Fourier Series

The Fourier series for the function f(x) in the interval c< x < c+2l is

f(x)=$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos\left(\frac{n\pi x}{l}\right)$+$\sum_{n=1}^{\infty} b_n sin\left(\frac{n\pi x}{l}\right) x$

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)dx,$$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)cos\left(\frac{n\pi x}{l}\right)dx,$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)sin\left(\frac{n\pi x}{l}\right)dx,$$

$where a_0, a_n, b_n\ are\ known\ as\ Euler's\ Formulae.$

- Conditions for a Fourier Expansion:
- With the help of Euler's formula we cannot conclude that the series will converges to f(x). Dirichlet's conditions under which the expansion of f(x) as a trigonometric series converge to f(x) at every point of continuity.
- 1.f(x) is single valued and finite in the interval (c,c+2π).
- 2.f(x) is periodic with period 2π.
- 3.f(x) has a finite number of discontinuous in any one period
- 4.f(x) has at the most a finite number of maxima and minima in (c,c+2π).
- Remarks: If f(x) is discontinuous at x, then the series converges to $\frac{1}{2}[f(x^{+}) +$

- Application:

- Studies of condition of resonance
- Unbounded growth of the response when frequency coincides with the natural frequency.

- Determine the response of an un damped 1-degree of freedom system of natural frequency and mass m, initially at rest in equilibrium.

- Estimate the strength of buildings/structures/bridges with the help of vibrations/signal analysis and breakdown your response to understandable sine waves using Fourier series.

- <u>Periodic function:</u>
- A function f(x) is said to be periodic if $f(x)=f(x+T)$ for all x
- Ex: sinx,cosx are periodic functions having period 2π
- <u>Even(or)Odd function in (-l,l)</u>
- Put $x=-x$,
- i.e $f(-x)=f(x)$ then f(x) is even
- i.e $f(-x)=-f(x)$ then f(x) is odd
- <u>Even(or)Odd function in (0,2l)</u>
- Put $x=2l-x$,
- i.e $f(2l-x)=f(x)$ then f(x) is even
- i.e $f(2l-x)=-f(x)$ then f(x) is odd

- If the function is even then $b_n = 0, and$ **apply Cosine series formula**
- f(x)=$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos\left(\frac{n\pi x}{l}\right)$
- $a_0 = \frac{2}{l}\int_0^l f(x)dx,$
- $a_n = \frac{2}{l}\int_0^l f(x)cos\left(\frac{n\pi x}{l}\right)dx,$

- If the function is Odd then $a_0, a_1 = 0$,and **apply Sine series formula**
- f(x)=$\sum_{n=1}^{\infty} b_n sin\left(\frac{n\pi x}{l}\right)$

- $b_n = \frac{2}{l}\int_0^l f(x)sin\left(\frac{n\pi x}{l}\right)dx,$

- Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.
- Solution:$l = \pi$
- f(x)=$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx + \sum_{n=1}^{\infty} b_n sinnx$ ------ –(1)
- $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx,$
- $= \frac{1}{\pi}\int_0^{2\pi} e^{-x}dx = \frac{1}{\pi}[-e^{-x}]_0^{2\pi}$
- $= \frac{-1}{\pi}[e^{-2\pi} - e^{-0}]$
- $a_0 = \frac{1}{\pi}[1 - e^{-2\pi}]$ ------ –(2)
- $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)cosnxdx,$
- $a_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}cosnxdx$ 0 for x=0 and 2π
- $= \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}(-cosnx + nsinnx\right]_0^{2\pi}$
- $= \frac{-1}{\pi(n^2+1)}(e^{-2\pi}(cos2n\pi) - e^{-0}(cos0))$
- $= \frac{-1}{\pi(n^2+1)}(e^{-2\pi} - 1)$ ------ –(3)
- $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sinnxdx,$
- $b_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}sinnxdx$ 0 for x=0 and 2π
- $= \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}(-sinnx - ncosnx\right]_0^{2\pi}$
- $= \frac{-n}{\pi(n^2+1)}(e^{-2\pi}(cos2n\pi) - e^{-0}(cos0))$
- =$\frac{n}{\pi(n^2+1)}(1 - e^{-2\pi})$ ------ –(4)

Substitute equations 2,3,4 in (1)

$$f(x) = \frac{1}{\pi}[1 - e^{-2\pi}]\left\{\frac{1}{2} + \frac{1}{(n^2+1)}cosnx + \frac{n}{(n^2+1)}sinnx\right\}$$

