ATME COLLEGE OF ENGINEERING

13th KM Stone, Bannur Road, Mysuru - 570 028

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

NOTES

SIGNALS AND DIGITAL SIGNAL PROCESSING

SUB CODE: BEE502

SEMESTER: V

INSTITUTIONAL VISION AND MISSION

VISION:

- Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

MISSION:

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torchbearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence.

Department Vision and Mission

Vision:

To create Electrical and Electronics Engineers who excel to be technically competent and fulfill the cultural and social aspirations of the society.

Mission:

- To provide knowledge to students that builds a strong foundation in the basic principles of electrical engineering, problem solving abilities, analytical skills, soft skills and communication skills for their overall development.
- To offer outcome based technical education.
- To encourage faculty in training & development and to offer consultancy through research & industry interaction.

Program Educational Objectives (PEOs)

PEO1: To produce competent and ethical Electrical and Electronics Engineers who will exhibit the necessary technical and managerial skills to perform their duties in society.

PEO2: To make Graduates continuously acquire and enhance their technical and socio-economic skills.

PEO3: To aspire Graduates on R & D activities leading to offering solutions and excel in various career paths.

PEO4: To produce quality engineers who have the capability to work in teams and contribute to real time projects.

Program Outcomes (POs)

Engineering Graduates will be able to:

PO1: Engineering Knowledge: Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3: Design / Development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4: Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6: The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9: Individual and team work: Function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11: Project management and finance: Demonstrate knowledge and understanding of the engineering management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12: Life-long learning: Recognize the need for and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

The students will develop an ability to produce the following engineering traits:

PSO1: Apply the concepts of Electrical & Electronics Engineering to evaluate the performance of power systems and also to control industrial drives using power electronics.

PSO2: Demonstrate the concepts of process control for Industrial Automation, design models for environmental and social concerns and also exhibit continuous self- learning.

MODULE 1: INTRODUCTION

Structure

1.0 Objective

- To study the classification of signals.
- To study the operation on signals.
- To study the types of signal.
- To study the properties of a system.

1.1 Signal and System definition

A **signal** is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon.

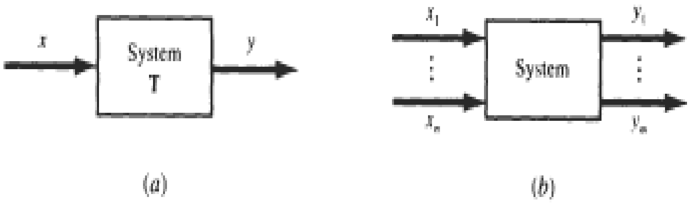
For instance, in a RC circuit the signal may represent the voltage across the capacitor or the current flowing in the resistor. Mathematically, a signal is represented as a function of an independent variable's'. Usually 't' represents time. Thus, a signal is denoted by **x(t)**.

A **system** is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.

Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of **x** into y. This transformation is represented by the mathematical notation

$$y = Tx$$

where **T** is the operator representing some well-defined rule by which **x** is transformed into y. Relationship (1.1) is depicted as shown in Fig. 1-1(a). Multiple input and/or output signals are possible as shown in Fig. 1-1(b). We will restrict our attention for the most part in this text to the single-input, single-output case.



1.1 System with single or multiple input and outputsignals

1.2 Classification of signals

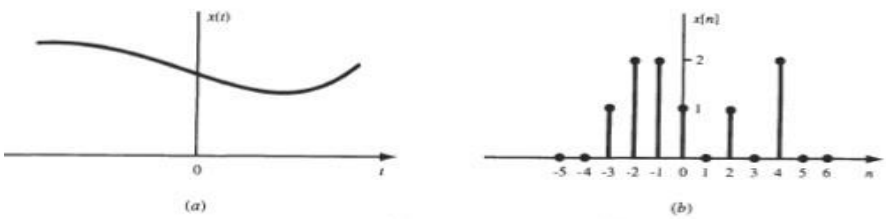
Basically seven different classifications are there:

- Continuous and Discrete time Signals
- Even and Odd Signals
- Random and Deterministic signal
- Power and Energy Signal
- Periodic and non periodic signal

1.2.1 Continuous-Time and Discrete-Time Signals

A signal x(t) is a continuous-time signal if t is a continuous variable. If t is a discrete variable, that is, x(t)isdefinedatdiscretetimes,thenx(t)isadiscrete-timesignal.Sincea

Discrete-time signal is defined at discrete times, a discrete-time signal is often identified as a sequence of numbers, denoted by {x,) or x[n], where n = integer. Illustrations of a continuous-time signal x(t) and of a discrete-time signal x[n] are shown in Fig. 1-2.



1.2 Graphical representation of (a) continuous-time and (b) discrete-time signals

1.2.2 Deterministic and Random Signals:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time **'t'.**

Random signals are those signals that take random values at any given time and must be characterized statistically.

1.2.3 Even and Odd Signals

A signal x(t) or x[n] is referred to as an even signal if

x (- t) = x(t) x [-n] = x [n]

A signal x(t) or x[n] is referred to as an odd signal if

x(-t) = - x(t) x[- n] = - x[n]

Examples of even and odd signals are shown in Fig. 1.3.

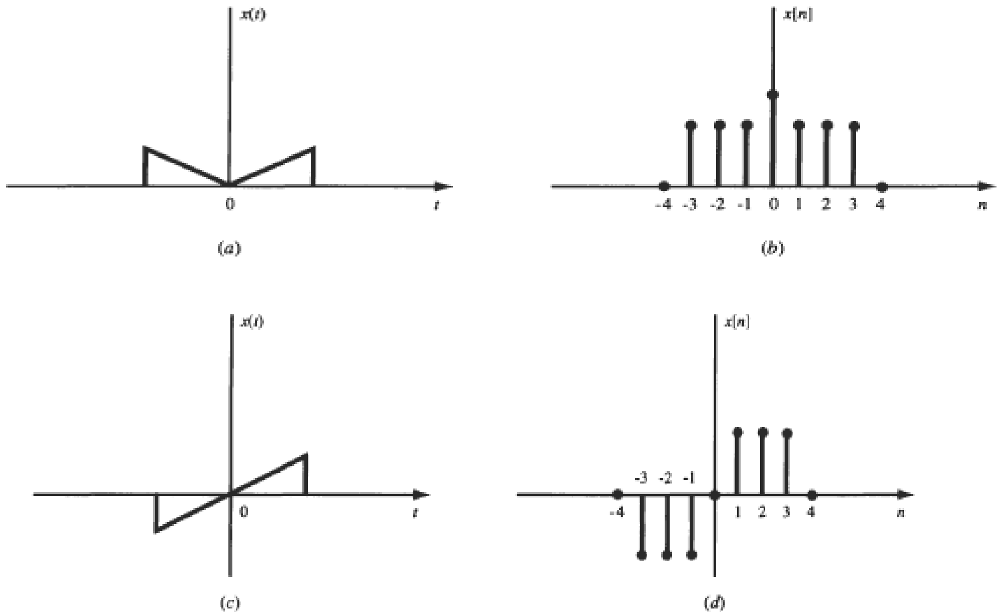


Fig. 1.3: Examples of even signals (a and b) and odd signals (c andd).

Any signal x(t) or x[n] can be expressed as a sum of two signals, one of which is even and one of which is odd. That is,

$$x(t) = x_o(t) + x_e(t)$$

Where,

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

Similarly for x[n],

$$x[n] = x_o[n] + x_e[n]$$

.

Where,

$$x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$$

$$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$

Note that the product of two even signals or of two odd signals is an even signal and that the product of an even signal and an odd signal is an odd signal.

1.2.4 Periodic and Non-periodic Signals

A continuous-time signal x(t) is said to be periodic with period T if there is a positive nonzero value of T for which

$$x(t+T) = x(t) \qquad \text{all } t$$

An example of such a signal is given in Fig. 1-4(a). From Eq. (1.9) or Fig. 1-4(a) it follows that

$$x(t+mT) = x(t)$$

for all t and any integer m. The fundamental period T, of x(t) is the smallest positive value of T for which Eq. (1.9) holds. Note that this definition does not work for a constant.

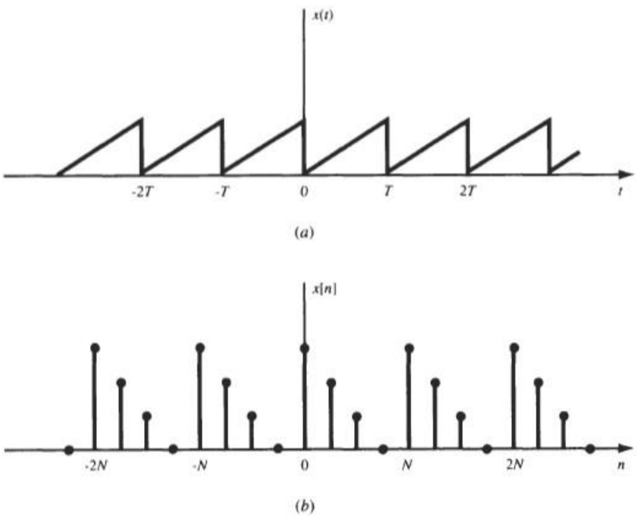


Fig.1.4: Examples of periodic signals.

signal x(t) (known as a dc signal). For a constant signal x(t) the fundamental period is undefined since x(t) is periodic for any choice of T (and so there is no smallest positive value). Any continuous-time signal which is not periodic is called a non-periodic (or periodic)signal.

Periodic discrete-time signals are defined analogously. A sequence (discrete-time signal) x[n] is periodic with period N if there is a positive integer N for which

$$x[n+N] = x[n] \qquad \text{all } n$$

An example of such a sequence is given in Fig. 1-4(b). From Eq. (1.11) and Fig. 1-4(b) it follows that

$$x[n+mN] = x[n]$$

for all n and any integer m. The fundamental period $N_o$of x[n] is the smallest positive integer N for which Eq.(1.11) holds. Any sequence which is not periodic is called a nonperiodic (or a periodic sequence.

Note that a sequence obtained by uniform sampling of a periodic continuous-time signal may not be periodic. Note also that the sum of two continuous-time periodic signals may not be periodic but that the sum of two periodic sequences is always periodic.

1.2.5 Energy and Power Signals

Consider v(t) to be the voltage across a resistor R producing a current i(t). The instantaneous power p(t) per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t) \qquad \text{…………(1.13)}$$

Total energy E and average power P on a per-ohm basis are

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules}$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts} \qquad \text{……(1.14)}$$

For an arbitrary continuous-time signal x(t), the normalized energy content E of x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt \qquad \text{…………………(1.15)}$$

The normalized average power P of x(t) is defined as

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt \qquad (1.16)$$

Similarly, for a discrete-time signal x[n], the normalized energy content ***E*** of x[n] is defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad (1.17)$$

The normalized average power P of x[n] is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Based on definitions (1.15) to (1.18), the following classes of signals are defined:

1. x(t) (or x[n]) is said to be an energy signal (or sequence) if and only if 0 < E < m, and so P =0.
2. x(t) (or x[n]) is said to be a power signal (or sequence) if and only if 0 < P < m, thus implying that E =m.
3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

Note that a periodic signal is a power signal if its energy content per period is finite, and then the average power of this signal need only be calculated over a period.

1.3 Basic Operations on signals

The operations performed on signals can be broadly classified into two kinds

- Operations on dependent variables
- Operations on independent variables

1.3.1 Operations on dependent variables

The operations of the dependent variable can be classified into five types: amplitude scaling, addition, multiplication, integration and differentiation.

Amplitude scaling

Amplitude scaling of a signal *x*(*t*) given by equation 1.19, results in amplification of *x*(*t*) if *a* >1, and attenuation if *a* <1.

$$y(t) = ax(t)$$

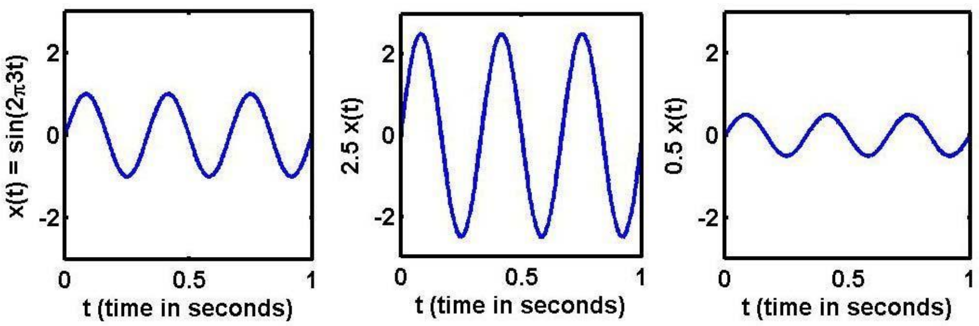


Fig. 1.5: Amplitude scaling of sinusoidal signal

Addition

The addition of signals is given by equation below.

$$y(t) = x1(t) + x2 (t)$$

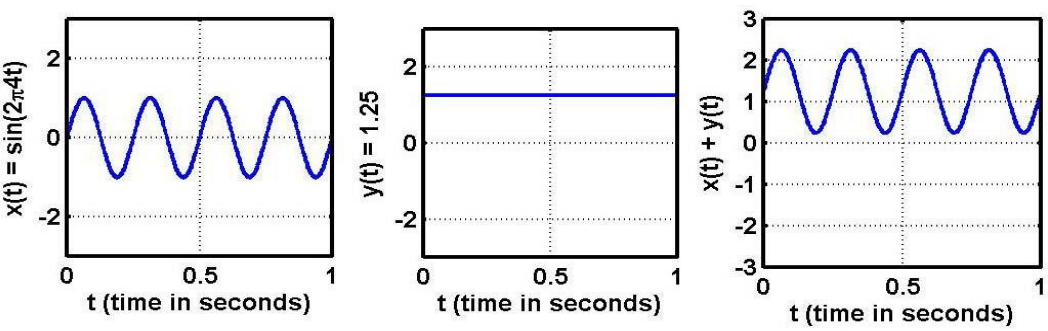


Fig.1.6: Example of the addition of a sinusoidal signal with a signal of constant amplitude

Physical significance of this operation is to add two signals like in the addition of the background music along with the human audio. Another example is the undesired addition of noise along with the desired audio signals.

