Nyquist Stability Analysis

The stability criterion for application in the frequency domain was developed by H. Nyquist in 1932 and is based upon theory of complex variables due to Cauchy's. This theorem is concerned with the mapping of contours in the complex s-plane.

Two methods of designing a control system are design using root locus and design using bode plot.

In design using root locus, the system is designed to satisfy the specified time domain specifications.

In design using bode plot, the system is designed to satisfy the specified frequency domain specifications.

Stability of a system can be accessed by determining the gain margin and phase margin. These gain and phase margins can be found by drawing the frequency response plots.

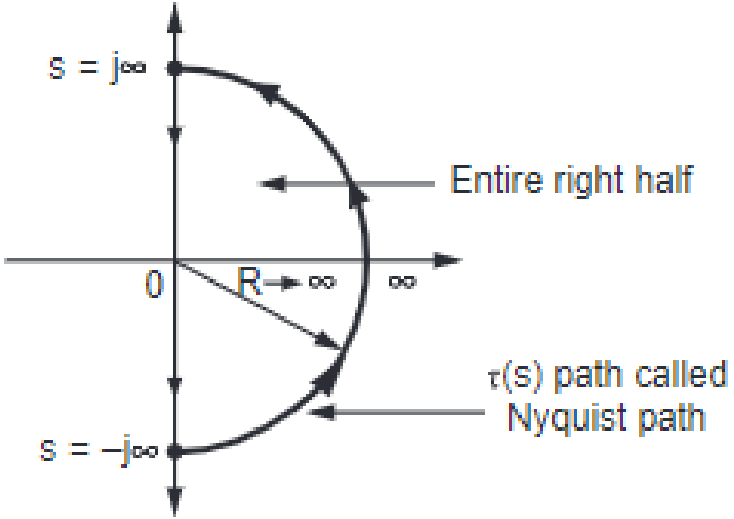
Polar plots, Bode plots, Nyquist plots are the examples of frequency response plots

Bode plots show the frequency response of a system. There are two Bode plots one for gain (or magnitude) and one for phase.

The Nyquist plot combines gain and phase into one plot in the complex plane. It is drawn by plotting the complex gain $G(j\omega)$ for all frequencies ω. That is, the plot is a curve in the plane parameterized by ω.

Application of Mapping Theorem in Nyquist Stability Analysis

For analyzing the stability of control system, we consider a closed contour (Nyquist Path or Nyquist Contour) in the s-plane enclosing the entire right half s-plane. The contour consists of entire jω axis form $\omega = -\infty$ to $\omega = +\infty$ and semi circular path of infinite radius as shown in Fig. (a). The radius of the semicircle is treated as Nyquist Encirclement

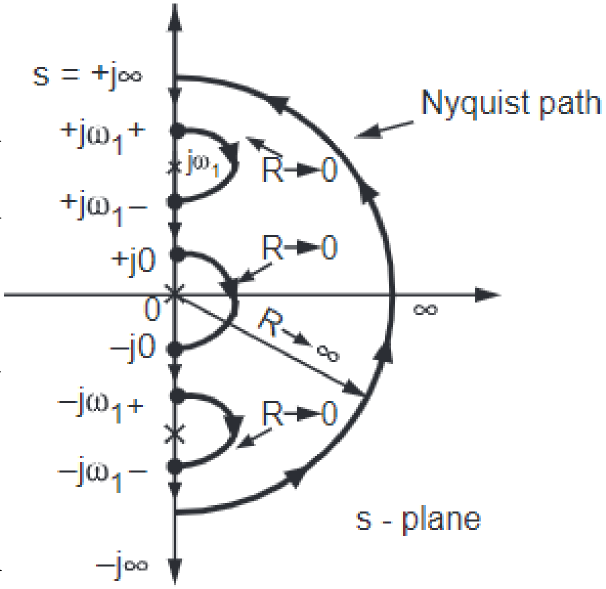


The Nyquist method is concerned with mapping of F(s)=1+G(s)H(s), and finding the number of encirclement about the critical point i.e. , the origin of F(s) Plane. It is rather difficult to plot the function F(s)=1+G(s)H(s), whereas the open loop transfer function G(s)H(s) is readily available and can be directly mapped from s-plane to G(s)H(s)-plane. The plot of F(s)=1+G(s)H(s) and G(s)H(s) is shown in Fig. (a) it can be seen that the vector 1+G(s)H(s) is the sum of unit vector and the vector G(s)H(s).

1+G(s)H(s) is identical to the vector drawn from (-1+j0) point to the terminal point of vector G(s) H(s) as shown in Fig. Therefore the critical point origin on 1+G(s)H(s) Plane is same as critical point (-1+j0) on G(s)H(s) plane. **Therefore the stability of a closed loop system is investigated by examining encirclement of the (-1,+j0) point by the G(S)H(S) contour**

This Nyquist path obviously encloses all the poles and zeros lying on right of s-plane. This closed contour in the s-plane is then mapped onto G(s)H(s)-Plane as a closed curve and the total number of encirclement about the critical point (-1,+ j0) is determined.

If there is a pole at the origin or on the imaginary axis(the jω-axis), mapping about this point becomes indeterminate and in such cases, a detour is taken around these point and the mapping is done as shown in Fig. (b) (In such a case Nyquist path is modified in such a way to bypass these poles by selecting semicircles of radius tending to zero around them).



Generalised Nyquist Contour

Once the number of encirclement is determined, the number of closed loop zeros can be obtained from Nyquist stability criterion

Nyquist Encirclement

A point is said to be encircled by a contour if it is found inside the contour.

Nyquist Mapping

The process by which a point in s-plane is transformed into a point in F(s) plane is called mapping and F(s) is called mapping function.

Nyquist stability Criterion

Nyquist criterion expressed Mathematically a Z = P +N

Z = Number of Zeros of 1+G(s)H(s) on the Right half of S-plane

N= Number of Encirclement of the critical point (-1,+ j0) point of F-plane by Nyquist plot

Clockwise encirclement are taken positive and anticlockwise encirclement are as negative

P = Number of Poles of G(s)H(s) on the right half of the S-Plane

For closed loop stability, No Zeros of 1+G(s)H(s) must be in the Right half of S-plane, i.e. Z=0 for stability, so Nyquist criteria obtained by substituting Z=0 in N=Z-P i.e N = -P

i.e. $$\boxed{N = -P}$$

Nyquist stability criterion states that for absolute stability of the system, the number of encirclements of new origin of F-plane by Nyquist plot must be equal to number for poles of 1+G(s)H(s) i.e. poles of G(s)H(s) which are in the right half of s-plane and in clockwise direction.

e.g. If $G(s)H(s) = \frac{10}{s(s+1)}$

Then P = No. of poles of G(s)H(s) which are located in right half of s-plane.

= 0 as there is no pole of G(s)H(s) in right half of s-plane

$\therefore$ For stability, $N = -P = 0$

The Nyquist plot obtained by mapping Nyquist path from S-plane to F-plane should not encircle origin of F-plane.

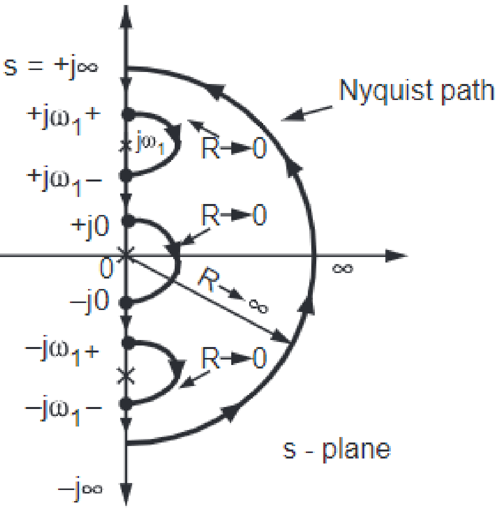
Note : Now for ease of mapping Nyquist path from s-plane to F -plane, instead of considering mapping function as 1+ G(s)H(s), it is considered as G(s)H(s) only

So in all the problems solved hereafter, N is the number of encirclements of a critical point -1+ j0 and not the encirclements of origin as mapping function used is G(s)H(s) and not the function 1 + G(s)H(s).

Generalized Nyquist Path and its Mapping

Depending upon the situation of poles of G(s)H(s) Nyquist path should be selected.

F(s) has two poles on imaginary axis at $\pm j\omega$ while one pole at origin. Then Nyquist path should be selected as shown in figure



Generalised Nyquist Contour

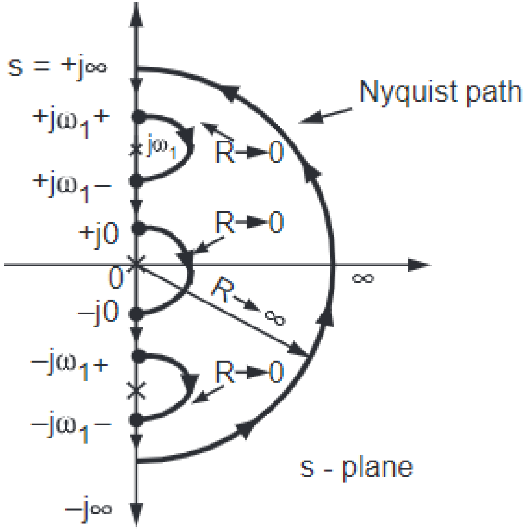
The points by which path is modified are :

- $+j\omega_1^+ \rightarrow$ A point just above $+j\omega_1$, very very close to $+j\omega_1$.
- $+j\omega_1^- \rightarrow$ A point which is very very close to $+j\omega_1$, but just below it.
- $-j\omega_1^+ \rightarrow$ A point which is very very close to $-j\omega_1$, but just above it.
- $-j\omega_1^- \rightarrow$ A point which is very very close to $-j\omega_1$ but just below it.
- $+j0 \rightarrow$ A point which is very very close to origin but just above it on positive imaginary axis hence denoted as $+j0$
- $-j0 \rightarrow$ A point which is very very close to origin but just below it on negative imaginary axis hence denoted as $-j0$.

The various sections of Nyquist path are,

Section	Start	End	Comment
I	$s = +j\infty$	$s = +j\omega_1^+$	Along the imaginary axis.
II	$s = +j\omega_1^+$	$s = +j\omega_1^-$	Along a semicircle of radius tending to zero.
III	$s = +j\omega_1^-$	$s = +j0$	Along the imaginary axis.
IV	$s = +j0$	$s = -j0$	Along semicircle of radius tending to zero.
V	$s = -j0$	$s = -j\omega_1^+$	Along the imaginary axis.
VI	$s = -j\omega_1^+$	$s = -j\omega_1^-$	Along semicircle of radius tending to zero.
VII	$s = -j\omega_1^-$	$s = -j\infty$	Along the imaginary axis.
VIII	$s = -j\infty$	$s = +j\infty$	Along the semicircle of $R \rightarrow \infty$ encircling entire right half.

Generalized Nyquist Path and its Mapping



Generalised Nyquist Contour

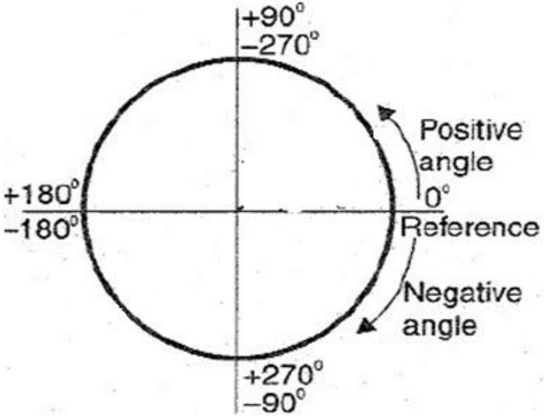
Key Point : The section V, VI and VII are mirror images of section III, II and I respectively. Hence their mapping in F-plane will be also mirror images *about real axis.*

The various sections of Nyquist path are,

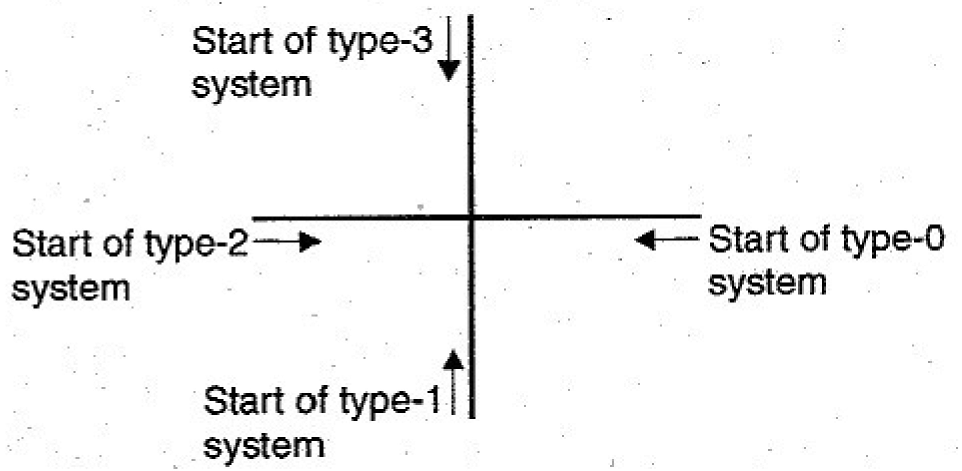
Section	Start	End	Comment
I	$s = +j\infty$	$s = +j\omega_1^+$	Along the imaginary axis.
II	$s = +j\omega_1^+$	$s = +j\omega_1^-$	Along a semicircle of radius tending to zero.
III	$s = +j\omega_1^-$	$s = +j0$	Along the imaginary axis.
IV	$s = +j0$	$s = -j0$	Along semicircle of radius tending to zero.
V	$s = -j0$	$s = -j\omega_1^+$	Along the imaginary axis.
VI	$s = -j\omega_1^+$	$s = -j\omega_1^-$	Along semicircle of radius tending to zero.
VII	$s = -j\omega_1^-$	$s = -j\infty$	Along the imaginary axis.
VIII	$s = -j\infty$	$s = +j\infty$	Along the semicircle of $R \to \infty$ encircling entire right half.

