

Module-3

Performance of three phase induction motors

Phasor Diagram of Induction Motor:

The phasor diagram of loaded induction motor is similar to the loaded transformer. The only difference is the secondary of induction motor is rotating and short circuited while transformer secondary is stationary and connected to load. The load on induction motor is mechanical while load on transformer is electrical. Still by finding electrical equivalent of mechanical load on the motor, the phasor diagram of induction motor can be developed.

Let Φ = Magnetic flux links with both primary and secondary.

There is self induced e.m.f. E_1 in the stator while a mutually induced e.m.f. E_{2r} in the rotor.

Let R_1 = Stator resistance per phase.

X_1 = Stator reactance per phase

The stator voltage per phase V_1 has to counter balance self induced e.m.f. E_1 and has to supply voltage drops $I_1 R_1$ and $I_1 X_1$. So on stator side we can write,

$$\overline{V_1} = -\overline{E_1} + \overline{I_1 R_1} + j\overline{I_1 X_1} = \overline{E_1} + \overline{I_1} (\overline{R_1} + j\overline{X_1}) = -\overline{E_1} + \overline{I_1} \overline{Z_1}$$

The rotor induced e.m.f. in the running condition has to supply the drop across impedances as rotor short circuited.

$$\therefore \overline{E_{2r}} = \overline{I_{2r} R_2} + j\overline{I_{2r} X_2} = \overline{I_{2r}} (\overline{R_2} + j\overline{X_2}) = \overline{I_{2r}} \overline{Z_{2r}}$$

The value of E_{2r} depends on the ratio of rotor turns to stator turns.

The rotor current in the running condition is I_{2r} which lags E_{2r} by rotor p.f. angle Φ_{2r} .

The reflected rotor current I_{2r}' on stator side is the effect of load and is given by,

$$I_{2r}' = K I_{2r}$$

The induction motor draws no load current I_o which is phasor sum of I_c and I_m . The total stator current drawn from supply is,

$$\overline{I_1} = \overline{I_o} + \overline{I_{2r}'}$$

The Φ_1 is angle between V_1 and I_1 and $\cos\Phi_1$ gives the power factor of the induction motor.

Thus using all above relations the phasor diagram of induction motor on load can be obtained.

The steps to draw phasor diagram are,

1. Takes Φ as reference phasor.
2. The induced voltage E_1 lags Φ by 90° .
3. Show $-E_1$ by reversing voltage phasor.
4. The phasor E_{2r} is in phase with E_1 . So I_{2r} show lagging E_{2r} i.e. E_1 direction by Φ_{2r} .
5. Show $I_{2r} R_2$ in phase with I_{2r} and $I_{2r} X_{2r}$ leading the resistive drop by 90° , to get exact location of.

6. Reverse I_{2r} to get I_{2r}' .
7. I_m is in phase with Φ while I_c is at leading with. Add I_m and I_c to get I_o .
8. Add I_o and I_{2r}' to get I_1 .
9. From tip of $-E_1$ phasor, add $I_1 R_1$ in phase with I_1 and $I_1 X_1$ at 90° leading to I_1 to V_1 get phasor.
10. Angle between V_1 and I_1 is Φ_1 .

The phasor diagram is shown in the Fig. 1.

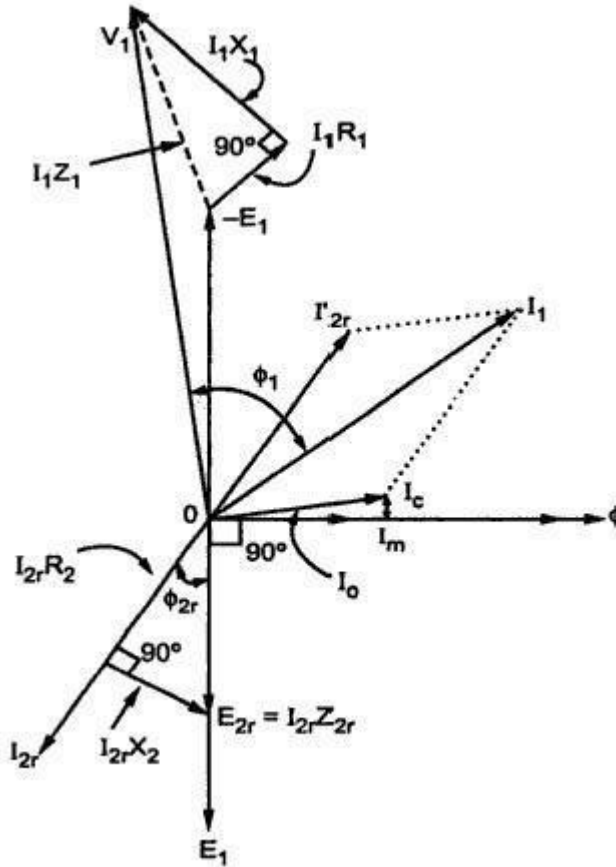


Fig. 1 On load phasor diagram of induction motor

Equivalent Circuit of Induction Motor :

We have already seen that the induction motor can be treated as generalized transformer. Transformer works on the principle of electromagnetic induction. The induction motor also works on the same principle. The energy transfer from stator to rotor of the induction motor takes place entirely with the help of a flux mutually linking the two. Thus stator acts as a primary while the rotor acts as a rotating secondary when induction motor is treated as a transformer.

- If
- E_1 = Induced voltage in stator per phase
 - E_2 = Rotor induced e.m.f. per phase on standstill
 - k = Rotor turns / Stator turns

then $k = E_2 / E_1$

Thus if V_1 is the supply voltage per phase to stator, it produces the flux which links with both stator and rotor. Due to self induction E_1 , is the induced e.m.f. in stator per phase while E_2 is the induced e.m.f. in rotor due to mutual induction, at standstill. In running condition the induced e.m.f. in rotor becomes E_{2r} which is $s E_2$.

Now E_{2r} = Rotor induced e.m.f. in running condition per phase
 R_2 = Rotor resistance per phase
 X_{2r} = Rotor reactance per phase in running condition
 R_1 = Stator resistance per phase
 X_1 = Stator reactance per phase

So induction motor can be represented as a transformer as shown in the Fig. 1.

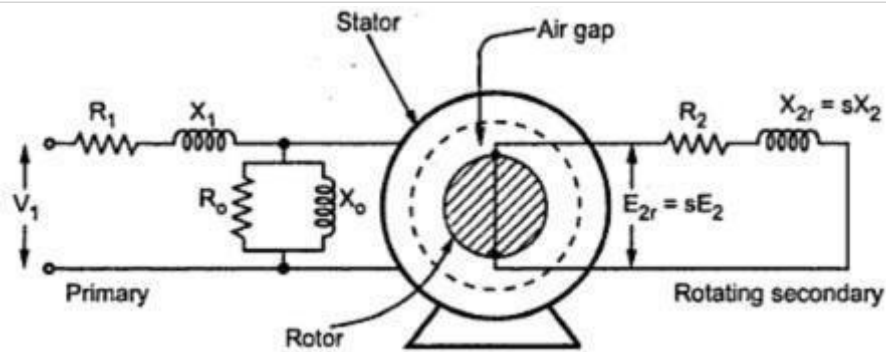


Fig. 1 Induction motor as a transformer

When induction motor is on no load, it draws a current from the supply to produce the flux in air gap and to supply iron losses.

1. I_c = Active component which supplies no load losses
2. I_m = Magnetizing component which sets up flux in core and air gap

These two currents give us the elements of an exciting branch as,

R_o = Representing no load losses = V_1 / I_c

and X_o = Representing flux set up = V_1 / I_m

Thus, $\bar{I}_o = \bar{I}_c + \bar{I}_m$

The equivalent circuit of induction motor thus can be represented as shown in the Fig. 2.

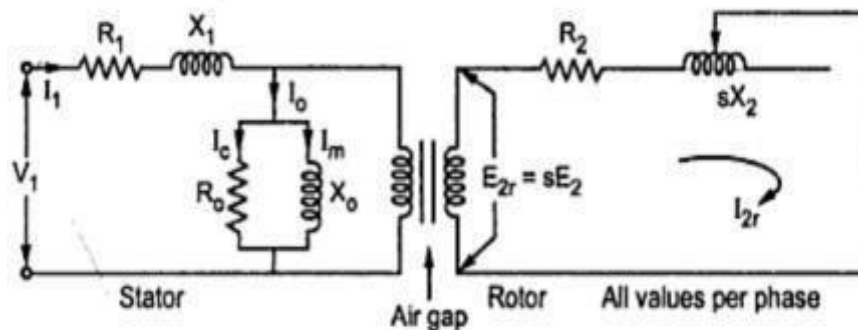


Fig. 2 Basic equivalent circuit

The stator and rotor sides are shown separated by an air gap.

I_{2r} = Rotor current in running condition

$$= E_{2r}/Z_{2r} = (s E_2)/\sqrt{(R_2^2 + (s X_2)^2)}$$

It is important to note that as load on the motor changes, the motor speed changes. Thus slip changes. As slip changes the reactance X_{2r} changes. Hence $X_{2r} = sX_2$ is shown variable.

Representing of rotor impedance :

It is shown that, $I_{2r} = (sE_2)/\sqrt{(R_2^2 + (s X_2)^2)} = E_2/\sqrt{((R_2/s)^2 + X_2^2)}$

So it can be assumed that equivalent rotor circuit in the running condition has fixed reactance X_2 , fixed voltage E_2 but a variable resistance R_2/s , as indicated in the above equation.

Now $R_2/s = R_2 + (R_2/s) - R_2$

$$\therefore \frac{R_2}{s} = R_2 + R_2 \left(\frac{1}{s} - 1 \right) = R_2 + R_2 \left(\frac{1-s}{s} \right)$$

So the variable rotor resistance R_2/s has two parts.

1. Rotor resistance R_2 itself which represents copper loss.
2. $R_2(1 - s)/s$ which represents load resistance R_L . So it is electrical equivalent of mechanical load on the motor.

Key Point : Thus the mechanical load on the motor is represented by the pure resistance of value $R_2(1 - s)/s$.

So rotor equivalent circuit can be shown as,

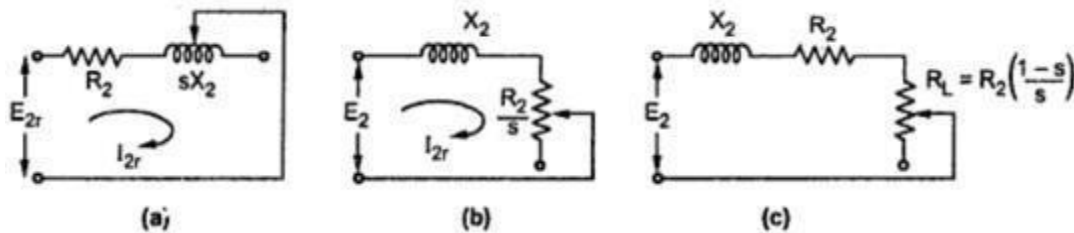


Fig. 3 Rotor equivalent circuit

Now let us obtain equivalent circuit referred to stator side.

Equivalent circuit referred to stator :

Transfer all the rotor parameters to stator,

$$k = E_2/E_1 = \text{Transformation ratio}$$

$$E_2' = E_2 / k$$

The rotor current has its reflected component on the stator side which is I_{2r}' .

$$I_{2r}' = k I_{2r} = (k s E_2) / \sqrt{(R_2^2 + (s X_2)^2)}$$

$$X_2' = X_2 / K^2 = \text{Reflected rotor reactance}$$

$$R_2' = R_2 / K^2 = \text{Reflected rotor resistance}$$

$$R_L' = R_L/K^2 = (R_2/K^2)(1-s/s)$$

$$= R_2' (1-s/s)$$

Thus R_L' is reflected mechanical load on stator.

So equivalent circuit referred to stator can be shown as in the Fig. 4

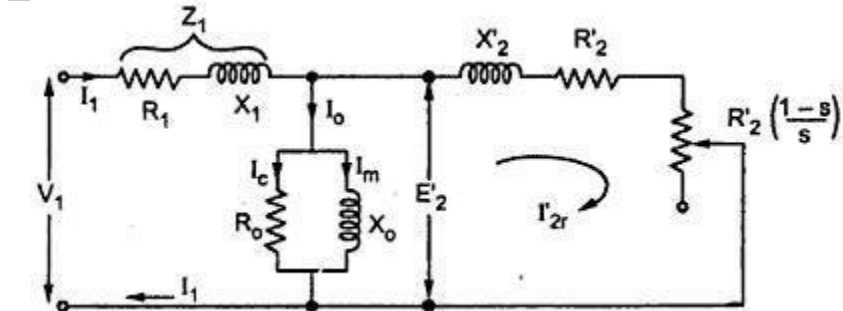


Fig. 4 Equivalent circuit referred to stator

The resistance $R_2' (1-s)/s = R_L'$ is fictitious resistance representing the mechanical load on the motor.

Approximate Equivalent Circuit

Similar to the transformer the equivalent circuit can be modified by shifting the exciting current (R_o and X_o) purely across the supply, to the left of R_1 and X_1 . Due to this, we are neglecting the drop across R_1 and X_1 due to I_o , which is very small. Hence the circuit is called approximate equivalent circuit. The circuit is shown in the Fig.5.

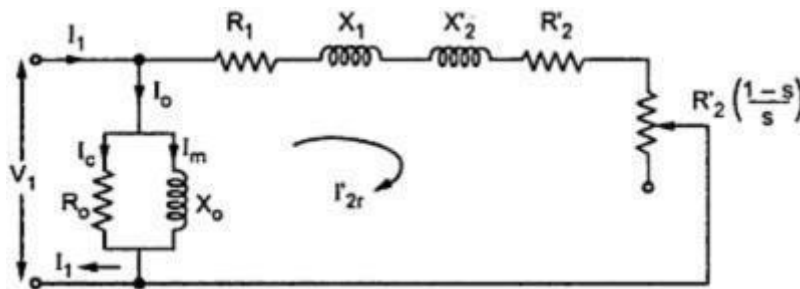


Fig. 5 Approximate equivalent circuit

Now the resistance R_1 and R_2' while reactance X_1 and X_2' can be combined. So we get,

$$R_{1e} = \text{Equivalent resistance referred to stator} = R_1 + R_2'$$

$$X_{1e} = \text{Equivalent reactance referred to stator} = X_1 + X_2'$$

$$R_{1e} = R_1 + (R_2/K^2)$$

and

$$X_{1e} = X_1 + (X_2/K^2)$$

While $\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}$ phasor diagram

and $\bar{I}_o = \bar{I}_c + \bar{I}_m$

Thus the equivalent circuit can be shown in the Fig.6.