Multiplication

The multiplication of signals is given by the simple equation of 1.22.

$$y(t) = x1(t).x2\ (t)$$

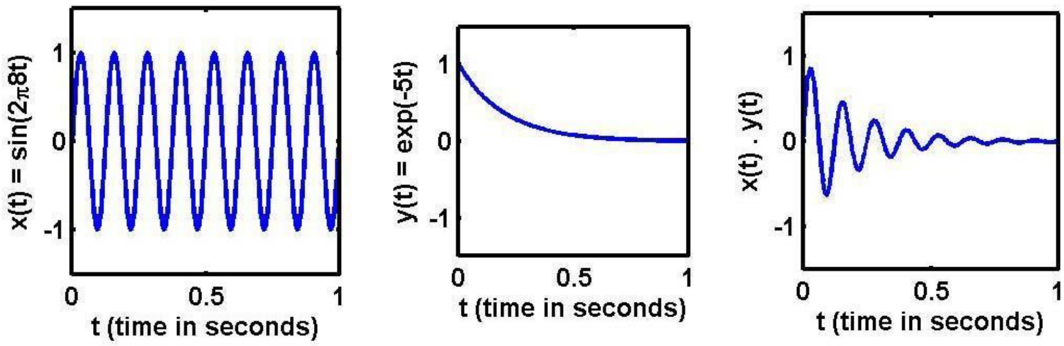


Fig. 1.7: Example of multiplication of two signals

Differentiation

The differentiation of signals is given by the equation below for the continuous.

$$y(t) = \frac{d}{dt} x(t)$$

The operation of differentiation gives the rate at which the signal changes with respect to time, and can be computed using the following equation, with Δt being a small interval of time.

$$\frac{d}{dt} x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

If a signal doesn"t change with time, its derivative is zero, and if it changes at a fixed rate with time, its derivative is constant. This is evident by the example given in figure 1.8.

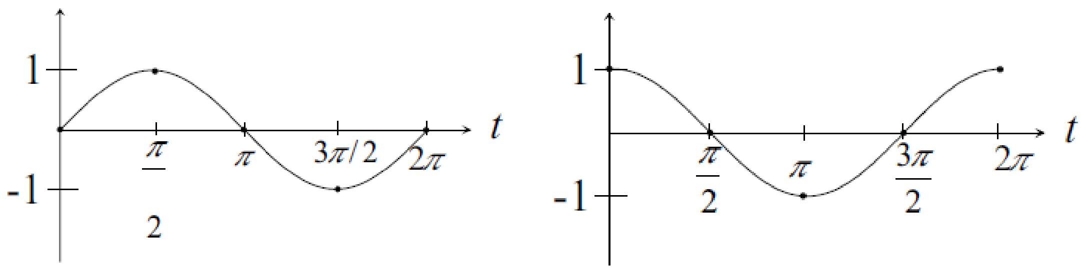


Fig. 1.8: Differentiation of Sine -Cosine

Integration

The integration of a signal $x(t)$, is given by equation below.

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

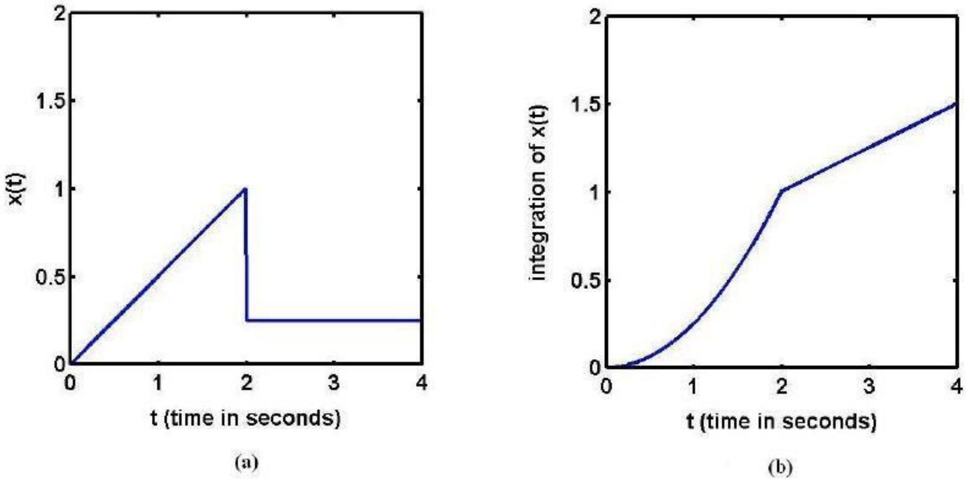


Fig. 1.9: Integration of x(t)

1.3.2 Operations on independent variables

Time scaling

Time scaling operation is given by equation,

$$y(t)=x(at)$$

This operation results in expansion in time for a<1 and compression in time for a>1, as evident from the examples of figure 1.10.

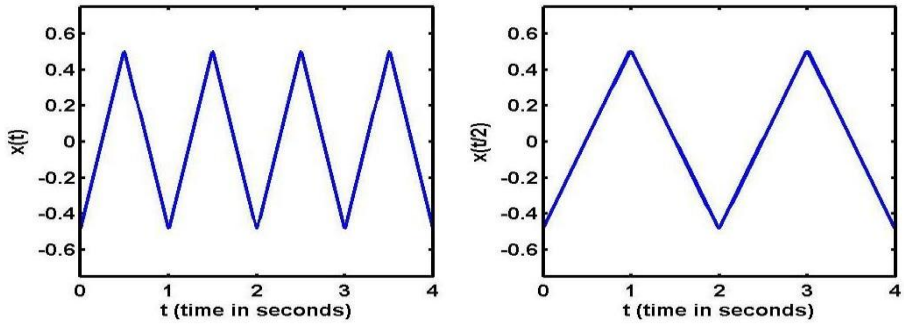


Fig. 1.10: Time scaling of a continuous signal

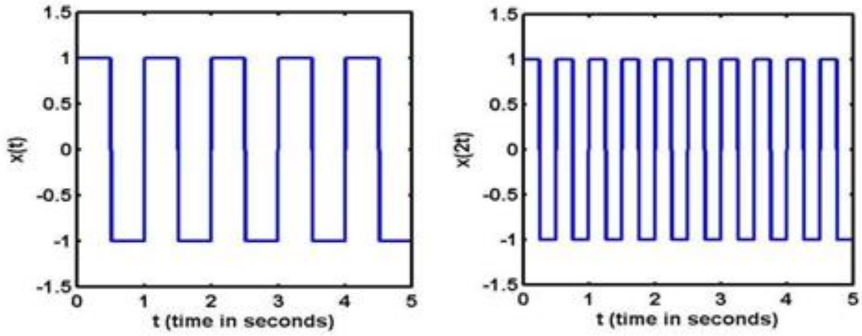


Fig. 1.11: Examples of time scaling of a continuous time signal

An example of this operation is the compression or expansion of the time scale that results in the „*fast-forward*' or the „*slow motion*' in a video, provided we have the entire video in some stored form.

Time reflection

Time reflection is given by equation , and some examples is shown in fig 1.12

$$y(t)= x(-t)$$

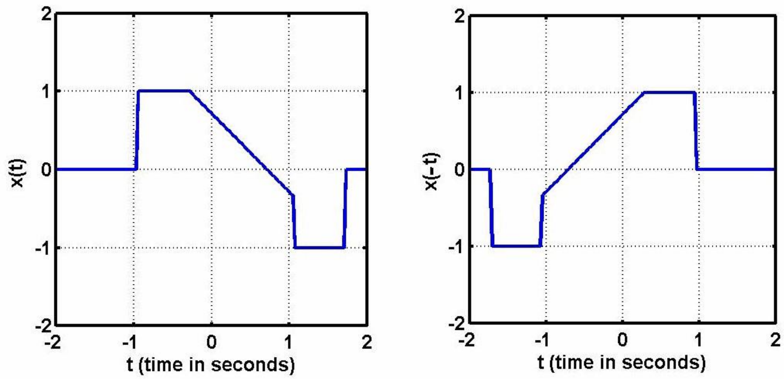


Fig. 1.12: Example of time reflection of a continuous time signal

Time shifting

The equation representing time shifting is given by equation (1.28), and example of this operation is shown in figure 1.13.

$$y(t) = x(t - t_0)$$

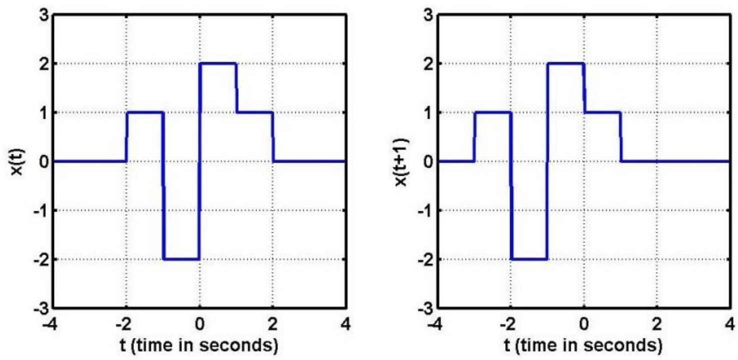


Fig. 1.13: Examples of time shift of a continuous time signal

Precedence rule

The combined transformation of shifting and scaling is contained in equation, an example for precedence rule is shown in fig. 1.14

$$y(t) = x(at - t_0)$$

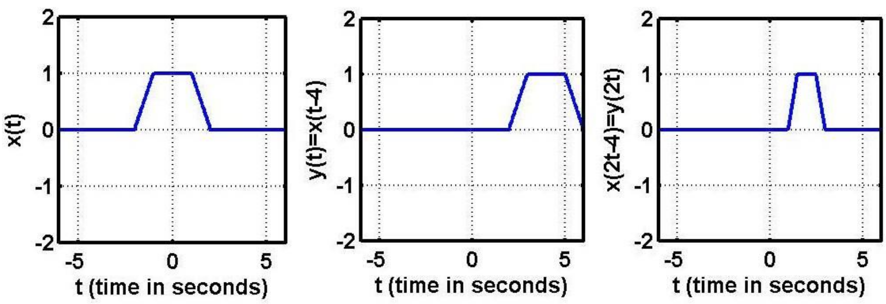


Fig. 1.14: Examples of simultaneous time shifting and scaling. The signal has to be shifted first and then timescale.

1.4 Elementary signals

Exponential signals:

The exponential signal given by equation, is a monotonically increasing function if $a > 0$, and is a decreasing function if $a < 0$.

$$x(t) = e^{at}$$

It can be seen that, for an exponential signal,

$$x(t + a^{-1}) = e.x(t)$$

$$x(t - a^{-1}) = e^{-1}.x(t)$$

Hence, equation (1.30), shows that change in time by $\pm 1/\ a$ seconds, results in change in magnitude by $e\pm1$. The term $1/\ a$ having units of time, is known as the time-constant. Let us consider a decaying exponential signal

$$x(t) = e^{-at} \qquad \text{for } t \geq 0.$$

This signal has an initial value $x(0) = 1$, and a final value $x(\infty) = 0$. The magnitude of this signal at five times the time constant is,

$$x(5/a) = 6.7x10^{-3}$$

While at ten times the time constant, it is as low as,

$$x(10/a) = 4.5x10^{-5}$$

It can be seen that the value at ten times the time constant is almost zero, the final value of the signal. Hence, in most engineering applications, the exponential signal can be said to have reached its final value in about ten times the time constant. If the time constant is 1 second, then final value is achieved in 10 seconds!! We have some examples of the exponential signal in figure1.15.

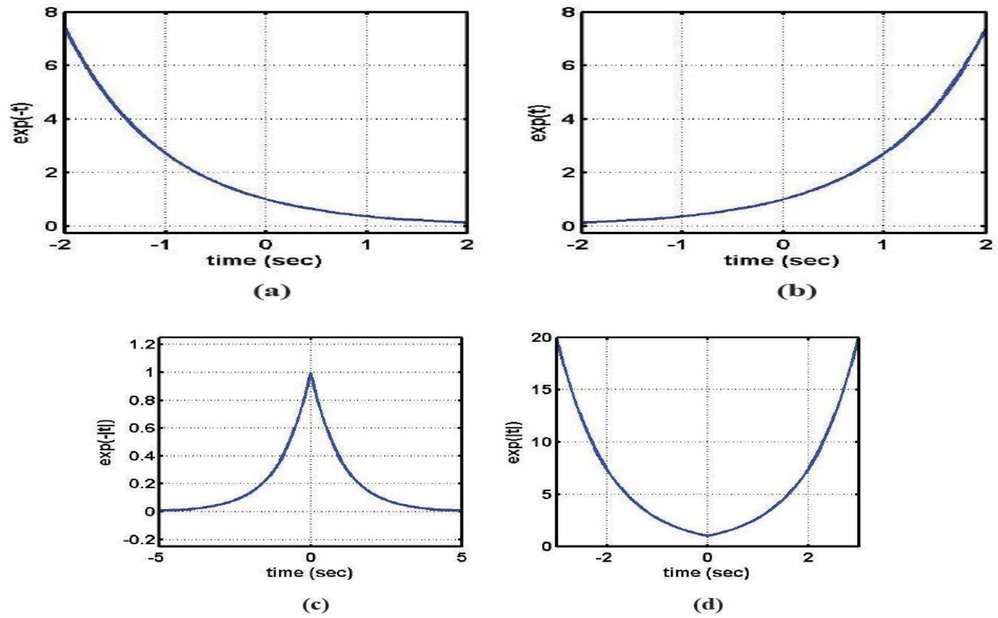


Fig 1.15: The continuous time exponential signal (a) e^{-t}, (b) e^{t}, (c) $e^{-|t|}$, and (d) $e^{|t|}$

The sinusoidal signal:

The sinusoidal continuous time periodic signal is given by equation 1.34, and examples are given in figure 1.15

$$x(t) = A\sin(2\pi ft)$$

The different parameters are:

Angular frequency $\omega = 2\pi f$ in radians, Frequency f in Hertz, (cycles per second) Amplitude A in Volts (or Amperes) Period T in seconds