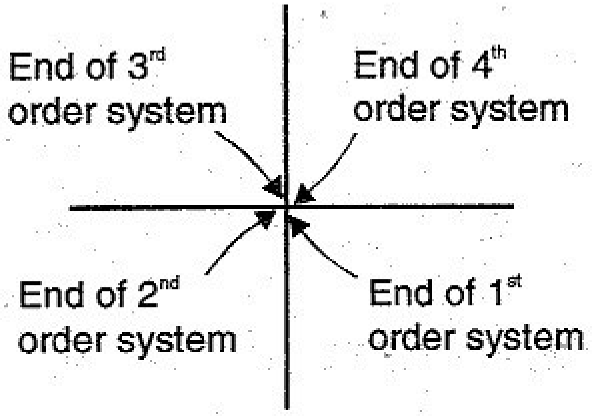
Now Mapping of these sections in S-Plane can be achieved by drawing the polar plots for various sections and joing one after the other, the mapped locus called Nyquist Plot,

From the encirclement of the critical point (-1,+ j0) by Nyquist Plot, stability of the system can be predicted count the number of encirclements of the critical point (-1,+ j0) and check whether it satisfies N = -P



Polar Graph





Start of Polar Plot

End of Polar Plot

Notes on general polar plots

Polar plot is a plot of magnitude of G(jω) versus the phase of G(jω) in polar coordinates

For physical realizable systems, the order of the denominator is larger than or equal to that of the numerator of the transfer function

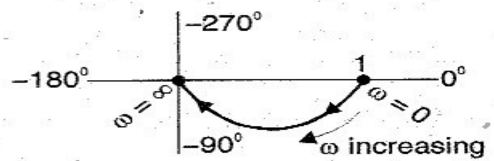
Type 0 systems: finite starting point on the positive real axis, the terminal point is the origin tangent to one of the axis ˆ

Type 1 Systems : starting at infinity asymptotically parallel to -ve imaginary axis, Also the curve converges to zero tangent to one of the axis

Type 2 systems : the starting magnitude is infinity and asymptotic to -180, Also the curve converges to zero tangent to one of the axis

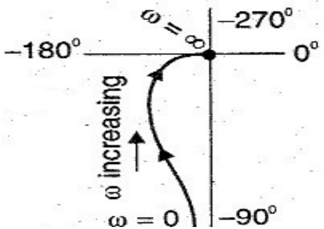
Type : 0, Order : 1

$$G(s) = \frac{1}{1+sT}$$



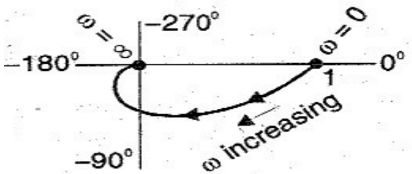
Type : 1, Order : 2

$$G(s) = \frac{1}{s(1+sT)}$$



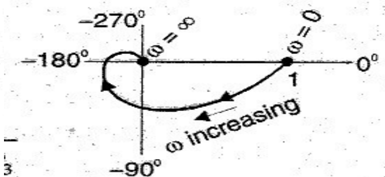
Type : 0, Order : 2

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$



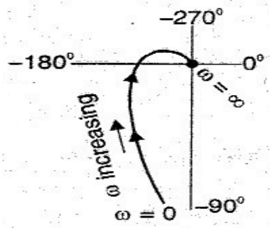
Type : 0, Order : 3

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$$



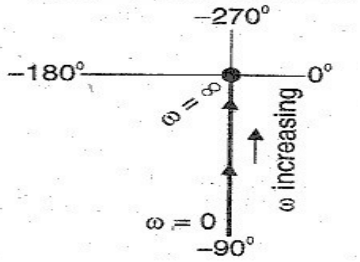
Type : 1, Order : 3

$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$



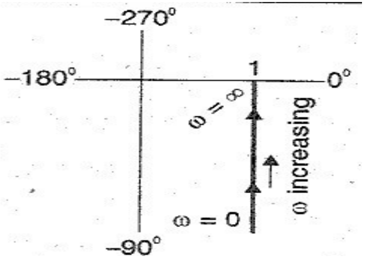
Type : 1, Order : 1

$$G(s) = \frac{1}{s}$$



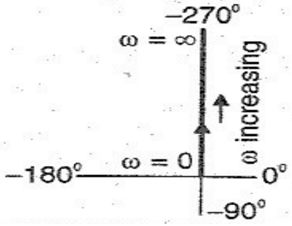
$$G(s) = \frac{1+sT}{sT}$$

$$G(j\omega) = \frac{1+j\omega T}{j\omega T}$$



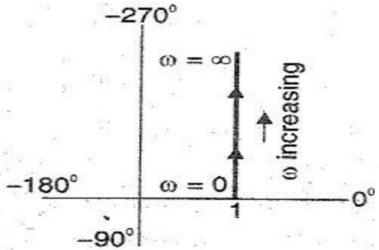
$$G(s) = s$$

$$G(j\omega) = j\omega$$



$$G(s) = 1+sT$$

$$G(j\omega) = 1+j\omega T$$



Step 1 : Count how many number of poles of G(s)H(s) are in the right half of s-plane i.e. with positive real part. This is the value of P.

Step 2 : Decide the stability criterion as $N = -P$ i.e. how many times Nyquist plot should encircle '$-1 + j0$' point for absolute stability.

Step 3 : Select Nyquist path as per the function G(s)H(s).

Step 4 : Analyse the sections as starting point and terminating point of plot.
Last section analysis not required.

Step 5 : Mathematically find out ω_{pc} and intersection of Nyquist plot with negative real axis by rationalizing $G(j\omega)H(j\omega)$.

Step 6 : With the knowledge of step 4 and 5, sketch the Nyquist plot.

Step 7 : Count the number of encirclements N of $-1 + j0$ by Nyquist plot. If this matches with the criterion decided in step 2 system is stable, otherwise unstable.

$$\text{G.M.} = \frac{1}{|OQ|} \quad \text{where}$$

Q = Intersection point of Nyquist plot with negative real axis obtained in step 5.

$$\text{i.e. G.M.} = 20 \text{Log}_{10} \frac{1}{|OQ|} \text{ dB}$$

Draw Nyquist Plot, Investigate the stability of a negative feedback control system whose open loop transfer function is given by $G(s)\,H(s) = 100 / (S+1)\,(S+2)\,(S+3)$

Step 1 Count number of poles of G(s)H(s) are in right half of S-Plane

i.e. with positive Real Part, This is the value of 'P'

P = 0, As No Open Loop poles in the right half of S-Plane

Step 2 Decide the stability criterion as **N = -P**

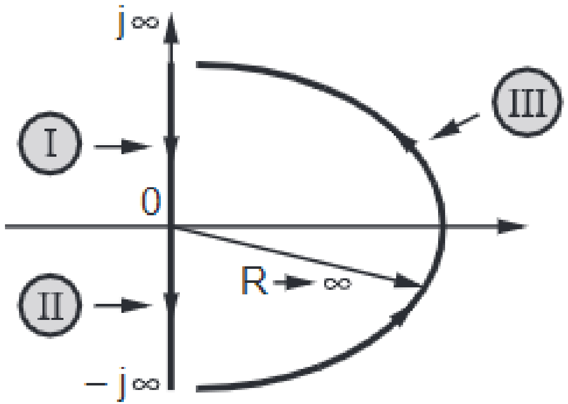
i.e. How many times Nyquist plot should encircle " **-1+j0** " point for absolute stability

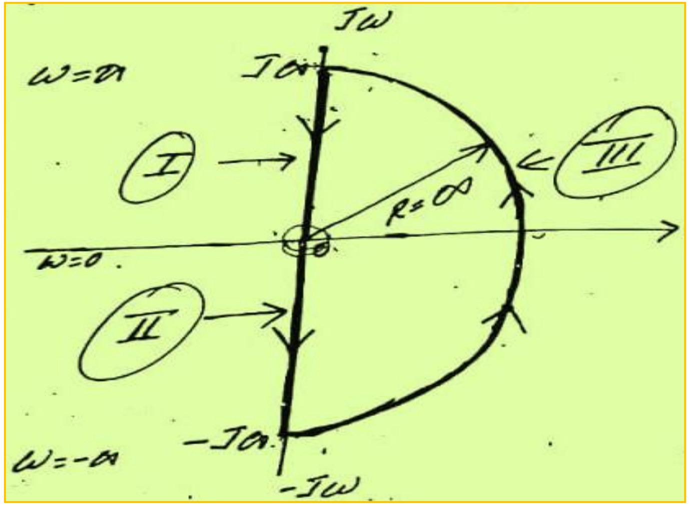
Here P = 0, No Encirclement of critical Point " -1+j0 " i.e. (-1, 0) by Nyquist plot for stability

Step 3 Select Nyquist Path as per the function G(s)H(s). Identify the various segments on the contour with reference to Nyquist path

Select Nyquist contour which encloses the entire right half s-plane except singular points. The Nyquist contour encloses all the right half s-plane poles and zeros of G(s)H(s). The poles on imaginary axis are singular points and so they are avoided by taking a detour around it.

Here Select the proper Nyquist contour – Include the entire right half of s-plane by drawing a semicircle of radius R, with R tends to infinity





Step 4 Analyze the Section I, II and III as a starting point and terminating point. Last section III analysis is not required for closed loop stability

Express the magnitude and phase equations in terms of ω and Estimate the magnitude and phase for different values of ω

$$G(j\omega)H(j\omega) = \frac{100}{(1+j\omega)(2+j\omega)(3+j\omega)}$$

Let S=jω in G(s)H(s) & calculate magnitude and phase $M\angle\emptyset = |G(j\omega)H(j\omega)| * \angle G(j\omega)H(j\omega)$

Step 4 $\quad G(s)H(s) = \frac{100}{(s+1)(s+2)(s+3)}$

Put $s \to j\omega$

$$G(j\omega)H(j\omega) = \frac{100+j0}{(j\omega+1)(j\omega+2)(j\omega+3)}$$

$$|G(j\omega)H(j\omega)| = \frac{\sqrt{100^2+0^2}}{\sqrt{1^2+\omega^2} \times \sqrt{2^2+\omega^2} \times \sqrt{3^2+\omega^2}}$$

$\angle G(j\omega)H(j\omega)$

$$\angle G(j\omega)H(j\omega) = \frac{\tan^{-1}\left(\frac{0}{100}\right)}{\tan^{-1}\left(\frac{\omega}{1}\right) \times \tan^{-1}\left(\frac{\omega}{2}\right) \times \tan^{-1}\left(\frac{\omega}{3}\right)}$$

$$\tan^{-1}(0) = 0^\circ$$

$$\tan^{-1}(\infty) = 90^\circ$$

$$\phi = \tan^{-1}\left(\frac{0}{100}\right) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right)$$

$$\phi = 0^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right)$$

$M \angle \varnothing = \left| G(j\omega)H(j\omega) \right| \angle G(j\omega)H(j\omega)$

$$M = \frac{100}{\sqrt{1^2+\omega^2} \times \sqrt{2^2+\omega^2} \times \sqrt{3^2+\omega^2}}$$

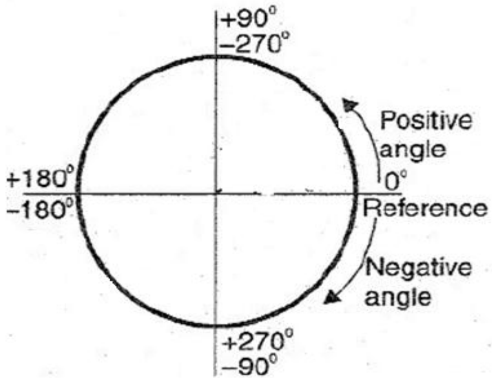
$$\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right)$$

Analyze the Section I, II and III as a starting point and terminating point

Section I	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (a)	S= j∞	ω = ∞	0	-270°
At Terminating point (b)	S= 0	ω = 0	16.6	0°

Section I : $s = +j\infty$ to $s = 0$

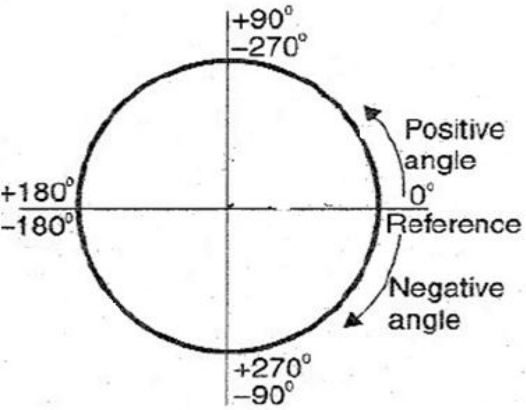
Starting point	$\omega \to \infty$	$0\angle -270^\circ$	$0^\circ - (-270^\circ) = +270^\circ$ Anticlockwise
Terminating point	$\omega \to 0$	$16.667\angle 0^\circ$	



Terminating point phase angle - Starting point phase angle = Positive value, then sketch polar plot of **Section I** in Anticlockwise rotation

Terminating point phase angle - Starting point phase angle = Negative value, then sketch polar plot of **Section I** in clockwise rotation

- Mapping of these sections in F-Plane can be achieved by drawing the polar plots for various sections and joing one after the other, the mapped locus called Nyquist Plot,

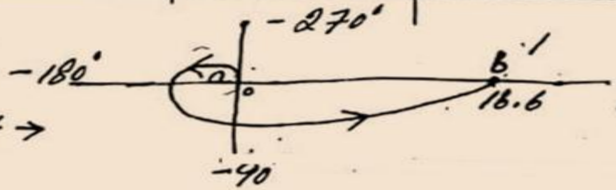


analyse Section (I) $s = +j\infty$ to $s = 0$

Starting point	$\omega \to \infty$	$M\angle\phi = 0\angle -270^\circ$
Terminating point	$\omega \to 0$	$M\angle\phi = 16.6\angle 0^\circ$

$0^\circ - (-270^\circ) \Rightarrow +270^\circ$ anticlockwise rotation

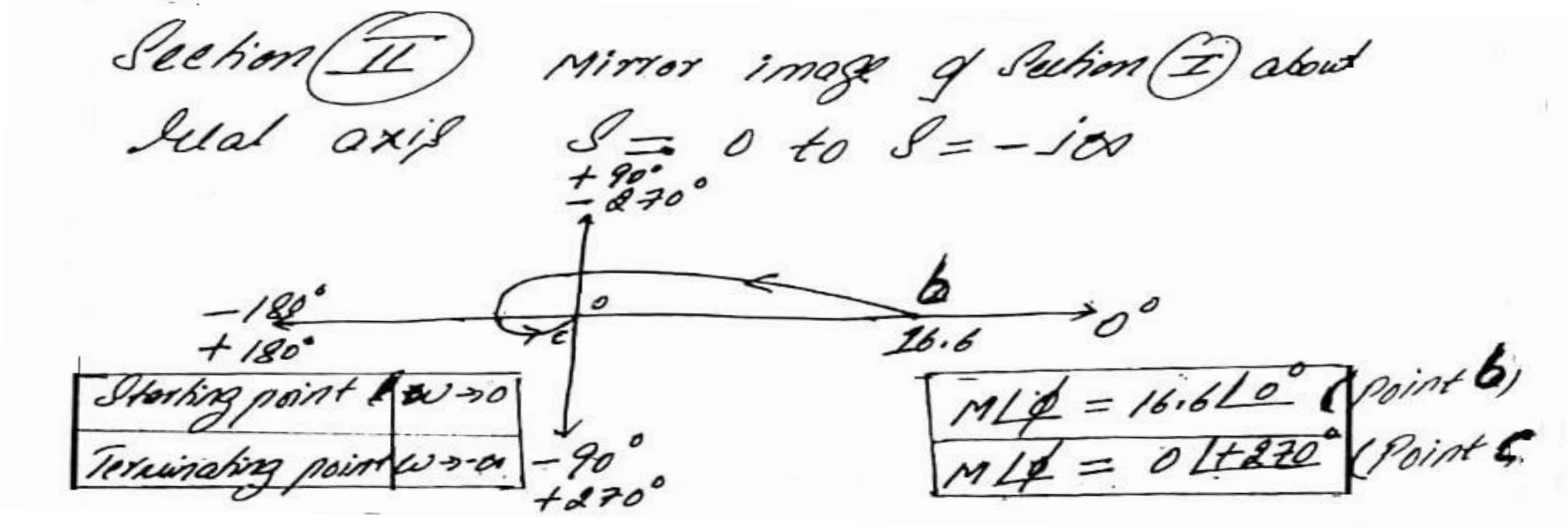
Section (I) polar plot →



Point $a = 0\angle +270^\circ$
Point $b = 16.6\angle 0^\circ$

Section II : Mirror Image of Section I on real axis

Section II	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (b)	S= 0	ω = 0	16.6	0˚
At Terminating point (c)	S= -j∞	ω = -∞	0	+270˚