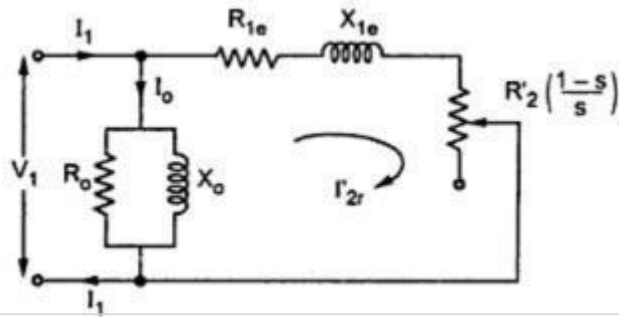


Fig. 6

Power Equations from Equivalent Circuit

With reference to approximate equivalent circuit shown in the Fig. 6, we can write various power equations as,

$$P_{in} = \text{input power} = 3 V_1 I_1 \cos \Phi$$

where V_1 = Stator voltage per phase

I_1 = Current drawn by stator per phase

$\cos \Phi$ = Power factor of stator

$$\text{Stator core loss} = I_m^2 R_0$$

$$\text{Stator copper loss} = 3 I_1^2 R_1$$

where R_1 = Stator resistance per phase

$$P_2 = \text{Rotor input} = (3 I_{2r}^2 R_2')/s$$

$$P_c = \text{Rotor copper loss} = 3 I_{2r}^2 R_2'$$

Thus $P_c = s P_2$

$$P_m = \text{Gross mechanical power developed}$$

$$\therefore P_m = P_2 - P_c = \frac{3 (I_{2r}')^2 R_2'}{s} - 3 (I_{2r}')^2 R_2' = 3 (I_{2r}')^2 R_2' \left(\frac{1-s}{s} \right)$$

T = Torque developed

$$\therefore T = \frac{P_m}{\omega} = \frac{3 (I_{2r}')^2 R_2' \left(\frac{1-s}{s} \right)}{\frac{2\pi N}{60}}$$

where N = Speed of motor

But $N = N_s(1-s)$, so substituting in above

$$T = \frac{3(I_{2r}')^2 R_2'}{\frac{2\pi N_s}{60}} = 9.55 \times \frac{3(I_{2r}')^2 R_2'}{N_s} \text{ N-m}$$

and $I_{2r}' = V_1 / ((R_{1e} + R_L') + j X_{1e})$

where $R_L' = R_2' (1-s)/s$

$$I_{2r}' = V_1 / \sqrt{(R_{1e} + R_L')^2 + X_{1e}^2}$$

Key Point : Remember that in all the above formula all the values per phase values.

Maximum Power Output

Consider the approximate equivalent circuit as shown in the Fig.7

In this circuit, the exciting current I_0 is neglected hence the exciting no load branch is not shown.

$$\therefore I_1 = I_{2r}'$$

The total impedance is given by,

$$Z_T = (R_{1e} + R_L') + j X_{1e} \quad \text{where } R_L' = R_2' (1-s)/s$$

$$I_1 = V_1 / \sqrt{(R_{1e} + R_L')^2 + (X_{1e})^2}$$

The power supplied to the load i.e. P_{out} per phase is,

Per phase $P_{out} = I_1^2 R_L'$ watts per phase

$$\therefore \text{Total} = 3 I_1^2 R_L'$$

$$\therefore P_{out} = 3 \frac{V_1^2}{[(R_{1e} + R_L')^2 + (X_{1e})^2]} (R_L')$$

To obtain maximum output power, differentiate the equation of total P_{out} with respect to variable R_L' and equal to zero.

$$\therefore \frac{d}{dR_L'} \left[\frac{3 V_1^2 (R_L')}{[(R_{1e} + R_L')^2 + (X_{1e})^2]} \right] = 0$$

$$\therefore [(R_{1e} + R_L')^2 + (X_{1e})^2] [3 V_1^2] - 3 V_1^2 (R_L') [2 (R_{1e} + R_L')] = 0$$

$$\therefore (R_{1e} + R_L')^2 + (X_{1e})^2 - 2 (R_L') (R_{1e} + R_L') = 0 \quad \dots \text{Taking } 3 V_1^2 \text{ common}$$

$$\therefore R_{1e}^2 + (R_L')^2 + 2 R_{1e} R_L' + X_{1e}^2 - 2 R_{1e} R_L' - 2 (R_L')^2 = 0$$

$$\therefore R_{1e}^2 + X_{1e}^2 = (R_L')^2$$

But $Z_{1e} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$ = Leakage impedance referred to stator
 $\therefore Z_{1e} = R_L'$

Thus the mechanical load on the induction motor should be such that the equivalent load resistance referred to stator is equal to the total leakage impedance of motor referred to stator.

Slip at maximum P_{out} : This can be obtained as,

$$\begin{aligned} R_L' &= Z_{1e} = R_2'(1-s)/s & \text{where } R_L' &= R_2/K^2 \\ \therefore s Z_{1e} &= R_2' - sR_2' \\ \therefore s(Z_{1e} + R_2') &= R_2' \end{aligned}$$

$$s = \frac{R_2'}{(R_2' + Z_{1e})}$$

This is slip at maximum output.

Expression for maximum P_{out} : Using the condition obtained in expression of total P_{out} , we can get maximum P_{out} .

$$\begin{aligned} \therefore (P_{out})_{max} &= 3 I_1^2 Z_{1e} & \text{as } R_L' &= Z_{1e} \\ &= 3 \frac{V_1^2}{(R_{1e} + Z_{1e})^2 + (X_{1e})^2} \cdot Z_{1e} & \text{as } R_L' &= Z_{1e} \\ &= 3 \frac{V_1^2}{(R_{1e}^2 + 2 R_{1e} Z_{1e} + Z_{1e}^2 + X_{1e}^2)} \cdot Z_{1e} \end{aligned}$$

$$\text{But } R_{1e}^2 + X_{1e}^2 = Z_{1e}^2$$

$$\therefore (P_{out})_{max} = 3 \frac{V_1^2}{2 Z_{1e}^2 + 2 R_{1e} Z_{1e}} \cdot Z_{1e} = 3 \frac{V_1^2}{2 Z_{1e} + (R_{1e} + Z_{1e})} \cdot Z_{1e}$$

$$\therefore (P_{out})_{max} = \frac{3 V_1^2}{2 (R_{1e} + Z_{1e})} \text{ watts}$$

Maximum Torque

In case of induction motor, the speed of the motor decreases with increase in load. Thus the maximum power output is not obtained at a slip which corresponds to maximum torque. In the previous section we have seen the condition for maximum power output. In this section we will find the condition which gives maximum torque.

The expression for torque is given by,

$$T = \frac{3(I'_{2r})^2 R'_2}{s \omega_s} = \frac{3(I'_{2r})^2 R'_2}{s \left(\frac{2\pi N_s}{60} \right)}$$

The condition for maximum torque can be obtained from maximum power transfer theorem. When $I'_{2r}{}^2 R'_2/s$ is maximum consider the approximate equivalent circuit of induction motor as shown in The Fig. 8.

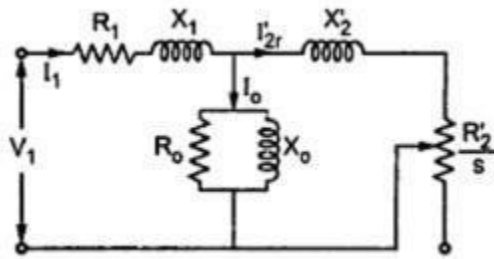


Fig. 8

The value of R_o is assumed to be negligible. Hence the circuit will be reduced as shown below.

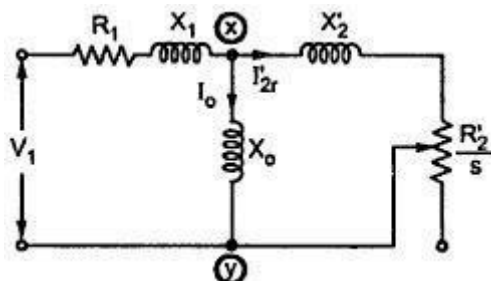


Fig. 9

The thevenin's equivalent circuit for the above network is shown in the Fig.10 across the terminals x and y.

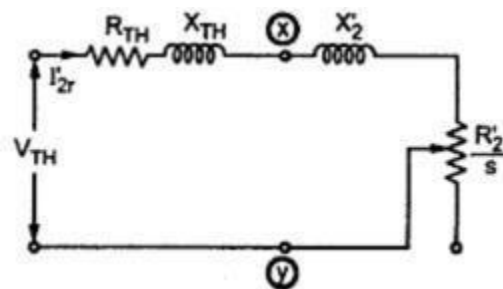


Fig. 10

$$\text{Now, } Z_{TH} = R_{TH} + jX_{TH} = (R_1 + jX_1) \parallel jX_o$$

$$V_{TH} = \frac{V_1(jX_o)}{R_1 + j(X_{TH} + X_o)}$$

The mechanical torque developed by rotor is maximum if there is maximum power transfer to the resistor R_2'/s . This takes place when R_2'/s equals to impedance looking back into the supply source.

$$\frac{R_2'}{s} = R_{TH} + j(X_{TH} + X_2')$$

$$\frac{R_2'}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}$$

$$\therefore s = s_m = \frac{R_2'}{\sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}}$$

This is the slip corresponding to the maximum torque. The maximum torque is given by,

$$T_m = \frac{3}{\omega_s} \cdot \frac{R_2'}{s_m} \cdot (I'_{2r})^2$$

$$\frac{R_2'}{s_m} = \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}$$

$$\therefore I'_{2r} = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2'}{s_m}\right)^2 + (X_{TH} + X_2')^2}}$$

$$I'^2_{2r} = \frac{V_{TH}^2}{\left(R_{TH} + \frac{R_2'}{s_m}\right)^2 + (X_{TH} + X_2')^2}$$

Substituting,

$$T_m = \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2} \cdot \frac{V_{TH}^2}{\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}\right]^2 + (X_{TH} + X_2')^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2} \cdot \frac{V_{TH}^2}{2R_{TH}^2 + 2R_{TH}\sqrt{R_{TH}^2 + (X_{TH} + X_2')^2} + 2(X_{TH} + X_2')^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2} \cdot \frac{R_{TH}^2}{2\left[R_{TH}^2 + (X_{TH} + X_2')^2 + R_{TH}\sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}\right]}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2} \cdot \frac{0.5 V_{TH}^2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2')^2} \left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}\right]}$$

$$\therefore T_m = \frac{3}{\omega_s} \cdot \frac{0.5 V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}}$$

From the above expression, it can be seen that the maximum torque is independent of rotor resistance.

Synchronous Watt:

The torque produced in the induction motor is given by,

$$T = \frac{\frac{3(I_{2r}')^2 R_2'}{s}}{\frac{2\pi N_s}{60}} = \frac{P_2}{\frac{2\pi N_s}{60}} \text{ N-m}$$

Thus torque is directly proportional to the rotor input. By defining new unit of torque which is synchronous watt we can write,

$$T = P_2 \text{ synchronous-watts}$$

If torque is given in synchronous-watts then it can be obtained in N-m as,

$$1 \text{ syn-watt} = \frac{60}{2\pi N_s} \text{ N-m}$$

$$\text{i.e. } 1 \text{ N-m} = \frac{2\pi N_s}{60} \text{ syn-watt}$$

Key Point : Unit synchronous watt can be defined as the torque developed by the motor such that the power input to the rotor across the air gap is 1 W while running at synchronous speed.

Losses in Induction Motor:

The various power losses in an induction motor can be classified as,

- i) Constant losses
- ii) Variable losses

i) Constant losses :

These can be further classified as core losses and mechanical losses.

Core losses occur in stator core and rotor core. These are also called iron losses. These losses include eddy current losses and hysteresis losses. The eddy current losses are minimized by using laminated construction while hysteresis losses are minimized by selecting high grade silicon steel as the material for stator and rotor.

The iron losses depends on the frequency. The stator frequency is always supply frequency hence stator iron losses are dominate. As against this in rotor circuit, the frequency is very small which is slip times the supply frequency. Hence rotor iron losses are very small and hence generally neglected, in the running condition.

The mechanical losses include frictional losses at the bearings and windings losses. The friction changes with speed but practically the drop in speed is very small hence these losses are assumed to be the part of constant losses.

ii) Variable losses :

This include the copper losses in stator and rotor winding due to current flowing in the winding. As current changes as load changes as load changes, these losses are said to be variable losses.

Generally stator iron losses are combined with stator copper losses at a particular load to specify total stator losses at particular load condition.

Rotor copper loss = $3 I_{2r}^2 R_2$ Analysed separately

where I_{2r} = Rotor current per phase at a particular load
 R_2 = Rotor resistance per phase

Power Flow in an Induction Motor

Induction motor converts an electrical power supplies to it into mechanical power. The various stages in this conversion is called power flow in an inductor motor.

The three phase supply given to the stator is the net electrical input to the motor. If motor power factor is $\cos \Phi$ and V_L , I_L are line values of supply voltage and current drawn, then net electrical supplied to the motor can be calculated as,

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

Where

P_{in} = Net input electrical power.

This is nothing but the stator input.

The part of this power is utilized to supply the losses in the stator which are stator core as well as copper losses.

The remaining power is delivered to the rotor magnetically through the air gap with the help of rotating magnetic field. This is called rotor input denoted as P_2 .

So

$$P_2 = P_{in} - \text{stator losses (core + copper)}$$

The rotor is not able to convert its entire input to the mechanical as it has to supply rotor losses. The rotor losses are dominantly copper losses as rotor iron losses are very small and hence generally neglected. So rotor losses are rotor copper losses denoted as P_c .

So

$$P_c = 3 \times I_{2r}^2 \times R_2$$

where I_{2r} = Rotor current per phase in running condition

R_2 = Rotor resistance per phase.

After supplying these losses, the remaining part of P_2 is converted into mechanical which is called gross mechanical power developed by the motor denoted as P_m .

∴

$$P_m = P_2 - P_c$$

Now this power, motor tries to deliver to the load connected to the shaft. But during this mechanical transmission, part of P_m is utilized to provide mechanical losses like friction and windage.

And finally the power is available to the load at the shaft. This is called net output of the motor denoted as P_{out} . This is also called shaft power.

∴

$$P_{out} = P_m - \text{Mechanical losses.}$$

The rating of the motor is specified in terms of value of P_{out} when load condition is full load condition.

The above stages can be shown diagrammatically called power flow diagram of an induction motor.

This is shown in the Fig.1.

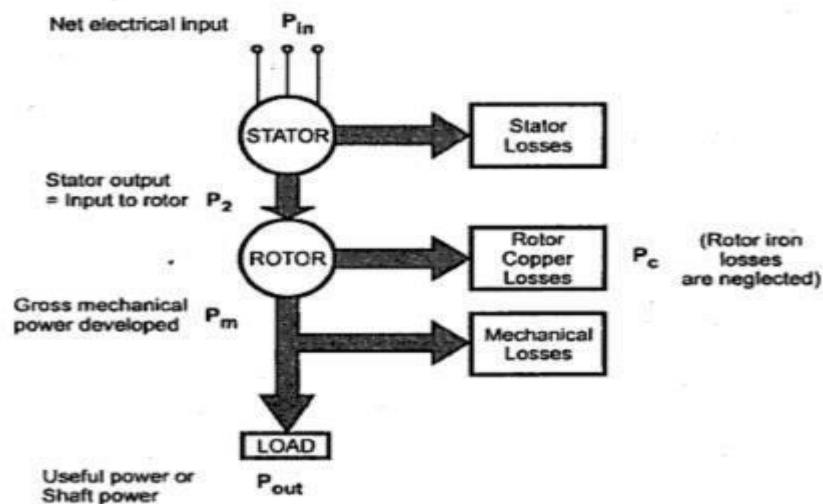


Fig. 1 Power flow diagram

From the power flow diagram we can define,

$$\text{Rotor efficiency} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{gross mechanical power developed}}{\text{rotor input}}$$

$$= P_m / P_2$$

$$\text{Net motor efficiency} = \frac{\text{net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_{in}}$$

Relation between P_2 , P_c , and P_m :

The rotor input P_2 , rotor copper loss P_c and gross mechanical power developed P_m are related through the slip s . Let us derive this relationship.

Let T = Gross torque developed by motor in N-m.

We know that the torque and power are related by the relation,

$$P = T \times \omega$$

where P = Power

and ω = angular speed

$$= (2\pi N)/60, N = \text{speed in r.p.m.}$$

Now input to the rotor P_2 is from stator side through rotating magnetic field which is rotating at synchronous speed N_s .