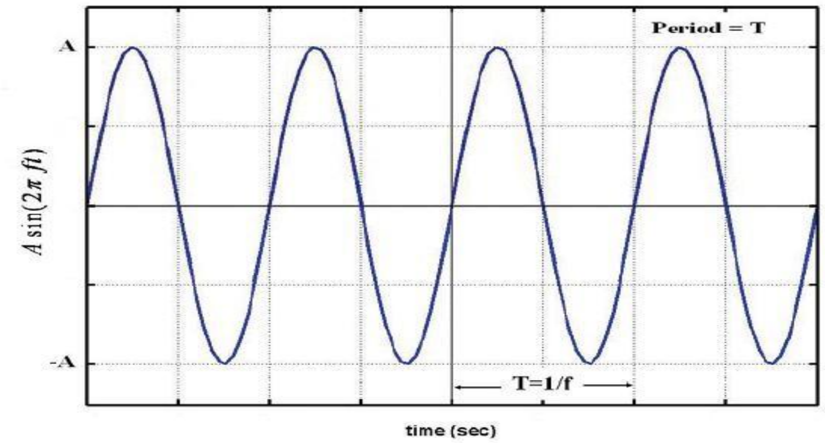


Fig. 1.16: Sinusoidal Signal

The complex exponential:

We now represent the complex exponential using the Euler's identity (equation (1.35)),

$$e^{j\theta} = \left(\cos\theta + j\sin\theta\right)$$

to represent sinusoidal signals. We have the complex exponential signal given by equation (1.36)

$$e^{j\omega t} = \left(\cos(\omega t) + j\sin(\omega t)\right)$$
$$e^{-j\omega t} = \left(\cos(\omega t) - j\sin(\omega t)\right)$$

Since sine and cosine signals are periodic, the complex exponential is also periodic with the same period as sine or cosine. From equation (1.36), we can see that the real periodic sinusoidal signals can be expressed as:

$$\cos(\omega t) = \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right)$$

$$\sin(\omega t) = \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right)$$

Let us consider the signal $x(t)$ given by equation (1.38). The sketch of this is given in fig 1.15

$$x(t) = A(t)e^{j\theta(t)}$$

The unit impulse:
The unit impulse usually represented as $\delta(t)$, also known as the dirac delta function, is given by,

$$\delta(t) = 0 \quad for \quad t \neq 0; \quad and \quad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

Figure 1.17, has the plot of the impulse function

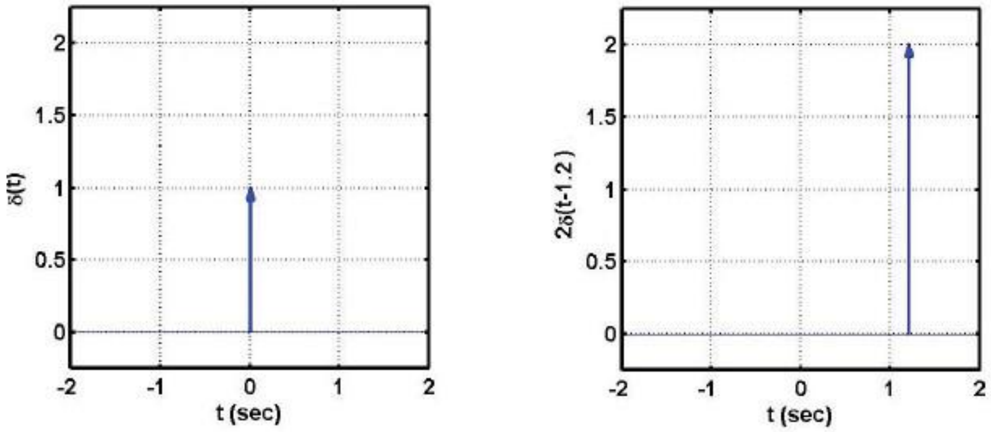


Fig. 1.17: Impulse Signal

The unit step:
The unit step function, usually represented as $u(t)$, is given by,

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

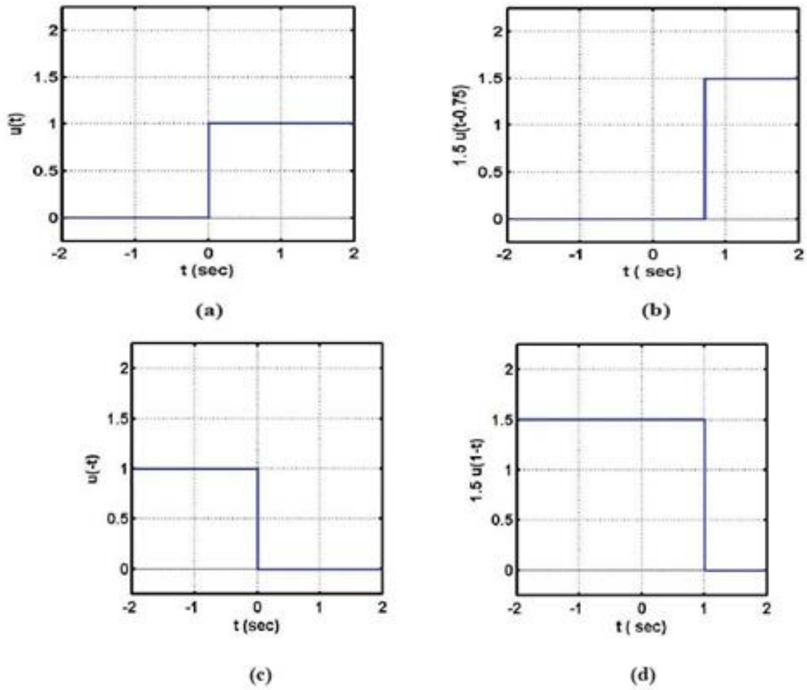


Fig 1.18: Step Signals

The unit ramp:
The unit ramp function, usually represented as $r(t)$, is given by,

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

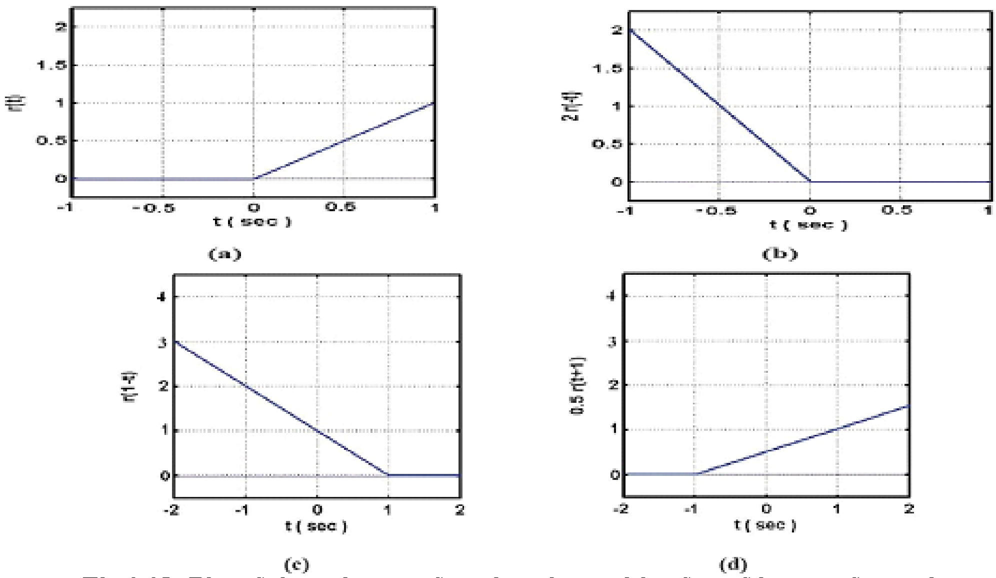


Fig 1.19: Plot of the unit ramp function along with a few of its transformations

The signum function:
The signum function, usually represented as $\text{sgn}(t)$, is given by

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

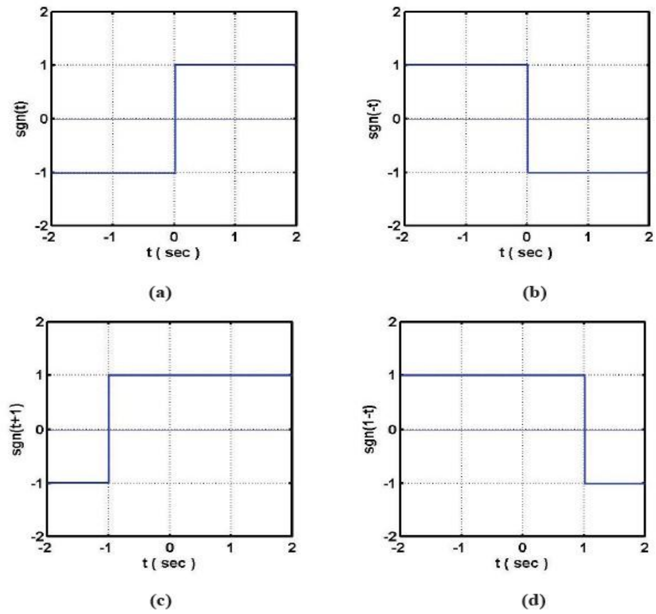


Fig 1.20: Plot of the unit signal function along with a few of its transformations

1.5 System viewed as interconnection of operation:

A system is an interconnection of operations that transforms an input signal into an output signal. The properties of these output signals are entirely different from that of the input signal.
If H is continuous time system
x(t) is input signal
Then output y(t)=H{x(t)}
Similarly For discrete time system
y(n)=H{x(n)}

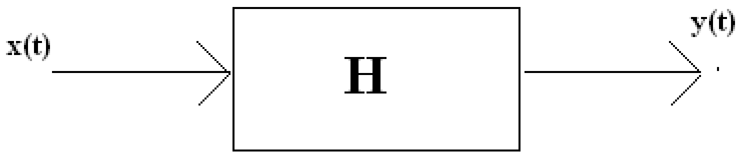


Fig. 1.21: operation of a system

1.6 Properties of system:

In this article discrete systems are taken into account. The same explanation stands for continuous time systems also.

The discrete time system:
The discrete time system is a device which accepts a discrete time signal as its input, transforms it to another desirable discrete time signal at its output as shown in figure 1.20

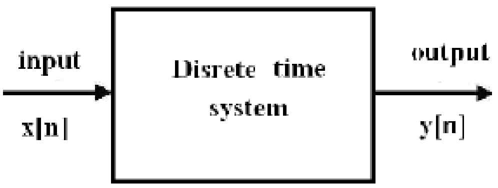


Fig 1.21: DT system

Stability
A system is stable if „bounded input results in a bounded output‟. This condition, denoted by BIBO, can be represented by:

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty \quad implies \quad \sum_{n=-\infty}^{\infty} |y[n]| < \infty \quad for \quad all \quad n$$

Hence, a finite input should produce a finite output, if the system is stable. Some examples of stable and unstable systems are given in figure 1.21

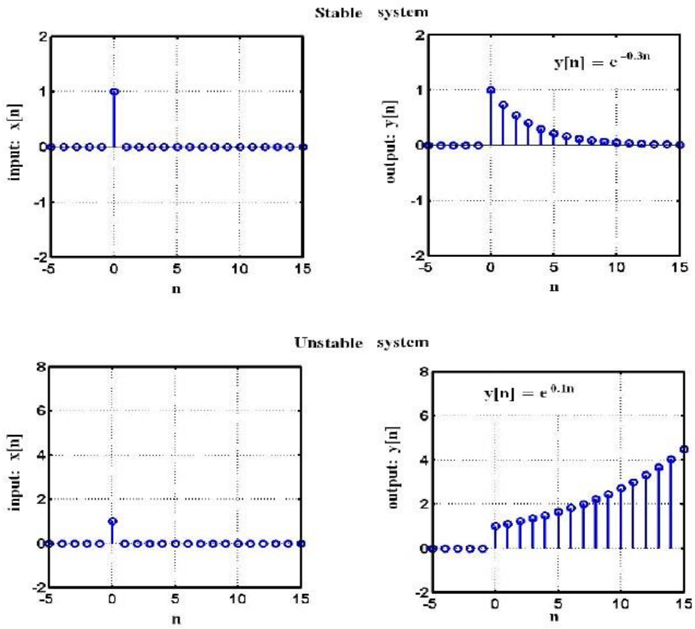


Fig 1.22: Examples for system stability

Memory

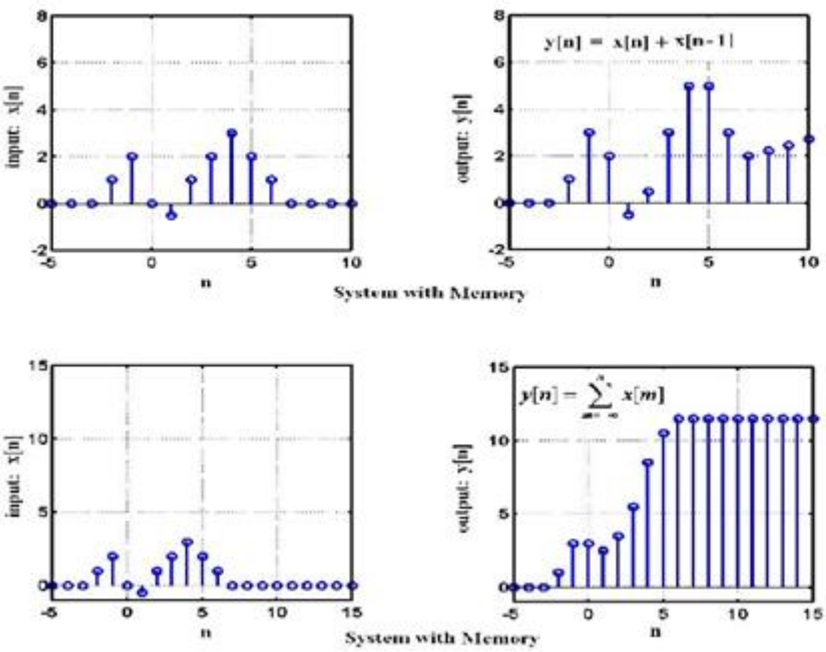
The system is memory-less if its instantaneous output depends only on the current input. In memory-less systems, the output does not depend on the previous or the future input.