- Find a Fourier series to represent
- $x - x^2\ from\ x = -\pi\ to\ x = \pi$
- $and\ hence\ deduce\ that\ \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - - = \frac{\pi^2}{12}$
- Solution: $l = \pi$
- f(x)= $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx + \sum_{n=1}^{\infty} b_n sinnx - -(1)$
- $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx,$
- $= \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)dx$
- $= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$
- $= \frac{1}{\pi}\left(\frac{\pi^2}{2} - \frac{\pi^3}{3} - (\frac{(-\pi)^2}{2} - \frac{(-\pi)^3}{2})\right)$
- $a_0 = \frac{1}{\pi}\left[-2\frac{\pi^3}{3}\right] = -2\frac{\pi^2}{3}$
- $\frac{a_0}{2} = \frac{-\pi^2}{3} - - - - - - -(2)$
- $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cosnxdx,$
- $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)cosnxdx,$
- $= \frac{1}{\pi}\left[(x - x^2)\frac{(sinnx)}{n} - (1 - 2x)\frac{(-cosnx)}{n^2} - 2\frac{(-sinnx)}{n^3}\right]_{-\pi}^{\pi}$

0 for x=π and -π

- $= \frac{1}{\pi}\left[(1 - 2\pi)\frac{(cosn\pi)}{n^2} - (1 + 2\pi)\frac{\cos(-n\pi)}{n^2}\right]$
- $= \frac{1}{\pi}\left[(-4\pi)\frac{(-1)^n}{n^2}\right] = \left[-4\frac{(-1)^n}{n^2}\right] - - - - - - - - - - - - -(3)$
- $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)sinnxdx,$
- $= \frac{1}{\pi}\left[(x - x^2)\frac{(-cosnx)}{n} - (1 - 2x)\frac{(-sinnx)}{n^2} - 2\frac{(cosnx)}{n^3}\right]_{-\pi}^{\pi}$
- $= \frac{1}{\pi}\left[(\pi - \pi^2)\frac{(-cosn\pi)}{n} - 2\frac{(cosn\pi)}{n^3} - \left[(-\pi - \pi^2))\frac{(-\cos(-n\pi)}{n} - 2\frac{(\cos(-n\pi)}{n^3}\right]\right]$
- $= \frac{1}{\pi}\left[(-2\pi)\frac{(-1)^n}{n}\right]$
- $= \frac{-2(-1)^n}{n} -(4)$
- $f(x) = \frac{-\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}cosnx - \sum_{n=1}^{\infty}\frac{2(-1)^n}{n}sinnx - - -(5)$
- Put x=0 in (5) we get , $\frac{-\pi^2}{3} - 4\left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - -\right) = 0$
- $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - -$-

- Find the Fourier series expansion for f(x) , if
- $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi' \end{cases}$
  $and\ hence\ deduce\ that\ \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - - = \frac{\pi^2}{8}$
- Solution: $l = \pi$.let us consider
- $\varphi(x) = xand\ = 2\pi - x$
- Put $\mathrm{x} = 2\pi - x\ in\ \varphi(x)\ we\ get$
- $\varphi(2\pi - x) = 2\pi - x = \aleph(x)$ hence the given function is f(x) even.
- Now we apply Cosine half range series formula
- $\mathrm{f(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx - - -(1)$
- $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx,$
- $= \frac{2}{\pi}\int_0^{\pi} xdx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{\pi^2}{\pi} = \pi$
- $\frac{a_0}{2} = \frac{\pi}{2} - - - - - - -(2)$
- $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)cosnxdx,$
- $= \frac{2}{\pi}\int_0^{\pi} xcosnxdx,$
- $= \frac{2}{\pi}\left[x\frac{sinnx}{n} - \frac{(-cosnx}{n^2}\right]_0^{\pi}$
- $= \frac{2}{\pi}\left[\frac{cosn\pi - 1}{n}\right]$
- $= \frac{2}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right] - - -(3)$
- Sub (2) and (3) in (1), we get
- $f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2}\right] cosnx - - -(4)$
- Put x=0, we get ,
- $0 = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2}\right] cosn(0)$
- $-\frac{\pi^2}{4} = -\left(\frac{2}{1^2} + \frac{2}{3^2} + \frac{2}{5^2} - -\right)$
- $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - - = \frac{\pi^2}{8}$