So torque developed by the rotor can be expressed in terms of power input and angular speed at which power is inputted i.e. ω_s as,

$$P_2 = T \times \omega_s \text{ where } \omega_s = (2\pi N_s)/60 \text{ rad/sec}$$

$$P_2 = T \times (2\pi N_s)/60 \text{ where } N_s \text{ is in r.p.m. (1)}$$

The rotor tries to deliver this torque to the load. So rotor output is gross mechanical power developed P_m and torque T . But rotor gives output at speed N and not N_s . So from output side P_m and T can be related through angular speed ω and not ω_s .

$$P_m = T \times \omega \text{ where } \omega = (2\pi N)/60$$

$$P_m = T \times (2\pi N)/60 \text{ (2)}$$

The difference between P_2 and P_m is rotor copper loss P_c .

$$P_c = P_2 - P_m = T \times (2\pi N_s/60) - T \times (2\pi N/60)$$

$$P_c = T \times (2\pi/60)(N_s - N) = \text{rotor copper loss (3)}$$

Dividing (3) by (1),s

$$\frac{P_c}{P_2} = \frac{T \times \frac{2\pi}{60} (N_s - N)}{T \times \frac{2\pi}{60} \times N_s} = \frac{N_s - N}{N_s}$$

$P_c/P_2 = s$ as $(N_s - N)/N_s = \text{slip } s$

Rotor copper loss $P_c = s \times \text{Rotor input } P_2$

Thus total rotor copper loss is slip times the rotor input.

Now

$$P_2 - P_c = P_m$$

$$P_2 - sP_2 = P_m$$

$$(1 - s)P_2 = P_m$$

Thus gross mechanical power developed is $(1 - s)$ times the rotor input

The relationship can be expressed in the ratio form as,

$$P_2 : P_c : P_m \quad \text{is} \quad 1 : s : 1 - s$$

The ratio of any two quantities on left hand side is same as the ratio of corresponding two sides on the right hand side.

$$\text{For example, } \frac{P_c}{P_m} = \frac{s}{1-s}, \quad \frac{P_2}{P_c} = \frac{1}{s} \quad \text{and so on.}$$

This relationship is very important and very frequently required to solve the problems on the power flow diagram.

Key Point : The torque produced by rotor is gross mechanical torque and due to mechanical losses entire torque cannot be available to drive load. The load torque is net output torque called shaft torque or useful torque and is denoted as T_{sh} . It is related to P_{out} as,

$$T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\left(\frac{2\pi N}{60} \right)}$$

and $T_{sh} < T$ due to mechanical losses.

Derivation of k in Torque Equation

We have seen earlier that

$$T = (k s E_2^2 R_2) / (R_2^2 + (s X_2)^2)$$

and it mentioned that $k = 3/(2\pi n_s)$. Let us see its proof.

The rotor copper losses can be expressed as,

$$P_c = 3 \times I_{2r}^2 \times R_2$$

but $I_{2r} = (s E_2) / \sqrt{R_2^2 + (s X_2)^2}$, hence substituting above

$$P_c = 3 \times \left[\frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \right]^2 \times R_2$$

$$\therefore P_c = \frac{3 s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Now as per $P_2 : P_c : P_m$ is $1 : s : 1-s$,

$$P_c / P_m = s / (1-s)$$

Now $P_m = T \times \omega$

$$= T \times (2\pi N / 60)$$

$$\therefore T \times \frac{2\pi N}{60} = \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$\therefore T = \frac{60}{2\pi N} \times \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Now $N = N_s(1-s)$ from definition of slip, substituting in above,

$$\begin{aligned} \therefore T &= \frac{60}{2\pi N_s (1-s)} \times \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (s X_2)^2} \\ &= \frac{3}{2\pi \left(\frac{N_s}{60} \right)} \times \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \end{aligned}$$

but $N_s / 60 = n_s$ in r.p.m.

So substituting in the above equation,

$$T = \frac{3}{2\pi n_s} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Comparing the two torque equations we can write,

$$k = \frac{3}{2\pi n_s} \text{ where } n_s \text{ is in r.p.s.}$$

Efficiency of an Induction Motor:

The ratio of net power available at the shaft (P_{out}) and the net electrical power input (P_{in}) to the motor is called as overall efficiency of an induction motor.

$$\therefore \% \eta = \frac{P_{out}}{P_{in}} \times 100$$

The maximum efficiency occurs when variable losses become equal to constant losses. When motor is on no load, current drawn by the motor is small. Hence efficiency is low. As load increases, current increases so copper losses also increases. When such variable losses achieve the same value as that of constant losses, efficiency attains its maximum value. If load is increased further, variable losses becomes greater than constant losses hence deviating from condition for maximum, efficiency starts decreasing. Hence the nature of the curve of efficiency against output power of the motor is shown in the Fig. 1.

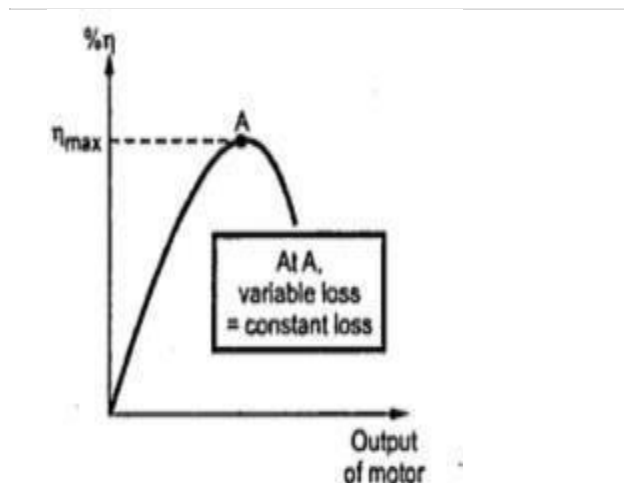
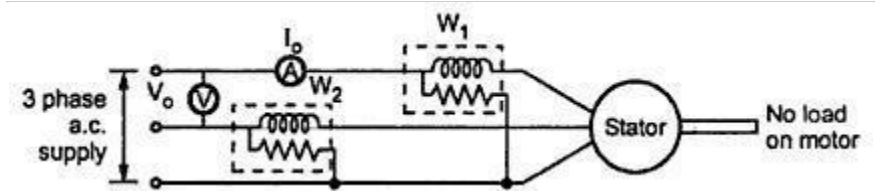


Fig. 1 Efficiency curve for an induction motor

No Load Test:

In this test, the motor is made to run without any load i.e. no load condition. The speed of the motor is very close to the synchronous speed but less than the synchronous speed. The rated voltage is applied to the stator. The input line current and total input power is measured. The two wattmeter method is used to measure the total input power. The circuit diagram for the test is shown in the Fig. 1.

**Fig. 1 No load test**

As the motor is on no load, the power factor is very low which is less than 0.5 and one of the two wattmeters read negative. It is necessary to reverse the current coil or pressure coil connections of such a wattmeter to get the positive reading. This reading must be taken negative for the further calculations.

The total power input W_o is the algebraic sum of the two wattmeter readings. The observation table is,

| V_o volts Rated line voltage | I_o Amp No load current | $W_o = W_1 + W_2$ (Algebraic sum) in watts |
|-----------------------------------|------------------------------|---|
| | | |

The calculations are,

$$W_o = \sqrt{3} V_o I_o \cos \Phi_o$$

| | | |
|--------------|--|----------------------------------|
| \therefore | $\cos \Phi_o = \frac{W_o}{\sqrt{3} V_o I_o}$ | where V_o, I_o are line values |
|--------------|--|----------------------------------|

This is no load power factor.

Thus we are now in a position to obtain magnitude and phase angle of no load current I_o , which is required for the circle diagram.

From the knowledge of I_o and Φ_o , the parameters of the equivalent circuit can be obtained as,

$$I_c = I_o \cos \Phi_o = \text{Active component of no load current}$$

$$I_m = I_o \sin \Phi_o = \text{Magnetising component of no load current}$$

$$R_o = V_o (\text{per phase}) / I_c (\text{per phase}) = \text{No load branch resistance}$$

$$X_o = V_o (\text{per phase}) / I_m (\text{per phase}) = \text{No load branch reactance}$$

The power input W_o consists of following losses,

1. Stator copper loss i.e. $3 I_o^2 R_1$ where I_o is no load per phase current and R_1 is stator resistance per phase.
2. Stator core loss i.e. iron loss.
3. Friction and windage loss.

The no load rotor current is very small and hence rotor copper loss is negligibly small. The rotor frequency is s times supply frequency and on no load it is very small. Rotor iron losses are proportional to this frequency and hence are negligibly small.

Key Point : Under no load condition, I_o is also very small and in many practical cases it is also neglected.

Thus W_o consists of stator iron loss and friction and windage loss which are consists for all load conditions. Hence W_o is said to give fixed losses of the motor.

$\therefore W_o = \text{No load power input}$

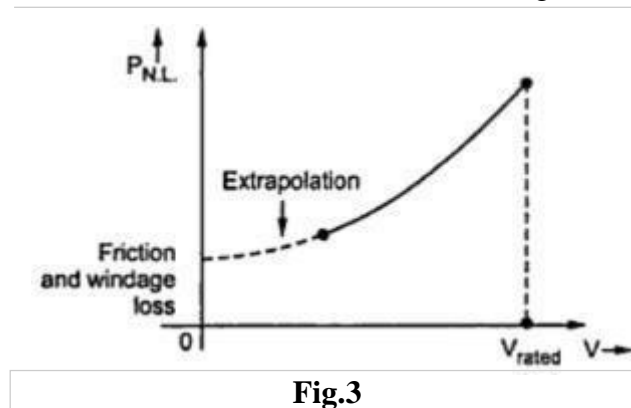
| | | |
|--------------|---|-----------------------------------|
| \therefore | $W_o = \text{Fixed Loss}$ | ... Neglecting stator copper loss |
|--------------|---|-----------------------------------|

Separating No Load Losses:

The no load losses are the constant losses which include core loss and friction and windage loss. The separation between the two can be carried out by the no load test conducted from variable voltage, rated frequency supply.

When the voltage is decreased below the rated value, The core loss reduces as nearly square of voltage. The slip does not increase significantly the friction and windage loss almost remains constant.

The voltage is continuously decreased till the machine slip suddenly begins to increase and the motor tends to stall. At no load, this takes place at a sufficiently reduced voltage. The graph showing no load losses P_{NL} versus as shown in the Fig. is extrapolated to $V = 0$ which gives friction and windage loss as iron or core loss is zero at zero voltage.



Blocked Rotor Test

In this test, the rotor is locked and it is not allowed to rotate. Thus the slip $s = 1$ and $R_L' = R_2' (1-s)/s$ is zero. If the motor is slip ring induction motor then the windings are short circuited at the slip rings.

The situation is exactly similar to the short circuit test on transformer. If under short circuit condition, if primary is excited with rated voltage, a large short circuit current can flow which is dangerous from the windings point of view. So similar to the transformer short circuit test, the

reduced voltage (about 10 to 15 % of rated voltage) just enough such that stator carries rated current is applied. Now the applied voltage V_{sc} , the input power W_{sc} and a short circuit current I_{sc} are measured.

As $R_L' = 0$, the equivalent circuit is exactly similar to that of a transformer and hence the calculations are similar to that of short circuit test on a transformer.

V_{sc} = Short circuit reduced voltage (line value)

I_{sc} = Short circuit current (line value)

W_{sc} = Short circuit input power

Now $W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \phi_{sc}$ Line values

$$\therefore \cos \phi_{sc} = \frac{W_{sc}}{\sqrt{3} V_{sc} I_{sc}}$$

This gives us short circuit power factor of a motor.

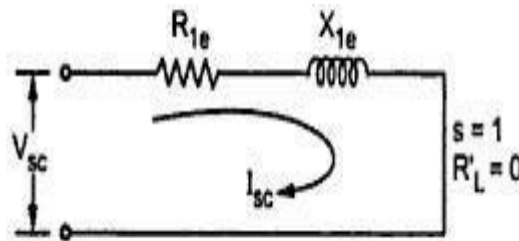


Fig. 1

Now the equivalent circuit is as shown in the Fig. 1.

$$\therefore W_{sc} = 3 (I_{sc})^2 R_{1e}$$

where I_{sc} = Per phase value

$$\therefore R_{1e} = \frac{W_{sc}}{3 (I_{sc})^2}$$

This is equivalent resistance referred to stator.

$Z_{1e} = V_{sc} \text{ (per phase)} / I_{sc} \text{ (per phase)} = \text{Equivalent impedance referred to stator.}$

$$\therefore R_{1e} = \frac{W_{sc}}{3 (I_{sc})^2}$$

$$\therefore X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$$

= Equivalent reactance referred to stator

During this test, the stator carries rated current hence the stator copper loss is also dominant. Similarly the rotor also carries short circuit current to produce dominant rotor copper loss. As the

voltage is reduced, the iron loss which is proportional to voltage is negligibly small. The motor is at standstill hence mechanical loss i.e. friction and windage loss is absent. Hence we can write,

$$W_{sc} = \text{Stator copper loss} + \text{Rotor copper loss}$$

But it is necessary to obtain short circuit current when normal voltage is applied to the motor. This is practically not possible. But the reduced voltage test results can be used to find current I_{SN} which is short circuit current if normal voltage is applied.

If $V_L = \text{Normal rated voltage (line value)}$

$V_{sc} = \text{Reduced short circuit voltage (line voltage)}$

| | |
|------|--|
| then | $I_{SN} = \left(\frac{V_L}{V_{sc}} \right) \times I_{sc}$ |
|------|--|

where $I_{sc} = \text{Short circuit current at reduced voltage}$

Thus, $I_{SN} = \text{Short circuit current at normal voltage}$

Now power input is proportional to square of the current.

So $W_{SN} = \text{Short circuit input power at normal voltage}$

This can be obtained as,

| |
|--|
| $W_{SN} = \left(\frac{I_{SN}}{I_{sc}} \right)^2 W_{sc}$ |
|--|

But at normal voltage core loss cannot be negligible hence,

$$W_{SN} = \text{Core loss} + \text{Stator and rotor copper loss}$$

Circle Diagram :

Introduction

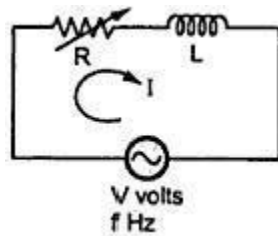
In a particular circuit, if one of the circuit elements is variable, then depending upon its value, the circuit characteristics varies. As the value of the variable element is changed, the circuit parameters like current, power factor, power losses etc. also change. The locus of the extremity of the current phasor, obtained for various values of a variable element is called a locus diagram.

From the equivalent circuit of an induction motor, the motor can be treated as series R-L circuit where the element resistance of the circuit is variable which varies as slip s . Thus for variable load conditions, the resistance changes and hence the current drawn by the motor also changes. The locus diagram of such a current phasor is circular in nature and hence called circle diagram of a three phase induction motor. Using this diagram, all the performance characteristics of an induction motor like power factor, efficiency, stator losses, rotor losses, maximum output, maximum torque etc. can be predicted. Thus, a circle diagram is a graphical approach of predetermining the operation characteristics of an induction motor.

Let us prove that the locus diagram obtained for a current phasor is a circle, for a series R-L circuit with an element R as variable.

Circle Diagram for a Series R-L Circuit:

Consider a series R-L circuit with a variable R as shown in the Fig. 1. It is excited by an alternating source of V volts. The frequency of the source is f Hz.