Examples of memory less systems:

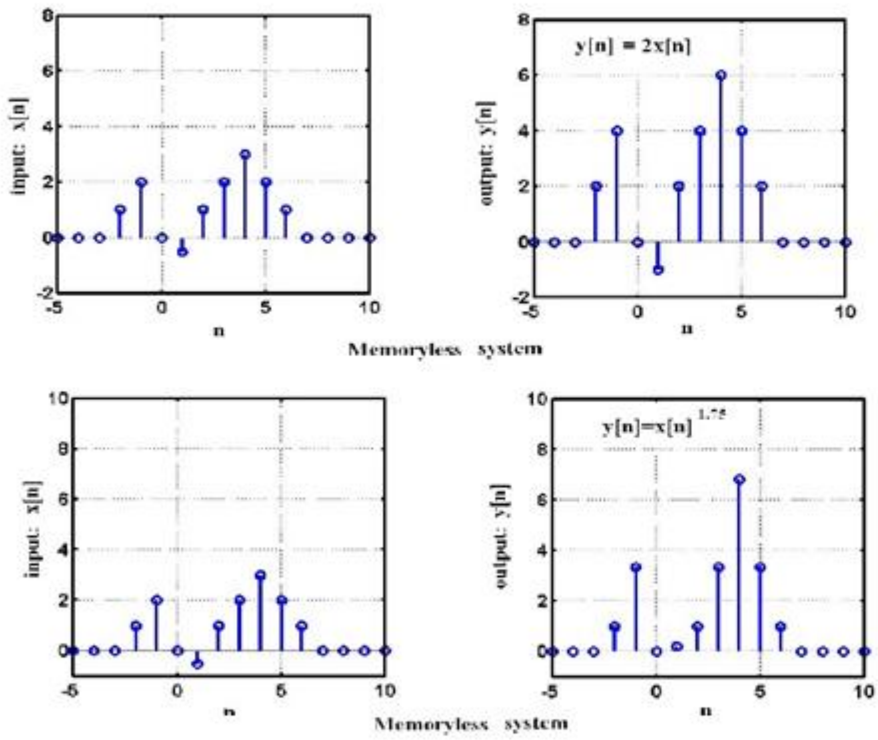
$$y[n] = ax[n]$$

$$y[n] = ax^2[n]$$

$$i[n] = a_0 + a_1 v[n] + a_2 v^2[n] + a_3 v^3[n] + ...$$



a)



b)

Fig. 1.23: a) System with memory b) System without memory

Causality:

A system is causal, if its output at any instant depends on the current and past values of input. The output of a causal system does not depend on the future values of input. This can be represented as:

For a causal system, the output should occur only after the input is applied, hence,

All physical systems are causal (examples in figure 7.5). Non-causal systems do not exist. This classification of a system may seem redundant. But, it is not so. This is because, sometimes, it may be necessary to design systems for given specifications. When a system design problem is attempted, it becomes necessary to test the causality of the system, which if not satisfied, cannot be realized by any means. Hypothetical examples of non-causal systems are given in figure below.

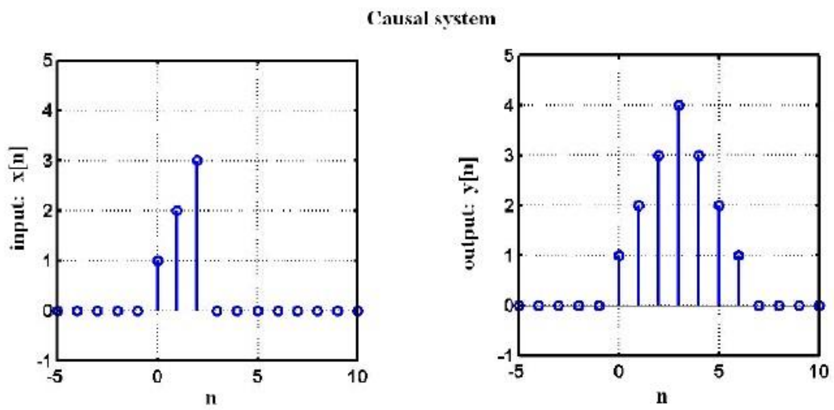


Fig. 1.24: Example for an causal system

Inevitability:

A system is said to be invertible if the input of the system can be recovered from the system output

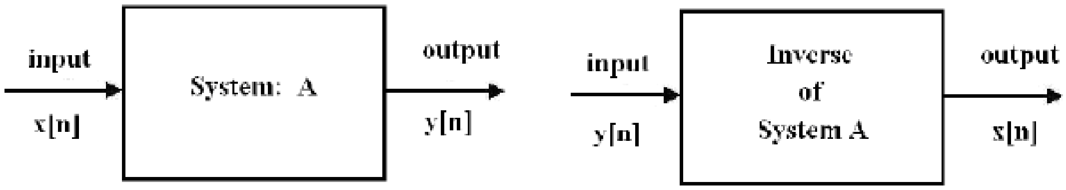


Fig. 1.25: invertible system

Linearity:

A system is said to be linear if it satisfies the principle of superposition.

$$\begin{aligned}
&\text{if } x_1(t) \xrightarrow{H} y_1(t) \\
&\& \ x_2(t) \xrightarrow{H} y_2(t) \\
&\text{then the system is linear if} \\
&ax_1(t) + bx_2(t) \xrightarrow{H} ay_1(t) + y_2(t)
\end{aligned}$$

Time invariance:

A system is time invariant, if its output depends on the input applied, and not on the time of application of the input. Hence, time invariant systems, give delayed outputs for delayed inputs.

$$\begin{aligned}
&\text{if } x_1(t) \xrightarrow{H} y_1(t) \\
&\text{then } x_1(t - t_0) \xrightarrow{S} y_1(t - t_0)
\end{aligned}$$

1.7 Outcomes

1. Knowledge on classification of signals.
2. Learnt the basic operation on signals.
3. Knowledge on types of signals.
4. Understand the properties of system.

1.8 Further reading

1. https://www.youtube.com/watch?v=Gpp_C6f13kw
2. http://nptel.ac.in/courses/117101055/downloads/Lec-4.pdf
3. https://web.iit.edu/sites/web/files/departments/academic-affairs/academic-resource-center/pdfs/signal_systems_prop.pdf
4. https://gradestack.com/Circuit-Theory-and/Introduction-to-Different/Different-Types-of/19344-3926-40307-study-wtw

MODULE 1 (b) : Time-Domain Representations For LTI Systems

Outline

2.0 Objectives:

1. Study the convolution of both continuous time and discrete time signals
2. Representing the continuous time system by differential equation and discrete time system by difference equation
3. Representation of the system by block diagrams

2.1 Introduction:

The Linear time invariant (LTI) system:

Systems which satisfy the condition of linearity as well as time invariance are known as linear time invariant systems. Throughout the rest of the course we shall be dealing with LTI systems. If the output of the system is known for a particular input, it is possible to obtain the output for a number of other inputs. We shall see through examples, the procedure to compute the output from a given input-output relation, for LTI systems.

2.2 Convolution:

A continuous time system as shown below, accepts a continuous time signal x(t) and gives out a transformed continuous time signal y(t).

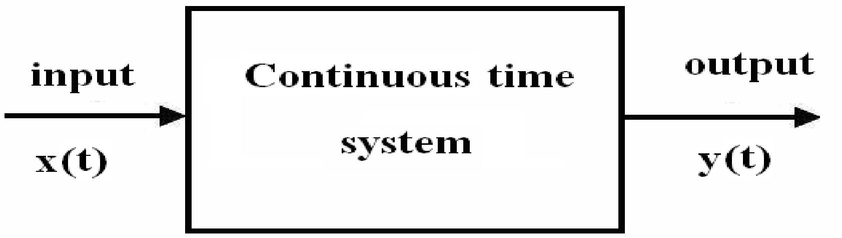


Figure 2.1: The continuous time system

Some of the different methods of representing the continuous time system are:

i) Differential equation
ii) Block diagram
iii) Impulse response
iv) Frequency response
v) Laplace-transform
vi) Pole-zero plot

It is possible to switch from one form of representation to another, and each of the representations is complete. Moreover, from each of the above representations, it is possible to obtain the system properties using parameters as: stability, causality, linearity, inevitability etc. We now attempt to develop the convolution integral.

2.2.1 Convolution Sum:

We now attempt to obtain the output of a digital system for an arbitrary input x[n], from the knowledge of the system impulse response h[n].

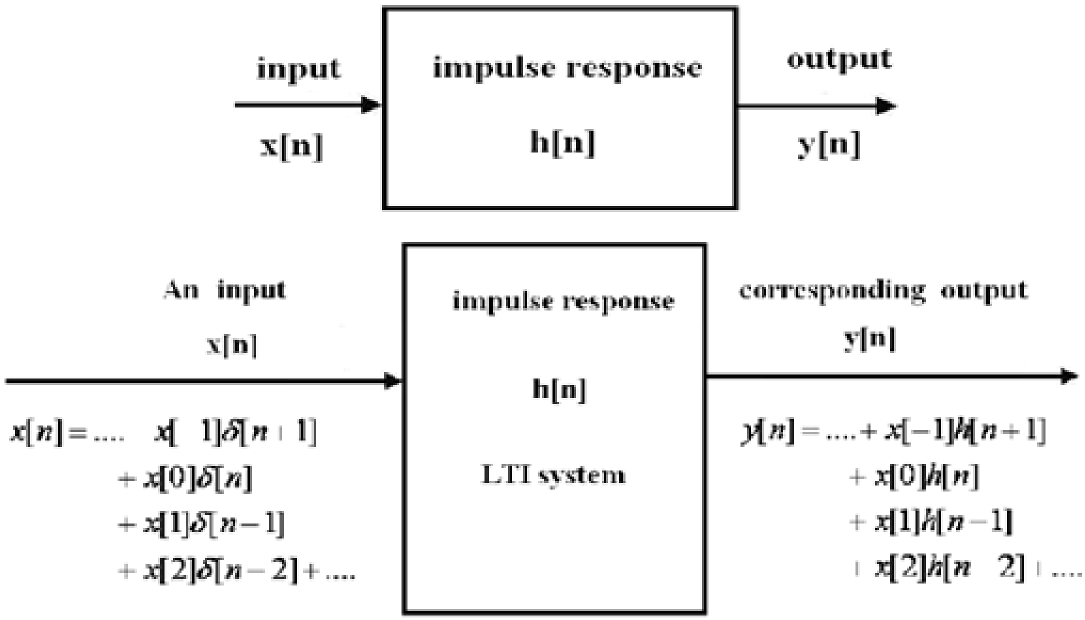


Fig. 2.2: Impulse response of a LTI system

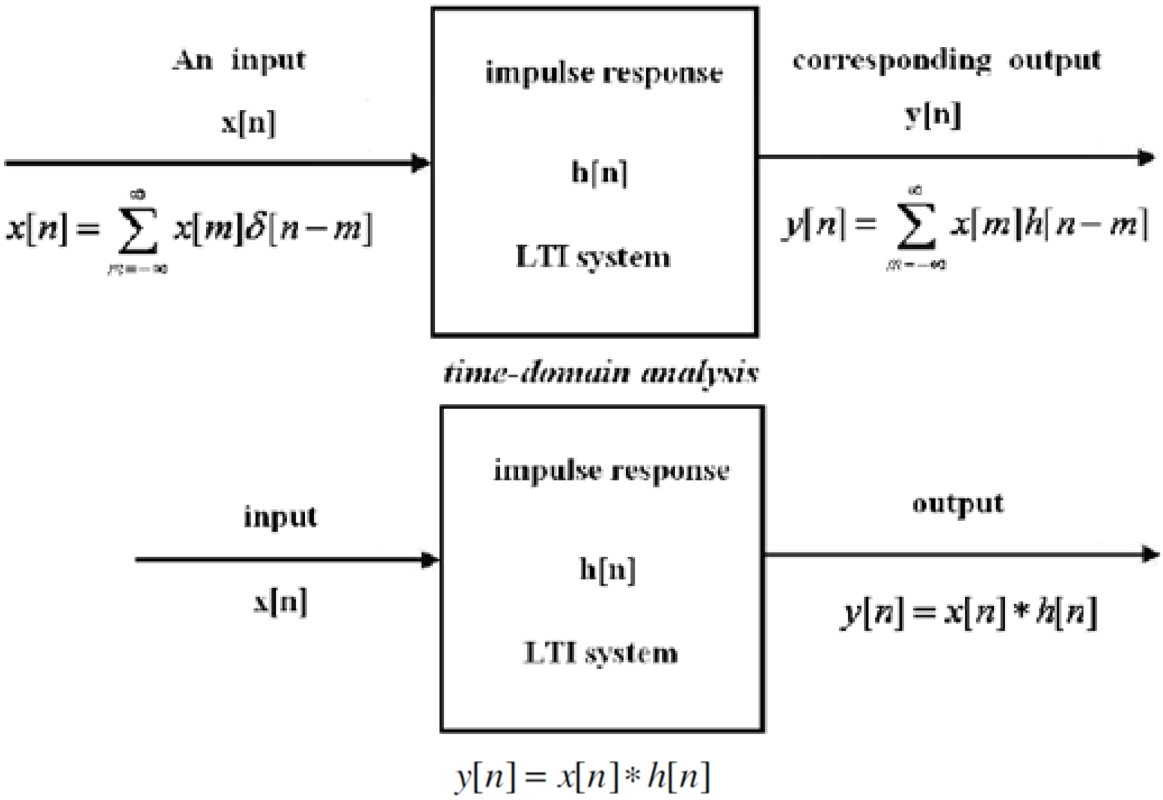


Fig. 2.3: Time domain analysis of a LTI system

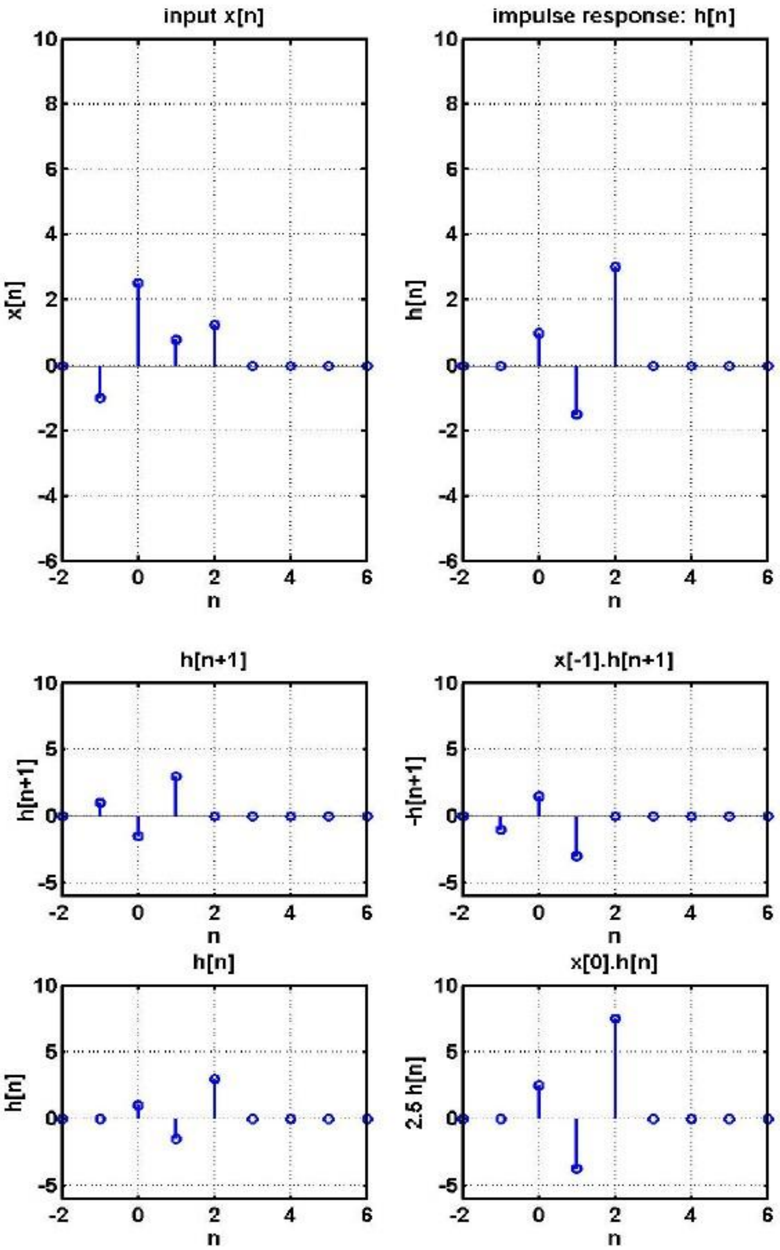
Methods of evaluating the convolution sum:
Given the system impulse response h[n], and the input x[n], the system output y[n], is given by the convolution sum:

