Module-2

Block diagram: Block diagram of a closed loop system, procedure for drawing block diagram and block diagram reduction to find transfer function. **Signal flow graphs:** Construction of signal flow graphs, basic properties of signal flow graph, signal flow graph algebra, construction of signal flow graph for control systems. (10 Hours)
Revised Bloom's Taxonomy Level: L1 – Remembering, L2 – Understanding, L3 – Applying, L4 – Analysing

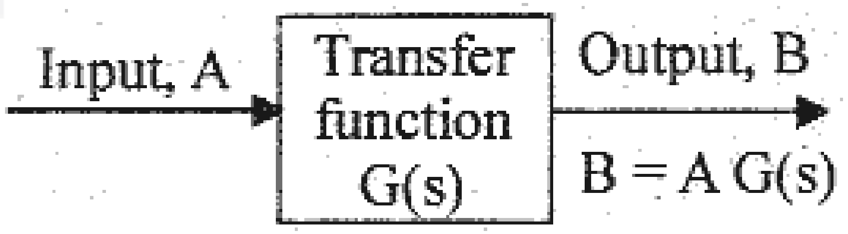
BLOCK DIAGRAM

A control system may consist of a number of components. In control engineering to show the functions performed by each component, we commonly use a diagram called the block diagram. A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components.

The elements of a block diagram are block, branch point and summing point.

BLOCK

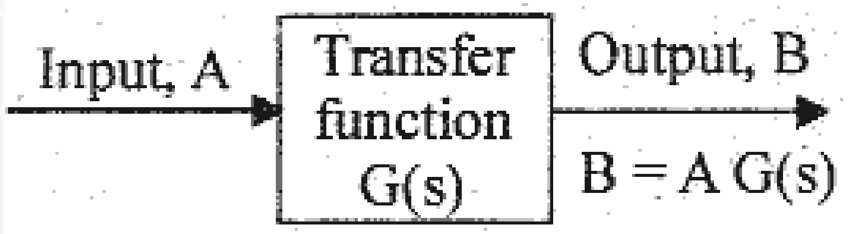
In a block diagram all system variables are linked to each other through functional blocks. The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Figure shows the block diagram of functional block.



Functional block

BLOCK

Figure shows the block diagram of functional block.

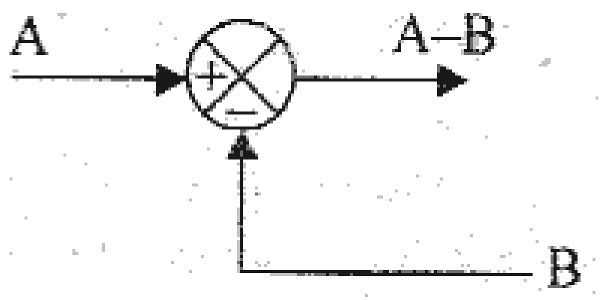


Functional block

The arrowhead pointing towards the block indicates the input, and the arrowhead leading away from the block represents the output. Such arrows are referred to as signals. The output signal from the block is given by the product of input signal and transfer function in the block..

SUMMING POINT

Summing points are used to add two or more signals in the system. Referring to figure, a circle with a cross is the symbol that indicates a summing operation

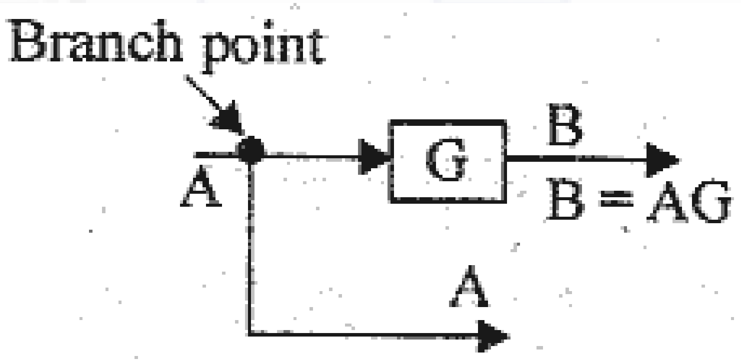


Summing point

The plus or minus sign at each arrowhead indicates whether the signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

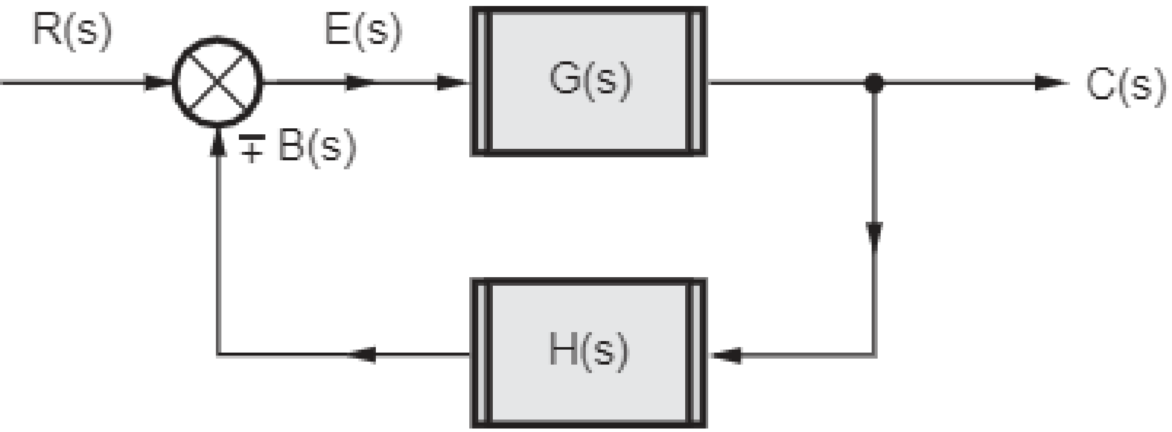
BRANCH POINT

A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.



Branch point

Simple or Canonical form of Closed loop System



R(s) → Laplace of reference input r(t)

C(s) → Laplace of controlled output c(t)

E(s) → Laplace of error signal e(t)

B(s) → Laplace of feedback signal b(t)

G(s) → Equivalent forward path transfer function

H(s) → Equivalent feedback path transfer function.

$$E(s) = R(s) \mp B(s)$$

$$B(s) = C(s)H(s)$$

$$C(s) = E(s)G(s)$$

$B(s) = C(s)H(s)$ and substituting in equation (1)

$$E(s) = R(s) \mp C(s)H(s)$$

$$E(s) = \frac{C(s)}{G(s)}$$

$$\frac{C(s)}{G(s)} = R(s) \mp C(s)H(s)$$

$$C(s) = R(s)G(s) \mp C(s)G(s)H(s)$$

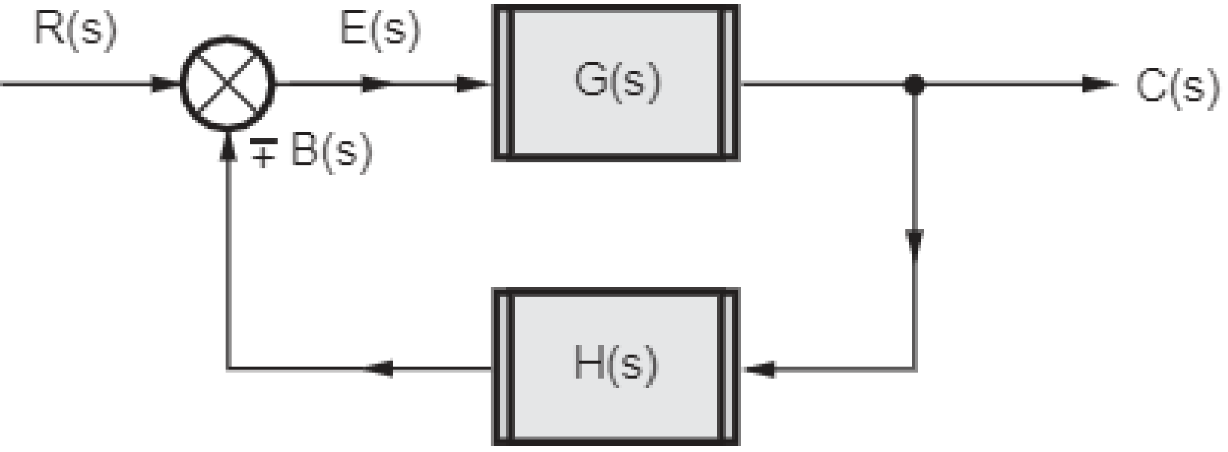
Closed Loop T.F.

$$\therefore C(s)\,[1 \pm G(s)H(s)] = R(s)\,G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

- Use + sign for negative feedback and Use − sign for positive feedback.

Simple or Canonical form of Closed loop System



$G(s) \rightarrow$ Equivalent forward path transfer function

$H(s) \rightarrow$ Equivalent feedback path transfer function.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

Closed Loop T.F.

- Use + sign for negative feedback and Use – sign for positive feedback.

CONSTRUCTING BLOCK DIAGRAM FOR CONTROL SYSTEMS

A control system can be represented diagrammatically by block diagram. The differential equations governing the system are used to construct the block diagram. By taking Laplace transform the differential equations are converted to algebraic equations. The equations will have variables and constants. From the working knowledge of the system the input and output variables are identified and the block diagram for each equation can be drawn. Each equation gives one section of block diagram. The output of one section will be input for another section. The various sections are interconnected to obtain the overall block diagram of the system..

BLOCK DIAGRAM REDUCTION

The block diagram can be reduced to find the overall transfer function of the system. The following rules can be used for block diagram reduction. The rules are framed such that any modification made on the diagram does not alter the input-output relation

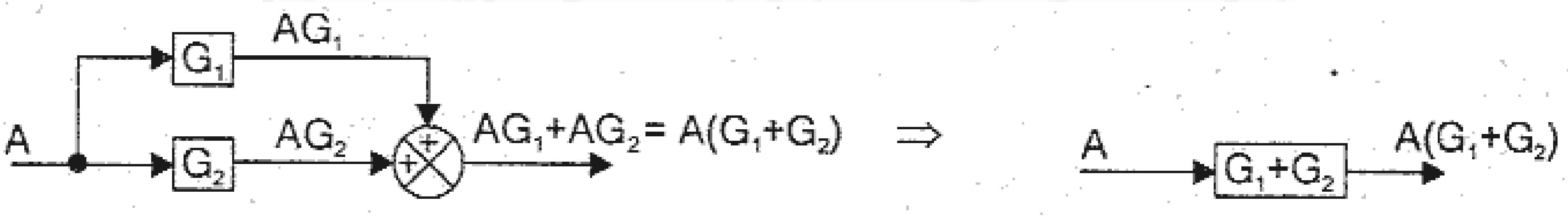
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-l: Combining the blocks in cascade

The transfer functions of the blocks which connected in series get multiplied with each other

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-2: Combining Parallel blocks (or combining feed forward paths)



The transfer functions of the blocks which are connected in parallel get added algebraically (considering the sign)

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-3: Moving the branch point (Take off Point) Ahead/Beyond/After of the block

Add a block in series with signal which are Take-off point, add reciprocal of Transfer Function in Block

While shifting a take off point beyond the block, add a block in series with all the signals which are taking off from that point, having T.F. as reciprocal of the T.F. of the block beyond which take off point is to be shifted.

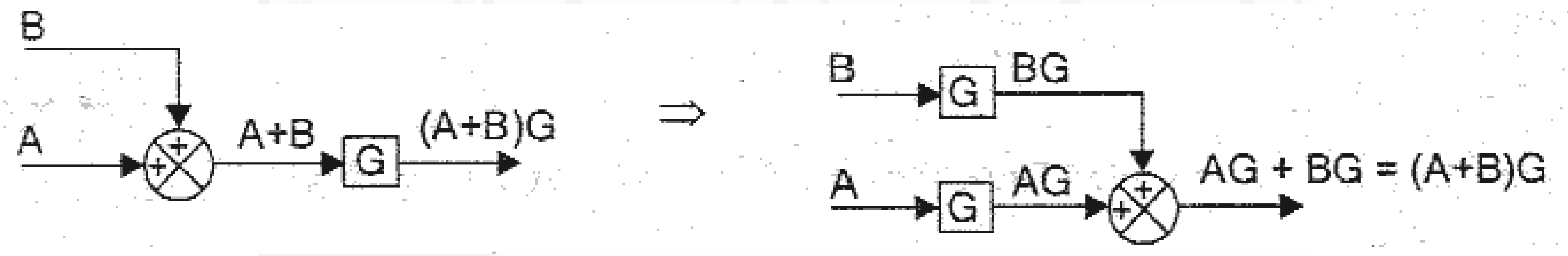
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-4: Moving the branch point before/Behind the block

While shifting a take off point behind the block, add a block having T.F. same as that of the block behind which take off point is to be shifted, in series with all the signals taking off from that take off point.

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-5: Moving the summing point beyond/ahead/before/in the front of the block



Thus while shifting a summing point after a block ,add a block having T.F. same as that of block after which summing point is to be shifted, in series with all the signals at that summing point

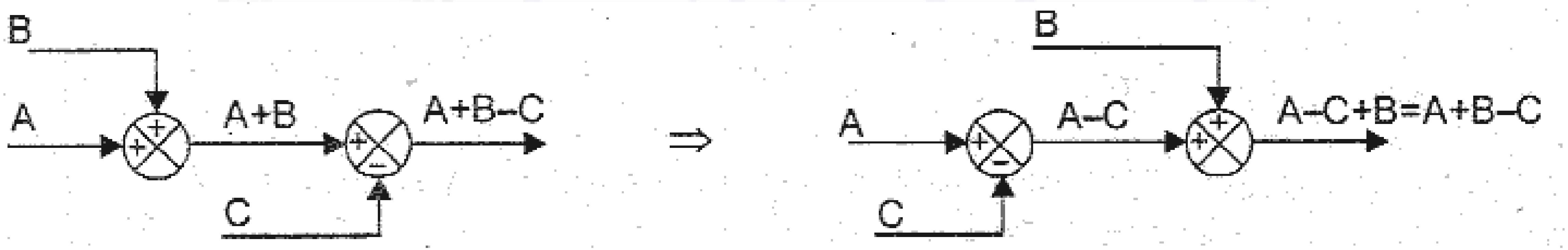
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-6: Moving the summing point behind the block

Thus while shifting a summing point behind the block i.e. before the block, add a block having T.F. as reciprocal of the T.F. of the block before which summing point is to be shifted, in series with all the signals at that summing point.

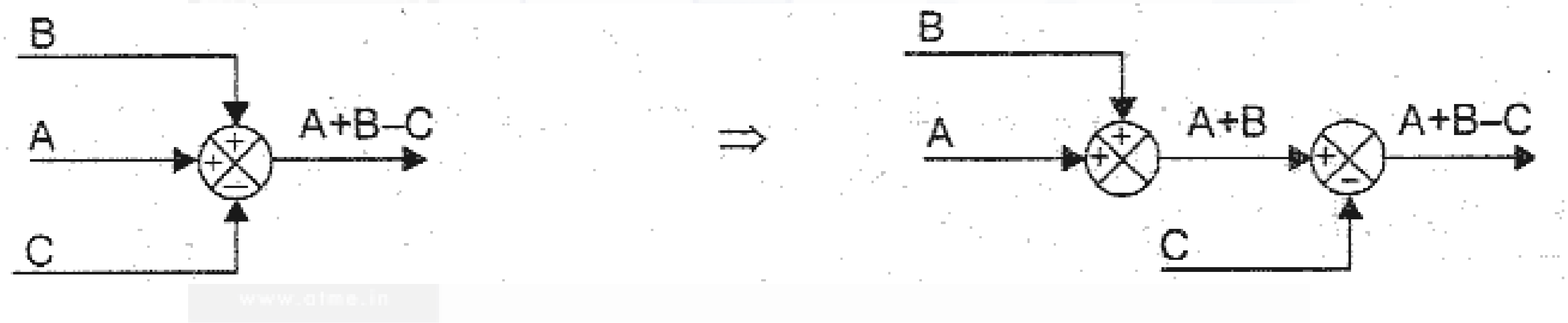
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-7: Interchanging summing point



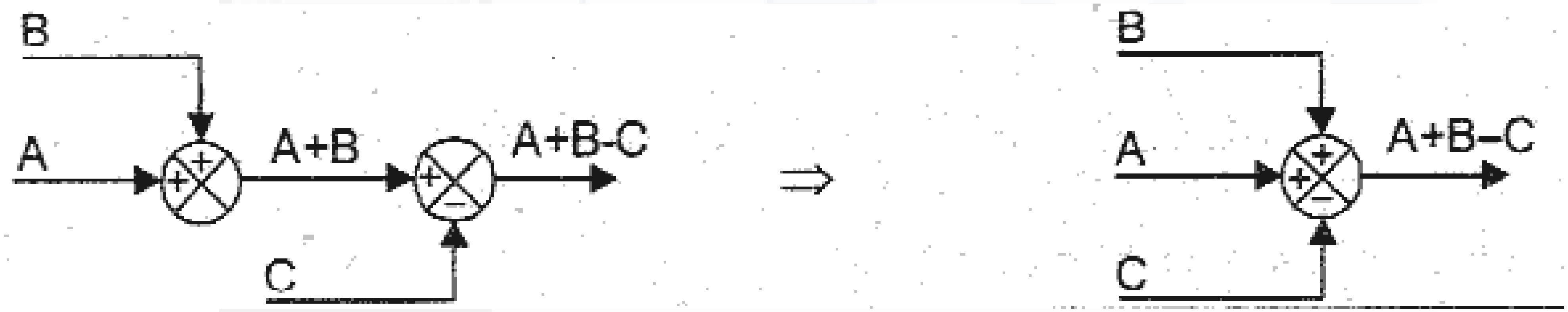
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-8: Splitting summing points