**Fig. 1**

Let I = Current flowing through the circuit

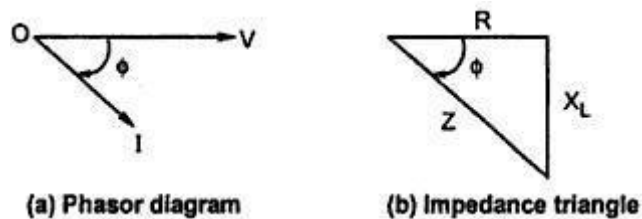
Z = Impedance of the circuit

$$Z = R + j X_L \quad \text{where } X_L = 2\pi fL$$

Now R is variable while X_L is fixed.

$$\begin{aligned} \therefore I &= \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}} \\ &= \frac{V}{X_L} \cdot \frac{X_L}{\sqrt{R^2 + X_L^2}} \quad \dots \text{Multiply and divide by } X_L \end{aligned}$$

The phasor diagram is shown in the Fig. 2(a). The current I lags voltage V by angle ϕ as the circuit is inductive. The impedance triangle is shown in the Fig. 2(b).

**Fig. 2**

From the impedance triangle we can write,

$$\sin \Phi = X_L/Z$$

Substituting in the expression for I,

$$I = (V/X_L) \sin \Phi \dots \dots \dots (1)$$

This is the equation of a circle in polar co-ordinates with a diameter equal to (V/X_L) .

When the resistance $R = 0$, then $\Phi = 90^\circ$ hence $\sin \Phi = 1$.

$$\therefore I = I_m = (V/X_L)$$

This is the maximum value of current.

As R resistance, the phase angle decreases thus decreasing \sin . Effectively current I also decrease. When $R \rightarrow \infty$ the $\Phi \rightarrow 0^\circ$ and current becomes zero.

The locus obtained of extremities of a current phasor plotted for various values of R is a semicircle. The semicircle is shown in the Fig. 3. The voltage axis is taken as vertical axis as a reference, with respect to which the various current phasors are plotted.

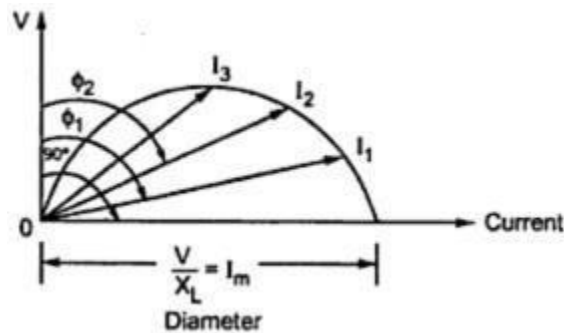


Fig. 3 Circle diagram

The power factors at various conditions are $\cos\Phi_1$, $\cos\Phi_2$ etc. As Φ varies only from 0° to 90° , the diagram is semicircle, infact it is a half part of a circle hence it is known as circle diagram.

This theory of series R-L circuit can be easily extended to a three phase induction motor. Circle Diagram of a 3 Phase Induction Motor

The equivalent circuit of a 3 phase induction motor is shown in the Fig.1.

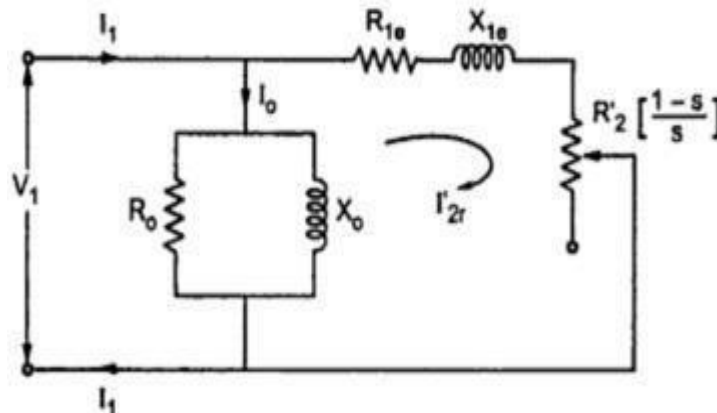


Fig. 1 Equivalent circuit of a 3 phase induction motor

All the values shown are per phase values. The circuit is similar to series R-L circuit. The reactance X_{1e} is fixed while the total resistance $R_{1e} + (R_2'(1-s)/s)$ is variable. This is because the slip s varies as load varies. The voltage across the parallel exciting branch is V_1 . Hence we can write the expression for the rotor current referred to stator as,

$$I_{2r}' = \frac{V_1}{\sqrt{(R_{1e} + R_L)^2 + X_{1e}^2}}$$

Where

$R_L' = R_2' (1-s)/s =$ Variable equivalent load resistance

$R_{1e} = R_1 + R_2' =$ Equivalent resistance of motor referred to stator

$X_{1e} = X_1 + X_2' =$ Equivalent reactance of motor referred to stator

Dividing and multiplying by,

$$I_{2r}' = \frac{V_1}{X_{1e}} \times \frac{X_{1e}}{\sqrt{(R_{1e} + R_L')^2 + (X_{1e})^2}}$$

$$\therefore I_{2r}' = I_{\max} \sin \dots\dots\dots(1)$$

$$\text{where } \sin \Phi = X/Z = X_{1e}/\sqrt{(R_{1e} + R_L')^2 + X_{1e}^2}$$

$$\text{and } I_{\max} = V_1/X_{1e}$$

The I_{2r}' will be at its maximum when $R_{1e} + R_L' = 0$ i.e., there exists an ideal short circuit. Hence current I_{\max} is called ideal short circuit current of an induction motor.

The equation (1) represents equation of a circle with V_1/X_{1e} as its diameter. Thus locus of extremity of I_{2r}' is a circle, as shown in the Fig.2.

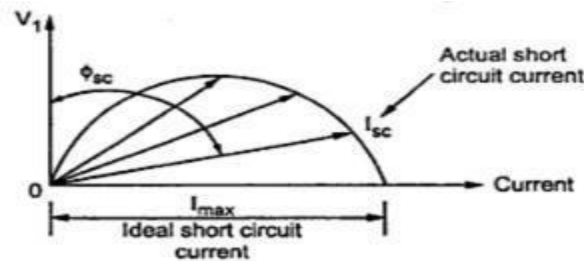


Fig. 2 Circle diagram of I_{2r}'

But the total stator current I_1 per phase is a vector addition of current I_0 and I_{2r}' .

$$I_1 = I_0 + I_{2r}' \dots\dots\dots \text{Vector addition}$$

For an induction motor, I_0 has a fixed value and phase angle Φ_0 which is decided by its active component and magnetising component I_m .

$$\bar{I}_0 = \bar{I}_c + \bar{I}_m$$

As I_0 has fixed magnitude and phase, the locus of extremities of I_1 , which is $I_0 + I_{2r}'$ is also a circle with a diameter still as V_1/X_{1e} . The only change will be that the diameter V_1/X_{1e} will no longer be along X-axis i.e. current axis but will get shifted at the tip of the I_0 phasor. All the I_{2r}' phasors are to be drawn from I_0 phasor to get I_1 , as I_1 has fixed magnitude and phase angle Φ_0 .

Key Point : Thus the current locus for a stator current is also a semicircle which is truly called circle diagram of a three phase induction motor. This diagram once obtained can be used to predict the performance of an induction motor under variable load conditions.

The circle diagram is shown in the Fig. 3.

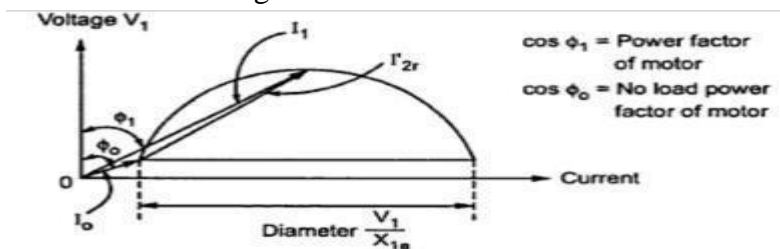


Fig. 3 Circle diagram of a three phase induction motor

Let us see, how to obtain the data for plotting the circle diagram.

Obtaining Data to Plot Circle Diagram

The data required to draw the circle diagram is obtained by conducting two testes which are,

1. No load test or open circuit test
2. Blocked rotor test or short circuit test

Construction of Circle Diagram:

By using the data obtained from the no load test and the blocked rotor test, the circle diagram can be drawn using the following steps :

Step 1 : Take reference phasor V as vertical (Y-axis).

Step 2 : Select suitable current scale such that diameter of circle is about 20 to 30 cm.

Step 3 : From no load test, I_o and Φ_o are obtained. Draw vector I_o , lagging V by angle Φ_o . This is the line OO' as shown in the Fig. 1.

Step 4 : Draw horizontal line through extremity of I_o i.e. O' , parallel to horizontal axis.

Step 5 : Draw the current I_{SN} calculated from I_{sc} with the same scale, lagging V by angle Φ_{sc} , from the origin O . This is phasor OA as shown in the Fig. 1.

Step 6 : Join $O'A$ is called output line.

Step 7 : Draw a perpendicular bisector of $O'A$. Extend it to meet line $O'B$ at point C . This is the centre of the circle.

Step 8 : Draw the circle, with C as a center and radius equal to $O'C$. This meets the horizontal line drawn from O' at B as shown in the Fig. 1.

Step 9 : Draw the perpendicular from point A on the horizontal axis, to meet $O'B$ line at F and meet horizontal axis at D .

Step 10 : Torque line.

The torque line separates stator and rotor copper losses.

Note that as voltage axis is vertical, all the vertical distances are proportional to active components of currents or power inputs, if measured at appropriate scale.

Thus the vertical distance AD represents power input at short circuit i.e. W_{SN} , now which consists of core loss and stator, rotor copper losses.

$$\begin{aligned}\text{Now } FD &= O'G \\ &= \text{Fixed loss}\end{aligned}$$

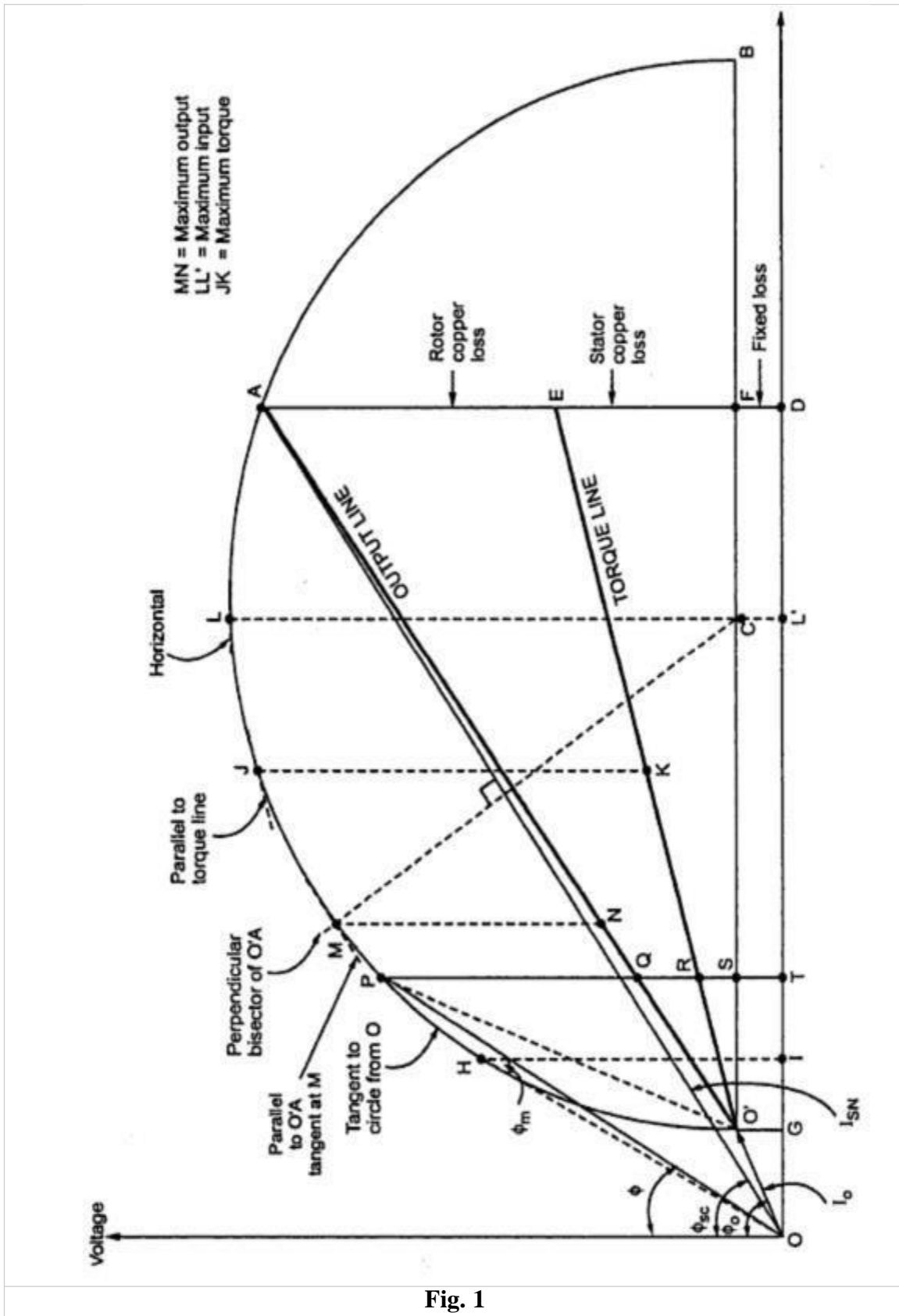
Where $O'G$ is drawn perpendicular from O' on horizontal axis. This represents power input on no load i.e. fixed loss.

$$\text{Hence } AF \propto \text{Sum of stator and rotor copper losses}$$

Then point E can be located as,

$$AE/EF = \text{Rotor copper loss} / \text{Stator copper loss}$$

The line $O'E$ under this condition is called torque line.



Power scale : As AD represents W_{SN} i.e. power input on short circuit at normal voltage, the power scale can be obtained as,

$$\text{Power scale} = W_{SN}/l(AD) \text{ W/cm}$$

where $l(AD)$ = Distance AD in cm

Location of Point E : In a slip ring induction motor, the stator resistance per phase R_1 and rotor resistance per phase R_2 can be easily measured. Similarly by introducing ammeters in stator and rotor circuit, the currents I_1 and I_2 also can be measured.

$$\therefore K = I_1/I_2 = \text{Transformation ratio}$$

$$\text{Now } AF/EF = \text{Rotor copper loss} / \text{Stator copper loss} = (I_2^2 R_2)/(I_1^2 R_1) = (R_2/R_1)(I_2^2/I_1^2) = (R_2/R_1).(1/K^2)$$

$$\text{But } R_2' = R_2/K^2 = \text{Rotor resistance referred to stator}$$

$$\therefore AE/EF = R_2'/R_1$$

Thus point E can be obtained by dividing line AF in the ratio R_2' to R_1 .

In a **squirrel cage motor**, the stator resistance can be measured by conducting resistance test.

$$\therefore \text{Stator copper loss} = 3I_{SN}^2 R_1 \text{ where } I_{SN} \text{ is phase value.}$$

$$\therefore \text{Neglecting core loss, } W_{SN} = \text{Stator Cu loss} + \text{Rotor Cu loss}$$

$$\therefore \text{Rotor copper loss} = W_{SN} - 3I_{SN}^2 R_1$$

$$\therefore AE/EF = (W_{SN} - 3I_{SN}^2 R_1)/(3I_{SN}^2 R_1)$$

Dividing line AF in this ratio, the point E can be obtained and hence O'E represents torque line.

Predicting Performance Form Circle Diagram:

Let motor is running by taking a current OP as shown in the Fig. 1. The various performance parameters can be obtained from the circle diagram at that load condition.

