Power System Analysis 1 – BEE601

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INTRODUCTION

The electrical power system normally operates in a balanced three-phase sinusoidal steady-state mode. However, there are certain situations that can cause unbalanced operations. The most severe of these would be a fault or short circuit. Examples may include a tree in contact with a conductor, a lightning strike, or downed power line

TYPES OF FAULTS

Two broad classifications of faults are

1. Symmetrical faults
2. Unsymmetrical faults

Statistics of fault

	Types of fault	% of occurrence
Unsymmetrical faults	Single Line to ground faults(L-G)	70%
	Line to Line faults(L-L)	15%
	Double Line to ground faults(L-L-G)	10%
symmetrical faults	Triple line faults (Balanced three phase faults)	5%

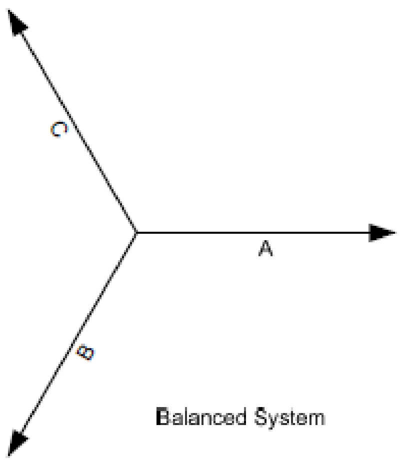
In a power system, the most severe fault is three phase fault and less severe fault is open conductor fault. **The various faults in the order of decreasing severity are**,

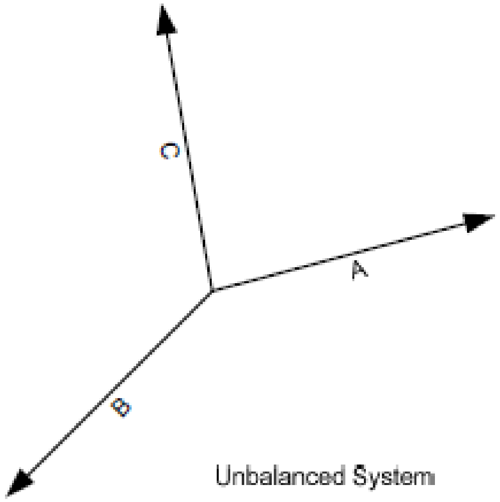
1) Three phase fault
2) Double line-to-ground fault
3) Line-to-line fault
4) Single line-to-ground fault
5) Open conductor fault

Symmetrical faults:.

That fault which gives rise to symmetrical fault currents (i.e. equal fault currents with 120° displacement) is called a symmetrical fault. The most common example of symmetrical fault is when all the three conductors of a 3-phase line are brought together simultaneously into a short-circuit condition.

Fault current is same in all the Three Phases and hence **system remains balanced** even after fault occurrence, symmetrical fault conditions are analyzed on a single phase basis using thevenin's theorem or using bus matrix impedance matrix. these faults are relatively rare, but are the easiest to analyze so we'll consider them first





Unsymmetrical faults:

Those faults on the power system which give rise to unsymmetrical currents (i.e. unequal fault currents in the lines with unequal phase displacement) are known as unsymmetrical faults.

On the occurrence of an unsymmetrical fault, the currents in the three lines become un equal and so is the phase displacement among them. It may be noted that the term 'un-symmetry' applies only to the fault itself and the resulting line currents. However, the system impedances and the source voltages are always symmetrical through its main elements viz. generators, transmission lines, synchronous reactors etc. There are three ways in which unsymmetrical faults may occur in a power system

I. Single Line to ground faults(L-G)
II. Line to Line faults(L-L)
III. Double Line to ground faults(L-L-G)

Fault current is not same in all the Three Phases, System is **no longer balanced**; these faults are very common, but more difficult to analyze. Analyzed using symmetrical components

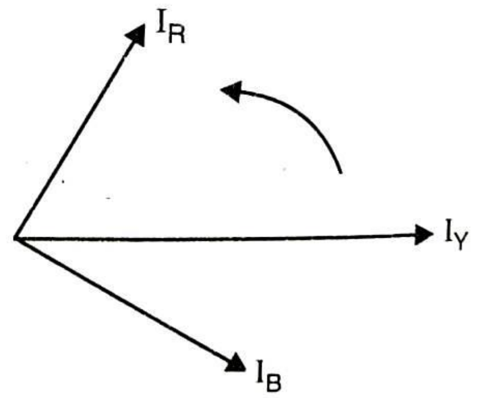
Symmetrical Components

The analysis of unsymmetrical polyphase network by the method of symmetrical components was introduced by Dr. C. Fortesque. He proved that

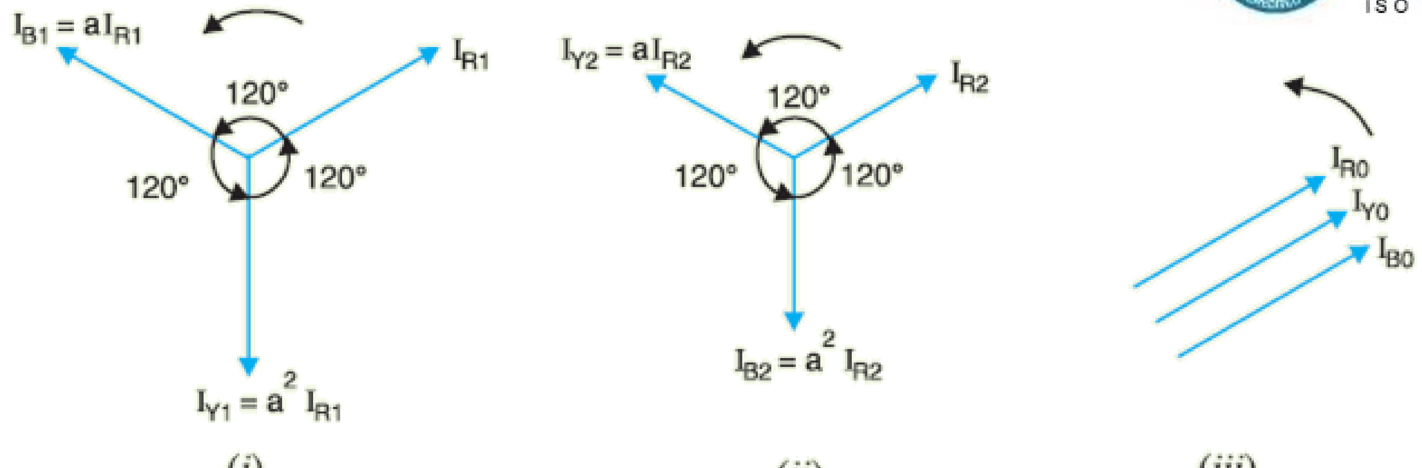
An unbalanced system of N related vectors can be resolved into N system of balanced vectors.

The N-Sets of balanced vectors of original vectors are called symmetrical components. Each set consist of N-Vectors which are equal in length and having equal phase angles between adjacent vectors.

I.e. **unbalanced 3-phase voltages (or currents) could be transformed into 3 sets of balanced voltages (or currents) called symmetrical components**.

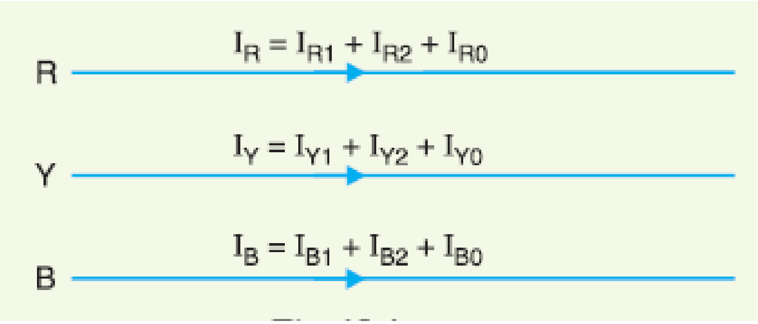


Unbalanced System



Symmetrical components of unbalanced current vectors

i. Positive (or Normal) Phase sequence components
ii. Negative Phase sequence components
iii. Zero Phase sequence components



$$\vec{I_R} = \vec{I_{R1}} + \vec{I_{R2}} + \vec{I_{R0}}$$

$$\vec{I_Y} = \vec{I_{Y1}} + \vec{I_{Y2}} + \vec{I_{Y0}}$$

$$\vec{I_B} = \vec{I_{B1}} + \vec{I_{B2}} + \vec{I_{B0}}$$

$$\vec{I_R} = \vec{I_{R1}} + \vec{I_{R2}} + \vec{I_{R0}}$$

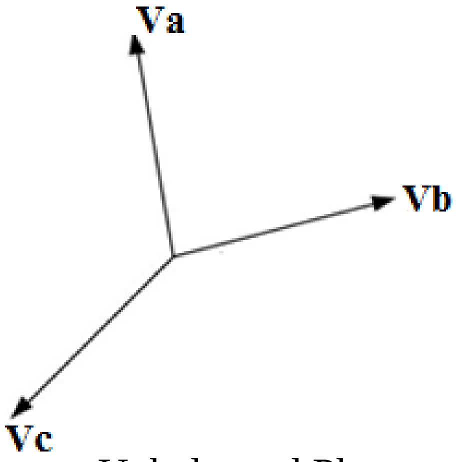
$$\begin{aligned}\vec{I_Y} &= \vec{I_{Y1}} + \vec{I_{Y2}} + \vec{I_{Y0}} \\ &= a^2\, \vec{I_{R1}} + a\, \vec{I_{R2}} + \vec{I_{R0}}\end{aligned}$$

$$\begin{aligned}\vec{I_B} &= \vec{I_{B1}} + \vec{I_{B2}} + \vec{I_{B0}} \\ &= a\, \vec{I_{R1}} + a^2\, \vec{I_{R2}} + \vec{I_{R0}}\end{aligned}$$

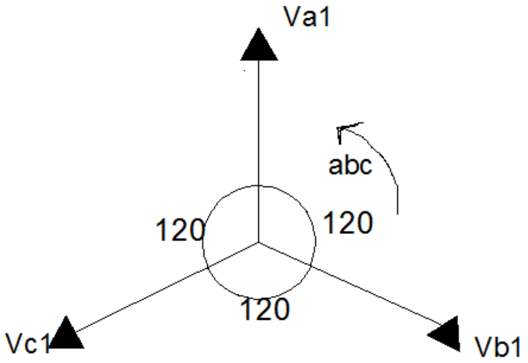
** a balanced system of 3 phase current implies that three current are equal in magnitude having 120° displacement from each other

In a three phase system, the three unbalanced vectors [either Voltages Vectors Va, Vb & Vc or Current Vectors Ia, Ib & Ic] can be resolved into three balanced system of vectors. The vectors of the balanced system are called symmetrical components of the original system. The symmetrical components of three phase syst

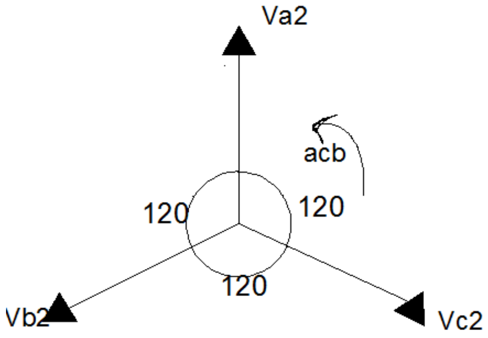
1. Positive sequence components
2. Negative sequence components
3. Zero sequence components



Unbalanced Phasor



Positive sequence component



Negative sequence component

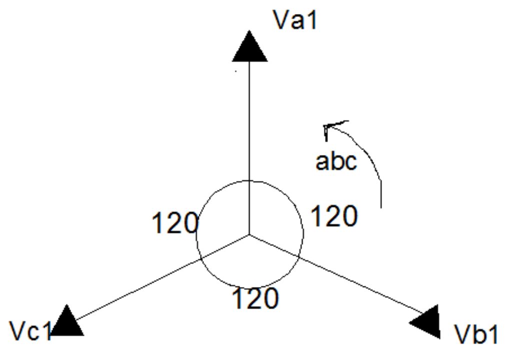
Zero sequence component

Let Va, Vb, and Vc be the 3-Phase unbalanced voltage vectors with phase sequence abc.

Each voltage vector can be resolved into positive, negative and zero sequence components

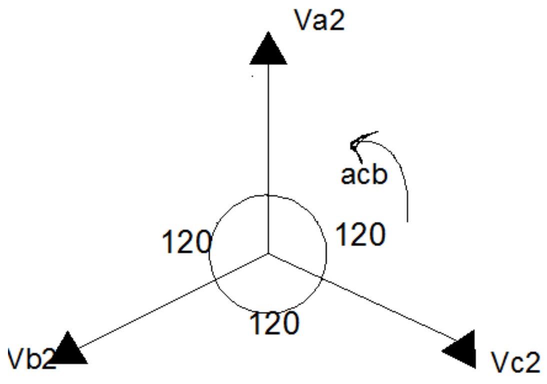
The positive sequence components of 3-Phase unbalanced vectors consists of three vectors equal in magnitude, displaced form each other by 120° in phase, and having the same phase sequence as the original vectors

Va1, Vb1& Vc1 = Positive sequence components of Va, Vb, and Vc respectively with phase sequence abc.



The negative sequence components of 3-Phase unbalanced vectors consists of three vectors equal in magnitude, displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original vectors

Va2, Vb2& Vc2 = Negative sequence components of Va, Vb, and Vc respectively with phase sequence acb.



The zero sequence components of 3-Phase unbalanced vectors consists of three vectors equal in magnitude and with zero phase displacement from each other.

Va0, Vb0 & Vc0 = Zero sequence components of Va, Vb, and Vc respectively

The vector diagrams of positive, negative and zero sequence components are shown in below figure 1.

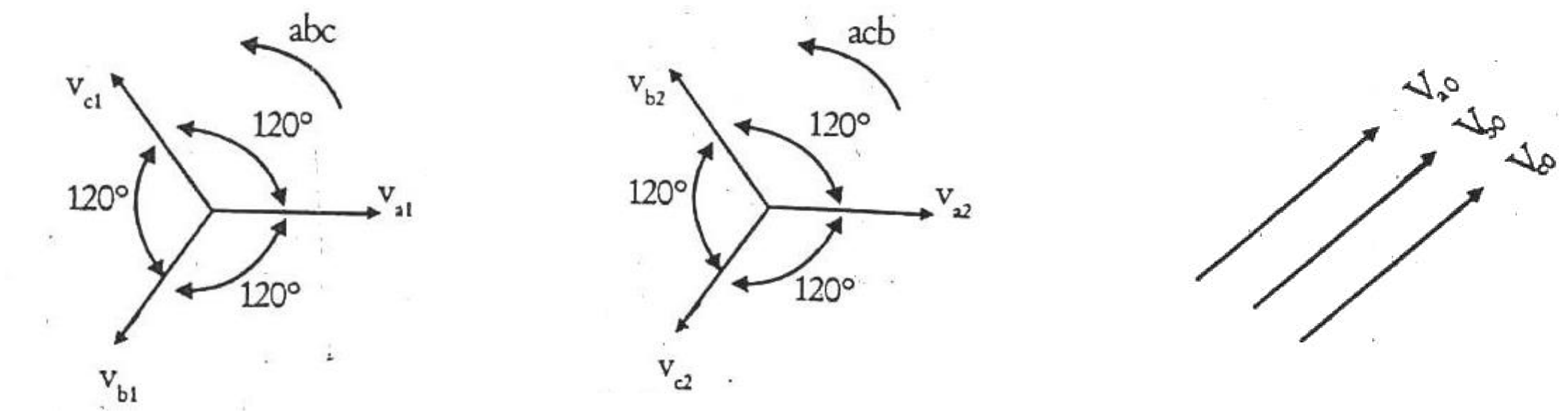


Fig a:Positive sequence component

Fig b:Negative sequence component

Fig c: Zero sequence component

From the vector diagram of symmetrical components the following conclusions can be made.

On rotating the vector Va1, by 120° in anticlockwise direction we get Vc1
On rotating the vector Va1, by 240° in anticlockwise direction we get Vb1.
On rotating the vector Va2, by 120° in anticlockwise direction we getVb2
On rotating the vector Va2, by 240° in anticlockwise direction we get Vc2

The j and a operator

Therefore, on rotating the symmetrical component of one vector by 120° or multiples of 120° we get the symmetrical components of other vectors. Hence we can define an operator which causes a rotation of 120° in the anticlockwise direction, such an operator is denoted by the letter "a"

The Operator "a" can be defined as written as $a = 1\angle 120° = -0.5+j0.866$

$a = 1\angle 120° = -0.5+j0.866$

$a^2 = 1\angle 240° = -0.5-j0.866$

$a^3 = 1\angle 360° = 1\angle 0°$

$1 + a + a^2 = 1+(-0.5+j0.866)+(-0.5-j0.866) = 0$

$1 + a + a^2 = 0$

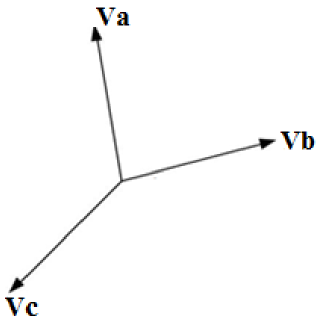
Recall the operator j. In polar form, $j = 1\angle 90°$. Multiplying by j has the effect of rotating a phasor 90° without affecting the magnitude

Properties of the vector j

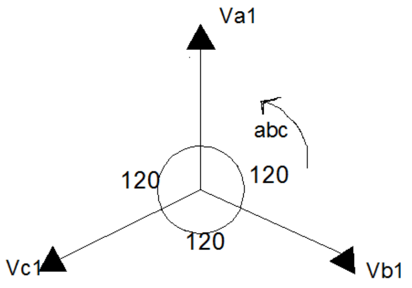
$$1 = 1.0 + j0.0$$
$$j = 1\angle 90^o$$
$$j^2 = 1\angle 180^o = -1$$
$$j^3 = 1\angle 270^o = -j$$
$$-j = 1\angle -90^o$$

Computation of unbalanced vectors from their symmetrical components

Each of the original unbalanced vector is the sum of its positive, negative and zero sequence component. Therefore the original unbalanced three phase voltage vectors can be expressed in terms of their components as shown below.

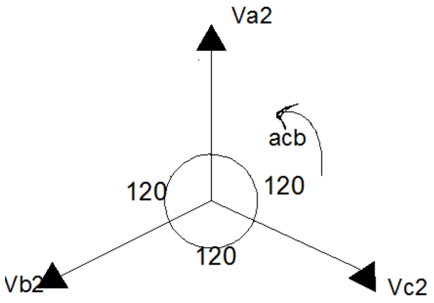


$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{b0} + V_{b1} + V_{b2}$$
$$V_c = V_{c0} + V_{c1} + V_{c2}$$



From the vector diagram shown in figure(1), we get the following relations between various symmetrical components

$$V_{b0} = V_{a0} \;; \quad V_{b1} = a^2 V_{a1} \;; \quad V_{b2} = a V_{a2}$$
$$V_{c0} = V_{a0} \;; \quad V_{c1} = a V_{a1} \;; \quad V_{c2} = a^2 V_{a2}$$



the Above equations can be written as

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$
$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

The equations can be arranged in the matrix form as shown below

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

The Above equation can be used to compute the unbalanced voltage vectors from the knowledge of symmetrical components.

Computation of symmetrical components of unbalanced vectors

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

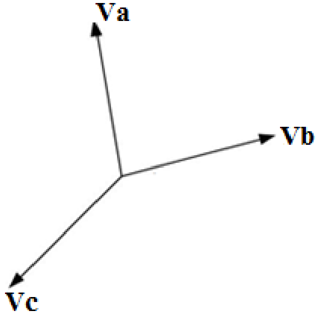
The Above Matrix equation can be written in the vector notation as: V = A*Vsy.

$$\text{where, } \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} ;\ \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{ and } \mathbf{V}_{sy} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

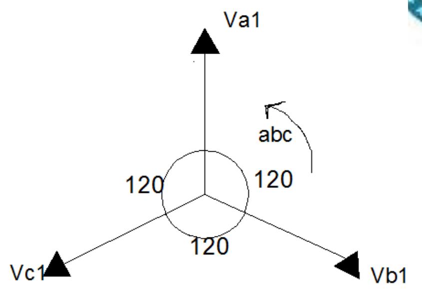
On pre multiplying the above equation by A^{-1} we get

$$A^{-1}\, V = Vsy$$

$$Vsy = A^{-1}\, V$$



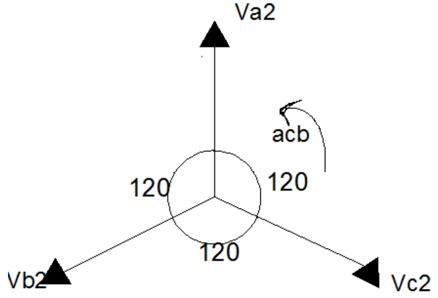
$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$



On substituting for A^{-1} we get

$$V_{sy} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} V$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

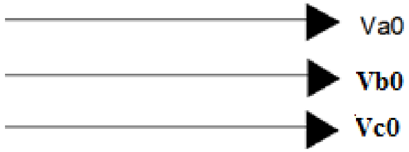


The matrix equation can be expressed as three independent linear equations as shown below.

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

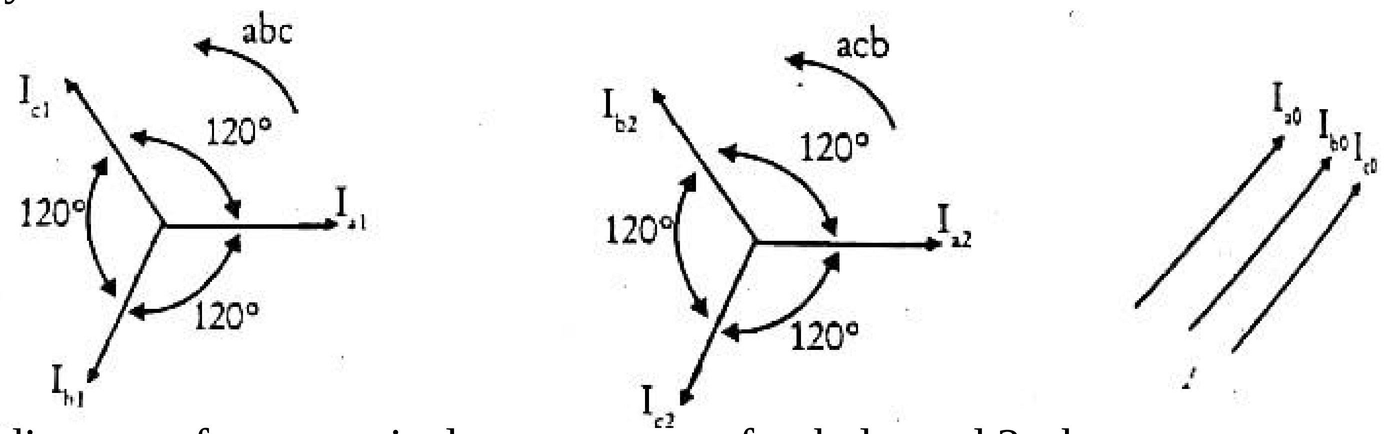
$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2V_c)$$

$$V_{a2} = \frac{1}{3}(V_a + a^2V_b + aV_c)$$



symmetrical components of unbalanced current vectors

The symmetrical components of unbalanced current vectors can be obtained by an analysis similar to that of voltage vectors. All the equations developed for voltages can be used for current if we replace V by I.



Vector diagram of symmetrical components of unbalanced 3-phase current vectors

The following equations are used to compute the unbalanced current vectors from the knowledge of their symmetrical components

$$I_a = I_{a0} + I_{a1} + I_{a2}$$
$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

The following equations are used to compute the symmetrical components of unbalanced current vectors

$$I_{a0} = \frac{1}{3}\left[I_a + I_b + I_c\right]$$

$$I_{a1} = \frac{1}{3}\left[I_a + aI_b + a^2 I_c\right]$$

$$I_{a2} = \frac{1}{3}\left[I_a + a^2 I_b + a I_c\right]$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Numerical 1

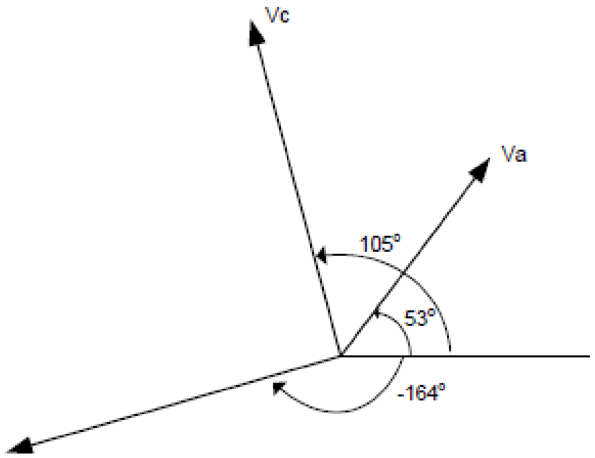
The voltage across a 3 phase unbalanced load are Va = 5∠53°, Vb = 7∠ − 164°, Vc = 7∠105°, find the symmetrical components of Voltages. The phase sequence is abc

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\therefore V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2 V_c]$$

$$V_{a2} = \frac{1}{3}[V_a + a^2 V_b + a V_c]$$



Unbalanced condition

Solve for the zero sequence component:

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$= \frac{1}{3}(5\angle 53^o + 7\angle -164^o + 7\angle 105^o)$$

$$= 3.5\angle 122^o$$

We Know That $V_{b0} = V_{a0}$ Therefore $V_{b0} = 3.5\angle 122^o$

$V_{c0} = V_{a0}$ $V_{c0} = 3.5\angle 122^o$

Solve for the positive sequence component:

$$V_{a1} = \frac{1}{3}\left(V_a + aV_b + a^2V_c\right)$$

$$= \frac{1}{3}\left(5\angle 53^o + \left(1\angle 120^o \cdot 7\angle -164^o\right) + \left(1\angle 240^o \cdot 7\angle 105^o\right)\right)$$

$$= 5.0\angle -10^o$$

We Know That

$$V_{b1} = a^2 V_{a1}$$

$$V_{c1} = a V_{a1}$$

$a = 1\angle 120° = -0.5+j0.866$

$a^2 = 1\angle 240° = -0.5-j0.866$

$a^3 = 1\angle 360° = 1\angle 0°$

Therefore

$$V_{b1} = 5.0\angle -130^o$$

$$V_{c1} = 5.0\angle 110^o$$

Solve for the negative sequence component:

$$V_{a2} = \frac{1}{3}\left(V_a + a^2 V_b + aV_c\right)$$

$$= \frac{1}{3}\left(5\angle 53^o + \left(1\angle 240^o \cdot 7\angle -164^o\right) + \left(1\angle 120^o \cdot 7\angle 105^o\right)\right)$$

$$= 1.9\angle 92^o$$

We Know That $V_{b2} = a V_{a2}$

$V_{c2} = a^2 V_{a2}$

$a = 1\angle 120° = -0.5+j0.866$

$a^2 = 1\angle 240° = -0.5-j0.866$

$a^3 = 1\angle 360° = 1\angle 0°$

Therefore $V_{b2} = 1.9\angle -148^o$

$V_{c2} = 1.9\angle -28^o$

Numerical 2

The Symmetrical Components of Phase Voltages in A 3 Phase Unbalanced System are

$V_{a0} = 10\angle 180°$ V, $V_{a1} = 50\angle 0°$ V and $V_{a2} = 20\angle 90°$ V,

Determine the phase voltages Va, Vb, and Vc

The phase voltages Va, Vb, and Vc are given by following Equation

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$
$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

Given that $V_{a0} = 10\angle 180°$ V $= -10+j0$

$V_{a1} = 50\angle 0°$ V $= 50+j0$

$V_{a2} = 20\angle 90°$ V $= 0+j20$

$$\therefore aV_{a1} = 1\angle 120° \times 50\angle 0° = 50\angle 120° = -25 + j43.30$$

$$a^2V_{a1} = 1\angle 240° \times 50\angle 0° = 50\angle 240° = -25 - j43.30$$

$$aV_{a2} = 1\angle 120° \times 20\angle 90° = 20\angle 210° = -17.32 - j10$$

$$a^2V_{a2} = 1\angle 240° \times 20\angle 90° = 20\angle 330° = 17.32 - j10$$

The phase voltages Va

$$V_a = V_{a0} + V_{a1} + V_{a2} = -10 + 50 + j20 = 40 + j20 = 44.72\angle 27^\circ \ V$$

The phase voltages Vb

$$V_b = V_{aC} + a^2 V_{a1} + aV_{a2} = -10 - 25 - j43.30 - 17.32 - j10$$
$$= -52.32 - j53.30 = 74.69\angle -134^\circ \ V$$

The phase voltages Vc

$$V_c = V_{a0} + a\,V_{a1} + a^2\,V_{a2} = -10 - 25 + j43.30 + 17.32 - j10$$
$$= -17.68 + j33.3 = 37.70\angle 118^\circ \ V$$

Prove that a balanced set of three phase voltages will have only positive sequence components of voltages only.

