ATME College of Engineering

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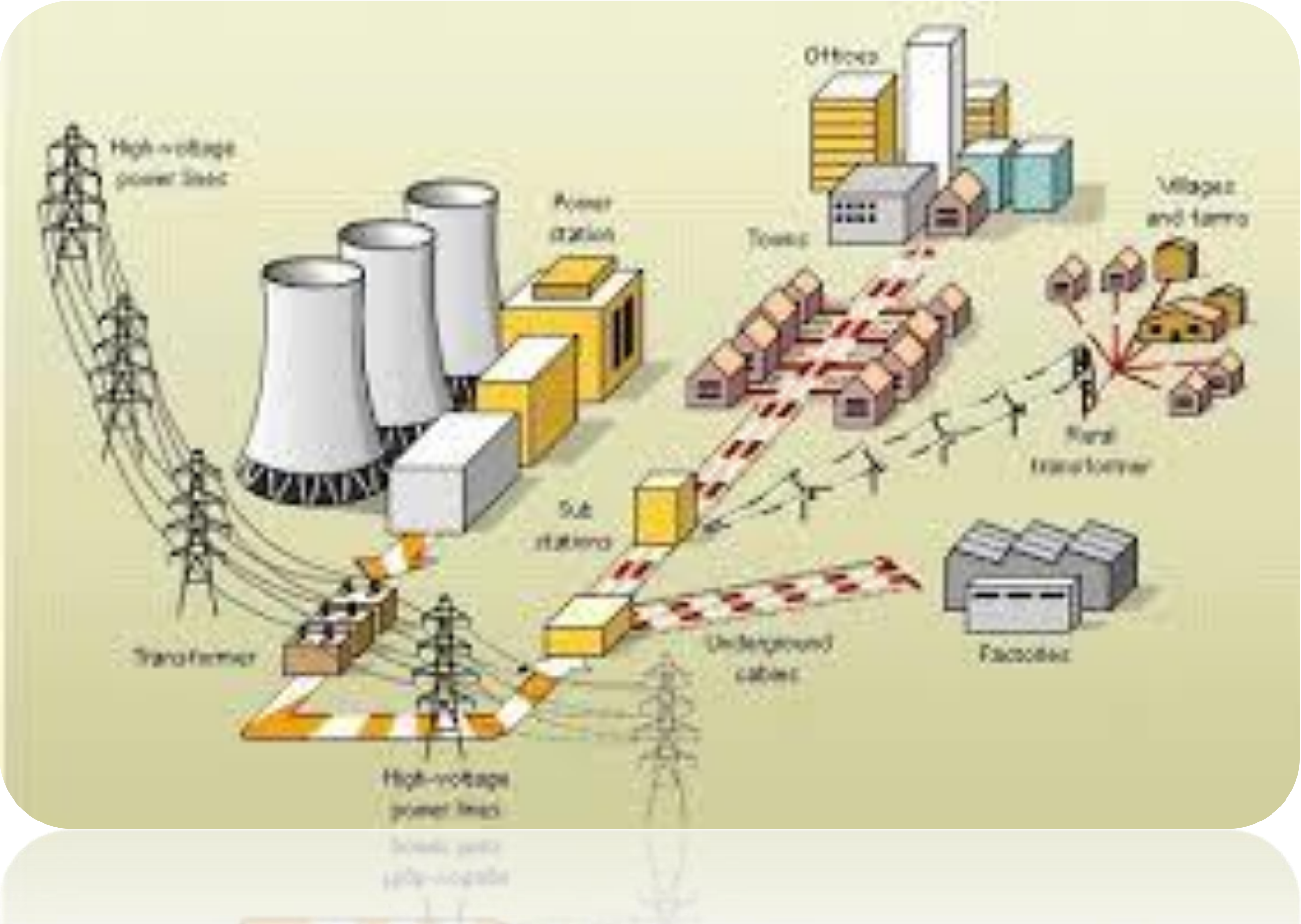
Power System Analysis 1 – BEE601



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STABILITY ANALYSIS

Understanding Stability, Types of Stability, Stability limits

and

Power Angle Equation of Synchronous Machines

Contents

1. Definition of various terms: Stability, Steady State Stability, Transient Stability, Dynamic Stability, Steady State Stability Limit and Transient State Stability Limit

2. Power Angle Equation of Synchronous Machines: Salient and Non-Salient Pole

3. Methods of improving steady state stability

4. Transient Stability: Dynamics of Machines – Swing equation

5. Swing Curve

6. Equal Area Criterion

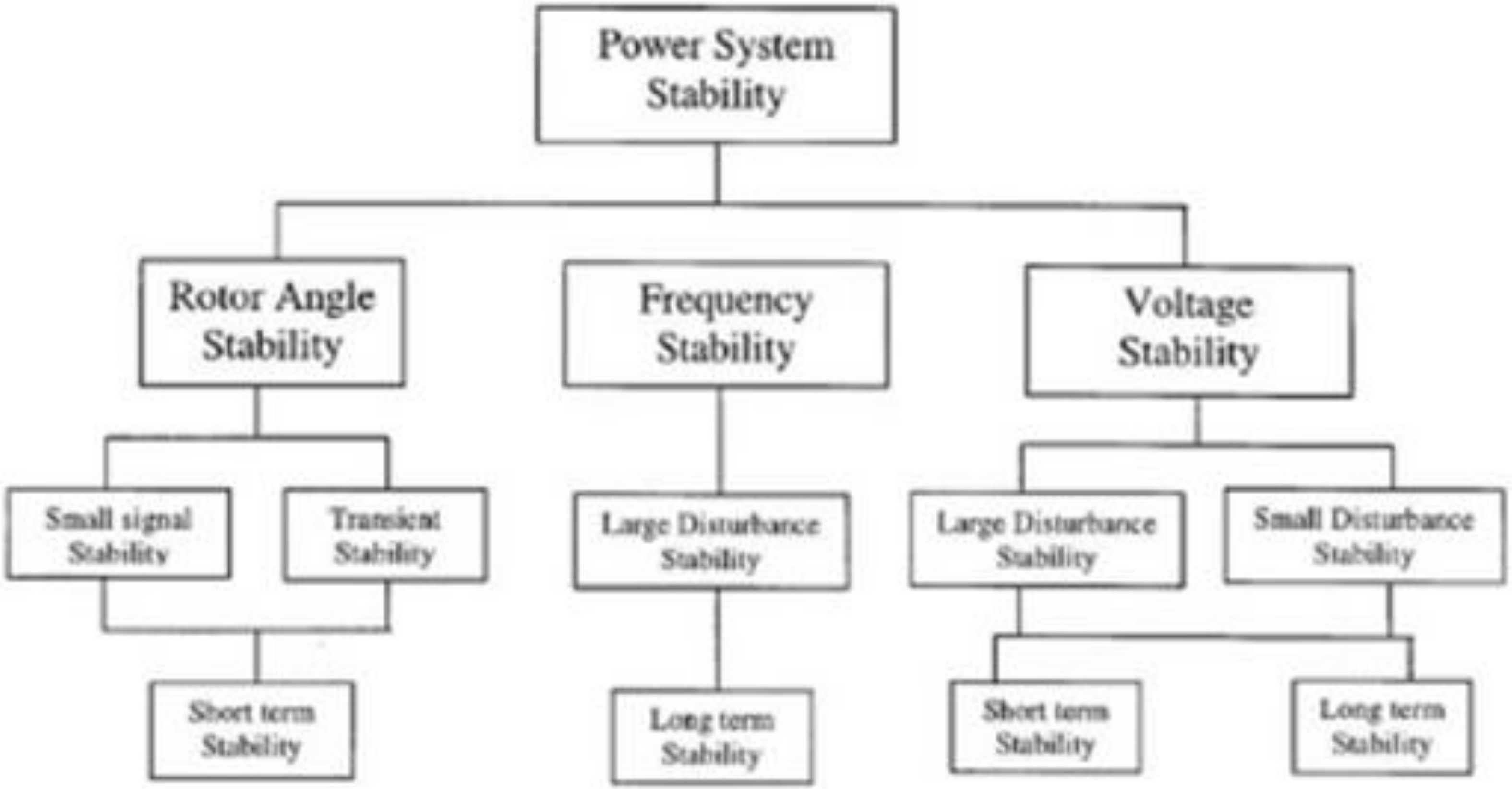
7. Methods of improving transient stability

Introduction

Power systems: Stability of a large interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance.

Instability: refers to a condition of loss of synchronism which results in large undesirable fluctuation of currents and voltages within the power system network.

Different types of Stability



Important definitions

Stability:

Stability is that attribute of the power system or part of the power system, which enables it to develop restoring forces between the elements thereof equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.

Types of Stability:

1. Steady State Stability
2. Transient Stability
3. Dynamic Stability

Steady State Stability:

This is the stability of the power system under consideration subjected to a **gradual or relatively slow** change in load.

Transient Stability:

This is the stability of the power system subjected to **a sudden large disturbance**. The large disturbance may be brought about by a sudden large change in load, faults in system or loss of generation in the system.

Dynamic stability:

This is the stability of the power system which denotes the artificial stability given to a system by the action of **automatic control device** like fast acting voltage regulators and governors.

Steady State Stability Limit (SSSL):

This refers to the **maximum flow of power** possible through a particular point in the system without loss of stability when the power is increased gradually.

Transient State Stability Limit (TSL):

This refers to the **maximum flow of power** possible through a particular point in the system without the loss of stability when a **sudden disturbance occurs.**

Infinite Bus:

A system having a constant voltage and a constant frequency regardless of the load on it is called an infinite bus-bar system or an infinite bus.