Draw perpendicular from point P to meet output line at Q, torque line at R, the base line at S and horizontal axis at T.

We know the power scale as obtained earlier.

Using the power scale and various distances, the values of the performance parameters can be obtained as,

$$\text{Total motor input} = PT \times \text{Power scale}$$

$$\text{Fixed loss} = ST \times \text{power scale}$$

$$\text{Stator copper loss} = SR \times \text{power scale}$$

$$\text{Rotor copper loss} = QR \times \text{power scale}$$

$$\text{Total loss} = QT \times \text{power scale}$$

$$\text{Rotor output} = PQ \times \text{power scale}$$

$$\text{Rotor input} = PQ + QR = PR \times \text{power scale}$$

$$\text{Slip } s = \text{Rotor Cu loss} = QR/PR$$

$$\text{Power factor } \cos = PT/OP$$

$$\text{Motor efficiency} = \text{Output} / \text{Input} = PQ/PT$$

$$\text{Rotor efficiency} = \text{Rotor output} / \text{Rotor input} = PQ/PR$$

$$\text{Rotor output / Rotor input} = 1 - s = N/N_s = PQ/PR$$

The torque is the rotor input in synchronous watts.

Maximum Quantities:

The maximum values of various parameters can also be obtained by using circle diagram.

1. Maximum Output : Draw a line parallel to O'A and is also tangent to the circle at point M. The point M can also be obtained by extending the perpendicular drawn from C on O'A to meet the circle at M. Then the maximum output is given by $l(MN)$ at the power scale. This is shown in the Fig. 1.

2. Maximum Input : It occurs at the highest point on the circle i.e. at point L. At this point, tangent to the circle is horizontal. The maximum input given $l(LL')$ at the power scale.

3. Maximum Torque : Draw a line parallel to the torque line and is also tangent to the circle at point J. The point J can also be obtained by drawing perpendicular from C on torque line and extending it to meet circle at point J. The l(JK) represents maximum torque in synchronous watts at the power scale. This torque is also called stalling torque or pull out torque.

4. Maximum Power Factor : Draw a line tangent to the circle from the origin O, meeting circle at point H. Draw a perpendicular from H on horizontal axis till it meets it at point I. Then angle OHI gives angle corresponding to maximum power factor angle.

$$\therefore \text{Maximum p.f.} = \cos \angle \{ \text{OHI} \}$$

$$= \text{HI/OH}$$

5. Starting Torque : The torque is proportional to the rotor input. At $s = 1$, rotor input is equal to rotor copper loss i.e. $I^2 R$.

∴ $T_{\text{start}} = I(\text{AE}) \times \text{Power scale} \dots \dots \dots \text{in synchronous watts}$

Full load Condition:

The full load motor output is given on the name plates in watts or h.p. Calculates the distance corresponding to the full load output using the power scale.

Then extend AD upwards from A onwards, equal to the distance corresponding to full load output, say A'. Draw parallel to the output line O'A from A' to meet the circle at point P'. This is the point corresponding to the full load condition, as shown in the Fig. 2.

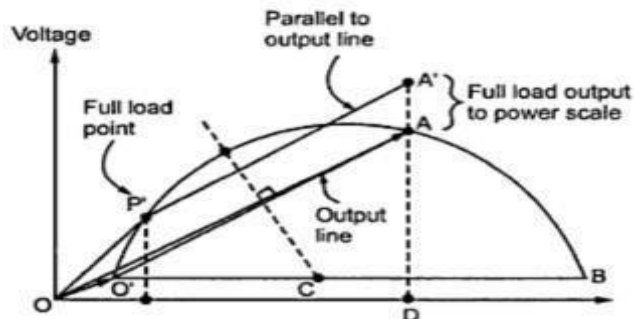


Fig. 2 Locating full load point

Once point P' is known, the other performance parameters can be obtained easily as discussed above.

Load Test on Three Phase Induction Motor:

By conducting the load test on three phase induction motor, the performance of the motor viz. slip, power factor, input, efficiency etc. at various loads can be studied.

The induction motor is loaded by any of the following methods :

1. Brake test
2. By connecting a d.c. generator

In case of loading by connecting a d.c. generator, the induction motor is connected to a d.c. generator. The generator is loaded by a lamp bank. Thus in turn an induction motor is loaded. The Fig. 1 shows the experimental set up for conducting load test on three phase induction motor using a d.c. generator.

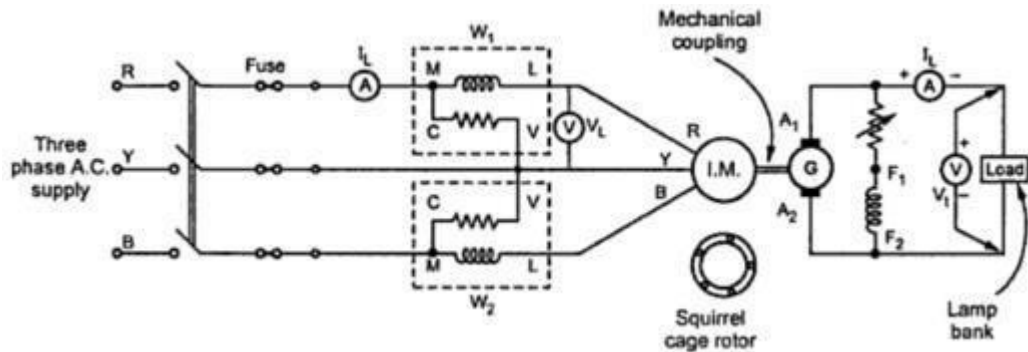


Fig. 1 Load test on three phase induction motor

On induction motor side, ammeter reads line current and voltmeter reads line voltage V_L . The two wattmeters are connected as per the two wattmeter method hence,

$$P_{in} = \text{Power input} = W_1 + W_2$$

On generator side, the ammeter reads load current and voltmeter reads terminal voltage V_t .

By varying the lamp bank, load on generator i.e. load on induction motor can be varied. The induction motor can be star or delta connected and can be squirrel cage or slip ring type. The speed readings are taken using tachometer. The load is increased till induction motor carries rated line current. The following observation table is prepared,

| Induction Motor | | | | | Generator | | |
|-----------------|------------|------------|------------|------------|------------|------------|-------------|
| Sr No. | I_L A | V_L V | W_1 W | W_2 W | I_L A | V_t V | N r.p.m. |
| 1 | | | | | | | |
| 2 | | | | | | | |
| ⋮ | | | | | | | |

Calculations : The output of induction motor is input to a d.c. generator.

$$\text{Output of d.c. generator} = V_t \times I_L \text{ W}$$

$$\text{Assume } \eta_{gen} = 80 \%$$

$$\therefore P_{\text{out of induction motor}} = P_{\text{in of d.c. generator}}$$

$$= P_{\text{out of d.c. generator}} / \eta_{\text{gen}} = (V_t I_L) / \eta_{\text{gen}} \quad W$$

$$P_{\text{in of induction motor}} = W_1 + W_2 \quad W$$

$$\cos \Phi = P_{\text{in}} / (\sqrt{3} V_L I_L) = (W_1 + W_2) / (\sqrt{3} V_L I_L) = \text{power factor}$$

$$\% \eta_{\text{motor}} = \frac{P_{\text{out of motor}}}{P_{\text{in of motor}}} \times 100$$

$$\% \text{ slip} = \frac{N_s - N}{N_s} \times 100$$

where $N_s = 120f / P$ for a given motor

For various loads above parameters are obtained.

As the load on the induction motor increases,

1. The output of motor increases.
2. The power factor increases.
3. The efficiency increase upto certain load and then decreases.
4. The speed decreases marginally.
5. The slip increases.
6. The input current increases.

The various performance characteristics can be obtained as shown in the Fig. 2.

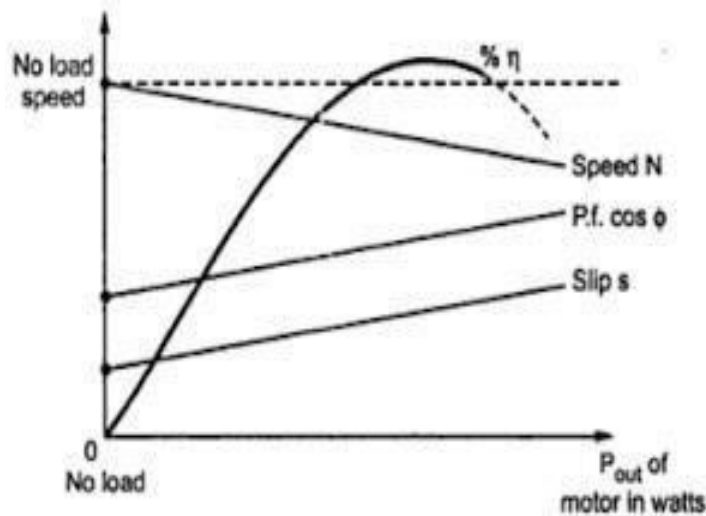


Fig. 2

The graphs indicate the behaviour of various performance parameters against output of the induction motor and not shown to the scale.

Effect of harmonics on Performance of 3-ph Induction Motor

The induction motor performance is affected by the harmonics in the time variation of the impressed voltage. But its effect on the performance of the motor is not predominant hence it is not considered here.

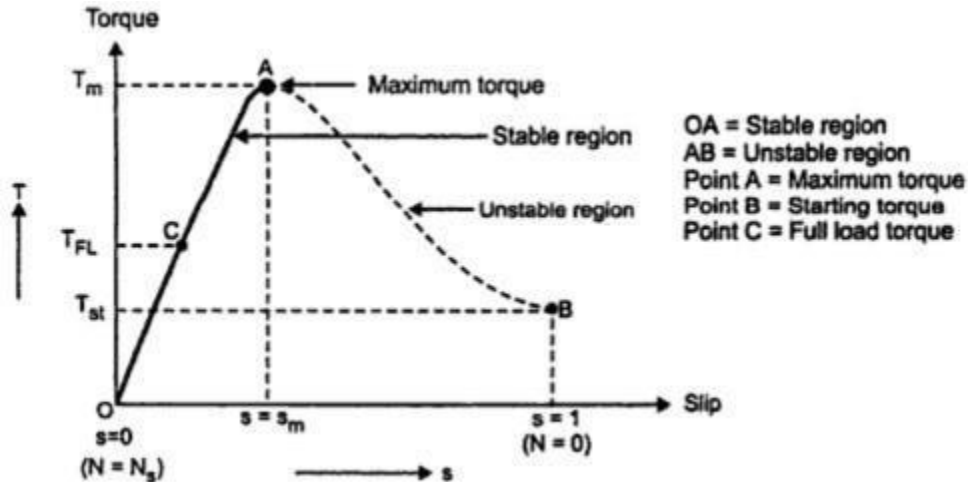


Fig. 1 Torque speed characteristics

The torque-slip characteristics as shown in Fig.1 is obtained when the space distribution of flux wave along the air gap periphery is sinusoidal. But the air gap flux is not purely sinusoidal as it contains odd harmonics (5^{th} , 7^{th} , 11^{th} etc). Hence at low speeds, the torque-slip characteristic is not smooth. The distribution of stator winding and variation of air gap reluctance due to stator and rotor slots are main causes of air gap flux harmonics.

The harmonics caused due to variation of air gap reluctance are called tooth or slot harmonics. Due to these harmonics produced in air gap flux, unwanted torque is developed along with vibration and noise.

Now even though stator currents are sinusoidal, the stator m.m.f. is not sinusoidal as stator winding has the number of slots not more than 3 to 4 per phase. If carry out analysis of stator m.m.f. with the help of Fourier series it can be seen that in addition to fundamental wave it contains odd harmonics m.m.f. waves.

The third harmonic flux waves produced by each of the three phases neutralize each other as it differs in time phase by 120° . Thus air gap flux does not contain third harmonics and its multiplies. The fundamental mmf wave produces flux which rotates at synchronous speed which given as $n_s = 2f_1/P$ rps where f_1 is supply frequency and P is number of poles. Similarly fifth harmonic mmf wave produces flux which rotates at $2f_1/5P = n_s/5$ rps and in direction opposite to the fundamental mmf wave. The seven harmonic mmf produces flux which rotates at $n_s/7$ rps and in the direction of fundamental m.m.f. wave.

Thus it can be seen that harmonic m.m.f. wave produces flux which rotates at $1/K$ times the fundamental speed and in the direction of fundamental wave if $K = 6m + 1$ and in the reversed

direction if $K = 6m - 1$ where m is any integer. The most important and predominant harmonics whose effects must be studied are 5th and 7th harmonics.

The electromagnetic torque that is developed in the induction motor is because of zero relative speed between stator and rotor fields. This fact can be explained as follows :

When rotor is revolving in the same direction of rotation as the stator field, the frequency of rotor currents is sf_1 and the rotor field produced will have speed of sn_s rpm with respect to rotor in the forward direction. But there is mechanical rotation of rotor at n rpm which is superimposed on this. The speed of rotor field in space is thus given by sum of these speeds

$$sn_s + n = sn_s + n_s(1-s) = n_s$$

The stator and rotor fields are thus stationary with respect to each other which produces a steady torque maintaining the rotation. This torque existing at any mechanical speed n other than synchronous speed is called synchronous torque.

The fifth harmonic field rotates at $n_s/5$ rps and in a direction opposite to direction of rotor. Therefore slip of rotor with respect to fifth harmonic field speed is

$$\begin{aligned} s_5 &= \frac{n_s \text{ fifth harmonic} - n_r}{n_s \text{ fifth harmonic}} \quad \text{where } n_r \text{ is rotor speed.} \\ &= \frac{-\frac{n_s}{5} - n_r}{-\frac{n_s}{5}} = \frac{-\frac{n_s}{5} - n_s(1-s)}{-\frac{n_s}{5}} = 1 + 5(1-s) = 6 - 5s \end{aligned}$$

Here $-n_s/5$ represents fifth harmonic field rotating opposite to the rotor. The frequency of rotor currents induced by fifth harmonic rotating field is

$$\begin{aligned} f_{2 \text{ fifth harmonic}} &= s_5 \times \text{Stator frequency} \\ &= (6 - 5s) \times f_1 \end{aligned}$$

Now speed of fifth harmonic rotor field with respect to rotor is given by

$$\frac{2(f_{2 \text{ fifth harmonic}})}{5p} = \frac{2}{5p} f_1 (6 - 5s) = \frac{n_s}{5} (6 - 5s)$$

Now, speed of fifth harmonic rotor field with respect to stator

$$\begin{aligned} &= \frac{\text{Speed of fifth harmonic rotor field with respect to rotor}}{\text{field with respect to rotor}} + \text{Rotor speed} \\ &= -\frac{n_s}{5} (6 - 5s) + n_r = -\frac{6}{5} n_s + sn_s + n_s (1 - s) = -\frac{n_s}{5} \text{ rps} \end{aligned}$$

Negative sign is used before $n_s/5 (6 - 5s)$ which indicates 5th harmonic field rotates opposite to rotor movement. Thus it can be seen that speed of fifth harmonic stator field and rotor field is

equal and relative speed between the two is zero. Thus it produces 5th harmonic induction motor torque similar to torque produced by fundamental component.

Similar analysis can be made on 7th harmonic to show 7th harmonic torque produced similar to fundamental one. Thus each space harmonic can be considered to produce its own asynchronous torque. The induction motor can be considered as equivalent to number of induction motors in series having poles equal to number of harmonics multiplied by number of poles. The torque produced by fundamental component and the harmonic are shown in the Fig. 2.