- For a function$f(x) = |x|, -\pi < x < \pi$.
- $Obtain\ a\ Fourier\ series.$
- Solution:
- $f(x) = |x|\ is\ an\ even\ function.$
- Apply Cosine half range formula,$l = \pi$
- $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$
- $\mathrm{f(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx ---(1)$
- $a_0 = \frac{2}{\pi}\int_0^{\pi} xdx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \pi$
- $\frac{a_0}{2} = \frac{\pi}{2}.$
- $a_n = \frac{2}{\pi}\int_0^{\pi} xcosnxdx$
- $= \frac{2}{\pi}\left[x\frac{sinnx}{n} - \frac{cosnx}{n^2}\right]_0^{\pi}$ ($sinnx$=0 for x=π and 0)
- $= \frac{-2}{\pi}\frac{cosn\pi - cos0}{n^2} = \frac{-2((-1)^n - 1)}{\pi n^2}$
- $\mathrm{f(x)} = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{-2((-1)^n-1)}{\pi n^2} cosnx ---(2)$

- Obtain the Fourier series for $f(x) = 2x - x^2, in\ 0 \le x \le 2. show\ that\ f(x) = \frac{2}{3} - \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2} connπx.$
- Solution: L=1
- Put x=2-x to find the given function as even or odd.
- $f(2-x) = 2(2-x) - (2-x)^2$
- $= (2-x)(2-(2-x) = (2-x)x = 2x - x^2$
- $f(x) = even.$ Apply Cosine half range formula
- $\mathrm{f(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosn\pi x ---(1)$
- $a_0 = 2\int_0^1 (2x - x^2)dx = 2\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2(1 - \frac{1}{3}) = \frac{4}{3}$
- $\frac{a_0}{2} = \frac{2}{3} ----------------(2)$
- $a_n = 2\int_0^1 (2x - x^2)cosn\pi xdx$
- $= 2\left[(2x - x^2)\frac{sinn\pi x}{n\pi} - (2-2x)\left(\frac{-cosn\pi x}{n^2\pi^2}\right) + (-2)\left(\frac{-sinn\pi x}{n^3\pi^3}\right)\right]_0^1$ ($sinn\pi x$=0 for x=1 and 0)
- $= 2\left[-(2-2)\left(\frac{-cosn\pi}{n^2\pi^2}\right) + 2\left(\frac{-cosn0}{n^2\pi^2}\right)\ \right] = \frac{-4}{n^2\pi^2} --(3)$
- $f(x) = \frac{2}{3} - \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2} conn\pi x.$

- $Find\ the\ half\ range\ sine\ series\ for\ f(x) = x\cos x, (0,\pi).$
- Solution:
- $l = \pi$
- $\mathrm{f(x)} = \sum_{n=1}^{\infty} b_n sinnx ---(1)$
- $b_n = \frac{2}{\pi}\int_0^{\pi} xcosxsinnxdx$
- $= \frac{2}{\pi}\int_0^{\pi} \frac{x}{2}(\sin(n+1)\,x + \sin(n-1)\,x)dx$
- $= \frac{1}{\pi}\left[x\left(\frac{-\cos(n+1)x}{(n+1)} - \frac{-\cos(n-1)x}{n-1}\right) + (\frac{\sin(n+1)x}{(n+1)^2} +\right.$
- When n=1,
- $b_1 = \frac{2}{\pi}\int_0^{\pi} xcosxsinxdx$
- $= \frac{2}{\pi}\int_0^{\pi} \frac{x}{2}(\sin 2\,x)dx$
- $= \frac{1}{\pi}\left[x\left(\frac{-\cos 2x}{2} + \frac{\sin 2x}{4}\right)\right]_0^{\pi}$
- $= \frac{1}{\pi}\left(-\pi\left(\frac{\cos(2)\pi}{2} + 0\right) - 0\right)$
- $= \left(\frac{-1}{2}\right)$
- $\mathrm{f(x)} = \frac{-1}{2} sinx + \sum_{n=2}^{\infty}\left(\frac{2n(-1)^n}{n^2-1}\right) sinnx --(2)$