Steady State Stability Studies:

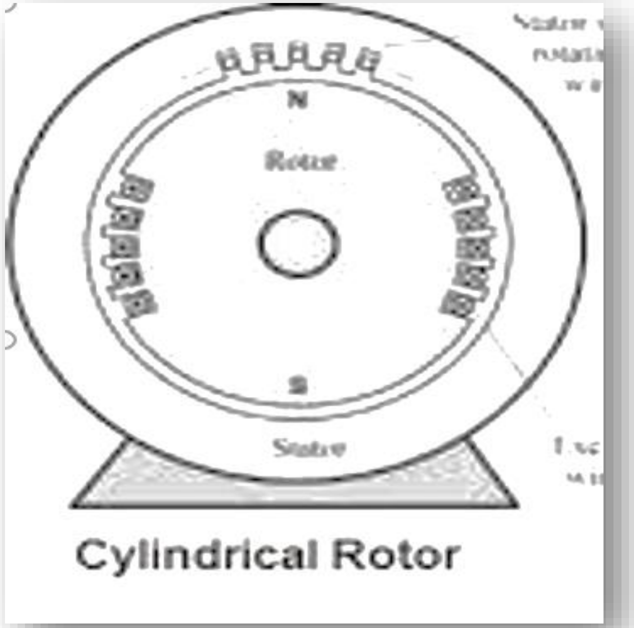
This is basically concerned with the study of stability of the power system under consideration subjected to a gradual change in load and thus obtain the maximum flow of power possible through a particular point in the system without loss of stability.

Power angle equation of synchronous machine:

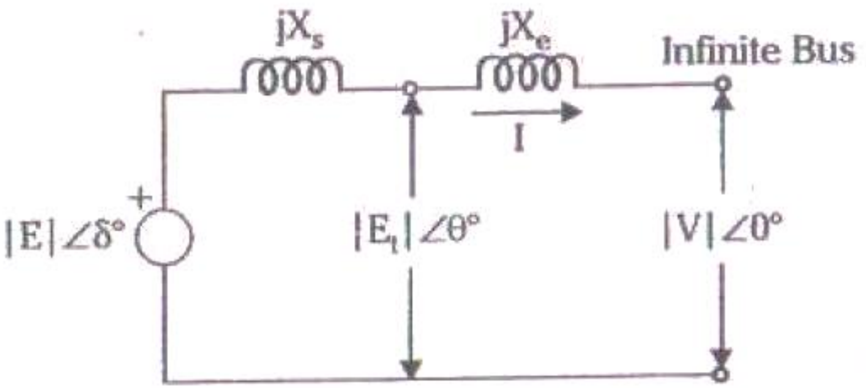
The synchronous machine is one of the most important element of a power system. It can be either a synchronous generator that feeds electrical power into the power system network or a synchronous motor that draws electrical power from the network. Further, the synchronous machines can be either Non-salient pole type or salient pole type.

Power-angle equation of a non salient pole synchronous machine:

- In non salient pole synchronous machine, the rotor consists of a cylindrical structure
- It has a number of slots at intervals along the outer periphery for accommodating the field coils
- Air gap is uniform all along the rotor periphery.
- Hence the flux linkage is also uniform thereby offering the same reactance for the flow of armature current at all places. This reactance is called as the **synchronous reactance (Xs)** of the machine.

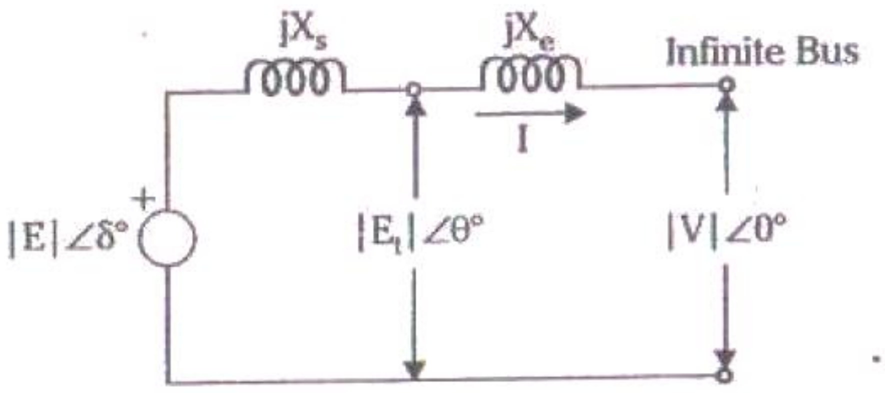


The single phase equivalent reactance diagram of a non salient pole synchronous generator connected through a transmission line to an infinite bus is shown.

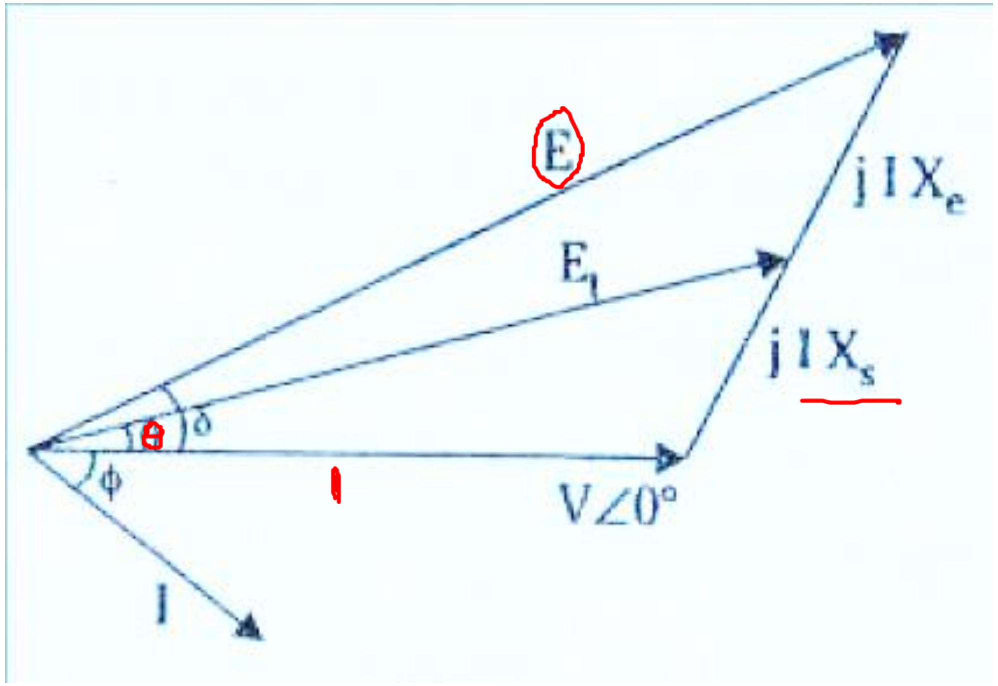


Let,

- ✓ $E\angle\delta$ - generated voltage in the machine
- ✓ where δ is load angle or torque angle or power angle
- ✓ X_s - synchronous reactance of the machine
- ✓ $E_t\angle\theta$ - voltage at the terminals of the machine
- ✓ X_e - reactance of the transmission line
- ✓ $V\angle 0$ - voltage at infinite bus which is taken as reference
- ✓ I - load current.



Let us suppose that the machine is operating for a large power factor load such that the load current lags the infinite bus voltage (reference) by an angle ϕ. The corresponding phasor diagram of the system.



Pf = Cosφ

For φ=0; cos 0= 1 (pf max, pf angle is less)

For φ=90; cos 90=0 (pf min, pf angle is high)

Let,

- ✓ $E\angle\delta$ - generated voltage in the machine
- ✓ where δ is load angle or torque angle or power angle
- ✓ X_s - synchronous reactance of the machine
- ✓ $E_t\angle\theta$ - voltage at the terminals of the machine
- ✓ X_e - reactance of the transmission line
- ✓ $V\angle 0$ - voltage at infinite bus which is taken as reference
- ✓ I - load current.

Referring to the phasor diagram,

$$E = V + I\,j(X_s + X_e)$$

or

$$I = (E - V) / j(X_s + X_e)$$

The net power delivered by the machine is given as

$$P = Re\left[V . I^*\right]$$

Substituting Eq. (6.1) in the above equation, we get

$$P = Re = \left[V . \left(\frac{E - V}{j(X_s + X_e)}\right)^*\right]$$

J=90

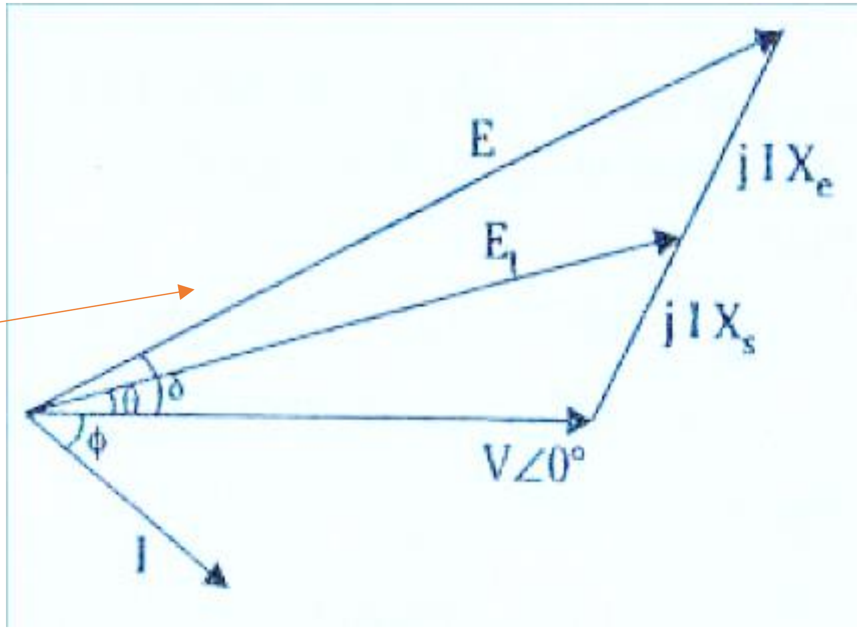
$$= Re\left[|V| \angle 0° \left(\frac{|E| \angle \delta \; - \; |V| \angle 0}{(X_s + X_e) \angle 90°}\right)^*\right]$$

$$= Re\left[|V| \angle 0° \left(\frac{|E| \angle -\delta \; - \; |V|}{(X_s + X_e) \angle -90°}\right)^*\right]$$

$$= \frac{|V||E|}{(X_s + X_e)} \cos(90° - \delta) - \frac{|V|^2}{(X_s + X_e)} \cancel{\cos 90°}$$