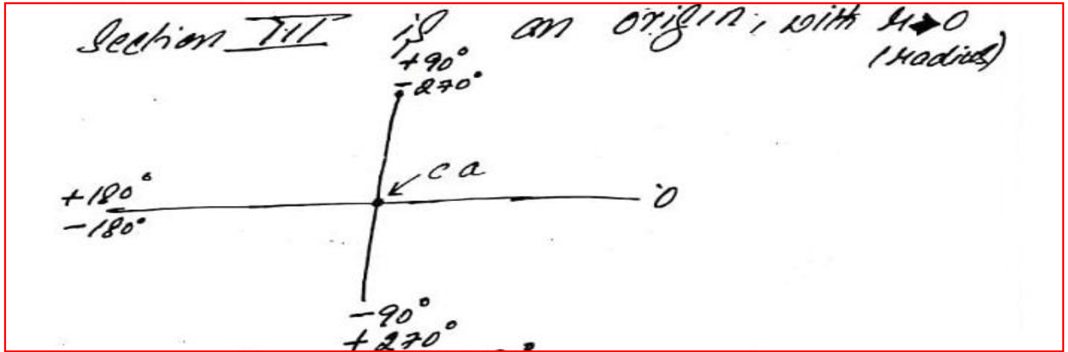


Section III : Last section III analysis is not required for closed loop stability

Section II	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (b)	S= -j∞	ω = -∞	0	+270˚
At Terminating point (a)	S= +j∞	ω = ∞	0	-270˚

Section III $S = -j\infty$ to $S = +j\infty$

Starting point	$\omega \rightarrow -\infty$	$M\angle\phi = 0\angle +270$	(point c)
Terminating point	$\omega \rightarrow +\infty$	$M\angle\phi = 0\angle -270$	(point a)



Step 5 Find “ωpc” Phase cross over frequency mathematically & “Q” intersection of Nyquist plot with negative real axis

Intersection with negative real axis.

$$G(j\omega)H(j\omega) = \frac{100(1-j\omega)(2-j\omega)(3-j\omega)}{(1+j\omega)(1-j\omega)(2+j\omega)(2-j\omega)(3+j\omega)(3-j\omega)}$$

...Rationalize

$$= \frac{100\left[(6-6\,\omega^2)+j\omega\,(\omega^2-11)\right]}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

Equating imaginary part to zero, $\omega = 0$ or $\sqrt{11}$

But at $\omega = 0$, the plot intersects positive real axis.

$\therefore \quad \omega^2 = 11 \quad$ i.e. $\omega_{pc} = \sqrt{11}$ rad/sec

$$\therefore \quad Q = \frac{100\times[6-6\times 11]}{(1+11)(4+11)(9+11)} = -\,\mathbf{1.667}$$

$$G(j\omega)H(j\omega) = \frac{100}{(1+j\omega)(2+j\omega)(3+j\omega)} \times \frac{(1-j\omega)(2-j\omega)(3-j\omega)}{(1-j\omega)(2-j\omega)(3-j\omega)}$$

$$= \frac{100(2-j\omega-2j\omega+j^2\omega^2)(3-j\omega)}{(1)^2-(j\omega)^2\times((2)^2-(j\omega)^2\times(3)^2-(j\omega)^2}$$

$j=\sqrt{-1}$
$j^2=-1$
$j^3=-j$

$$= \frac{100(2-j\omega-2j\omega-\omega^2)(3-j\omega)}{(1+\omega^2)\times(4+\omega^2)(9+\omega^2)}$$

$a^2-b^2=(a+b)(a-b)$

$$= \frac{100(6-3j\omega-6j\omega-3\omega^2-2j\omega+j^2\omega^2+2j^2\omega^2+j\omega^3}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

$$= \frac{100(6-11j\omega-6\omega^2+j\omega^3)}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

$$\Rightarrow \frac{600-600\omega^2+100j\omega^3-1100j\omega}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

$$\Rightarrow \frac{600-600\omega^2+j\omega(100\omega^2-1100)}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

Intersection with negative real axis.

$$G(j\omega)H(j\omega) = \frac{100(1-j\omega)(2-j\omega)(3-j\omega)}{(1+j\omega)(1-j\omega)(2+j\omega)(2-j\omega)(3+j\omega)(3-j\omega)}$$

...Rationalize

$$= \frac{100\left[(6-6\,\omega^2)+j\omega\,(\omega^2-11)\right]}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

Equating imaginary part to zero, $\omega = 0$ or $\sqrt{11}$

But at $\omega = 0$, the plot intersects positive real axis.

$\therefore \quad \omega^2 = 11$ i.e. $\omega_{pc} = \sqrt{11}$ rad/sec

$$\therefore \quad Q = \frac{100\times[6-6\times 11]}{(1+11)(4+11)(9+11)} = -\mathbf{1.667}$$

Equating imaginary part to zero

$$\frac{\omega(100\omega^2-1100)}{(1+\omega^2)(4+\omega^2)(9+\omega^2)} = 0$$

$\omega = 0$ (or) $(100\omega^2-1100)=0$

$\omega^2 = 11 = \sqrt{11}$

i.e $\omega_{pc} = \omega = \sqrt{11}$

But @ $\omega = 0$, the plot intersects positive real axis so $\omega \neq 0$ $\therefore$ $\omega^2 = 11$ i.e $\omega_{pc} = \sqrt{11}$ rad/sec

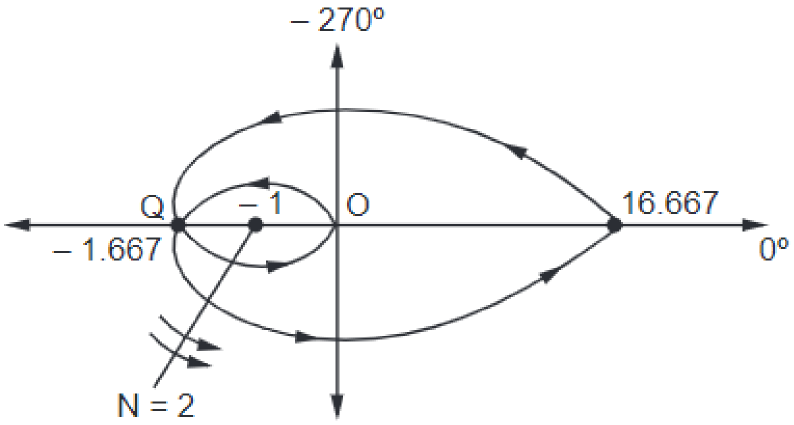
Substituting in real part $\omega^2 = 11$, $\omega = \sqrt{11}$

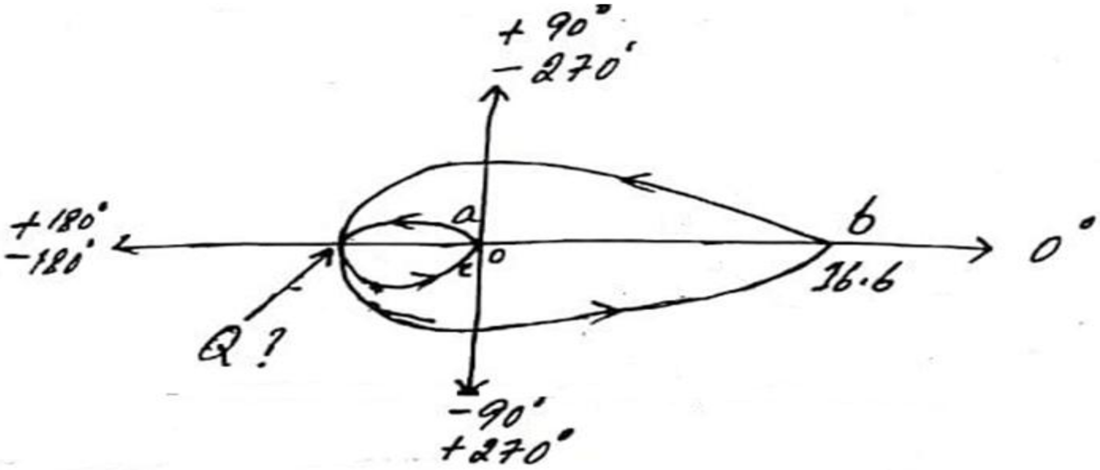
$$\text{point } Q = \frac{600-600\omega^2}{(1+\omega^2)(4+\omega^2)(9+\omega^2)}$$

$$Q = \frac{600-600(11)}{(1+11)(4+11)(9+11)} = \frac{600-6,600}{12\times 15\times 20}$$

$$Q = \frac{-6,000}{3600} = -1.667$$

➢ Step 6: Sketch the Nyquist plot, Drawn polar plots for various sections(i.e. Section I, II and III) are joined one after the other, The mapped locus is called Nyquist Plot





Step 7: find the value of N, Number of Encirclement of the critical point (-1,+ j0) point of F-plane by Nyquist plot

Clockwise encirclement are taken positive and anticlockwise encirclement are as negative

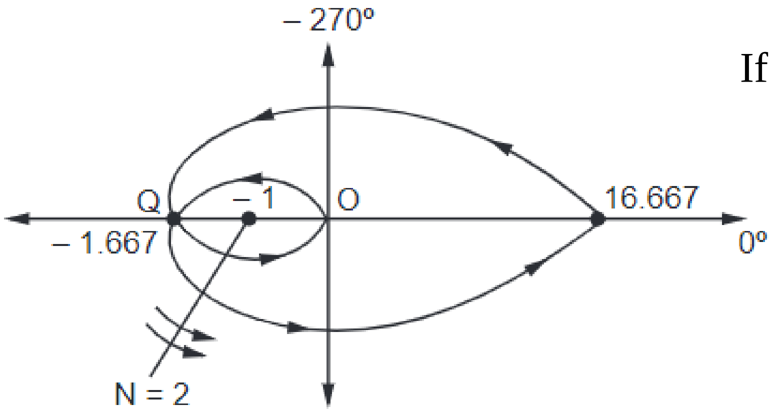


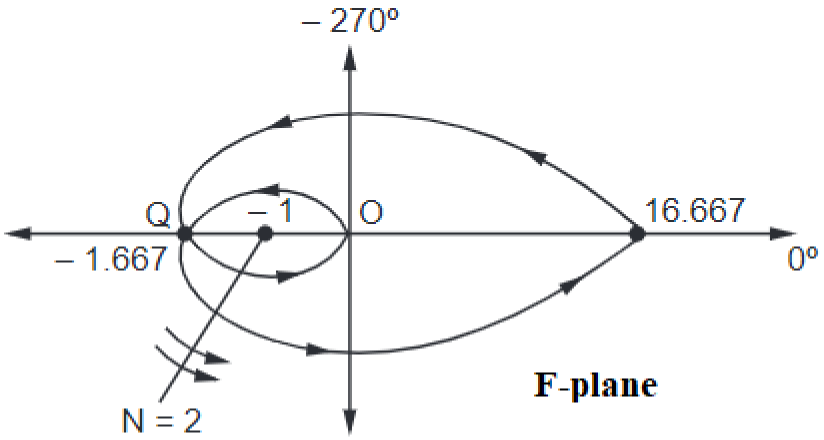
Fig. 12.13.20 (b)

If N = -P then system is stable, If N ≠ -P then system is unstable

Step 7 : N = +2 i.e. It does not satisfy N = 0 hence **system is unstable.**

$$\text{G.M.} = 20 \log \frac{1}{|OQ|} = 20 \log \frac{1}{1.667} = \mathbf{-4.437\ dB}$$

Step 8: Determine Gain Margin



$$\text{G.M.} = \frac{1}{|OQ|} \quad \text{where}$$

Q = Intersection point of Nyquist plot with negative real axis

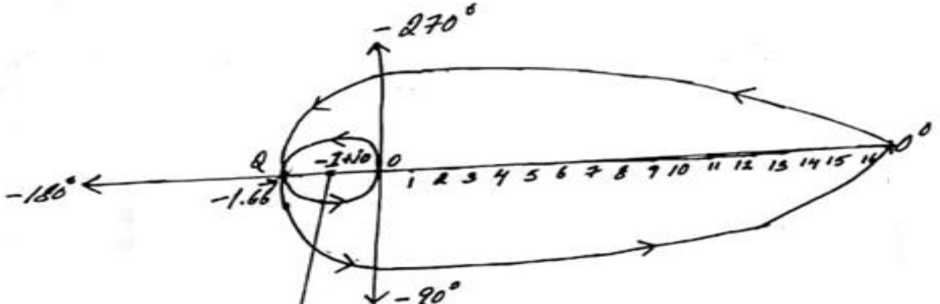
$$\text{i.e. G.M.} = 20\,\text{Log}_{10}\frac{1}{|OQ|}\ \text{dB}$$

$$\text{G.M.} = 20\log\frac{1}{|OQ|} = 20\log\frac{1}{1.667} = \mathbf{-\ 4.437\ dB}$$

Step 6: Sketch Nyquist plot & find the value of 'N' from the graph i.e number of encirclement of "$-1+j0$" by Nyquist plot, if this matches with the criterion decided in step 2, system is stable

if $N = -P$ Then System is Stable

if $N \neq -P$ The system is unstable



→ +1

→ +1

Total No of Encirclements } → $N = 2$.

Anticlockwise encirclements are treated as positive

clockwise encirclements are treated as negative.

here $N = +2$ i.e it does not satisfy $N = 0$ hence system is unstable

∴ Two closed loop poles are in the right half of s-plane hence the system is unstable.

Program

G(s) H(s) = 100 / (S+1) (S+2) (S+3)

```
clc
num = [100];
den =[1 6 11 6];
sys=tf(num, den);
nyquist(sys)
[re,im,w]=nyquist(sys)
```

clc #clears all the text from the Command Window, resulting in a clear screen
num = [100]; #Coefficients of the numerator
den =[1 6 11 6]; #Coefficients of the denominator
sys=tf(num,den);#creates a continuous-time transfer function with numerator(s) and denominator(s) specified by num and den
nyquist(sys) # creates a Nyquist plot of a dynamic system sys
[re,im,w]=nyquist(sys)# return the real and imaginary parts of the frequency response at the frequencies w (in rad/TimeUnit).

Draw Nyquist Plot, Investigate the stability of a negative feedback control system whose open loop transfer function is given by G(s) H(s) = K / S (S+2) (S+10) & hence calculate the range of value of K for Stability.