Solution:

A balanced three phase system of voltages is one where in all the phase voltages are of equal magnitude and symmetrically displaced by 120°. This is shown in fig 3.3

Let Va, Vb and Vc be the balanced system of three phase voltages. From fig. 3.3, it can be observed that,

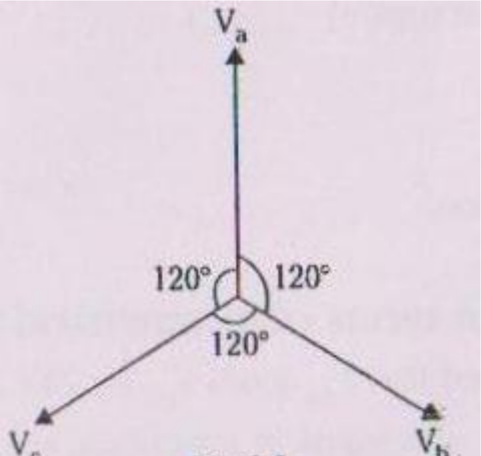


Fig. 3.3

$$V_a = V_a$$
$$V_b = a^2.V_a$$
$$V_c = a.V_a \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Using eq. 1 in the above matrix, we get...

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ a^2.V_a \\ a.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a + a^2.V_a + a.V_a \\ V_a + a^3.V_a + a^3.V_a \\ V_a + a^4.V_a + a^2.V_a \end{bmatrix}$$

putting $a^3=1$ and $a^4=a$, we get,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} V_a + a.V_a + a^2.V_a \\ V_a + V_a + V_a \\ V_a + a.V_a + a^2.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a + a.V_a + a^2.V_a \\ V_a + V_a + V_a \\ V_a + a.V_a + a^2.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a(1 + a + a^2) \\ 3V_a \\ V_a(1 + a + a^2) \end{bmatrix}$$

but,
$(1+a+a^2)=0,$

$$= (1/3) \begin{bmatrix} 0 \\ 3V_a \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

$$V_{a0}=0$$
$$V_{a1}=V_a$$
$$V_{a2}=0$$

This clearly indicates that a balanced set of three phase voltages will have only positive sequence voltages. The negative and zero sequence components are always absent in a balanced system. This holds good for a balanced set of currents as well.

Determine the sequence components of the three voltages, $V_a=200\angle 0°V$, $V_b=200\angle 245°V$ and $V_c=200\angle 105°V$

The positive sequence components of voltage is ,

$$V_{a1}=(1/3)\ (V_a + a.\ V_b+ a^2.V_c)$$
$$=(1/3)\ (200\angle 0°+200\angle 245°+120°+200\ \angle 105°+240°)$$
$$=(1/3)\ (200+ (199.24+j1743)+(193.19-j51.76))$$
$$= 0.9748-j11.44$$
$$=197.81\angle -3.3°\ V$$

The negative sequence component of voltage is,

$$V_{a2}=(1/3)\ (V_a + a^2.\ V_b+ a.V_c)$$
$$=(1/3)\ (200\angle 0°+200\angle (245°+240°)+200\ \angle (105°+120°))$$
$$=(1/3)\ (200+(-114.72+j163.83+(-141.42-j141.42))$$
$$=-18.71+j7.47$$
$$=20.15\angle 158.2°\ V$$

The zero sequence component of voltage is,

$$V_{a0}= (1/3)\ (V_a + V_b+ V_c)$$
$$=(1/3)\ (200\angle 0°+200\angle 245°+200\ \angle 105°)$$
$$=(1/3)(200+(-84.52-j181.26)+(-51.76-j193.18))$$
$$=21.21+j3.97$$
$$=21.6\angle 16.58°\ V$$

The positive, negative and zero sequence components of line are $20\angle10°$, $6\angle60°$ and $3\angle30°$ A respectively. Determine the line currents.

$I_{a1}= 20\angle10°$

$I_{a2}= 6\angle60°$

$I_{a0}= 3\angle30°$

we have, the line current,

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=3\angle30°+20\angle10° +6\angle60°$

$=27.25\angle21.88°$ A

$I_b= I_{a0}+a^2.I_{a1}+a.I_{a2}$

$= 3\angle30°+20\angle(10°+\angle240°)+6\angle(60°+120°)$

$=20.1\angle-120.7°$ A

$I_c= I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=3\angle30°+20\angle(10°+120°) +6\angle(60°+240°)$

$=13.7\angle122°$ A

In a three phase system, Ia1=100∠30° A, Ib2=40∠90° A and Ic0= 10∠-30° A. Find the line currents.

Solution:

The sequence components of currents given in the problem are not of phase a only. Hence it is first required to express the sequence components in terms of phase a.

Consider fig 3.4. The negative sequence components of line currents are depicted in the sketch.

$I_{a2}=a^2.I_{b2}=40\angle(90°+240°)=40\angle 330°$ A

also we have

$I_{a0}=10\angle -30°$ A

$I_{a1}=100\angle 30°$ A

$I_{a2}=40\angle 330°$ A

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=10\angle -30°+100\angle 30° +40\angle 330°$

$=132.24\angle 10.89°$ A

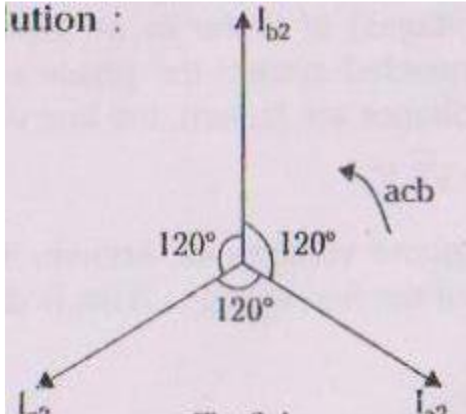


Fig. 3.4

$I_b= I_{a0}+a^2.I_{a1}+a.I_{a2}$

$= 10\angle -30°+100\angle(30°+240°)+40\angle(330°+120°)$

$=65.57\angle -82.4°$ A

$I_c= I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=10\angle -30°+100\angle(30°+120°)+40\angle(330°+240°)$

$=11.32\angle 167.48°$ A

Numerical

The Symmetrical Components of Phase-A fault Current in a 3 Phase Unbalanced System are

Ia0 = 350∠ 90° A, Ia1 = 600∠-90° A and Ia2= 250∠90° A, Determine the phase Currents Ia, Ib, and Ic

The following equations are used to compute the 3-Phase unbalanced current vectors Ia, Ib, and Ic from the knowledge of their symmetrical components

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_a = I_{a0} + I_{a1} + I_{a2}$$
$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

Given that $I_{a0} = 350\angle 90° = 0 + j350$

$$I_{a1} = 600\angle -90° = 0 - j600$$
$$I_{a2} = 250\angle 90° = 0 + j250$$

$$\therefore a I_{a1} = 1\angle 120^\circ \times 600\angle -90^\circ = 600\angle 30^\circ = 519.62 + j300$$

$$a^2 I_{a1} = 1\angle 240^\circ \times 600\angle -90^\circ = 600\angle 150^\circ = -519.62 + j300$$

$$a I_{a2} = 1\angle 120^\circ \times 250\angle 90^\circ = 250\angle 210^\circ = -216.51 - j125$$

$$a^2 I_{a2} = 1\angle 240^\circ \times 250\angle 90^\circ = 250\angle 330^\circ = 216.51 - j125$$

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = j350 - j600 + j250 = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2} = j350 - 519.62 + j300 - 216.51 - j125$$

$$= -736.13 + j525 = 904.16\angle 145° \ \text{A}$$

$$I_c = I_{a0} + a\, I_{a1} + a^2\, I_{a2}$$

$$I_c = I_{a0} + a\, I_{a1} + a^2\, I_{a2} = j350 + 519.62 + j300 + 216.51 - j125$$

$$= 736.13 + j525 = 904.16\angle 35° \ \text{A}$$

Numerical

Determine the symmetrical Components of Unbalanced three Phase currents Ia = 10∠0° A, Ib = 12∠230° A and Ic= 10∠130° A

The symmetrical Components of Ia given by the following Matrix equations

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\therefore I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + a\,I_b + a^2\,I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2\,I_b + a\,I_c\right)$$

Given that

$$I_a = 10\angle 0^\circ = 10 + j0$$

$$I_b = 12\angle 230^\circ = -7.71 - j9.19$$

$$I_c = 10\angle 130^\circ = -6.43 + j7.66$$

$$\therefore a I_b = 1\angle 120^\circ \times 12\angle 230^\circ = 12\angle 350^\circ = 11.82 - j2.08$$

$$a^2 I_b = 1\angle 240^\circ \times 12\angle 230^\circ = 12\angle 470^\circ = 12\angle 110^\circ = -4.10 + j11.28$$

$$a I_c = 1\angle 120^\circ \times 10\angle 130^\circ = 10\angle 250^\circ = -3.42 - j9.40$$

$$a^2 I_c = 1\angle 240^\circ \times 10\angle 130^\circ = 10\angle 370^\circ = 10\angle 10^\circ = 9.85 + j1.74$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}\,(10 - 7.71 - j9.19 - 6.43 + j7.66)$$

$$= \frac{1}{3}(-4.14 - j1.53) = -1.38 - j0.51 = 1.47\angle -160^\circ$$

$$I_{a1} = \frac{1}{3}\left(I_a + a\,I_b + a^2\,I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + a\,I_b + a^2\,I_c\right) = \frac{1}{3}\,(10 + 11.82 - j2.08 + 9.85 + j1.74)$$

$$= \frac{1}{3}(31.67 - j0.34) = 10.56 - j0.11 = 10.56\angle -0.6^\circ \approx 10.56\angle 0^\circ$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2\,I_b + a\,I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2\,I_b + a\,I_c\right) = \frac{1}{3}\,(10 - 4.10 + j11.28 - 3.42 - j9.40)$$

$$= \frac{1}{3}(2.48 + j1.88) = 0.83 + j0.63 = 1.04\angle 37^\circ$$

From the vector diagram symmetrical components, we get the following relations between various symmetrical components

$$V_{b0} = V_{a0} \;;\quad V_{b1} = a^2 V_{a1} \;;\quad V_{b2} = a V_{a2}$$

$$V_{c0} = V_{a0} \;;\quad V_{c1} = a V_{a1} \;;\quad V_{c2} = a^2 V_{a2}$$

The Zero Sequence components are

We Know That $I_{a0} = I_{b0} = I_{c0}$

$$I_{a0} = 1.47\angle{-160^\circ}\ \text{A}$$

$$I_{b0} = 1.47\angle{-160^\circ}\ \text{A}$$

$$I_{c0} = 1.47\angle{-160^\circ}\ \text{A}$$

The Positive Sequence components are

We Know That $I_{b1} = a^2 I_{a1}$; $I_{c1} = a I_{a1}$

$$I_{a1} = 10.56\angle 0° \text{ A}$$
$$I_{b1} = a^2 I_{a1} = 1\angle 240° \times 10.56\angle 0° = 10.56\angle 240° \text{ A}$$
$$I_{c1} = a I_{a1} = 1\angle 120° \times 10.56\angle 0° = 10.56\angle 120° \text{ A}$$

The Negative Sequence components are

We Know That $I_{b2} = a I_{a2}$; $I_{c2} = a^2 I_{a2}$

$$I_{a2} = 1.04\angle 37° \text{ A}$$
$$I_{b2} = a I_{a2} = 1\angle 120° \times 1.04\angle 37° = 1.04\angle 157° \text{ A}$$
$$I_{c2} = a^2 I_{a2} = 1\angle 240° \times 1.04\angle 37° = 1.04\angle 277° \text{ A}$$

Relation between sequence components of phase and line voltages in star connected systems.

Let Va, Vb and Vc be the phase voltages having a phase sequence abc as shown in fig 3.5.

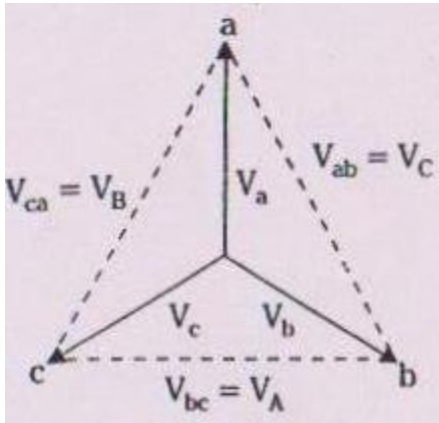


Fig. 3.5

The three line voltages of the system are Vbc, Vca and Vab. It is known from elementary vector algebra that.

$V_{bc} = V_c - V_b$

$V_{ca} = V_a - V_c$

$V_{ab} = V_b - V_a$

Let

$V_{bc} = V_A$(opposite to Vertex A)

$V_{ca} = V_B$(opposite to Vertex B)

$V_{ab} = V_C$(opposite to Vertex C)

therefore, we get

$V_A = V_{bc} = V_c - V_b$

$V_B = V_{ca} = V_a - V_c$

$V_C = V_{ab} = V_b - V_a$ 3.18

The positive sequence component of line voltage is given as

$V_{A1} = (1/3)(V_A + a.V_B + a^2.V_C)$

$= (1/3)((V_c - V_b) + a(V_a - V_c) + a^2.(V_b - V_a))$ .... in View of eq. 3.18

$= (1/3)((a(V_a + a.V_b + a^2.V_c) - a^2(V_a + a.V_b + a^2.V_c))$

$= (1/3)(a - a^2)(V_a + a.V_b + a^2.V_c)$

but, $(V_a + a.V_b + a^2.V_c) = 3.V_{a1}$

and, $(a - a^2) = j\sqrt{3}$

$V_{A1} = (1/3)(j\sqrt{3})(3.V_{a1})$

Thus, $V_{A1} = j\sqrt{3}.\ V_{a1}$ ....

Hence, positive sequence component of line voltage is √3 times the positive sequence component of phase voltage and leads the corresponding phase voltage by 90°.

The negative sequence component of line voltage is

$V_{A2}=(1/3)(V_A+a^2.V_B+a.V_C)$

$=(1/3)((V_c-V_b)+a^2(V_a-V_c)+a(V_b-V_c))$, view of eq. 3.18

$=(1/3)\ (a^2(V_a+a^2.V_b+a.V_c)-a(V_a+a^2.V_b+a.V_c))$

$=(1/3)\ (a^2-a)\ (V_a+a^2.V_b+a.V_c)$

$=(1/3)(-j\sqrt{3})(3.V_{a2})$ [because $(V_a+a^2.V_b+a.V_c)=3.V_{a2}$ and $(a^2-a)=-j\sqrt{3}$]

Thus,

$V_{A2}=-j\sqrt{3}.V_{a2}$ 3.20

Hence, negative sequence component of line voltage is √3 times the negative sequence component of phase voltage and lags the corresponding phase voltage by 90°.

Finally, the zero sequence component of line voltage is given as,

$V_{A0}=(1/3)(V_A+V_B+V_C)$

$=(1/3)((V_c-V_b)+(V_a-V_c)+(V_a-V_b))$, in view of eq. 3.18

$=0$

Thus, $V_{A0}=0$ 3.21

Therefore, it is evident from the above equation that zero sequence component of line voltage is zero.

In similar lines as above, it can be proved that

$V_{B1}=j\sqrt{3}\ Vb1$; $V_{B2}=-j\sqrt{3}Vb2$; $V_{B0}=0$

$V_{C1}=j\sqrt{3}Vc1$; $V_{C2}=-j\sqrt{3}Vc2$; $V_{C0}=0$

The positive and negative sequence components of phase voltages of a three phase system are Va1=230 ∠ 30° V and Va2=60∠ 60° V. Determine the positive and negative sequence components of line voltages and hence the line voltages.

The positive, negative and zero sequence line voltages is given by,

$V_{A1}=j\sqrt{3}.\ V_{a1}=\sqrt{3}\ (230\angle(30°+90°))=\ 398.37\angle 120°\ V$

$V_{A2}=-j\sqrt{3}.V_{a2}\ =\sqrt{3}\ (60\angle(60°-90°))=\ 103.92\angle -30°\ V$

It is known that zero sequence component of line voltage is zero. Thus $V_{A0}=0$.

Hence the line voltages of the system are,

$V_A=V_{A0}+V_{A1}+V_{A2}$

$=0+398.37\angle 120°+103.92\angle -30°$

$=312.72\angle 110°\ V$

$V_B=V_{A0}+a^2.V_{A1}+a.V_{A2}$

$=0+398.37\angle(120°+240°)+103.92\angle(-30°+120°)$

$=411.7\angle 14.62°\ V$

$V_C=V_{A0}+a.V_{A1}+a^2.V_{A2}$

$=0+398.37\angle(120°+120°)+103.92\angle(-30°+240°)$

$=491.13\angle -126°\ V$

Relation between sequence components of phase and line currents in delta connected systems.

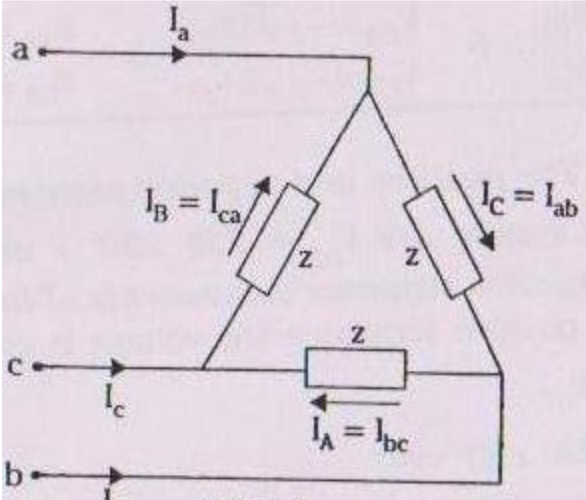


Fig. 3.8

The phase currents (currents in delta winding) are I_{ab}, I_{bc}, and I_{ca}. Let us designate $I_{ab}=I_C$, $I_{bc}=I_A$ and $I_{ca}=I_B$ (opposite to respective vertices).

Now, applying KCL to the system shown in fig 3.8, we get

$I_a=I_C-I_B$

$I_b=I_A-I_C$

$I_c=I_B-I_A$.........3.23

Then, the sequence component of line current are

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)\ ((I_C-I_B)+a(I_A-I_C)+a^2.(I_B-I_A))$

$=(1/3)((a(I_A+a.I_B+a^2.I_C)-a^2(I_A+a.I_B+a^2.I_C))$

$=(1/3)(a-a^2)(I_A+a.I_B+a^2.I_C)$

but, $(I_A+a.I_B+a^2.I_C) =3.I_{A1}$

and, $(a-a^2)=j\sqrt{3}$

$I_{a1}=(1/3)(j\sqrt{3})(3.I_{A1})$

$I_{a1}= j\sqrt{3}\ .I_{A1}$ ....

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)((I_C-I_B)+a^2(I_A-I_C)+a(I_B-I_C))$, view of eq. 3.18

$=(1/3)\ (a^2(I_A+a^2.I_B+a.I_C)-a(I_A+a^2.I_B+a.I_C))$

$=(1/3)\ (a^2-a)\ (I_A+a^2.I_B+a.I_C)$

$=(1/3)(-j\sqrt{3})(3.I_{A2})$ [because $(I_A+a^2.I_B+a.I_C)=3.I_{A2}$ and $(a^2-a)=-j\sqrt{3}$]

Thus,

$I_{a2}=-j\sqrt{3}.I_{A2}$ 3.25

From above equations, it can be inferred that the line currents in delta system is √3 times the currents. The positive sequence line current leads the respective phase current by 90° whereas the negative sequence line current lags the negative sequence phase currents by 90°.

Finally, the zero sequence components of the line current is

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)((I_C-I_B)+(I_A-I_C)+(I_A-I_B))$, in view of eq. 3.18

$=0$

Thus, $I_{a0}=0$ 3.26

$I_{b1}=j\sqrt{3}\ I_{B1}$; $I_{b2}=-j\sqrt{3}\ I_{B2}$; $I_{b0}=0$

$I_{c1}=j\sqrt{3}\ I_{C1}$; $I_{c2}=-j\sqrt{3}\ I_{C2}$; $I_{c0}=0$.

when the line currents Ia, Ib and Ic are leaving the delta connected windings as shown in fig, then it can be proved that

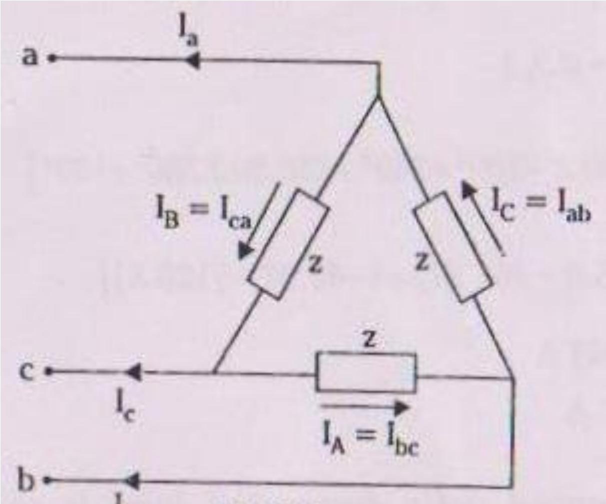


Fig. 3.9

$I_{a1}=-j\sqrt{3}\ I_{A1}$; $I_{a2}=j\sqrt{3}I_{A2}$; $I_{a0}=0$

$I_{b1}=-j\sqrt{3}\ I_{B1}$; $I_{b2}=j\sqrt{3}I_{B2}$; $I_{b0}=0$

$I_{c1}=-j\sqrt{3}\ I_{C1}$; $I_{c2}=j\sqrt{3}I_{C2}$; $I_{c0}=0$

In a three phase, three wire system, the line currents are Ia=100∠0°A and Ib=100∠-100° A. Determine the sequence components of line currents.

In three wire system always,

$I_a+I_b+I_c = 0$

$I_c= -(I_a+I_b)$

$= -(100\angle 0°+100\angle -100°)$

$=128.56\angle -130°$ A

Therefore, the sequence components of line currents are

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)(100\angle 0°+100\angle -100°+128.56\angle -130°)$

$=0$ A (as expected)

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(100\angle 0°+100\angle(-100°+120°)+128.56\angle(-130°+240°)$

$=108.5\angle 10°$ A

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(100\angle 0°+100\angle(-100°+240°)+128.56\angle(-130°+120°)$

$=20.5\angle -110°$ A

A balanced delta connected load is connected to a three phase symmetrical supply. The line currents are each 10A in magnitude. If fuse in one of the lines blows out, determine the sequence components of line current.

Let us assume that the fuse blows in line c. Then with the current in line a as reference, the diagram of the circuit is as shown in fig.

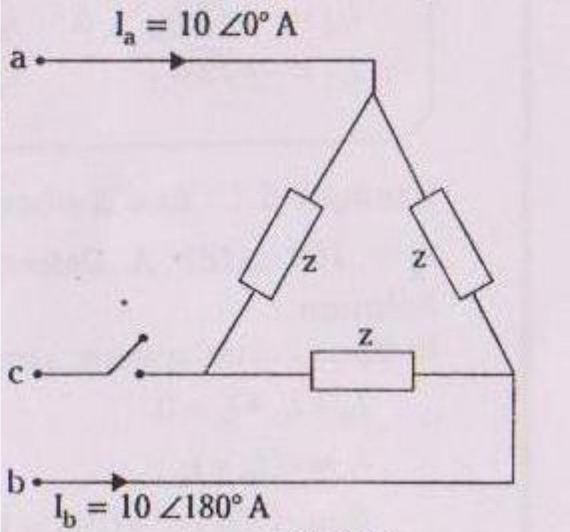


Fig. 3.10

Since this is a three wire system, we have

$I_a+I_b+I_c=0$

but,

$I_c=0$, as fuse blows out.

Therefore,

$I_b= -I_a$

if $I_a=10\angle 0°$ A, then

$I_b= -10\angle 0° = 10\angle 180°$ A

Hence, the positive sequence components of line current is

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(10\angle 0°+10\angle(180°+120°)+0)$

$=5.78\angle -30°$ A.

The negative sequence component of line current is

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(10\angle 0°+10\angle(180°+240°)+0)$

$=5.78\angle 30°$ A.

The zero sequence component of line current is absent in any three wire system.

Thus $I_{a0}=0$ A

A delta connected balanced resistive load is connected across an unbalanced three phase supply. Find the symmetrical components of line current and delta current.

$I_a+I_b+I_c=0$

or,

$I_c=-(I_a+I_b)$

$=-(10\angle 30°+15\angle -60°)$

$=18\angle 154°$ A.

Hence, symmetrical components of line currents are

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(10\angle 30°+15\angle(-60°+120°)+18\angle(154°+240°))$

$=13.94\angle 41.86°$ A

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(10\angle 30°+15\angle(-60°+240°)+18\angle(154°+120°))$

$=4.65\angle 248°$ A

$I_{a0}=(1/3)(I_a+I_b+I_c)=0$ A

Here, the line currents are entering the delta connected load. Therefore, the sequence components of delta currents are,

$I_{A1}=(I_{a1} / j\sqrt{3}) = (13.94\angle(41.86°-90°) /\sqrt{3}) = 8.05\angle -48.14°$ A

$I_{A2}=(I_{a2} / -j\sqrt{3}) = (4.65\angle(248°+90°) /\sqrt{3}) = 2.68\angle 338°$ A

Effect of neutral in the system

Let us consider both the cases independently.

Case i): Four wire system.

Now, the current can flow through the neutral wire. Applying KCL at node 'n', we get the current through the neutral as

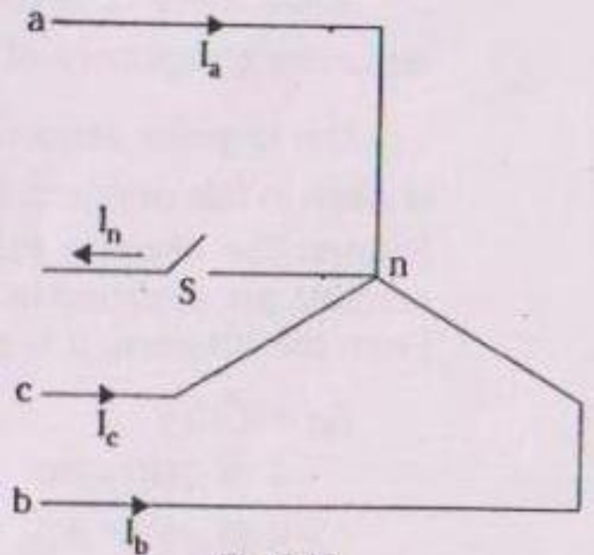


Fig. 3.13

$I_n = I_a + I_b + I_c$ ....

but, we have

$I_a = I_{a0} + I_{a1} + I_{a2}$

$I_b = I_{a0} + a^2.I_{a1} + a.I_{a2}$

and,

$I_c = I_{a0} + a.I_{a1} + a^2.I_{a2}$

Using these results in eq. 3.29 yields,

$I_n = 3.I_{a0} + I_{a1}(1 + a + a^2) + I_{a2}(1 + a + a^2)$

$= 3I_{a0} + 0 + 0$, as $(1 + a + a^2) = 0$

or $I_n = 3.I_{a0}$ 3.30

Case ii):

Three wire system

In this case, the neutral wire is not made available so that

$I_n = 0$ 3.31

Hence eq. 3.30 yields

$I_{a0} = 0$ 3.32

That is, zero sequence currents are absent in three wire system

In a three phase, three wire system, if Ia1=100∠30° A, Ib2=40∠90° A, find the line currents of the system.

Solution:

Since there is a three wire system, the zero sequence component of line current Ia0=0. The negative sequence component of phase 'b' is given in this problem. Hence, we determine Ia2 as follows. The negative sequence components of line currents are.

$I_{a2}=a^2 . I_{b2}$

$=40\angle(90°+240°)$

$=40\angle 330°$ A

$I_{a0}=0$

$I_{a1}=100\angle 30°$ A

$I_{a2}=40\angle 330°$ A

Hence the line currents of the system are

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=0+100\angle 30°+40\angle 330°$

$=124.89\angle 13.9°$ A

$I_b=I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=0+100\angle(30°+240°)+40\angle(330°+120°)$

$=60\angle -90°$ A

$I_c=I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=0+100\angle(30°+120°)+40\angle(330°+240°)$

$=124.89\angle 166.1°$ A

In a three phase system supplying power to a Y-load, the line currents when the neutral of the supply is not connected to the neutral of the load are Ia=20∠0° A and Ib=20∠-100° A. When the neutrals are connected, the current through the neutral wire is found to be 12∠-30° A. Determine the line currents under this situation.

case i) when neutral of load is isolated from neutral of supply

In this case, $I_{a0}=0$

$I_a=I_{a1}+I_{a2}=20\angle 0°$ 1

$I_b=a^2.I_{a1}+a.I_{a2}=20\angle -100°$ 2

$I_c=a.I_{a1}+a^2.I_{a2}=-(I_a+I_b)= -(20\angle 0°+20\angle -100°) = 25.7\angle 130°$ ......