=0

$$= \frac{|V||E|}{(X_s + X_e)} \sin \delta$$



1^Θ = cos Θ +j sin Θ
Real (1^Θ) = cos Θ
Here Θ = 90 -δ

Cos(- Θ)= -cos Θ

Power Angle curve

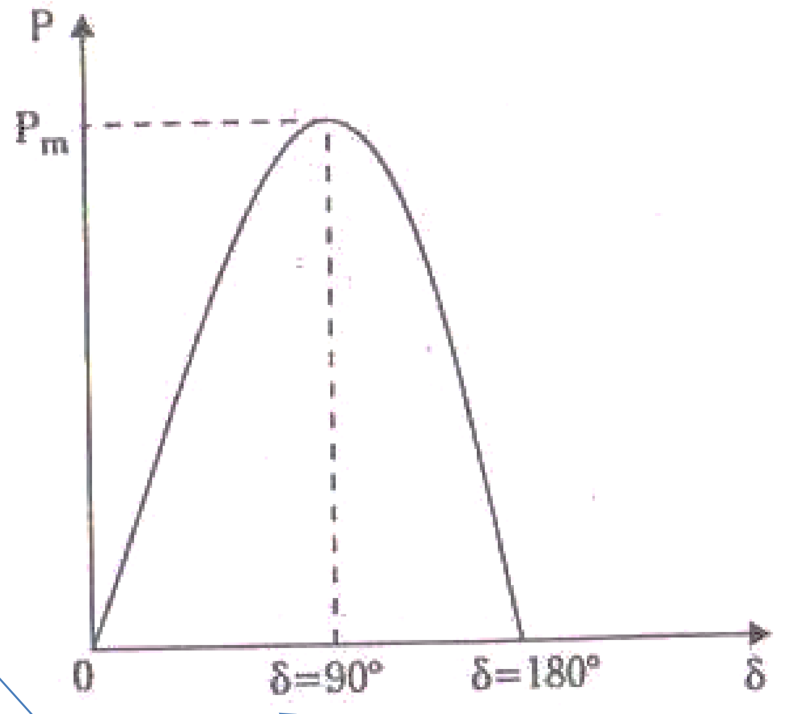
A graphical plot showing the variation of electrical power P against fixed values of E,V and reactance is called as the power angle curve. This is shown in fig. below.

$$P = \frac{|V||E|}{(X_s + X_e)} \sin\delta$$

The maximum power transfer occurs at δ = 90°.

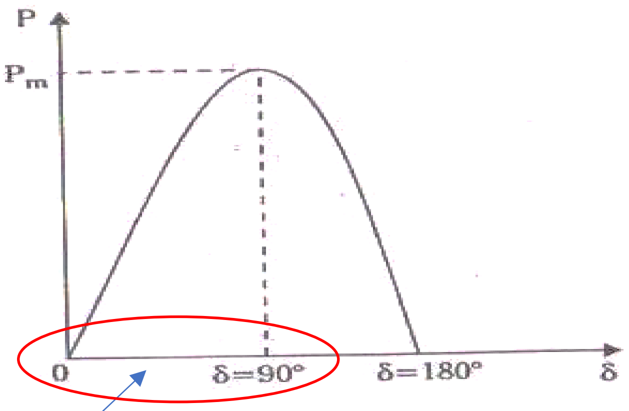
Where, Sin δ = sin 90 =1

The corresponding power is $P_{m} =$



- For values of δ > 90°, the power output of the machine reduces successively and finally the machine comes to standstill. Hence, P_m at which maximum power transfer occurs is called as the steady state stability limit (SSSL) of the machine.

- The machine operation is stable in the region **0° < δ < 90°** i.e the slope of the curve (dP / dδ) > 0. This term (dP / dδ) is called as **synchronizing power coefficient or machine stiffness.**



$$P = \frac{|V||E|}{(X_s + X_e)} \sin\delta$$

Sin 90 = 1,
Sin 180 =0

Sl No	Condition	Remarks
1.	(dP / dδ) > 0	Stability criterion.
2.	(dP / dδ) = 0.	SSSL reached
3.	(dP / dδ) < 0	System is unstable

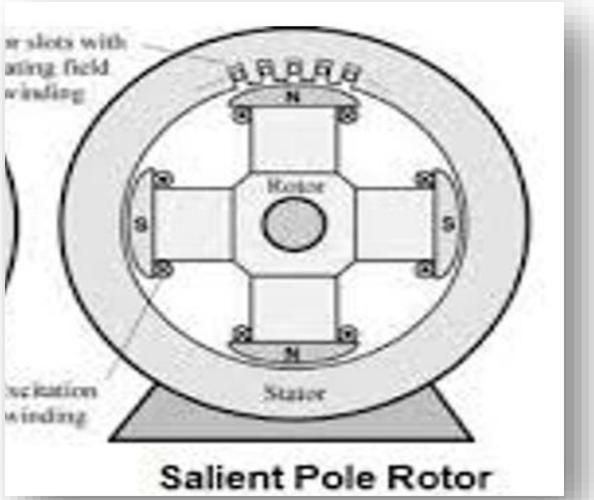
Note:

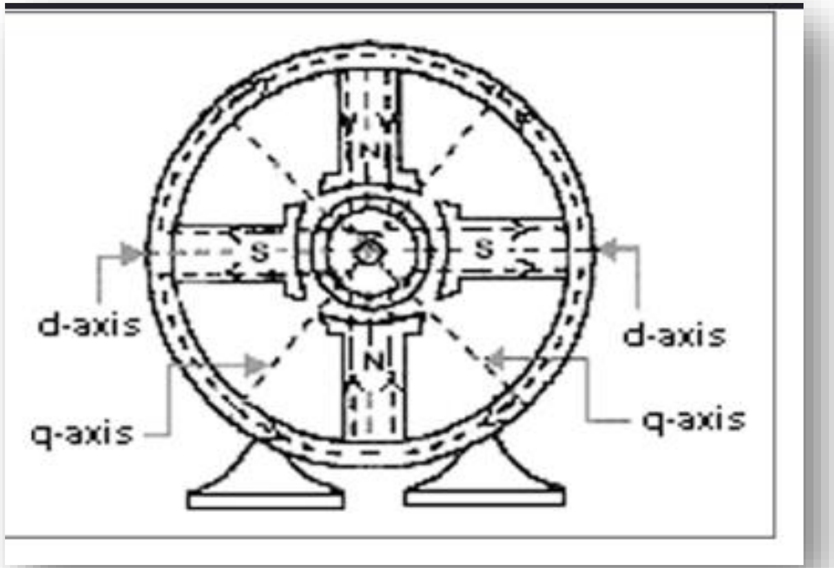
1) If the synchronous machine is connected directly to an infinite bus, then $X_e = 0$; then,
$P = [(|V|.|E|) / X_e]\sin\delta$

2) P_m, the maximum power is also called as pull out power of the machine.

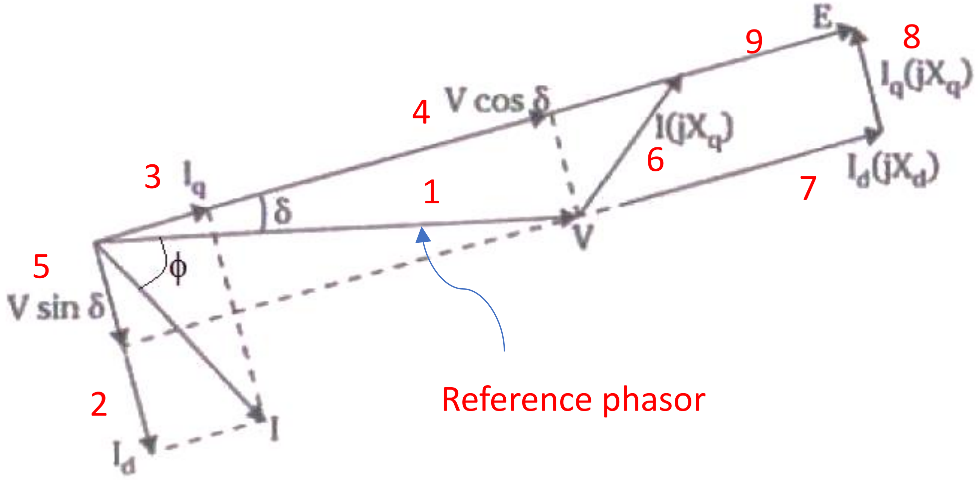
Power-angle equation of a salient pole synchronous machine:

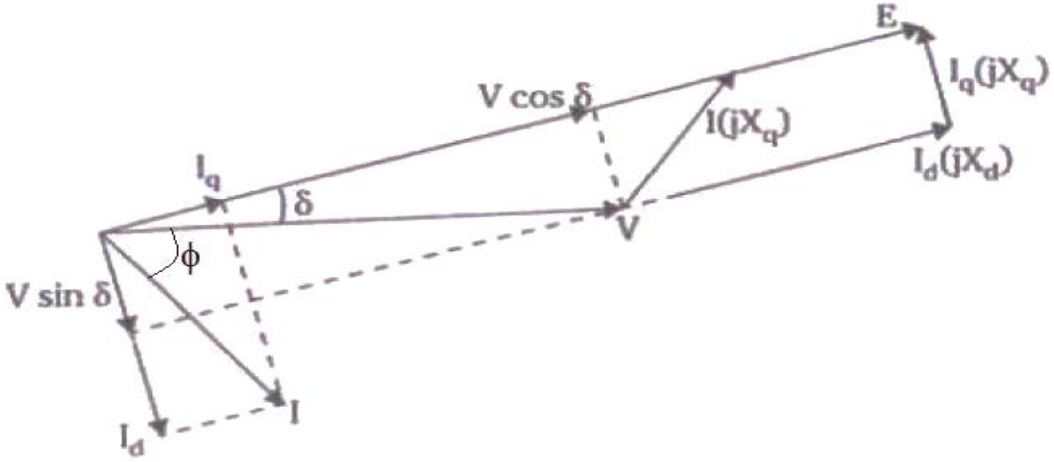
- A salient pole machine has a number of projecting (salient) poles.
- The air gap is not uniform along the rotor periphery. It is least along the axis of the main poles, called the direct axis and maximum along the axis of the inter poles, called the quadrature axis.
- Hence flux linkages is also non uniform leading to different values of reactances called direct axis reactance X_d and quadrature axis reactance X_q for the flow of armature current.





The phasor diagram of the machine neglecting its armature resistance is shown.





$E\angle\delta$ = Generated voltage in machine	$V\angle 0°$ = Voltage at infinite bus
δ = Load angle or power angle	X_d = synchronous reactance of the machine
X_q = reactance of the transmission line	I = current delivered at a lagging power factor of cosϕ

Note : Follow the step 1 to step 9 to write the phasor diagram

P=V*I

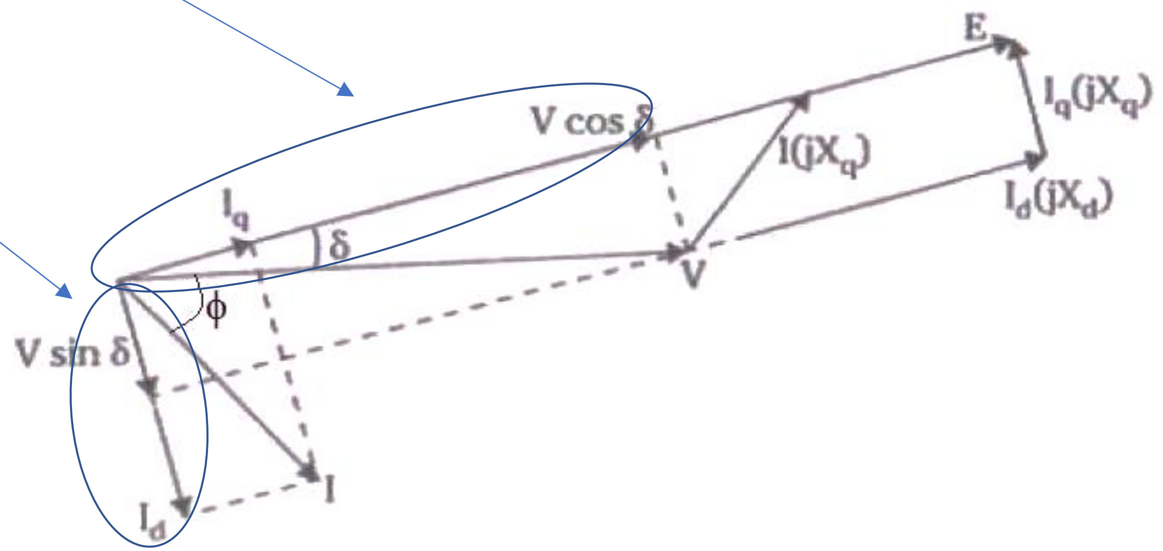
Power developed as P = | V | sin δ | I_d |+ | V | cos δ | I_q | (a)

$$|I_q \cdot X_q| = |V \sin \delta|$$

or $$|I_q| = \frac{|V| \sin \delta}{X_q} \qquad (1)$$

$$|I_d \cdot X_d| = |E - V \cos \delta|$$

or $$|I_d| = \frac{|E| - |V| \cos \delta}{X_d} \qquad (2)$$



Using eqn 1 , 2 in

$$P = |V| \cos \delta \frac{(|V| \sin \delta)}{X_q} + |V| \sin \delta \frac{(|E| - |V| \cos \delta)}{X_d}$$

$$= |V|^2 \frac{\sin 2\delta}{2 . X_q} + \frac{|V| . |E| \sin \delta}{X_d} - \frac{|V|^2 . \sin 2\delta}{2 . X_d}$$

Formula:
$\sin 2\theta = 2 \sin\theta \cos\theta$

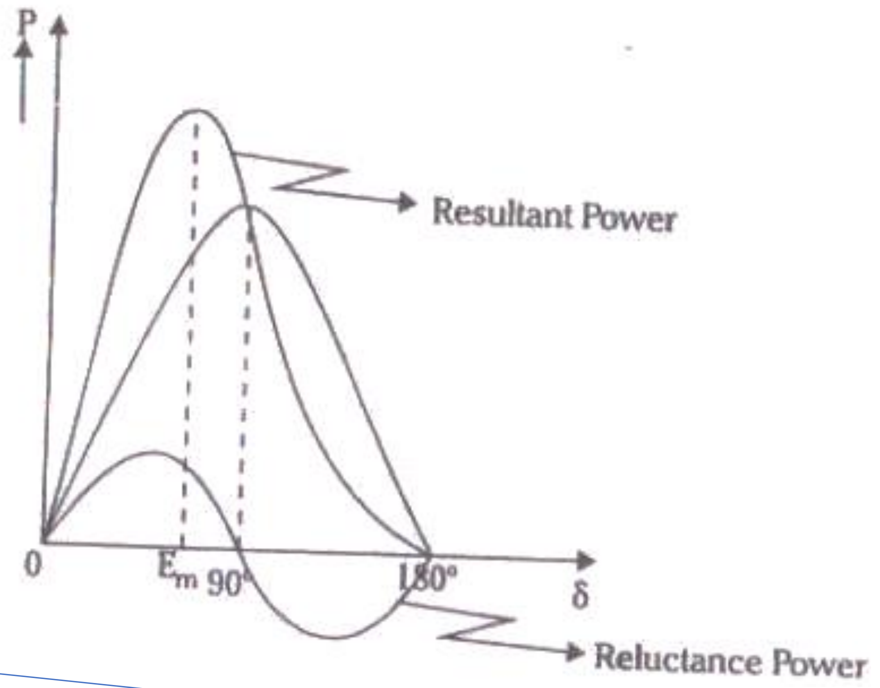
$$P = |V| \cos\delta \frac{(|V| \sin\delta)}{X_q} + |V| \sin\delta \frac{(|E| - |V| \cos\delta)}{X_d}$$

$$= |V|^2 \frac{\sin 2\delta}{2.X_q} + \frac{|V|.|E| \sin\delta}{X_d} - \frac{|V|^2.\sin 2\delta}{2.X_d}$$

$$= |V|^2 \frac{\sin 2\delta}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) + |V||E| \frac{\sin\delta}{X_d}$$

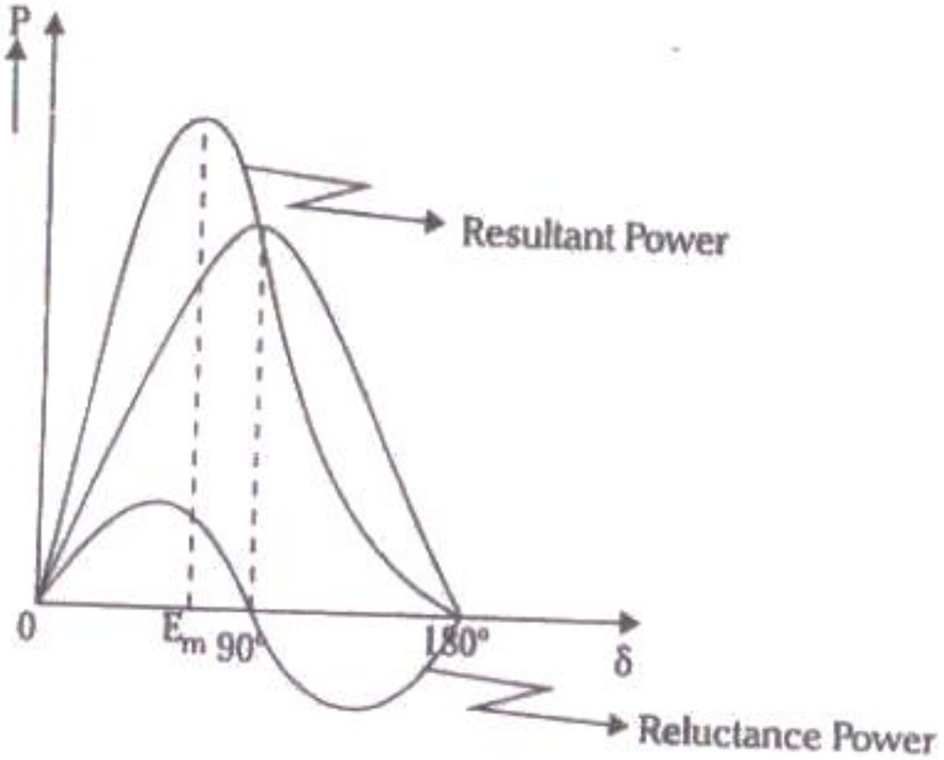
$$= \frac{|V|.|E|}{X_d}.\sin\delta + \frac{|V|^2.\sin 2\delta}{2}.\left(\frac{X_d - X_q}{X_d.X_q}\right)$$