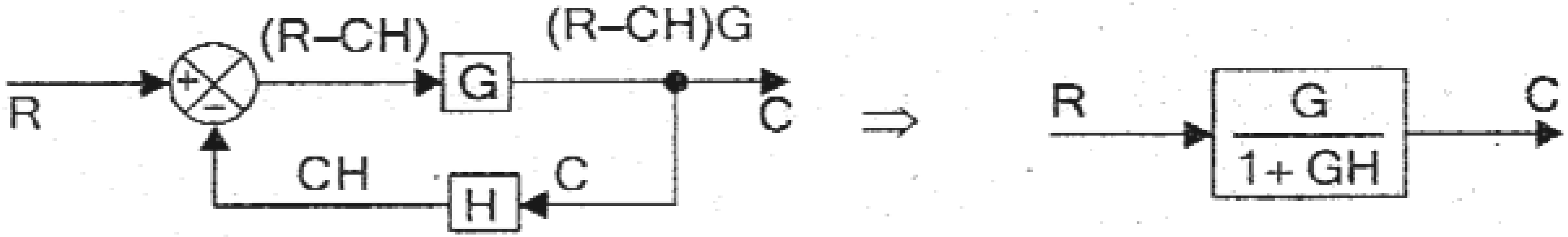
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-9: Combining summing points



RULES OF BLOCK DIAGRAM ALGEBRA

Rule-10: Elimination of (negative) feedback loop



Proof:

$$C = (R - CH)G \quad \Rightarrow \quad C = RG - CHG \quad \Rightarrow \quad C + CHG = RG$$

$$\therefore C(1 + HG) = RG \quad \Rightarrow \quad \frac{C}{R} = \frac{G}{1 + GH}$$

RULES OF BLOCK DIAGRAM ALGEBRA

Rule-11: Elimination of (positive) feedback loop

Procedure to solve BLOCK DIAGRAM Reduction Problem

- Step 1: Reduce the block connected in series
- Step 2: Reduce the block connected in parallel
- Step 3: Reduce the Minor internal Feedback loops
- Step 4: Try to shift branch point(Take off points) towards right and summing point towards left
- Step 5: Repeat step 1 to 4 till simple form is obtained
- Step 6: using standard function of simple closed loop system, obtain the closed loop transfer function C(s)/R(s) of overall system

1.Reduce the block diagram shown in fig 1 and find C/R.

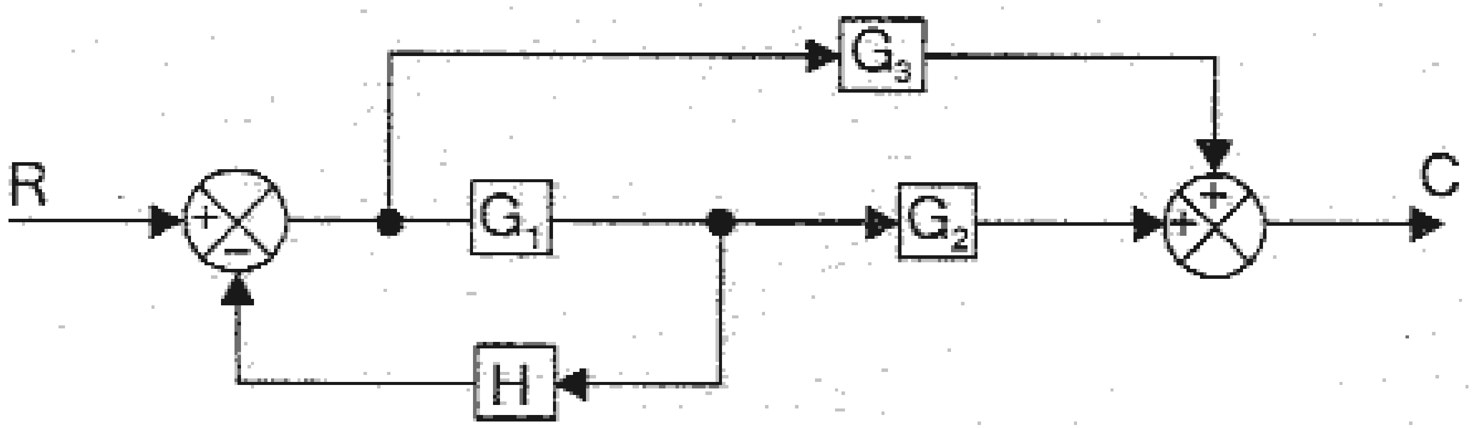


fig 1

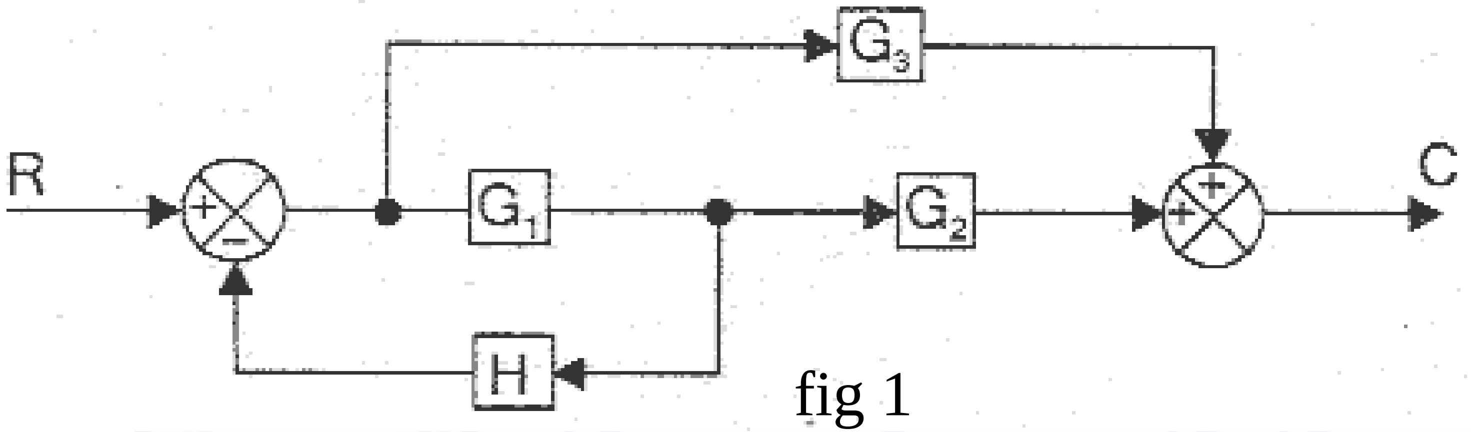
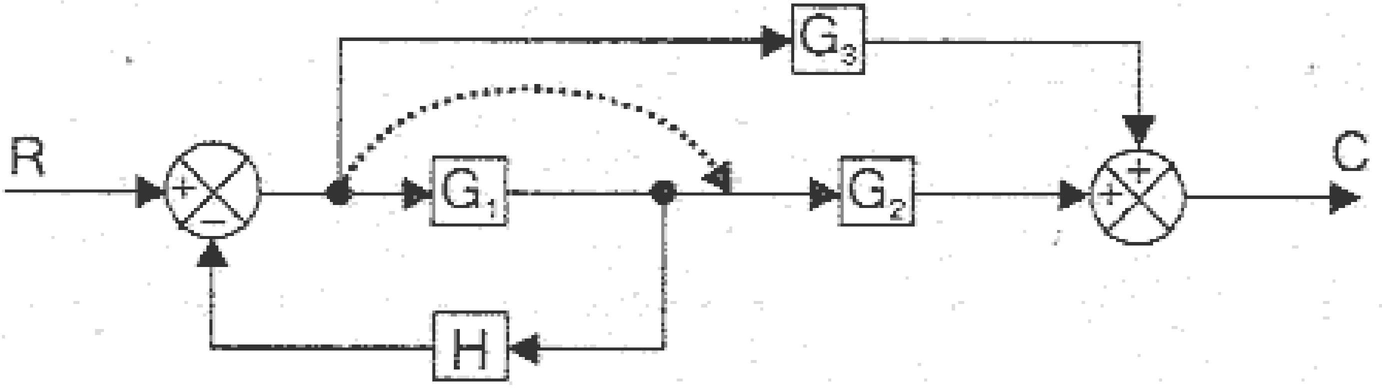
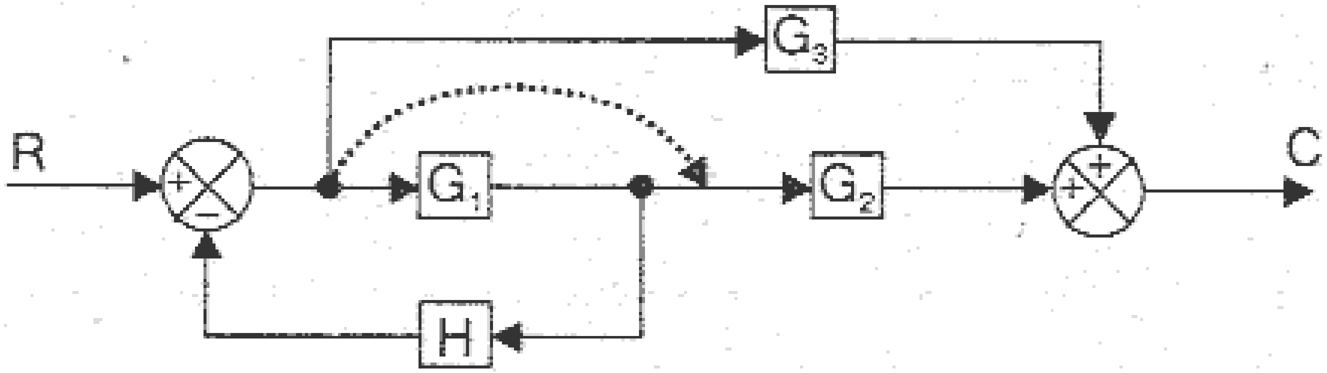


fig 1

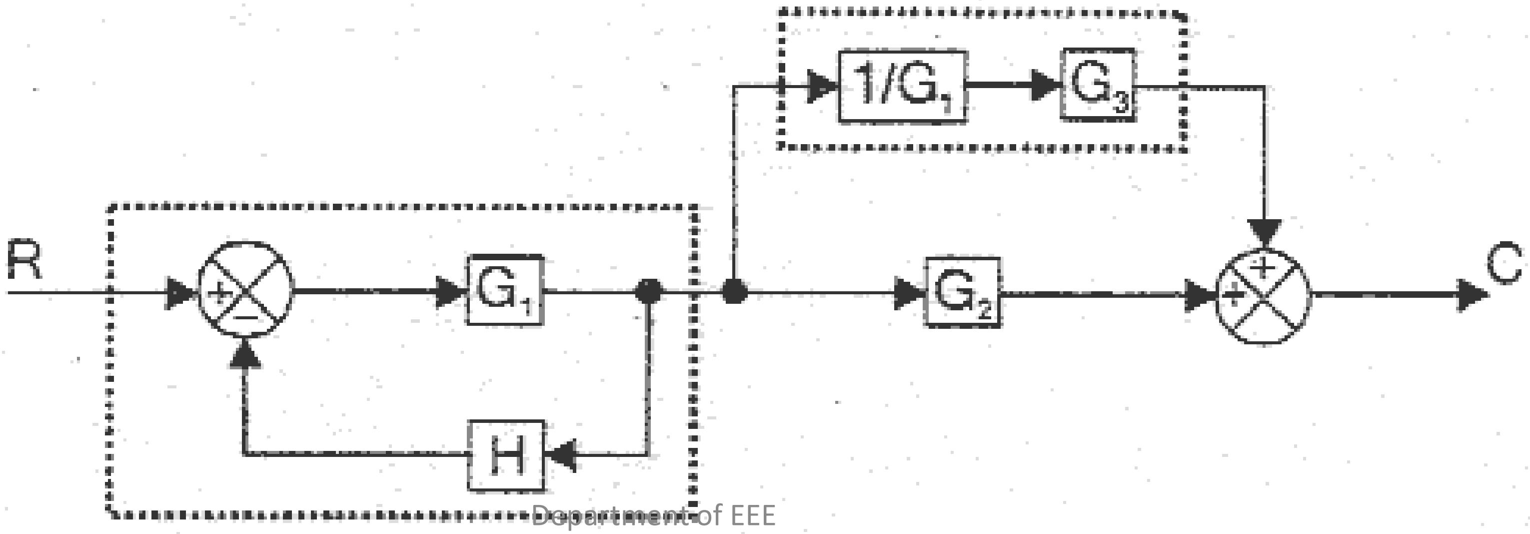
Step 1: Move the branch point after the block.



Step 1: Move the branch point after the block.



Step 2: Eliminate the feedback path and combining blocks in cascade.



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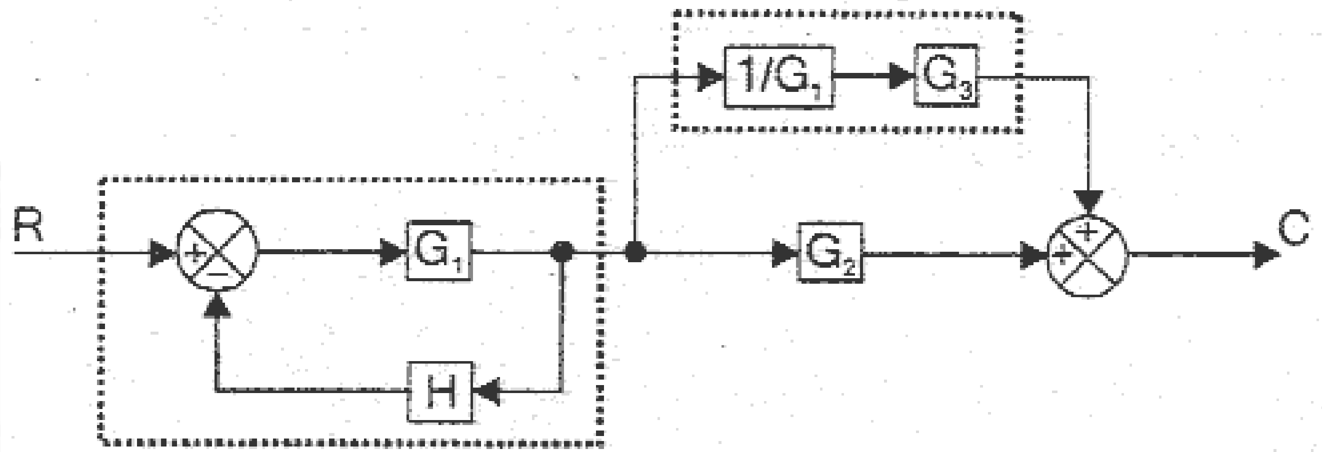
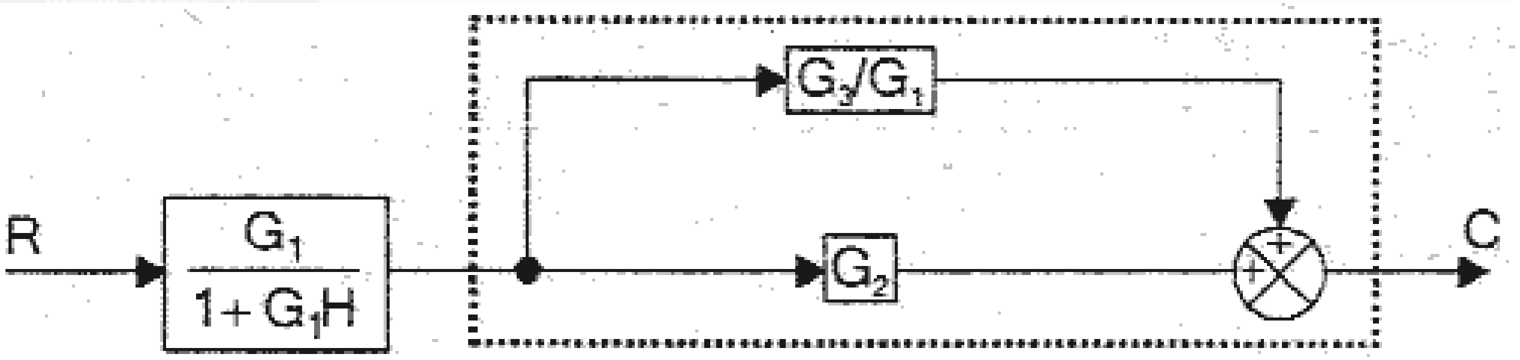