$I_c'=I_{a0}+(a.I_{a1}+a^2.I_{a2})$

$=4\angle -30°+ 25.7\angle 130°$, from result 3$=22\angle 126.48°$ A

case ii) when the neutrals are connected

Here, it is given that I_n, the neutral current is $12\angle -30°$.

therefore,

$3.I_{a0} =12\angle -30°$

$I_{a0}=4\angle -30°$

Let I_a', I_b' and I_c' be the new values of line currents in this case, we get

$I_a'=I_{a0}+(I_{a1}+I_{a2})$

$= 4\angle -30°+20\angle 0°$, from result 1

$=23.53\angle -4.87°$ A

$I_b'=I_{a0}+(a^2.I_{a1}+a.I_{a2})$

$=4\angle -30°+20\angle -100°$, from result 2

$=21.69\angle -90°$ A

Phase shift of symmetrical components in Y-Δ transformer bank

Positive and negative sequence voltages and currents undergo a phase angle change in passing through a Y-Δ transformer (or a bank of three single phase transformers). This phenomenon is called as phase shift.

Voltage relations

Consider a transformer connection as shown in fig 3.15

Let E_a^s, E_b^s and E_c^s be the phase voltages on the Y-side of the transformer and E_a^d, E_b^d and E_c^d be the voltages across the windings on the Δ side of the transformer. Here, superscripts "s" and "d" stand for star side and delta side respectively.

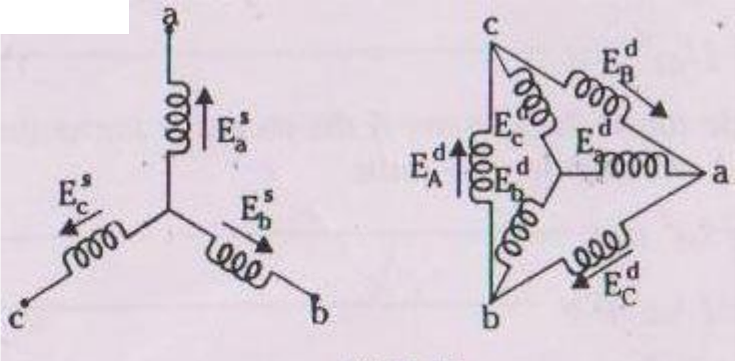


Fig. 3.15

$E_a^s = n.E_A^d$

$E_b^s = n.E_B^d$

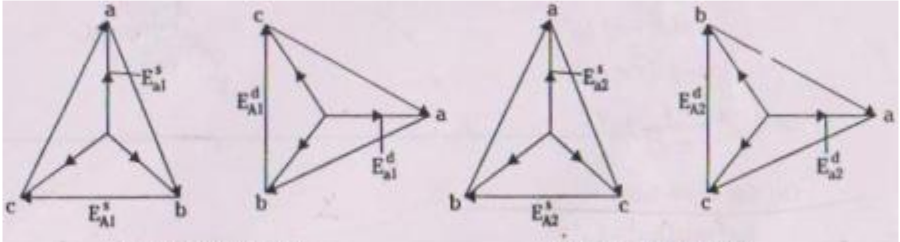
$E_c^s = n.E_C^d$

Hence, the sequence components are also related as,

$E_{a1}^s = n.E_{A1}^d$ 3.33

$E_{a2}^s = n.E_{A2}^d$ 3.34

$E_{a0}^s = n.\ E_{A0}^d = 0$ 3.35



(a) Positive Sequence Components

(b) Negative Sequence Components

Fig. 3.16

$E_{A1}^d = j\sqrt{3}\ E_{a1}^d$ ....

and $E_{A2}^d = -j\sqrt{3}\ E_{a2}^d$

$E_{a1}^s = n.E_{A1}^d = +j\sqrt{3}\ n.\ E_{a1}^d$ ....

and $E_{a2}^s = n.\ E_{A2}^d = -j\sqrt{3}.n.E_{a2}^d$

Current relations

consider a star delta transformer connection as shown in fig 3.17

Let I_a^s, I_b^s and I_c^s be the line currents in the star side, I_a^d, I_b^d and I_c^d the line currents in delta side I_A^d, I_B^d and I_C^d the currents in the delta windings.

If 'n' is the turns ratio, then from the theory of transformers, we can write

$I_A^d = n.I_a^s$ $\qquad$ $I_a^d = I_B^d - I_C^d = n(I_b^s - I_c^s)$

$I_B^d = n.I_b^s$ $\qquad$ $I_b^d = I_C^d - I_A^d = n(I_c^s - I_a^s)$

$I_C^d = n.I_c^s$ $\qquad$ $I_c^d = I_A^d - I_B^d = n(I_a^s - I_b^s)$

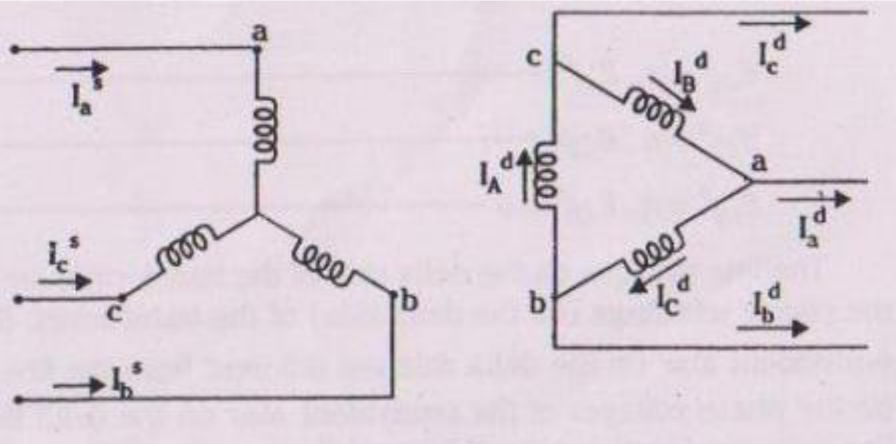


Fig. 3.17

Considering only positive sequence currents, the above relation becomes

$I_{a1}^d = n(I_{b1}^s - I_{c1}^s) = n(a^2.I_{a1} - a.I_{a1}^s) = n(a^2 - a)I_{a1}^s = -j\sqrt{3}\ n.I_{a1}^s$

$I_{a1}^s = (j/n.\sqrt{3})I_{a1}^d$

on the same lines

$I_{a2}^d = n(I_{b2}^s - I_{c2}^s) = n(a.I_{a2}^s - a2.I_{a2}^s) = n(a - a2)I_{a2}^s = j\sqrt{3}.n.I_{a2}^s$

$I_{a2}^s = (-j/n\sqrt{3})I_{a2}^d$

Note:

1)The turns ratio 'n' of a transformer is defined as,

n=number of primary turns / number of secondary turns

=primary voltage / secondary voltage

=secondary current / primary current

2)If each voltage is expressed in per unit with its own voltage as the base voltage, then equations can be written as,

$E_{a1}^{s}=jE_{a1}^{d}$ p.u

$E_{a2}^{s}= -jE_{a2}^{d}$ p.u

3) Similarly, the per unit for a Y-Δ transformer

$I_{a1}^{s}=jI_{a1}^{d}$ p.u

$I_{a2}^{s}= -jI_{a2}^{d}$ p.u

4)If delta side forms the primary and the star side forms the secondary, that is in the case of Δ-Y transformer

$I_{a1}^{d}=jI_{a1}^{s}$ p.u .

$I_{a2}^{d}= -jI_{a2}^{s}$ p.u

$E_{a1}^{d}=jE_{a1}^{s}$ p.u

$E_{a2}^{d}= -jE_{a2}^{s}$ p.u

Complex power in terms of symmetrical components

$S=P+jQ = V_a.I_a^*+V_b.I_b^*+V_c.I_c^*$

where,

S=Total complex power,(* indicates conjugate)

P=Active power

Q=Reactive power

In matrix from, the above equation can be expressed as,

$$S=P+jQ=\begin{bmatrix}V_a & V_b & V_c\end{bmatrix}\begin{bmatrix}I_a^*\\I_b^*\\I_c^*\end{bmatrix}$$

$$\begin{bmatrix}V_a & V_b & V_c\end{bmatrix}=\begin{bmatrix}V_a\\V_b\\V_c\end{bmatrix}^T=\left\{\begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}\begin{bmatrix}V_{a0}\\V_{a1}\\V_{a2}\end{bmatrix}\right\}^T$$

$$=\begin{bmatrix}V_{a0}\\V_{a1}\\V_{a2}\end{bmatrix}^T\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}$$

$([A].[B])^T=[A]^T.[B]^T$

$$\begin{bmatrix}I_a^*\\I_b^*\\I_c^*\end{bmatrix}=\begin{bmatrix}I_a\\I_b\\I_c\end{bmatrix}^*=\left\{\begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}\right\}^*$$

$$=\begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}^*\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

Now $a^*=a^2$

$(a^2)^*=a$, Using these, we get

$$\begin{bmatrix}I_a\\I_b\\I_c\end{bmatrix}^*=\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

$$S=(P+jQ)=\begin{bmatrix}V_{a0}\\V_{a1}\\V_{a2}\end{bmatrix}^T\begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}.\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

$$=\begin{bmatrix}V_{a0} & V_{a1} & V_{a2}\end{bmatrix}\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

$S=P+jQ = 3\{V_{a0}.I_{a0}^*+V_{a1}.I_{a1}^*+V_{a2}.I_{a2}^*\}$ VA

Note:

1)In terms of active and reactive powers, the above equation can be written as

$$P=3\{|V_{a0}|\ |I_{a0}|\ \cos\theta_0 + |V_{a1}|\ |I_{a1}|\ \cos\theta_1 + |V_{a2}|\ |I_{a2}|\ \cos\theta_2\}\ W$$

and

$$Q=3\{|V_{a0}|\ |I_{a0}|\ \sin\theta_0 + |V_{a1}|\ |I_{a1}|\ \sin\theta_1 + |V_{a2}|\ |I_{a2}|\ \sin\theta_2\}\ VAR$$

2)If VB is the base voltage and IB the base current of the system, then the complex power in pu is given as,

$$S_{p.u}=3(V_{a0}\ I_{a0}^* + V_{a1}\ I_{a1}^* + V_{a2}\ I_{a2}^*)\ (1\ /\ V_B\ I_B)$$

$$S_{p.u}=S\ /S_B$$

Where SB=base power of the system =3*VB*IB

3)If the symmetrical components of voltages and currents are given in pu directly, then the total 3 phase power is given as

$$S_{p.u}=V_{a0}\ I_{a0}^* + V_{a1}\ I_{a1}^* + V_{a2}\ I_{a2}^*$$

The sequence components of the phase voltages are $V_{a1}=200\angle 30°$, $V_{a2}=60\angle 60°$ and $V_{a0}=20\angle -30°$ V. The line currents are $I_{a1}=20\angle 10°$, $I_{a2}=5\angle 20°$ and $I_{a0}=3\angle -10°$ A. Determine the three phase power in kVA and p.u. If the base power is 1kVA.

The three phase complex power is given as

$S_{p.u}=3(V_{a0}\, I_{a0}^*+V_{a1}\, I_{a1}^*+V_{a2}\, I_{a2}^*)$

$=3\{(20\angle -30°)(3\angle 10°)+(200\angle 30°)(20\angle -10°)+(60\angle 60°)(5\angle -20°)\}$

$=(12.13+j4.62)$ kVA

We have $S_{p.u}=S\,/S_B = (12.13+j4.62)$ kVA / 1kVA = $(12.13+j4.62)$ p.u

In a three phase system, the sequence quantities are Va1=(0.9+j0.2)p.u; Va2=(0.1+j0.1)p.u; Va0=(0.1+j0.05)p.u and Ia1=(0.9-j0.1) p.u; Ia2=(0.2-j0.1)p.u; Ia0=(0.05-j0.02)p.u. Find the three phase complex power in p.u and in MAV on a base of 100MVA. Also compute the active and reactive powers.

$$S_{p.u}=(V_{a0}\, I_{a0}{}^{*}+V_{a1}\, I_{a1}{}^{*}+V_{a2}\, I_{a2}{}^{*})$$

$$=\{(0.1+j0.05)(0.05-j0.02)^{*}+(0.9+j0.2)(0.9-j0.1)^{*}+(0.1+j0.1)(0.2-j0.1)^{*}\}$$

$$=\{(0.1+j0.05)(0.05+j0.02)^{*}+(0.9+j0.2)(0.9+j0.1)^{*}+(0.1+j0.1)(0.2+j0.1)^{*}\}$$

$$=(0.817+j0.3126)\text{p.u}$$

Next, $S=S_{p.u}\times S_B$

$$=(0.817+j0.3126)\times 100\text{ MVA} =(81.7+j31.26)\text{MVA}$$

We have, $S=P+jQ=(81.7+j31.26)$MVA

Therefore,

The active power is P=81.7MW

The reactive power is Q=31.26MVAR

In a three phase four wire system, the sequence voltages and currents are: Va1=0.9∠10°p.u; Va2=0.25∠110°p.u; Va0=0.12∠300°p.u and Ia1=0.75∠25° p.u; Ia2=0.15∠170°p.u; Ia0=0.1∠330°p.u. Find the complex power in p.u. If the neutral gets disconnected, find the new power.

The total three phase power in pu is given as,

$$S_{p.u}=(V_{a0}\ I_{a0}^{*}+V_{a1}\ I_{a1}^{*}+V_{a2}\ I_{a2}^{*})$$

$$=(0.12\angle 300°)(0.1\angle 330°)^{*}+(0.9\angle 10°)(0.75\angle 25°)^{*}+(0.25\angle 110°)(0.15\angle 170°)^{*}$$

$$=(0.12\angle 300°)(0.1\angle -330°)+(0.9\angle 10°)(0.75\angle -25°)+(0.25\angle 110°)(0.15\angle -170°)$$

$$=(0.68-j0.212)\text{p.u}$$

When the neutral gets opened, then $I_{a0}=0$. Hence the new power is,

$$S_{p.u}'=(V_{a1}\ I_{a1}^{*}+V_{a2}\ I_{a2}^{*})$$

$$=(0.9\angle 10°)(0.75\angle 25°)^{*}+(0.25\angle 110°)(0.15\angle 170°)^{*}$$

$$=(0.9\angle 10°)(0.75\angle -25°)+(0.25\angle 110°)(0.15\angle -170°)$$

$$=(0.67-j0.206)\text{p.u.}$$

Some Facts about Sequence Currents

The following facts about positive, negative and zero phase sequence currents

1. A balanced 3-phase system consists of positive sequence components only; the negative and zero sequence components being zero.

2. The presence of negative or zero sequence currents in a 3-phase system introduces Unsymmetry and is indicative of an abnormal condition of the circuit in which these components are found.

3. The vector sum of the positive and negative sequence currents of an unbalanced 3-phase system is zero. The resultant only consists of three zero sequence currents

   i.e. Vector sum of all sequence currents in 3-phase unbalanced system = $I_{R0}+I_{y0}+I_{B0}$

4. In a 3-phase, 4 wire unbalanced system, the magnitude of zero sequence components is one-third of the current in the neutral wire

i.e. Zero sequence current = 1/3 [Current in neutral wire]

In the absence of path through the neutral of a 3-phase system, the neutral current is zero and the line currents contain no zero-sequence components. A delta-connected load provides no path to the neutral and the line currents flowing to delta-connected load can contain no zero-sequence components

5. In a 3-phase unbalanced system, the magnitude of negative sequence components cam exceed that of the positive sequence components. If the negative sequence components were the greater, the phase sequence of the resultant system would be reversed.
6. The current of a single phase load drawn from a 3-phase system comprises equal positive, negative and zero sequence components.

SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

The sequence impedances are impedances offered by the circuit elements (or power system components) to positive, negative and zero sequence currents.

In any element of a circuit, the voltage drop caused by current of a certain sequence depends on the impedance of the element to that sequence current.

The impedance offered by an equipment or circuit when positive sequence currents alone are flowing is called the positive sequence impedance. is represented by Z_1

Similarly, The impedance offered by an equipment or circuit to negative and zero sequence currents are Respectively called Negative sequence impedance Z_2 and Zero sequence impedance Z_0

For example, **the impedance which any piece of equipment offers to positive sequence current will not necessarily be the same as offered to negative sequence current or zero sequence current.**

Therefore, in unsymmetrical fault calculations, each piece of equipment will have three values of impedance-one corresponding to each sequence current viz.

(i) Positive sequence impedance (Z_1)

(ii) Negative sequence impedance (Z_2)

(iii) Zero sequence impedance (Z_0)

The following points may be noted :

1. In a 3-phase balanced system, each piece of equipment or circuit offers only one impedance-the one offered to positive or normal sequence current. This is expected because of the absence of negative and zero sequence currents in the 3-phase balanced system.

2. In a 3-phase unbalanced system, each piece of equipment or circuit will have three values of impedances viz. positive sequence impedance, negative sequence impedance and zero sequence impedance.

3. The positive and negative sequence impedances of linear, symmetrical and static circuits(e.g. transmission lines, cables, transformers and static loads) are equal and are the same as those used in the analysis of balanced conditions. This is due to the fact that impedance of such circuits is independent of the phase order, provided the applied voltages are balanced.

   It may be noted that positive and negative sequence impedances of rotating machines (e.g. synchronous and induction motors) are normally different.

4. **The zero sequence impedance depends upon the path taken by the zero sequence current.**
   As this path is generally different from the path taken by the positive and negative sequence currents, therefore, zero sequence impedance is usually different from positive or negative sequence impedance.

Sequence impedance of a symmetrical circuit

The voltage drops are computed as follows

$V_a = I_a.Z + I_n.Z_n$

$= I_a.Z + (I_a + I_b + I_c)Z_n$

$= I_a.(Z + Z_n) + I_b.Z_n + I_c Z_n$ 1

Similarly,

$V_b = I_a.Z_n + I_b(Z + Z_n) + I_c.Z_n$ 2

and $V_c = I_a.Z_n + I_b.Z_n + I_c(Z_n + Z_n)$ 3

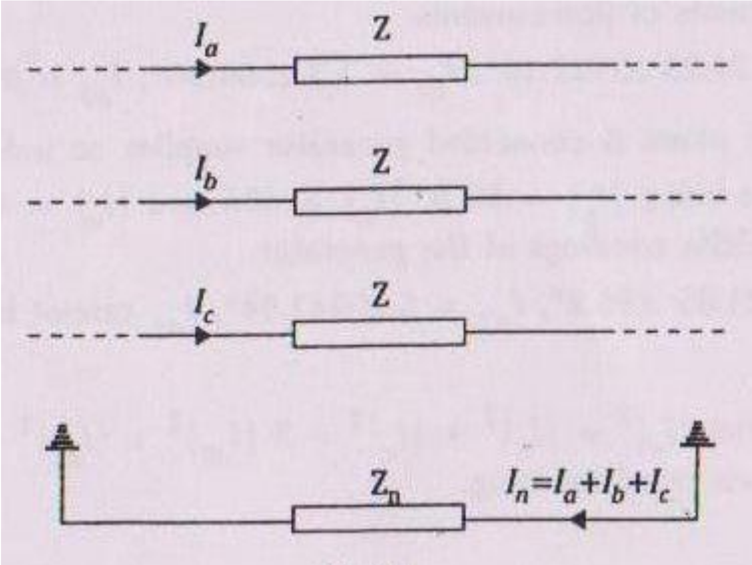


Fig. 4.1

In matrix form, the above equations can be expressed as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \text{.................} \; 4$$

Expressing the voltages and currents by their sequence components, we get.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

or,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z+3Z_n & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 5$$

This gives relationships

$V_{a0}=(Z+3Z_n)I_{a0}$

$V_{a1}=Z.I_{a1}$

$V_{a2}=Z.I_{a2}$ 6

Also, as per definition of sequence impedances, we have the three sequence impedances as,

$Z_0=V_{a0}/I_{a0}=Z+3Z_n$=Zero sequence impedance 7

$Z_1=V_{a1}/I_{a1}=Z$=positive sequence impedance 8

$Z_2=V_{a2}/I_{a2}=Z$=negative sequence impedance 9

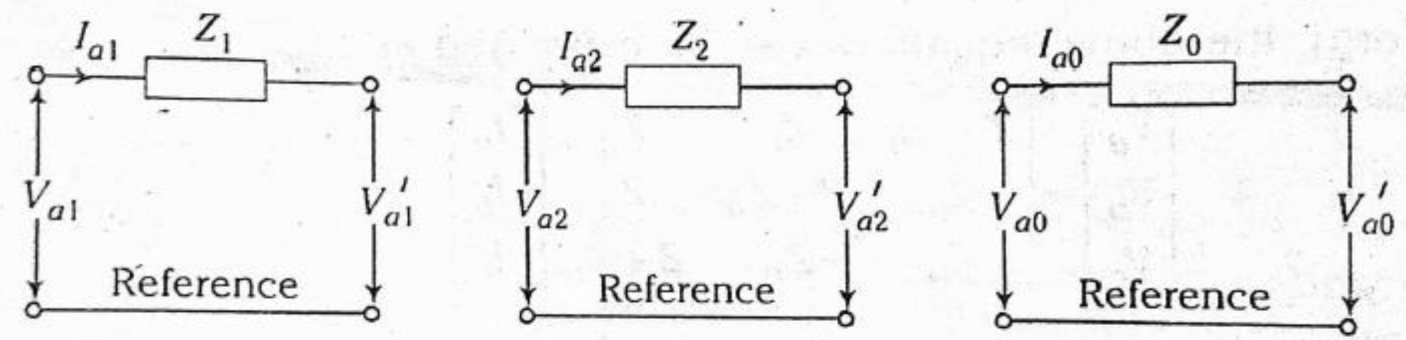
Note:

i) The positive sequence impedance is the same as the negative sequence impedance.

ii) Zero sequence impedance is much larger than the positive (or negative) sequence impedance. In the absence of the neutral, Zn= ∞

therefore,

Z0= ∞ and hence Ia0=Va0/Z0=Va0/∞=0 , as expected.



(a) Positive sequence network (b) Negative sequence network (c) Zero sequence network

Across a star connected symmetrical impedance load of 10Ω in each phase and a neutral impedance of 3.33Ω, an unbalanced three phase supply with Va=220∠0°, Vb=200∠110° and Vc=180∠-110° is applied. Determine the line currents using symmetrical components.

Solution:

since the circuit is symmetrical, we have

Z1=Z2=10 Ω

and, Z0=Z+3Zn=10+3(3.33)=20 Ω

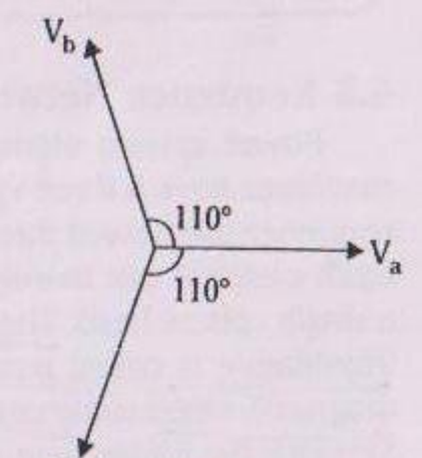


Fig. 4.2

It can be seen that phase sequence is acb. Hence in the equations for the determination of sequence components of phase voltages, the subscripts b and c should be interchanged.

$$V_{a1}=(1/3)\,(V_a + a.\,V_c + a^2.V_b)$$
$$=(1/3)\,(220\angle 0° + 180\angle(-110° + 120°) + 200\angle(110° + 240°))$$
$$=198.07\angle -0.33° \text{ volts.}$$
$$V_{a2}=(1/3)\,(V_a + a^2.\,V_c + a.V_b)$$
$$=(1/3)\,(220\angle 0° + 180\angle(-110° + 240°) + 200\angle(110° + 120°))$$
$$=9.56\angle -147.7° \text{ volts.}$$

$V_{a0} = (1/3)\ (V_a + V_c + V_b)$

$= 220\angle 0° + 180\angle -110° + 200\angle 110°$

$= 30.64\angle 11.77°$ volts.

Now,

Ia1=Va1/Z1

$= (198.07\angle -0.33°)/10$

$= 19.807\angle -0.33°$ A

$I_{a2} = V_{a2}/Z_2$

$= (9.56\angle -147.7°)/10$

$= 0.956\angle -147.7°$ A

$I_{a0} = V_{a0}/Z_0$

$= (30.64\angle 11.77°)/20$

$= 1.532\angle 11.77°$ A

The line currents are,

$I_a = I_{a0} + I_{a1} + I_{a2}$

$= 1.532\angle 11.77° + 19.807\angle -0.33 + 0.956\angle -147.7°$

$= 20.49\angle -0.87°$ A

$I_b = I_{a0} + a.I_{a1} + a^2.I_{a2}$ (for phase sequence acb)

$= 1.532\angle 11.77° + 19.807\angle(-0.33° + 120°) + 0.956\angle(-147.7° + 240°)$

$= 20.27\angle 114.4°$ A

$I_c = I_{a0} + a^2.I_{a1} + a.I_{a2}$

$= 1.532\angle 11.77° + 19.807\angle(-0.33° + 240°) + 0.956\angle(-147.7° + 120°)$

$= 18.85\angle -113.86°$ A

Prove that a three phase symmetrically wound alternator generators only positive sequence components of voltages.

Figure depicts an unbalanced synchronous generator. Ea, Eb, Ec are the induced emfs of the three phases. Since the windings are symmetrical, the induced emfs are perfectly balanced.

Let, $|E_a|=|E_b|=|E_c|=V_p$

Then, if follows that (assuming a abc phase sequence)

$E_a=V_p\angle 0°$

$E_b=V_p\angle -120°$

$E_c=V_p\angle 120°$

Hence the sequence components of voltages are,

$E_{a0}=(1/3)(E_a+E_b+E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle -120°+V_p\angle 120°)$

$=0$

$E_{a1}=(1/3)(E_a+a.E_b+a^2E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle(-120°+120°)+V_p\angle(120°+240°))$

$=V_p$

$=E_a$

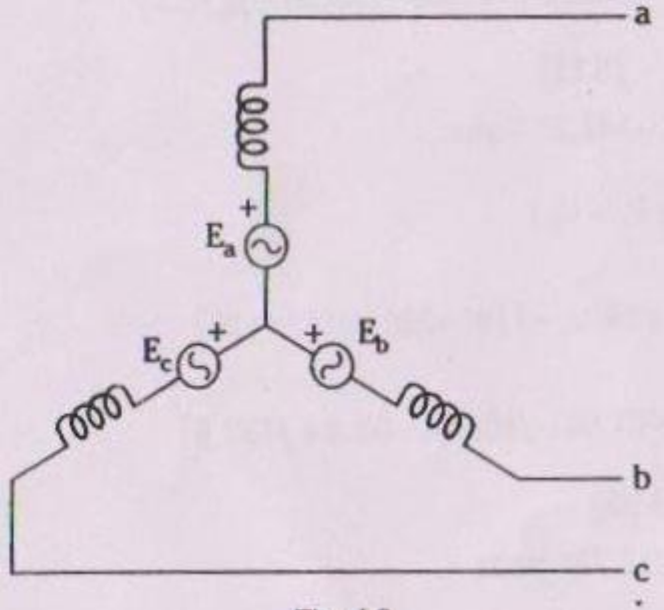


Fig. 4.3

$$E_{a2}=(1/3)(E_a+a^2.E_b+a.E_c)$$
$$=(1/3)(V_p\angle 0°+V_p\angle(-120°+240°)+V_p\angle(120°+120°))$$
$$=0$$

From the results obtained above, it can be inferred that a three phase symmetrically wound alternator generators only positive sequence components of voltages.