Step 1 Count number of poles of G(s)H(s) are in right half of S-Plane

i.e. with positive Real Part, This is the value of 'P'

P = 0, As No Open Loop poles in the right half of S-Plane

Step 2 Decide the stability criterion as **N = -P**

i.e. How many times Nyquist plot should encircle " **-1+j0** " point for absolute stability

Here N = 0, So Critical Point " -1+j0 " i.e. (-1, 0) Should Not get encircled by Nyquist plot for stability

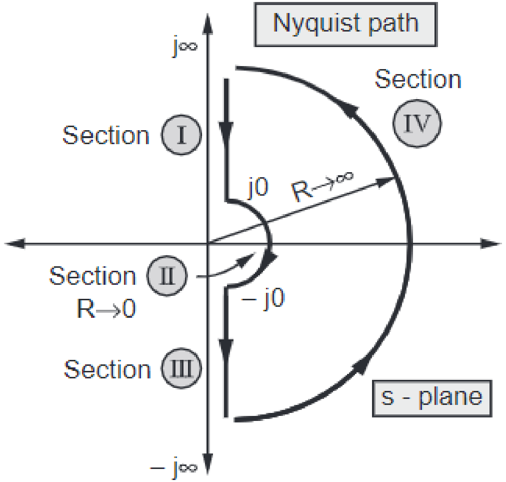
Step 3 Select Nyquist Path as per the function G(s)H(s). Identify the various segments on the contour with reference to Nyquist path

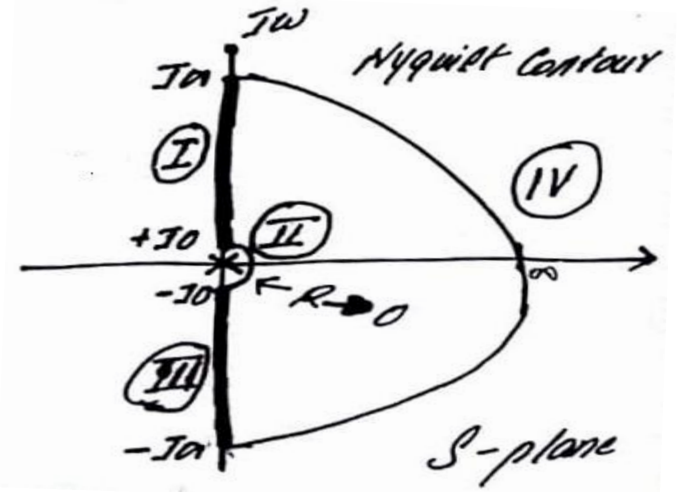
Select Nyquist contour which encloses the entire right half s-plane except singular points. The Nyquist contour encloses all the right half s-plane poles and zeros of G(s)H(s). The poles on imaginary axis are singular points and so they are avoided by taking a detour around it.

Here Select the proper Nyquist contour – Include the entire right half of s-plane by drawing a semicircle of radius R, with R tends to infinity

Pole is at orgin hence Nyquist contour is as shown

G(s)H(s) has one pole at orgin





Step 4 Analyze the Section I, II, III & IV as a starting point and terminating point. Last section IV analysis is not required for closed loop stability

Express the magnitude and phase equations in terms of ω and Estimate the magnitude and phase for different values of ω

Let S=jω in G(s)H(s) & calculate magnitude and phase $M\angle \varnothing = |G(j\omega)H(j\omega)| * \angle G(j\omega)H(j\omega)$

$G(s)\,H(s) = K / S\,(S+2)\,(S+10)$

$$G(j\omega)H(j\omega) = \frac{K}{j\,\omega\,(2 + j\,\omega)(10 + j\,\omega)}$$

$$M = |G(j\omega)H(j\omega)|$$

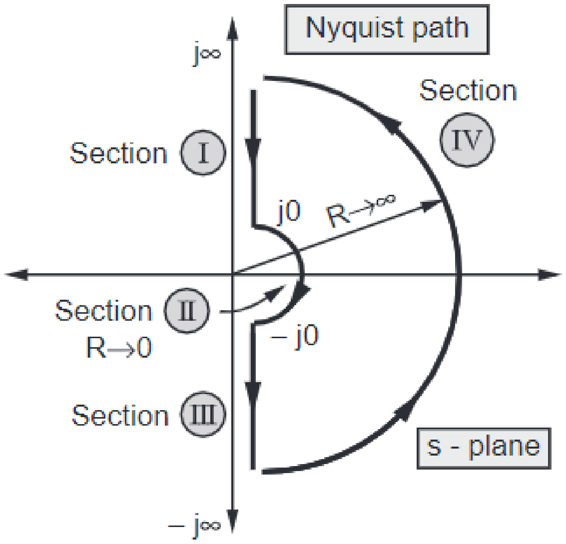
$$G(j\omega)H(j\omega) = \frac{K}{j\,\omega\,(2 + j\,\omega)(10 + j\,\omega)}$$

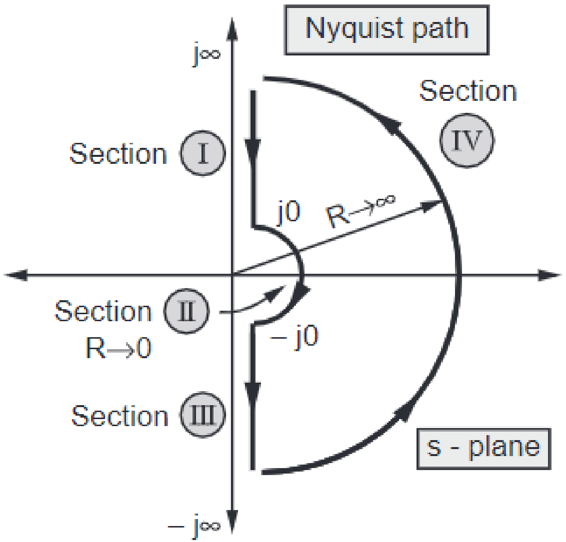
$$M = |G(j\omega)H(j\omega)|$$

$$= \frac{K}{\omega \times \sqrt{4+\omega^2} \times \sqrt{100+\omega^2}}$$

$$\phi = \frac{\tan^{-1}\left(\frac{0}{K}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega}{2}\right)\tan^{-1}\left(\frac{\omega}{10}\right)}$$

$$= -90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$





Section	Start	End	
I	$s = +j\infty$	$s = +j0$	along imaginary axis
II	$s = +j0$	$s = -j0$	along semicircle of radius tending to zero
III	$s = -j0$	$s = -j\infty$	along the imaginary axis
IV	$s = -j\infty$	$s = +j\infty$	along the semicircle of $R \to \infty$ encircling entire right half

$+j0 \rightarrow$ A point which is very very close to origin but just above it on positive imaginary axis hence denoted as $+j0$

$-j0 \rightarrow$ A point which is very very close to origin but just below it on negative imaginary axis hence denoted as $-j0$

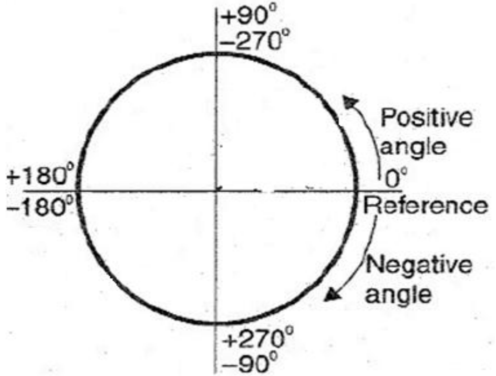
Analyze the Section I, II, III and IV as a starting point and terminating point

Section I	S=jω	ω	Magnitude $\lvert G(j\omega)H(j\omega) \rvert$	Phase angle $\angle G(j\omega)H(j\omega)$
At Starting point (a)	S= j∞	$\omega = \infty$	0	-270°
At Terminating point (b)	S= +j0	$\omega = +0$	∞	-90°

Section I : $s = +j\infty$ to $s = +j0$

i.e. $\omega \to \infty$ to $\omega \to +0$

Starting point	$\omega \to \infty$	$0 \angle -270^\circ$	$-90^\circ - (-270)^\circ$
Terminating point	$\omega \to +0$	$0 \angle +90^\circ$	$= +180^\circ$ Anticlockwise rotation



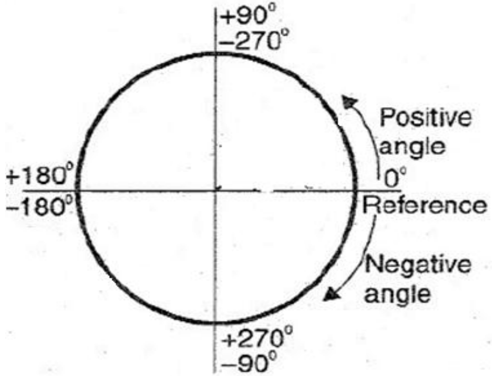
Terminating point phase angle - Starting point phase angle = Positive value, then sketch polar plot of **Section I** in Anticlockwise rotation

Terminating point phase angle - Starting point phase angle = Negative value, then sketch polar plot of **Section I** in clockwise rotation

Section I : $s = +j\infty$ to $s = +j0$

i.e. $\omega \rightarrow \infty$ to $\omega \rightarrow +0$

Starting point	$\omega \rightarrow \infty$	$0 \angle -270°$	$-90° - (-270)°$
Terminating point	$\omega \rightarrow +0$	$0 \angle +90°$	$= +180°$ Anticlockwise rotation



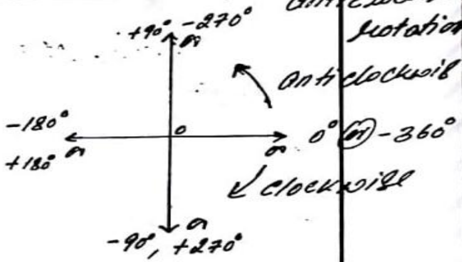
Section (I) $S = +j\infty$ to $S = +j0$

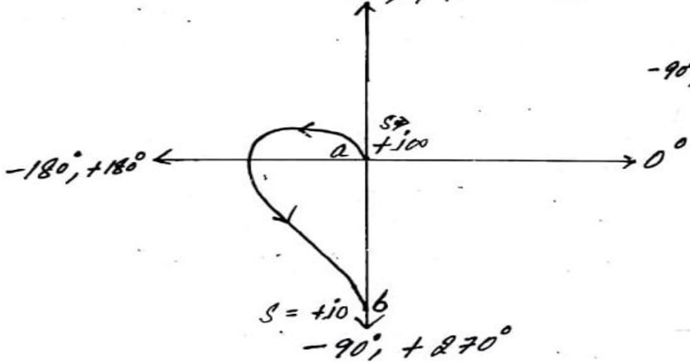
i.e $\omega \rightarrow \infty$ to $\omega \rightarrow +0$

Starting point	$\omega \rightarrow \infty$	$M \angle \phi \Rightarrow 0 \angle -270°$	$-90° - (-270°)$ $\Rightarrow +180°$ anticlockwise rotation
Terminating point	$\omega \rightarrow +0$	$M \angle \phi \Rightarrow \infty \angle -90°$	

Point a = $0 \angle -270°$

Point b = $\infty \angle -90°$





Section II : $s = +j\,0$ to $s = -j\,0$

i.e. $\omega \rightarrow +0$ to $\omega \rightarrow -0$

Starting point	$\omega \rightarrow +0$	$\infty \angle -90^\circ$	$90^\circ - (-90)^\circ = +180^\circ$ Anticlockwise rotation
Terminating point	$\omega \rightarrow -0$	$\infty \angle +90^\circ$	

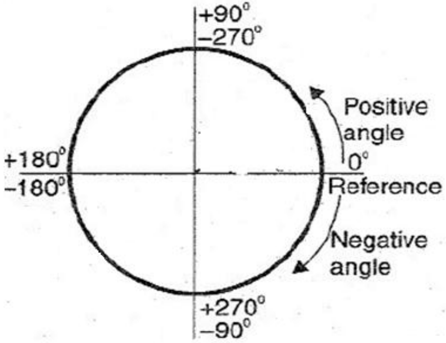
Section II $s = +j0$ to $s = -j0$

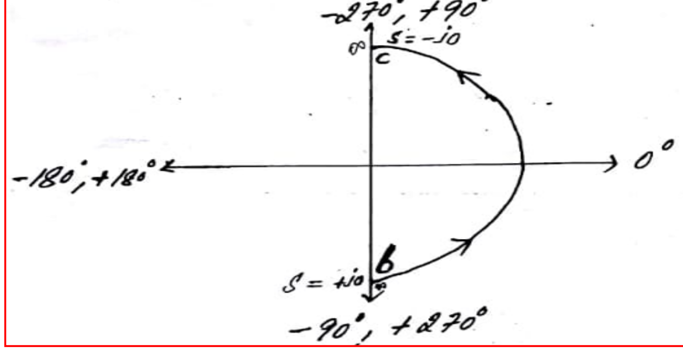
$\omega = \rightarrow +0$ to $\omega \rightarrow -0$

Starting point	$\omega \rightarrow +0$	$M\angle\phi = \infty\angle -90^\circ$	$90^\circ - (-90^\circ) \Rightarrow +180^\circ$ Anticlockwise rotation
Terminating point	$\omega \rightarrow -0$	$M\angle\phi = \infty\angle +90^\circ$	

Point b $= \infty\angle -90^\circ$

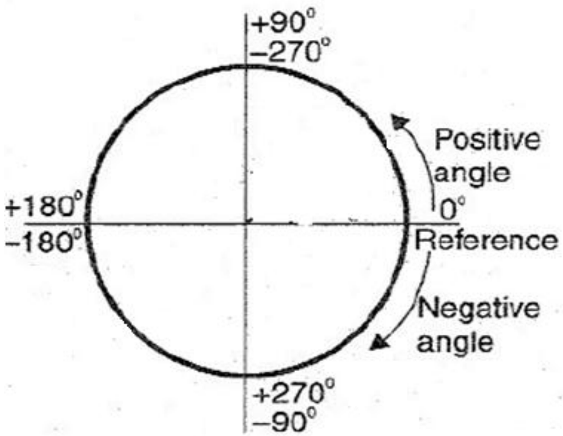
Point c $= \infty\angle +90^\circ$

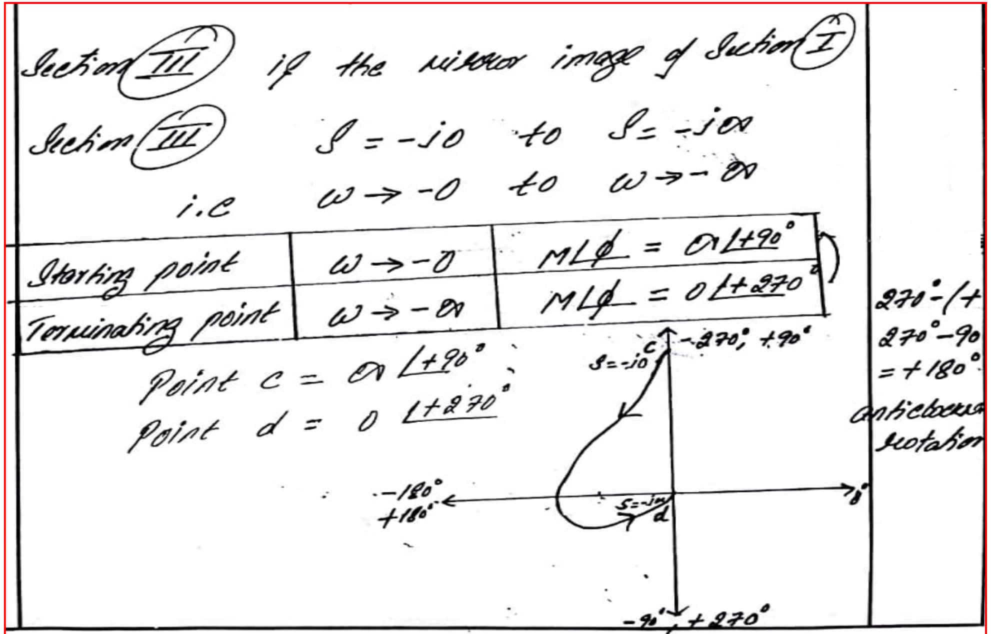




Section III : Mirror Image of Section I about real axis

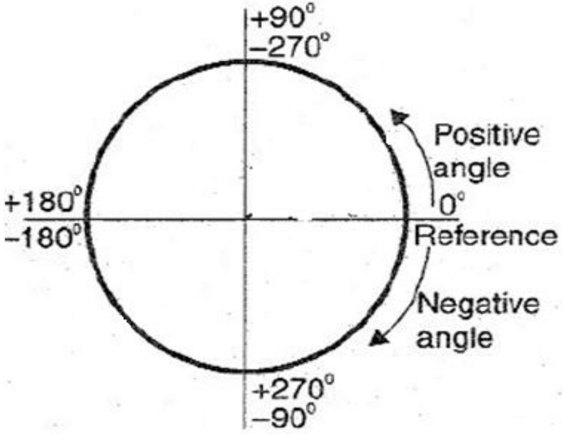
Section III	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (c)	S= -j0	ω = −0	∞	+90°
At Terminating point (d)	S= -j∞	ω = -∞	0	+270°