Step 3: Combining parallel blocks



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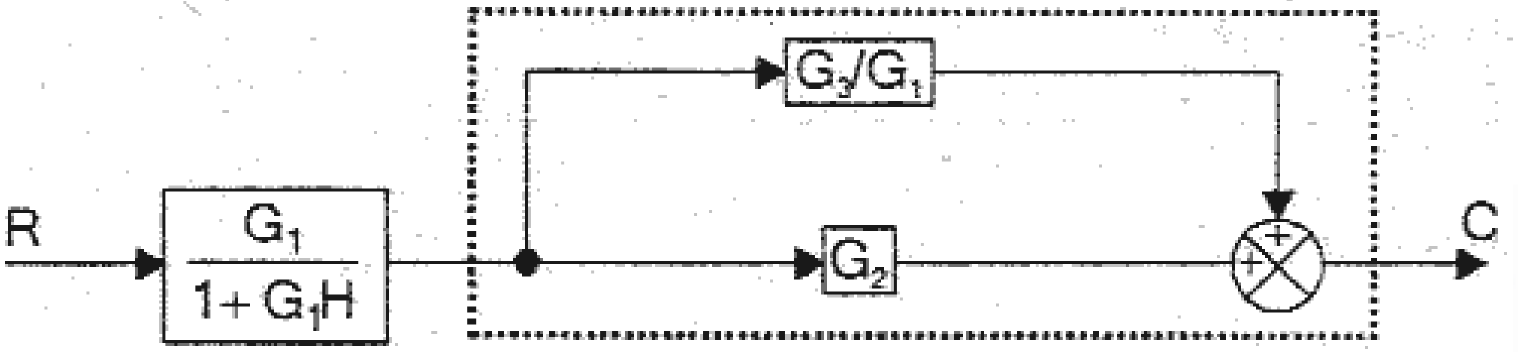
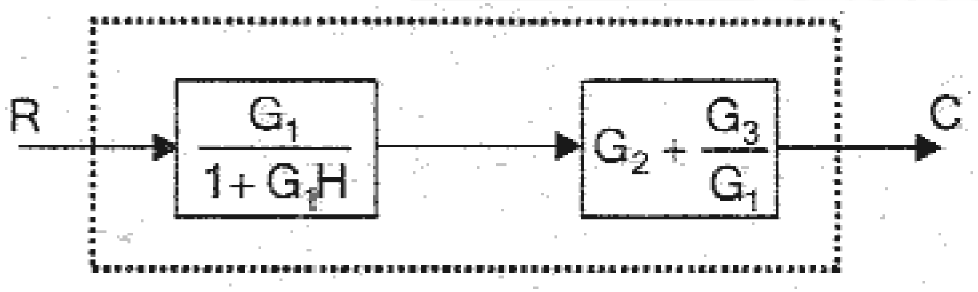
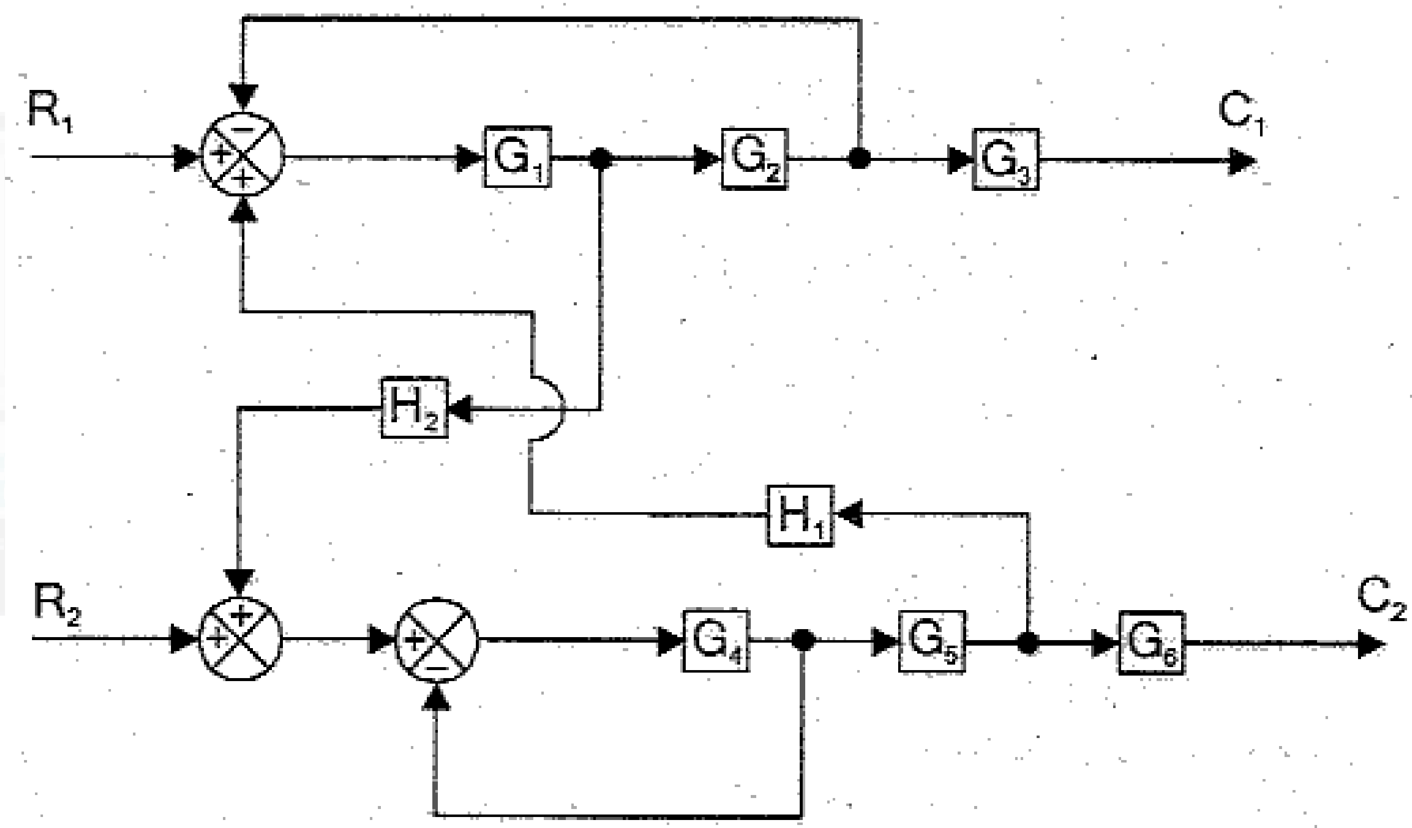


Step 4: Combining blocks in cascade



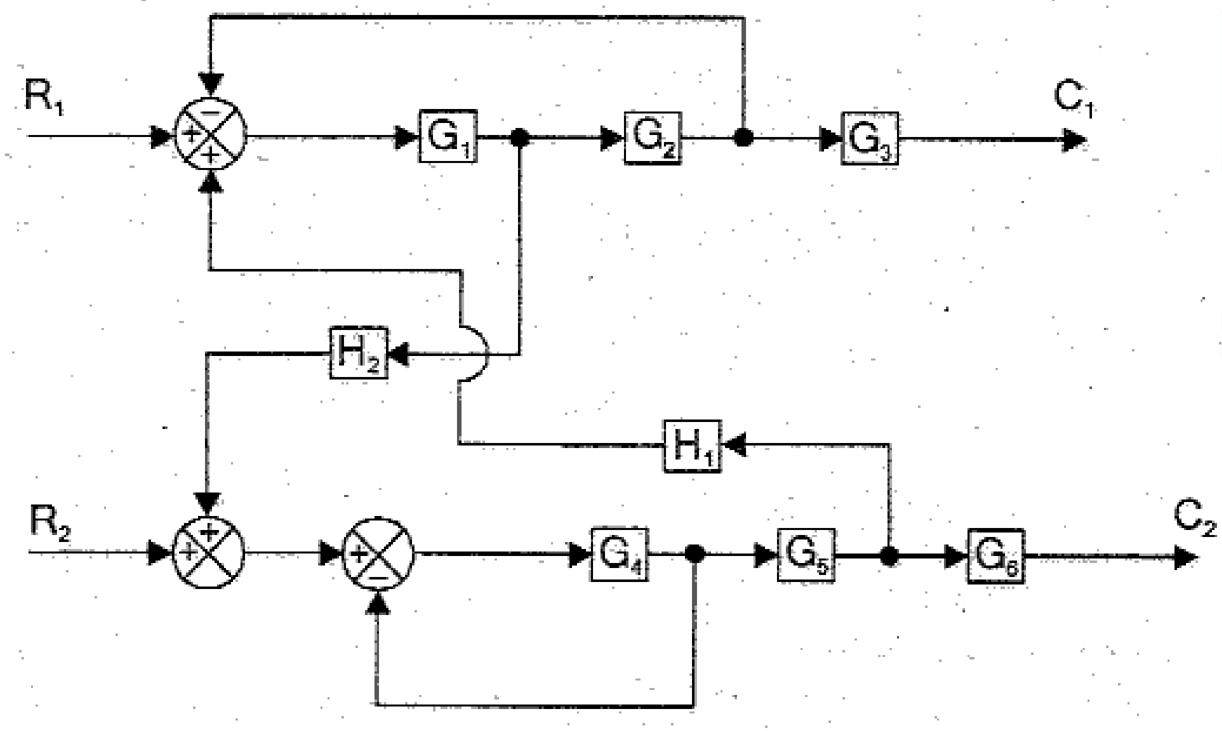
$$\frac{C}{R}=\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)=\frac{G_1G_2+G_3}{1+G_1H}$$

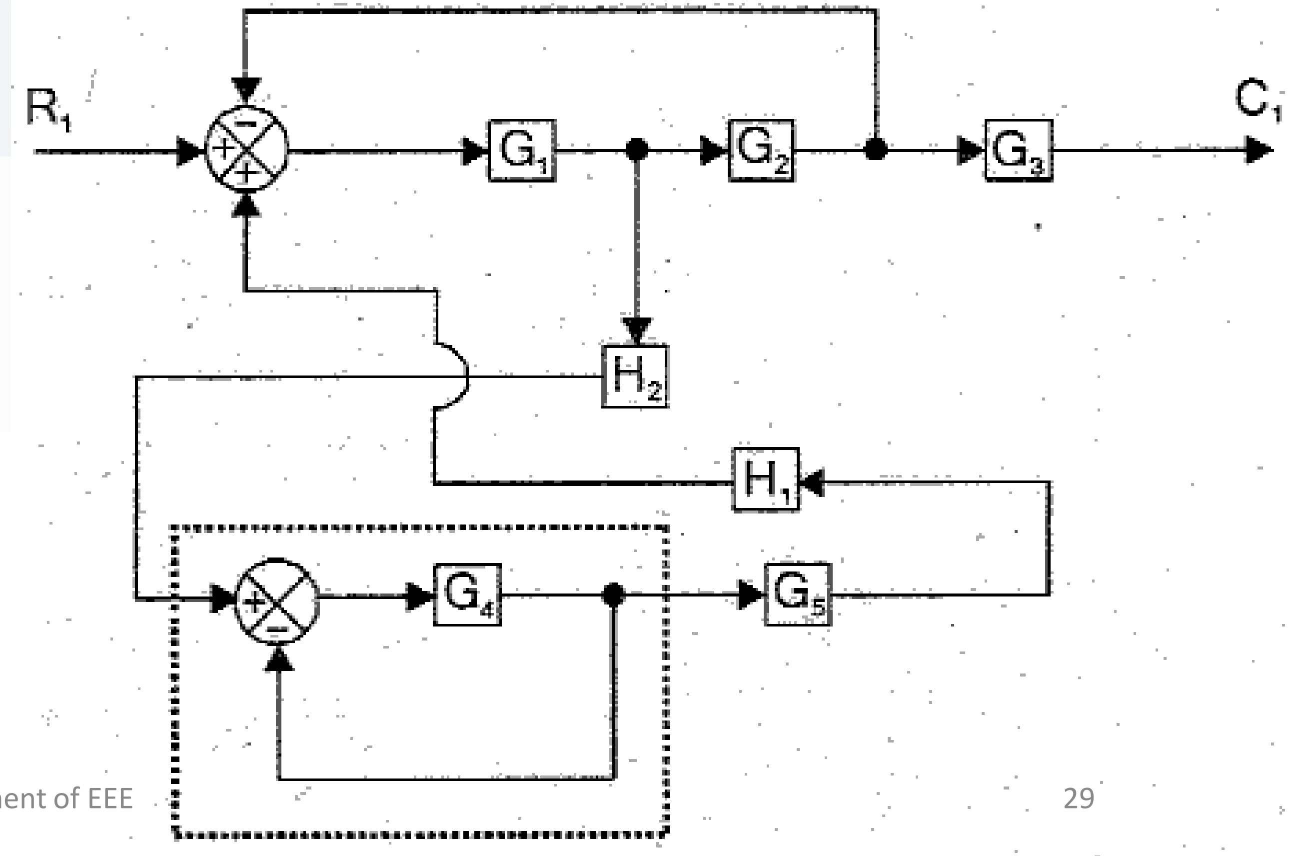
10.For the system represented by the block diagram shown in figure. Determine C1/R1 and C2/R1



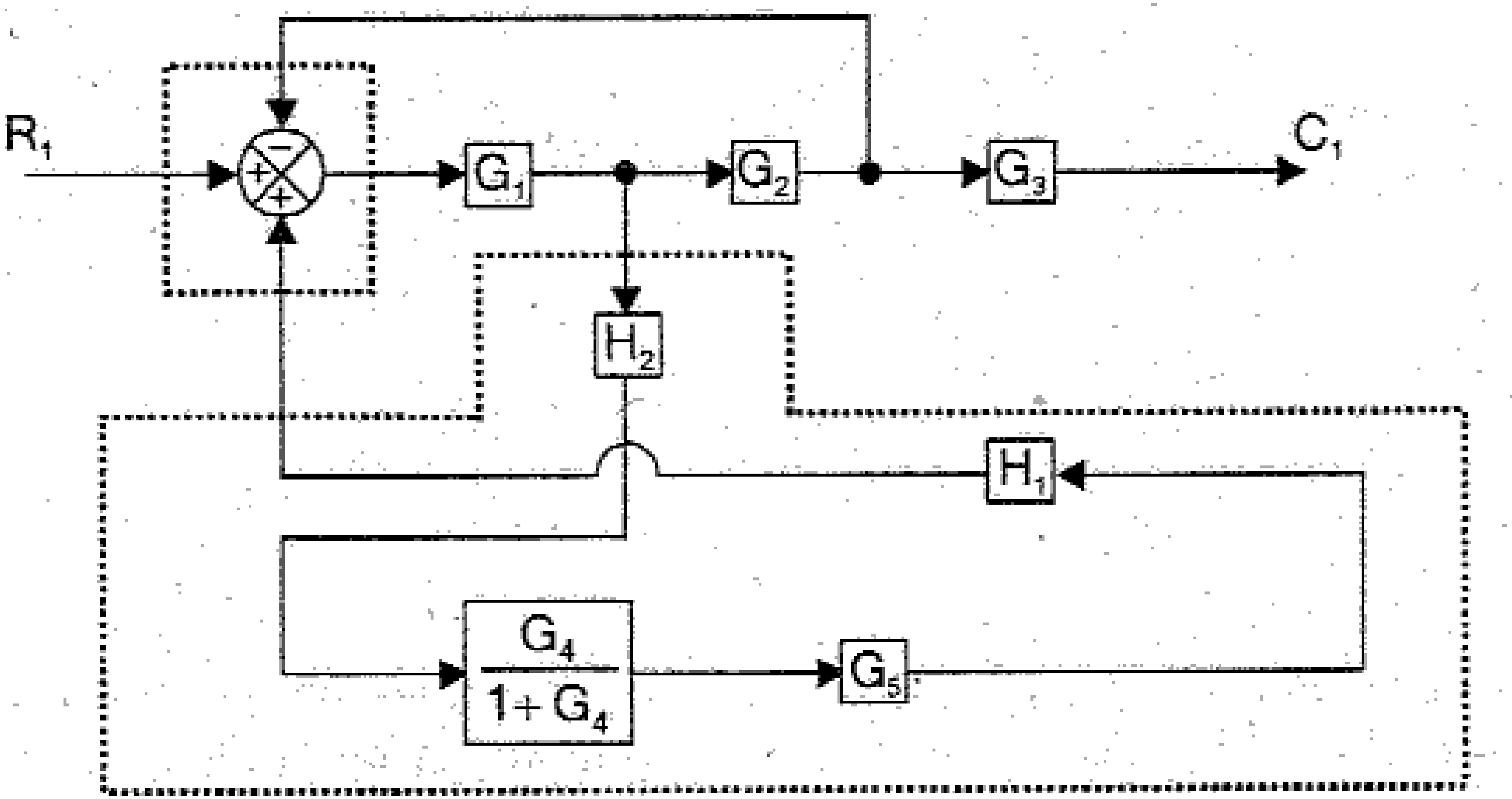
Case1 to Find C1/R1

Set R2=0 and consider only one output C1.Hence we can remove the summing point which adds R2 and need not consider G6, since G6 is on the open path. The resulting block diagram is shown