Thus, $$P = \frac{|V|.|E|}{X_d}.\sin\delta + \frac{|V|^2 (X_d - X_q)}{2.X_d.X_q}.\sin 2\delta$$



- As evident from above equation, there is a fundamental and a second harmonic component of power.
- The first term is the same as that of non-salient pole machine with $X_s = X_d$. This constitutes the major part of power transfer.
- The second term is quite small (10-20%)compared to the first term and is known as **reluctance power.**

- The power angle curve of the machine is shown in figure.
- It is noticed that the maximum power output (SSSL) occurs at $\delta < 90°$(about 70°).
- This value of δ at which the power flow is maximum can be computed by equating the synchronizing power coefficient i.e $dP / d\delta$ to zero.



Find SSSL of a system consisting of a generator of equivalent reactance of 1 pu connected to an infinite bus through a series reactance of 0.5 pu. The terminal voltage of the generator is held at 1.2 pu and voltage of inifinite bus is 1.0pu

$$I = (E - V) / j(X_s + X_e)$$

For infinite bus Xs =0

$X_s = 1PU$

Now, from the diagram, it can be observed that

$$I = \frac{|E_t| \angle\theta - V \angle 0^\circ}{j X_e}$$

$$= \frac{1.2 \angle\theta - 1\angle 0^\circ}{1 \angle 90^\circ}$$

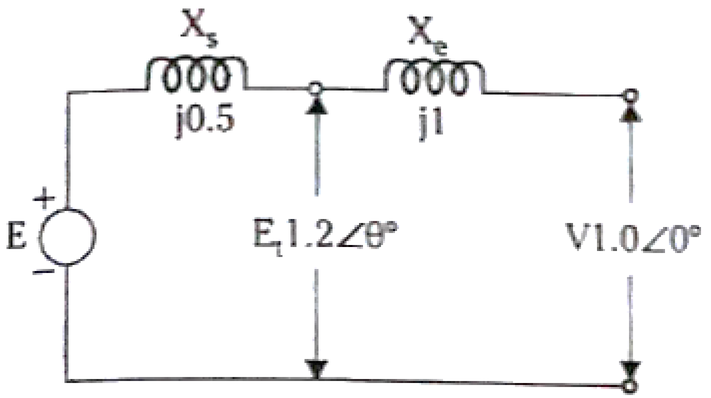
Also, $E = E_t + j.I.X_s$

$$= 1.2 \angle\theta + j\left(\frac{1.2 \angle\theta - 1\angle 0^\circ}{1 \angle 90^\circ}\right) 0.5$$

$$= 1.8 \angle\theta - 0.5$$

$$= (1.8 \cos\theta - 0.5) + j\, 1.8 \sin\theta$$

$$= (1.8 \cos\theta - 0.5) + j\, 1.8 \sin\theta$$



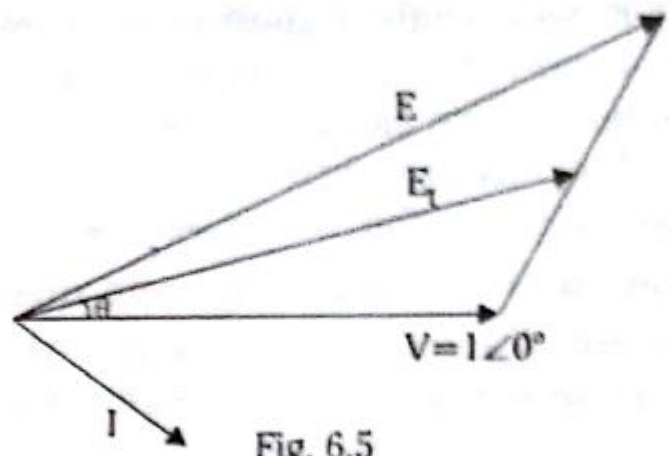


Fig. 6.5

SSSL is reached when generated emf E has an angle of 90° i.e. $\delta = 90°$. This imposes the condition that real part of Eq. (1) is zero. Thus

$$1.8\cos\theta - 0.5 = 0$$

or $\theta = 73.87°$.

Now $E_t = 1.2\angle 73.87° = 0.333 + j1.152$.

$$I = \frac{|E_t|\angle\theta - V\angle 0°}{jX_e}$$

$$I = \frac{1.2\angle 73.87° - 1.0}{1\angle 90°}$$

$$= \frac{0.333 + j1.152 - 1.0}{1\angle 90°}$$

$$= (1.152 + j0.667)$$

$$E = E_t + j.I.X_s$$

$$E = (0.333 + j1.152) + j0.5(1.152 + j0.667)$$

$$= -0.002 + j1.728$$

$$\simeq 1.728\angle 90°$$

Thus, the steady state stability limit is given by

$$\text{SSSL} = P_m = \frac{|V||E|}{X_s + X_e} = \frac{1 \times 1.728}{(0.5+1)}$$

$$= 1.152 \text{ p.u.}$$

Example 6.2 : A salient pole synchronous machine having $X_d = 0.6$p.u. and $X_q = 0.4$ p.u. per phase is operated from an infinite bus of voltage 1 p.u. If the excitation voltage is 1.1 p.u., find the SSSL and the angle at which it occurs.

$$P = \frac{|V||E|}{X_d} \cdot \sin\delta + \frac{|V|^2 (X_d - X_q)}{2.X_d.X_q} \cdot \sin 2\delta \quad = \frac{(1)(1.1)}{0.6} \sin\delta + \frac{(1)^2 (0.6-0.4)}{2\times 0.6\times 0.4} \cdot \sin 2\delta \quad = 1.83 \sin\theta + 0.4167 \sin 2\delta$$

When power is maximum, $\frac{dP}{d\delta} = 0$. Differentiating (1) and equating it to zero, we get

$$\frac{dP}{d\delta} = 1.83 \cos\delta + 0.8334 \cos 2\delta = 0$$

$$1.83 \cos\delta + 0.8334 (2\cos^2\delta - 1) = 0$$

or $$1.667 \cos^2\delta + 1.83 \cos\delta - 0.8334 = 0$$

This equation is quadratic in $\cos\delta$. Therefore, solving for $\cos\delta$, we get

$$\cos\delta = 0.3462$$

$$\delta = 69.7°$$

Substituting this value of δ in (1), we get the SSSL as

$$\text{SSSL} = P_m = 1.83 \sin(69.7°) + 0.4167 \sin(139.4°)$$

$$= 1.987 \text{ p.u.}$$

Thus, the SSSL of the machine is 1.987 p.u. It occurs at an angle of 69.7°.

Example 6.3 : A salient pole alternator has $X_d = 0.7$ p.u. and $X_q = 0.4$ p.u. If the machine is operating at normal voltage and full load at a power factor of 0.8 lag, to what value will the terminal voltage rise if the load is disconnected. Neglect the armature resistance.

In this problem, it is required to find the No-load emf E. Let the reference voltage by $V = 1\ \angle 0°$ p.u. and the power delivered by the machine is $P = 1$p.u.