- Expand $f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$ as Fourier series of sine terms.
- Solution:l=1
- $f(x) = \sum_{n=1}^{\infty} b_n sinn\pi x$
- $b_n = \frac{2}{1}\left[\int_0^{\frac{1}{2}} \left(\frac{1}{4} - x\right) sinn\pi x dx + \int_{\frac{1}{2}}^{1} \left(x - \frac{3}{4}\right) sinn\pi x dx \right]$
- $= 2\left\{\left[\left(\frac{1}{4} - x\right)\frac{-cosn\pi x}{n\pi} + \frac{-sinn\pi x}{n^2\pi^2}\right]_0^{\frac{1}{2}} + \left[\left(x - \frac{3}{4}\right)\frac{-cosn\pi x}{n\pi} - \frac{-sinn\pi x}{n^2\pi^2}\right]_{\frac{1}{2}}^{1} \right\}$
- $= 2\left\{\left(\frac{1}{4} - \frac{1}{2}\right)\frac{-cos\frac{n\pi}{2}}{n\pi} - \frac{sin\frac{n\pi}{2}}{n^2\pi^2} - \left(\left(\frac{1}{4} - 0\right)\frac{-cos0}{n\pi} + \frac{sin0}{n^2\pi^2}\right) + \left[\left(1 - \frac{3}{4}\right)\frac{-cosn\pi}{n\pi} + \frac{sinn\pi}{n^2\pi^2} - \left(\left(\frac{1}{2} - \frac{3}{4}\right)\frac{-cos\frac{n\pi}{2}}{n\pi} + \frac{sin\frac{n\pi}{2}}{n^2\pi^2}\right)\right]\right\}$
- $= 2\left[\left(-\frac{1}{4}\right)\frac{-cos\frac{n\pi}{2}}{n\pi} - \frac{2sin\frac{n\pi}{2}}{n^2\pi^2} - \left(\left(\frac{1}{4}\right)\frac{-1}{n\pi}\right) + \left(-\frac{1}{4}\right)\frac{(-1)^n}{n\pi} - \left(\left(\frac{1}{4}\right)\frac{cos\frac{n\pi}{2}}{n\pi}\right)\right]$
- $= 2\left[-\frac{2sin\frac{n\pi}{2}}{n^2\pi^2} + \frac{1-(-1)^n}{4n\pi}\right]$ when n is even, b_2=0, b_4=0 and so on
- $f(x) = \sum_{n=1}^{\infty}\left[-\frac{4sin\frac{n\pi}{2}}{n^2\pi^2} + \frac{1-(-1)^n}{2n\pi}\right] sinn\pi x$

- Find the half-range cosine series expansion
- of the function $f(x) = \begin{cases} 0, \ 0 \le x \le \frac{l}{2} \\ l - x, \frac{l}{2} \le x \le l \end{cases}$
- Solution:
- $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos\left(\frac{n\pi x}{l}\right)$
- $a_0 = \frac{2}{l}\int_0^l f(x)dx$
- $= \frac{2}{l}\left(\int_0^{\frac{l}{2}} 0dx + \int_{\frac{l}{2}}^{l}(l-x)dx\right)$
- $= \frac{2}{l}\left[lx - \frac{x^2}{2}\right]_{\frac{l}{2}}^{l} = \frac{2}{l}\left(l^2 - \frac{l^2}{2} - \frac{l^2}{2} + \frac{l^2}{8}\right)$
- $=\frac{2}{l}\left(\frac{l^2}{8}\right) = \frac{l}{4}$
- $\frac{a_0}{2} = \frac{l}{2}$

- $a_n = \frac{2}{l}\int_0^l f(x)cos\left(\frac{n\pi x}{l}\right)dx$
- $= \frac{2}{l}\left(\int_0^{\frac{l}{2}} 0dx + \int_{\frac{l}{2}}^{l}(l-x)cos\left(\frac{n\pi x}{l}\right)dx\right)$
- $= \frac{2}{l}\left[(l-x)\frac{sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}} - (-1)\frac{cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right]_{\frac{l}{2}}^{l}$
- $= \frac{2}{l}\left[(l-l)\frac{sin\left(\frac{n\pi l}{l}\right)}{\frac{n\pi}{l}} + \frac{cos\left(\frac{n\pi l}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} - \left[\left(l-\frac{l}{2}\right)\frac{sin\left(\frac{n\pi l}{2l}\right)}{\frac{n\pi}{l}} + \frac{cos\left(\frac{n\pi l}{2l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right]\right]$
- $= \frac{2}{l}\left[\frac{(-1)^n}{\left(\frac{n\pi}{l}\right)^2} - \left[\left(\frac{l}{2}\right)\frac{sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{l}} + \frac{cos\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{l}\right)^2}\right]\right]$
- $= 2\left[\frac{l(-1)^n}{(n\pi)^2} - \left[\left(\frac{l}{2}\right)\frac{sin\left(\frac{n\pi}{2}\right)}{n\pi} + \frac{lcos\left(\frac{n\pi}{2}\right)}{(n\pi)^2}\right]\right]$

