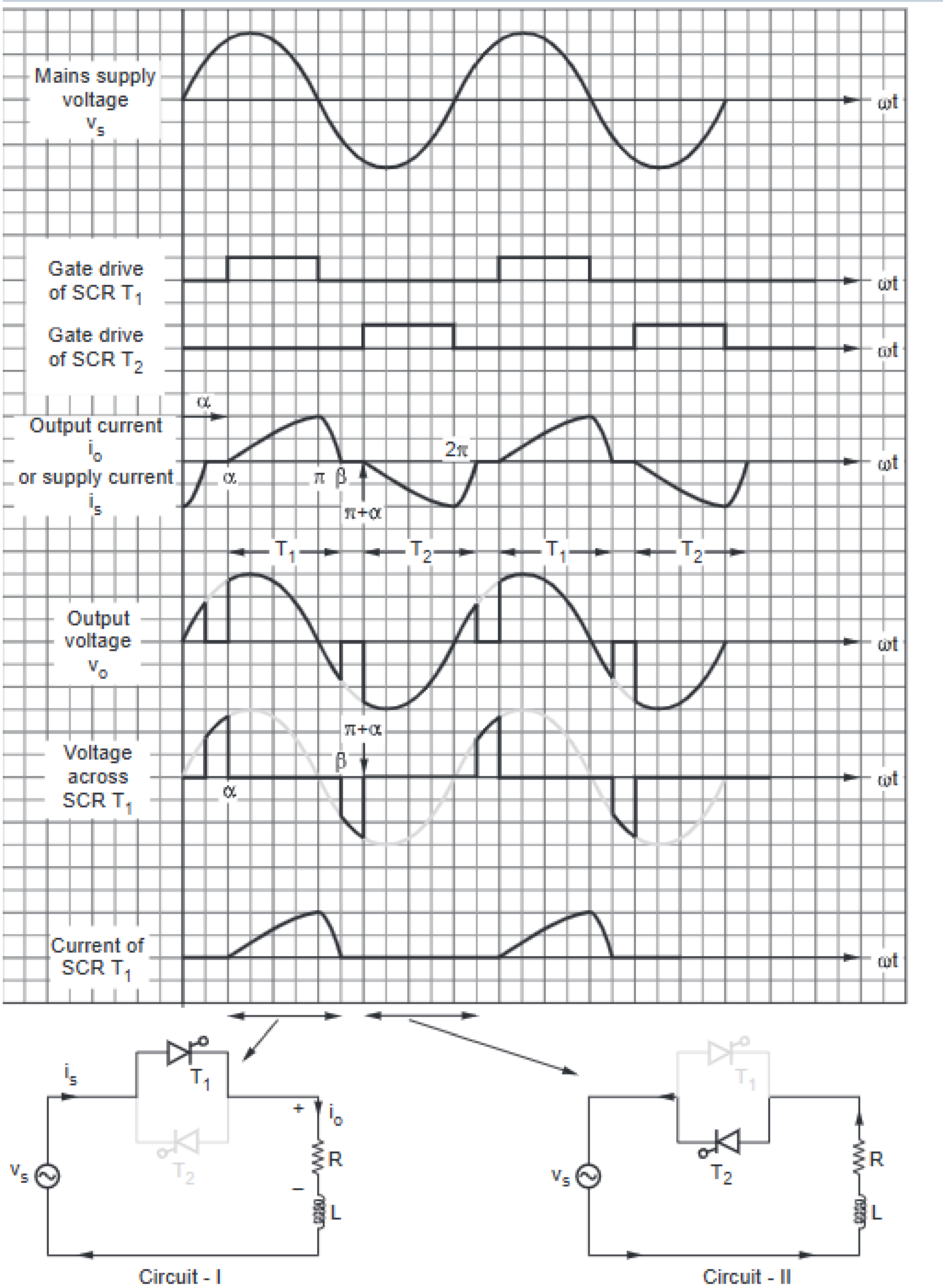


Fig 2 **Waveforms of 1ϕ full wave controller for inductive load (Discontinuous output current)**

The r.m.s. value is given as,

$$V_{o(r.m.s.)} = \left[\frac{1}{T}\int_0^T v_o^2\,(\omega t)\ d\omega t\right]^{\frac{1}{2}}$$

When SCRs T_1 and T_2 conduct, the supply voltage v_s appears across the load. The supply voltage can be expressed as,

$$v_s(\omega t) = V_m \sin \omega t$$

$$V_{o(r.m.s)} = \left[\frac{1}{\pi}\int_\alpha^\beta v_o^2(\omega t)\ d\omega t\right]^{\frac{1}{2}}$$