Now, the armature current is given by the relation,

$$I = \frac{P}{V\cos\phi}\angle -\cos^{-1}\phi$$

$$= \frac{1}{1\times 0.8}\angle -\cos^{-1} 0.8$$

$$= 1.25\ \angle -36.87°\ \text{pu.}$$

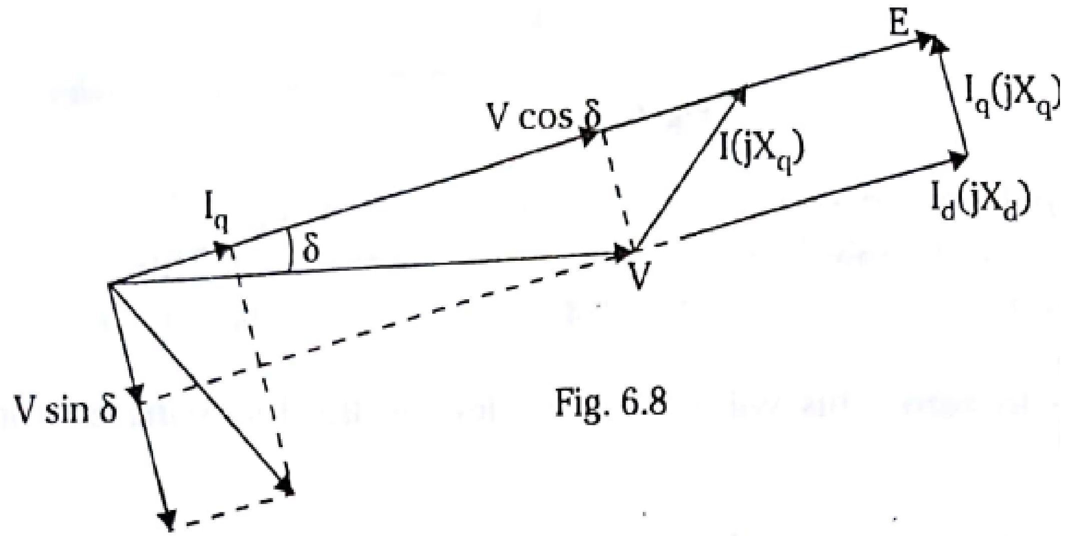


Fig. 6.8

From the figure, we can write

$$E' = V + I(jX_q)$$
$$= 1 + 1.25\angle -36.87° \times 0.4\angle 90°$$

$$= 1.36\angle 17.1° \text{ pu.} \qquad \therefore \quad \delta = 17.1°.$$

$$I_d = I\sin(\delta + \phi)$$
$$= 1.25\sin(17.1° + 36.87°)$$

$$= 1.01 \text{ pu.}$$

$$|E| = |E'| + I_d(X_q - X_q)$$
$$= 1.36 + 1.01(0.7 - 0.4)$$

$$= 1.663 \text{ pu.}$$

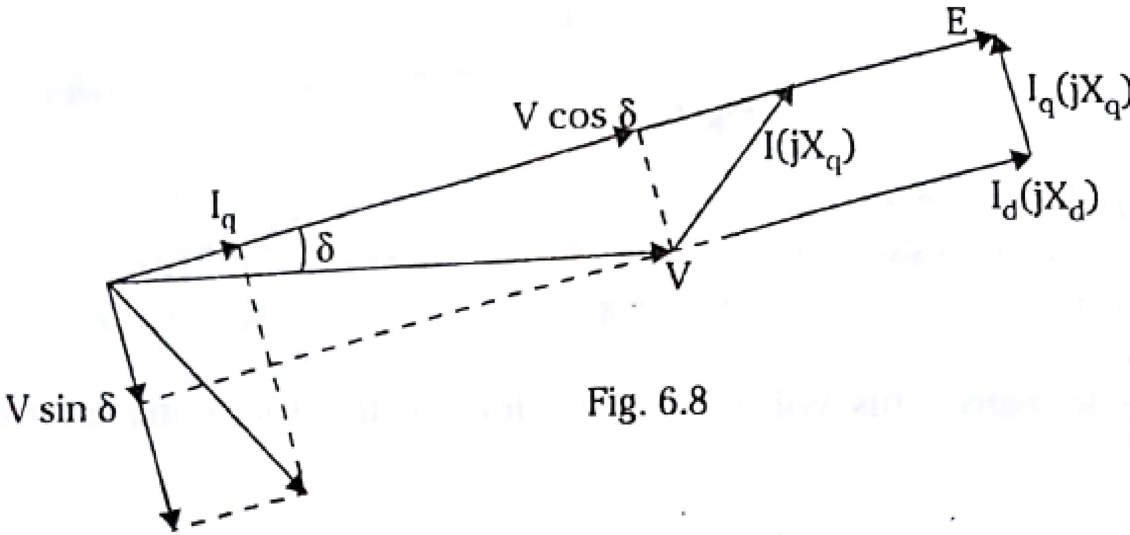


Fig. 6.8

Steady State Stability of a Two Machine System

Let $E_g \angle\delta°$ = Internal emf of the generator

$E_m \angle 0°$ = Back emf of the motor

X_g = reactance of generator

X_{T1}, X_{T2} = reactance of transformer 1 & 2 respectively

X_{TL} = reactance of transmission line

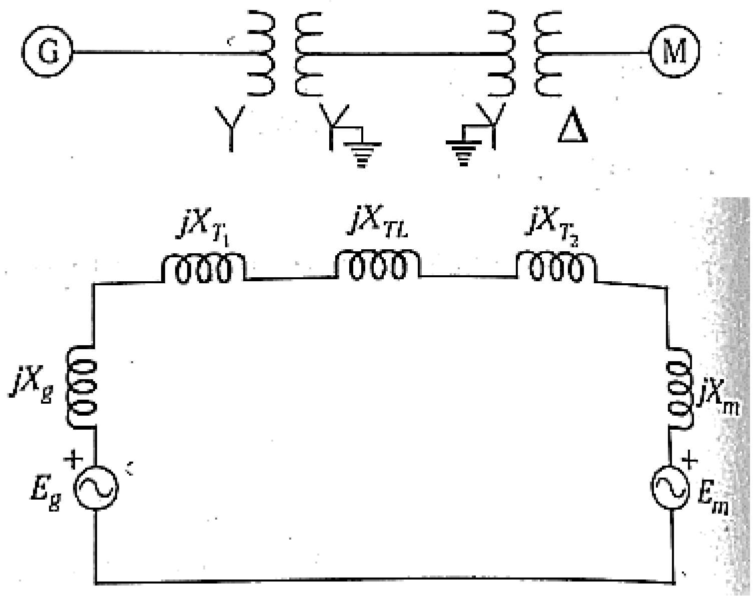
X_m = reactance of motor

Also, let us denote $X = X_g + X_{T1} + X_{TL} + X_{T2} + X_m$.
The phasor diagram of the system is shown in Fig. 6.13.
From the diagram, we can form the vector relation

$$E_g = E_m + j\,I\,X.$$

$$\text{or} \quad I = \frac{E_g - E_m}{j\,X}$$

$$P_g = P_{mot}$$



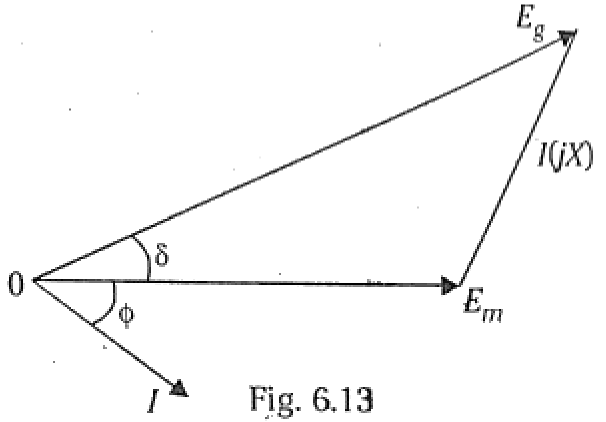


Fig. 6.13

Now, $P_g = Re\left(E_g\ I^*\right)$

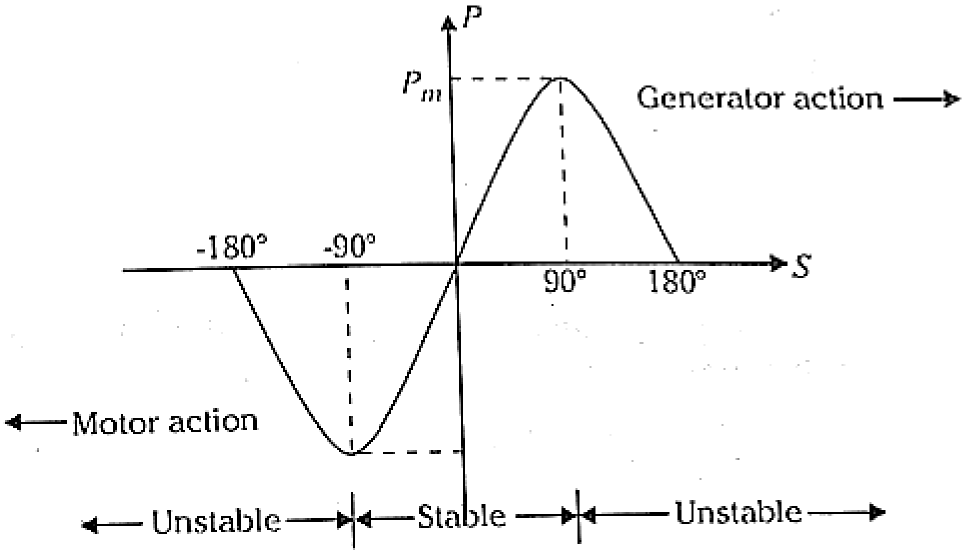
$$= Re\left[|E_g|\ \angle\delta \times \left(\frac{|E_g|\ \angle\delta - |E_m|\ \angle 0^\circ}{|X|\ \angle 90^\circ}\right)^*\right]$$

$$= Re\left[|E_g|\ \angle\delta \times \left(\frac{|E_g|\ \angle -\delta - |E_m|\ \angle 0^\circ}{|X|\ \angle -90^\circ}\right)\right]$$

$$= \frac{|E_g|^2}{|X|}\cos 90^\circ - \frac{|E_g|\,|E_m|}{|X|}\cos\left(90^\circ + \delta\right)$$

$$= -\frac{|E_g|\,|E_m|}{|X|}\left(-\sin\delta\right)$$

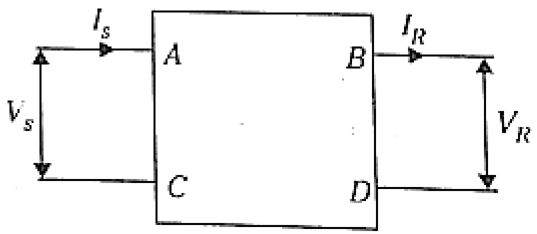
Thus, $P_g = \dfrac{|E_g|\,|E_m|}{|X|}\sin\delta$



$$P_m = \frac{|E_g|\,|E_m|}{|X|}$$

SSSL of a two terminal pair network represented by ABCD constants

The input voltage per phase (voltage at sending end) and the input current (current at sending end) of a transmission line can be expressed as :



$$V_s = A.V_R + B.I_R$$

$$I_s = C.V_R + D.I_R$$

where,

V_s = sending end voltage per phase = $|V_s| \angle\delta$

I_s = sending end current per phase

V_R = receiving end voltage per phase = $|V_R| \angle 0°$

I_R = receiving end current per phase

and A, B, C and D are called the **Generalised Circuit constants**. The values of A, B, C and D are determined as follows :