Section IV : Last section IV analysis is not required for closed loop stability

Section IV	**S=jω**	**ω**	**Magnitude $\lvert G(j\omega)H(j\omega)\rvert$**	**Phase angle $\angle G(j\omega)H(j\omega)$**
At Starting point (d)	S= -j∞	ω = -∞	0	+270˚
At Terminating point (a)	S= +j∞	ω = ∞	0	-270˚



Section (IV) is on origin. with Radius Tends to zero

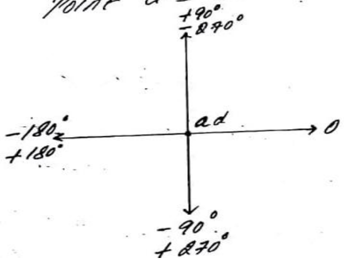
$S = -j\infty$ to $S = +j\infty$

$\omega \to -\infty$ to $\omega \to +\infty$

Starting point	$\omega \to -\infty$	$M\angle\phi = 0\angle{+270^\circ}$
End point	$\omega \to \infty$	$M\angle\phi = 0\angle{-270^\circ}$

Point $d = 0\angle{+270^\circ}$

Point $a = 0\angle{-270^\circ}$



Step 5 Find "ωpc" Phase cross over frequency mathematically & "Q" intersection of Nyquist plot with negative real axis

Step 5 : $$G(j\omega)H(j\omega) = \frac{K}{j\omega(10+j\omega)(2+j\omega)}$$

Rationalizing ,

$$G(j\omega)H(j\omega) = \frac{K(-j\omega)(10-j\omega)(2-j\omega)}{(j\omega)(-j\omega)(10+j\omega)(10-j\omega)(2+j\omega)(2-j\omega)}$$

$$\therefore\ G(j\omega)H(j\omega) = \frac{-Kj\omega[20-12j\omega-\omega^2]}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$= \frac{-12K\omega^2}{D} - \frac{Kj\omega(20-\omega^2)}{D}$$

where $D = \omega^2(4+\omega^2)(100+\omega^2)$

Equating imaginary part to zero,

$$\omega(20-\omega^2) = 0 \quad \text{i.e.} \quad \omega^2 = 20 \quad \text{i.e.} \quad \omega_{pc} = \sqrt{20}$$

Substituting in real part,

$$\text{Point Q} = \frac{-12K\times 20}{20\times(20+4)\times(100+20)} = -\frac{K}{240}$$

Step 5 : Find ω_{pc} phase cross over frequency of 'Q' Intersection of Nyquist plot with negative real axis.

Rationalizing $G(j\omega)H(j\omega) = \frac{K}{j\omega(10+j\omega)(2+j\omega)}$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(10+j\omega)(2+j\omega)} \times \frac{(-j\omega)(10-j\omega)(2-j\omega)}{(-j\omega)(10-j\omega)(2-j\omega)}$$

$$\Rightarrow \frac{K(-j\omega)(10-j\omega)(2-j\omega)}{(j\omega)(-j\omega)(10+j\omega)(10-j\omega)(2+j\omega)(2-j\omega)}$$

$$\Rightarrow \frac{-Kj\omega[20-j10\omega-2j\omega+j^2\omega^2]}{\omega^2[((10)^2-(j\omega)^2)\times((2^2-(j\omega)^2)]}$$

$j^2=-1$
$j=\sqrt{-1}$
$j^3=-j$

$$\Rightarrow \frac{-Kj\omega[20-12j\omega-\omega^2]}{\omega^2(100+\omega^2)(4+\omega^2)}$$

$$\Rightarrow \frac{-20Kj\omega+12Kj^2\omega^2+Kj\omega^3}{\omega^2(100+\omega^2)(4+\omega^2)}$$

$$\Rightarrow \frac{-12K\omega^2-20Kj\omega+Kj\omega^3}{\omega^2(100+\omega^2)(4+\omega^2)} \Rightarrow \frac{-12K\omega^2+Kj\omega(\omega^2-20)}{\omega^2(100+\omega^2)(4+\omega^2)}$$

$$\therefore G(j\omega)H(j\omega) = \frac{-12k\omega^2}{\omega^2(4+\omega^2)(100+\omega^2)} + \frac{k\,j\omega(\omega^2-20)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

Equating imaginary part to zero

$$\omega(\omega^2-20) = 0$$

$\omega \neq 0$ The plot intersect positive real axis

$$\therefore \omega^2 - 20 = 0$$

$$\omega^2 = 20 \quad \text{i.e.} \quad \omega = \sqrt{20}$$

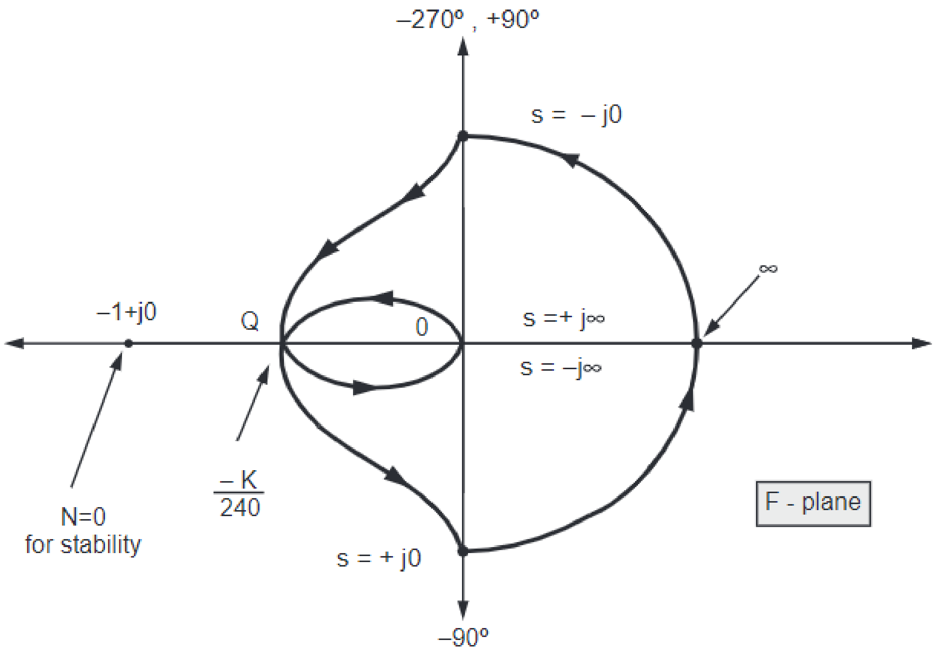
$$\omega_{pc} = \sqrt{20}$$

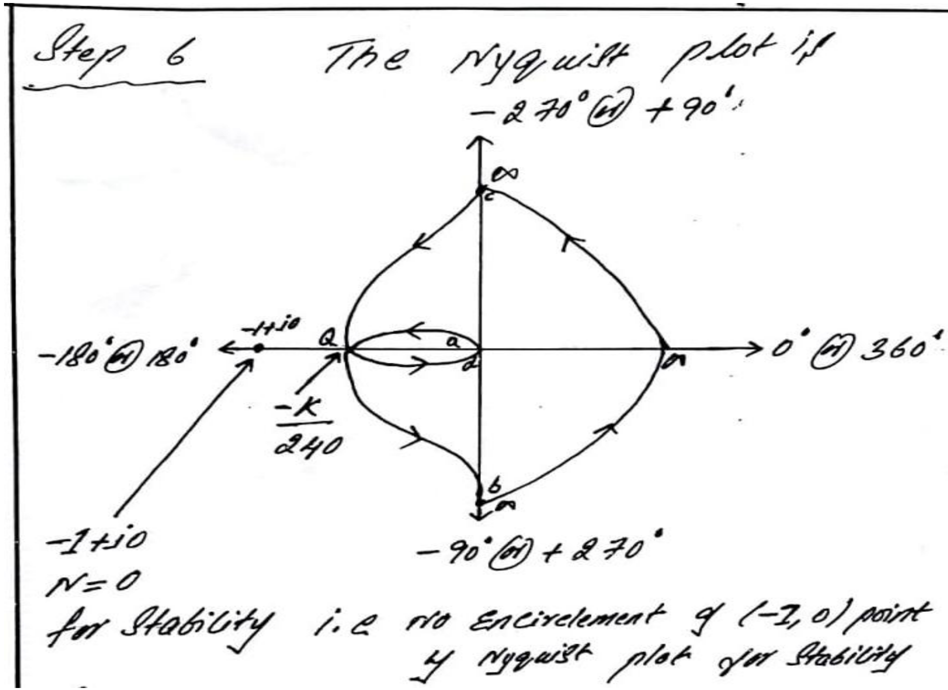
Substituting the value ω in real part

$$\text{point } Q \Rightarrow \frac{-12k\omega^2}{\omega^2(4+\omega^2)(100+\omega^2)} = \frac{-12k \times 20}{20(20+4)(100+20)}$$

$$\text{point } Q = -\frac{k}{240}$$

➢ Step 6: Sketch the Nyquist plot, Drawn polar plots for various sections(i.e. Section I, II III & IV) are joined one after the other, The mapped locus is called Nyquist Plot





Step 7: find the value of N, Number of Encirclement of the critical point (-1,+ j0) point of F-plane by Nyquist plot

Clockwise encirclement are taken positive and anticlockwise encirclement are as negative

If N = -P then system is stable, If N ≠ -P then system is unstable

Step 7 : Now for absolute stability, N = 0

i.e. it should be located on left side of point Q i.e. | OQ | < 1

$$\therefore \qquad \left| -\frac{K}{240} \right| < 1 \quad \therefore \quad K < 240$$

So range of values of K for stability is

$$\boxed{0 < K < 240}$$

Step 7 :

Now for absolute Stability, N=0

as per Step 2

i.e (-1+j0) point Should be located on left side of point Q i.e |Q| < 1

$$\left| -\frac{k}{240} \right| < 1 \;, \quad \frac{k}{240} < 1$$

$\therefore$ k < 240, So range of values of 'k' for Stability is 0 < k < 240

Draw Nyquist Plot, Investigate the stability of a negative feedback control system whose open loop transfer function is given by G(s) H(s) = 5 / s(1-s)

Step 1 Count number of poles of G(s)H(s) are in right half of S-Plane

i.e. with positive Real Part, This is the value of 'P'

P = 1, As No Open Loop poles in the right half of S-Plane

Step 2 Decide the stability criterion as **N = -P**

i.e. How many times Nyquist plot should encircle " **-1+j0** " point for absolute stability

Here N = -P, Therefore N = -1 for stability, Nyquist Plot Must Encircle critical Point " -1+j0 " i.e. (-1, 0) once in Clockwise direction for stability

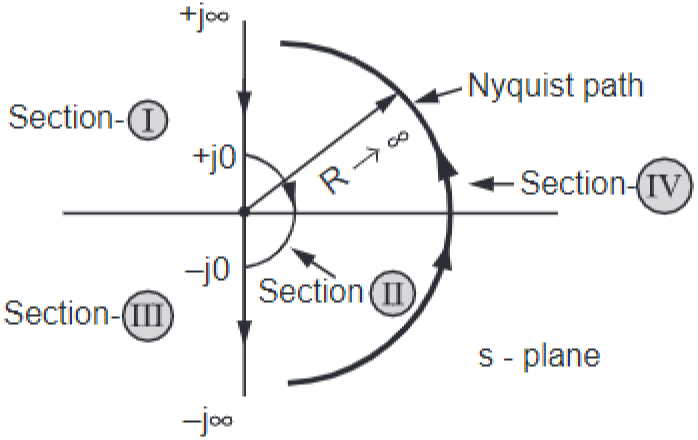
Step 3 Select Nyquist Path as per the function G(s)H(s). Identify the various segments on the contour with reference to Nyquist path

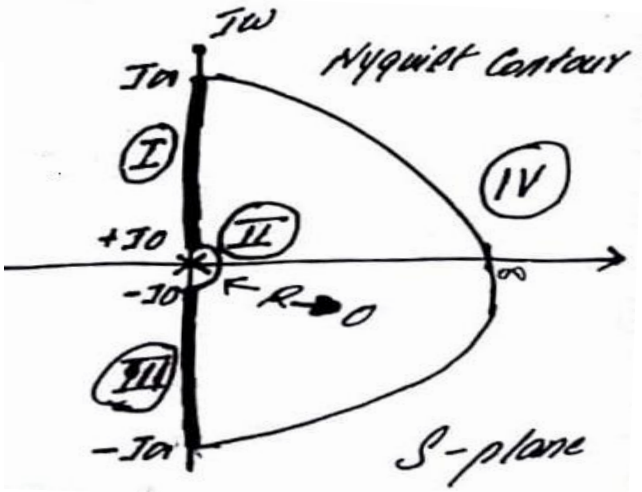
Select Nyquist contour which encloses the entire right half s-plane except singular points. The Nyquist contour encloses all the right half s-plane poles and zeros of G(s)H(s). The poles on imaginary axis are singular points and so they are avoided by taking a detour around it.

Here Select the proper Nyquist contour – Include the entire right half of s-plane by drawing a semicircle of radius R, with R tends to infinity

Pole is at orgin hence Nyquist contour is as shown

G(s)H(s) has one pole at orgin





Step 4 Analyze the Section I, II, III & IV as a starting point and terminating point. Last section IV analysis is not required for closed loop stability

Express the magnitude and phase equations in terms of ω and Estimate the magnitude and phase for different values of ω

Let S=jω in G(s)H(s) & calculate magnitude and phase $M\angle\varnothing = |\mathbf{G(j\omega)H(j\omega)}| * \angle\mathbf{G(j\omega)H(j\omega)}$

$$G(s)\,H(s) = 5 / s(1-s) \qquad G(j\omega) = \frac{5}{j\omega(1-j\omega)}$$

Note that $\angle 1 - j\omega = -90^\circ$ as $\omega \to +\infty$

$\angle +1 - j\omega = 0^\circ$ as $\omega \to +0$

$\angle 1 - j\omega = 0^\circ$ as $\omega \to -0$

Note that right half open loop pole is represented as 1–s and not as s–1.