Here observe that we have taken $T = \pi$, since r.m.s. values of positive and negative half cycles are same for symmetric waveforms. Since $v_o(\omega t) = v_s(\omega t)$ when SCR conducts, we can write above equation as,

$$V_{o(r.m.s)} = \left[\frac{1}{\pi}\int_\alpha^\beta V_m^2 \sin^2 \omega t\ d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi}\int_\alpha^\beta \frac{1-\cos 2\,\omega t}{2}\ d\omega t\right]^{\frac{1}{2}}$$

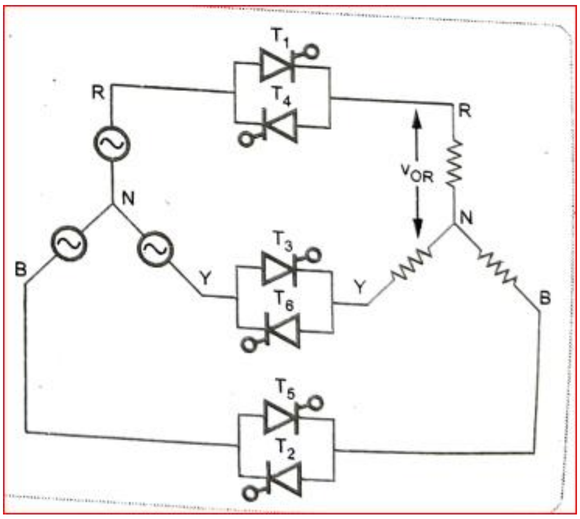
$$= \left[\frac{V_m^2}{2\pi}\int_\alpha^\beta (1-\cos 2\omega t)\ d\omega t\right]^{\frac{1}{2}}$$

Solving the above integration we get,

$$V_{o(r.m.s.)} = V_m\sqrt{\frac{\beta-\alpha+\frac{\sin 2\alpha}{2}-\frac{\sin 2\beta}{2}}{2\pi}}$$

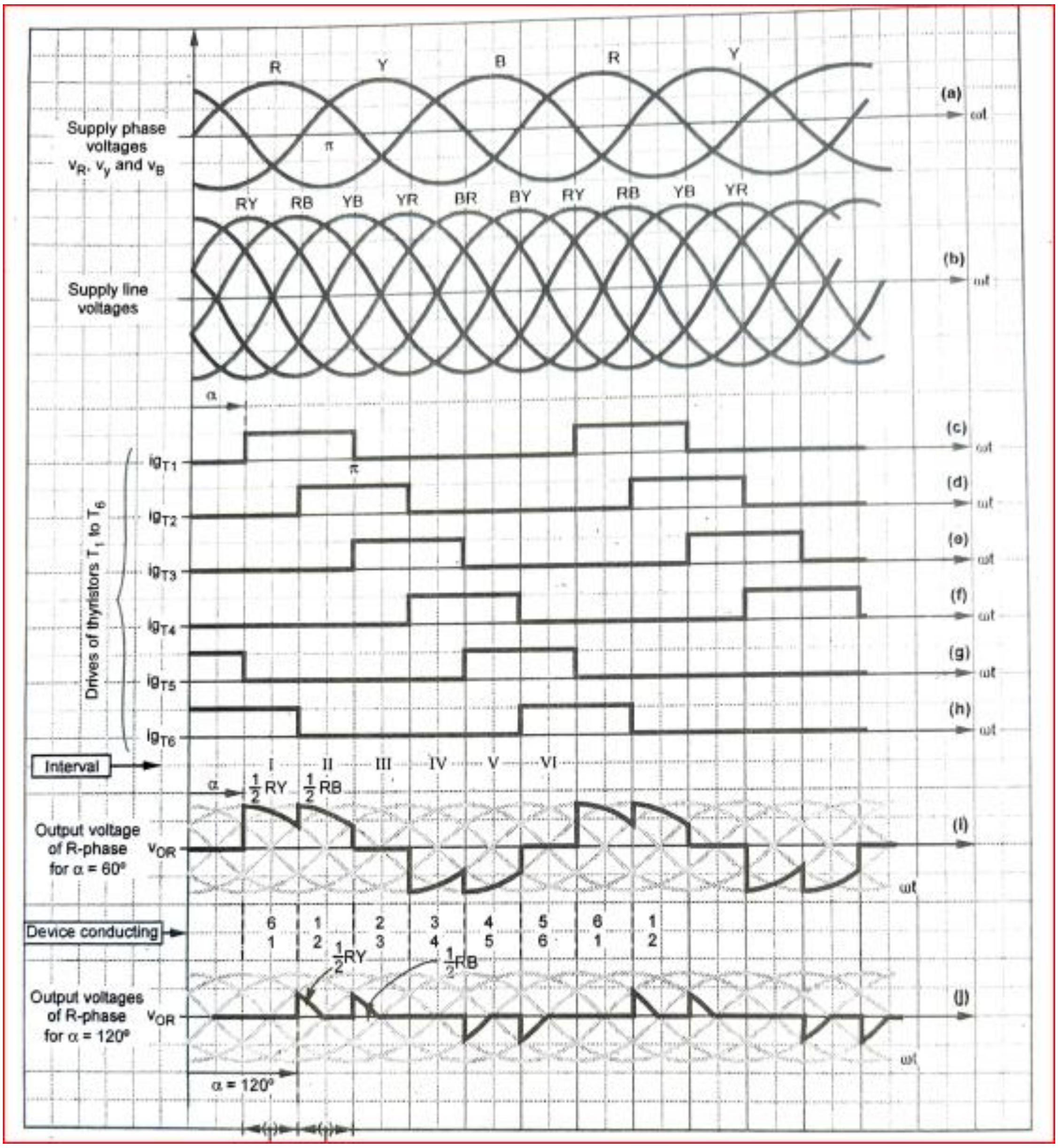
Since the output voltage, current and input current waveforms are symmetric, their average values (d.c. components) are zero.