<u>On open circuit</u>

$$I_R = 0.$$

$$\therefore \quad A = \frac{V_s}{V_R} = |A| \angle\alpha \quad \& \quad C_\gamma = \frac{I_s}{V_R} = |C| \angle\gamma$$

On short circuit

$$V_R = 0$$

$$\therefore \quad B = \frac{V_s}{I_R} = |B| \angle\beta^\circ \quad \& \quad D = \frac{I_s}{I_R} = |D| \angle\Delta$$

The power delivered by the system is

$$P = Re\left[V_R \; I_R^*\right]$$

$$I_R = \frac{V_s - A.V_R}{B}$$

$$= \frac{|V_s| \, \delta| - |A| \angle\alpha \, . \, |V_R| \angle 0^\circ}{|B| \angle\beta}$$

$$I_R = \frac{|V_s|}{|B|} \angle\delta - \beta - \frac{|A| \, . \, |V_R|}{|B|} \angle\alpha - \beta$$

$$\therefore \quad I_R^* = \frac{|Vs|}{|B|} \angle\beta - \delta - \frac{|A| \, . \, |V_R|}{|B|} \angle\beta - \alpha$$

$$P = Re\left[|V_R| \angle 0^\circ \times \left(\frac{|V_s|}{|B|} \angle\beta - \delta - \frac{|A| \, . \, |V_R|}{|B|} \angle\beta - \alpha\right)\right]$$

$$\text{Thus} \quad P = \frac{|V_R| \,.\, |V_s)}{|B|} \,.\, \cos(\beta - \delta) - \frac{|A| \,.\, |V_R|^2}{|B|} \cos(\beta - \alpha)$$

The maximum power that can be transferred occurs at $\delta = \beta$. Hence, the SSSL of the system is

$$\text{SSSL} = \frac{|V_R| \,.\, |V_s|}{|B|} - \frac{|A| \,.\, |V_R|^2}{|B|} \cos(\beta - \alpha)$$

Methods of improving SSSL:

For a two machine system, $SSSL = \frac{|E_g|.|E_m|}{|X|}$.

As indicated by the equation, the SSSL can be increased by

1. Increasing either of the voltages $|E_g|$ or $|E_m|$. This can be achieved by increasing the excitation to the generator or motor or both.
2. Reducing the reactance between the transmission and receiving points. If the transmission lines are of sufficiently high reactance, the stability limit cab be raised by using two parallel lines which incidentally also increases the reliability of the system.
   - Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the SSSL by decreasing the line reactance.
   - The use of bundled conductors is another method of reducing the line reactance and hence improving the SSSL.

Transient State Stability Studies:

This is basically concerned with the study of stability of the power system under consideration following a **sudden disturbance** (with sudden and large changes) on a power system, and thus obtain the maximum flow of power possible through a particular point in the system without loss of stability.

The sudden disturbances causes rotor speeds and rotor angular differences to undergo fast changes whose magnitudes are dependent on the severity of disturbance.

For a large disturbance, changes in angular differences δ may be so large that the machines may fall out of step. Thus, the transient stability of a system predominantly depends upon the dynamics of the synchronous machine.

Dynamics of a synchronous machine

The kinetic energy of a rotor is

$$K.E. = \frac{1}{2} . I\,\omega^2$$

Where

I = moment of inertia in kg.m^2

ω = angular speed in rad/sec.

The angular momentum is

$$M = I\,.\,\omega$$

$$\therefore \quad K.E. = \frac{1}{2} M\omega \text{ joules.}$$

$$H = \frac{\text{stored energy in megajoules}}{\text{machine rating in mega volt-amperes}}$$

Let G = rating of the machine in MVA, then

GH = Stored energy in mega joules

Hence, $GH = K.E.$

$$= \frac{1}{2} I\,\omega^2 = \frac{1}{2} M\omega \text{ megajoules (MJ)}$$

Where ω = $2\pi f$ elect-rad/sec

= 360 ϕ elec.deg/sec

$$\text{or} \quad GH = \frac{1}{2} M(360 f)$$

$$\therefore \quad M = \frac{GH}{180\,.\,f} \quad \text{MJ-sec./elect.deg.}$$

- 'M' is called as the inertia constant, relates two inertia constants of the machine.
- For stability studies it is necessary to determine 'M' which depends upon the size and speed of the machine.
- But 'H' has a characteristics value of range of values for each class of machines

Typical values of 'H' are indicated below:

- ✓Cylindrical rotor alternators: 4 -10
- ✓Salient pole alternators: 2-3
- ✓Salient pole sync motors: 0.5-2

The inertia constant M of the equivalent machine is the sum of the inertia constants of the individual machines, i.e.

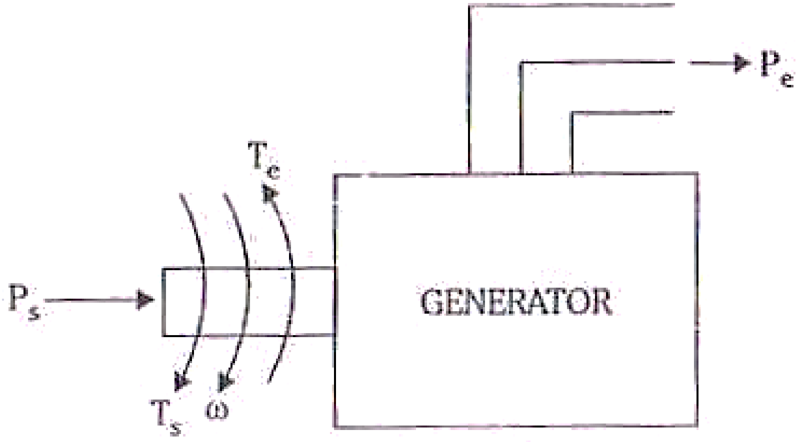
$$M_{eq} = M_1 + M_2 + \ldots\ldots + M_n$$

or $$H_{eq} \cdot G_{Base} = H_1 G_1 + H_2 G_2 + \ldots\ldots + H_n G_n$$

or $$H_{eq} = \frac{H_1 G_1}{G_{Base}} + \frac{H_2 G_2}{G_{Base}} + \ldots\ldots\ldots + \frac{H_n G_n}{G_{Base}}$$

Swing Equation:

- The load angle or the torque angle δ depends upon the loading of the machine.
- Larger the loading, larger is the value of the torque angle.
- If some load is added or removed from the shaft of the synchronous machine, the rotor will decelerate or accelerate respectively with respect to the synchronously rotating stator field and a relative motion begins.
- It is said that the rotor is swinging with respect to the stator field.
- The equation describing the relative motion of the rotor (load angle δ) with respect to the stator field as a function of time is called as swing equation.



Generator receives mechanical power P_s at torque T_s and rotor speed ω via shaft from the prime mover.
It delivers electrical power P_e to the power system network via the bus bars.
The generator develops electromechanical torque T_e in opposition to T_s.

Assuming that winding and friction losses to be negligible, the accelerating torque on the rotor is given by $T_a = T_s - T_e$

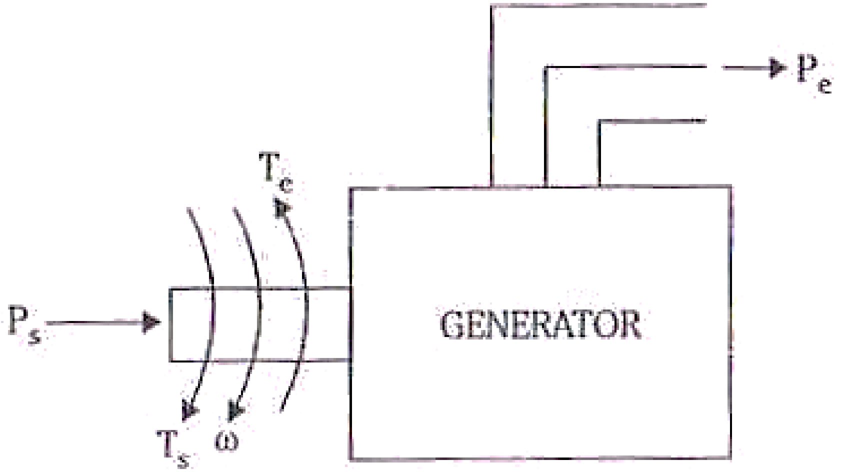
Multiplying by ω on both sides, we get $\omega T_a = \omega T_s - \omega T_e$.

$\boxed{P = T * \omega}$

But, $\omega T_a = P_a$ = accelerating power

$\omega T_s = P_s$ = mechanical power input

$\omega T_e = P_e$ = electrical power output assuming that power loss is negligible.