Step 4 :- analysis Section 1, 2, 3 & 4

$$G(s) = \frac{5}{s(1-s)} = \frac{5+j0}{j\omega(1-j\omega)}$$

Put $s = j\omega$

$$M = |G(j\omega)| \quad \phi = \angle G(j\omega)$$

$$M \Rightarrow \frac{\sqrt{5^2}}{\sqrt{\omega^2} \times \sqrt{1^2 + (-\omega^2)}} \Rightarrow \frac{5}{\omega \times \sqrt{1+\omega^2}}$$

$$\phi = \frac{\tan^{-1}\left(\frac{0}{5}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right) \times \tan^{-1}\left(\frac{-\omega}{1}\right)} = \frac{0^\circ}{90^\circ \times -\tan^{-1}\left(\frac{\omega}{1}\right)}$$

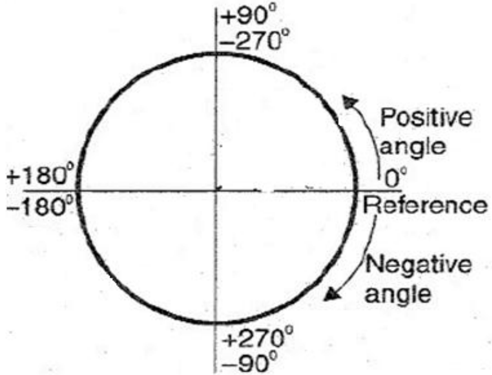
$\tan^{-1}(0) = 0^\circ$, $\tan^{-1}(\infty) = 90^\circ$

Analyze the Section I, II, III and IV as a starting point and terminating point

Section I	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (a)	S= j∞	ω = ∞	0	0°
At Terminating point (b)	S= +j0	ω = +0	∞	-90°

Section I : $s = +j\infty$ to $s = +j0$

Start	$\omega \to \infty$	$0 \angle \frac{0^\circ}{90^\circ, -90^\circ}$ $= 0 \angle 0^\circ$	$-90^\circ - 0^\circ = -90^\circ$ Clockwise
End	$\omega \to +0$	$\infty \angle \frac{0^\circ}{90^\circ, 0^\circ}$ $= \infty \angle -90^\circ$	

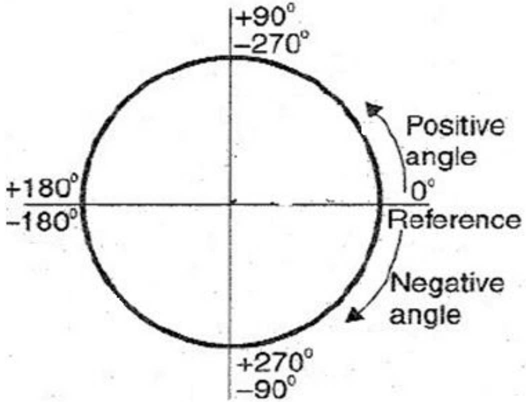


Terminating point phase angle - Starting point phase angle = Positive value, then sketch polar plot of **Section I** in Anticlockwise rotation

Terminating point phase angle - Starting point phase angle = Negative value, then sketch polar plot of **Section I** in clockwise rotation

Section I : s = + j∞ to s = + j0

Start	$\omega \to \infty$	$0 \angle \frac{0^\circ}{90^\circ, -90^\circ}$ $= 0 \angle 0^\circ$	$-90^\circ - 0^\circ = -90^\circ$ Clockwise
End	$\omega \to +0$	$\infty \angle \frac{0^\circ}{90^\circ, 0^\circ}$ $= \infty \angle -90^\circ$	



$$M = \frac{5}{\omega \times \sqrt{1+\omega^2}},$$

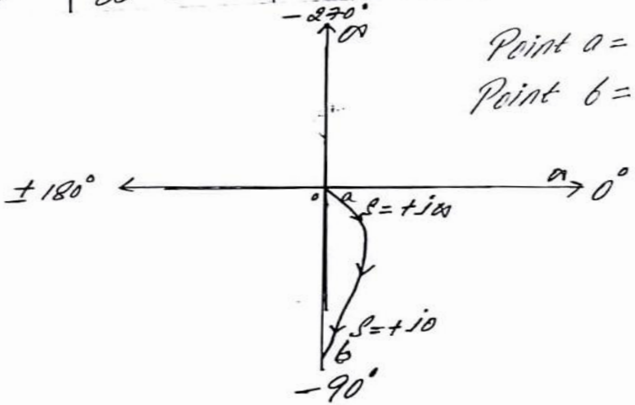
$$\phi = -90^\circ + \tan^{-1}\left(\frac{\omega}{1}\right)$$

Section (1) $s = +j\infty$ to $s = +j0$

Start	$\omega \to \infty$	$M\angle\phi \Rightarrow 0\angle 0^\circ$
End	$\omega \to +0$	$M\angle\phi \Rightarrow \infty\angle -90^\circ$

$-90^\circ - (0^\circ)$
$= -90^\circ$
Clockwise rotation.

Point a = $0\angle 0^\circ$
Point b = $\infty\angle -90^\circ$

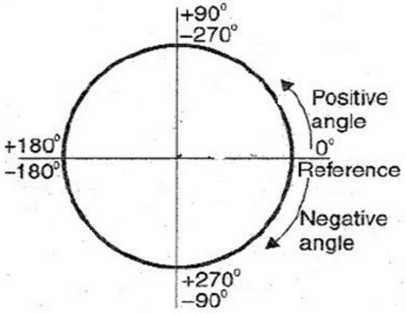


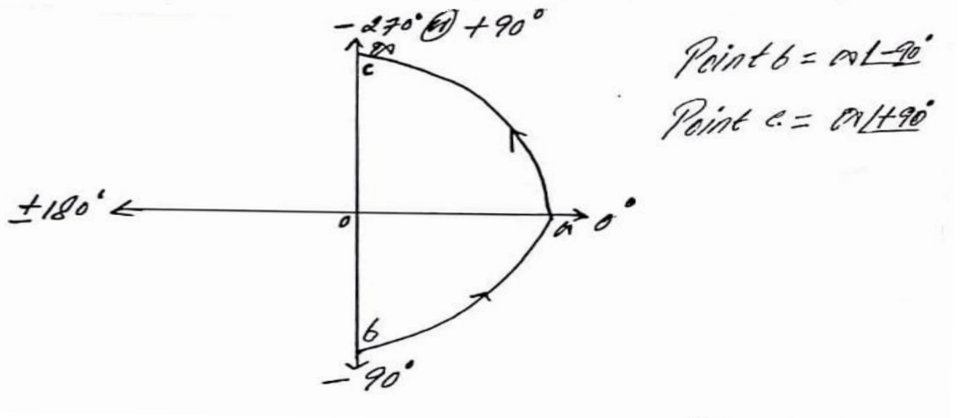
Section II : S= +j0 to S= -j0

Section II	**S=jω**	**ω**	**Magnitude $\lvert G(j\omega)H(j\omega)\rvert$**	**Phase angle $\angle G(j\omega)H(j\omega)$**
At Starting point (b)	S= +j0	ω = +0	∞	-90°
At Terminating point (c)	S= -j0	ω = -0	∞	$+90^\circ$

Section II : $s = +j0$ to $s = -j0$

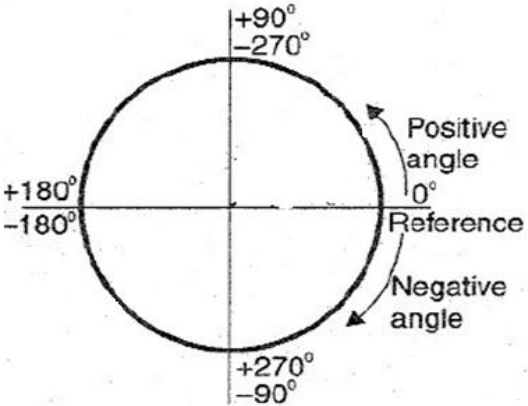
Start	$\omega \rightarrow +0$	$\infty \angle \frac{0^\circ}{90^\circ\ 0^\circ} = \infty \angle -90^\circ$	$90^\circ - (-90^\circ) = +180^\circ$ Anticlockwise
End	$\omega \rightarrow -0$	$\infty \angle \frac{0^\circ}{-90^\circ\ 0^\circ} = \infty \angle +90^\circ$	





Section III : Mirror Image of Section I about real axis

Section III	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (c)	S= -j0	ω = −0	∞	+90˚
At Terminating point (a)	S= -j∞	ω = -∞	0	0˚



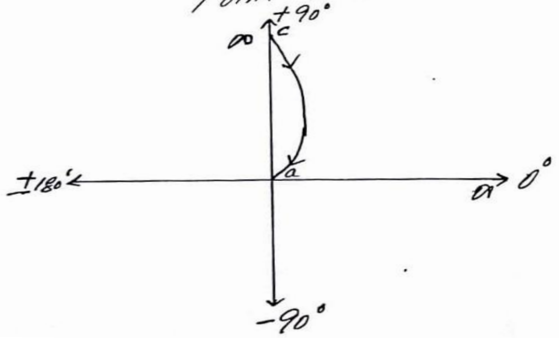
Section (3) Mirror image of Section (1) about real axis

Start $S = -j0$, $\omega \to -0$ $\quad M\angle\phi = \infty\angle{+90^\circ}$

End $S = -j\infty$ $\quad \omega \to -\infty$ $\quad M\angle\phi = 0\angle 0^\circ$

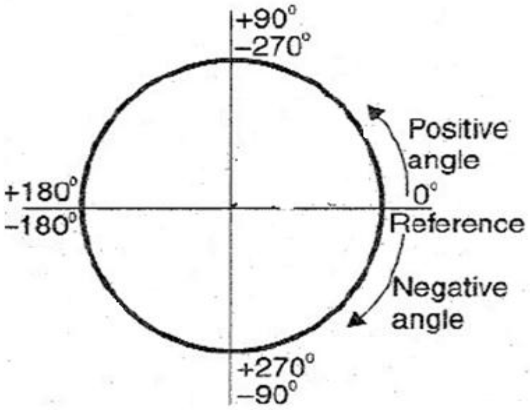
Point c $= \infty\angle{+90^\circ}$

Point a $= 0\angle 0^\circ$



Section IV : Last section IV analysis is not required for closed loop stability

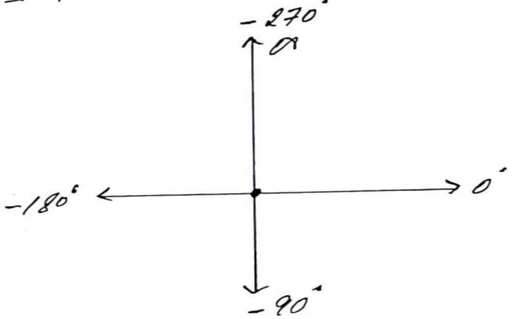
Section IV	S=jω	ω	Magnitude |G(jω)H(jω)|	Phase angle ∟G(jω)H(jω)
At Starting point (a)	S= -j∞	ω = −∞	0	0°
At Terminating point (a)	S= +j∞	ω = +∞	0	0°



Section (4) is on orgin with Radius tends to 0

Start $S = -J\infty$ $\omega \to -\infty$ $M\angle\phi = 0\angle 0^\circ$

End $S = +J\infty$ $\omega \to +\infty$ $M\angle\phi$ $0\angle 0^\circ$



Step 5 Find “ωpc” Phase cross over frequency mathematically & “Q” intersection of Nyquist plot with negative real axis

Step 5 : Intersection with negative real axis.

$$G(j\omega) = \frac{5(-j\omega)(1+j\omega)}{j\omega(-j\omega)(1+j\omega)(1-j\omega)} = \frac{-5j\omega(1+j\omega)}{\omega^2(1+\omega^2)}$$

$$= \frac{5\omega^2}{\omega^2(1+\omega^2)} - \frac{5j\omega}{\omega^2(1+\omega^2)}$$

$\therefore \quad \omega_{pc} = 0$ to make imaginary part zero.

But $\omega = 0$ is not on Nyquist path hence **there is no finite intersection with negative real axis.**

Step 5 intersection with negative real axis

$$G(j\omega) = \frac{5(-j\omega)(1+j\omega)}{j\omega(-j\omega)(1+j\omega)(1-j\omega)}$$

$$= \frac{-5j\omega(1+j\omega)}{\omega^2(1+\omega^2)}$$

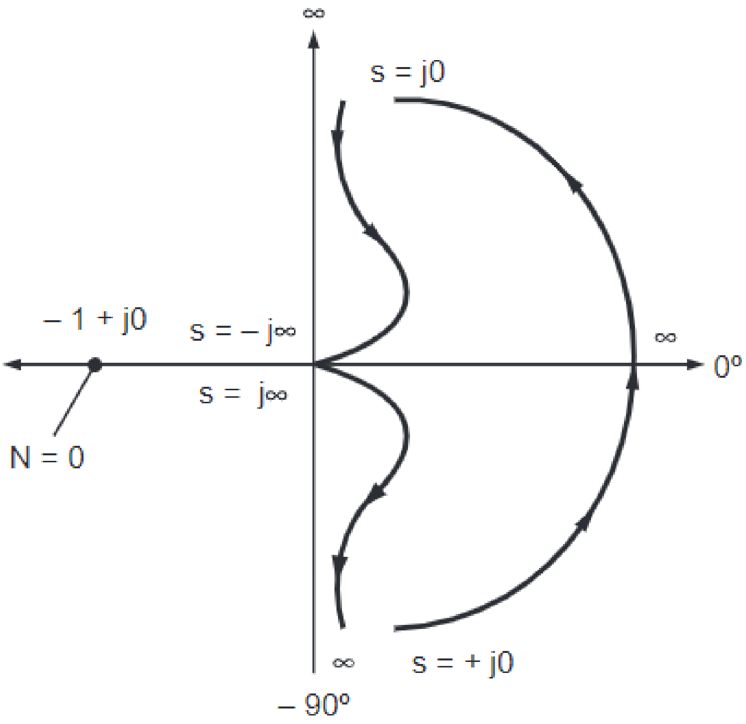
$$\Rightarrow \frac{5\omega^2}{\omega^2(1+\omega^2)} - \frac{5j\omega}{\omega^2(1+\omega^2)}$$

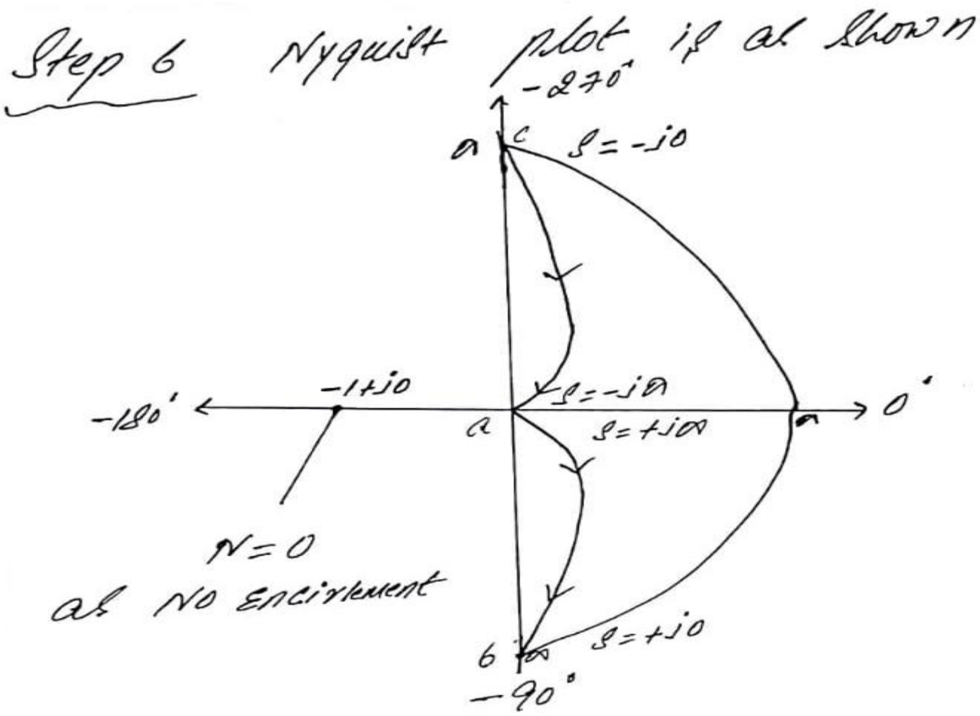
$\omega_{pc} = 0$ to make imaginary part zero

$$\frac{5j\omega}{\omega^2(1+\omega^2)} = 0, \quad \omega = 0$$

But $\omega = 0$ is Not on Nyquist path hence there is No finite intersection with negative real axis.