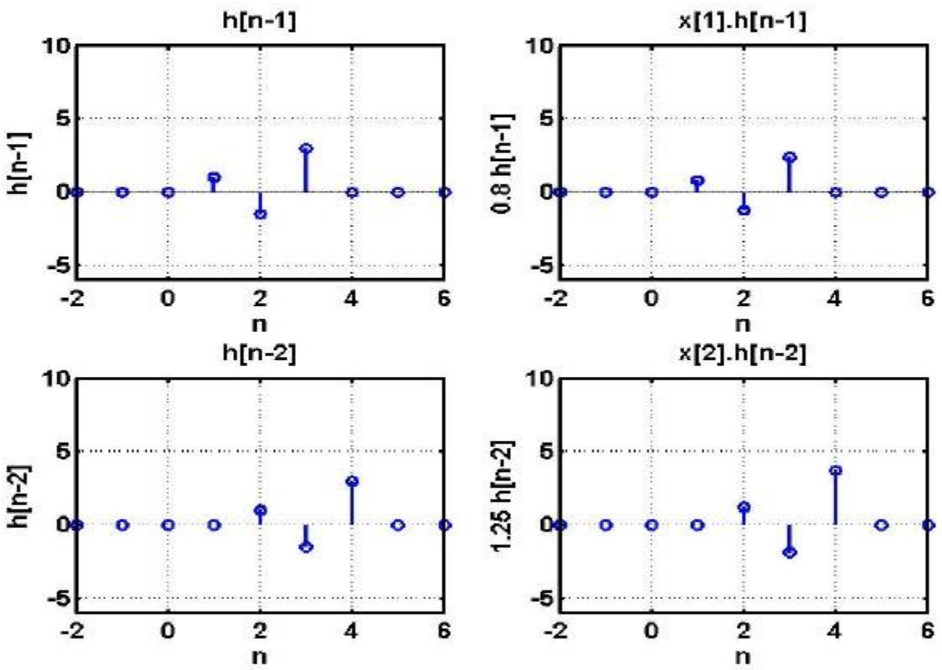
$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Problem:
To obtain the digital system output y[n], given the system impulse response h[n], and the system input x[n] as:

h[n]=[1,-1.5,3]
x[n]=[-1,2.5,0.8,1.25]
y[n]=[-1,4,-5.95,7.55,0.525,3.75]

1. Evaluation as the weighted sum of individual responses
The convolution sum of equation can be equivalently represented as:





Convolution as matrix multiplication:

Given

$$x[n] = [x_1 \quad x_2 \quad \ldots \quad x_L] \qquad \text{starting from } N_X$$

and

$$h[n] = [h_1 \quad h_2 \quad \ldots \quad h_M] \qquad \text{starting from } N_H$$

Step 1: Length of convolved sequence is NUM = (L+M-1)

Step 2: The convolved sequence starts at $i = N_X + N_H$

Step 3: The convolution is given by the following matrix multiplication

$$\begin{bmatrix} y[i] \\ y[i+1] \\ y[i+2] \\ y[i+3] \\ y[i+4] \\ y[i+5] \\ . \\ . \end{bmatrix} = \begin{bmatrix} x_1 & 0 & . & . & 0 \\ x_2 & x_1 & . & . & 0 \\ x_3 & x_2 & . & . & 0 \\ . & x_3 & . & . & 0 \\ . & . & . & . & x_1 \\ x_L & . & . & . & x_2 \\ 0 & x_L & . & . & . \\ 0 & 0 & . & . & x_L \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ . \\ . \\ h_M \end{bmatrix} = \begin{bmatrix} h_1 & 0 & . & . & 0 \\ h_2 & h_1 & . & . & 0 \\ h_3 & h_2 & . & . & 0 \\ . & h_3 & . & . & 0 \\ . & . & . & . & h_1 \\ h_M & . & . & . & h_2 \\ 0 & h_M & . & . & . \\ 0 & 0 & . & . & h_M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_L \end{bmatrix}$$

The dimensions of the above matrices are:

$$[NUM \quad by \quad 1] = [NUM \quad by \quad M][M \quad by \quad 1] = [NUM \quad by \quad L][L \quad by \quad 1]$$

For the given example:

x[n] is of length L=4, and starts at $N_X = -1$

h[n] is of length M=3 and starts at $N_H = 0$

Step 1: Length of convolved sequence is NUM = (L+M-1)=6

Step 2: The convolved sequence starts at i=(-1+0)=(-1)

$$\begin{bmatrix} y[-1] \\ y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 2.5 & -1 & 0 \\ 0.8 & 2.5 & -1 \\ 1.25 & 0.8 & 2.5 \\ 0 & 1.25 & 0.8 \\ 0 & 0 & 1.25 \end{bmatrix} \begin{bmatrix} 1 \\ -1.5 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -5.95 \\ 7.55 \\ 0.525 \\ 3.75 \end{bmatrix}$$

or

$$\begin{bmatrix} y[-1] \\ y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.5 & 1 & 0 & 0 \\ 3 & -1.5 & 1 & 0 \\ 0 & 3 & -1.5 & 1 \\ 0 & 0 & 3 & -1.5 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2.5 \\ 0.8 \\ 1.25 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -5.95 \\ 7.55 \\ 0.525 \\ 3.75 \end{bmatrix}$$

Evaluation using graphical representation:

Another method of computing the convolution is through the direct computation of each value of the output y[n]. This method is based on evaluation of the convolution sum for a single value of *n*, and varying *n* over all possible values.

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Step 1: Sketch x[m]

Step 2: Sketch h[-m]

Step 3: Compute y[0] using:

$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[-m]$$

which is the 'sum of the product of the two signals x[m] & h[-m]'

Step 4: Sketch h[1-m], which is right shift of h[-m] by 1.

Step 5: Compute y[1] using:

$$y[1] = \sum_{m=-\infty}^{\infty} x[m]h[1-m]$$

which is the 'sum of the product of the two signals x[m] & h[1-m]'

Step 6: Sketch h[2-m], which is right shift of h[-m] by 2.

Step 7: Compute y[2] using:

$$y[2] = \sum_{m=-\infty}^{\infty} x[m]h[2-m]$$

which is the 'sum of the product of the two signals x[m] & h[2-m]'

Step 8: Proceed this way until all possible values of y[n], for positive 'n' are computed

Step 9: Sketch h[-1-m], which is left shift of h[-m] by 1.

Step 10: Compute y[-1] using:

$$y[-1] = \sum_{m=-\infty}^{\infty} x[m]h[-1-m]$$

which is the 'sum of the product of the two signals x[m] & h[-1-m]'

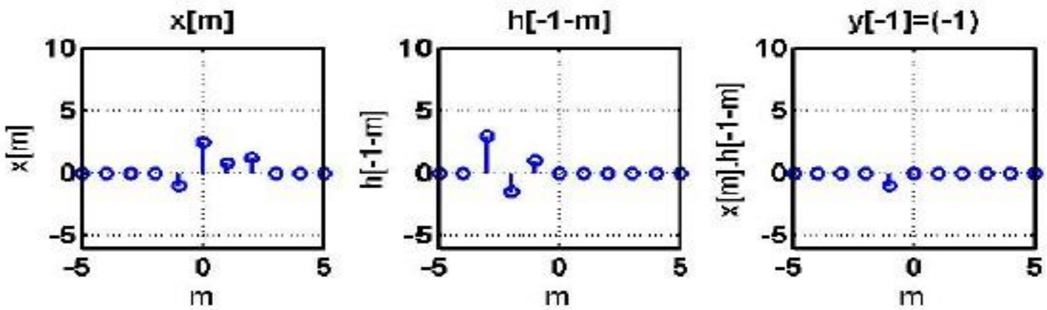
Step 11: Sketch h[-2-m], which is left shift of h[-m] by 2.

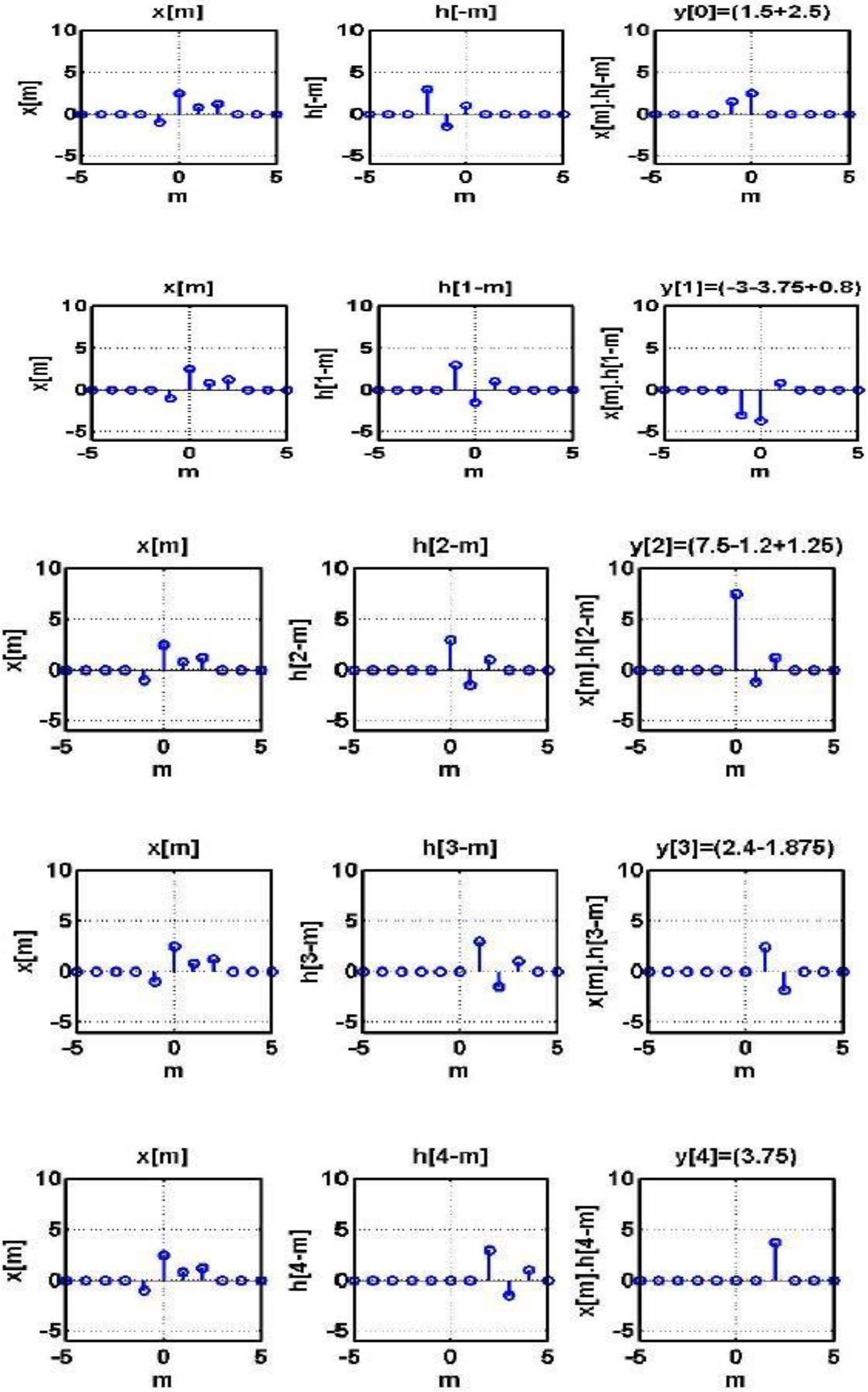
Step 12: Compute y[-2] using:

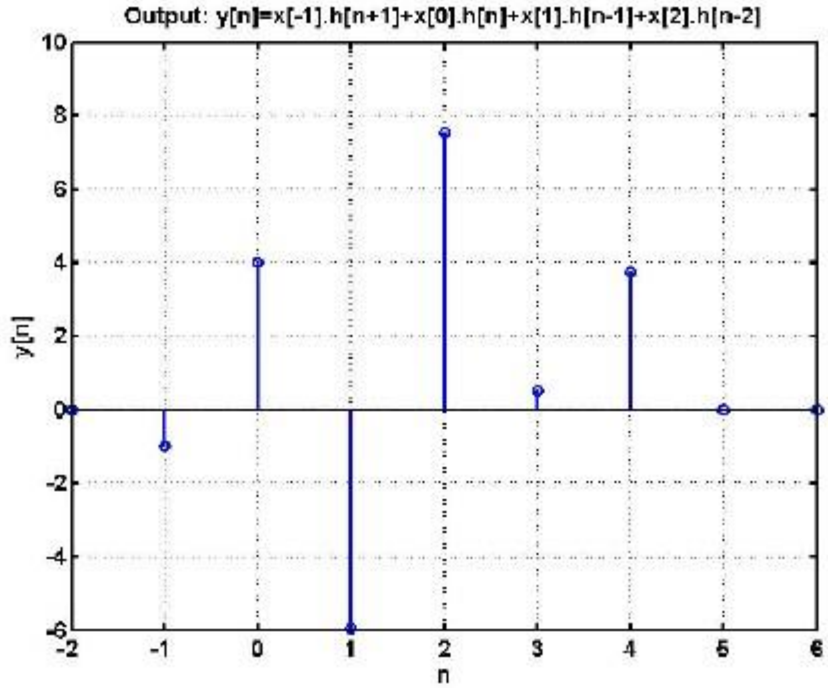
$$y[-2] = \sum_{m=-\infty}^{\infty} x[m]h[-2-m]$$

which is the 'sum of the product of the two signals x[m] & h[-2-m]'