Sequence impedances and networks of synchronous generator

- Here, Ea, Eb and Ec are the induced emfs of the three phases.
- Ia, Ib and Ic are the currents flowing in the lines
- Since a synchronous machine is designed with symmetrical windings, it induces emfs of positive sequence only (This fact is proved in example).
- Figure shows the three phase positive sequence network model of the synchronous generator Zn does not appear in the model as In=0 for positive sequence currents.
- Ea1, Eb1 and Ec1 are the positive sequence generated voltages and Z1 is the positive sequence impedance.
- Because of the balanced and symmetrical nature of the system, the three phase system can be replaced by a single phase network as shown in figure

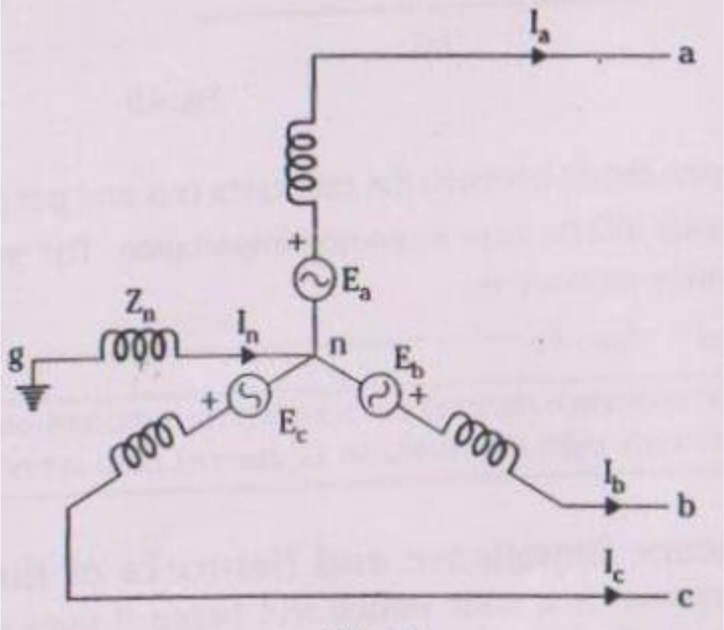
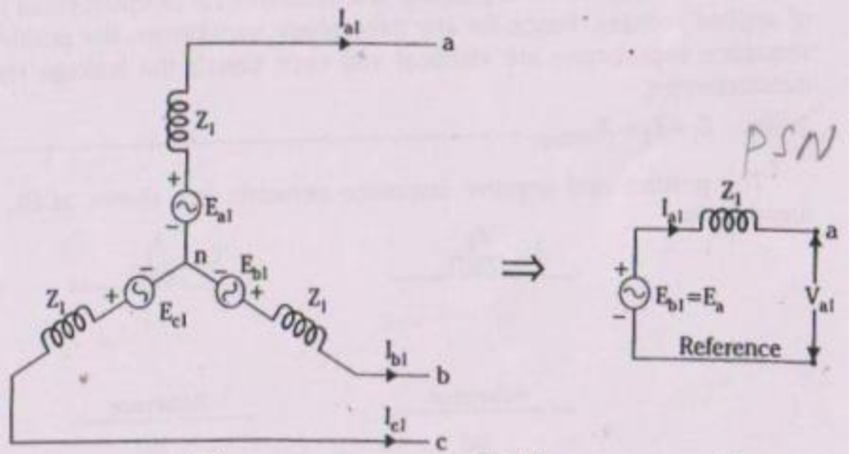


Fig. 4.4



(a) Fig. 4.5 (b)

Using the notation E for generated voltage and V for the terminal voltage, then the equation for positive sequence network is,

$$V_{a1}=E_{a1}-I_{a1}Z_1=E_a-I_{a1}Z_1$$

The negative sequence network models of a synchronous generator on a three phase and single-phase basis are shown in fig

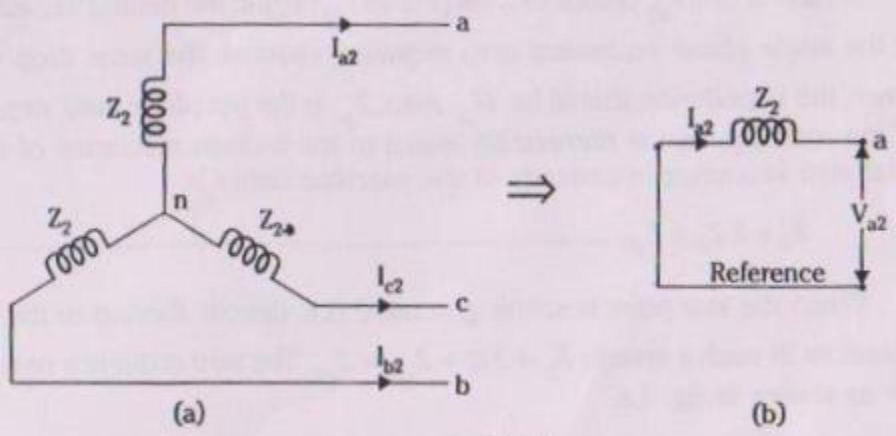


Fig. 4.6

The equation that holds good for the negative sequence network is,

$$V_{a2}=-I_{a2}Z_2$$

For a synchronous generator, star point is grounded through an impedance Zn, the zero sequence network models are as shown in fig

$$Z_0=3Z_n+Z_{g0}$$

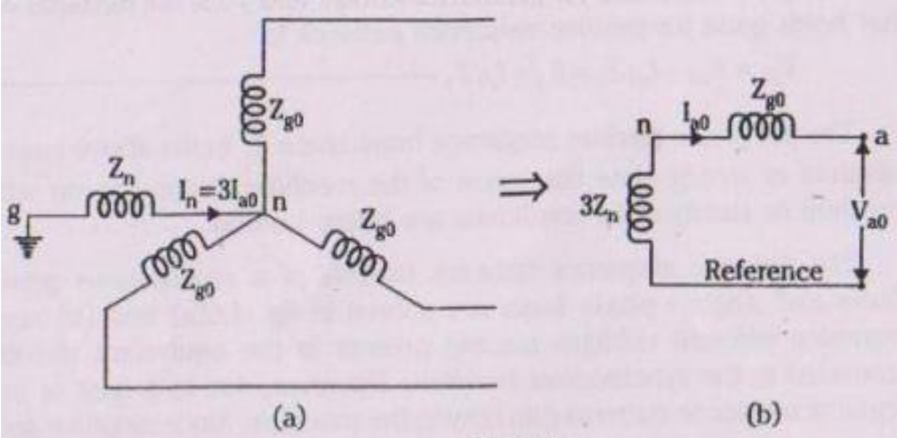


Fig. 4.7

When the star point is solidly grounded Zn=0.
Therefore in such a system Z0=3(0)+Zg0=Zg0.
The zero sequence networks in this case are as shown in fig

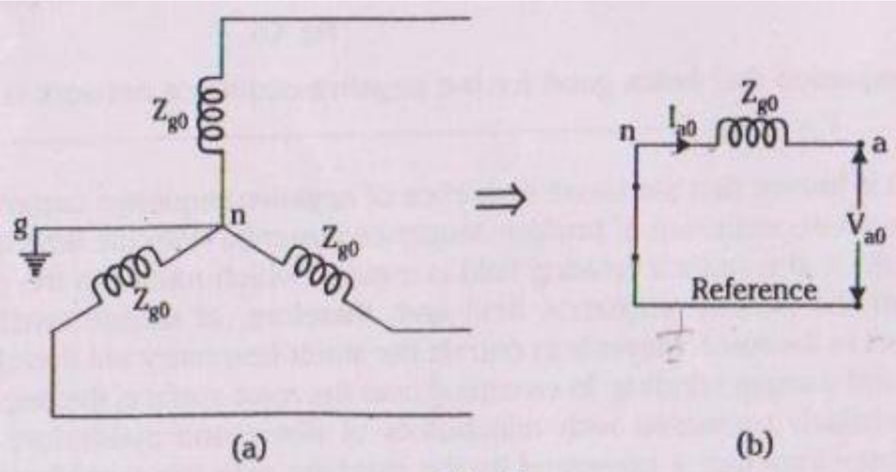


Fig. 4.8

$$V_{a0}= -I_{a0}.Z_0$$

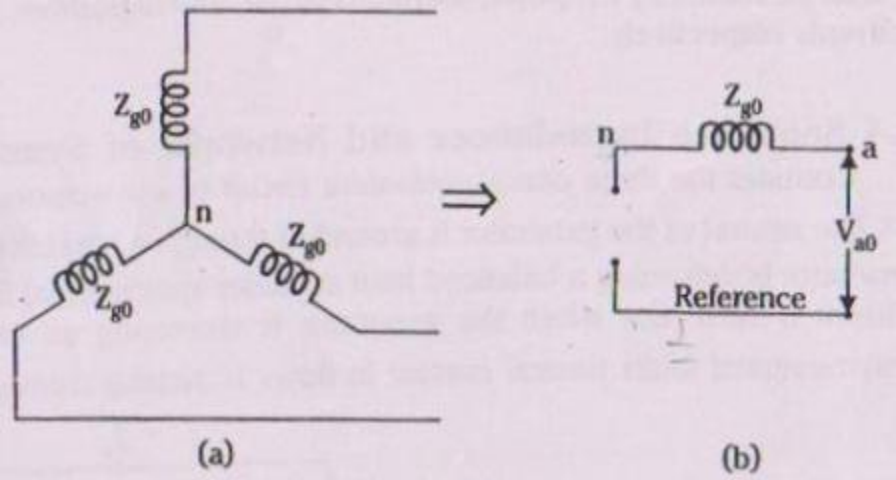


Fig. 4.9

When the star point is not grounded, Zn=∞,
Therefore eq. becomes Z0=∞+Zg0=∞.
The zero sequence networks for a ungrounded generator is as shown in figure.

Sequence Impedance and networks of three phase transformer

The positive and negative sequence networks are shown in figures.

Z1=Z2=X leakage

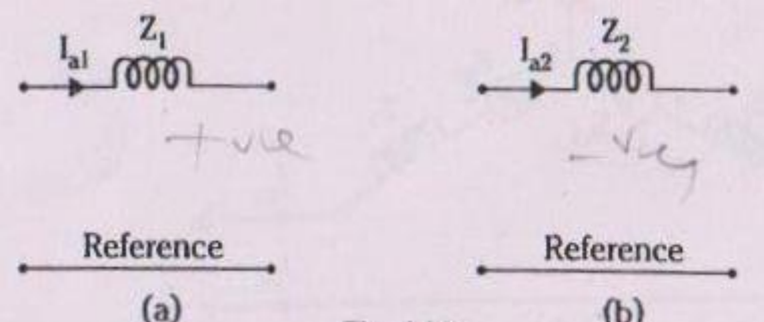


Fig. 4.10

Z0 is the per phase zero sequence impedance of the winding of the transformer.

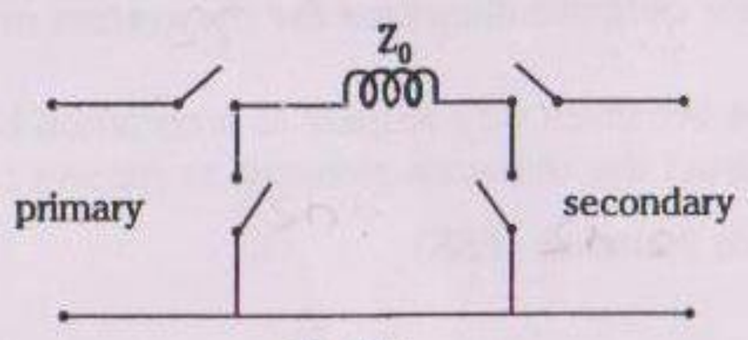
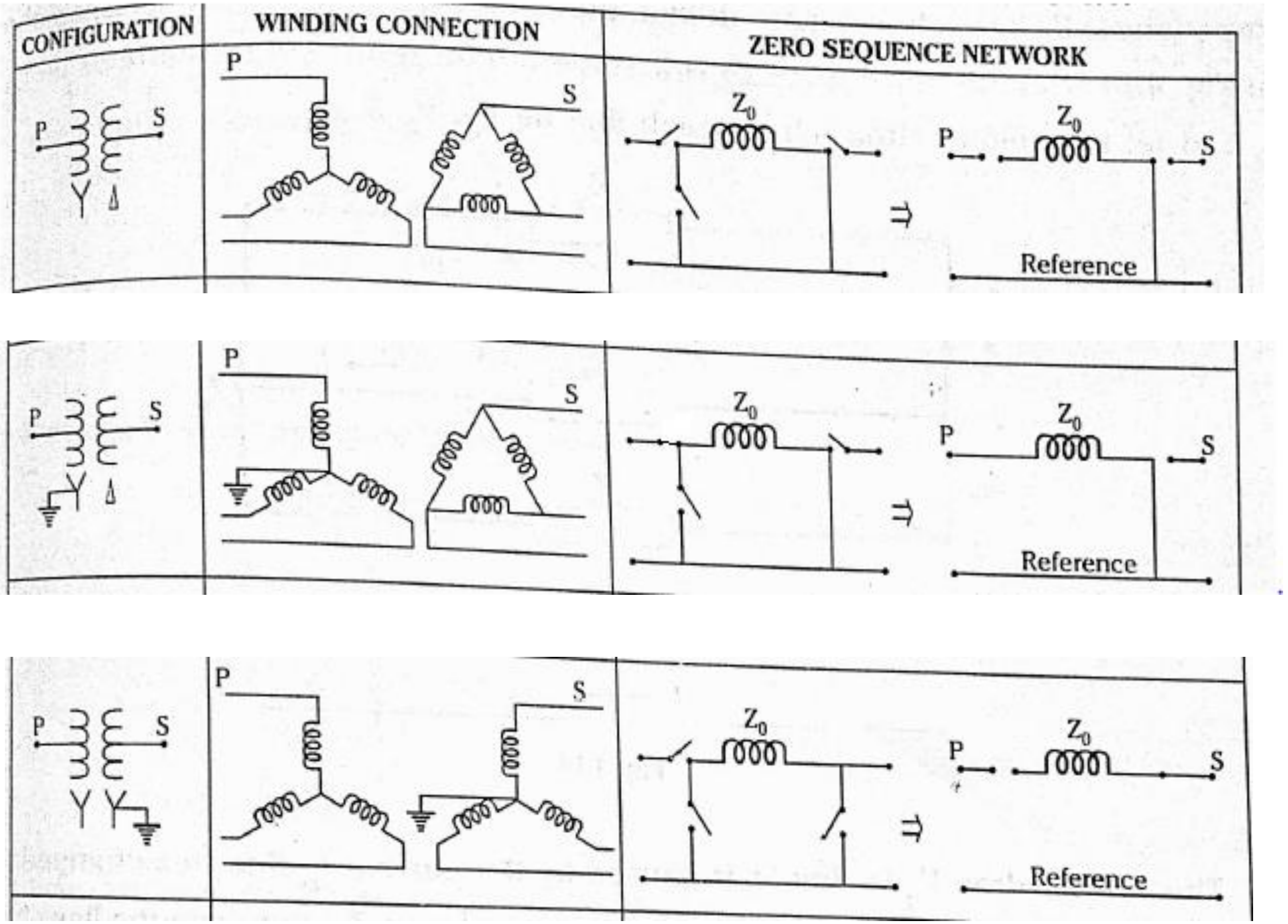
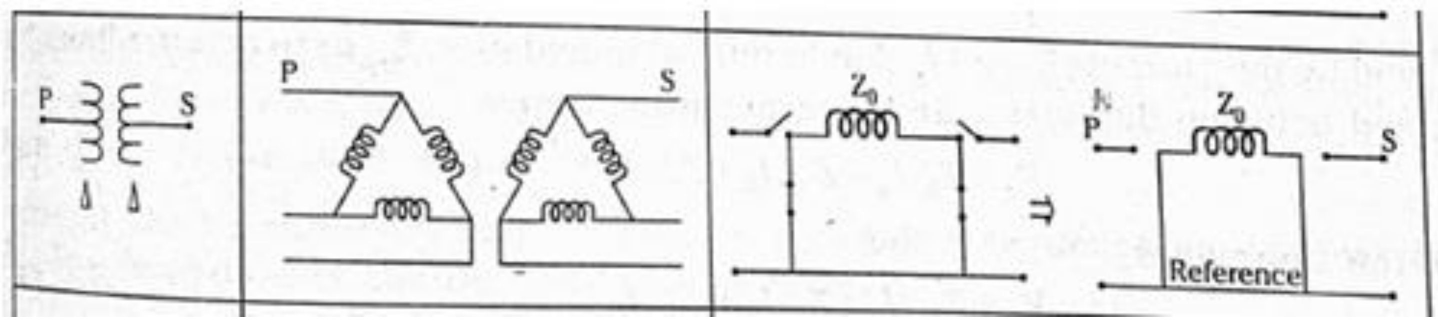
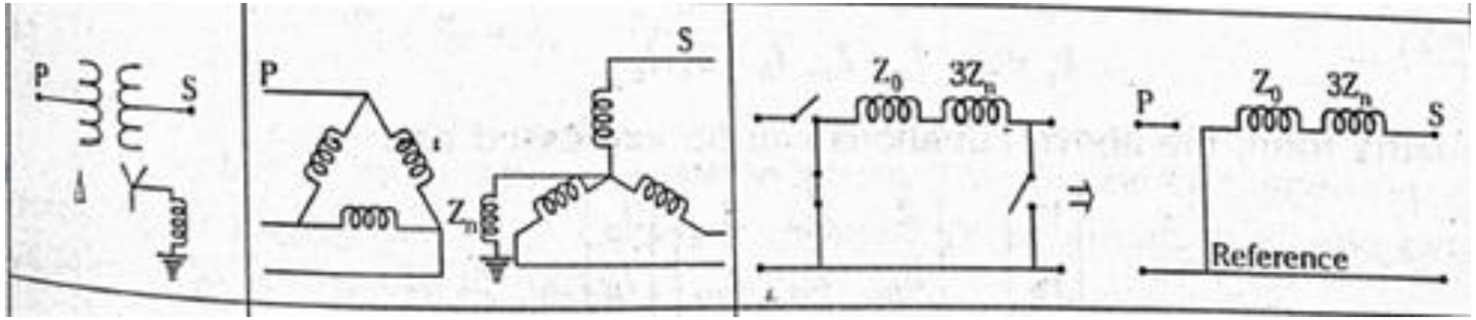


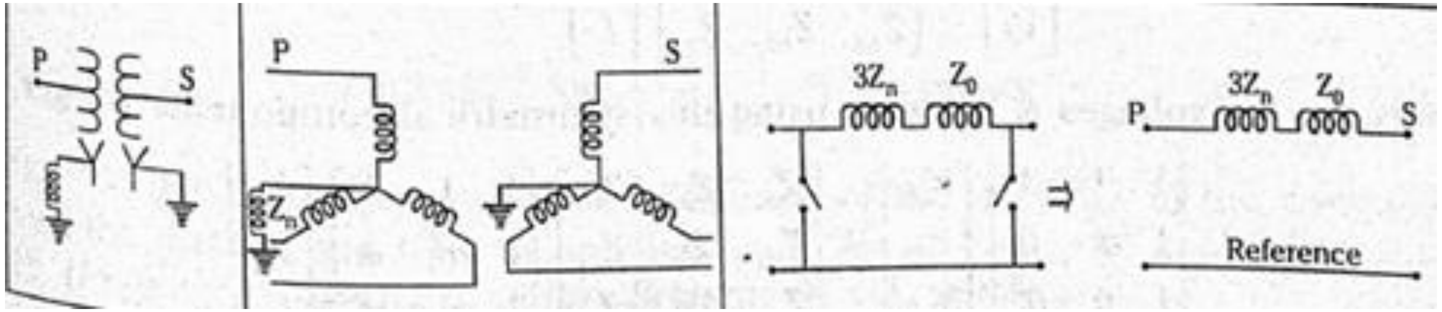
Fig. 4.11

Using the general rule, the zero sequence network for some of the configurations of three phase transformer are

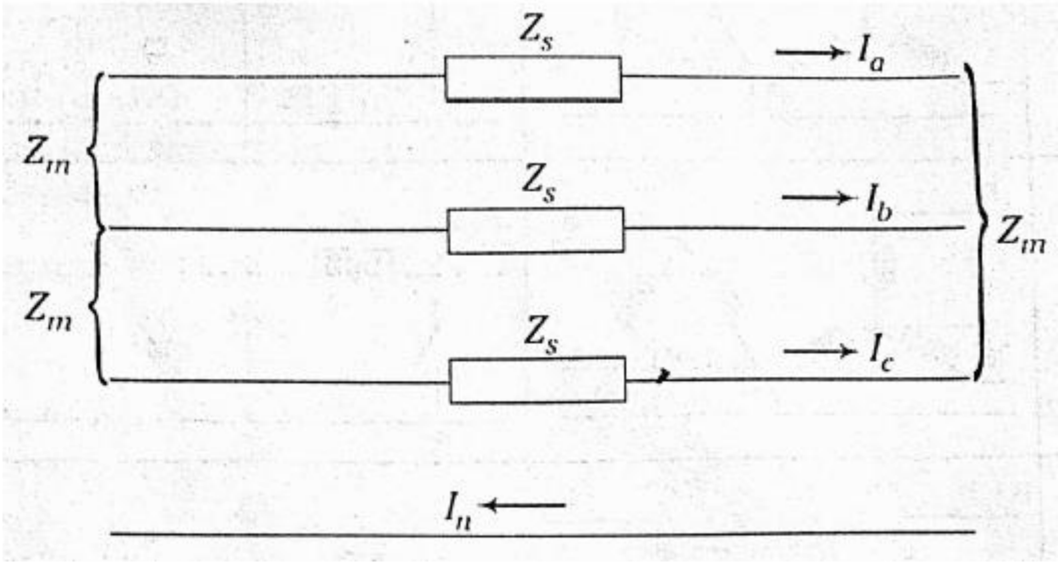



P
S
Z0
Reference


P
S
Zn
Z0
3Zn
Reference


P
S
Zn
3Zn
Z0
Reference

Sequence impedances and networks of transmission lines



- A Symmetrical circuit of three phase transmission line has a self impedance, Zs in each line and mutual impedance in between the lines.
- Neutral impedance Zn is assumed to be zero
- Let Currents are Ia, Ib, Ic, and voltage drop across each line be Va, Vb, Vc
- The voltage drop Va in line a is caused by the current Ia due to self impedance Zs, by the currents Ib, Ic due to Mutual impedance Zm between the lines b and a, between the lines c and a respectively.

$$V_a = Z_s.I_a + Z_m.I_b + Z_m.I_c$$

On the same lines,

$$V_b = Z_m.I_a + Z_s.I_b + Z_m.I_c$$

$$V_c = Z_m.I_a + Z_m.I_b + Z_s.I_c$$

In matrix form, the above equations can be expressed as :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Expressing the voltages & currents using this symmetrical components we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

or

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

This gives the relationships

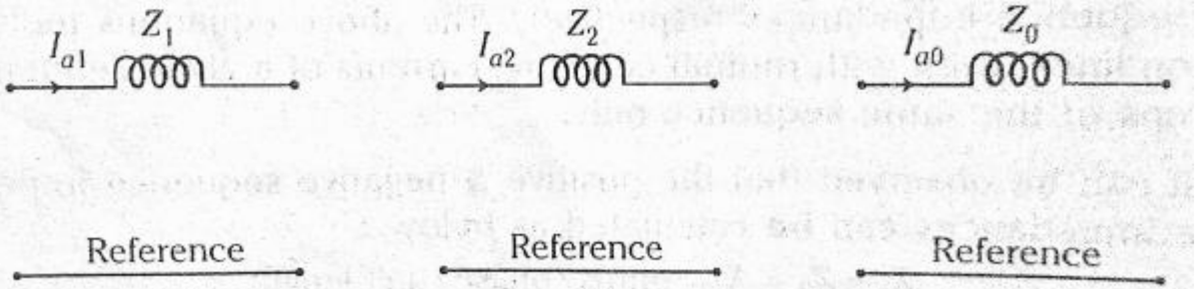
$$\left.\begin{aligned} V_{a0} &= (Z_s + 2Z_m).I_{a0} \\ V_{a1} &= (Z_s - Z_m).I_{a1} \\ V_{a2} &= (Z_s - Z_m).I_{a2} \end{aligned}\right\}$$

As per definitions of sequence impedances

$$Z_0 = \frac{V_{a0}}{I_{a0}} = (Z_s + 2Z_m)$$

$$Z_1 = \frac{V_{a1}}{I_{a1}} = (Z_s - Z_m)$$

$$Z_2 = \frac{V_{a2}}{I_{a2}} = (Z_s - Z_m)$$



(a) (b) (c)

Sequence impedances and networks of load

In balanced Y or Δ connected loads the positive, negative and zero sequence impedances are equal.

When the neutral point of star connected load is grounded through a impedance Zn then 3Zn is added to the zero sequence impedance of load to get the total zero sequence impedance of load.

The positive and negative sequence impedances of load are represented as a shunt impedance in their respective sequence network as shown in below fig

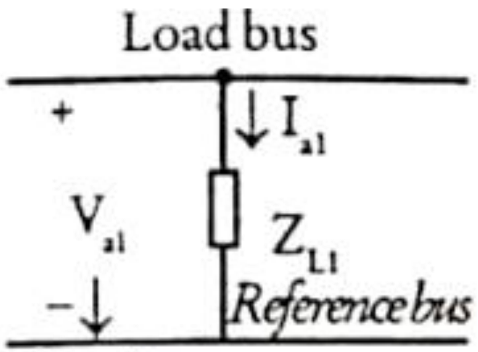


Fig a : Positive sequence network

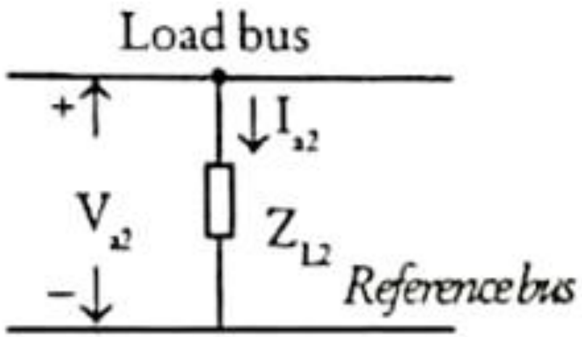


Fig b : Negative sequence network

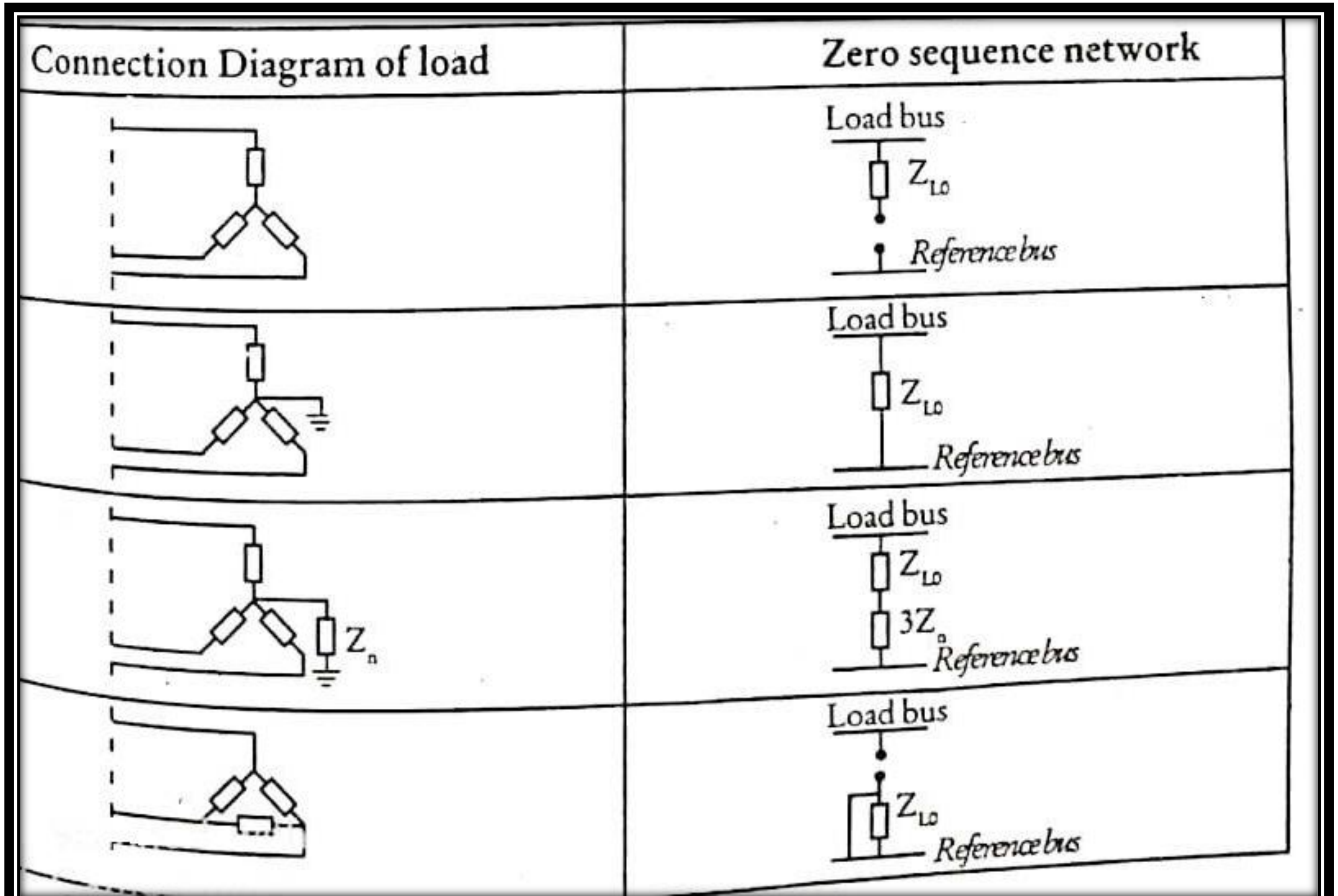
Z_{L1} = Positive sequence impedance of load

Z_{L2} = Negative sequence impedance of load

Z_{L0} = Zero sequence impedance of load.