3 phase Full Wave AC Voltage Controllers:



In the above diagram observe that the star connected 3 supply is applied to the star connected resistive load through antiparallel SCRs in each phase. Triacs can also be used instead of antiparallel SCRs. At any time current flows in two phases. Third phase is open. This is because SCRs from different phases conduct at a time.

Supply phase and line are shown first in the figure. The supply phase voltages are R, Y and B. The line voltages are RY, RB, YB, YR. BR, BY.



• Fig shows the vector diagram for line and phase voltages. In this figure observe that the phase shift between phase voltages R, Y and B is 120°. Line voltage RY is obtained by adding R and −Y. Observe that line voltage RY leads R phase voltage by 30°. This is shown in the waveforms of Fig. (a) and (b). Similarly line voltage RB lags R-phase voltage by 30°. The phase shift between individual line voltages is 60°.

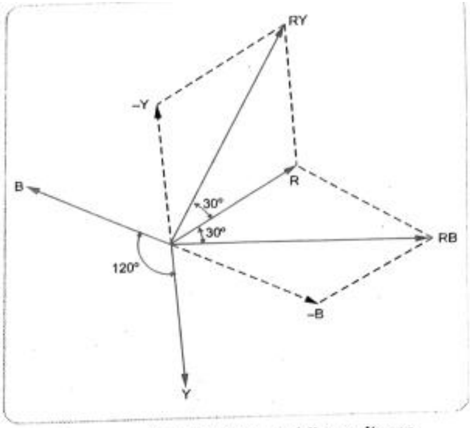


Fig. Supply phase and line voltages

Gate drives of thyristors T_1 - T_6

• The waveforms of Fig. 4.15.2 (c) to (h) shows the gate drives of all the six thyristors. Note that T_1 is applied the drive at $\alpha = 60°$. Note that 'α' is counted from zero crossing of the supply phase voltage. The firing pulse of T_1 ends at π. Similarly other firing pulses are generated.

• Note that in the first interval thyristor T_1 and T_6 are applied the gate drive. In the second interval thyristor T_1 and T_2 are applied the gate drive. Like this the cycle repeats after VI^{th} interval.

Output voltage waveform

• Consider an equivalent circuit-I of Fig. when thyristors T_1 and T_6 are conducting. Observe that the line voltage RY appears across load. Since load is resistive and balanced, half of the line voltage appears across each of the load phase. Therefore $v_{OR} = \frac{1}{2} RY$. The waveform of Fig. shows line voltages of half amplitude. And voltage $\frac{1}{2}$ RY appears as output in interval. Similarly in interval-II, thyristors T_1 and T_2 conduct. The line voltage RB appears across the load as shown in equivalent circuit-II of Fig. 4.15.2. Since the load is balanced, half of the line voltage appears across each load phase.Therefore $v_{OR} = \frac{1}{2} RB$ in II^{nd} interval. Like this the remaining waveforms can be completed.

The rms value of the output is given as follows for different ranges of α,

For $0 \le \alpha \le 60°$, $$V_{o(rms)} = \sqrt{6} V_s \sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)}$$

For $60° \le \alpha \le 90°$, $$V_{o(rms)} = \sqrt{6} V_s \sqrt{\frac{1}{\pi}\left(\frac{\pi}{12} + \frac{3 \sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{8}\right)}$$

For $90° \le \alpha \le 150°$, $$V_{o(rms)} = \sqrt{6} V_s \sqrt{\frac{1}{\pi}\left(\frac{5\pi}{24} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

Advantages. Disadvantages and Applications of 3 phase Controllers

Advantages

i) 3 phase controllers can deliver more power.

ii) 3 phase controllers contain less harmonics in output for high powers.

Diadvantages:

i) control of 3 phase controllers is complex.

ii) 3 phase are not suitable for resistive loads.

Applications

i) Induction control.

ii) Large fans control.