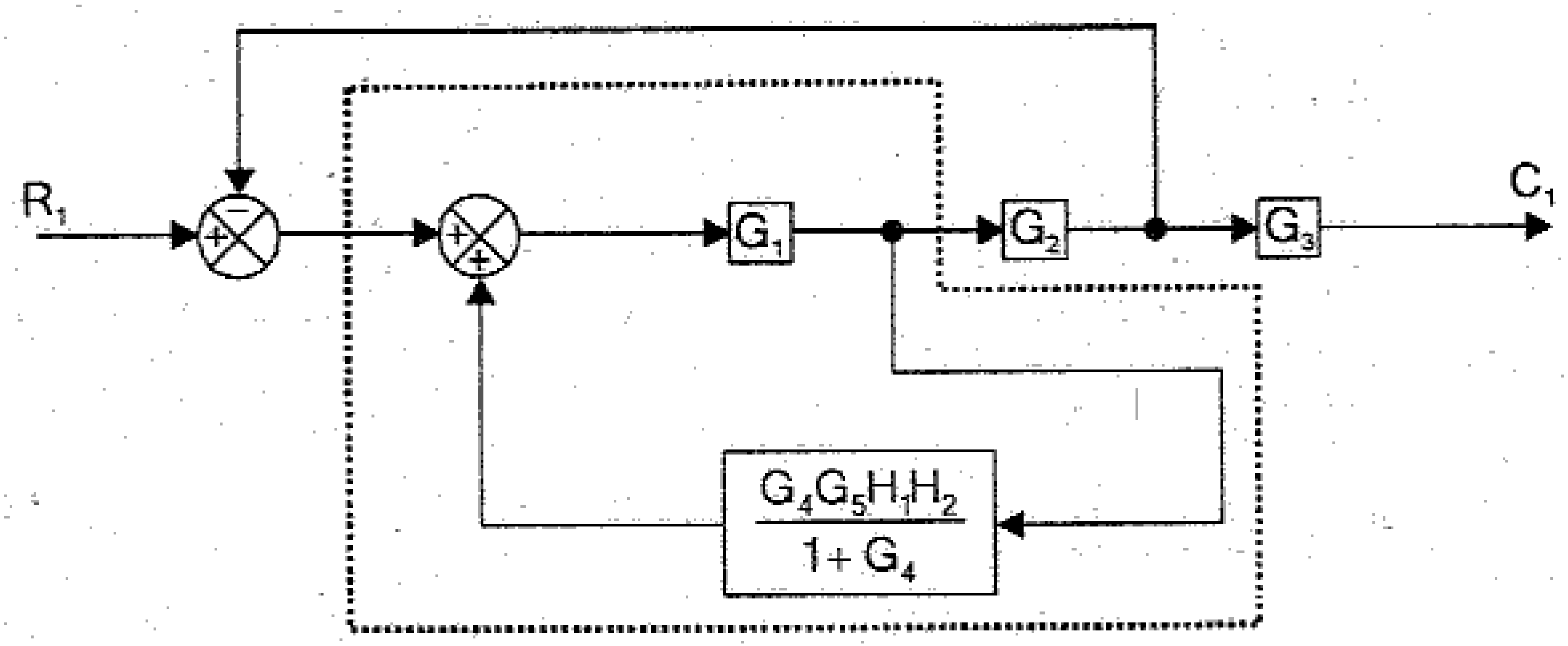




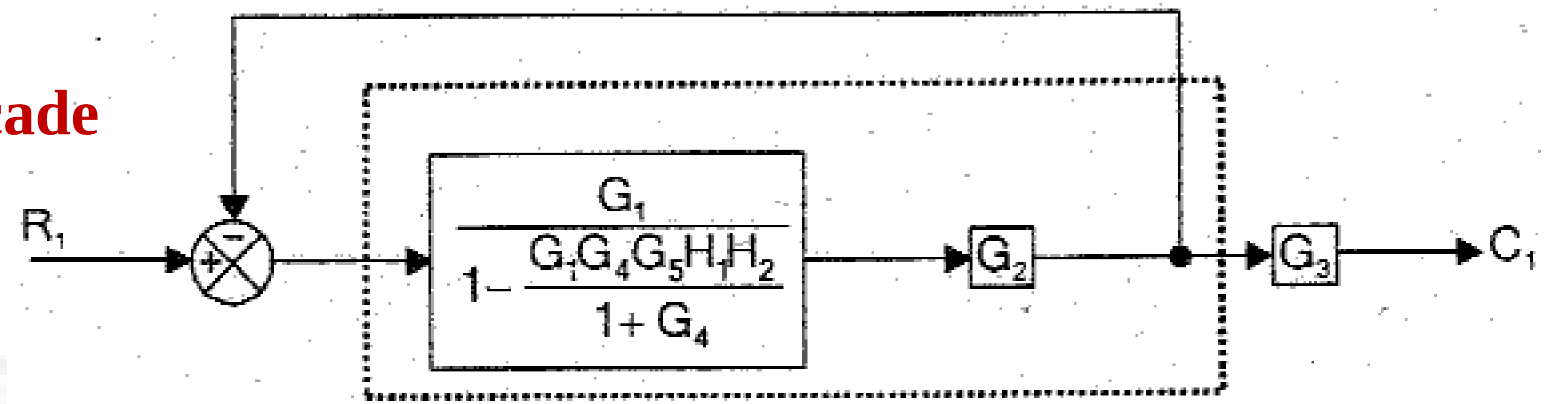
Combine the blocks in cascade & Splitting the summing point



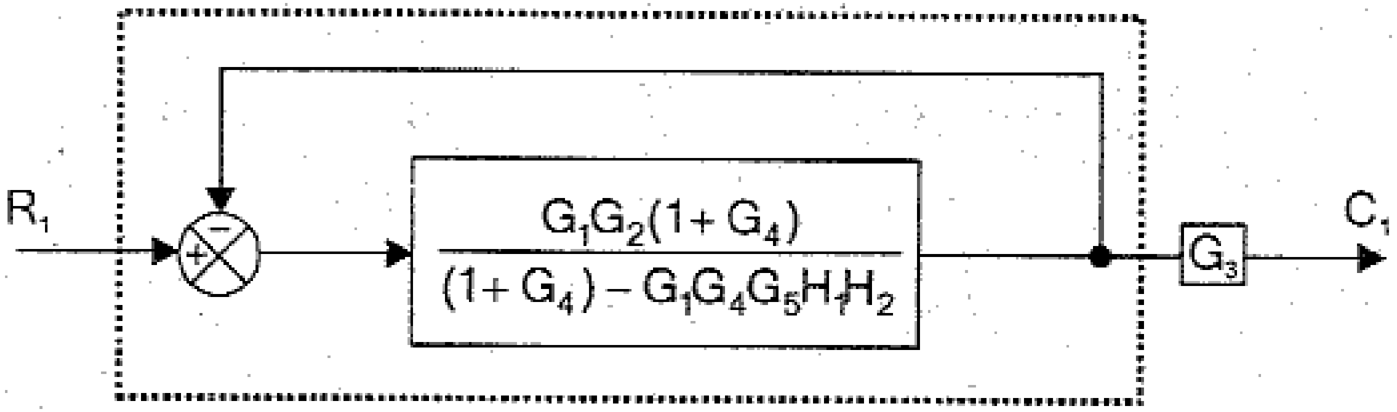
Eliminating feedback path

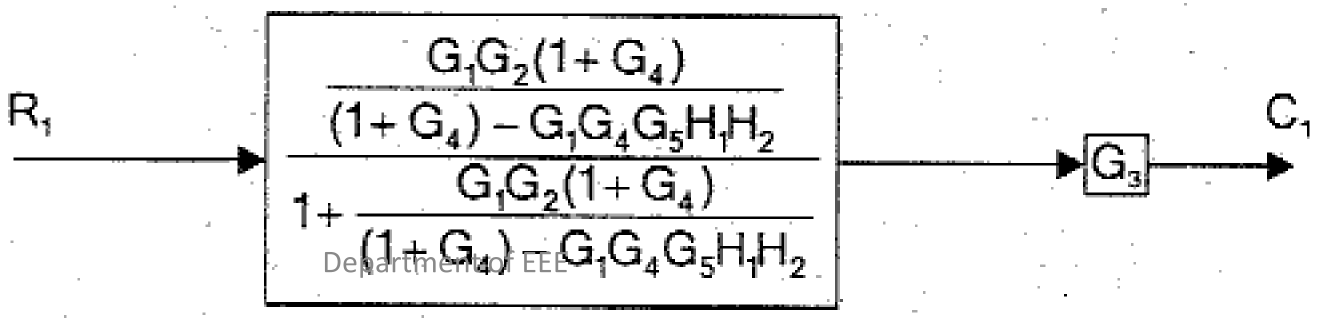


Combining blocks in cascade

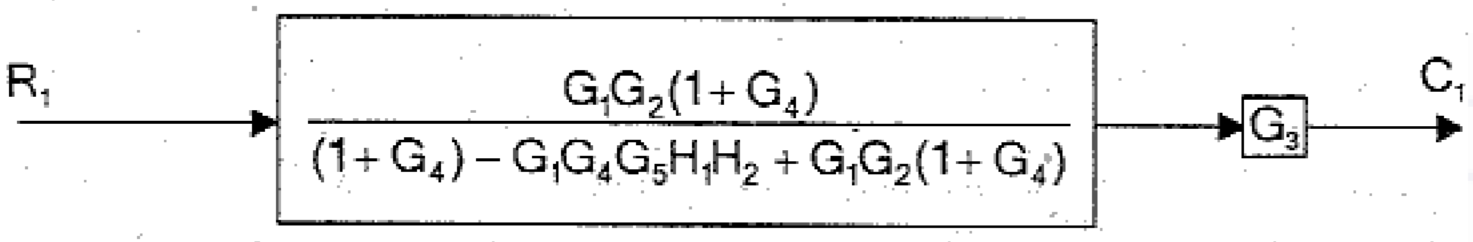


Eliminating feedback path





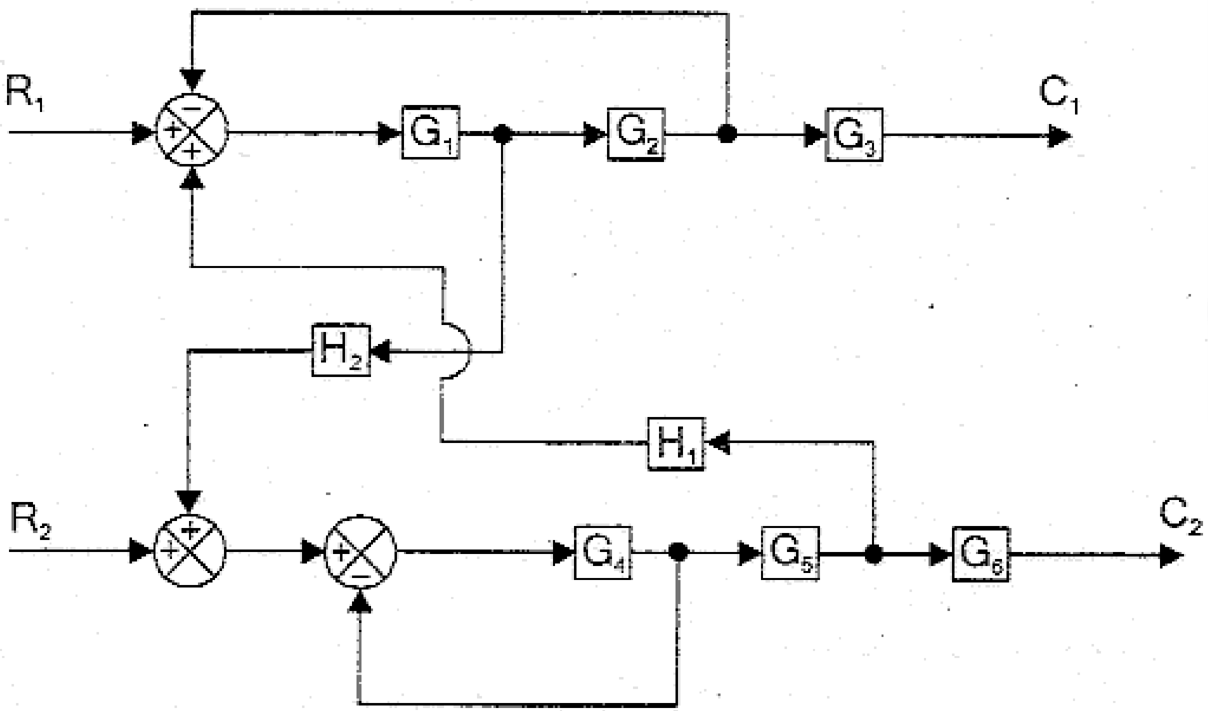
Combining blocks in cascade

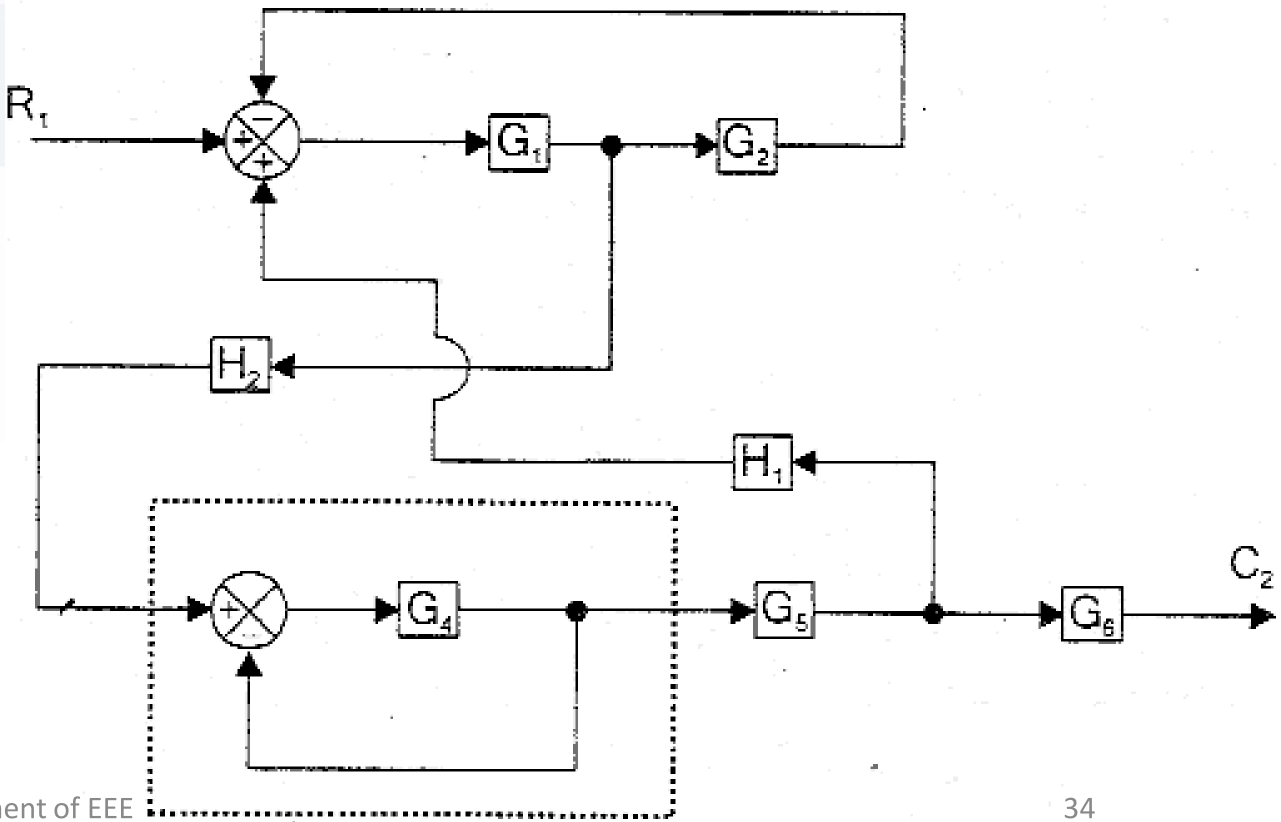


$$\frac{C_1}{R_1} = \frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)(1+G_4) - G_1G_4G_5H_1H_2}$$