Zero Sequence networks of load

The zero sequence network of the 3-phase load depends on the type of connection, i.e. Y or Δ connection. The zero sequence current will flow in network only if a return path exists for it. The zero sequence network for various types of loads are shown in below table



Positive sequence network:

The positive sequence network for a given power system shows all the paths for the flow of positive sequence currents in the system. It is represented by one-line diagram and is composed of impedances offered to the positive sequence currents.

While drawing the positive sequence network of a given power system, the following points may be kept in view:

I. **Each generator in the system is represented by the generated voltage in series with appropriate impedance**
II. **Current limiting impedances between the generator's neutral and ground pass no positive sequence current and hence are not included in the positive sequence network.**
III. **All resistances and magnetizing currents for each transformer are neglected**
IV. **For transmission lines, the shunt capacitances and resistances are generally neglected.**
V. **Motor loads are included in the network as generated e.m.f. in series with appropriate**

Negative sequence network:

The negative sequence network for a given power system shows all the paths for the flow of negative sequence currents in the system. It is also represented by one line diagram and is composed of impedances offered to the negative sequence currents. The negative sequence network can be readily obtained from positive sequence network with the following modifications :

1. **Omit the e.m.f. of 3-phase generators and motors in the positive sequence network. It is because these devices have only positive sequence-generated voltages.**
2. **Change, if necessary, the impedances that represent rotating machinery in the positive sequence network. It is because negative sequence impedance of rotating machine is generally different from that of positive sequence impedance.**
3. **Current limiting impedances between generator’s neutral and ground pass no negative sequence current and hence are not included in the negative Sequence Networks Unsymmetrical Faults.**
4. **For static devices such as transmission lines and transformers, the negative sequence impedances have the same value as the corresponding positive sequence impedances**

Zero sequence network:

The zero sequence network for a given power system shows all the paths for the flow of zero sequence currents. The zero sequence network of a system depends upon the nature of connections of the 3-phase windings of the components in the system.

The following points may be noted about zero Sequence Networks Unsymmetrical Faults:

1. **The zero sequence currents will flow only if there is a return path i.e. path from neutral to ground or to another neutral point in the circuit.**

1. **In the case of a system with no return path for zero sequence currents, these currents cannot exist.**

Reference Bus for Sequence Networks:

While drawing the sequence networks, it is necessary to specify the reference potential. which all sequence voltage drops are to be taken.

For this purpose, the Student may keep in mind the following points :

1. **For positive or negative Sequence Networks Unsymmetrical Faults, the neutral of the generator is taken as the reference bus.** This is logical because positive or negative sequence components represent balanced sets (they have phase difference of 120°, so they will not pass through neutral reactance) and hence all the neutral points must be at the same potential for either positive or negative sequence currents.

2. **For zero sequence network, the reference bus is the ground at the generator.** (three Zero Sequence Current is Identical (both in Magnitude and Phase) and being in phase, do not sum to zero at the star point, but they flow back along the neutral earth connection, return through the ground

A 250 MVA, 11kV, 3 phase generator is connected to a large system through a transformer and a line as shown in fig below.

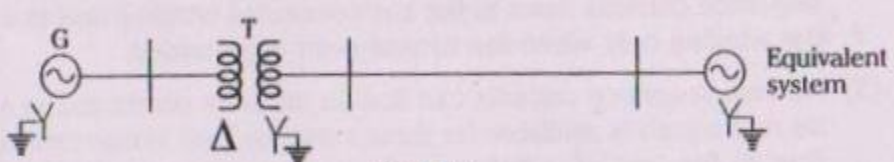


Fig. 4.13

Generator: $X_1 = X_2 = 0.15$ p.u, $X_0 = 0.1$ p.u

Transformer: $X_1 = X_2 = X_0 = 0.12$ p.u

Line: $X_1 = X_2 = 0.25$ p.u, $X_0 = 0.75$ p.u

Equivalent system: $X_1 = X_2 = X_0 = 0.15$ p.u

Draw the sequence network diagrams for the system and indicate all per unit values.

Solution:

all reactance's are give with respect to a common base in this problem. Hence we can directly construct the sequence network as follows:

Positive sequence network (PSN):

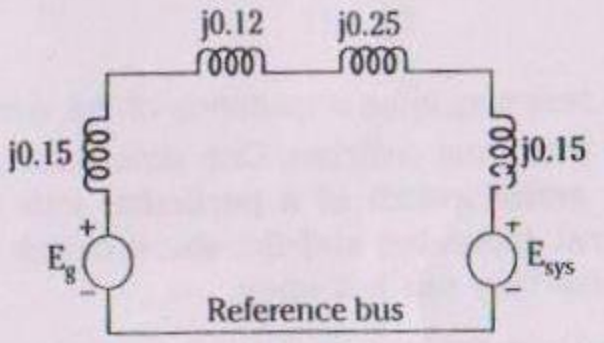


Fig. 4.14

Negative sequence network (NSN):

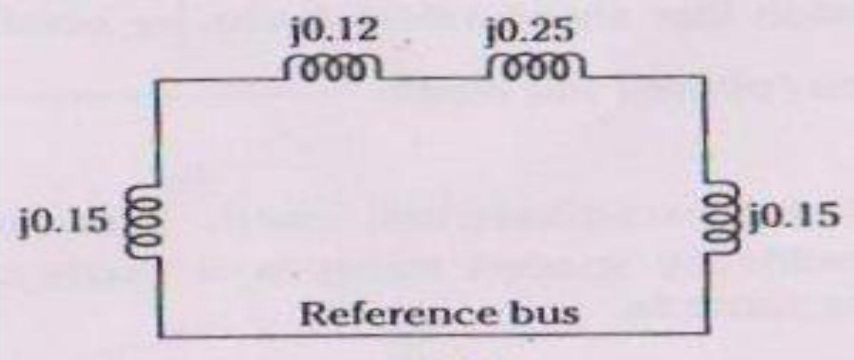
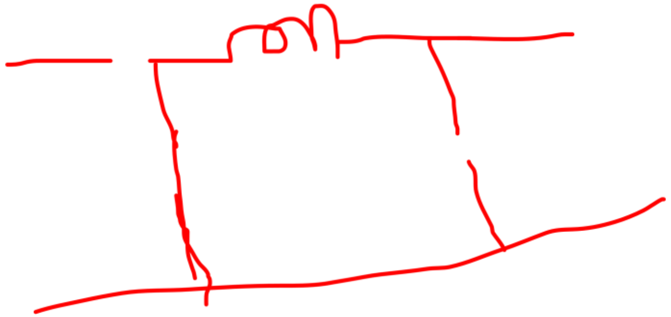


Fig. 4.15



Zero sequence network (ZSN):

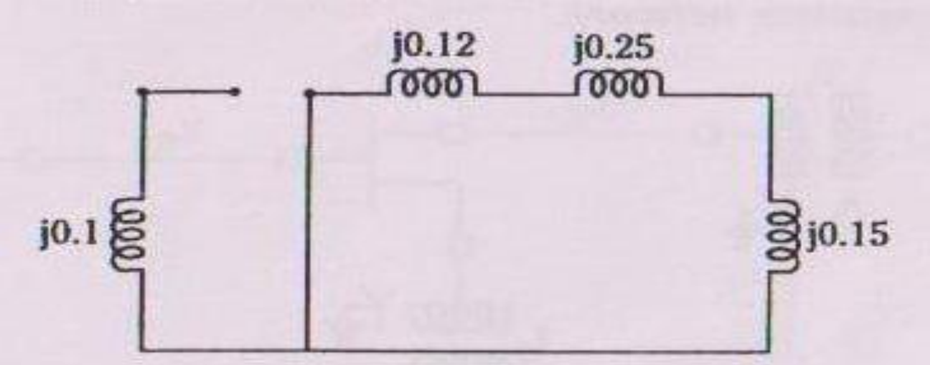


Fig. 4.16

Draw the positive, negative and zero sequence networks for the power system shown in fig.

$LT = HT \times \frac{LT}{HT}$

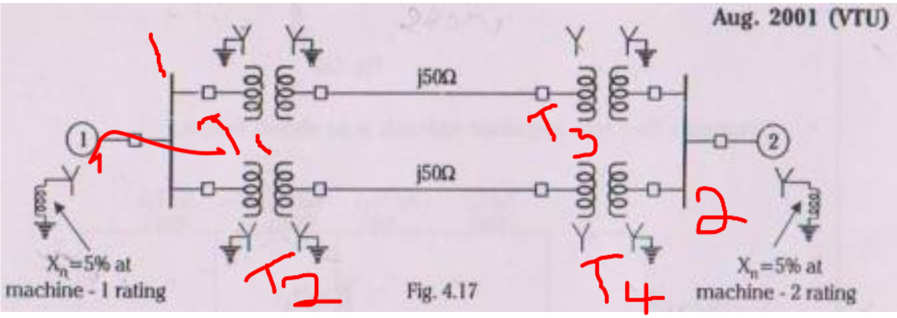


Fig. 4.17

Choose a base of 50MVA, 220kV in the 50Ω transmission lines and mark all reactance's in p.u. The ratings of the generators and transformers are:

Generator 1: 25MVA, 11kV, X''=20%.

Generator 2: 25MVA, 11kV, X''=20%.

Three phase transformer (each): 20MVA, 11 Y/220 Y kV, X=15%.

The negative sequence reactance of each synchronous machine is equal to the sub-transient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactance of lines are 250% of their positive sequence reactance's.

Solution:

base values: We choose a given,

base MVA=50, base kV on 50Ω transmission lines=220

base kV on generator 1=220(11/220)=11, base kV on generator 2=220(11/220)=11 sequence reactance's of generators: Since the ratings of the machines are the same, their reactance's are also the same. Positive sequence reactance=X''_{G1}=subtransient reactance on new base.

$$X''_{G1,\,new} = X''_{G1,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$

$$= j0.2 \times (50 / 25) \times (11^2 / 11^2)$$

$$= j\,0.4 \text{ p.u}$$

Negative sequence reactance=X''_{G1}=subtransient reactance on new base.

$=j0.4$ p.u (as per given data)

Zero sequence reactance=XG0=8% on new base

$$= j0.08 \times (50 / 25) \times (11^2 / 11^2)$$

$$=j0.16 \text{ p.u}$$

p.u value of generator neutral reactance=XGn=5% on new base.

$$= j0.05 \times (50 / 25) \times (\cancel{11}^2 / \cancel{11}^2)$$

$$=j0.1 \text{ p.u.}$$

Since the ratings of all transformers are identical, their sequence reactance's should be one and the same. Also, all the three sequence reactance's of transformers are the same.

Hence positive sequence reactance=Negative sequence reactance =Zero sequence reactance

$$X_{p.u,\,new} = X_{p.u,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$

$$= j0.15 \times (50 / 20) \times (\cancel{220}^2 / \cancel{220}^2)$$

$$= j\,0.375 \text{ p.u}$$

Sequence reactance of transmission lines:

Since the transmission line is a static apparatus, its positive and negative sequence reactance's are one and the same.

Hence, $X_{TL1} = X_{TL2} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j50 \times 50 / 220^2 = j0.052$ p.u

However, it is given that the zero sequence reactance's of transmission lines are equal to 250% of their positive sequence reactance's.

Therefore,

XTL .Zero=2.5(j0.052)=j0.13p.u

Positive sequence network (PSN):

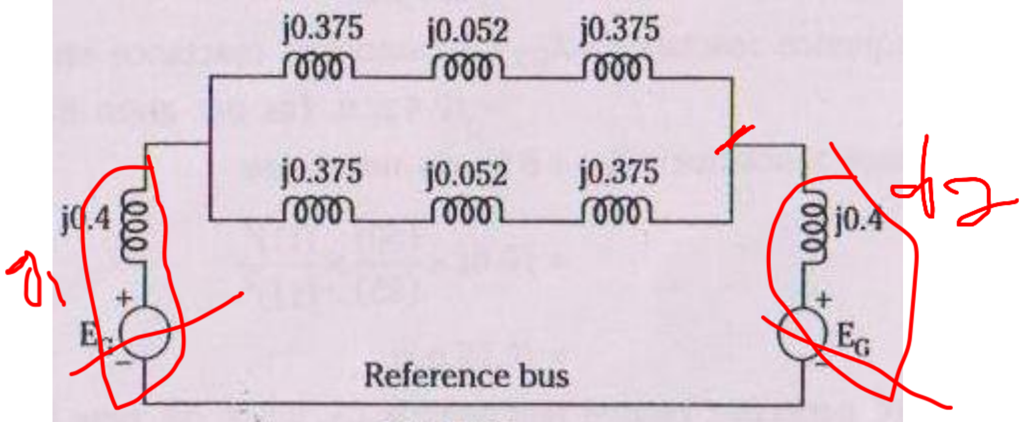


Fig. 4.18

Negative sequence network (NSN):

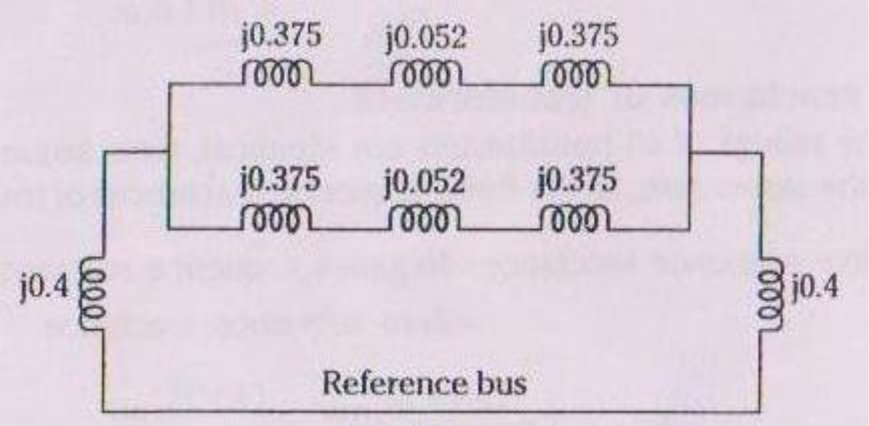


Fig. 4.19

Zero sequence network (ZSN):

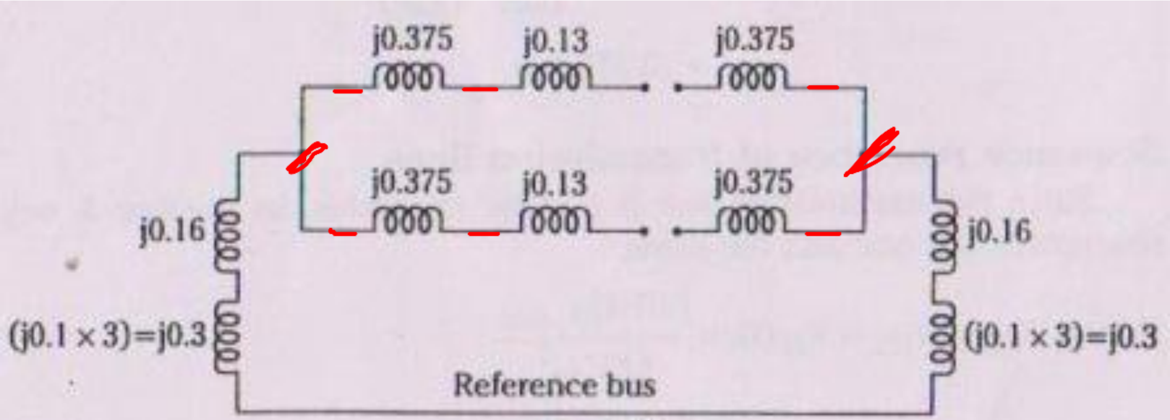


Fig. 4.20

For the power system whose one line diagram is shown in figure. Sketch the zero sequence network.

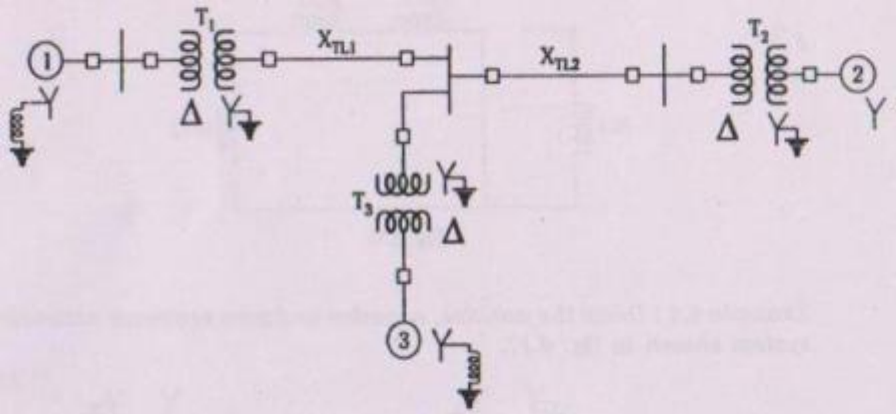


Fig. 4.21

Solution:

The zero sequence network is as shown below.

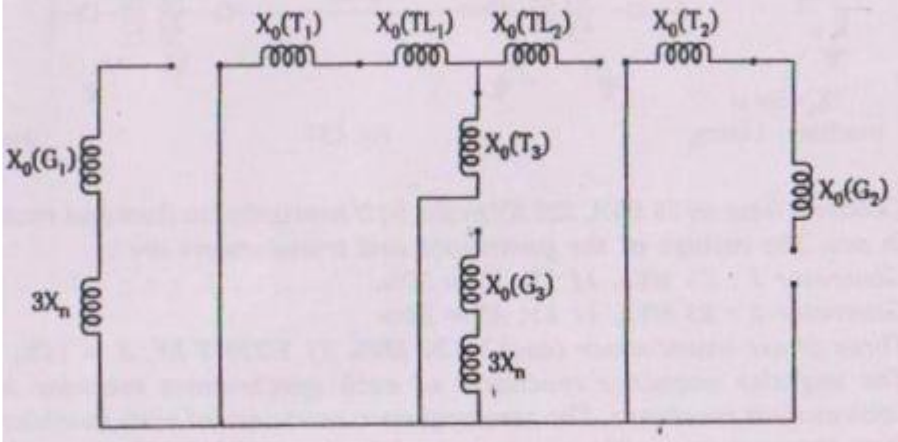


Fig. 4.22

Draw the sequence networks of the simple power system shown in figure

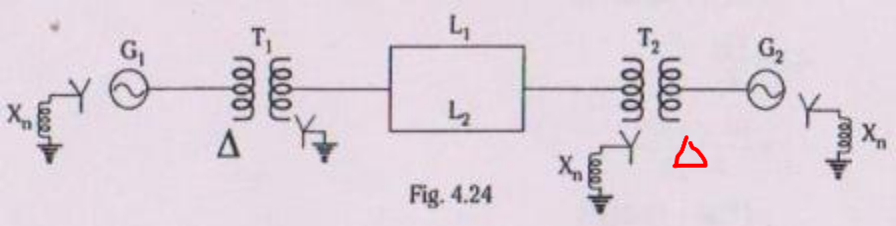


Fig. 4.24

Positive sequence network (PSN):

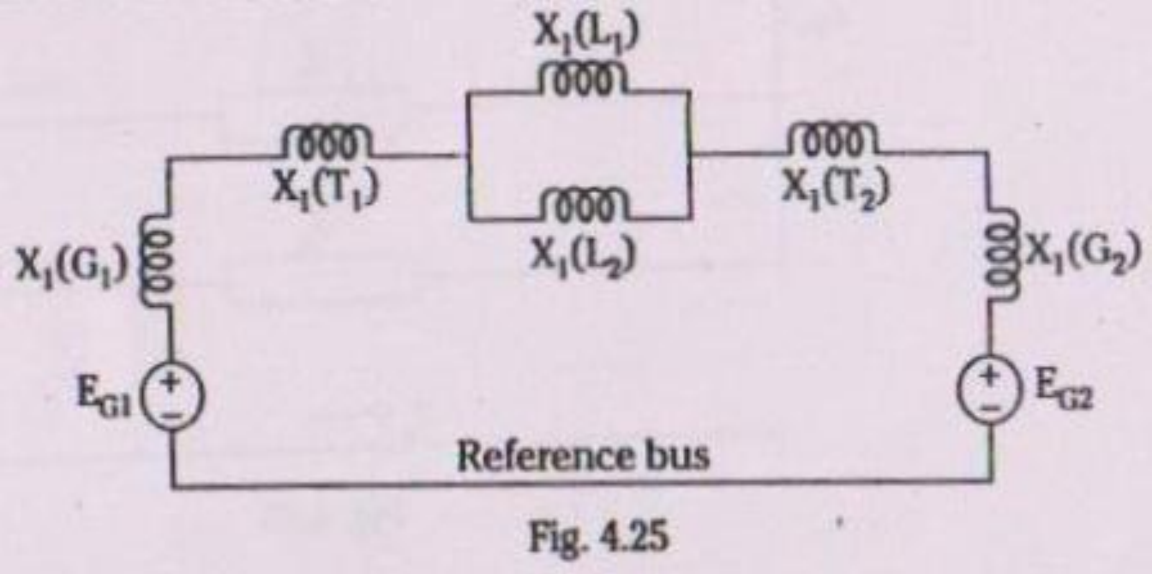


Fig. 4.25

Negative sequence network (NSN):

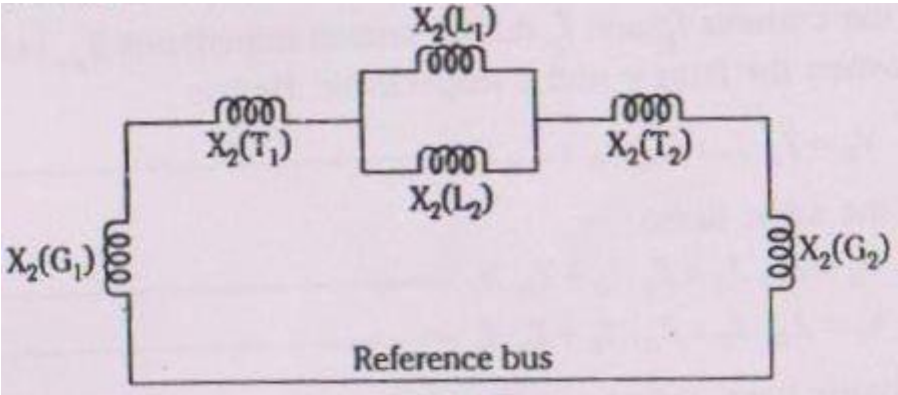


Fig. 4.26

Zero sequence network (ZSN):

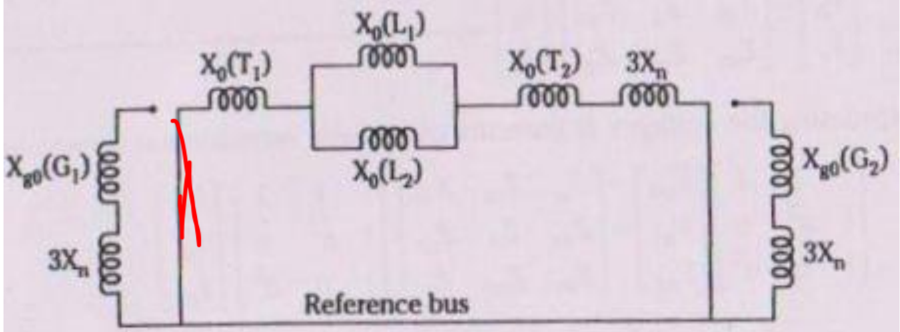


Fig. 4.27

The one line diagram of a power system is shown in figure.
The ratings of the devices are as follows:

G1 & G2: 104MVA, 11.8kV, X1=X2=0.2p.u; X0=0.1p.u.

T1 & T2: 125MVA, 11Y-220Y kV, X1=X2=X0=0.1p.u.

T3 & T4:120MVA, 230Y-6.9Y kV, X1=X2=X0=0.12p.u.

M1: 175MVA, 6.6kV, X1=X2=0.3p.u; X0=0.15p.u.

M2: 50MVA, 6.9kV, X1=X2=0.3p.u; X0=0.1p.u.

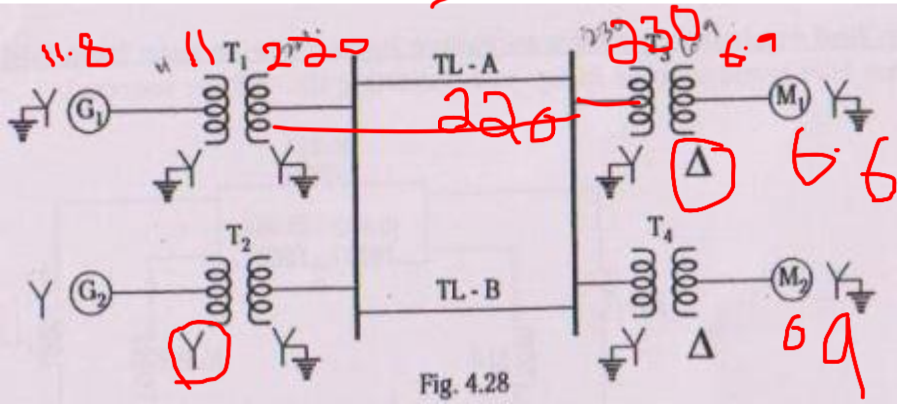


Fig. 4.28

Transmission line reactance's: X1=X2=30Ω; X0=60Ω.

Draw the sequence impedance diagram in p.u on a base of 200MVA, 220kV in the transmission lines. Also, find the equivalent positive sequence impedance as seen from the mid point of line-B.