Therefore, we get, $P_a = P_s - P_e$

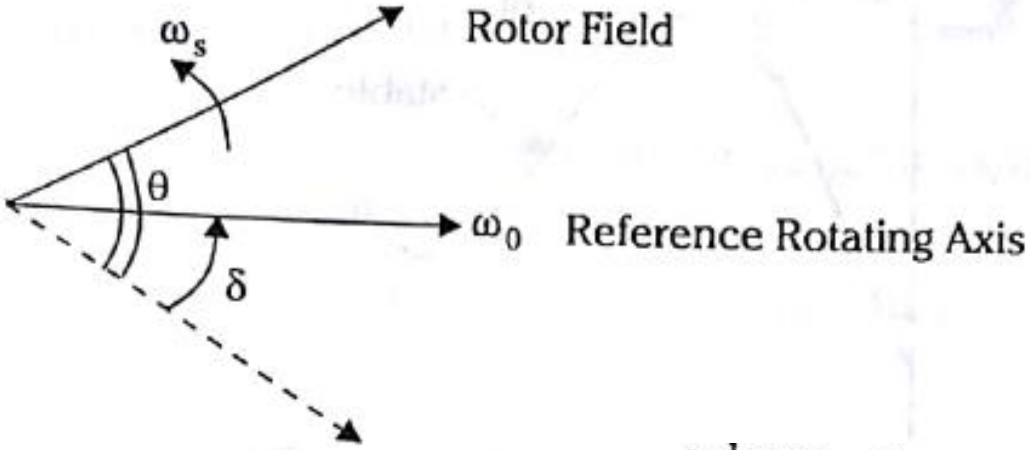
Under steady state conditions, $P_s = P_e$, so that $P_a = 0$.

When $P_s - P_e$ balance is disturbed, the machine undergoes dynamics governed by,

$$P_a = T_a \, \omega = I.\alpha.\omega = M.\frac{d^2\theta}{dt^2}$$

where $\alpha = d^2\theta_m / dt^2$ is the angular acceleration of the rotor.

Since the. angular position Θ of the rotor is continually varying with time, it is more convenient to measure the angular position and velocity with respect to a synchronously rotating axis



$$\delta = \theta - \omega_0 t$$

where, ω_0 = angular velocity of the reference rotating axis.
δ = rotor angular displacement with respect to the stator field.

Differentiating wrt t

$$\frac{d\delta}{dt} = \frac{d\theta}{dt} - \omega_0$$

$$\frac{d^2\delta}{dt^2} = \frac{d^2\theta}{dt^2}$$

Combining the above equations

$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e$$

$$P_a = T_a\,\omega = I.\alpha.\omega = M.\frac{d^2\theta}{dt^2}$$

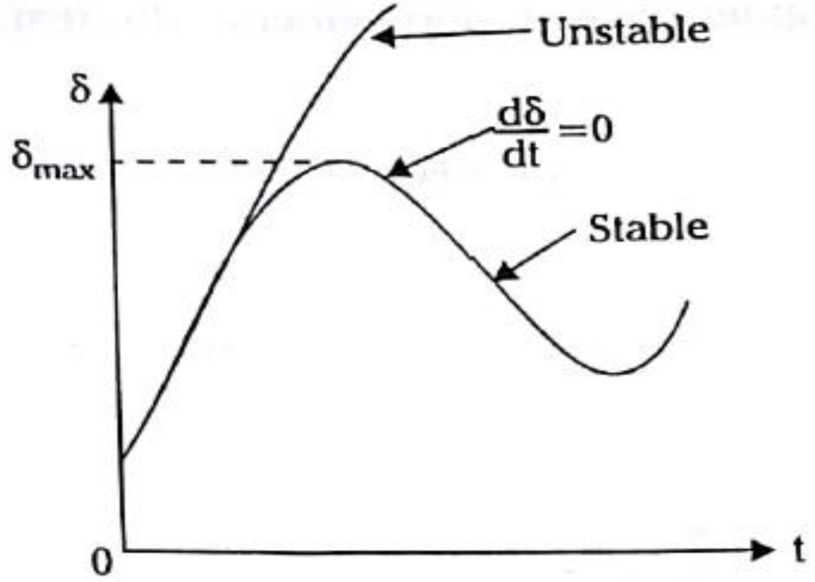
This equation is called as the swing equation of the synchronous machine. When the machine is connected to the infinite bus bars, then

$$P_e = \frac{|E|\,|V|}{X}\,.\,\sin\delta = P_m\,\sin\delta..$$

$$\text{or}\quad M\frac{d^2\delta}{dt^2} = P_s - P_m\,\sin\delta$$

Swing Curve

- The solution of swing equation gives the relation between rotor angle 'δ' as a function of time *"t "*.
- The plot of 'δ' versus ·*"t"* is called as swing curve.
- The exact solution of the swing equation is very tedious task.
- Normally, step-by-s tep method or any numerical solution techniques like Euler's method, Runge-Kutta's method are used for solving the swing equation.
- The swing curve is used to determine the stability of the system



For the stability of the system, $\frac{d\delta}{dt} = 0$.

The system will be unstable if $\frac{d\delta}{dt} > 0$ for a sufficiently long time (normally more than sec).

Example 6.9 : ***A two pole, 50 Hz, 11 kV turbo alternator has a rating of 100 MW, power factor 0.85 lagging. The rotor has a moment of inertia of 10,000 kg.m². Calculate H and M.***

The MVA rating of the alternator $= G = \dfrac{100}{0.85} = 117.65$ MVA.

$$\text{Kinetic Energy} = GH = \frac{1}{2} I \omega^2$$

Here,

$I = 10{,}000$

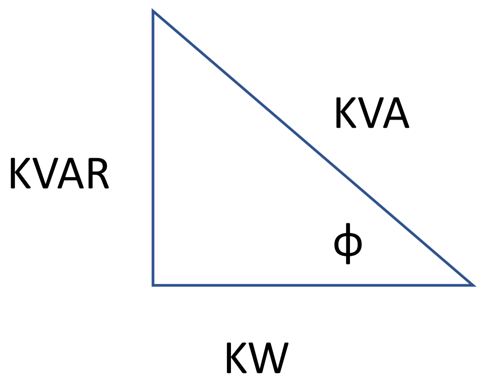
$\omega = 2\pi \times 50 = 100\pi$ rad/sec.

$G = 117.65$ MVA.

$$\text{Kinetic Energy} = GH = \frac{1}{2} I \omega^2$$

$$(117.65) \times H = \frac{1}{2} \times \frac{10{,}000 \times (100\pi)^2}{10^6} \text{ MJ}$$

or $H = 41.9$ MJ/MVA.



Cos ϕ = KW/KVA

KVA= KW/ Cos ϕ

Also, $M = \dfrac{GH}{180.f}$

$$= \frac{117.65 \times 4.19}{180 \times 50}$$

$= 0.0548$ MJ.sec / Elecd-deg.

Example 6.10 : ***Two power stations A and B are located close together. Station A has four identical generators each rated 100 MVA, 9 MJ/MVA whereas station B has three sets each rated 200 MVA, 4 MJ/MVA. Calculate the inertia constant of the equivalent machines of both stations on 150 MVA base.***

Since the stations are located close together, the generators of both the stations can be regarded as a single - equivalent large machine.

We have the inertia constant of the single equivalent machine.

$$H_{eq} = 4\left(\frac{H_1 G_1}{G_{Base}}\right) + 3\left(\frac{H_2 G_2}{G_{Base}}\right)$$

$G_1 = 100$ MVA, $H_1 = 9$ MJ/MVA

$G_2 = 200$ MVA, $H_2 = 4$ MJ/MVA

$G_{Base} = 150$ MVA.

$$H_{eq} = 4\left(\frac{9 \times 100}{150}\right) + 3\left(\frac{4 \times 200}{150}\right)$$

$$= 40 \text{ MJ / MVA}.$$

Example 6.11 : *A turbo generator, 6 pole, 50 Hz, of capacity 80 MW working at 0.8 pf has an inertia of 10 MJ/MVA.*
(a) Calculate the energy stored in the rotor at synchronous speed.
(b) Find rotor acceleration if the mechanical input is suddenly raised to 75 MW for an electrical load of 60 MW.
(c) Supposing the above acceleration is maintained for a duration of 6 cycles, calculate the change in torque angle and the rotor speed at the end of 6 cycles.

Solution :

(a) Energy stored in rotor $= GH = \dfrac{80}{0.8} \times 10 = 1000$ MJ.

Pa =Ps-Pe

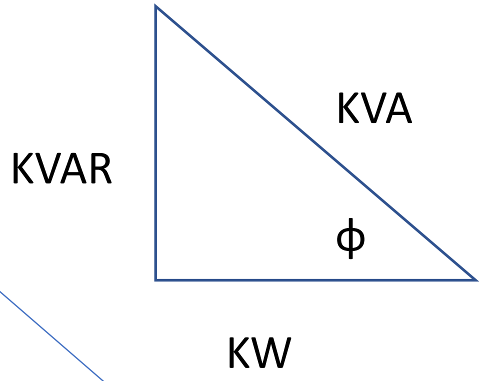
(b) The accelerating power, $P_a = (75-60) = 15 \text{ MW} = M \cdot \dfrac{d^2\delta}{dt^2}$

We also know that, $M = \dfrac{GH}{180 . f} = \dfrac{1000}{180 \times 50} = 0.111$ MJ.sec / Elect - deg.

$$0.111 \frac{d^2\delta}{dt^2} = 15.$$

$$M \frac{d^2\delta}{dt^2} = P_a$$

or angular acceleration $= \alpha = \dfrac{d^2\delta}{dt^2} = \dfrac{15}{0.111} = 135$ Elect - deg / sec^2;



Cos ϕ = KW/KVA
KVA= KW/ Cos ϕ

(c) We have,

$$\frac{d^2\delta}{dt^2} = \alpha.$$

Integrating this twice, we get

$$\delta = \frac{1}{2}\alpha \, . \, t^2$$

6 cycles correspond to $\frac{6}{50} = 0.12$ sec.

Change in δ at end of 6 cycles $= \frac{1}{2} \times 135 \times (0.12)^2$ $= 0.972$ Elect. degree.