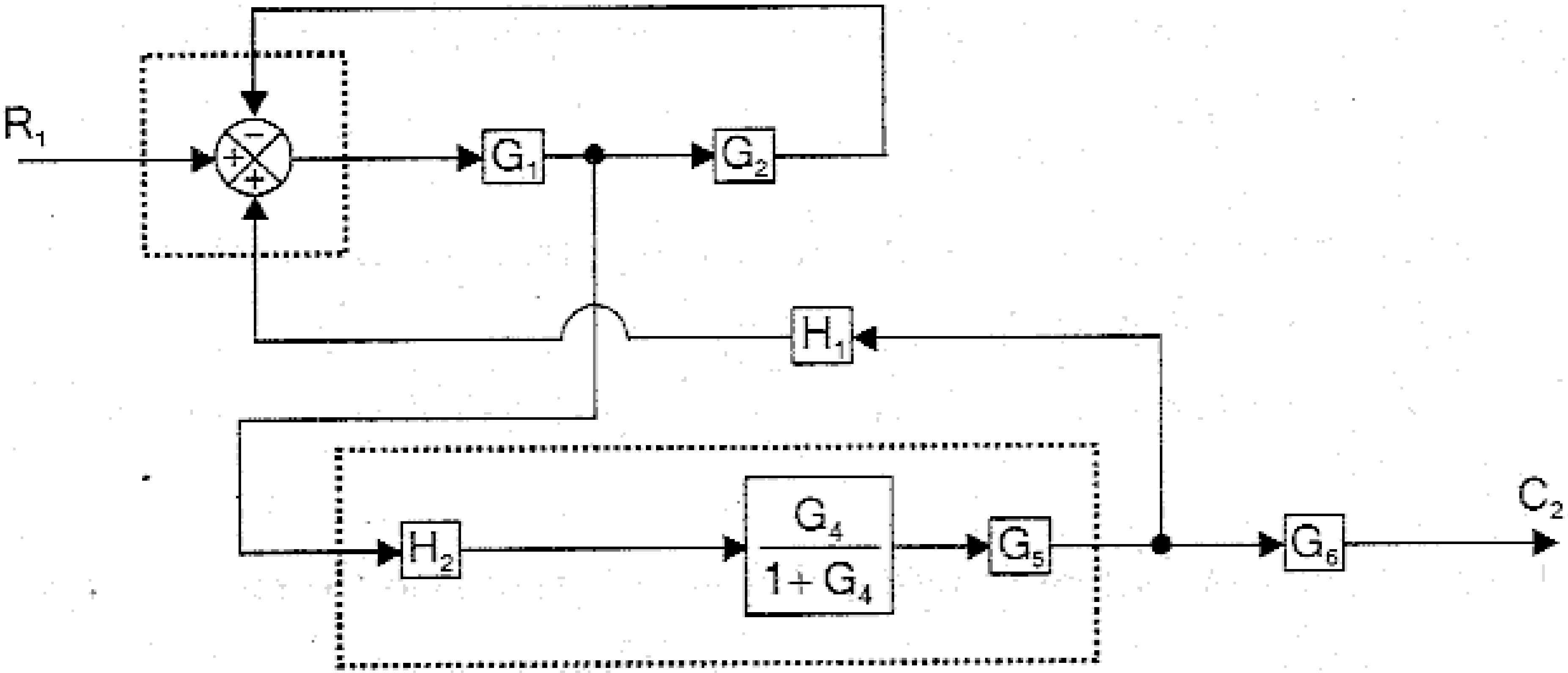
Case2 to Find C2/R1

Set R2=0 and consider only one output C2.Hence we can remove the summing point which adds R2 and need not consider G3, since G3 is on the open path. The resulting block diagram is shown

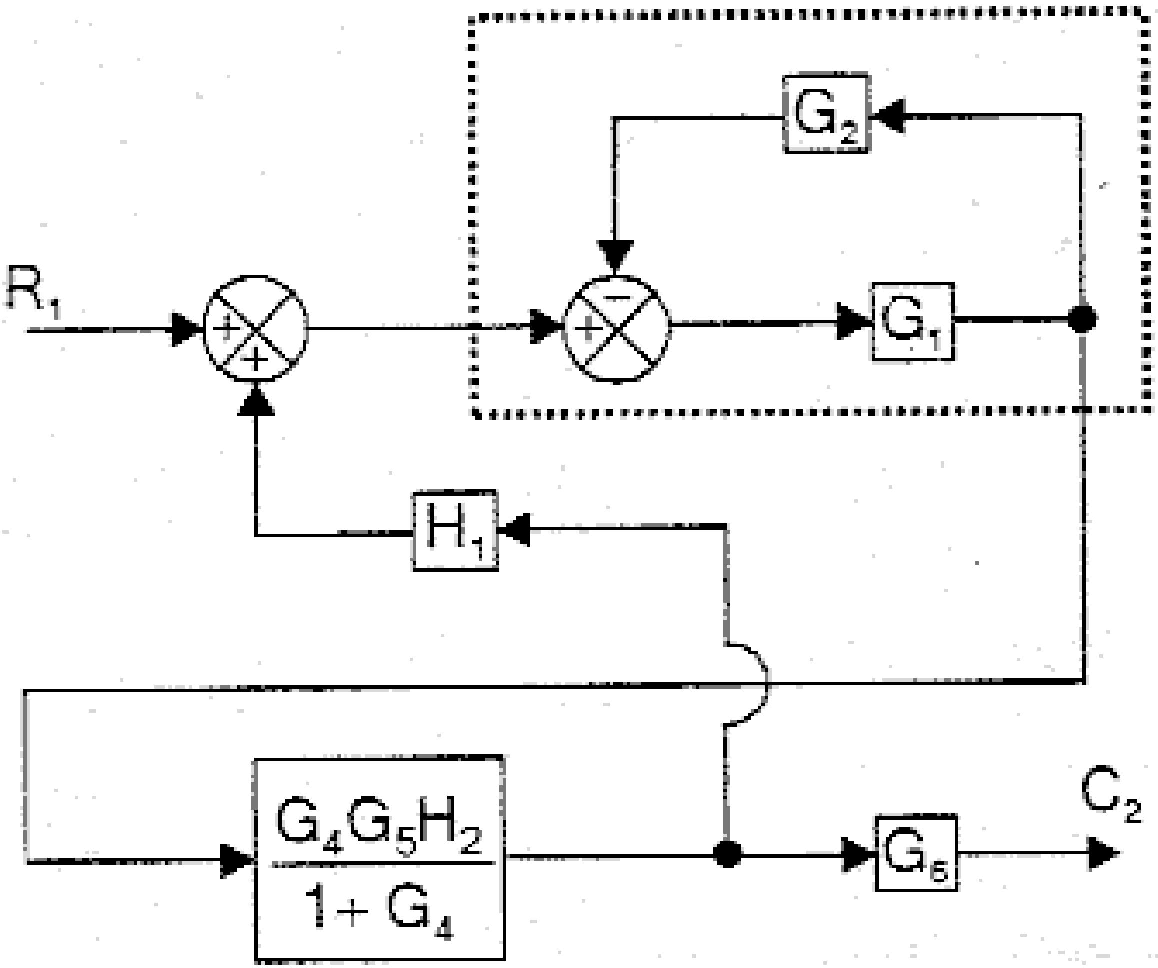




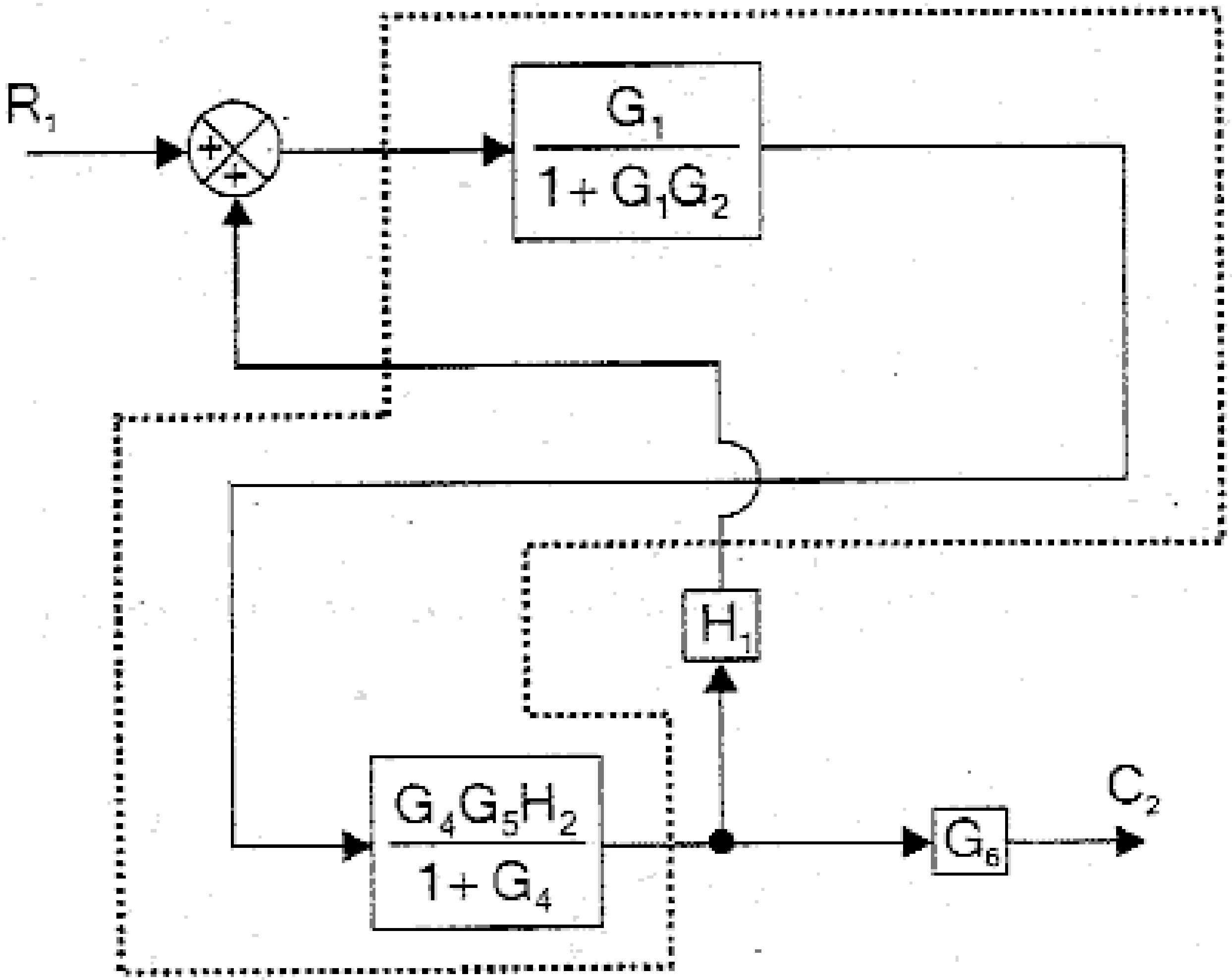
Combine the blocks in cascade & Splitting the summing point



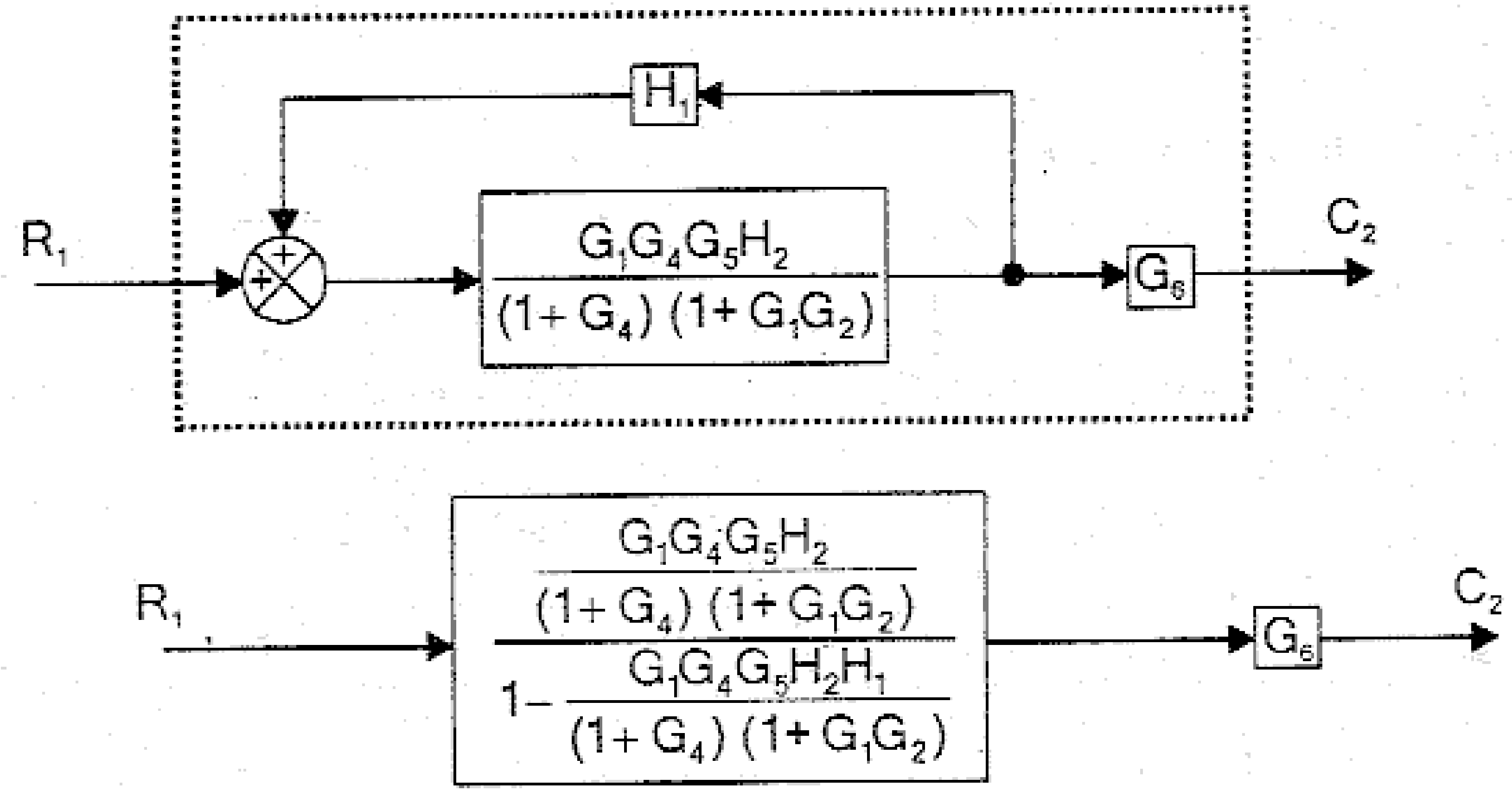
Eliminating feedback path



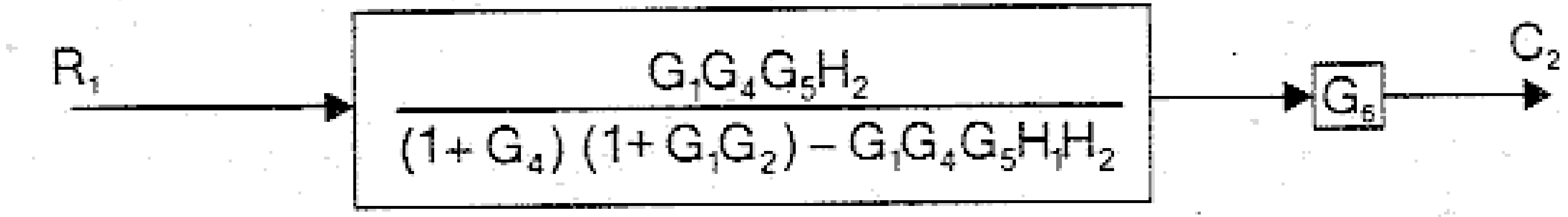
Combining blocks in cascade



Eliminating feedback path



Combining blocks in cascade



$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)(1+G_1G_2)-G_1G_4G_5H_1H_2}$$

BLOCK DIAGRAM

Advantages of Block Diagram

The various advantages of block diagram representation are,

1) Very simple to construct the block diagram for complicated systems.
2) The function of individual element can be visualised from block diagram.
3) Individual as well as overall performance of the system can be studied by using transfer functions shown in the block diagram.
4) Overall closed loop T.F. can be easily calculated by using block diagram reduction rules.

Disadvantages

The various disadvantages of block diagram representation are,

1) Block diagram does not include any information about the physical construction of the system.
2) Source of energy is generally not shown in the block diagram. So number of different block diagrams can be drawn depending upon the point of view of analysis. So block diagram for given system is not unique.

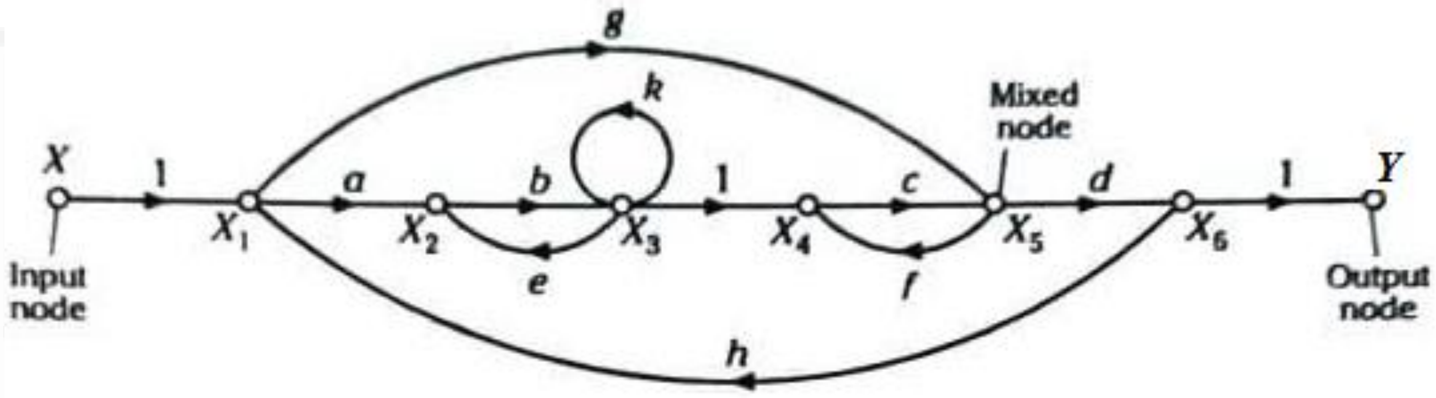
SIGNAL FLOW GRAPH

The signal flow graph is used to represent the control system graphically and it was developed by S.J. Mason.