$LT = HT \times \frac{LT}{HT}$

11/220

Solution:

Base values : We choose from give,

base MVA=200 MVA

base kV on the transmission lines=220KV

base kV on the generators G_1 & G_2=$22\cancel{0}(11/22\cancel{0})$=11

base kV on the motors M_1 & M_2=220(6.9/230)=6.6

Reactances of G_1 & G_2:

$X_1 = X_2 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (200 / 104) \times (11.8^2 / 11^2)$

$= j\ 0.44$ p.u

$X_0 = j0.1 \times (200 / 104) \times (11.8^2 / 11^2)$

$= j0.22$p.u

Reactances of transformers T_1 & T_2:(calculated on primary side of them),

$X_1 = X_2 = X_0 = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.1 \times (200 / 125) \times (\cancel{11}^2 / \cancel{11}^2)$

$= j\ 0.16$ p.u

230/69

Reactance of transmission lines:

$X_{1TL}=X_{2TL}= X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j30 \times 200 / 220^2 = j0.124 p.u$

$X_{0TL}= j60 \times 200 / 220^2 = j0.248 p.u$

Reactances of transformers T_3 & T_4:(calculated on primary side of them),

$X_1 = X_2 = X_0 = X_{p.u, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.12 \times (200 / 120) \times (230^2 / 220^2)$

$= j\ 0.22\ p.u$

Reactances of M_1:

$X_1 = X_2 = X_{p.u, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.3 \times (200 / 175) \times \cancel{(6.6^2 / 6.6^2)}$

$= j\ 0.342\ p.u$

$X_0 = j0.15 \times (200 / 175) \times \cancel{(6.6^2 / 6.6^2)}$

$= j0.171 p.u$

Reactances of M_2:

$X_1 = X_2 = X_{p.u, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.3 \times (200 / 50) \times (6.9^2 / 6.6^2)$

$= j\ 1.31\ p.u$

$X_0 = j0.1 \times (200 / 50) \times (6.9^2 / 6.6^2)$

$= j0.4372\ p.u$

Positive sequence network (PSN):

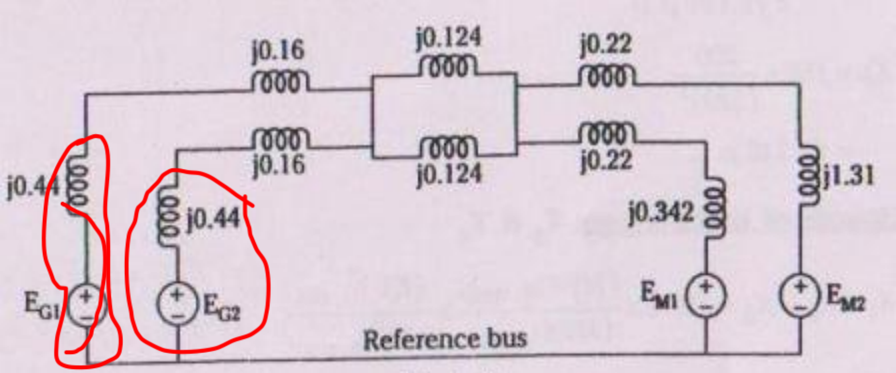


Fig. 4.29

Negative sequence network (NSN):

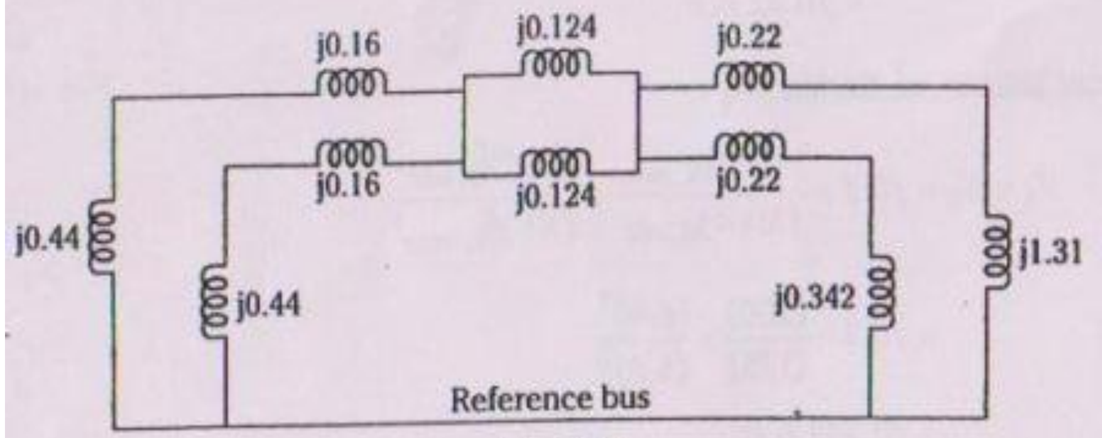


Fig. 4.30

Zero sequence network (ZSN):

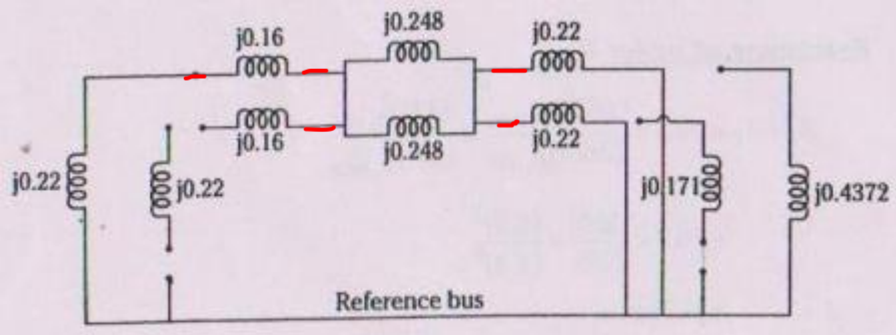


Fig. 4.31

To find equivalent positive sequence impedance as seen from mid point of line-B: The PSN is redrawn as in fig

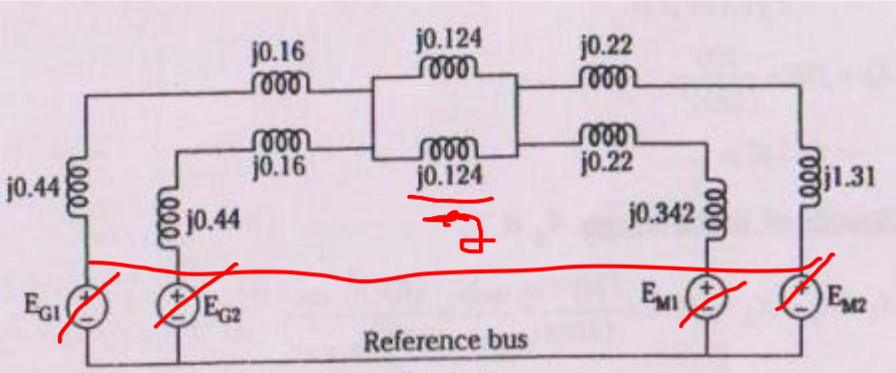


Fig. 4.29

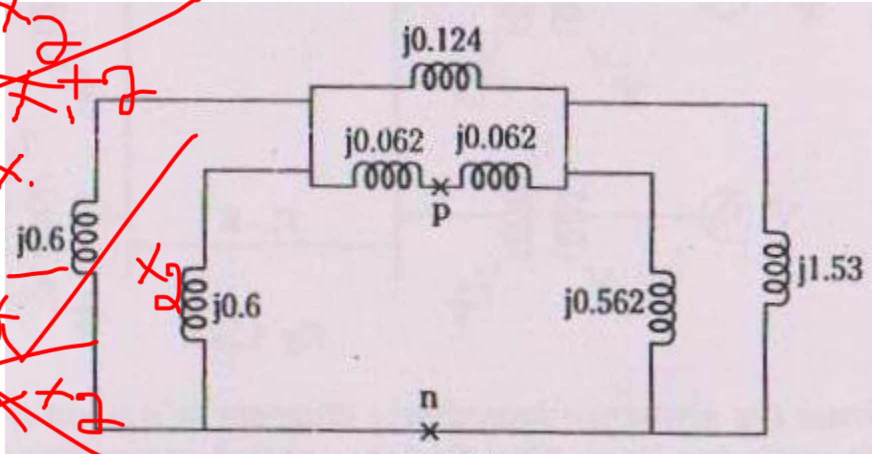


Fig. 4.32

The mid point of line-B is marked as 'P' and point 'n' is a point in the reference. Using elementary circuit theory, the network of fig

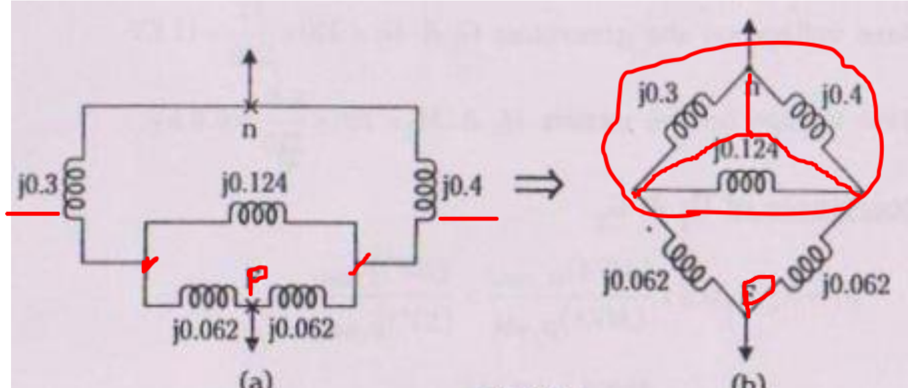


Fig. 4.33

Consider the delta connected network of fig, it cab be converted to its equivalent star connection as shown below.

$$X_3 = \frac{2 \times 3}{1+2+3}$$

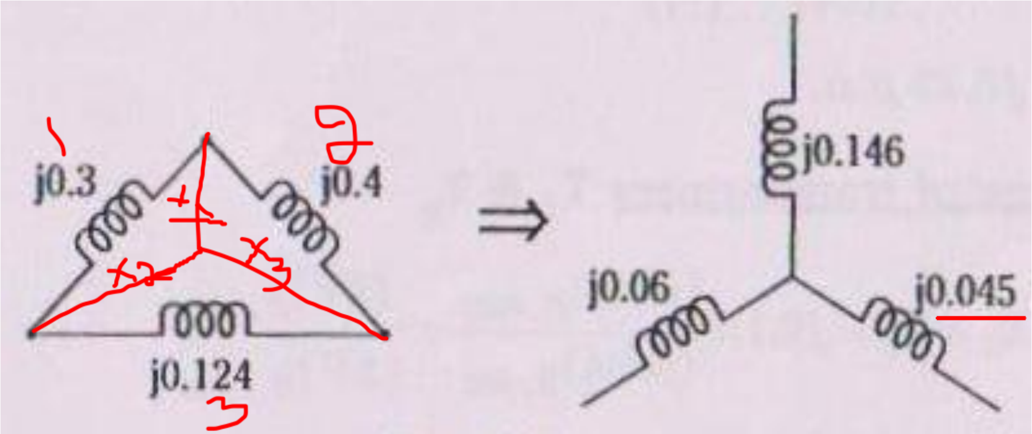


Fig. 4.34

$$X_1 = \frac{1 \times 2}{1+2+3}$$

$$X_2 = \frac{1 \times 3}{1+2+3}$$

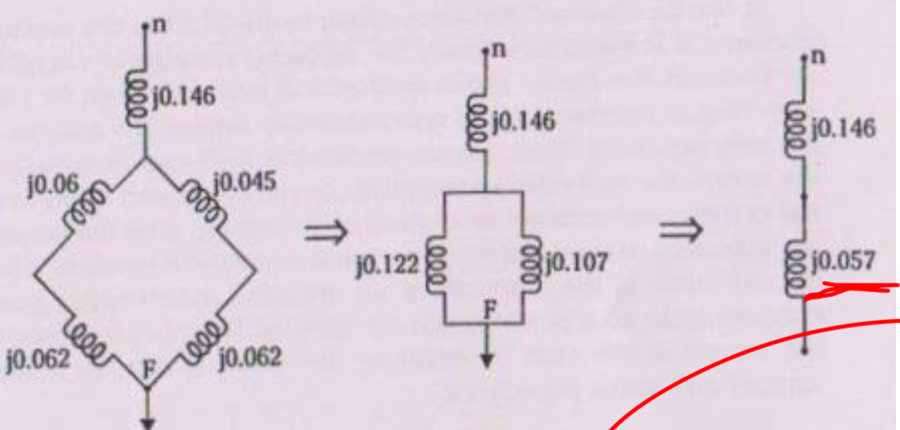


Fig. 4.35

equivalent positive sequence impedance as seen from the point P is, (j0.146+j0.057)=j0.203 p.u.

Draw the positive, negative and zero sequence reactance diagram of the power system shown in below fig. 1

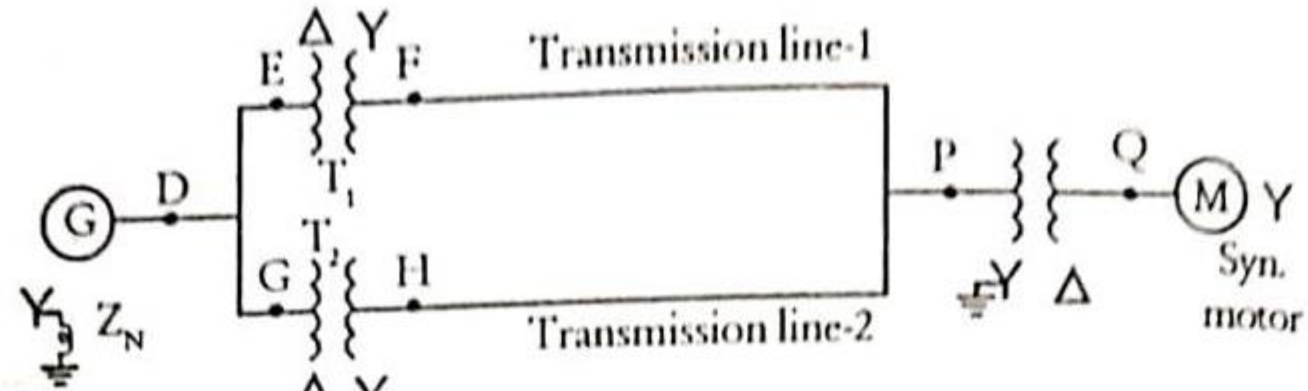


Fig 1.

XG,1 = Positive sequence reactance of Generator G

XM,1= Positive sequence reactance of motor M

XT1, 1 = Positive sequence reactance of transformer T1

XT2,1 = Positive sequence reactance of transformer T2

XT3,1= Positive sequence reactance of transformer T3

XTL1,1 = Positive sequence reactance of transmission line-1

XTL2,1 = Positive sequence reactance of transmission line-2

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Positive sequence reactance diagram of the power system shown in below fig. 2

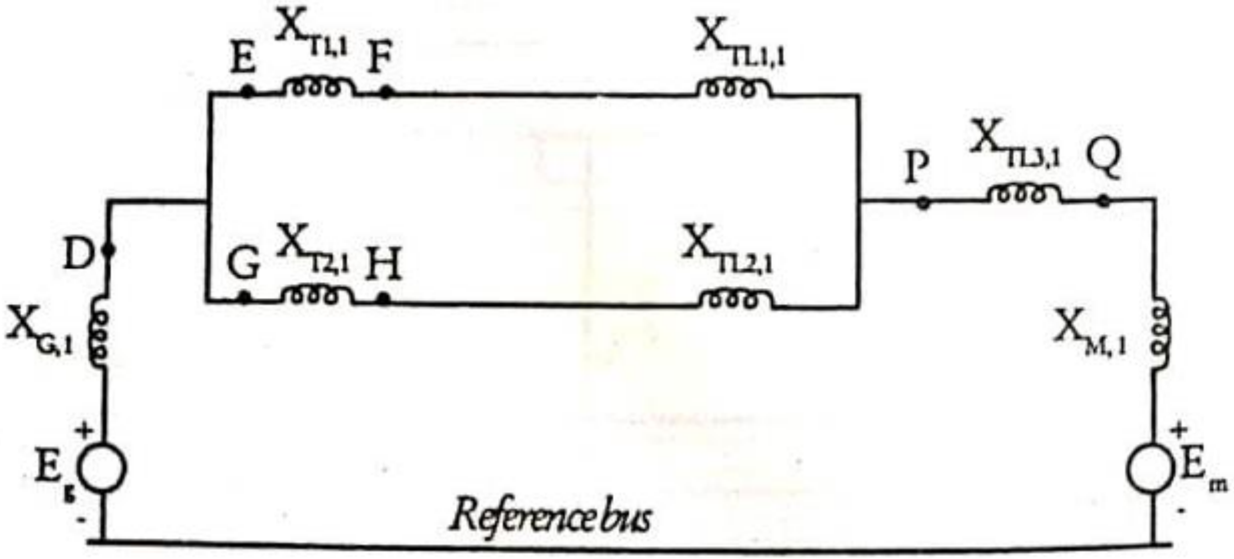


fig. 2

Note: The suffix “1" denotes Positive sequence reactances

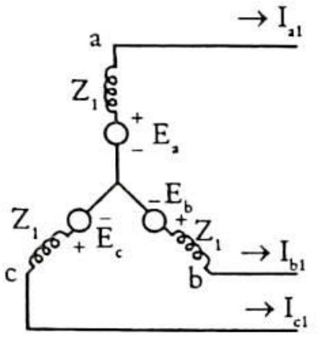


Fig (a) : Positive-sequence current paths.

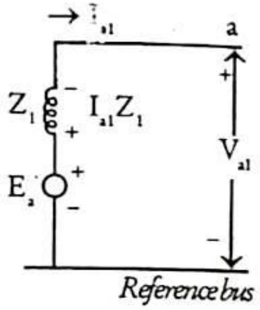


Fig (a) : Positive-sequence network

Negative sequence reactance diagram of the power system shown in below fig. 3

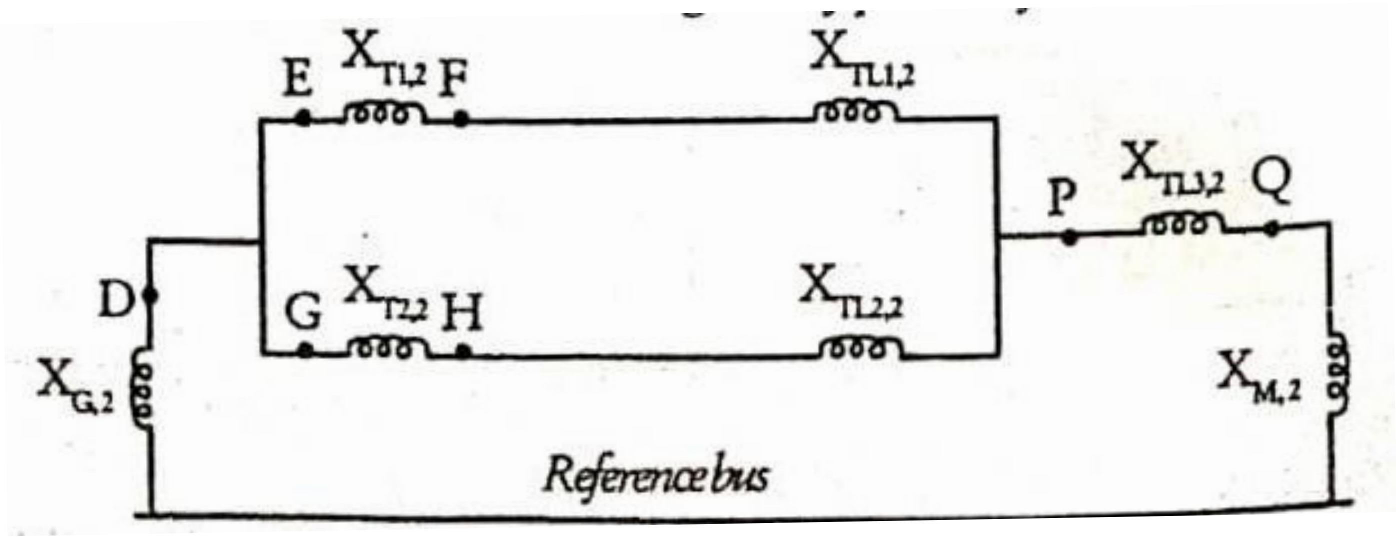


fig.3

Note: The suffix “2 " denotes Negative sequence reactances

Zero sequence reactance diagram of the power system shown in below fig. 4

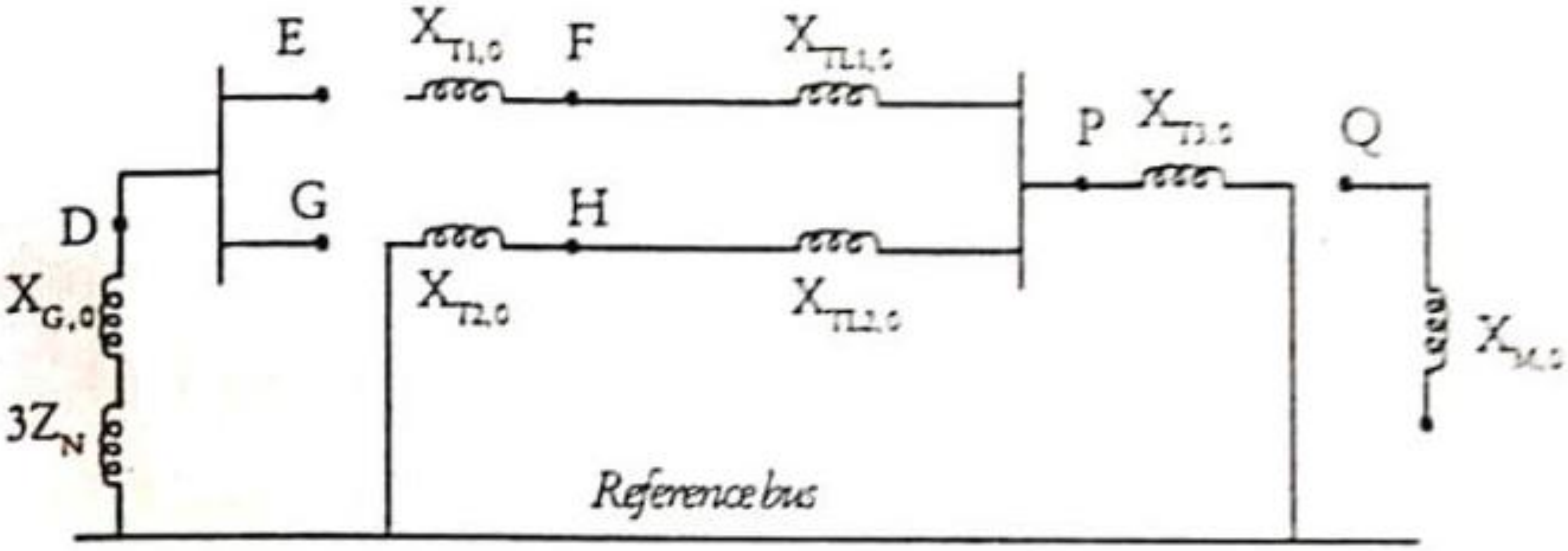
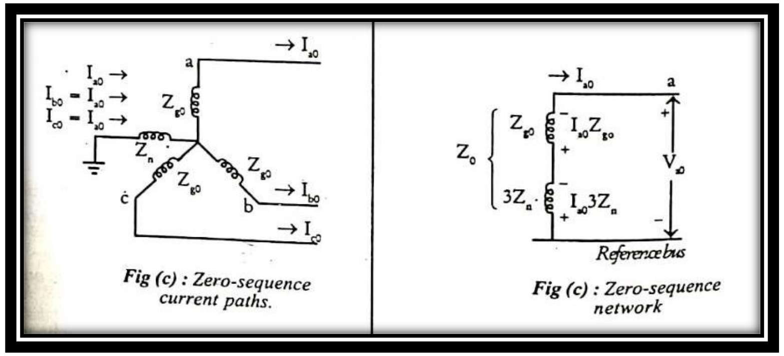
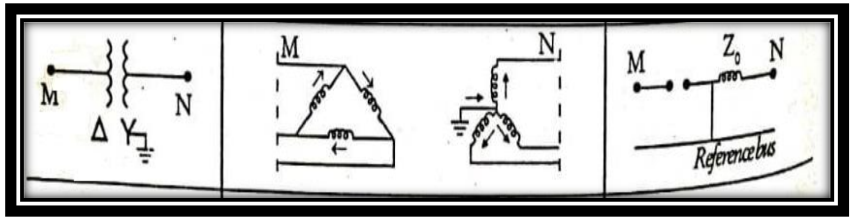
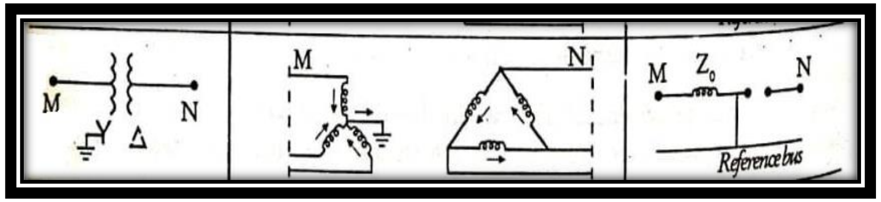


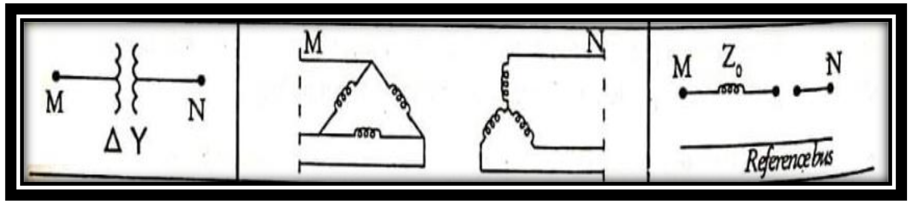
fig.4

Note: The suffix “ 0 " denotes Zero sequence reactances









For the power system shown in Fig 1. Draw the positive, negative and zero sequence reactance diagram

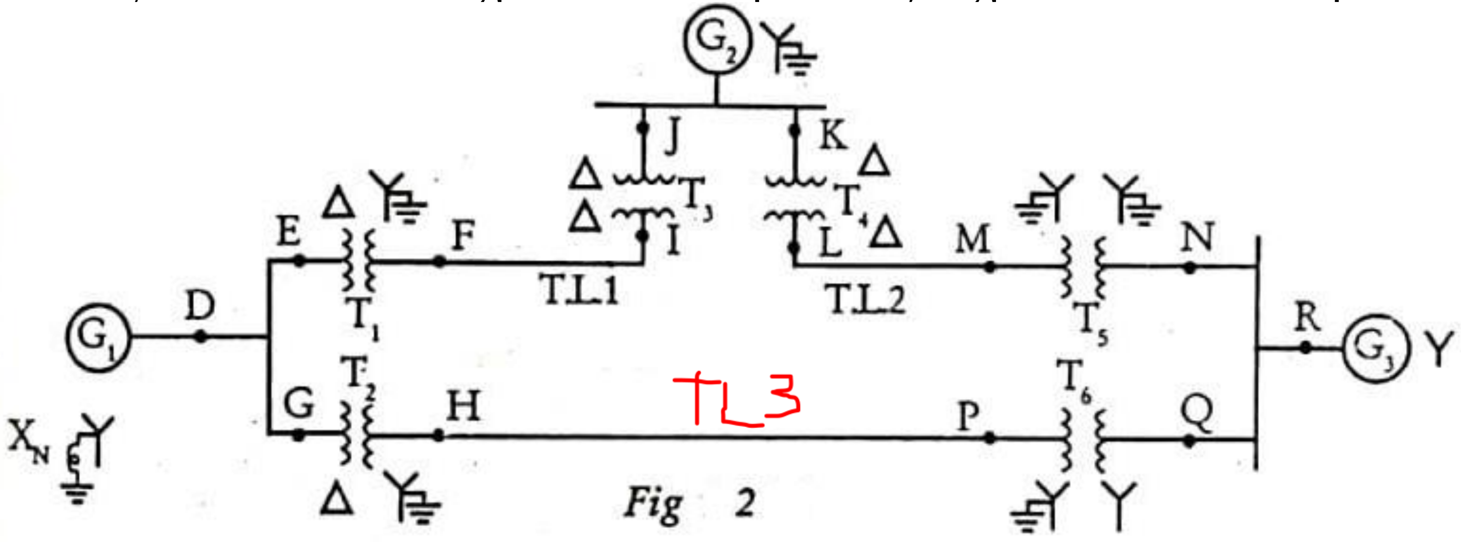


Fig 2

XG1,1 = Positive sequence reactance of Generator G1
XG2,1 = Positive sequence reactance of Generator G 2
XG3,1 = Positive sequence reactance of Generator G 3
XT1, 1 = Positive sequence reactance of transformer T1
XT2,1 = Positive sequence reactance of transformer T2
XT3,1= Positive sequence reactance of transformer T3
XT4,1= Positive sequence reactance of transformer T4
XT5,1= Positive sequence reactance of transformer T5
XT6,1= Positive sequence reactance of transformer T6
XTL1,1 = Positive sequence reactance of transmission line-1
XTL2,1 = Positive sequence reactance of transmission line-2
XTL3,1 = Positive sequence reactance of transmission line-3