➢ Step 6: Sketch the Nyquist plot, Drawn polar plots for various sections(i.e. Section I, II III & IV) are joined one after the other, The mapped locus is called Nyquist Plot





Step 7: find the value of N, Number of Encirclement of the critical point (-1,+ j0) point of F-plane by Nyquist plot

Clockwise encirclement are taken positive and anticlockwise encirclement are as negative

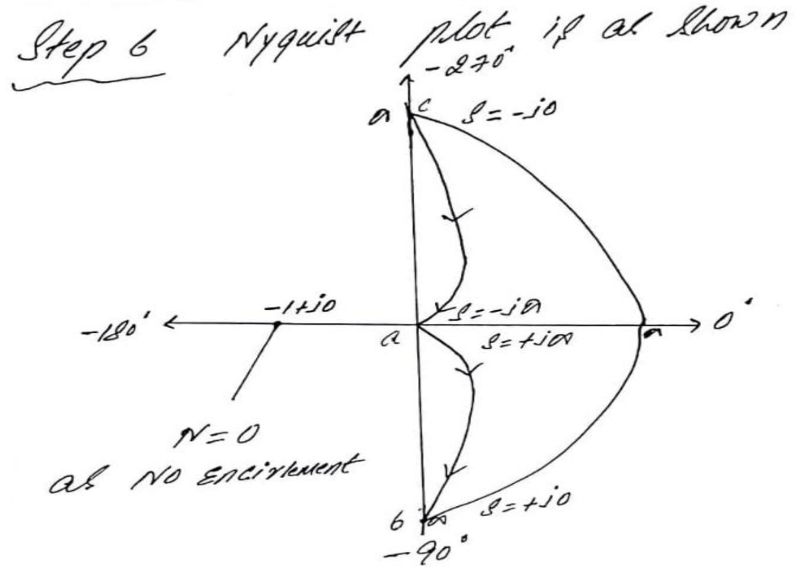
If N = -P then system is stable, If N ≠ -P then system is unstable

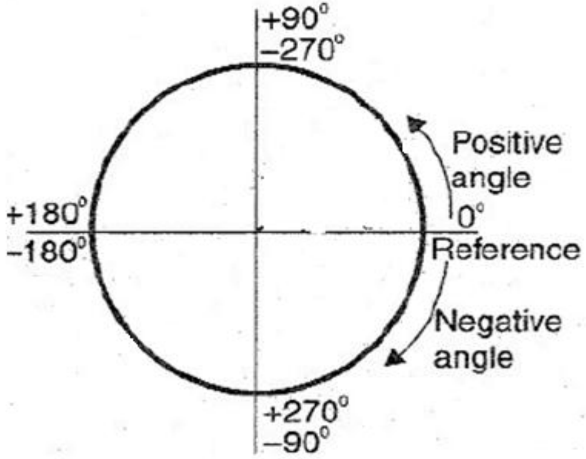
Step 7 : N = 0

– 1 + j0 point is not encircled.

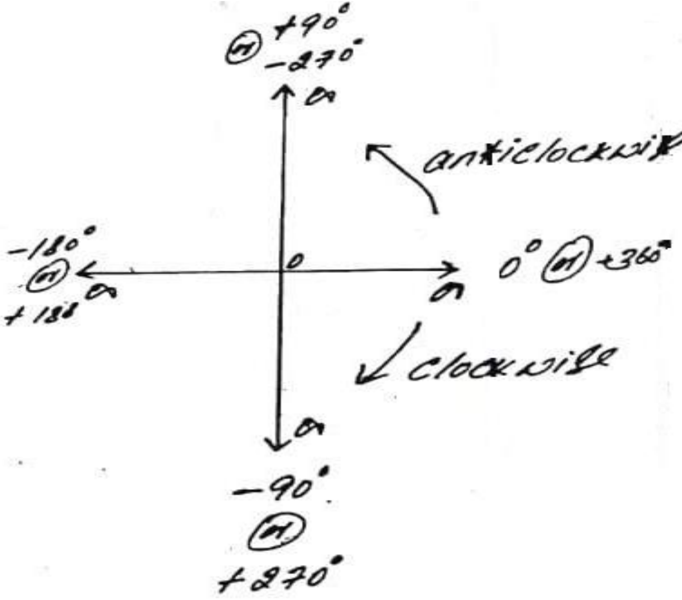
As it does not match with stability criteria of N = – 1.

Hence system is unstable.





+90°
–270°
Positive
angle
0°
+180°
–180°
Reference
Negative
angle
+270°
–90°



+90°
–270°
anticlockwise
–180°
+180
0
0°
+360°
clockwise
–90°
+270°

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = M(s)$$

Put $s = j\omega$, $M(j\omega) = \dfrac{G(j\omega)}{1+G(j\omega)}$

Let $G(j\omega) = X + jY$, where, X = Real part of $G(j\omega)$.

Y = Imaginary part of $G(j\omega)$.

$$\therefore M(j\omega) = \frac{X+jY}{1+X+jY} = \frac{\sqrt{X^2+Y^2}\angle \tan^{-1}\dfrac{Y}{X}}{\sqrt{(1+X)^2+Y^2}\angle \tan^{-1}\dfrac{Y}{1+X}} = \frac{\sqrt{X^2+Y^2}}{\sqrt{(1+X)^2+Y^2}}\angle\left(\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right)$$

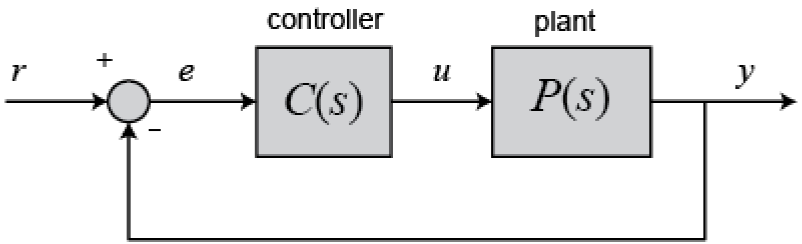
$x+jy$, $|\text{magnitude}| = \sqrt{x^2+y^2}$, $\angle$ Phase angle $= \tan^{-1}(y/x)$ $j^2 = -1$, $j = \sqrt{-1}$, $j^3 = -j$

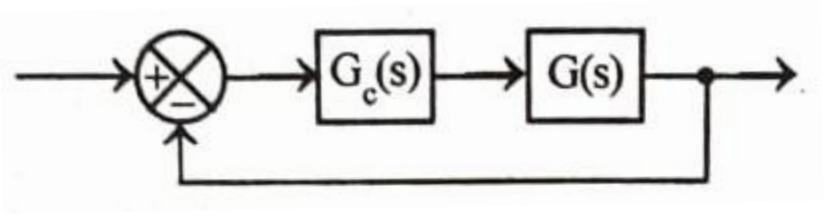
Controller

A controller is the most important component of the control system. It is responsible for the performance of the control system. It is a device or an algorithm that works to maintain the value of the controlled variable at set point

P, PI, PD and PID controller

A controller with transfer function Gc(s) can be introduced in cascade with open loop transfer function G(s) as shown in fig. to modify the transient and steady state response of the system





P-controller and its characteristics

The proportional controller is a device that produces an output signal which is proportional to the input signal.

Proportional controller (Kp) reduces the rise time, increases the overshoot, and reduces the steady-state error

The proportional controller improves the steady state tracking accuracy, disturbance signal rejection and relative stability. It also decreases the sensitivity of the system to parameter variations.

Transfer function of P-controller, $\frac{U(s)}{E(s)} = K_p$

Kp= Proportional gain

PI-controller and what are its effect on system performance

The PI-controller is a device that produces an output signal consisting of two terms-one proportional to input signal and the other proportional to the integral of input signal.

Integral control (Ki) tends to decrease the rise time, increase both the overshoot and the settling time, and reduces the steady-state error

The introduction of PI-controller in the system reduces the steady state error and increases the order and type number of the system by one

T.F of PI Controller $$G_c(s) = K_p + \frac{K_i}{s} = \frac{K_p\,(s + K_i / K_p)}{s}$$

PD-controller and what are its effect on system performance

The PD-controller is a device that produces an output signal consisting of two terms-one proportional to input signal and the other proportional to the derivative of input signal.

The PD-controller increases the damping of the system which results in reducing the peak overshoot

Derivative control (Kd) tends to reduce both the overshoot and the settling time

T.F of PD Controller $G_c(s) = K_p + K_d s = K_d\,(s + K_p / K_d)$

PID controller and what are its effect on system performance

The PID controller is a device which produces an output signal consisting of three terms- one proportional to input signal, another one proportional to integral of input signal and the third one proportional to derivative of input signal

The PID controller stabilises the gain, reduces the steady state error and peak overshoot of the system.

T.F of PID Controller $G_c(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$

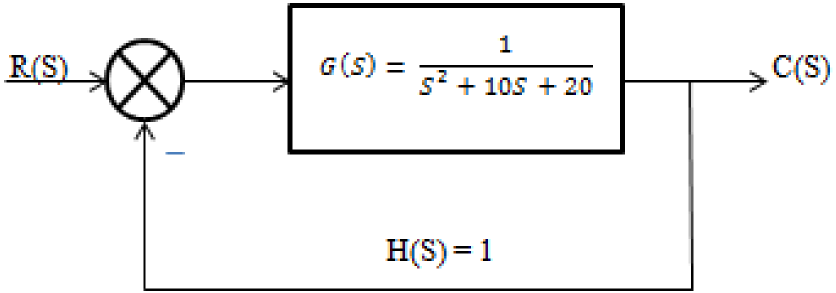
When you are designing a PID controller for a given system, follow the steps shown below to obtain a desired response.

1. Obtain an open-loop response and determine what needs to be improved
2. Add a proportional control to improve the rise time
3. Add a derivative control to reduce the overshoot
4. Add an integral control to reduce the steady-state error
5. Adjust each of the gains Kp Ki and Kd until you obtain a desired overall response

Consider the system with open loop transfer function & unity feedback system. $G(S) = 1 / (S^2+10S+20)$

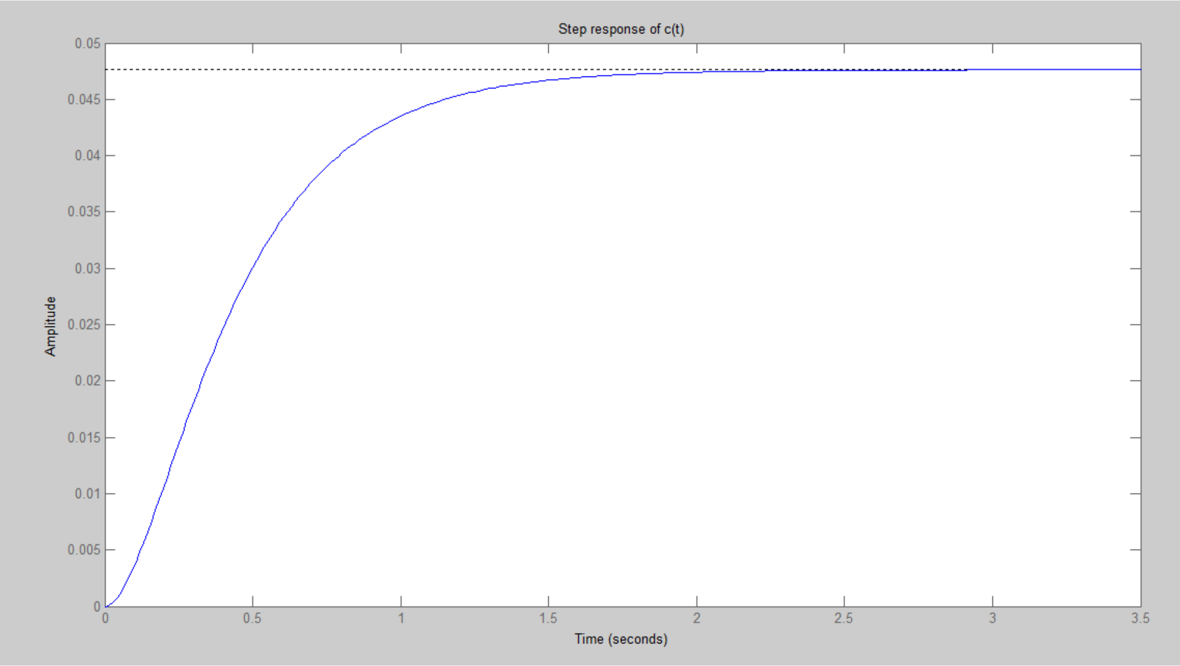
Closed loop transfer function can be obtained by using the relation $CS/R(S)=G(S)1+GSH(S)$