Step 13: Proceed this way until all possible values of y[n], for negative 'n' are computed




x[m]
h[-m]
y[0]=(1.5+2.5)
x[m].h[-m]
x[m]
h[1-m]
y[1]=(-3-3.75+0.8)
x[m].h[1-m]
x[m]
h[2-m]
y[2]=(7.5-1.2+1.25)
x[m].h[2-m]
x[m]
h[3-m]
y[3]=(2.4-1.875)
x[m].h[3-m]
x[m]
h[4-m]
y[4]=(3.75)
x[m].h[4-m]
m



2.2.2 Evaluation from direct convolution sum:

While small length, finite duration sequences can be convolved by any of the above three methods, when the sequences to be convolved are of infinite length, the convolution are easier performed by direct use of the „convolution sum" of equation (…).

$$since: \quad u[m] = \begin{cases} 0 & for \quad m < 0 \\ 1 & for \quad m \geq 0 \end{cases}$$

$$u[n-m] = \begin{cases} 0 & for \quad (n-m) < 0 \\ 1 & for \quad (n-m) \geq 0 \end{cases}$$
$$= \begin{cases} 0 & for \quad (-m) < n \\ 1 & for \quad (-m) \geq n \end{cases}$$
$$= \begin{cases} 0 & for \quad m > n \\ 1 & for \quad m \leq n \end{cases}$$

Example: A system has impulse response $h[n]$ □□exp(□0.8n)$u[n]$. Obtain the unit step response.
Solution:

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]x[m]$$
$$= \sum_{m=-\infty}^{\infty} \{\exp(-0.8(m))u[m]\}\{u[n-m]\}$$

$$= \sum_{m=0}^{\infty} \{\exp(-0.8(m))\}\{u[n-m]\}$$

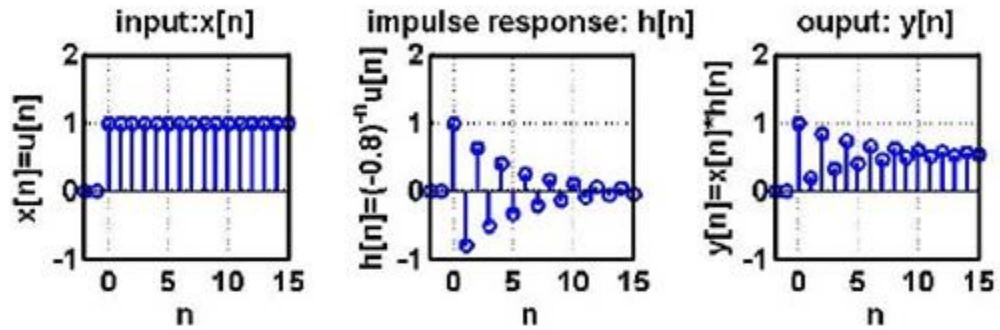
$$= \sum_{m=0}^{n} \{\exp(-0.8(m))\}$$

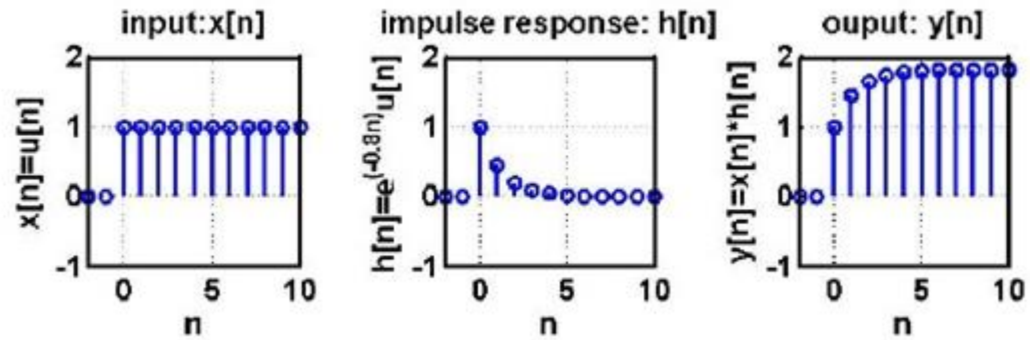
$$= \sum_{m=0}^{n} \{\exp(-0.8(m))\}$$

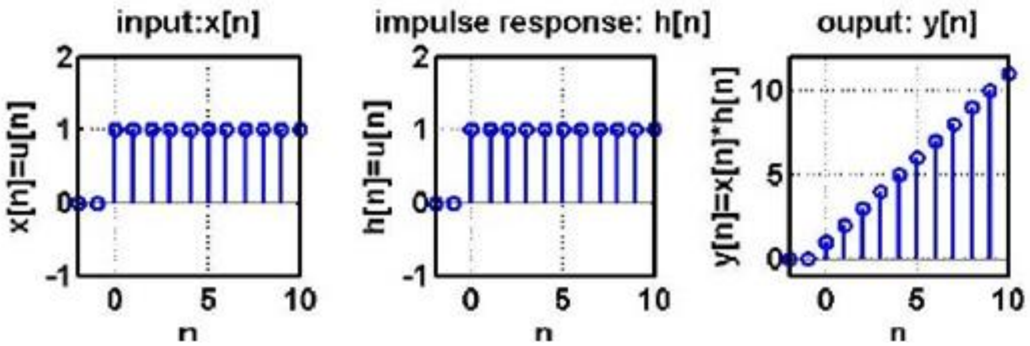
$$= \frac{(1-(-0.8)^{n+1})}{(1-(-0.8))}$$

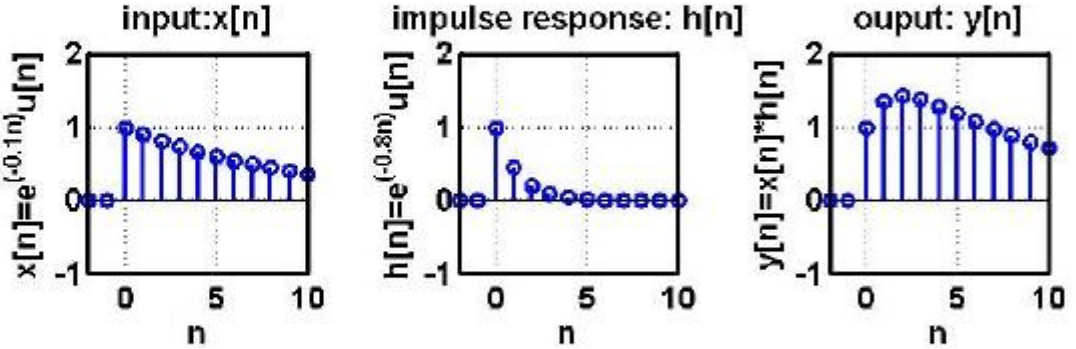
$$y[n] = \sum_{m=-\infty}^{\infty} \{(-0.8)^{(n-m)} u[n-m]\}$$

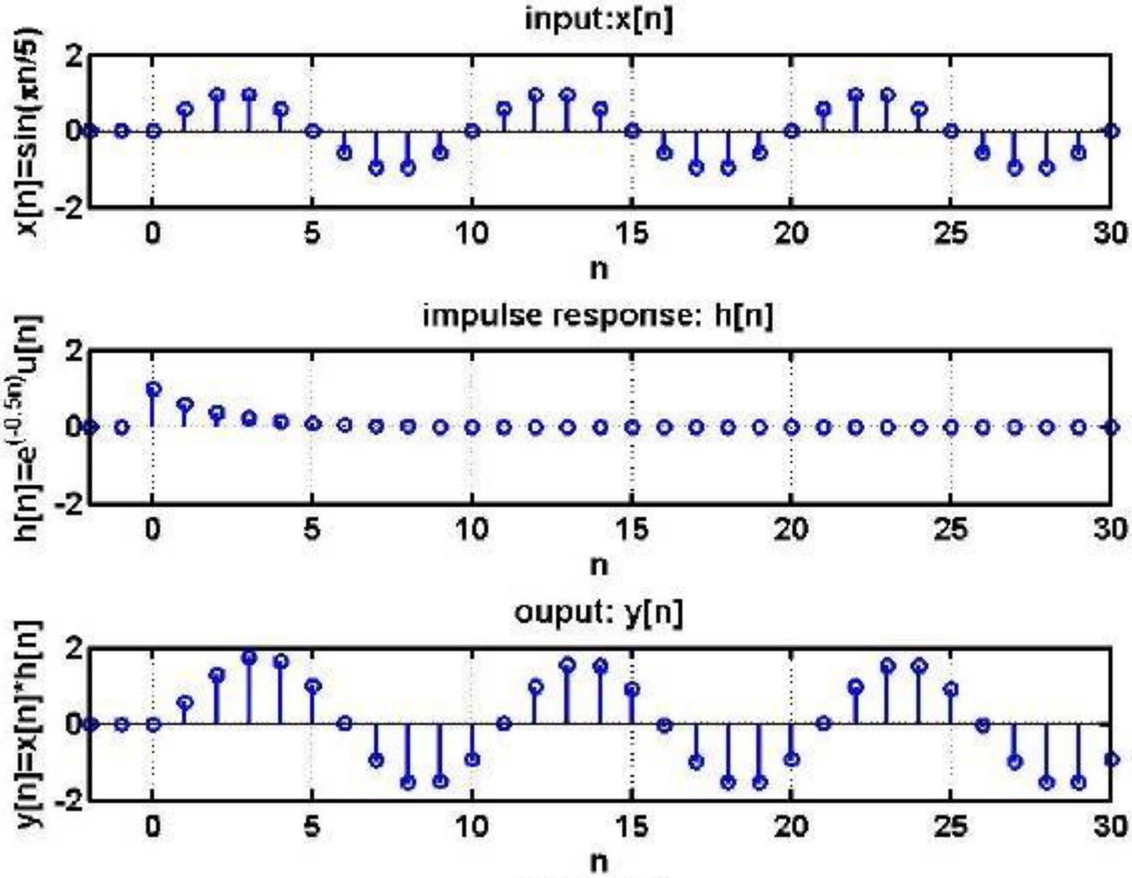
$$= \sum_{m=0}^{\infty} \{\exp(-0.8(n-m)) u[n-m]\}$$

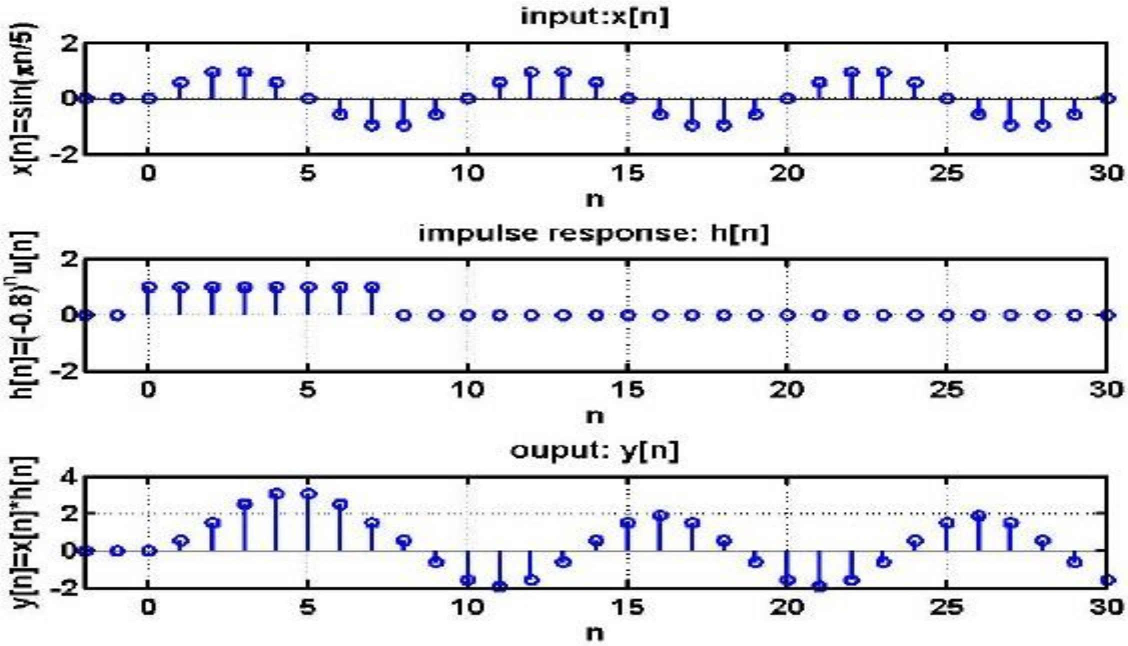







input:x[n]
x[n]=e^(-0.1n)u[n]
impulse response: h[n]
h[n]=e^(-0.8n)u[n]
ouput: y[n]
y[n]=x[n]*h[n]
n


input:x[n]
x[n]=sin(πn/5)
impulse response: h[n]
h[n]=e^(-0.5n)u[n]
ouput: y[n]
y[n]=x[n]*h[n]
n


input:x[n]
x[n]=sin(πn/5)
impulse response: h[n]
h[n]=(-0.8)^n u[n]
ouput: y[n]
y[n]=x[n]*h[n]
n

2.2.3 Convolution Integral:

We now attempt to obtain the output of a continuous time/Analog digital system for an arbitrary input x(t), from the knowledge of the system impulse response h(t), and the properties of the impulse response of an LTI system.