- $f(x) = \frac{l}{2} + 2\left[\frac{l(-1)^n}{(n\pi)^2} - \left[\left(\frac{l}{2}\right)\frac{sin\left(\frac{n\pi}{2}\right)}{n\pi} + \frac{lcos\left(\frac{n\pi}{2}\right)}{(n\pi)^2}\right]\right]$

The following table gives the vibrations of periodic current over a period

t in sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A amp	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

T	A	$\frac{2\pi t}{T}$	$\cos\frac{2\pi t}{T}$	$\sin\frac{2\pi t}{T}$	$A\cos\frac{2\pi t}{T}$	$A\sin\frac{2\pi t}{T}$
0	1.98	0	1	0	1.98	0
$\frac{T}{6}$	1.30	60	0.5	0.866	0.650	1.126
$\frac{T}{3}$	1.05	120	-0.5	0.866	-0.525	0.909
$\frac{T}{2}$	1.30	180	-1.0	0	-1.30	0
$\frac{2T}{3}$	-0.88	240	-0.5	-0.866	0.440	0.762
$\frac{5T}{6}$	-0.25	300	0.5	-0.866	-0.125	0.217
	4.5				1.12	3.014

$$a_0 = \frac{2}{N}\sum A = \frac{2}{6}\sum 4.5 = 1.5$$

$$a_1 = \frac{2}{N}\sum A\cos\frac{2\pi t}{T} = \frac{2}{6} * 1.12$$

$$= 0.373$$

$$b_1 = \frac{2}{N}\sum A\sin\frac{2\pi t}{T} = \frac{2}{6}(3.014)$$

$$= 1.005$$

The following values of y and x are given. Find the Fourier series of y up to second harmonic.

X	0	2	4	6	8	10	12
Y	9	18.2	24.4	27.8	27.5	22	9

X	Y	$\theta = \frac{2\pi}{12}$	$\cos\theta$	$\sin\theta$	$\cos 2\theta$	$\sin 2\theta$	$y\cos\theta$	$y\sin\theta$	$y\cos 2\theta$	$y\sin 2\theta$
0	9	0	1	0	1	0	9	0	9	0
2	18.2	60	0.5	0.866	-0.5	0.866	9.1	15.7612	-9.1	15.7612
4	24.4	120	-0.5	0.866	-0.5	-0.866	-12.2	21.1304	-12.2	-21.1304
6	27.8	180	-1.0	0	1	0	-27.8	0	27.8	0
8	27.5	240	-0.5	-0.866	-0.5	0.866	-13.75	-23.815	-13.75	23.815
10	22	300	0.5	-0.866	-0.5	-0.866	11	-19.052	-11	-19.052
	128.9						-24.65	-5.9754	-9.25	-0.6062

- $a_0 = \frac{2}{N}\sum y = \frac{2}{6} * 128.9 = 42.967$
- $a_1 = \frac{2}{N}\sum ycos\theta = \frac{2}{6} * -24.65 = -8.217$
- $b_1 = \frac{2}{N}\sum ysin\theta = \frac{2}{6}(-9.25) = -3.083$
- $a_2 = \frac{2}{N}\sum ycos2\theta = \frac{2}{6} * -5.9754 = -1.9918$
- $b_2 = \frac{2}{N}\sum ysin2\theta = \frac{2}{6}(-0.6062) = -0.202$
- Amplitude of the first harmonic= $\sqrt{{a_1}^2 + {b_1}^2} = \sqrt{(-8.217)^2 + (-3.083)^2} = 8.776$

Obtain the constant term and the first harmonic coefficients in the Fourier cosine series for y using the following table

X	0	1	2	3	4	5
Y	4	8	15	7	6	2

X	Y	θ	cosθ	cos2θ	ycosθ	ysinθ
0	4	0	1	1	4	4
1	8	30	0.866	0.5	6.928	4
2	15	60	0.5	-0.5	7.5	-7.5
3	7	90	0	-1	0	-7
4	6	120	-0.5	-0.5	-3	-3
5	2	150	-0.866	0.5	-1.732	1
	4.5				13.696	-8.5

$$a_0 = \frac{2}{N}\sum y = \frac{2}{6} * 42 = 14$$

$$a_1 = \frac{2}{N}\sum ycos\theta = \frac{2}{6} * 13.696$$

$$= -2.833$$

$$a_2 = \frac{2}{N}\sum ycos2\theta = \frac{2}{6} * -8.5$$

$$= -2.833$$