$$CS/ R(S)=1 / (S^2+10S+21)$$




```
clc
% prg c(t)
num = [1];
den = [1 10 21];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

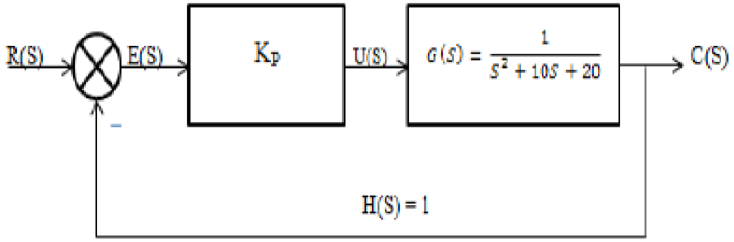
Response of second order system



Time domain specification parameter	Simulated values
Delay Time	0.383s
Rise Time	0.94s
Peak Time	2.34s
Settling Time	2.37s
Peak Overshoot	----
Steady State Error	0.9524

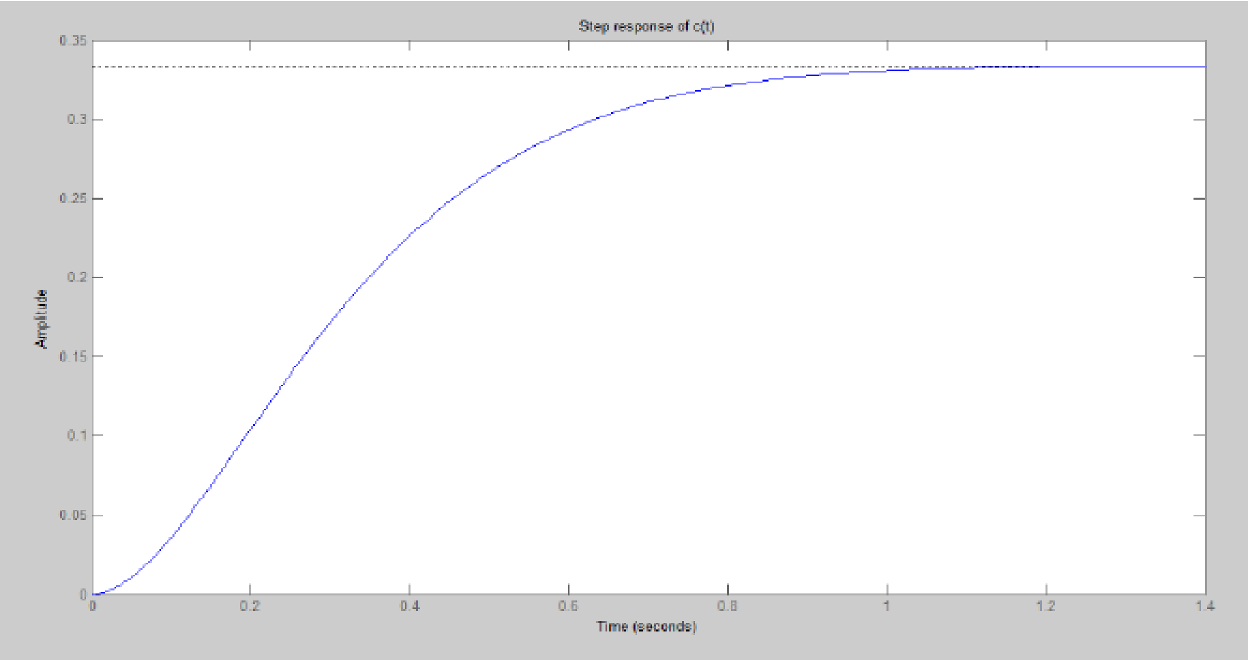
Effect of proportional (P) controller on the response

- A controller is the most important component of the control system. It is responsible for the performance of the control system. It is a device or an algorithm that works to maintain the value of the controlled variable at set point.
- In Proportional Controller the actuating signal is proportional to the error signal. The error signal is the difference between reference input signal and feedback signal obtained from output.
- Closed Loop Transfer Function $\frac{C(S)}{R(S)} = \frac{K_P}{S^2+10S+20+K_P}$
- The proportional controller <u>Reduces the rise time</u>
- <u>Reduces the steady state error</u>
- <u>Increases the peak overshoot</u>




```
clc
% prg c(t)
num = [10];
den = [1 10 30];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Response of P controller



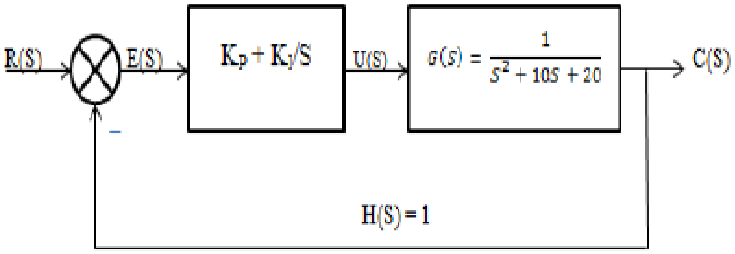
Time domain specification parameter	Simulated values
K_P	10
Delay Time	0.294s
Rise Time	0.663s
Peak Time	1.29s
Settling Time	1.29s
Peak Overshoot	----
Steady State Error	0.66

Effect of proportional Integral (PI) controller on the response

- In PI controller the actuating signal consists of proportional-error signal along with the integral of the error signal.
- Closed Loop Transfer Function

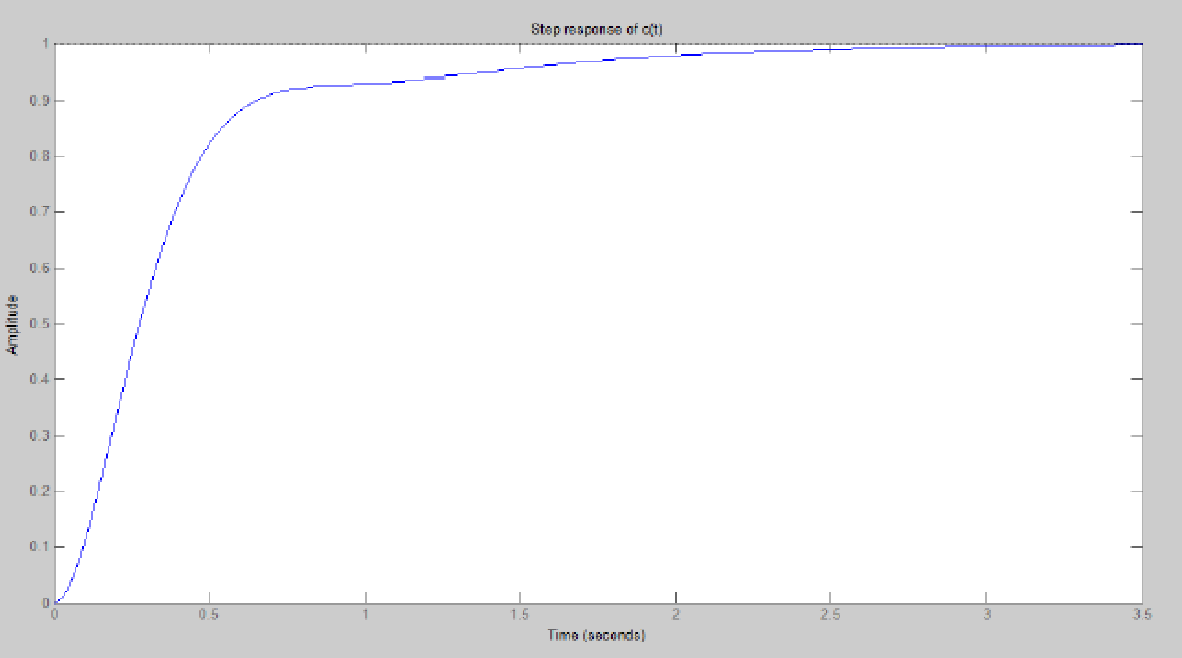
$$\frac{C(S)}{R(S)} = \frac{SK_P + K_I}{S^3 + 10S^2 + (20 + K_P)S + K_I}$$

- The PI controller reduces the steady state error.
- Increases the settling time.
- Improves the transient response.




```
clc
% prg c(t)
num = [30 50];
den = [1 10 50 50];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

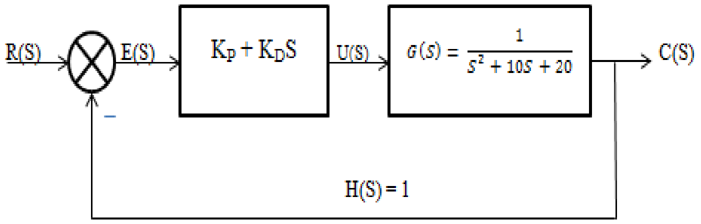
Response of PI controller



Time domain specification parameter	Simulated values
K_P	30
K_I	50
Delay Time	0.271s
Rise Time	0.645s
Peak Time	3.41s
Settling Time	3.41s
Peak Overshoot	---
Steady State Error	0.004

Effect of proportional Derivative (PD) controller on the response

- In PD controller the actuating signal consists of proportional error signal and also the derivative of error signal.
- Closed Loop Transfer Function

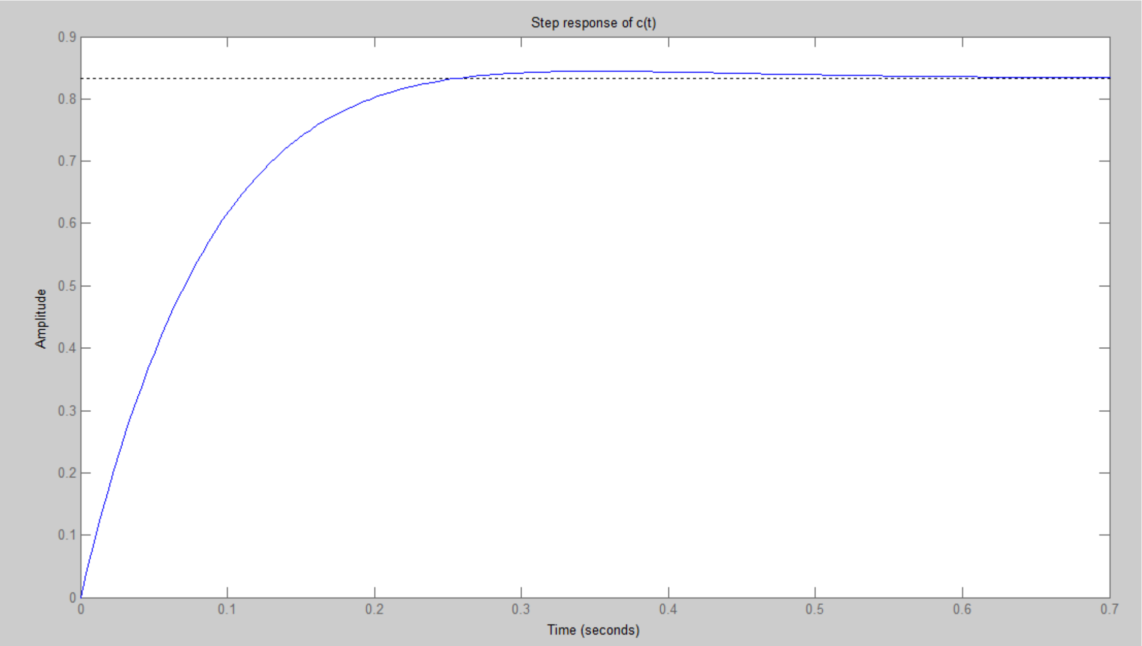


$$\frac{C(S)}{R(S)} = \frac{K_P + K_D S}{S^2 + (10 + K_D)S + 20 + K_P}$$

- The derivative controller reduces the peak overshoot and settling time.
- To control the steady state error the derivative gain K_D must be high.
- The PD controller reduces the response times of the system and can make it susceptible to noise.

```
clc
% prg c(t)
num = [10 100];
den = [1 20 120];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

Response of PD controller

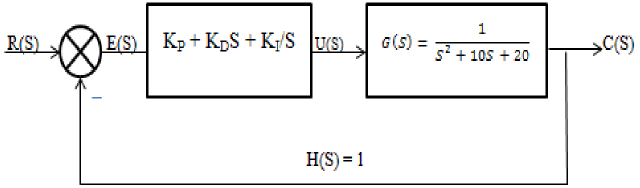


Time domain specification parameter	Simulated values
K_P	100
K_D	10
Delay Time	0.054s
Rise Time	0.159s
Peak Time	0.335s
Settling Time	0.504s
Peak Overshoot	---
Steady State Error	0.166

Effect of proportional Integral Derivative (PID) controller on the response

- **For PID controller, the actuating signal consists of proportional error signal and also the derivative and integral of error signal.**

- **Closed Loop Transfer Function**



$$\frac{C(S)}{R(S)} = \frac{S^2 K_D + SK_P + K_I}{S^3 + (10 + K_D)S^2 + (20 + K_P)S + K_I}$$

```
clc
% prg c(t)
num = [50 500 400];
den = [1 60 520 400];
G = tf(num, den)
step(G)
title('Step response of c(t)')
figure
```

- **The PID controller removes the steady state error and reduces the settling time while maintaining reasonable transient response.**

SYSTEM COMPENSATION

After carrying out the stability analysis of feedback control system, if it is found that the system performance is not satisfactory then certain modification or redesign has to be carried out

In redesigning control system, an additional components have to be incorporated. The additional component or device compensates for the performance deficiency and is called compensator.

The Process of redesign or addition of device is called compensation.

The only difference between controller and compensator is that controller additional elements like differentiator that subtracts the measured value of the output from the demanded value and mechanism for adjusting set point whereas compensator only includes elements that modify the dynamic behaviour of the control system

What is compensation?

The compensation is the design procedure in which the system behaviour is altered to meet the desired specifications, by introducing additional device called compensator.

The two types of compensation schemes employed in control system are series compensation and feedback or parallel compensation.

In order to satisfy the performance specifications such as time domain specifications, frequency domain specifications a compensator is introduced in open loop transfer function

The different types of compensators are used which may be mechanical, electrical hydraulic, pneumatic or any other type of device.

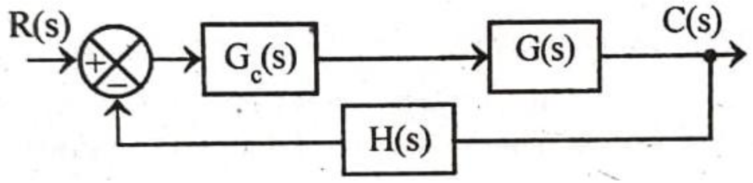
Types of System Compensation

There are four types of system compensation. They are

1. Cascade or Series compensation
2. Feedback. compensation
3. Input compensation and
4. Output compensation.

In cascade compensation the compensating element whose transfer function Gc(s) is placed in series with the forward transfer function G(s). It is also referred as series compensation

The series compensation is a design procedure in which a compensator is introduced in series with plant to alter the system behaviour and to provide satisfactory performance (i.e., to meet the desired specifications). The block diagram of series compensation scheme is shown in fig)



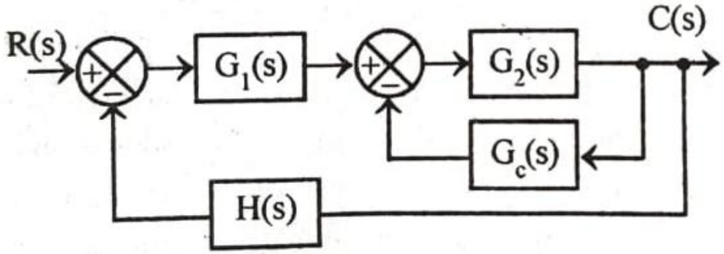
Gc(s) = Transfer function of series compensator

G(s) = Open loop transfer function of the plant.

H(s) = Feedback path transfer function.

What is feedback compensation?

The feedback compensation is a design procedure in which a compensator is introduced in the feedback path so as to meet the desired specifications. It is also called parallel compensation. The block diagram of feedback compensation scheme is shown in fig



$G_c(s)$ = Transfer function of feedback compensator

Feedback compensation $H(s)$ = Feedback path transfer function.

$G_1(s), G_2(s)$ = Open loop transfer function of the components of the plant.

Feedback compensation may be used to improve system stability, to reduce steady state error and improve speed of response of the system

Similarly the compensating device may be placed at the Input or along the output side as shown in Fig.(c) and (d) to improve the performance of the system. The selection of a particular compensation depends upon nature of the signals, power levels at various points, availability of the components and the cost considerations.