A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. By taking Laplace transform, the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain. The signal flow graph of the system can be constructed using these equations.

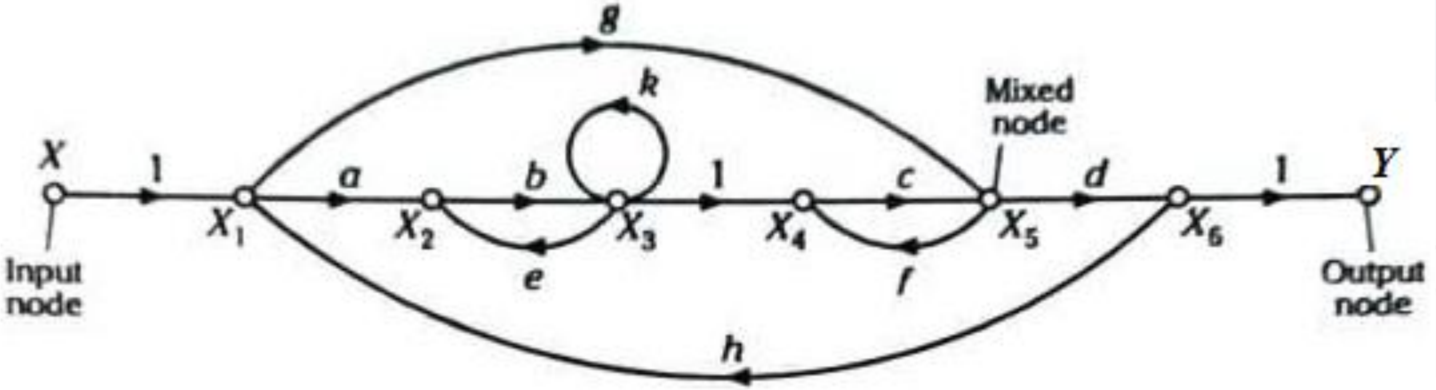
The signal flow graph approach and the block diagram approach yield the same information. The advantage in signal flow graph method is that, using Mason's gain formula the overall gain of the system can be computed easily. This method is simpler than the tedious block diagram reduction techniques.

The signal flow graph depicts the flow of signals from one point of a system to another and gives the relationships among the signals.



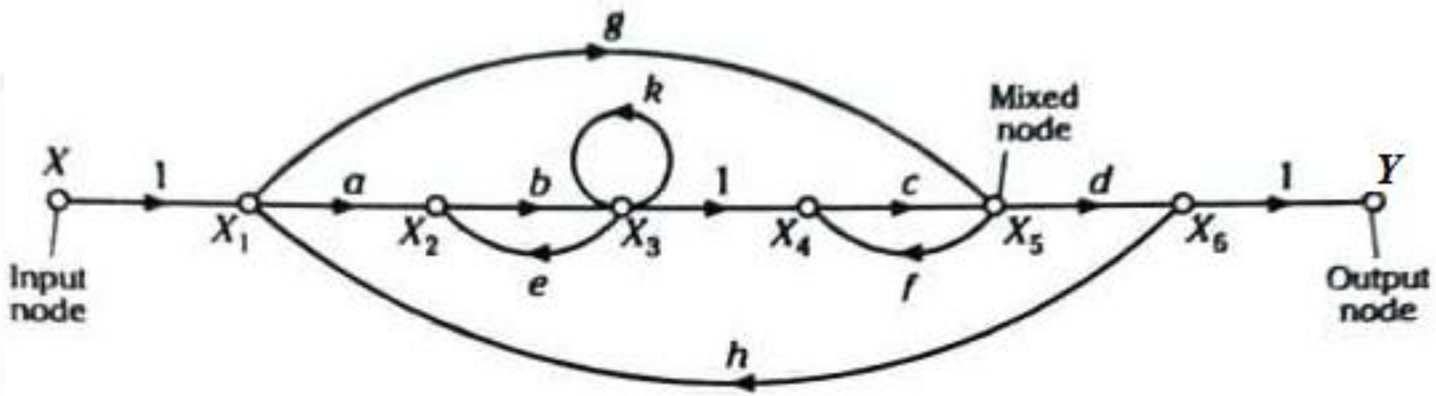
A signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable and each branch connected between two nodes acts as a signal multiplier. Each branch has a gain or transmittance. When the signal pass through a branch, it gets multiplied by the gain of the branch.

In a signal flow graph, the signal flows only one direction. The direction of signal flow is indicated by an arrow placed on the branch and the gain (multiplication factor) is indicated along the branch.



DEFINITIONS

Node: A node is a point representing a variable or signal.

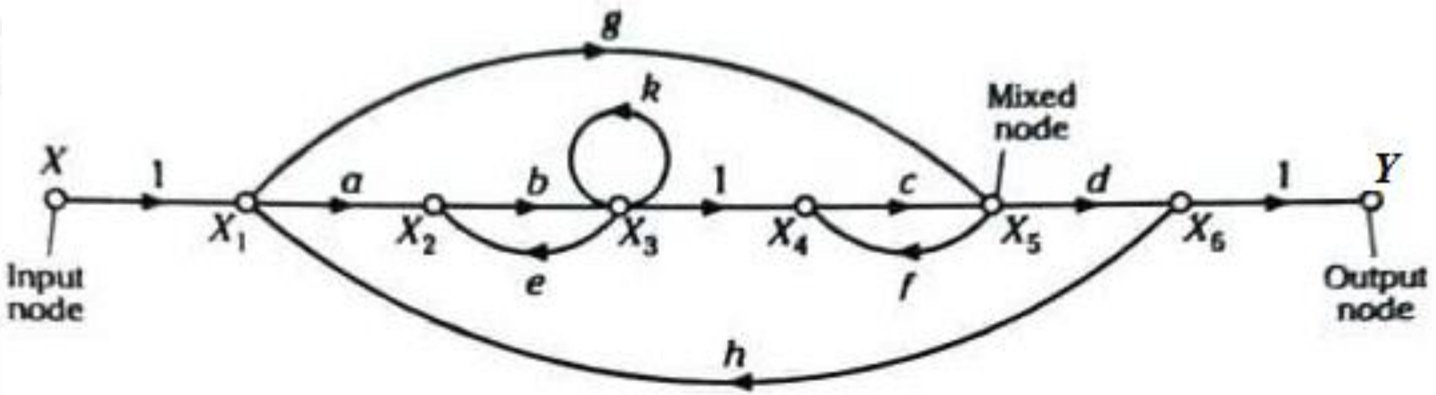


Branch: A branch is directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow and the gain of a branch is the transmittance

Transmittance : The gain acquired by the signal when it travels from one node to another called transmittance. The transmittance can be real or complex.

DEFINITIONS

Input node (Source) : It is a node that has only outgoing branches. Node Representing Variable X is Input Node

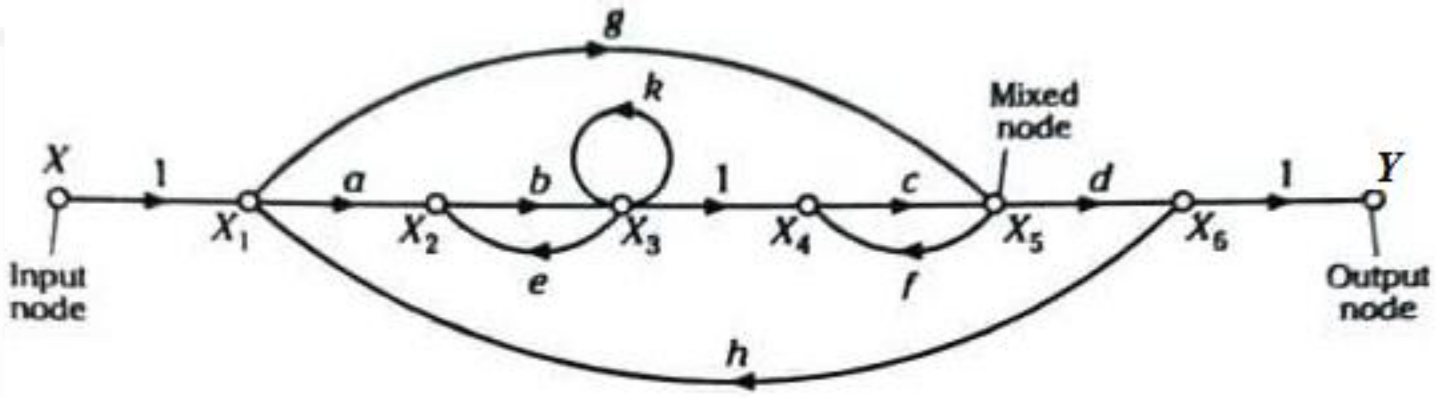


Output node (Sink) : It is a node that has only incoming branches, Node Representing Variable Y is output Node

Mixed node: It is a node that has both incoming and outgoing branches. for example X5 Node is a Mixed Node

DEFINITIONS

Path: Path is a traversal of connected branches in the direction of the branch arrows. The path Should not cross a node more than once.

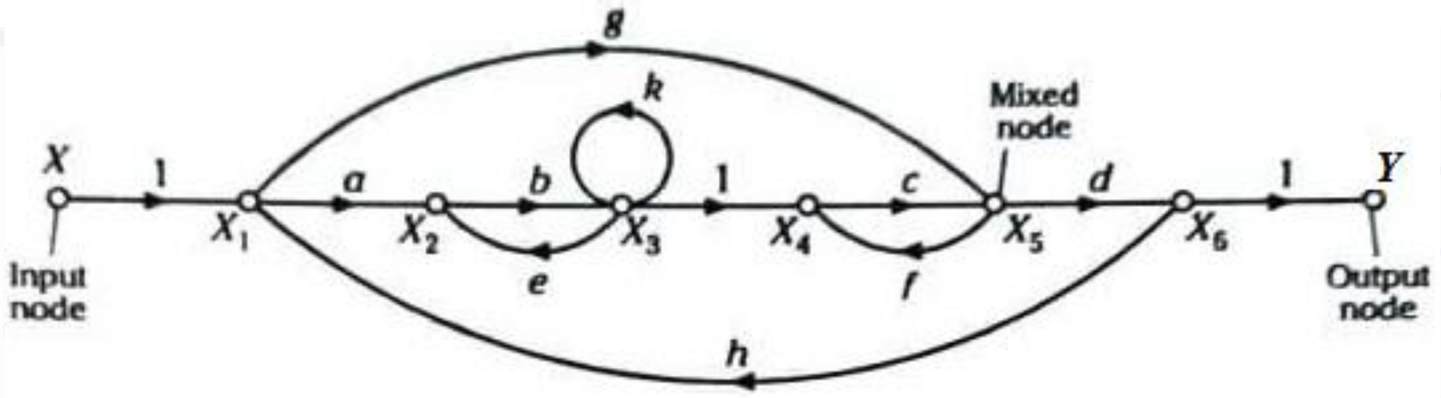


Open path : A open path starts at a node and ends at another node.

Closed path: Closed path starts and ends at same node.

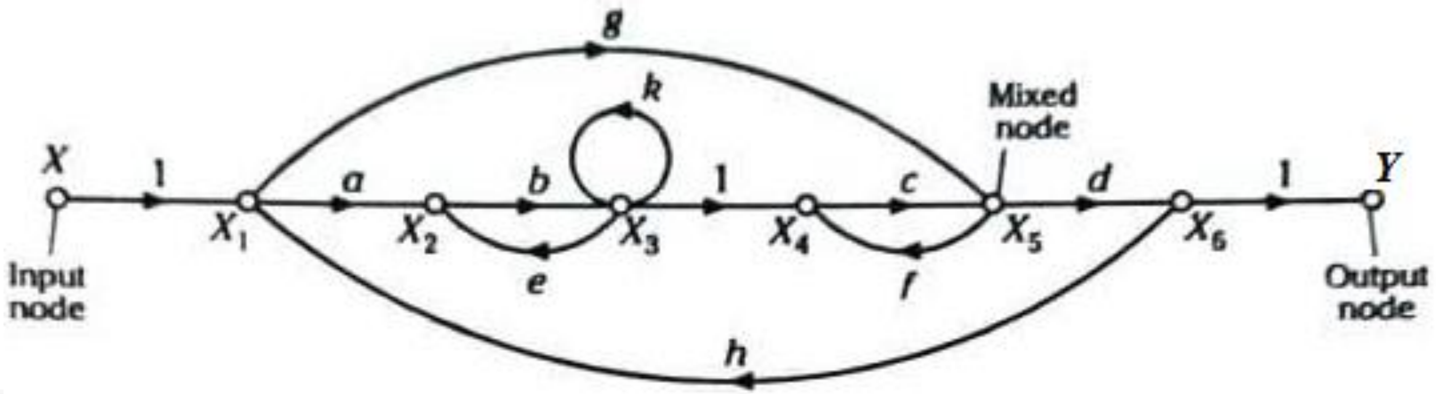
Forward path : It is a path from an input node to an output node that does not cross any node more than once.

Forward path gain It is the product of the branch transmittances (gains) of a forward path. There are two forward paths in the given example and the forward path gains are P1 = abcd, P2 = gd.



Individual loop : It is a closed path starting from a node and after passing through a certain part of a graph arrives at same node without crossing any node more than once.

Loop gain: It is the product of the branch transmittances (gains) of a loop. There are 5 Loops in given example and loop gain are L1= be, L2 = cf, L3 = abcdh, L4 = gdh, L5= k



Loop gain: It is the product of the branch transmittances (gains) of a loop. There are 5 Loops in given example and loop gain are L1= be, L2 = cf, L3 = abcdh, L4 = gdh, L5= k

Non-touching Loops: If the loops does not have a common node then they are said to be nontouching loops. L1 and L2 are Non touching loops and L1 and L4 are Non touching loops

Self-loop : The loop starting at node X3 and ending at same node represents self-loop. L5 is a self-loop.

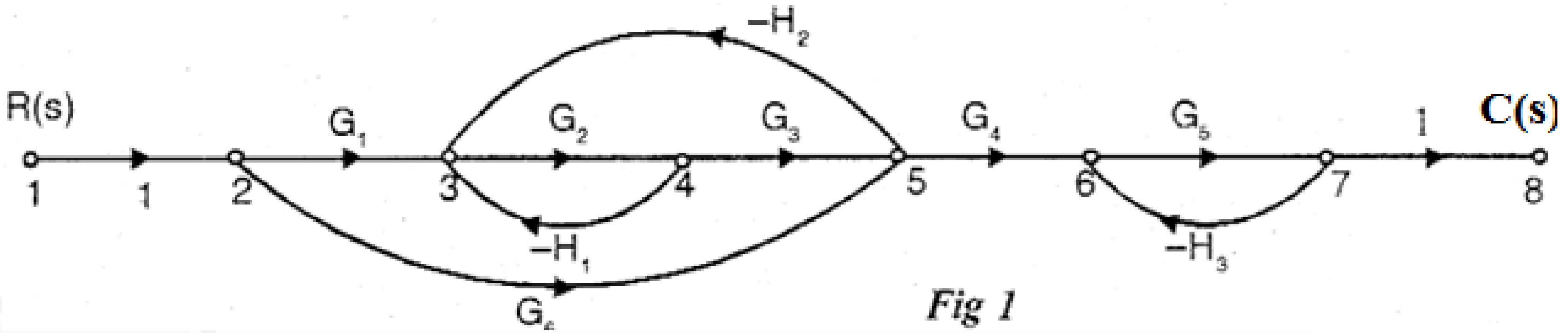


Fig 1

Forward Path Gains

Forward Path-1, Gain of forward path-1, P1 =G1G2G3G4G5

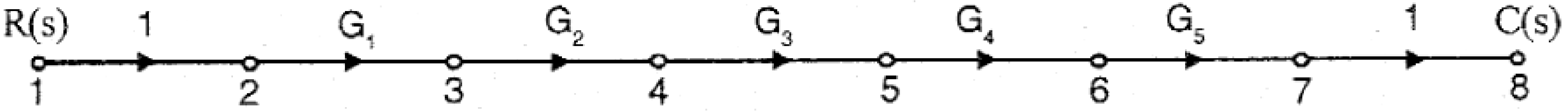


Fig 2 : Forward path-1.