Positive sequence reactance diagram of the power system shown in below fig. 2

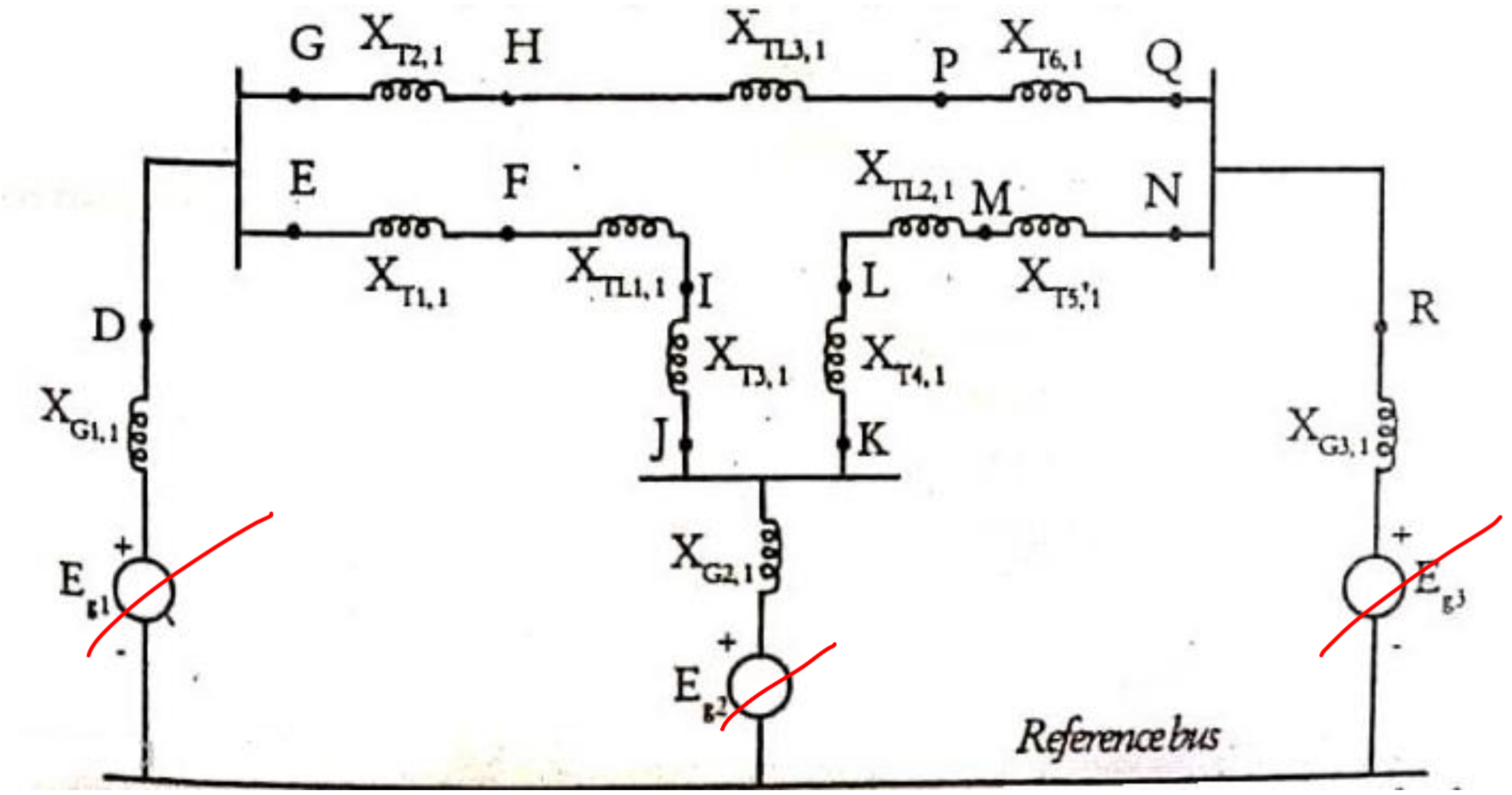


fig. 2

Note: The suffix “1" denotes Positive sequence reactances

Negative sequence reactance diagram of the power system shown in below fig. 3

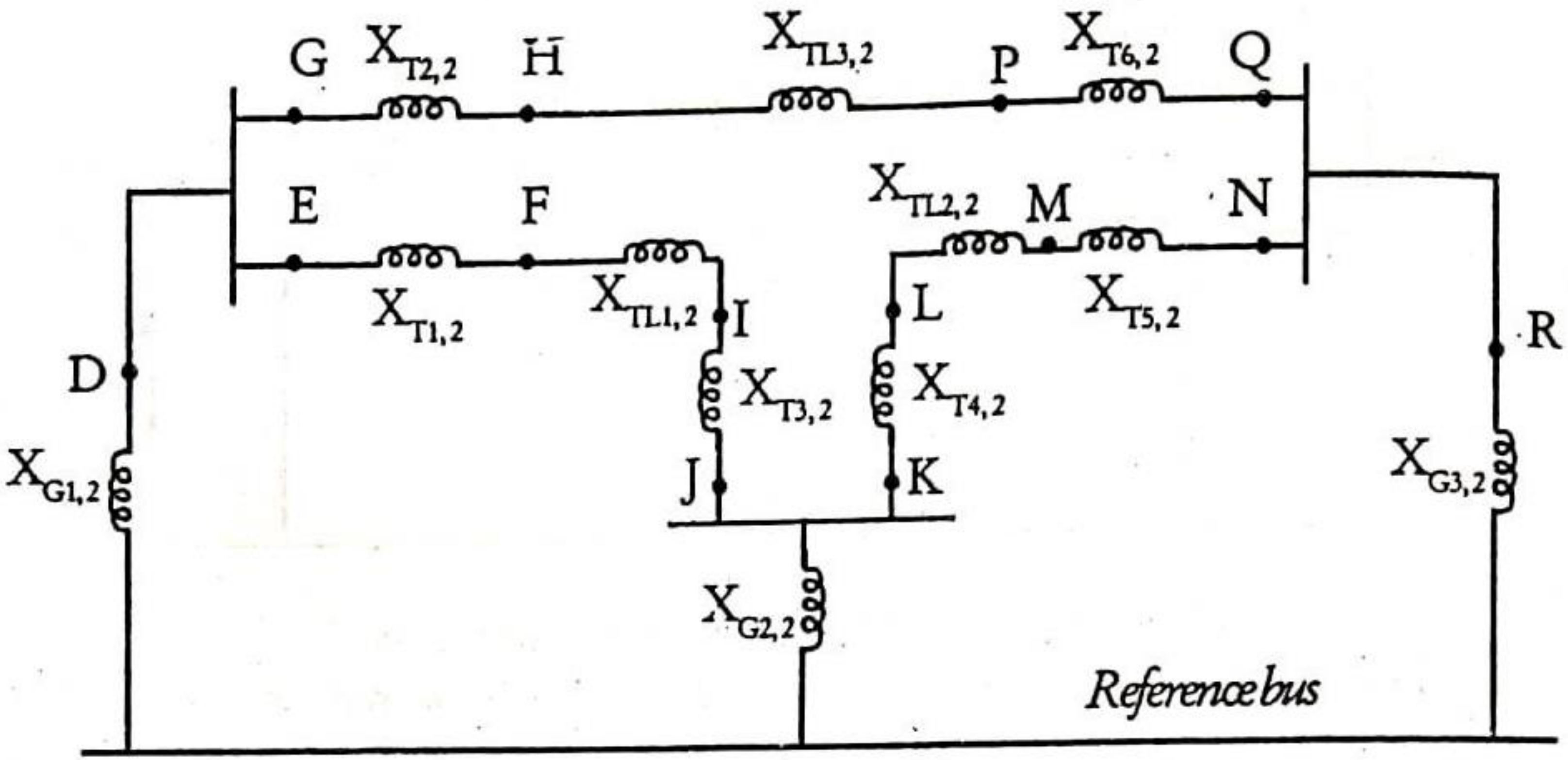
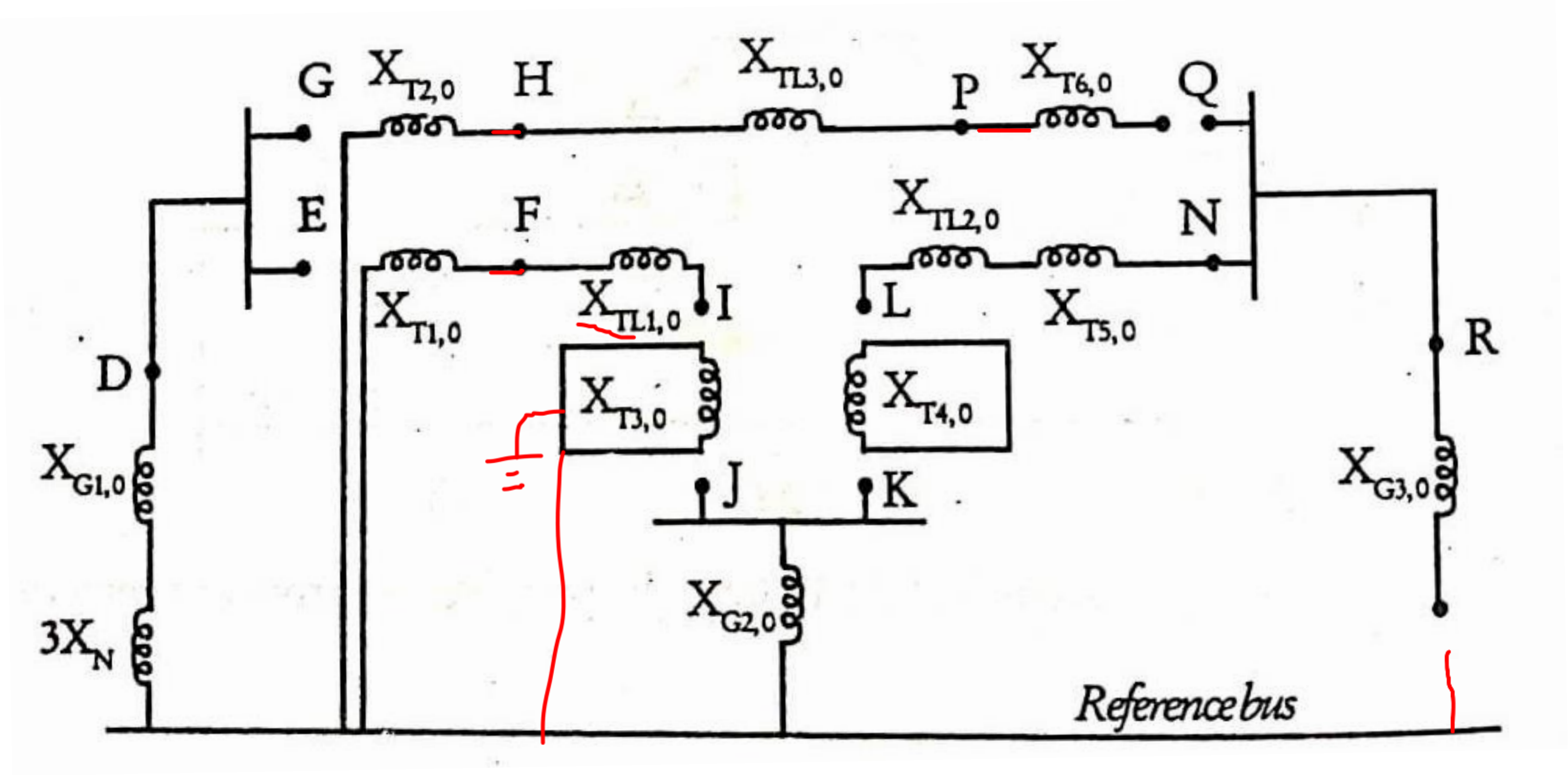


fig. 3

Note: The suffix “ 2” denotes Positive sequence reactances

Zero sequence reactance diagram of the power system shown in below fig. 4



Note: The suffix " 0" denotes Positive sequence reactances

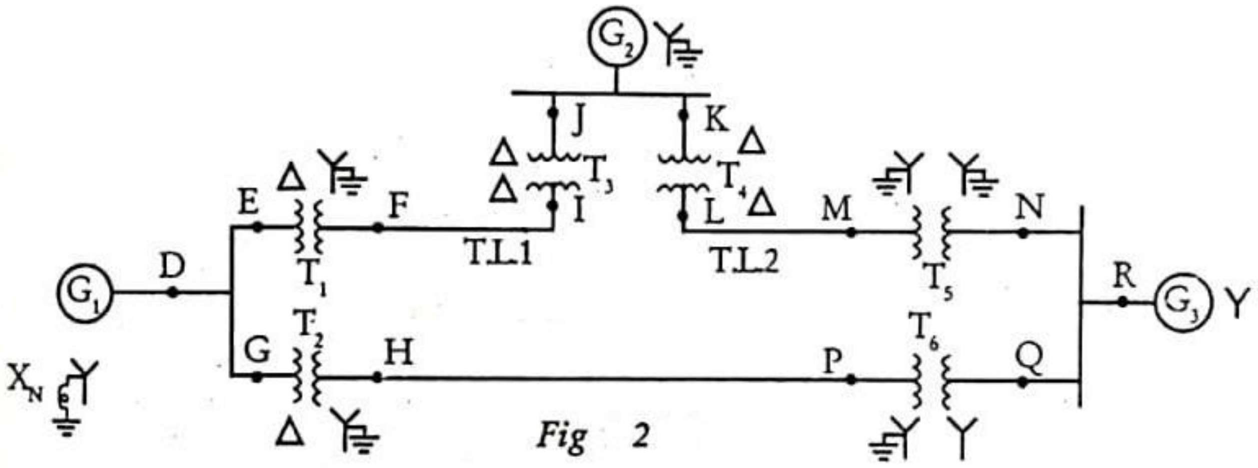


Fig 2

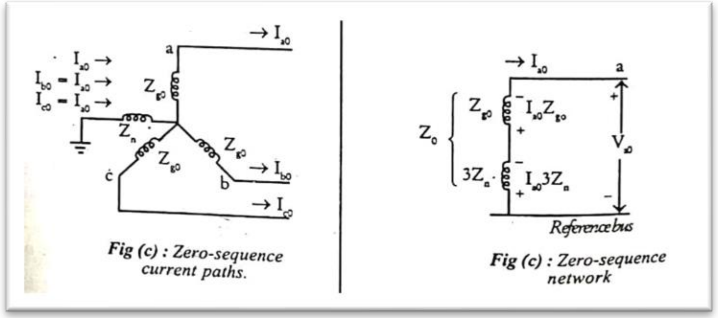
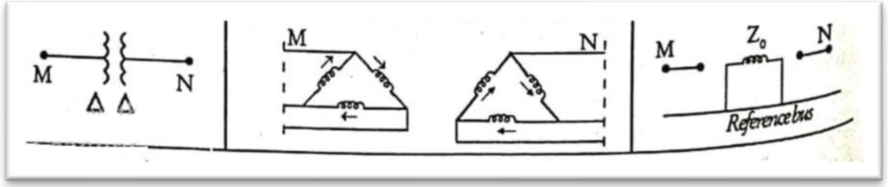
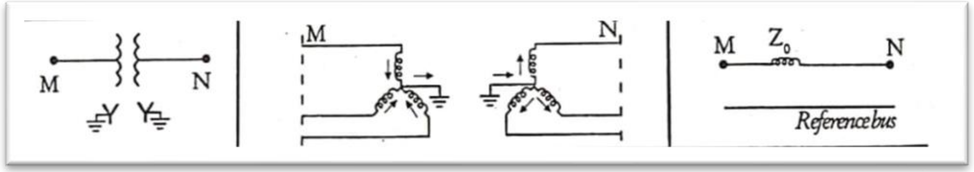
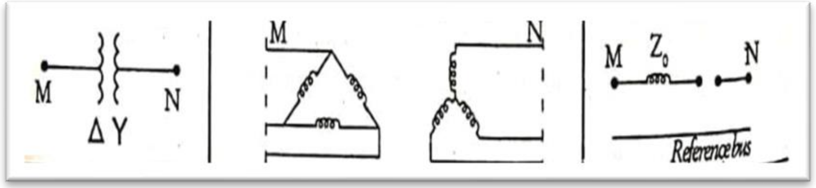


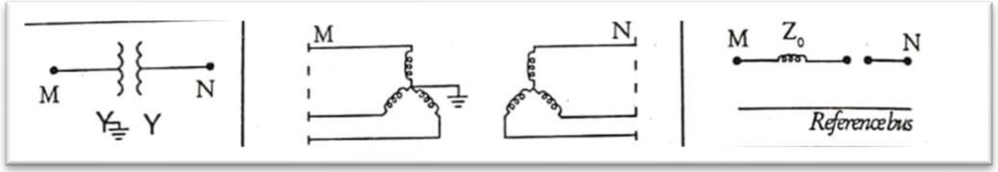
Fig (c) : Zero-sequence current paths.

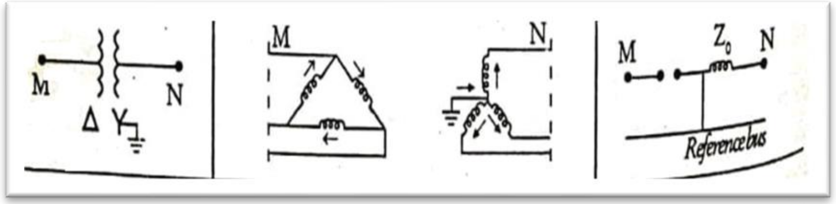
Fig (c) : Zero-sequence network

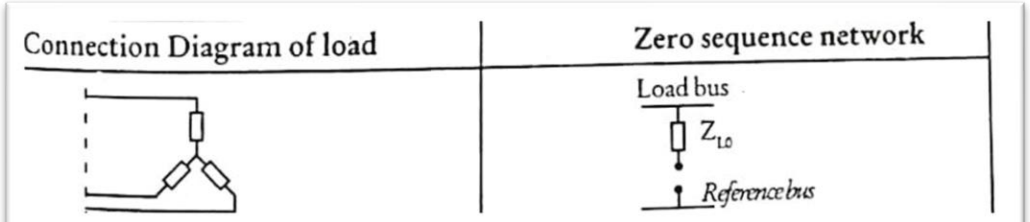












In Synchronous Generator

Positive sequence impedance > Negative sequence impedance

Zero sequence impedance = Variable item

= may be taken equal to +ve sequence impedance if its value is not given

In Transmission lines

Positive sequence impedance = Negative sequence impedance

= Impedance of the line

Zero sequence impedance = Variable item

= may be taken as 3 times the +ve sequence impedance if its value is not given

In Transformers

Positive sequence impedance = Negative sequence impedance

= Impedance of the Transformers

Zero sequence impedance = Positive sequence impedance, if there is circuit for earth current

= infinity, if there is no through circuit for earth current

Determine the positive, negative and zero sequence networks for the system shown in fig 1.. Assume zero sequence reactances for the generator and synchronous motors as 0.06 p.u. Current limiting reactors of 2.5 Ω are connected in the neutral of the generator and motor No.2.

The zero sequence reactance of the transmission line is j300Ω.

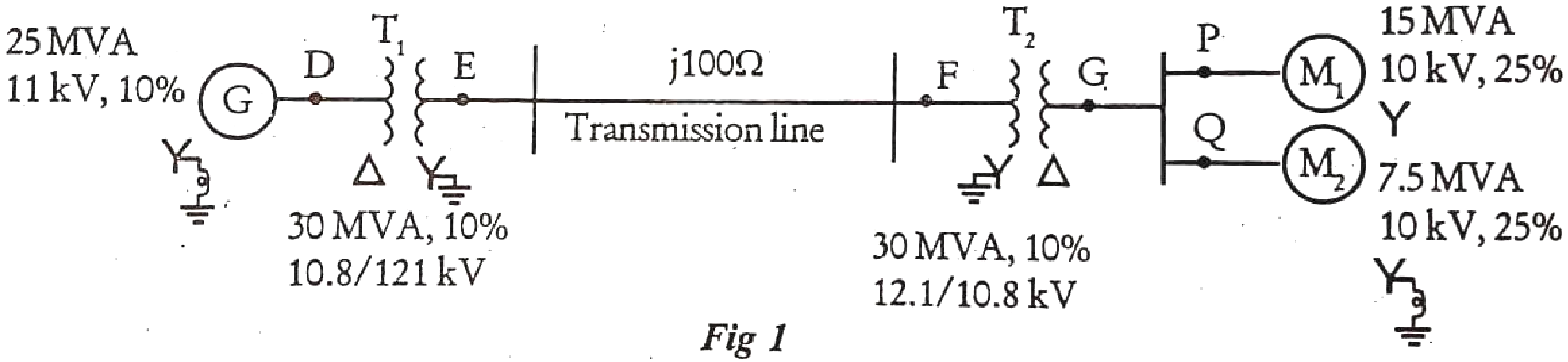


Fig 1

Choose the generator ratings as new base values for entire system.

Base megavolt ampere, MVA_b, New = 25MVA

Base kilovolt, kV_b, new = 11kV

Sequence reactances of Generator G

Since the generator rating and the new base values are same, the generator p.u. reactances does not Change.

Also for generator the positive and negative sequence reactances are same.

Positive sequence reactance of generator, $X_{G,1} = 10\% = 10/100 = 0.1$ p.u.

Negative sequence reactance of generator, $X_{G,2} = 0.1$p.u.

Zero sequence reactance of generator, $X_{G,0} = 0.06$ p.u.

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\Omega$$

p.u. value of generator Neutral reactance X_{GN} = Actual Neutral reactance /Base Impedance

$= 2.5 / 4.84 = 0.1517$ p.u

Sequence reactances of Transformer T1

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$X_{pu,old} = 10\% = 0.1,\quad kV_{b,old} = 10.8\ kV,\quad MVA_{b,old} = 30\ MVA$

$kV_{b,new} = 11\ kV,\quad MVA_{b,new} = 25\ MVA$

Sequence reactances of Transformer T1

$$\left.\begin{array}{l}\text{New p.u. reactance} \\ \text{of transformer } T_1\end{array}\right\} = 0.1 \times \left(\frac{10.8}{11}\right)^2 \times \left(\frac{25}{30}\right) = 0.08 \text{ p.u.}$$

In transformer the specified reactance is positive sequence reactance.

Also we assume that the positive, negative and zero sequence reactances of the transformer are equal.-

Positive sequence reactance of transformer T1, $X_{T1,1} = 0.08$ p.u.

Negative sequence reactance of transformer T1, $X_{T1,2} = 0.08$ p.u.

Zero sequence reactance of transformer T1, $X_{T1,0} = 0.08$ p.u

Sequence reactances of Transmission line

$$\left.\begin{array}{l}\text{The base kV on HT side}\\ \text{of transformer } T_1\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

kV B, new = 11 * (121 / 10.8) = 123.24kV

$$\text{Base impedance, } Z_b = \frac{\left(kV_{b,new}\right)^2}{MVA_{b,new}} = \frac{(123.24)^2}{30} = 506.27\Omega$$

p.u. value of Transmission line = Actual reactance /Base Impedance

= 100 / 506.27 = 0.198 p.u

the specified reactance is positive sequence reactance.

Also negative sequence reactances of the Transmission line are equal positive sequence reactance

Therefore

Positive sequence reactance of Transmission line $X_{TL,1} = 0.198$ p.u.

Negative sequence reactance of Transmission line $X_{TL,2} = 0.198$ p.u.

p.u. value of Zero sequence reactance of Transmission line $X_{TL,0}$

= Zero sequence reactance in ohms /Base Impedance

$= 300 / 506.27 = 0.593$ p.u

Sequence reactances of Transformer T2

The ratings and winding connections of transformer T1 and T2 are identical and so the sequence reactance of T1 and T2 are same.

Positive sequence reactance of transformer T2, $X_{T2,1} = 0.08$ p.u.

Negative sequence reactance of transformer T2, $X_{T2,2} = 0.08$ p.u

Zero sequence reactance of transformer T2, $X_{T2,0} = 0.08$ p.u

Sequence reactances of Synchronous motor M1

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of transformer } T_2\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

kV B, new = 123.24 * (10.8/121) = 11kV

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,\,new}}{MVA_{b,old}}$$

$X_{pu,old} = 25\% = 0.25$ $\quad kV_{b,old} = 10\ kV$ $\quad MVA_{b,old} = 15\ MVA$

$kV_{b,\,new} = 11\ kV$ $\quad MVA_{b,new} = 25\ MVA$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_1\end{array}\right\} = 0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = 0.344 \text{ p.u.}$$

The reactance specified in single line diagram is positive sequence reactance.

Also the negative sequence reactance of synchronous motor is same as that of positive sequence reactance

Positive sequence reactance of motor M1, $X_{M1,1} = 0.344$ p.u

Negative sequence reactance of motor M1, $X_{M1,2} = 0.344$ p.u

Zero sequence reactance of Motor M1on New Bases

$$X_{M1,0} = X_{pu,\,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = 0.083 \text{ p.u.}$$

Sequence reactances of Synchronous motor M2

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here, $X_{pu,old} = 25\% = 0.25$; $kV_{b,old} = 10\text{ kV}$; $MVA_{b,old} = 7.5\text{ MVA}$

$kV_{b,new} = 11\text{ kV}$; $MVA_{b,new} = 25\text{ MVA}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_2\end{array}\right\} = 0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = 0.689\text{ p.u.}$$

The reactance specified in single line diagram is positive sequence reactance.

Also the negative sequence reactance of synchronous motor is same as that of positive sequence reactance

Positive sequence reactance of motor M2, $X_{M2,1} = 0.689$ p.u

Negative sequence reactance of motor M2, $X_{M2,2} = 0.689$ p.u

Zero sequence reactance of Motor M2 on New Bases

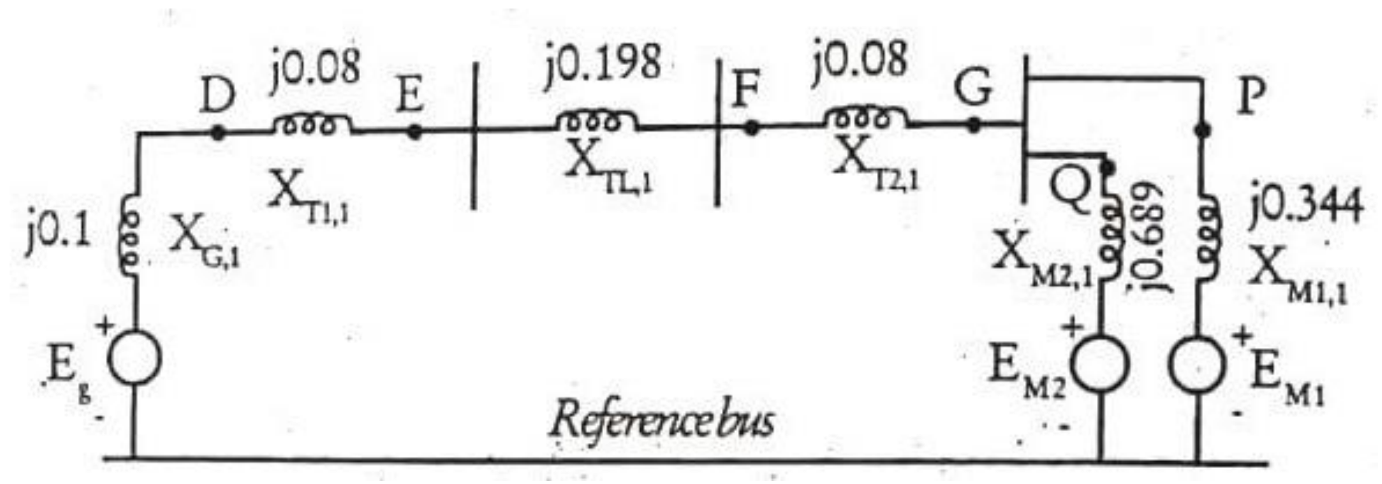
$$X_{M2,0} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = 0.165 \text{ p.u.}$$

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\Omega$$

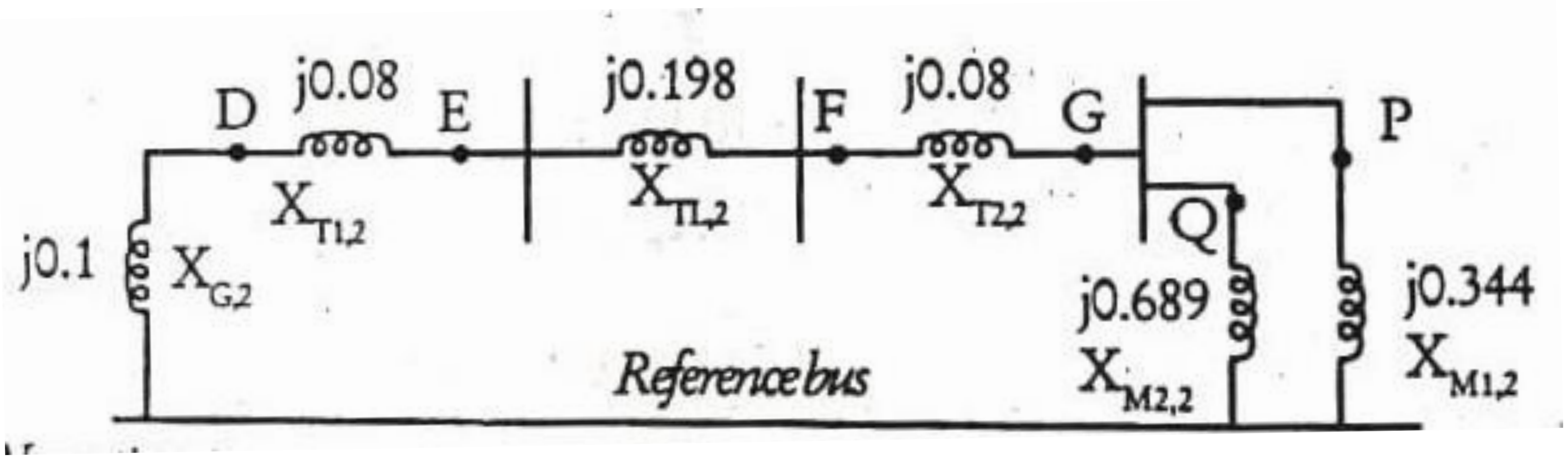
$$\left.\begin{array}{l}\text{p.u. value of motor}\\ \text{neutral reactance}\end{array}\right\} X_{MN} = \frac{\text{Actual neutral reactance}}{\text{Base impedance}} = \frac{2.5}{4.84} = 0.517 \text{ p.u.}$$

Positive sequence Network



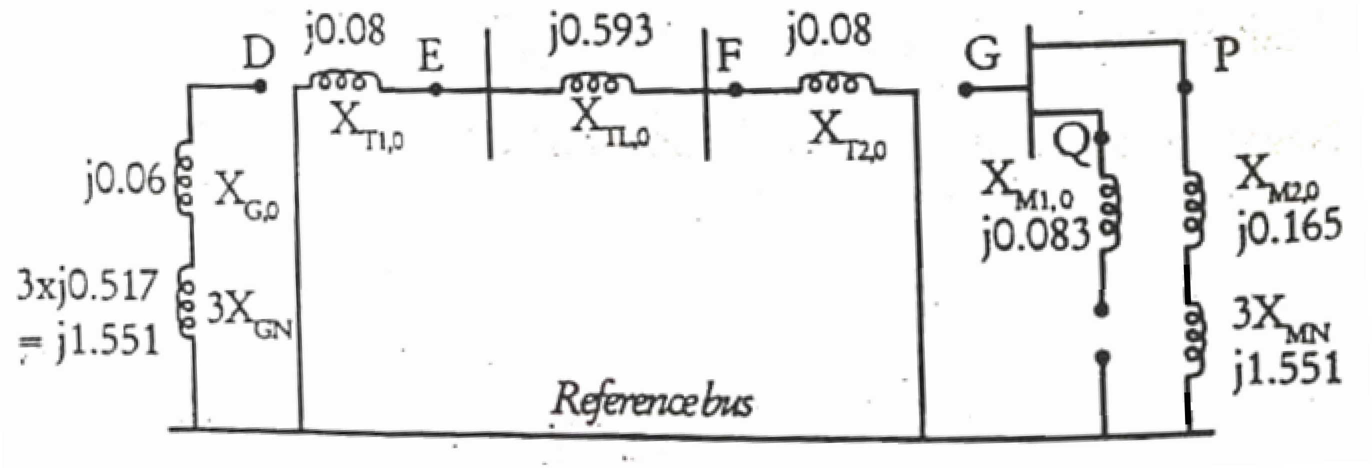
Positive sequence reactance diagram of the power system shown in fig 1