The output y(t) is given by, using the notation, y(t)=R{x(t)}.

$$
\begin{aligned}
y(t) &= R\{x(t)\} \\
&= R\left\{ \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \right\} \\
&= \int_{-\infty}^{\infty} x(\tau)R\{\delta(t-\tau)\}d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= x(t) * h(t)
\end{aligned}
$$

Methods of evaluating the convolution integral: (Same as Convolution sum)

Given the system impulse response h(t), and the input x(t), the system output y(t), is given by the convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Some of the different methods of evaluating the convolution integral are: Graphical representation, Mathematical equation, Laplace-transforms, Fourier Transform, Differential equation, Block diagram representation, and finally by going to the digital domain.

2.3 Differential equation and Difference equation representation:

General form of differential equation is

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

where$a_k$and $b_k$are coefficients, $x(.)$ is input and $y(.)$ is output and order of differential or difference equation is (M,N).

Example of Differential equation

• Consider the RLC circuit as shown in figure below. Let $x(t)$ be the input voltage source and $y(t)$ be the output current. Then summing up the voltage drops around the loop gives

$$Ry(t) + L\frac{d}{dt}y(t) + \frac{1}{C}\int_{-\infty}^{t} y(\tau)d\tau = x(t)$$

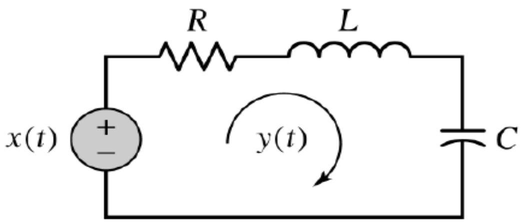


Fig. 2.4: RLC Circuit

2.3.1 Solving differential equation:

A wide variety of continuous time systems are described the linear differential equations:

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

- Just as before, in order to solve the equation for $y(t)$, we need the ICs. In this case, the ICs are given by specifying the value of y and its derivatives 1 through $N-1$ at $t = 0-$
- Note: the ICs are given at $t = 0-$ to allow for impulses and other discontinuities at $t = 0$.
- Systems described in this way are
- linear time-invariant (LTI): easy to verify by inspection
- Causal**:** the value of the output at time t depends only on the output and the input at times $0 \leq t \leq t$
- Asinthecaseofdiscrete-timesystem,thesolution$y(t)$canbedecomposedinto$y(t)$= $y_h(t)+y_p(t)$, where homogeneous solution or zero-input response (ZIR), $yh(t)$ satisfies the equation

- The zero-state response (ZSR) or particular solution $y_p(t)$ satisfies the equation

$$y_h^N(t) + \sum_{i=0}^{N-1} a_i y_h^{(i)}(t) = \sum_{i=0}^{m} b_i x^{(M-i)}(t), \qquad t \geq 0$$

with ICs $y_p(0^-) = y_p^{(1)}(0^-) = \ldots = y_p^{(N-1)}(0^-) = 0.$

Homogeneous solution (ZIR) for CT

- A standard method for obtaining the homogeneous solution or (ZIR) is by setting all terms involving the input to zero.

$$\sum_{i=0}^{N} a_i y_h^{(i)}(t) = 0, \qquad t \geq 0$$

and homogeneous solution is of the form

$$y_h(t) = \sum_{i=1}^{N} C_i e^{r_i t}$$

where r_i are the N roots of the system's characteristic equation

$$\sum_{k=0}^{N} a_k r^k = 0$$

and $C_1, \ldots, C_N$ are solved using ICs.

Homogeneous solution (ZIR) for DT

- The solution of the homogeneous equation

$$\sum_{k=0}^{N} a_k y_h[n-k] = 0$$

is

$$y_h[n] = \sum_{i=1}^{N} c_i r_i^n$$

where r_i are the N roots of the system's characteristic equation

$$\sum_{k=0}^{N} a_k r^{N-k} = 0$$

and $C_1, \ldots, C_N$ are solved using ICs.

Example 1 (ZIR)

- Solution of

$$\frac{d^2}{dt^2} y(t) + 5\frac{d}{dt} y(t) + 6y(t) = 2x(t) + \frac{d}{dt} x(t)$$

is

$$y_h(t) = c_1 e^{-3t} + c_2 e^{-2t}$$

- Solution of $y[n] - 9/16 y[n-2] = x[n-1]$ is $y_h[n] = c_1(3/4)^n + c_2(-3/4)^{n}$

Example 2 (ZIR)

- Consider the first order recursive system described by the difference equation $y[n] - \rho y[n-1] = x[n]$, find the homogeneous solution.
- The homogeneous equation (by setting input to zero) is $y[n] - \rho y[n-1] = 0$.
- The homogeneous solution for $N = 1$ is $y_h[n] = c_1 r_1^n$.
- r_1 is obtained from the characteristics equation $r_1 - \rho = 0$, hence $r_1 = \rho$
- The homogeneous solution is $y_h[n] = c_1 \rho^n$

Example 3 (ZIR)

- Consider the RC circuit described by $y(t) + RC\frac{d}{dt}y(t) = x(t)$
- The homogeneous equation is $y(t) + RC\frac{d}{dt}y(t) = 0$
- Then the homogeneous solution is

$$y_h(t) = c_1 e^{r_1 t}$$

where r_1 is the root of characteristic equation $1 + RCr_1 = 0$

- This gives $r_1 = -\frac{1}{RC}$

- The homogeneous solution is

$$y_h(t) = c_1 e^{\frac{-t}{RC}}$$

- Multiply both the sides of the equation by $(1/2)^n$ we get $c_p = 1/(1-2\rho)$.
- Then the particular solution is

$$y_p[n] = \frac{1}{1-2\rho}(\frac{1}{2})^n$$

Particular solution (ZSR)

- Particular solution or ZSR represents solution of the differential or difference equation for the given input.
- To obtain the particular solution or ZSR, one would have to use the method of integrating factors.
- y_p is not unique.
- Usually it is obtained by assuming an output of the same general form as the input.
- If $x[n] = \alpha_n$ then assume $y_p[n] = c\alpha^n$ and find the constant c so that $y_p[n]$ is the solution of given equation

Examples

Example 1 (ZSR)

- Consider the first order recursive system described by the difference equation $y[n] - \rho y[n-1] = x[n]$, find the particular solution when $x[n] = (1/2)^n$.
- Assume a particular solution of the form $y_p[n] = c_p(1/2)^n$.
- Put the values of $y_p[n]$ and $x[n]$ in the equation then we get $c_p(\frac{1}{2})^n - \rho c_p(\frac{1}{2})^{n-1} = (\frac{1}{2})^n$

- For $\rho = (1/2)$ particular solution has the same form as the homogeneous solution

- However no coefficient c_p satisfies this condition and we must assume a particular solution of the form $y_p[n] = c_p n(1/2)^n$.

- Substituting this in the difference equation gives $c_p n(1-2\rho) + 2\rho c_p = 1$

- Using $\rho = (1/2)$ we find that $c_p = 1$.

Example 2 (ZSR)

- Consider the RC circuit described by $y(t) + RC\frac{d}{dt}y(t) = x(t)$

- Assume a particular solution of the form $y_p(t) = c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t)$.

- Replacing $y(t)$ by $y_p(t)$ and $x(t)$ by $\cos(\omega_0 t)$ gives

$$c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t) - RC\omega_0 c_1 \sin(\omega_0 t) + RC\omega_0 c_2 \cos(\omega_0 t) = \cos(\omega_0 t)$$

- The coefficients c_1 and c_2 are obtained by separately equating the coefficients of $\cos(\omega_0 t)$ and $\sin(\omega_0 t)$, gives

$$c_1 = \frac{1}{1+(RC\omega_0)^2} \qquad and \qquad c_2 = \frac{RC\omega_0}{1+(RC\omega_0)^2}$$

- Then the particular solution is

$$y_p(t) = \frac{1}{1+(RC\omega_0)^2}\cos(\omega_0 t) + \frac{RC\omega_0}{1+(RC\omega_0)^2}\sin(\omega_0 t)$$

Complete solution

- Find the form of the homogeneous solution y_h from the roots of the characteristic equation

- Find a particular solution y_p by assuming that it is of the same form as the input, yet is independent of all terms in the homogeneous solution

- Determine the coefficients in the homogeneous solution so that the complete solution $y = y_h + y_p$ satisfies the initial conditions

2.3.2 Difference equation representation:

- A wide variety of discrete-time systems are described by linear difference equations:

$$y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k], \qquad n = 0, 1, 2, \ldots$$

where the coefficients $a_1, \ldots, a_N$ and $b_0, \ldots, b_M$ do not depend on n. In order to be able to compute the system output, we also need to specify the initial conditions (ICs) $y[-1], y[-2] \ldots y[-N]$

- Systems of this kind are
  - linear time-invariant (LTI): easy to verify by inspection
  - causal: the output at time n depends only on past outputs $y[n-1], \ldots, y[n-N]$ and on current and past inputs $x[n], x[n-1], \ldots, x[n-M]$

- Systems of this kind are also called Auto Regressive Moving-Average (ARMA) filters. The name comes from considering two special cases.
- auto regressive (AR) filter of order N, $AR(N)$: $b_0 = \ldots = b_M = 0$

$$y[n] + \sum_{k=1}^{N} a_k y[n-k] = 0 \qquad n = 0, 1, 2, \ldots$$

In the AR case, the system output at time n is a linear combination of N past outputs; need to specify the ICs $y[-1], \ldots, y[-N]$.

- moving-average (MA) filter of order $N, AR(N)$: $a_0 = \ldots = a_N = 0$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] \qquad n = 0, 1, 2, \ldots$$

In the MA case, the system output at time n is a linear combination of the current input and M past inputs; no need to specify ICs.

- An ARMA(N, M) filter is a combination of both.

- Let us first rearrange the system equation:

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \qquad n = 0, 1, 2, \ldots$$

- at $n = 0$

$$y[0] = \underbrace{-\sum_{k=1}^{N} a_k y[-k]}_{depends\ on\ ICs} + \underbrace{\sum_{k=0}^{M} b_k x[-k]}_{depends\ on\ input\ x[0] \to x[-M]}$$

- at $n = 1$

$$y[1] = \sum_{k=1}^{N} a_k y[1-k] + \sum_{k=0}^{M} b_k x[1-k]$$

After rearranging

$$y[1] = -a_1 y[0] \underbrace{-\sum_{k=1}^{N-1} a_{k+1} y[-k]}_{depends\ on\ ICs} + \underbrace{\sum_{k=0}^{M} b_k x[1-k]}_{depends\ on\ input x[1] \ldots x[1-M]}$$

- at $n = 2$

$$y[2] = \sum_{k=1}^{N} a_k y[2-k] + \sum_{k=0}^{M} b_k x[2-k]$$

After rearranging

$$y[2] = -a_1 y[1] - a_2 y[0] \underbrace{-\sum_{k=1}^{N-1} a_{k+1} y[-k]}_{depends\ on\ ICs} + \underbrace{\sum_{k=0}^{M} b_k x[1-k]}_{depends\ on\ input\ x[2] \ldots x[2-M]}$$

Example of Difference equation

- An example of II order difference equation is

$$y[n] + y[n-1] + \frac{1}{4} y[n-2] = x[n] + 2x[n-1]$$

- Memory in discrete system is analogous to energy storage in continuous system
- Number of initial conditions required to determine output is equal to maximum memory of the system

Initial Conditions

Initial Conditions summaries all the information about the systems past that is needed to determine the future outputs.

- In discrete case, for an N^{th} order system the N initial value are

$$y[-N], y[-N+1], \ldots, y[-1]$$

- The initial conditions for N^{th}-order differential equation are the values of the first N derivatives of the output

$$y(t)|_{t=0}, \frac{d}{dt}y(t)|_{t=0}, \frac{d^2}{dt^2}y(t)|_{t=0}, \ldots \frac{d^{N-1}}{dt^{N-1}}y(t)|_{t=0}$$

Solving difference equation

- Consider an example of difference equation $y[n] + ay[n-1] = x[n], \quad n = 0, 1, 2 \ldots$ with $y[-1] = 0$ Then

$$\begin{aligned}
y[0] &= -ay[-1] + x[0] \\
y[1] &= -ay[0] + x[1] \\
&= -a(-ay[-1] + x[0]) + x[1] \\
&= a^2y[-1] - ax[0]) + x[1] \\
y[2] &= -ay[1] + x[2] \\
&= -a(-a^2y[-1] - ax[0] + x[1]) + x[2] \\
&= a^3y[-1] + a^2x[0] - ax[1] + x[2]
\end{aligned}$$

and so on

- We get $y[n]$ as a sum of two terms:
$y[n] = (-a)^{n+1}y[-1] + \sum_{i=0}^{n}(-a)^{n-i}x[i], \qquad n = 0, 1, 2, \ldots$
- First term $(-a)^{n+1}y[-1]$ depends on IC's but not on input

- Second term $\sum_{i=0}^{n}(-a)^{n-i}x[i]$ depends only on the input, but not on the IC's

- This is true for any ARMA (auto regressive moving average) system: the system output at time n is a sum of the AR-only and the MA-only outputs at time n.

- Consider an ARMA (N,M) system $y[n] = -\sum_{i=1}^{N} a_i y[n-i] + \sum_{i=0}^{M} b_i x[n-i], \quad n = 0, 1, 2, \ldots$ with the initial conditions $y[-1], \ldots, y[-N]$.

- Output at time n is:

$$y[n] = y_h[n] + y_p[n]$$

  where $y_h[n]$ and $y_p[n]$ are homogeneous and particular solutions

- First term depends on IC's but not on input

- Second term depends only on the input, but not on the IC's

- Note that $y_h[n]$ is the output of the system determined by the ICs only (setting the input to zero), while $y_p[n]$ is the output of the system determined by the input only (setting the ICs to zero).

- $y_h[n]$ is often called the zero-input response (ZIR) usually referred as homogeneous solution of the filter (referring to the fact that it is determined by the ICs only)

- $y_p[n]$ is called the zero-state response (ZSR) usually referred as particular solution of the filter (referring to the fact that it is determined by the input only, with the ICs set to zero).

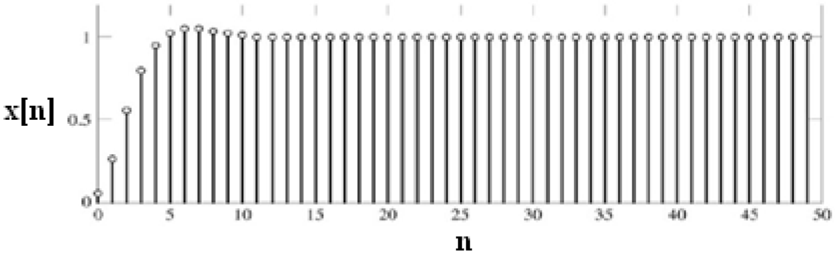


Fig. 2.5: Step response of a system

- Consider the output decomposition $y[n] = y_h[n] + y_p[n]$ of an ARMA (N, M) filter

$$y[n] = -\sum_{i=1}^{N} a_i y[n-i] + \sum_{i=0}^{M} b_i x[n-i], \qquad n = 0, 1, 2, \ldots$$

with the ICs $y[-1], \ldots, y[-N]$.