Forward Path-2, Gain of forward path-2, P2 = G4G5G6

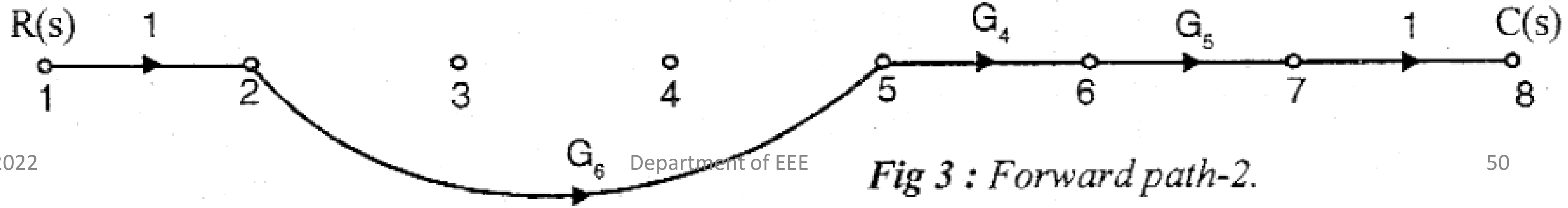


Fig 3 : Forward path-2.

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Individuals Loop

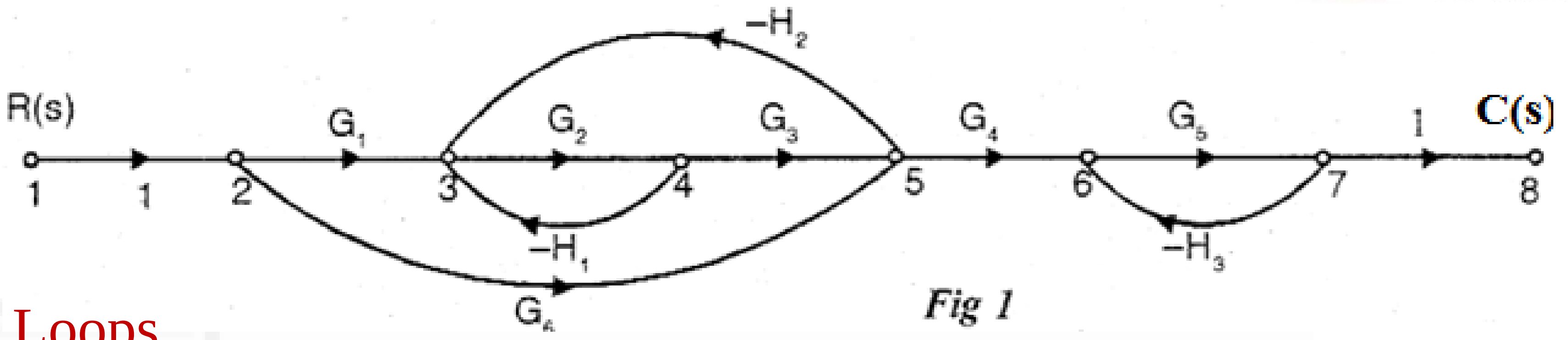


Fig 1

There are 3 Individuals Loops

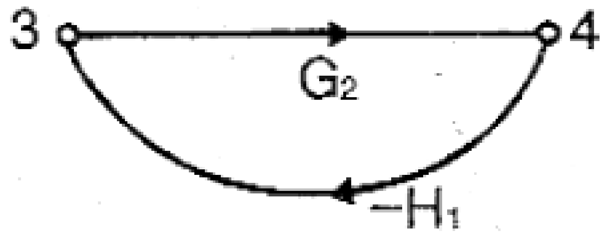


Fig 4 : Loop-1.

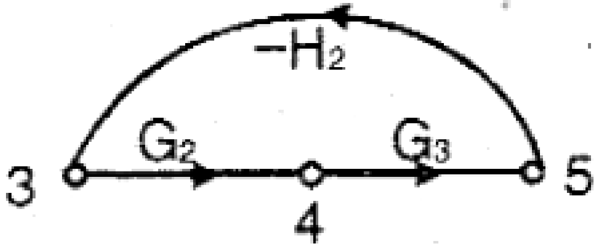


Fig 5 : Loop-2.

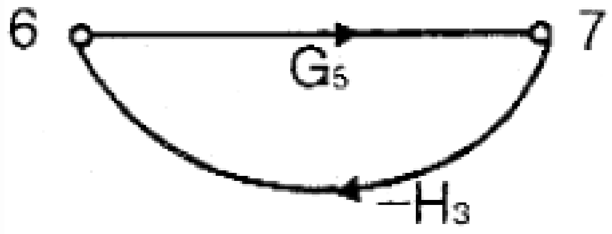


Fig 6 : Loop-3.

Loop Gain of Individuals Loop-1, $L1 = -G2H1$

Loop Gain of Individuals Loop-2, $L2 = -G2G3H2$

Loop Gain of Individuals Loop-3, $L3 = -G5H3$

Gain Products of Two Non-Individuals Loop

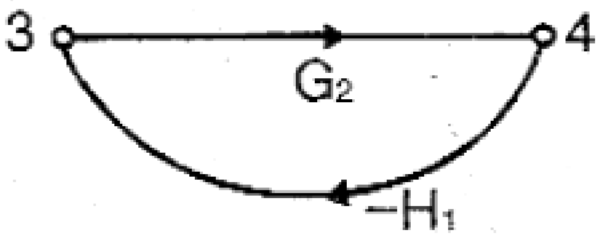


Fig 4 : Loop-1.

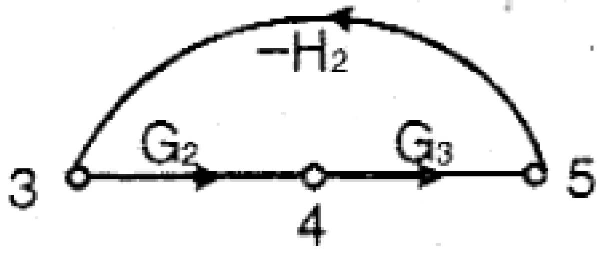


Fig 5 : Loop-2.

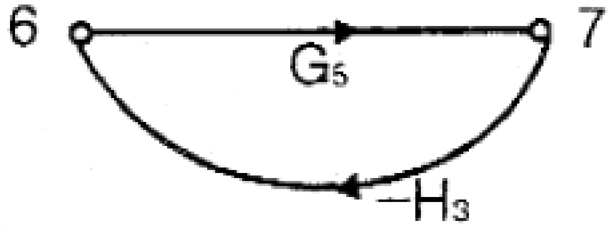


Fig 6 : Loop-3.

There are Two combinations of Two Non-touching loops

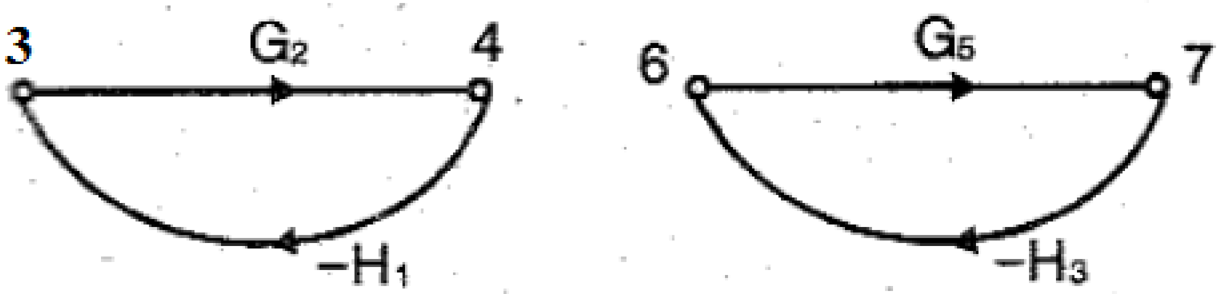


Fig.7 First combinations of 2 Non-touching loops

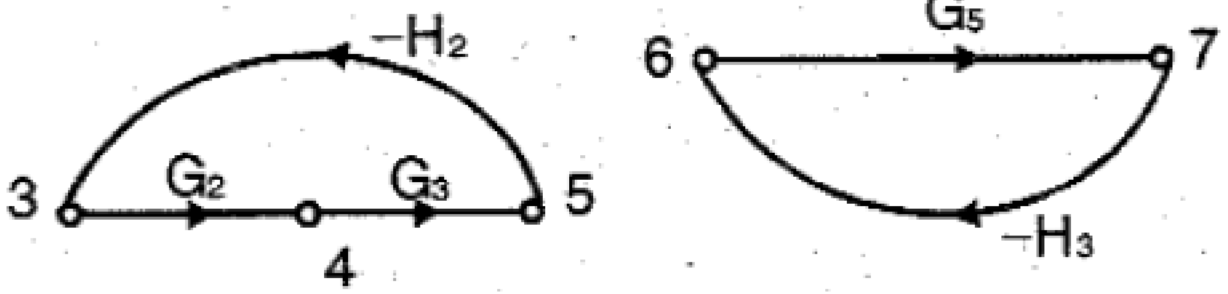


Fig.8 Second combinations of 2 Non-touching loops

Gain Product of 1st combinations of 2 Non-touching loops L13 = L1*L3 =(-G2H1)*(-G5H3) = G2G5H1H3

Gain Product of 2nd combinations of 2 Non-touching loops L23 = L2*L3 =(-G2G3H2)*(-G5H3) = G2G3G5H2H3

The basic properties of signal flow graph

(i) Signal flow graph is applicable to linear systems only.

(ii) A node in the signal flow graph represents the variable or signal.

(iii) A node adds the signals of all incoming branches and transmits the sum to all outgoing branches.

(iv) A mixed node which has both incoming and outgoing signals can be treated as an output node by adding an outgoing branch of unity transmittance

(v) A branch indicates functional dependence of one signal on the other

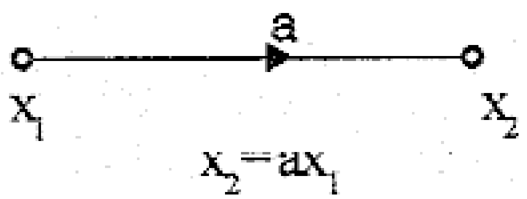
(vi) The signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.

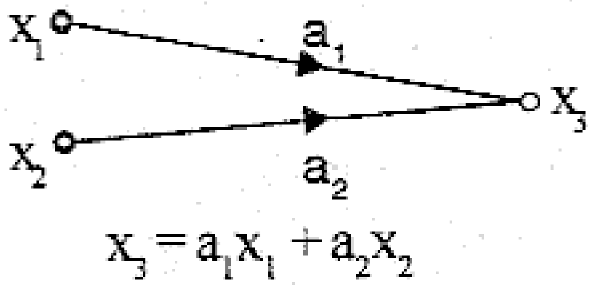
(vii) The signal flow graph of system is not unique. By rearranging the system equations different types of signal flow graphs can be drawn for a given system.

SIGNAL FLOW GRAPH ALGEBRA

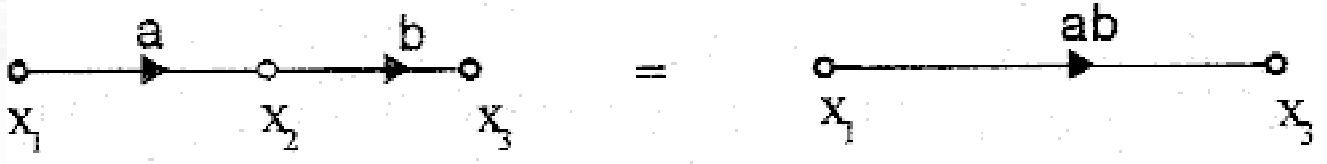
Signal flow graph for a system can be reduced to obtain the transfer function of the system using the following rules. The guideline in developing the rules for signal flow graph algebra is that the signal at a node is given by sum of all incoming signals.

Rule 1 : Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.

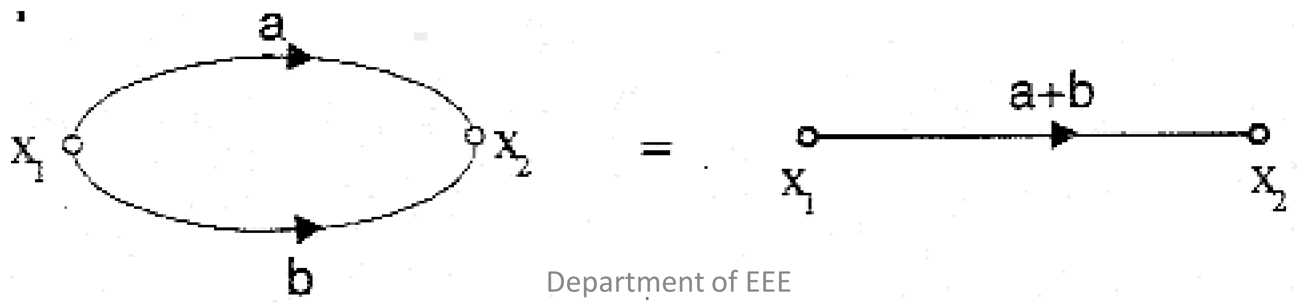




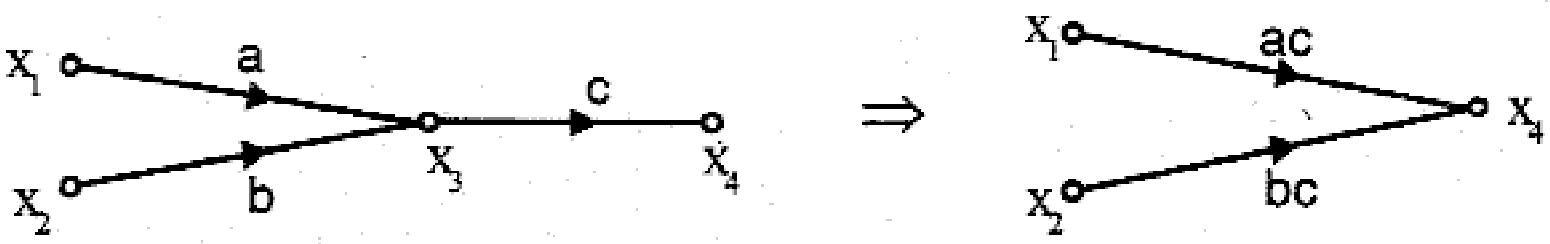
Rule 2 : Cascaded branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.



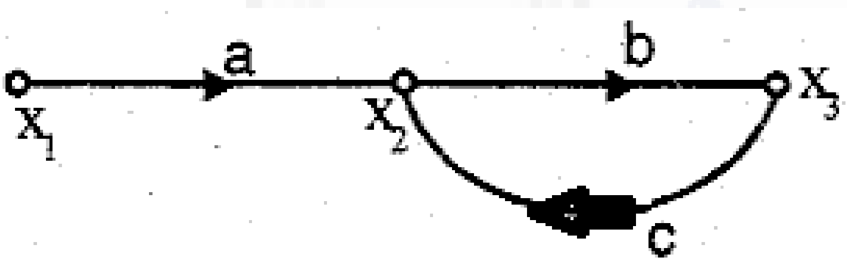
Rule 3 : Parallel branches may be represented by single branch whose transmittance is the sum of individual branch transmittances.



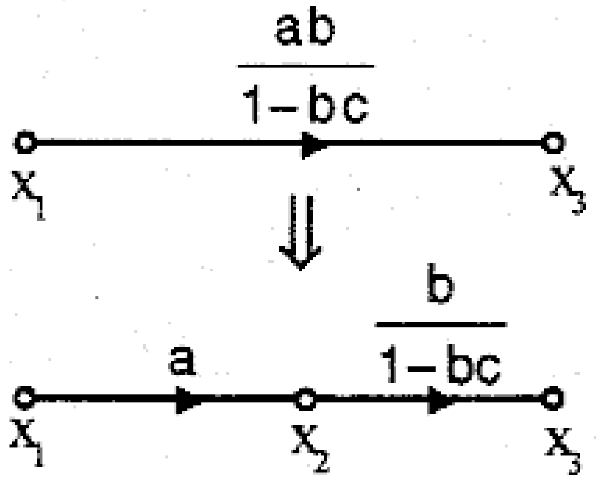
Rule 4: A mixed node can be eliminated by multiplying the transmittance of outgoing branch (from the mixed node) to the transmittance of all incoming branches to the mixed node



Rule 5 : A loop may be eliminated by writing equations at the input and output node and rearranging the equations to find the ratio of output to input. This ratio gives the gain resultant branch.



$\Rightarrow$



$$x_2 = ax_1 + cx_3 \quad ; \quad x_3 = bx_2$$

Put, $x_2 = ax_1 + cx_3$ in the equation for x_3.

$$\therefore\ x_3 = b\,(ax_1 + cx_3) \quad \Rightarrow \quad x_3 = abx_1 + bcx_3 \quad \Rightarrow \quad x_3 - bc\,x_3 = ab\,x_1 \quad \Rightarrow \quad x_3(1 - bc) = ab\,x_1$$

$$\therefore\ \frac{x_3}{x_1} = \frac{ab}{1 - bc}$$

S.J.Mason bas developed a simple procedure to determine the transfer function of the system represented as a signal flow graph. He has developed a formula called by his name **Mason's gain formula** which can be directly used to find the transfer function of the system.