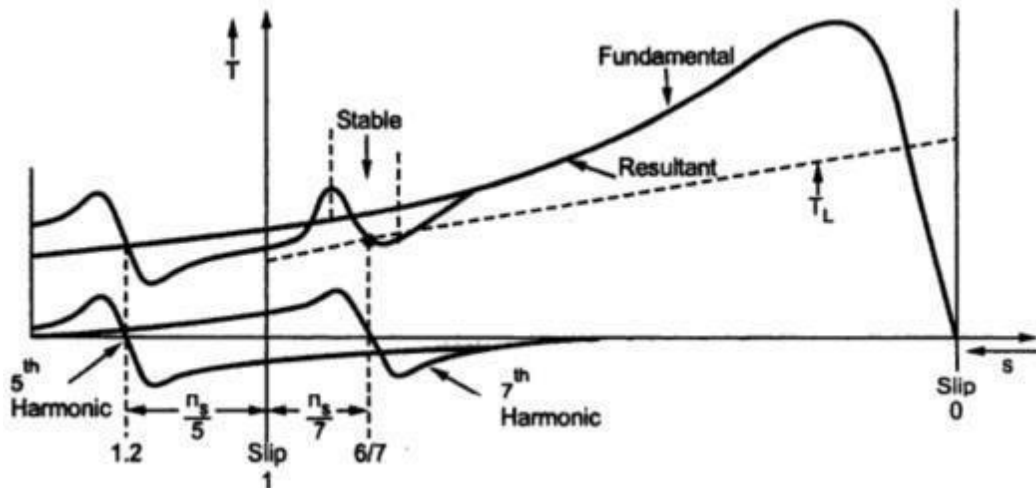


Fig. 2 Presence of harmonics

Crawling:

As fifth harmonic field rotates opposite to the rotor rotation, the torque produced by fifth harmonic opposes fundamental torque and it acts as braking torque on motor. The seventh harmonic field rotates in the direction of rotor rotation, the torque produced by seventh harmonic aids the fundamental torque. The resultant torque is shown in the Fig. 2 which shows the addition of fundamental, fifth harmonic and seventh harmonic torque. The fifth harmonic torque is zero at $-n_s/5$ rps while seventh harmonic torque is zero at $+n_s/7$.

There are two dips which can be seen in the resultant torque, one is near the slip 1.2 and other near slip 6/7. The slip near $s = 6/7$ is more important as torque here decreases with increase in speed. The load torque is shown in figure. The rotor will run at $n_s/7$ with X as the operating point. Thus stable operation is obtained near sub-synchronous speed $n_s/7$. This is called crawling or synchronous crawling. Due to crawling there is much higher stator current accompanied by noise and vibration. The torque obtained from induction motor here is called synchronous crawling.

When two harmonic fluxes of same order one because of stator and the rotor because of rotor interact with each other at one particular speed and produces harmonic synchronous torque just like that produced in synchronous motor. These torques are caused by tooth harmonics. The

stable operation at synchronous speed caused by slot harmonics is called synchronous crawling which is associated with vibration and noise.

Cogging:

A special behaviour is shown by squirrel cage induction motor during starting for certain combination of number of stator and rotor slots. If number of stator slots S_1 are equal to number of rotor slots S_2 or integral multiple of rotor slots S_2 then variation of reluctance as a function of space will have pronounced effect producing strong forces than the accelerating torque. Due to this motor fails to start. This phenomenon is called cogging. Such combination of stator and rotor slots should be avoided while designing the motor.

Let the slots of stator and rotor be 24. The stator-slotting produces its tooth harmonics of order $2S_1/P \pm 1$ whereas the rotor-slotting produces its tooth harmonics of order $2S_2/P \pm 1$ where S_1 and S_2 are number of stator and rotor slots. The plus sign refers to the harmonic field rotation in the direction of rotor.

Here $S_1 = S_2$ so stator and rotor slot harmonics are same and given by,

$$\text{Let } P = 4 \\ (2 \times 24 / 4) \pm 1 = 11 \text{ or } 23$$

The harmonics of order 11 produce backward rotating field for both stator and rotor. The harmonics of order 13 produces forward rotating field.

The two harmonics fields of same order say 11th harmonic would be stationary with respect to each other only when

$$n_r - (n_s - n_r / 11) = -n_s / 11 \\ n_r = 0$$

As the harmonic field due to 11th harmonic rotates backward with respect to stator hence negative sign is used for $n_s/11$.

Similarly, for 13th harmonic produced by stator and rotor would be stationary with respect to each other when

$$= (n_s - n_r / 13) + n_r = n_s / 13 \\ n_r = 0$$

Hence it can be seen that harmonic synchronous torque is produced at zero rotor speed. The 11th and 13th harmonic fields produced by stator and rotor are stationary with respect to each other. The harmonic synchronous torque is produced at zero rotor speed and the motor will remain at rest. This is called cogging. The torque speed characteristic with harmonic synchronous torque as $n_s/7$ is shown in the Fig.3.

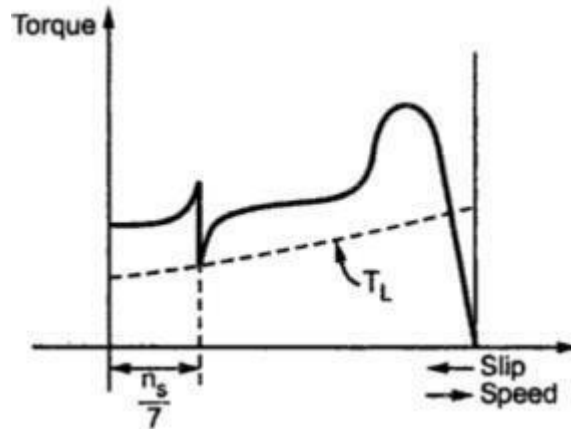


Fig. 2 Cogging

The stator slot harmonics of order $2S_1/P \pm 1$ may interact with rotor slot harmonics of order $2S_2/P \pm 1$ to develop the harmonic synchronous torques.

$$2S_1/P + 1 = 2S_2/P + 1$$

$$S_1 = S_2$$

And $2S_1/P - 1 = 2S_2/P + 1$

$$S_1 - S_2 = P$$

It can be thus seen that if $S_1 = S_2$ or $S_1 - S_2 = P$ then cogging will be definitely observed in the induction motor.

The cogging and crawling is not predominately in slip ring induction motor as these motors are started with higher starting torques with external resistance in rotor circuit.

The crawling effect can be reduced by taking proper care during the design. Still if crawling is observed then it can be overcome by applying a sudden external torque to the driven load in the direction of rotor. If there is reduction in supply voltage then torque also decreases ($T \propto V_1^2$). Hence asynchronous crawling may be observed which is absent under rated voltage conditions. Thus asynchronous torques cannot be avoided but can be reduced by proper choice of coil span and by skewing the stator or rotor slots.

Key Point : The synchronous harmonics torques can be totally eliminated by proper combination of stator and rotor slots.

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High Torque Rotors:

Special Rotor Constructions and Applications

In case of slip ring induction motor an external resistance can be added in the rotor circuit during starting which gives higher starting torque and lower starting line current at an improved power factor. This resistance is then gradually cut from the rotor circuit which would otherwise result in decrease of full load speed, poor speed regulation, more rotor losses and hence reduced efficiency. With lower rotor resistance it gives constant speed, low slip, less losses and high efficiency. This is the major advantage of slip ring induction motor that it gives high rotor resistance at starting and low rotor resistance at normal operating speed.

In case of squirrel cage induction motor there is no provision made for adding external resistance. If the resistance is designed in such a way that it gives better running performance then it has high starting current and consequently low starting torque. This is major disadvantage of squirrel cage induction motor although it is having the other qualities of low cost, ruggedness and maintenance free operation. Thus the designer had found different ways of improving the starting performance of the motor without affecting the running performance of the motor.

In squirrel cage induction motor high starting torque can be obtained by the use of deep bar or double cage rotors. Both these types of rotors make use of skin effect in which distribution of current is not uniform but the alternating current has the tendency to concentrate near the surface of the conductor. Due to this effect, effective area of cross section of the conductor is reduced and hence resistance of the conductor is increased when carrying alternating current.

The solid conductor can be considered to be made up of large number of strands each carrying a small part of current. The inductance of each strand will vary according to the

position. The strands in proximity of the centre are surrounded by greater magnetic flux and has greater inductance than near the surface. Due to high reactance at the centre, the alternating current flows near the surface of the conductor. The skin effect depends upon nature of material, diameter of wire, shape of wire and frequency.

Thus the current in the rotor during starting is having the frequency of supply. While under running condition the frequency of rotor current reduces to slip frequency. This variation in frequency changes the rotor resistance as it depends on skin effect. During starting it gives high resistance whereas it gives low resistance during running condition which is desirable. Thus the variation in rotor resistance can be achieved by deep bar or double cage construction of rotor and induction motor. Both these types of construction make use of skin effect phenomenon.

Deep Bar Rotor Construction:

There is no constructional difference between stator of deep bar motor and that of ordinary induction motor. The rotor consists of deep bars, short circuited by two end rings one on each side. The deep and narrow rotor bar of rectangular cross section is shown in the Fig. 1(a). The other rotor bar shapes are shown in the Fig. 1(b). The magnetic leakage flux lines are shown by dotted lines, Now consider that the bar consists of many number of layers of different depths. The top and bottom layers are shown in the Fig. 1.

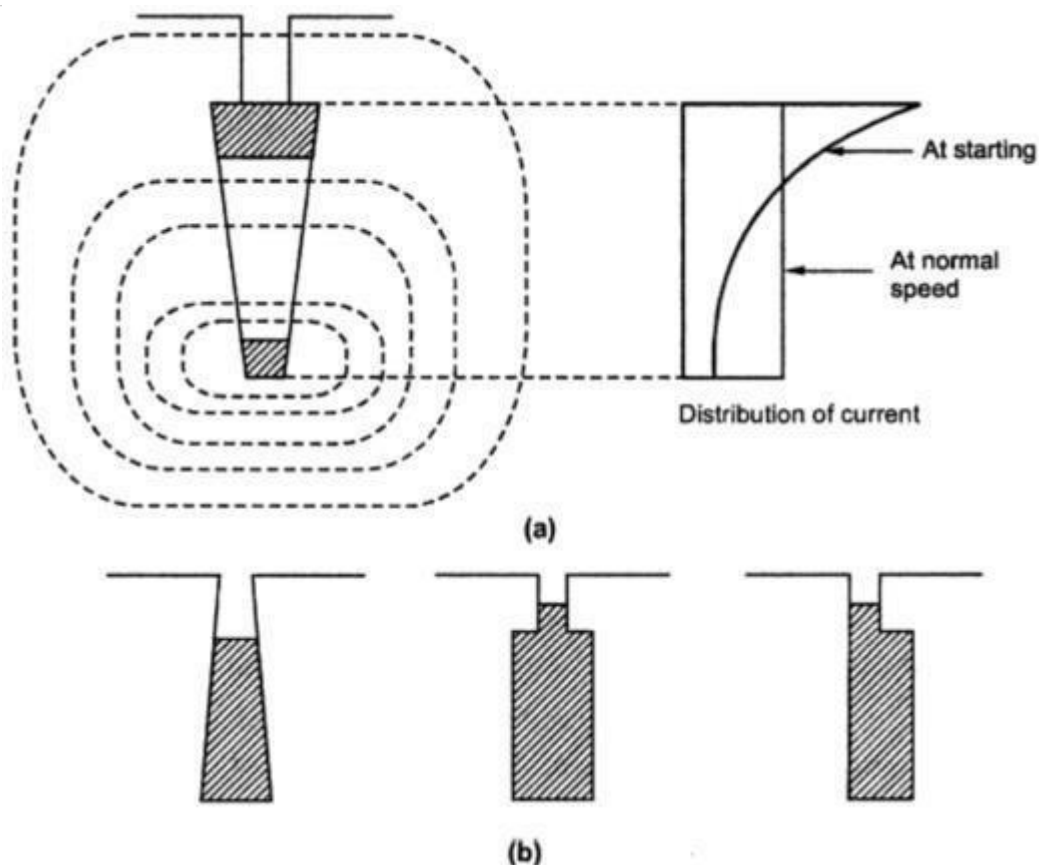


Fig. 1 Deep bar rotor

The leakage inductance of the bottom strips is greater than that of top strips as more flux links with bottom strip compared to top strip. All the strips are in parallel electrically. The bottom strip has greater leakage inductance than the top strip. During starting the rotor frequency is equal to the stator frequency and hence leakage reactance of bottom strip is largest and current in it is least. The top strip has low leakage reactance and current in it is large. Thus the current in low reactance top strip will be greater than that in high reactance lower strip and the current will be forced towards the top of the slot and phase of current in upper strip will lead that of the current in lower one. Thus there is non-uniform distribution of current which is shown in the Fig. 1. Due to this non-uniform distribution of current, and use to skin effect, effective area of cross section decreases. Hence rotor resistance increases resulting in high starting torque.

As leakage reactance is proportional to frequency, the non-uniform distribution of current depends upon the rotor frequency. The Fig. 2 shows a curve indicating a.c. effective resistance to d.c. resistance with change in frequency for a copper bar of 2.5 cm deep. The skin effect is maximum when rotor is at standstill.

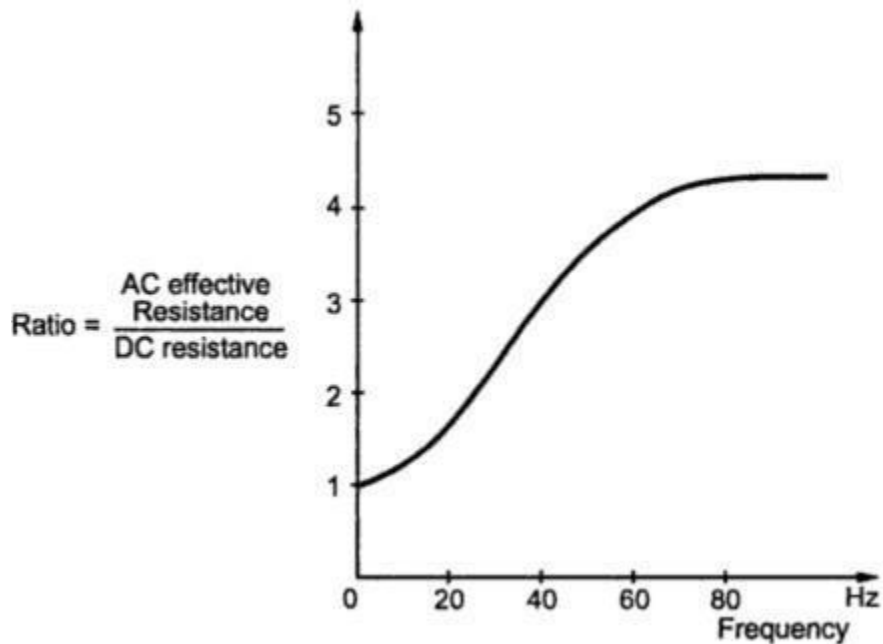


Fig. 2

With the increase in rotor speed, the rotor frequency decreases and skin effect also decreases. The reactances of different strips at this low frequency become almost equal and the current density over the conductor cross section becomes uniform so its a.c. resistance is equal to d.c. resistance. Thus with deep bar rotor has a low starting current with high starting torque without affecting running performance of motor. The net reactance of deep bar rotor at standstill is higher than that in a normal bar design, the breakdown or pull out torque in deep bar rotor is lower. The torque-slip characteristics of deep bar motor and normal induction motor is shown in the Fig.3.