- The output of an ARMA filter at time n is the sum of the ZIR and the ZSR at time n.

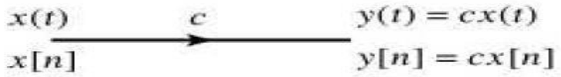
Example of difference equation

- example: A system is described by $y[n] - 1.143y[n-1] + 0.4128y[n-2] = 0.0675x[n] + 0.1349x[n-1] + 0.675x[n-2]$

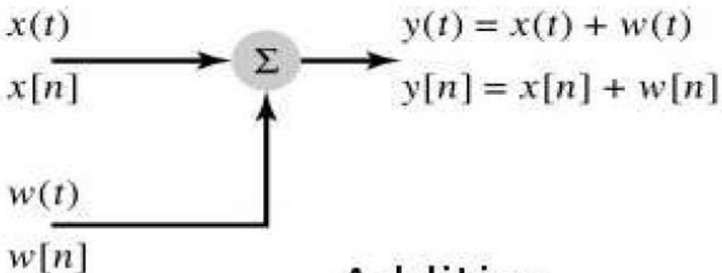
- Rewrite the equation as $y[n] = 1.143y[n-1] - 0.4128y[n-2] + 0.0675x[n] + 0.1349x[n-1] + 0.675x[n-2]$

2.4 Block Diagram representation:

- A block diagram is an interconnection of elementary operations that act on the input signal
- This method is more detailed representation of the system than impulse response or differential/difference equation representations
- The impulse response and differential/difference equation descriptions represent only the input-output behavior of a system, block diagram representation describes how the operations are ordered
- Each block diagram representation describes a different set of internal computations used to determine the system output
- Block diagram consists of three elementary operations on the signals:
  - Scalar multiplication: $y(t) = cx(t)$ or $y[n] = x[n]$, where c is a scalar
  - Addition: $y(t) = x(t) + w(t)$ or $y[n] = x[n] + w[n]$.
- Block diagram consists of three elementary operations on the signals:
  - Integration for continuous time LTI system: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$
    Time shift for discrete time LTI system: $y[n] = x[n-1]$
- Scalar multiplication: $y(t) = cx(t)$ or $y[n] = x[n]$, where c is a scalar



Scalar Multiplication



Addition

- Addition: $y(t) = x(t) + w(t)$ or $y[n] = x[n] + w[n]$
- Integration for continuous time LTI system: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$
  Time shift for discrete time LTI system: $y[n] = x[n-1]$

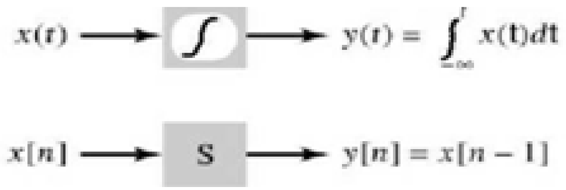


Fig. 2.6: Time shift and Integration symbol

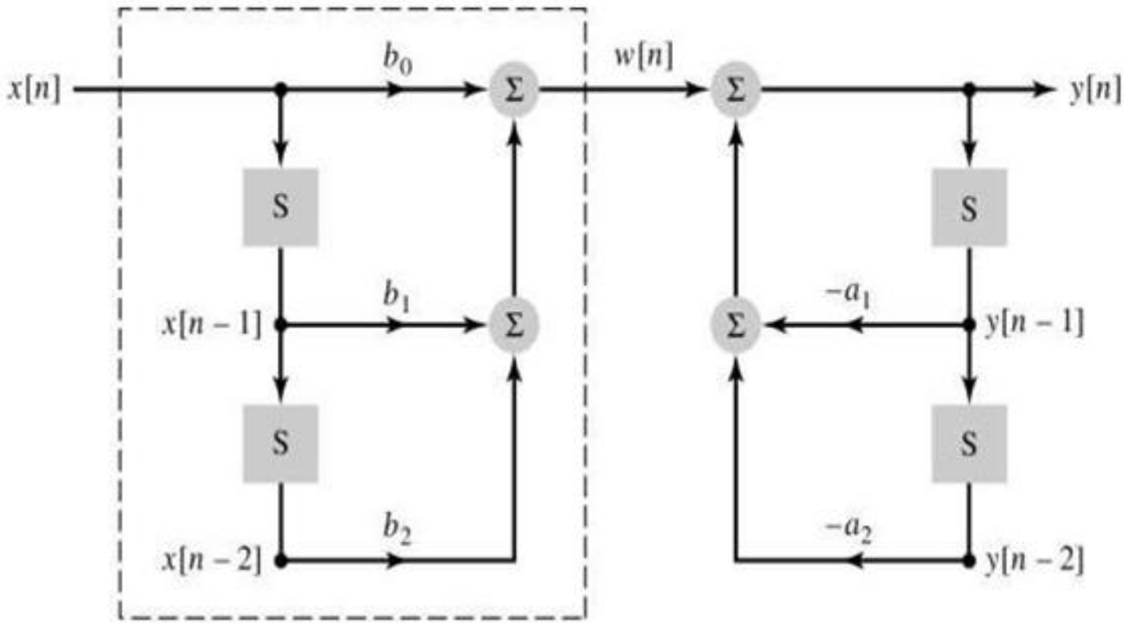


Fig. 2.7: Direct form-I representation

Example 1

- Consider the system described by the block diagram as in Figure 1.10
- Consider the part within the dashed box
- The input $x[n]$ is time shifted by 1 to get $x[n-1]$ and again time shifted by one to get $x[n-2]$. The scalar multiplications are carried out and

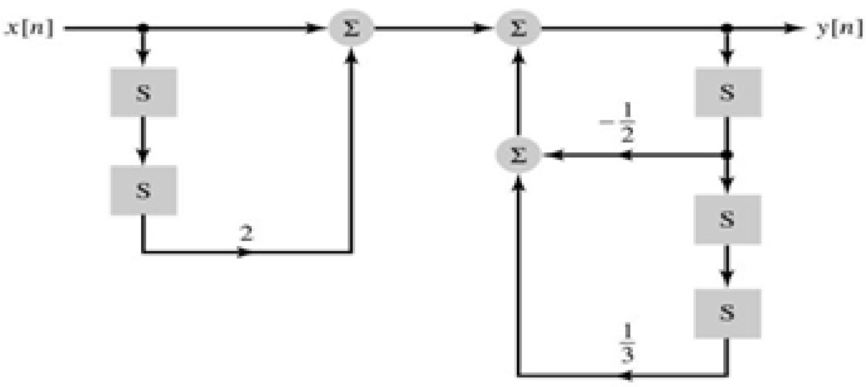
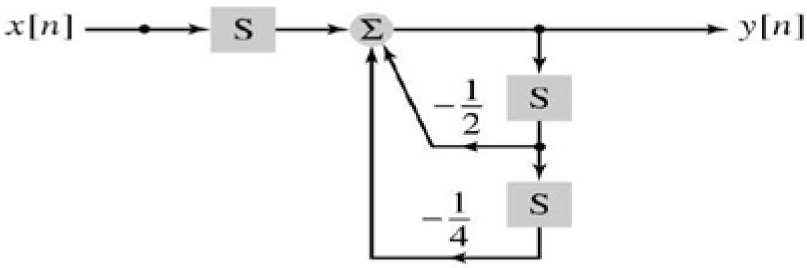


Fig. 2.8: **Direct form-I representation of Example 1**



they are added to get $w[n]$ and is given by

$$w[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2].$$

- Write $y[n]$ in terms of $w[n]$ as input $y[n] = w[n] - a_1y[n-1] - a_2y[n-2]$
- Put the value of $w[n]$ and we get $y[n] = -a_1y[n-1] - a_2y[n-2] + b_0x[n] + b_1x[n-1] + b_2x[n-2]$
  and $y[n] + a_1y[n-1] + a_2y[n-2] = b_0x[n] + b_1x[n-1] + b_2x[n-2]$
- The block diagram represents an LTI system

Example 2

- Consider the system described by the block diagram and its difference equation is $y[n] + (1/2)y[n-1] - (1/3)y[n-3] = x[n] + 2x[n-2]$

Example 3

- Consider the system described by the block diagram and its difference equation is $y[n] + (1/2)y[n-1] + (1/4)y[n-2] = x[n-1]$

- Block diagram representation is not unique, direct form II structure of Example 1
- We can change the order without changing the input output behavior Let the output of a new system be $f[n]$ and given input $x[n]$ are related by

$$f[n] = -a_1 f[n-1] - a_2 f[n-2] + x[n]$$

- The signal $f[n]$ acts as an input to the second system and output of second system is

$$y[n] = b_0 f[n] + b_1 f[n-1] + b_2 f[n-2].$$

- The block diagram representation of an LTI system is not unique

Continuous time

- Rewrite the differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

as an integral equation. Let $v^{(0)}(t) = v(t)$ be an arbitrary signal, and set

$$v^{(n)}(t) = \int_{-\infty}^{t} v^{(n-1)}(\tau) d\tau, \qquad n = 1, 2, 3, \ldots$$

where $v^{(n)}(t)$ is the n-fold integral of $v(t)$ with respect to time

- Rewrite in terms of an initial condition on the integrator as

$$v^{(n)}(t) = \int_{0}^{t} v^{(n-1)}(\tau) d\tau + v^{(n)}(0), \quad n = 1, 2, 3, \ldots$$

- If we assume zero ICs, then differentiation and integration are inverse operations, ie.

$$\frac{d}{dt} v^{(n)}(t) = v^{(n-1)}(t), \quad t > 0 \text{ and } n = 1, 2, 3, \ldots$$

- Thus, if $N \geq M$ and integrate N times, we get the integral description of the system

$$\sum k = 0^N a_k y^{(N-k)}(t) = \sum k = 0^M b_k x^{(N-k)}(t)$$

- For second order system with $a_0 = 1$, the differential equation can be

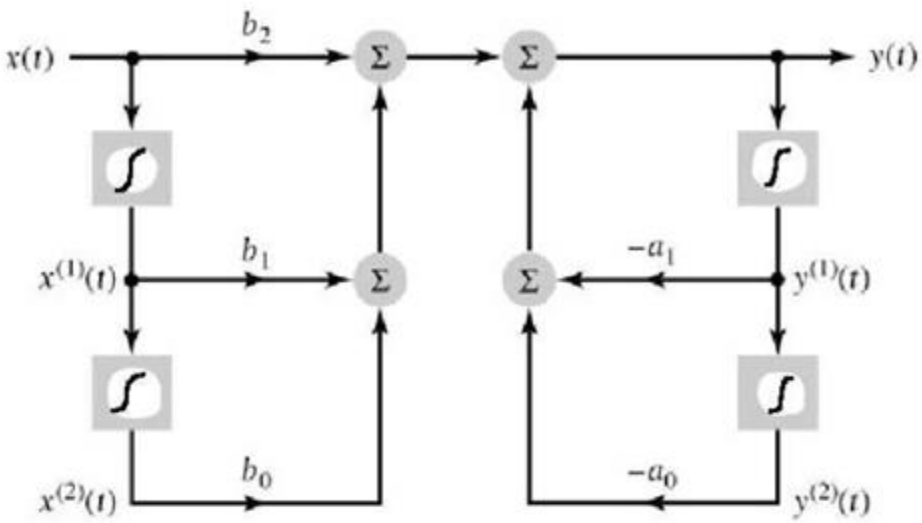


Fig. 2.9: Direct form-I representation

Where

$$y(t) = -a_1 y^{(1)}(t) - a_0 y^{(2)}(t) + b_2 x(t) + a_1 x^{(1)}(t) + b_0 x^{(2)}(t)$$

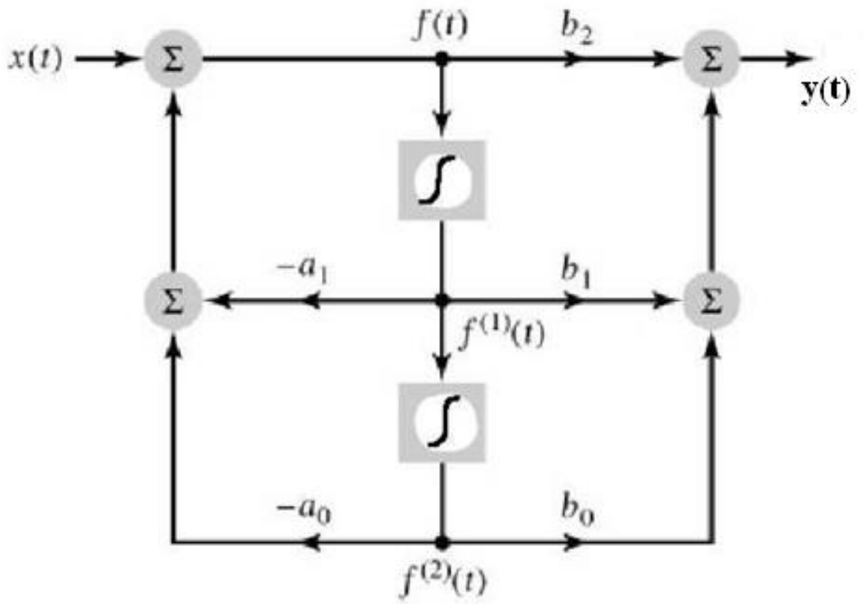


Fig. 2.10: Direct form-II representation

2.5 Outcomes:

1. Knowledge on the impulse response of a LTI system
2. Knowledge on representing continuous and discrete time system by differential and difference equations
3. Knowledge on block diagram representation of continuous and discrete time systems.

2.6 Further reading

1. http://mathworld.wolfram.com/Convolution.html
2. http://colah.github.io/posts/2014-07-Understanding-Convolutions/
3. https://www.khanacademy.org/math/differential-equations/laplace-transform/convolution-integral/v/introduction-to-the-convolution
4. http://fivedots.coe.psu.ac.th/Software.coe/240-381/slide/DSPCh6.pdf
5. https://www.youtube.com/watch?v=YnV4DlBzvls