The Mason's gain formula is used to determine the transfer function of the system from the signal flow graph of the system

Let, R(s) = Input to the system, C(s) = Output of the system C(s)

Transfer function of the system, T(s) = C(s)/R(s)

Mason's gain formula states the overall gain of the system (transfer function] as follows,

Overall gain, $$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Determinant of SFG Δ = 1 – (Sum of individual loop gains)

$+$ (Sum of gain products of all possible combinations of two non - touching loops)

$-$ (Sum of gain products of all possible combinations of three non - touching loops)

$+$ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

$$\text{Overall T.F.} = \frac{\Sigma T_K \Delta_K}{\Delta}$$

where K = Number of forward paths

T_K = Gain of K^{th} forward path

Δ = System determinant to be calculated as :

Δ = 1–[Σ all individual feedback loop gains [including self loops]] + [Σ Gain × Gain product of all possible combinations of two non-touching loops] – [Σ Gain × Gain × Gain product of combinations of three non touching loops] +....

Δ_K = Value of above Δ by eliminating all loop gains and associated products which are touching to the K^{th} forward path.

CONSTRUCTING SIGNAL FLOW GRAPH FOR CONTROL SYSTEMS

A control system can be represented diagrammatically by signal flow graph. The differential equations governing the system are used to construct the signal flow graph

1. Take Laplace transform of the differential equations governing the system in order to convert them to algebraic equations in s-domain.

2. The constants and variables of the s-domain equations are identified.

3. From the working knowledge of the system, the variables are identified as input, output and intermediate variables.

4. For each variable a node is assigned in signal flow graph and constants are assigned as the gain or transmittance of the branches connecting the nodes.

5. For each equation a signal flow graph is drawn and then they are interconnected to give overall signal flow graph of the system.

PROCEDURE FOR CONVERTING BLOCK DIAGRAM TO SIGNAL FLOW GRAPH

1. Assume nodes at input, output, at every summing point, at every branch point and in between cascaded blocks.

2. Draw the nodes separately as small circles and number the circles in the order 1, 2, 3, 4, etc.

3. From the block diagram find the gain between each node in the main forward path and connect all the corresponding circles by straight line and mark the gain between the nodes.

4. Draw the feed forward paths between various nodes and mark the gain of feed forward path along with sign

5. Draw the feedback paths between various nodes and mark the gain of feedback paths along with sign.

Find the overall transfer function of the system whose signal flow graph is shown in figure

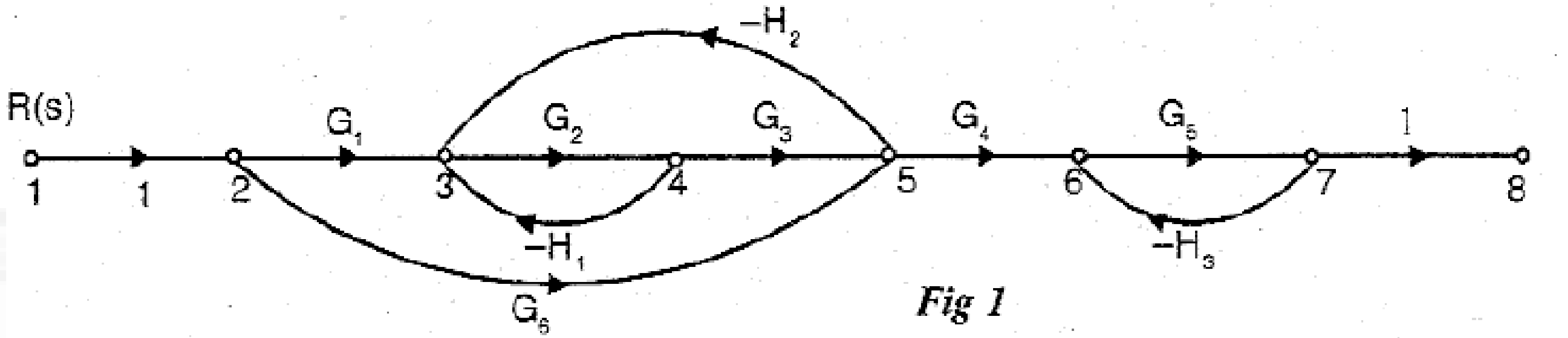


Fig 1

Forward Path Gains

Forward Path-1, Gain of forward path-1, **P1 =G1G2G3G4G5**

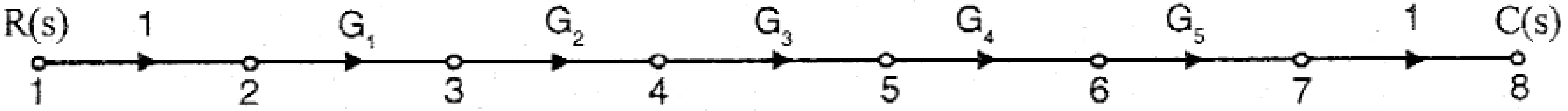


Fig 2 : Forward path-1.

Forward Path-2, Gain of forward path-2, **P2 = G4G5G6**

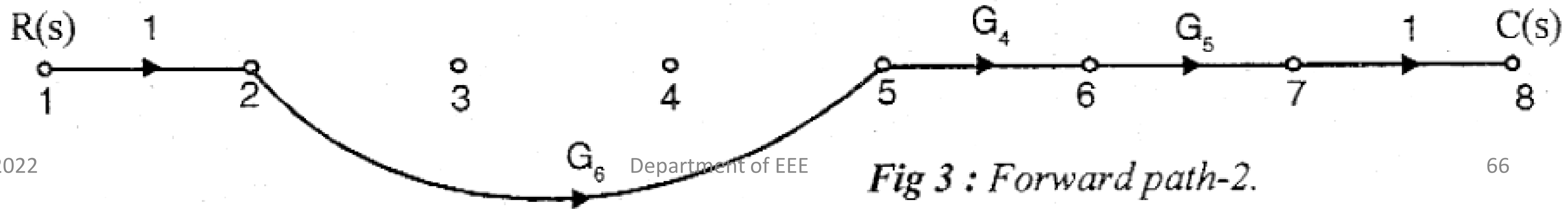


Fig 3 : Forward path-2.

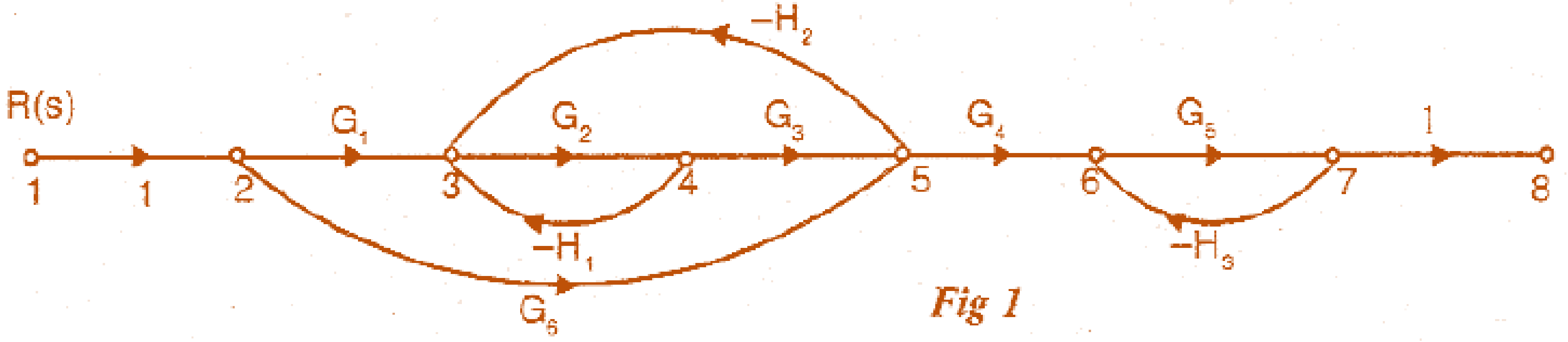


Fig 1

Individuals Loop

There are 3 Individuals Loops

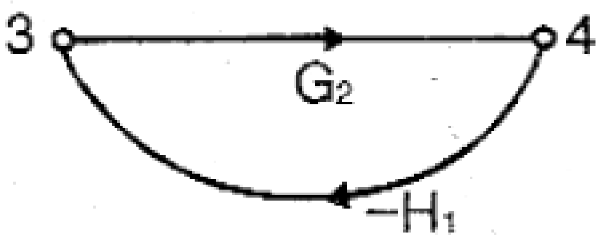


Fig 4 : Loop-1.

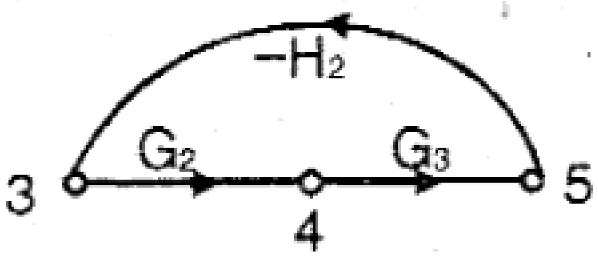
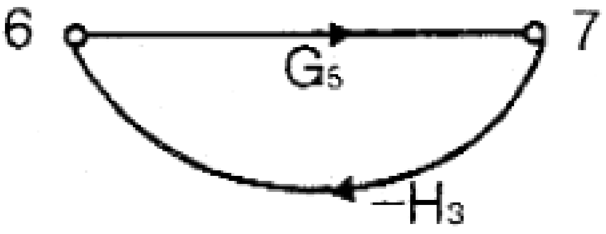


Fig 5 : Loop-2.



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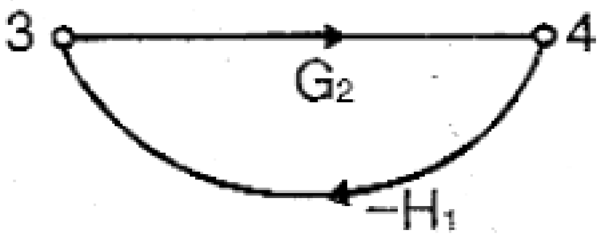
Loop Gain of Individuals Loop-2, **L2 = -G2G3H2**

Loop Gain of Individuals Loop-3, **L3 = -G5H3**

Gain Products of Two Non-Individuals Loop



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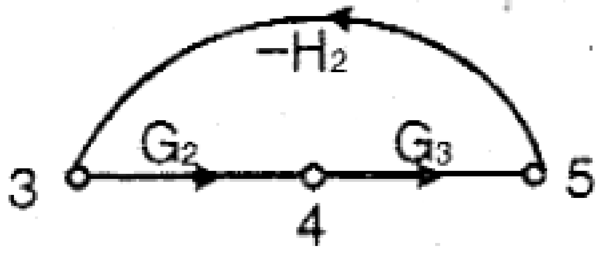

Fig 5 : Loop-2.

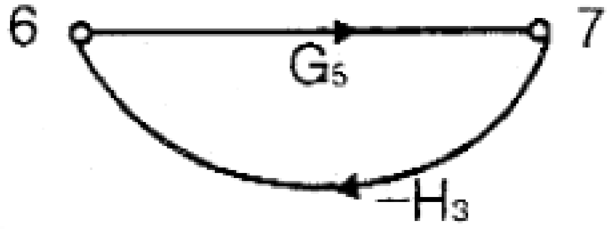

Fig 6 : Loop-3.

There are Two combinations of Two Non-touching loops

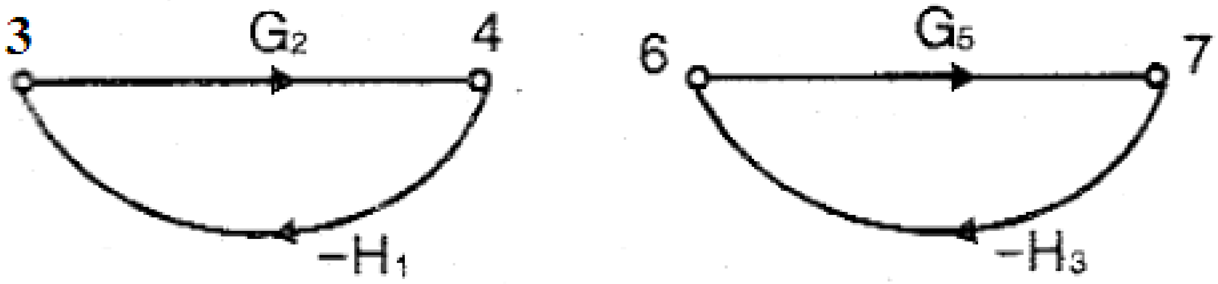

Fig.7 First combinations of 2 Non-touching loops

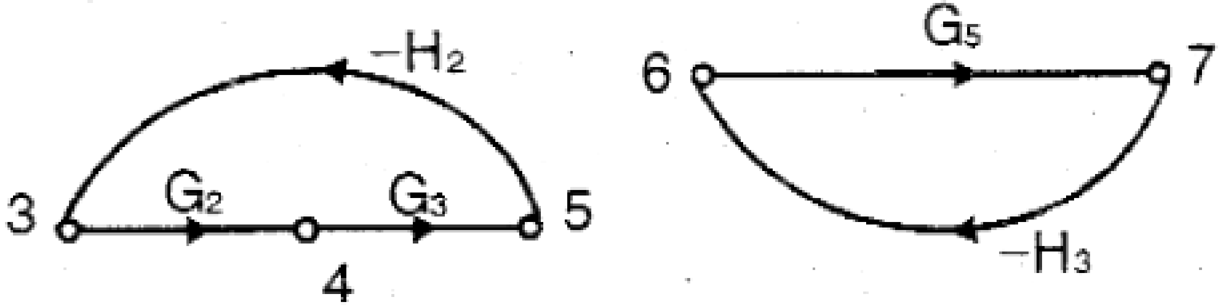

Fig.8 Second combinations of 2 Non-touching loops

Gain Product of 1st combinations of 2 Non-touching loops **L13 = L1*L3** =(-G2H1)*(-G5H3) = G2G5H1H3

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Determinant of SFG Δ = 1 – (Sum of individual loop gains)

$+$ (Sum of gain products of all possible combinations of two non - touching loops)

$-$ (Sum of gain products of all possible combinations of three non - touching loops)

$+$ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Overall gain, $T = \frac{1}{\Delta}\sum_{K} P_K \Delta_K$ Number of Forward path is 2 and so K =2

$$T = \frac{1}{\Delta}\sum_{K} P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

Calculation of **Determinant of SFG** Δ and ΔK

$$\Delta = 1 - (L1+L2+L3) + (L13+L23)$$

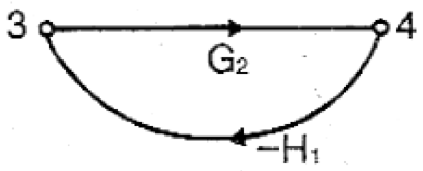
$$= 1 - (-G2H1 - G2G3H2 - G5H3) + (G2G5H1H3 + G2G3G5H2H3)$$

$$= 1 + G2H1 + G2G3H2 + G5H3) + G2G5H1H3 + G2G3G5H2H3$$

Number of Forward path is 2 and so K =2

$\Delta 1 = 1$, Since **there is no part of graph which is not touching with 1st forward path**

Fig 2 : Forward path-1.



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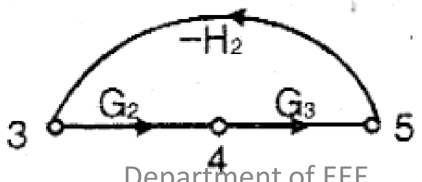

Fig 5 : Loop-2.

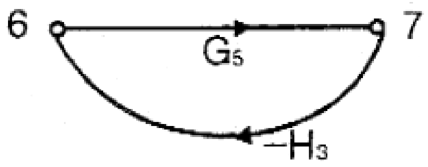

Fig 6 : Loop-3.

Calculation of **Determinant of SFG** Δ and ΔK

Number of Forward path is 2 and so K =2

$\Delta 2 = 1 - L1 = \mathbf{1-(-G2H1)}$

Since there is part of graph is which is not touching with 2nd forward path

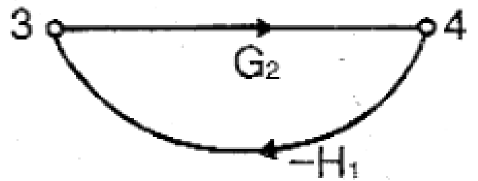

Fig 4 : Loop-1.

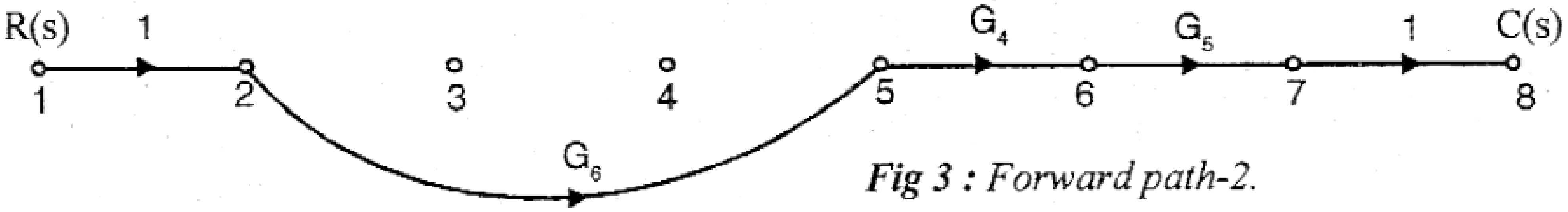

Fig 3 : Forward path-2.