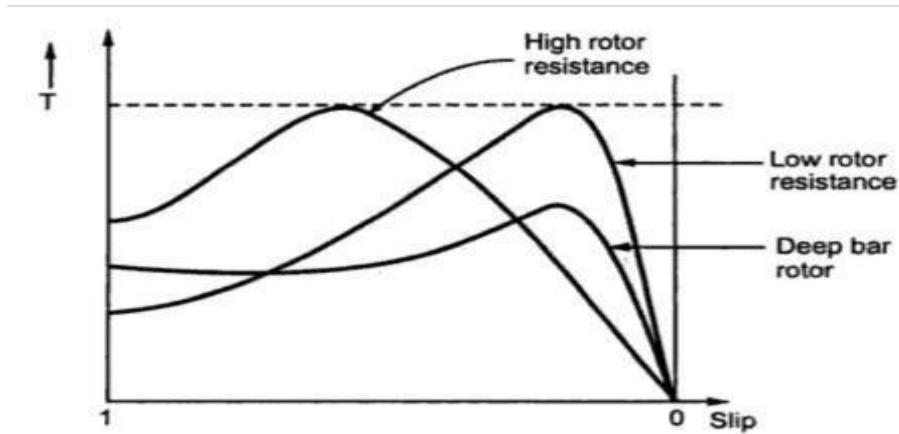


Fig. 3 Torque slip characteristics of deep bar rotor

The equivalent circuit of induction motor is applicable to deep bar rotor also wherein proper value of r_2' and x_2' must be determined for satisfactory running performance. During starting their values should correspond to effective value at stator frequency. During running their values should correspond to their effective values at low rotor frequency.

Double Cage Rotor Construction:

This is another way of obtaining improved starting performance without affecting its running performance. Though it is more expensive it gives better performance than deep bar rotor construction.

The stator of double cage rotor induction motor is same as that of ordinary induction motor whereas its rotor consists of two cages or two layers of bars short circuited by end rings since the upper cage is having smaller cross-sectional area than the lower cage, the upper cage is having higher resistance than that of lower cage. With equal cross sectional areas of two cages the upper cage is made up of high resistance material like brass, aluminium, bronze etc. and the lower cage is made up of low resistance material like copper. The upper cage and lower cage are separated by a narrow slit or constriction. This is shown in the Fig. 4.

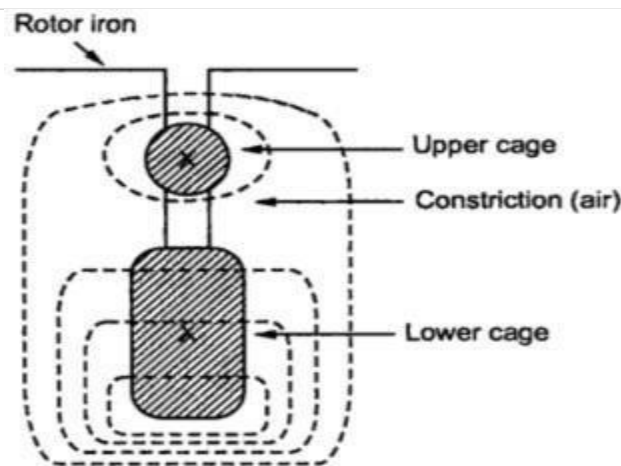


Fig. 4 Double cage rotor construction

The slot leakage flux pattern is also shown in the Fig. 4 for the double cage rotor. Similar to deep bar rotor construction the rotor bars in the upper cage have less leakage flux linkage and therefore has lower reactance. The dimension of air construction controls the self leakage flux linking upper and lower bars. If air constriction would have been absent then the main flux would return via iron path between the two slots and thereby missing the bars in the lower cages which will not contribute to production of torque in that case. Hence it can be seen that the upper cage has high resistance and low reactance whereas the lower cage has low resistance and high reactance.

During starting the rotor frequency is same as stator frequency or supply frequency. The division of rotor current in upper and lower cage is inversely proportional to their leakage impedances. At the time of starting the leakage reactance of lower cage is very high and consequently its leakage impedance is several times greater than that of upper cage whose leakage reactance is small. Hence most rotor current flows in upper cage having lower leakage impedance. The upper cage having high resistance sharing the rotor current results in low starting current at improved power factor giving high starting torque.

When rotor speeds up, the rotor frequency decreases which decreases the leakage reactance of lower cage. At normal operating speed the reactance difference between the two cages is negligibly small. Hence the division of rotor current in this case is mainly decided by the resistances of the two cages. As resistance of upper cage is very high most of the current flows through the lower cage giving excellent operating characteristics under running condition. It can be noted that starting current is confined mainly with upper cage so if there is frequent starting of motor then it would cause overheating and burning of upper cage.

The torque-slip characteristics of double cage induction motor are shown in the Fig. 5.

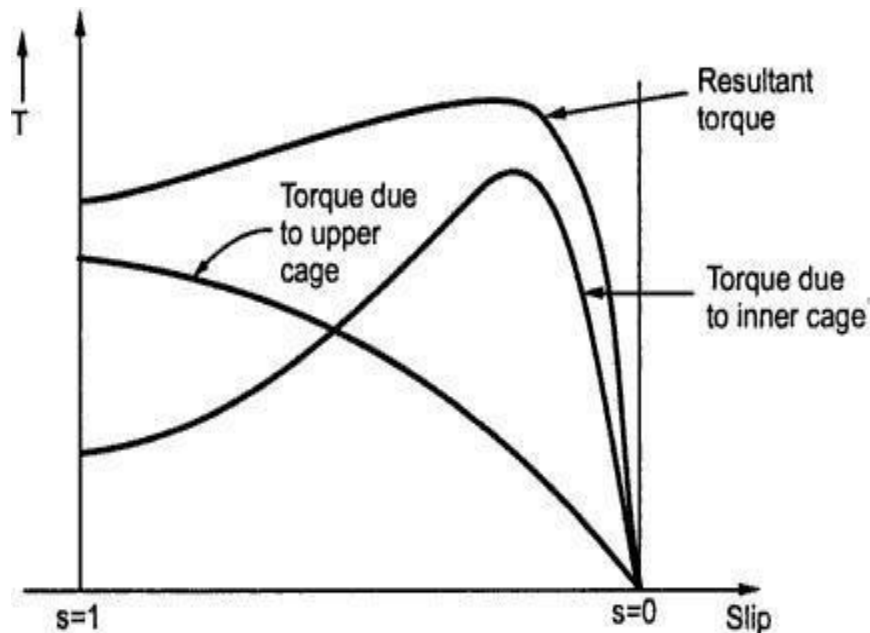


Fig. 5 Torque slip characteristics of double cage induction motor

Another type of double cage rotor construction is also possible which is shown in the Fig. 6. The slot-leakage flux pattern for this type of construction is also shown.

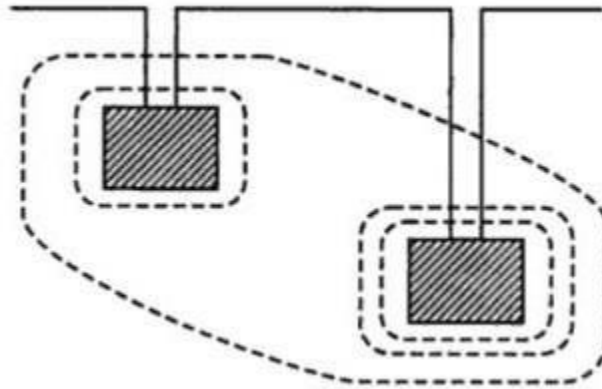


Fig. 6

The approximate equivalent circuit of double cage rotor induction motor is shown in the Fig. 7. Though the two cages are somewhat coupled magnetically, they can be treated as independent for simplicity and it gives approximately same results. The two cages are assumed to be parallel while drawing the equivalent circuit.

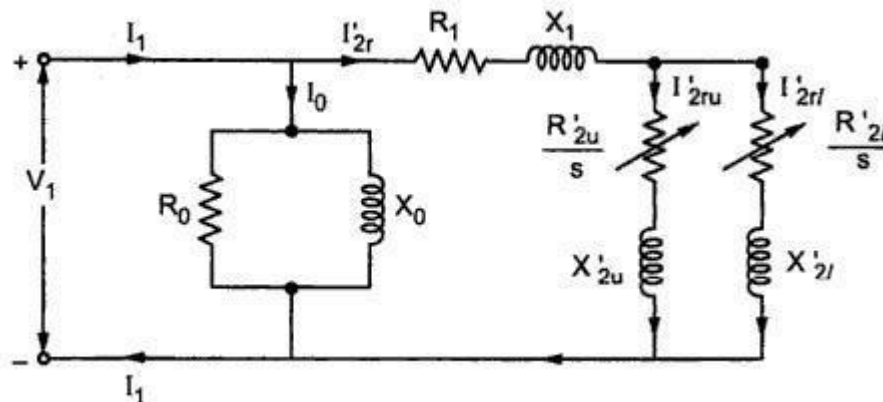


Fig. 8 Equivalent circuit of double cage induction motor

I'_{2ru} and I'_{2rl} are the currents in the upper and lower cages respectively referred to the stator. R'_{2u} and R'_{2l} are the resistance of upper and lower cages referred to the stator whereas X'_{2u} and X'_{2l} are the leakage reactances of the two cages referred to the stator of the motor.

Comparison of Single Cage and Double Cage Motors

| | Single cage | Double cage |
|---|---|---|
| 1 | Starting current is high hence not suitable for direct on line starting. | Starting current is low hence suitable for direct on line starting. |
| 2 | Starting torque is low. | Starting torque is high. |
| 3 | Effective rotor resistance is low hence at start rotor heating is not severe. | Effective rotor resistance is high hence at start rotor heating is large. |
| 4 | As rotor resistance is low rotor copper losses are less and efficiency is more. | The rotor copper losses are high due to high rotor resistance and efficiency is less. |
| 5 | The breakdown torque or maximum torque is more. | The breakdown torque or maximum torque is smaller as two cages produce maximum torques at different speeds. |
| 6 | The leakage reactance is low. | The effective leakage reactance is high. |
| 7 | The power factor is high. | The power factor is low. |
| 8 | The torque-slip characteristics are fixed and constant. | With proper choice of resistances and reactances of inner and outer cages, wide range of torque-slip characteristics can be obtained. |
| 9 | For same rating, cost is low. | For same rating, cost is high due to double cages. |

Application

- i) Squirrel cage type of motors having moderate starting torque and constant speed characteristics preferred for driving fans, blowers, water pumps, grinders, lathe machines, printing machines, drilling machines.
- ii) Slip ring induction motors can have high starting torque as high as maximum torque. Hence they are preferred for lifts, hoists, elevators, cranes, compressor.

Induction Generator :

The torque-slip or the torque-speed characteristics of the induction motor are shown in the Fig. 1. The operating mode of induction machine as a generator or motor or braking depends on value of slip s .

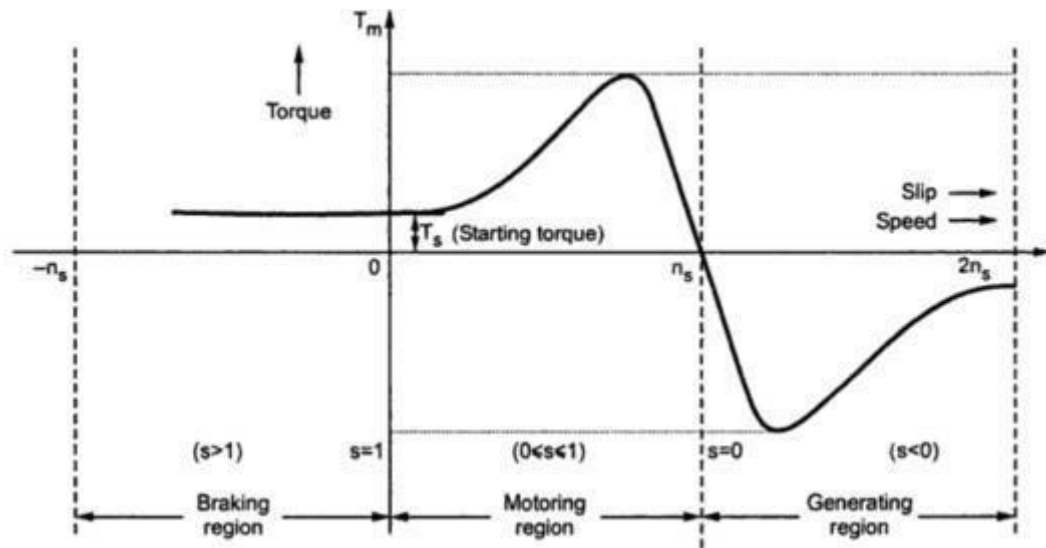


Fig. 1

When the slip lies in the region 0 and 1 i.e. when $0 \leq s \leq 1$, the machine runs as a motor which is the normal operation. The rotation of rotor is in the direction of rotating field which is developed by stator currents. In this region it takes electrical power from supply lines and supplies mechanical power output. The rotor speed and corresponding torque are in same direction.

When the slip is greater than 1, the machine works in braking mode. The motor is rotated in opposite direction to that of rotating field. In practice two of the stator terminals are interchanged which changes the phase sequence which in turn reverses the direction of rotation of magnetic field. The motor comes to quick stop under the influence of counter torque which produces braking action. This method by which the motor comes to rest is known as plugging. Only care is taken that the stator must be disconnected from the supply to avoid the rotor in other direction.

To run the induction machines as a generator, its slip must be less than zero i.e. negative. The negative slip indicates that the rotor is running at a speed above the synchronous speed. When running as a generator it takes mechanical energy and supplies electrical energy from the stator. As the speed of induction generator is not in synchronism with the line frequency, it is often called asynchronous generator.

Thus when the slip of the induction motor is negative i.e. when the induction motor runs faster than synchronous speed, the induction motor runs as a generator called induction generator.

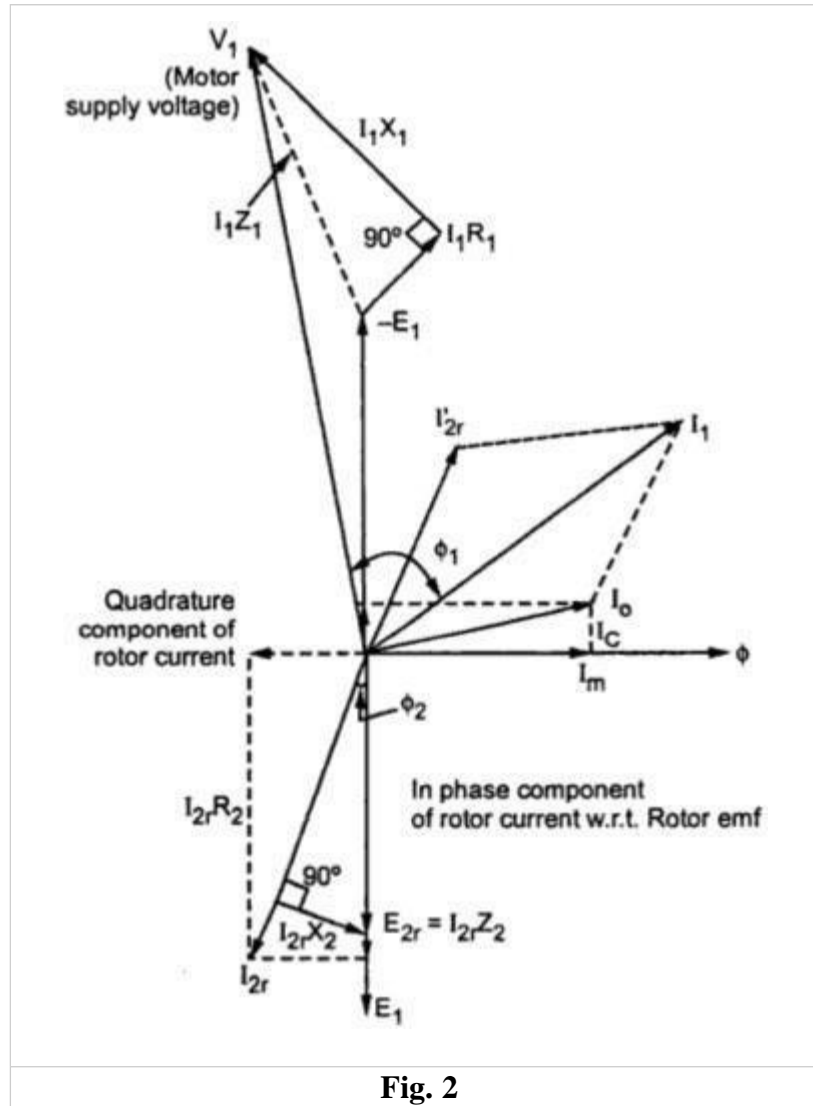
The stator of induction generator must be connected to a voltage source to produce the necessary rotating magnetic field revolving at synchronous speed. When rotor is rotated above

synchronous speed and the stator is disconnected from the supply, the generating action will not take place.