Negative sequence Network



Negative sequence reactance diagram of the power system shown in fig 1

Zero sequence Network



Zero sequence reactance diagram of the power system shown in fig 1

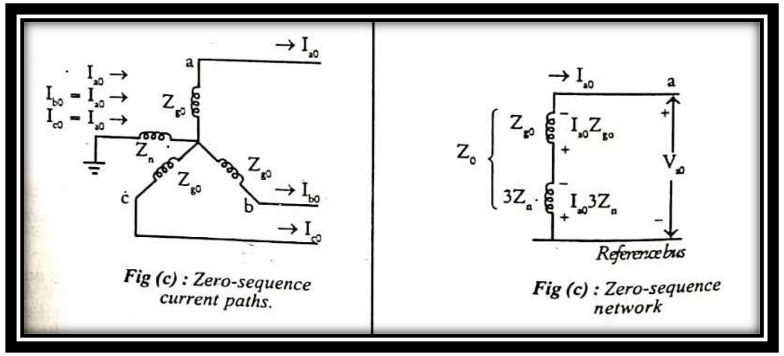
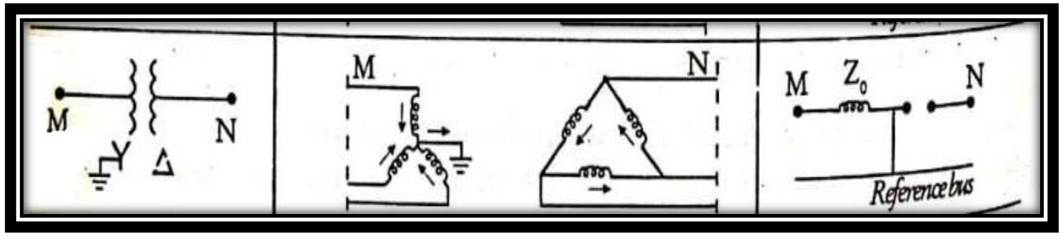
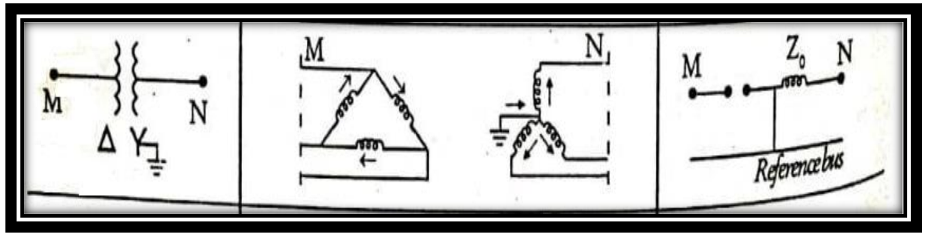
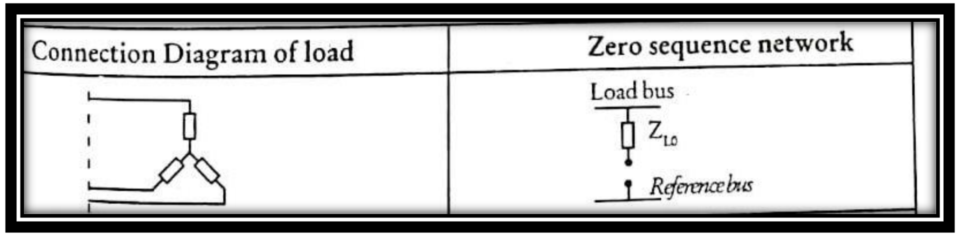


Fig (c) : Zero-sequence current paths.

Fig (c) : Zero-sequence network







The Sequence components of voltages in a three phase connected impedance of (8+j6) Ω in each phase with mutual impedance of j2 Ω are Va1 = 230 <0° V, Va2 = 60 <-30 ° V, Va0 = 40<-90 ° V. Determine the line currents and also voltage drop across the neutral impedance.

Solution: The circuit is symmetrical, then

$Z1 = Z2 = (8+j6)\ \Omega$

$Z0 = Z + 3Zn = 8 + j6 + (3 \times j2) = (8 + j12)\ \Omega$

The sequence components of line currents are:

Ia1= Va1/Z1 = (230 <0°) / (8+j6) = (18.4 – j13.8) A = 23 < -36.86 ° A

Ia2 = Va2/ Z2 = (Va2 = 60 <-30) / (j8+j6) = (2.36 – j5.51) A = 6< -66.8 A

Ia0 = Va0 / Z0 = (40<-90) / (8 + j12) = (-2.4-j3.2) A = 4< -126.86 A

The Line currents are:

Ia = Ia0+Ia1+Ia2 = -2.4-j3.2 + 18.4 – j13.8 + 2.36 – j5.51 = (18.36-j22.51) A = 29<-50.8 ° A

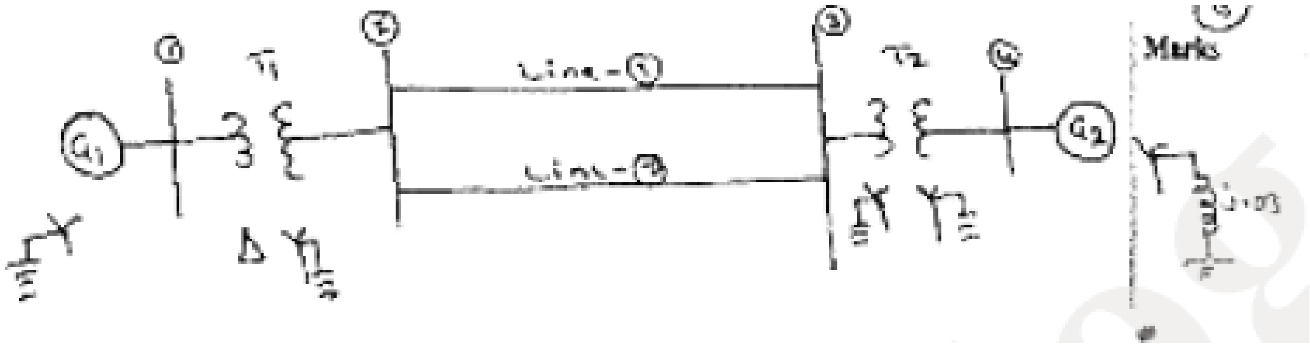
Ib = Ia0+ a ^{2}Ia1+ a Ia2= 4< -126.86 + 23 <(-36.86+ 240) + 6< (-66.8+120) =(-19.95-j7.44) A = 21.3 < -159.5 A

Ic = Ia0+ aIa1+ a ^{2}Ia2 = 4< -126.86 + 23 <(-36.86+ 120) + 6< (-66.8+240) = (-5.61+j20.3)A = 21.06< 105.45 A

WKT, Current flowing through the neutral is 3Ia0. Hence the drop in the neutral impedance is (3Ia0) x (Zn)

Vn = (3Ia0) x (Zn) = 3(4< -126.86) x 2<90 = 24<-36.86 V

Figure shows a power system network. Draw positive, negative and zero sequence network. The system data is as follows. Take base values of 11KV, 100MVA in generator 1 circuit.



Equipment	MVA rating	Voltage rating	X1(PU)	X2(PU)	X0(PU)
Generator-1	100	11KV	0.25	0.25	0.05
Generator-2	100	11KV	0.2	0.2	0.05
Transformer- T1	100	11/220KV	0.06	0.06	0.06
Transformer- T2	100	11/220KV	0.07	0.07	0.07
Line-1	100	220KV	0.1	0.1	0.3
Line-2	100	220KV	0.1	0.1	0.3

Sixth Semester B.E. Degree Examination, June/July 2019
Power System Analysis – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that per unit impedance of two winding transformer will remain same referred to primary as well as secondary. **(06 Marks)**

b. A 300 MVA, 20 KV, 3-phase generator has subtransient reactance of 20%. The generator supplies two synchronous motors through a 64 KVA transmission line having transformers at both ends as shown in Fig.Q1(b). T_1 is a 3-phase transformer and T_2 is composed of 3-single phase transformers of rating 100 MVA each, 127/13.2 KV, 10% reactance, series reactance of transmission line is 0.5 ohm/km. Draw the reactance diagram with all reactances marked in per unit. Select generator rating on base values.

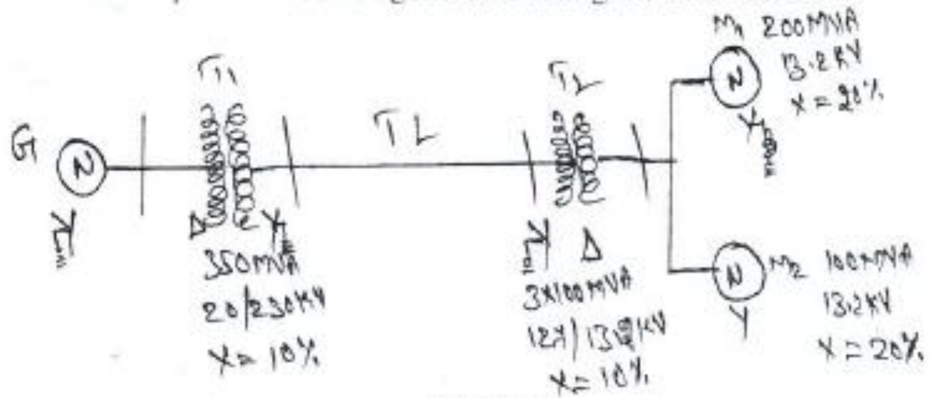


Fig.Q1(b) **(10 Marks)**

Module-3

5 a. Obtain the relationship between line and phase sequence components of voltages in star connection. Give the relevant phasor diagrams. **(08 Marks)**

b. Draw the positive, negative and zero sequence network for the power system shown in Fig.Q5(b). Choose a base of 50 MVA, 220 KV in the 50Ω transmission lines and marks all reactances in PU. The ratings of the generator and transformers are:

G_1: 25 MVA, 11 KV, X″ = 20%; G_2 : 25 MVA, 11 KV, X″ = 20%

3ϕ transformers (each) : 20 MVA, 11/220 KV, X = 15%

The negative sequence reactance of each synchronous machine is equal to the sub-transient reactance. The zero sequence reactance of a each machine is 8%. Assume that the zero sequence reactances of lines are 250% of their positive sequence reactances.

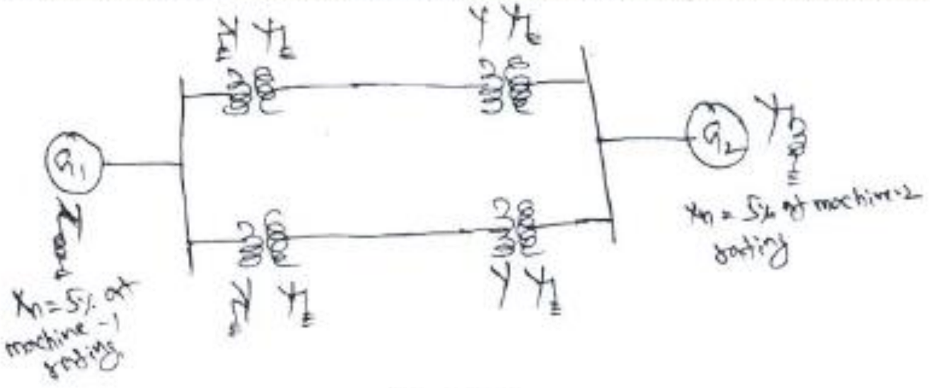
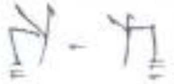
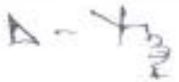


Fig.Q5(b) **(08 Marks)**

OR

6 a. Draw the zero sequence impedance networks of a transformer for the following connections:

i)  ii)  iii) 

(06 Marks)

b. The positive, negative and zero sequence components of line currents are $20\angle 10^\circ$, $6\angle 60^\circ$ and $3\angle 30^\circ$ A respectively. Determine the line currents. **(04 Marks)**

c. In a 3ϕ, 4 wire system, the sequence voltages and currents are:

$V_{a1} = 0.9\angle 10^\circ$ PU ; $V_{a2} = 0.25\angle 110^\circ$ PU ; $V_{a0} = 0.12\angle 300^\circ$ PU ;

$I_{a1} = 0.75\angle 25^\circ$ PU ; $I_{a2} = 0.15\angle 170^\circ$ PU ; $I_{a0} = 0.1\angle 330^\circ$ PU

Find the complex power in PU. If the neutral gets disconnected, find the new power.

(06 Marks)

Sixth Semester B.E. Degree Examination, June/July 2019

Power System Analysis and Stability

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. ***Missing data, if any, may be suitably assumed.***

PART – A

1 a. For the given one line diagram shown in Fig.Q1(a), draw impedance diagram and reactance diagram.

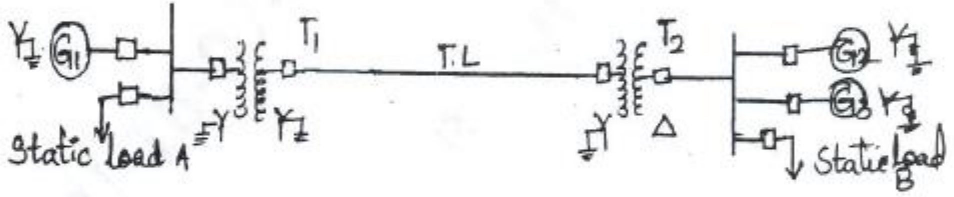


Fig.Q1(a) **(05 Marks)**

b. List any five advantages of PU system. **(05 Marks)**

c. The one line diagram of an unloaded power system is shown in Fig.Q1(c). Reactances of the 2 sections of the transmission line are shown on the diagram. The generator and transformer are rated as follows:

Gen 1 : 20 MVA, 13.8 KV, X″ = 0.2 pu
Gen 2 : 30 MVA, 18 KV, X″ = 0.2 pu
Gen 3 : 30 MVA, 20 KV, X″ = 0.2 pu
Tr : T_1 : 25 MVA, 220 KV/13.8, X″ = 0.1 pu
Tr : T_2 : 1-ϕ units each rated 10 MVA, 127/18 KV, X = 10%
Tr : T_3 : 35 MVA, 220 Y/22Y KV, X = 10%.

Draw the impedance diagram with all reactances marked in pu. Choose a base of 50 MVA, 13.8 KV.

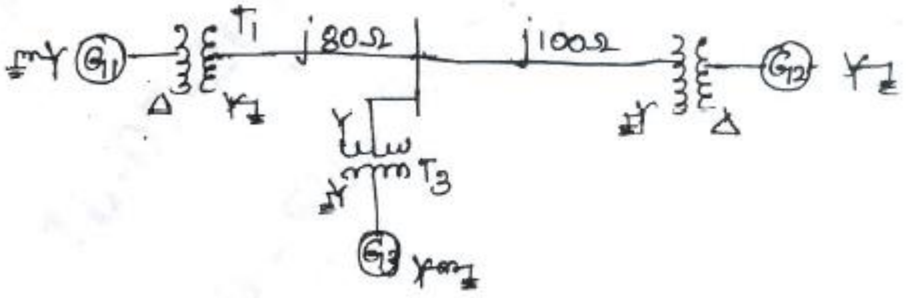


Fig.Q1(c) **(10 Marks)**

3 a. Prove that a balanced set of 3-ϕ voltages will have only positive sequence components of voltages. **(06 Marks)**

b. One conductor of a 3-ϕ line is open. The current flowing to the Δ-connected load through line 'a' is 10A. With the current in line 'a' as reference and assuming that line 's' is open, find the symmetrical components of the line currents. **(07 Marks)**

c. Obtain the relation between sequence components of phase and line currents in delta connected systems. **(07 Marks)**

4 a. For the following configurations of a 3-ϕ transformer, draw the winding connection and zero sequence network. **(08 Marks)**

i) Y - Δ ii) Δ - Δ iii) Y - Y⏚ iv) ⏚Y - Δ

b. In a 3-ϕ, 4 wire system, the sequence voltages and currents are,

$V_{a_1} = 0.9\angle 10^\circ$ pu; $V_{a_2} = 0.25\angle 110^\circ$ pu; $V_{a_0} = 0.12\angle 300^\circ$ pu

$I_{a_1} = 0.75\angle 25^\circ$ pu; $I_{a_2} = 0.15\angle 170^\circ$ pu; $I_{a_0} = 0.1\angle 330^\circ$ pu

Find the complex power in pu. If the neutral gets disconnected, find the new power. **(08 Marks)**

c. Draw a zero sequence network for the given one line diagram shown in Fig.Q4(c).

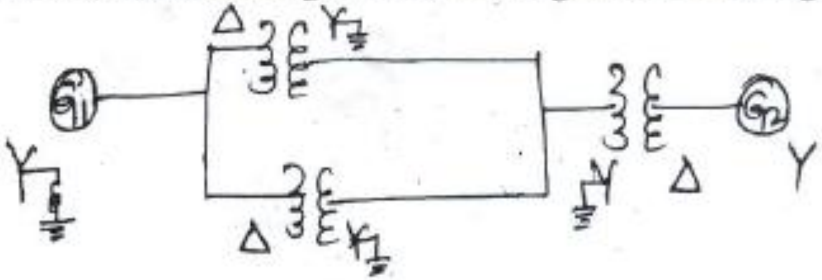


Fig.Q4(c) **(04 Marks)**

Sixth Semester B.E. Degree Examination, Dec.2018/Jan.2019

Power System Analysis – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that the per unit impedance of a transformer is the same when referred to either primary or secondary side. **(04 Marks)**

b. Draw the circuit model of synchronous generator, transmission lines and transformer. **(04 Marks)**

c. The OLD of an unloaded power system is as shown in Fig.Q1(c). Reactance of Tr. Line are shown in figure. Draw the per unit impedance diagram. Choose a base of 50 MVA, 13.8 KV in G_1 circuit. The ratings are as under.

$G_1 \rightarrow$ 20 MVA, 13.8 KV, $X'' = 0.2$Pu, $T_1 \rightarrow$ 25MVA, 220/13.8 KV, X = 10%

$G_2 \rightarrow$ 30 MVA, 18 KV, $X'' = 0.2$Pu, $T_2 \rightarrow 3, 1\phi$ Tr% each 10 MVA, 127/18 KV, X = 10%

$G_3 \rightarrow$ 30 MVA, 20 KV, $X'' = 0.2$Pu, $T_3 \rightarrow$ 35MVA, 220/22 KV, X = 10%. **(08 Marks)**

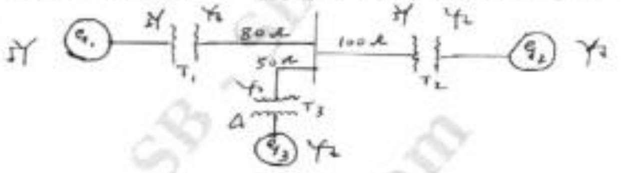


Fig.Q1(c)

OR

2 a. What is per unit quantity? Mention its advantage. **(04 Marks)**

b. How is the per unit impedance value in a given base are changed to per unit impedance value on new base. **(04 Marks)**

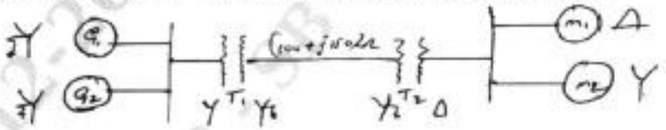
c. Draw the impedance diagram for the power system shown in Fig. Q2(c). The ratings of the components are as under,

$G_1 \rightarrow$ 25 MVA, 11 KV, x = 15% $G_2 \rightarrow$ 30 MVA, 12.5 KV, x = 20%

$M_1 \rightarrow$ 15 MVA, 11 KV, x = 12% $M_2 \rightarrow$ 25 MVA, 11.5 KV, x = 15%

$T_1 \rightarrow$ 30 MVA, 13/132 KV, x = 25% $T_2 \rightarrow$ 35 MVA, 132/11 KV, x = 20%

Choose a base of 132 KV on $(100 + j150)\Omega$ Tr. Line at 30 MVA base. **(08 Marks)**



Module-3

5 a. Derive an expression for the 3ϕ, complex power in terms of symmetrical components. **(08 Marks)**

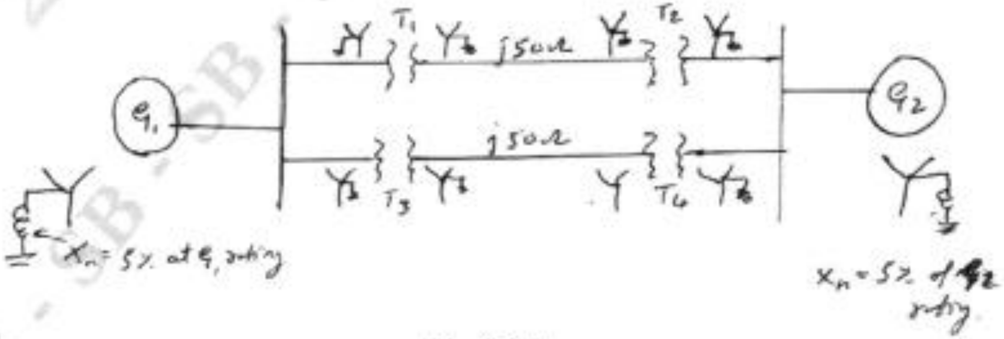
b. Draw the zero sequence network for different combination of 3ϕ transformer bank. **(04 Marks)**

c. A balanced Δ connected load is connected to a 3ϕ symmetrical supply. The line currents are each 10A in magnitude. If fuse in one of the line is blown out. Determine the sequence component of the line current. **(04 Marks)**

OR

6 a. Derive an expression for symmetrical components of voltage in terms of phase voltage. **(06 Marks)**

b. Draw the positive, negative and zero sequence network for the power system shown in Fig.Q6(b). Choose a base of 50MVA, 220KV in the 50Ω transmission line and mark all reactance in per unit. The ratings are as under :
$G_1 \rightarrow$ 25 MVA, 12 KV, $X'' = 20\%$, $G_2 \rightarrow$ 25MVA, 11KV, $X'' = 20\%$ T_1 to $T_4 \rightarrow$ 20MVA, 11/220 KV, $X = 15\%$.
The negative sequence reactance of each synchronous machine is equal to the subtransient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactance of line are 250% of their positive sequence reactance. **(10 Marks)**



Sixth Semester B.E. Degree Examination, Dec.2018/Jan.2019

Power System Analysis and Stability

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Draw the PU Reactance diagram for power system shown in Fig.Q1(a). Select the base values of 20 MVA, 6.6 KV in the generator 1 circuit. The ratings of various components are:

Generator 1: 10 MVA, 6.6 KV, $X'' = 0.1$ p.u

Generator 2: 20 MVA, 11.5 KV, $X'' = 0.1$ p.u

Transformer T_1 = 10 MVA, 3 phase, 6.6/115 KV, $X = 0.15$ p.u.

Transformer T_2 = 3 single phase units each rated 10 MVA, 7.5/75 KV, $X = 0.1$ pu.

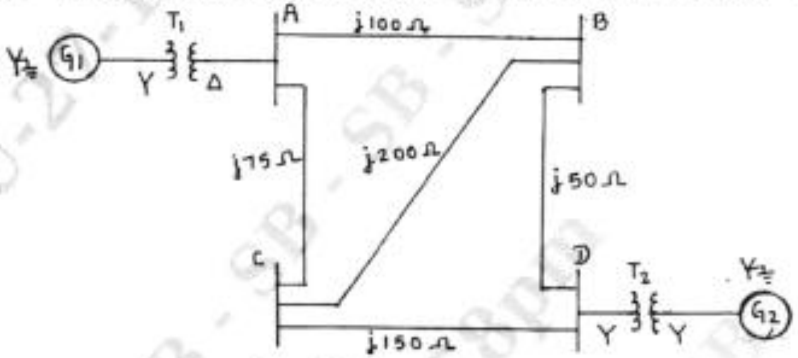


Fig.Q1(a) **(12 Marks)**

b. What are the advantages of per unit quantities? Show that:

$$\text{PU reactance}_{new} = \text{PU reactance}_{given}\ \frac{\text{base MVA}_{new}}{\text{base MVA}_{old}} \times \frac{\text{base KV}^2_{old}}{\text{base KV}^2_{new}}.$$ **(08 Marks)**

3 a. Derive the relation between sequence components of phase and line voltages in star connected systems. **(08 Marks)**

b. A delta connected balanced resistive load is connected across an unbalanced 3 phase supply as shown in Fig.Q3(b). With currents in lines A and B specified, find the symmetrical components of line currents. Also, find the symmetrical components of delta currents (phase-currents).

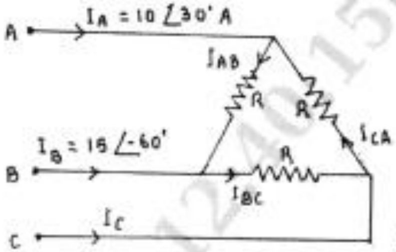


Fig.Q3(b) **(12 Marks)**

4 a. Draw the positive, negative and zero sequence network for the power system shown in Fig.Q4(a). Choose a base of 50 MVA, 220 KV in the j50Ω transmission line and mark all reactances in per unit. The ratings of generators and transformers are

Gen 1: 25 MVA, 11 KV, $X'' = 20\%$

Gen 2 : 25 MVA, 11 KV, $X'' = 20\%$

Transformers (each) : 20 MVA, 11Y/220Y KV, X = 15%. The negative sequence reactance of each synchronous machine is equal to the sub transient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactance of lines are 250% of their positive sequence reactance.

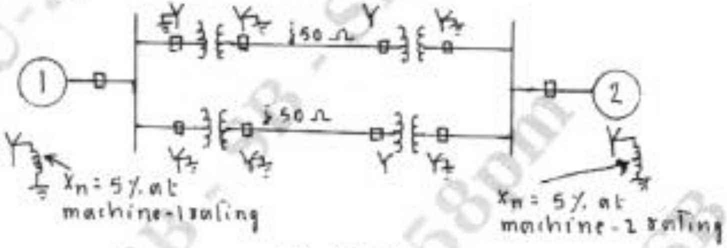


Fig.Q4(a) **(14 Marks)**

b. Derive an expression for complex power in terms of symmetrical components. **(06 Marks)**

Sixth Semester B.E. Degree Examination, June/July 2016
Power System Analysis and Stability

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, selecting atleast TWO questions from each part.*

PART – A

1 a. Show that per unit impedance of two winding transformer will remain same referred to primary as well as secondary. **(06 Marks)**

b. List the advantages of per unit system. **(04 Marks)**

c. A 300 MVA, 20 KV, 3 phase generator has subtransient reactance of 20%. The generator supplies two synchronous motors through a 64 km transmission line having transformers at both ends as shown in Fig. Q1(c), T_1 is a 3 phase transformer and T_2 is composed of 3 single phase transformers of rating 100 MVA each, 127/13.2 KV, 10% reactance. Series reactance of transmission line is 0.5 ohm/km. Draw the reactance diagram with all reactances marked in per unit. Select generator rating as base values. **(10 Marks)**

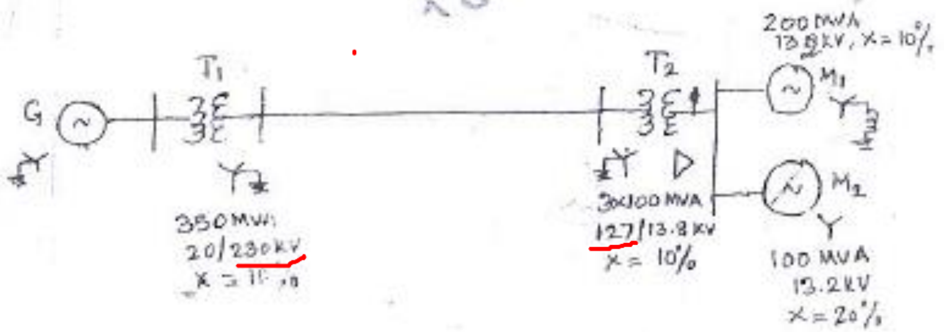


Fig. Q1(c)