When rotor of induction machine is driven above synchronous speed, the rotor conductors cut the flux of rotating field in opposite direction to that when it is operating as a motor. The rotor currents are also reversed. Due to the transformer action, currents are induced in the stator and the induction motor can be runs as a generator.

The construction of induction generator is same as that of motor with the difference that the direction of rotation of the motor and a generator is opposite for the same current direction.

The action of induction machine as a generator can be explained from the phasor diagram.



Consider the phasor diagram of the induction motor on load.

Let us consider the speed of the induction machine is less than synchronous speed so that machine takes current I_1 from supply. This current I_1 is phasor sum of no load current I_0 and I_{2r}' which is opposite of I_{2r} and referred as reflected rotor current in stator. The rotor current can

be resolved into two components, one in phase with rotor emf and the other one is quadrature component.

The rotor current I_{2r} is given by,

$$I_{2r} = \frac{sE_2}{R_2 + jsX_2} = \frac{E_2}{\frac{R_2}{s} + jX_2}$$

Rationalizing the denominator we get,

$$\begin{aligned} I_{2r} &= \frac{E_2 \left(\frac{R_2}{s} - jX_2 \right)}{\left(\frac{R_2}{s} + jX_2 \right) \left(\frac{R_2}{s} - jX_2 \right)} = \frac{E_2 \left(\frac{R_2}{s} \right)}{\frac{R_2^2}{s^2} + X_2^2} - j \frac{E_2 X_2}{\frac{R_2^2}{s^2} + X_2^2} \\ &= \frac{sER_2}{R_2^2 + s^2 X_2^2} - j \frac{E_2 s^2 X_2}{R_2^2 + s^2 X_2^2} \end{aligned}$$

Let the real part of above current be denoted by A while the imaginary part of the current be denoted by B. Thus the total rotor current I_{2r} be assumed as $A - jB$.

Now let the speed of the induction machine is increased. With increase in speed of prime mover i.e. of rotor of induction motor slip goes on reducing and hence the rotor current also as it depends on it. Thus I_{2r} decreases. At synchronous speed, it completely vanishes. Hence its opposite current I_{2r}' also vanished and the resultant stator current is nothing but the no load current. The core losses are supplied from line whereas friction and windage losses are supplied mechanically.

When the speed is increased further the machines enters in generating region. At zero power factor no power is interchanged between machine and supply lines, But the machine generates power to meet its core losses. When the speed is increased, the current I_{2r} increases in magnitude but it changes the phase. The current supplied by the generator will be then vector sum of I_o and I_{2r}' which is reversed in phase as indicated in the phasor diagram.

The rotor current is now given by

$$I_{2r} = \frac{E_2 R_2 (-s)}{R_2^2 + s^2 X_2^2} - j \frac{X_2 (-s)^2 E_2}{R_2^2 + s^2 X_2^2}$$

$$\therefore I_{2r} = -A - jB$$

It can be seen that the in phase component reverses while the quadrature component remains in the same direction.

The phasor diagram of induction machine as generator is shown in Fig. 2.

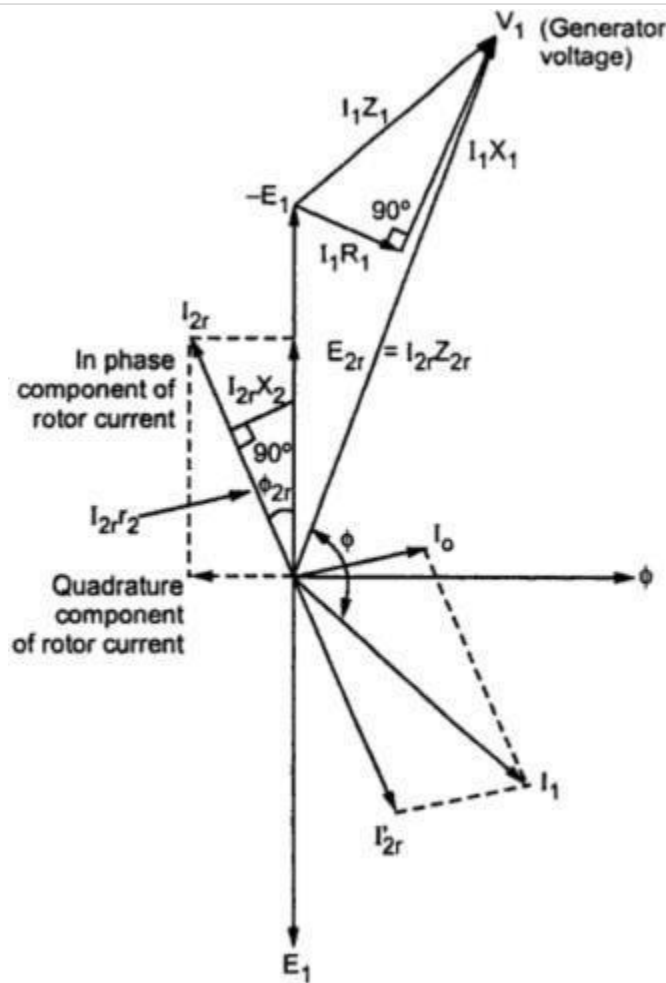


Fig. 2

The current I_{2r} leads the voltage $-E_{2r}$ which is opposite of E_{2r} . The angle between V_1 and I_1 is more than which shows that electrical power of the machine is negative i.e. it is supplying the power. Thus when the rotor is rotated above synchronous speed with the rotating field remaining in the same direction, then the direction of cutting of rotor is in opposite direction which results in reversal of rotor emf, current and torque. The machine is said to be operating in generating mode.

The induction generator is not self excited as it cannot generate its own exciting current. Thus it must be always connected to an a.c. supply. Generally it is operated in parallel with synchronous machines. It is shown in the Fig.3.

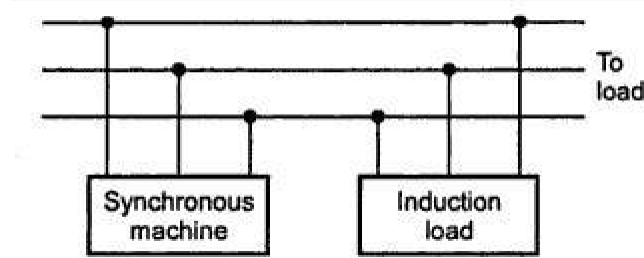


Fig. 3

Consider an example of a load which requires a lagging current which can not be supplied by induction generator alone as it supplies leading current.

But this current requirement is fulfilled with the help of synchronous generators operating in parallel with induction generator. Consider the following phasor diagram.

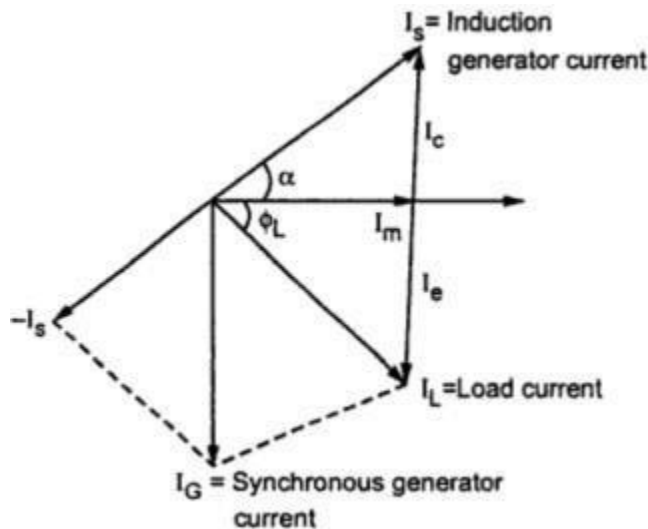


Fig. 4

The load current I_L can be resolved into two components one in phase component I_m and the other quadrature component I_e . The speed of the induction generator is adjusted in such a way that it supplies current I_c which is leading one. The induction generator current I_s is nothing but vector sum of I_c and I_m .

The synchronous generator which is in parallel with the induction generator must supply the remaining part of load current. For this the induction generator current I_s is subtracted vectorially from I_L (subtracting vectorially means reversing I_s and adding it with I_L). This current is nothing but algebraic sum of currents I_c and I_e . The synchronous generator supplies no power. The total current supplied by synchronous generator is lagging quadrature current.

If the load requires a leading current then theoretically the quadrature component of current can be supplied entirely by the induction generator. But for satisfactory operation it should be run in parallel with synchronous generator.

If the bank of delta connected capacitors is operated in parallel with induction generator then the reactive power requirement of induction generator is met by capacitors. This arrangement is shown in Fig. 5.

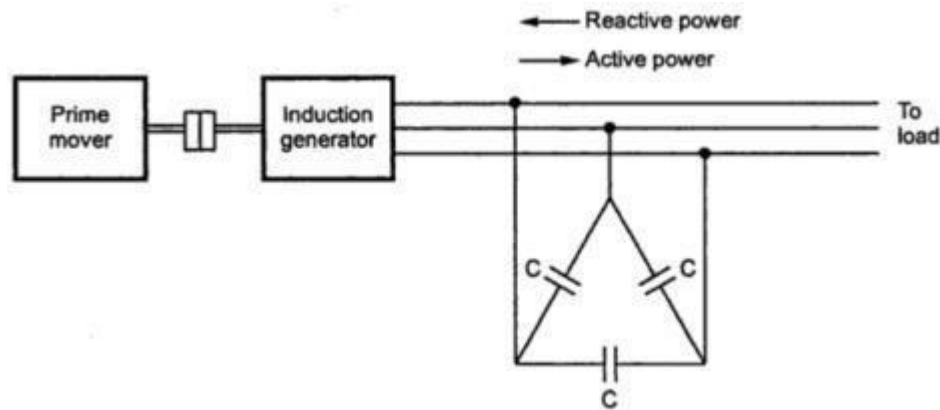


Fig. 5

The induction generator in this case is said to be isolated induction generator supplying a load. The external voltage source is not required in this case.

Unlike in synchronous generators, induction generators are not rotating at a definite speed at a given frequency. The speed varies with load as the load is proportional to slip. The frequency of the induction generator is same as the frequency of the line to which it is connected.

Circle Diagram of Induction Generator

Using circle diagram, the induction generators can also be analyzed.

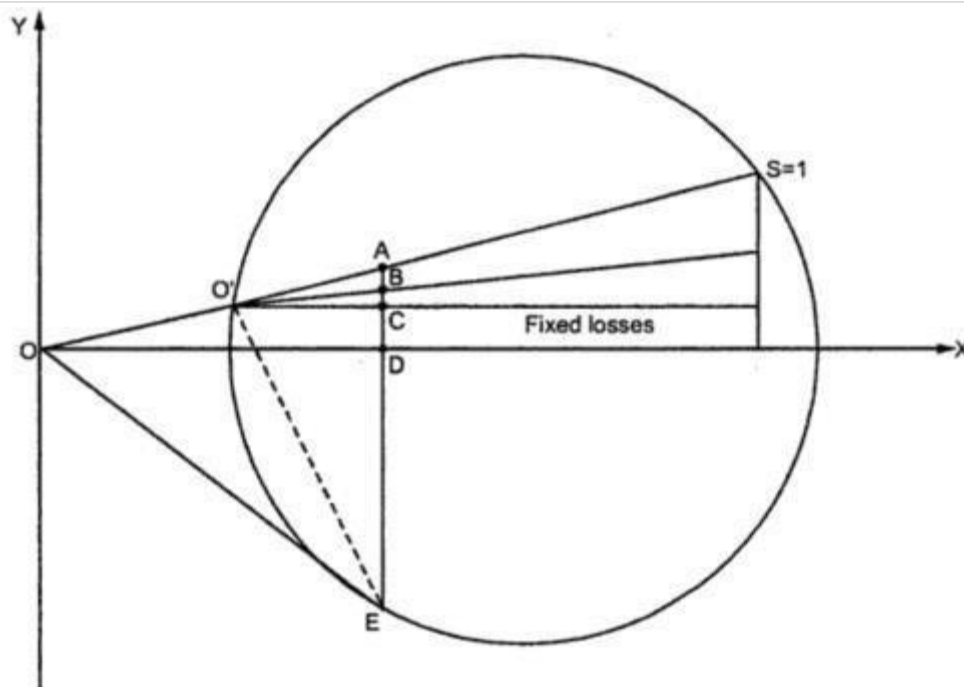


Fig. 1

As the antiphase component of current reverses, direction of current also changes. It will be below horizontal shown by OE. E is the operating point. As seen from circle diagram.

AB = Rotor Cu loss.

BC = Stator Cu loss

CD = Constant losses.

DE = Generator output.

BE = Rotor input.

The slip is given by,

$$s = \frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{AB}{BE}$$

Similarly other required quantities can be obtained from the circle diagram.

Comparison of Induction Generator and Synchronous Generator:

The distinct features of induction generator compared to synchronous generators are as follows :

- i) It will not require d.c. excitation.
- ii) It is not self excited but external a.c. supply of fixed frequency is required.
- iii) The frequency of induction generator is decided by the frequency of the excitation voltage which is supplying current to it.
- iv) Synchronization of generator is not required as no emf is generated until it is connected to the line.

Advantages:

The following are the advantages of induction generator.

- i) Synchronization for induction generator is required.
- ii) The construction is rugged for rotating parts.
- iii) Unlike in synchronous machine, there is no danger of hunting or drop out of synchronism for induction generators.
- iv) When it short circuited, it delivers small power as the excitation quickly reduces to zero.
- v) Induction generators are more suitable for high speeds.
- vi) With the help of excitation supply and frequency, the voltage and frequency of induction generator are controlled.

Disadvantages:

Although induction generators are having above mentioned advantages, it has following advantages.

- i) It must be run in parallel with the synchronous machine.
- ii) The load is not deciding the power factor of induction generator but the power factor depends on slip.s

Applications:

Because of distinct superiority of the synchronous generator, induction generators are rarely used to supply commercial power.

One application of induction generator is in railway for braking purposes. When the train is moving down a gradient, the induction generators runs above synchronism. As the torque in this region is negative, the braking action is achieved in the train. In addition to this the energy generated by induction generator is given to the line so that the load on main generating station is somewhat relieved. In this case no complicated control apparatus is required.

Importance of Induction Generators in Wind Mill:

The induction generator is extremely important in wind power electricity generation system. It is suitable because the stator frequency depends on that of the paralleled synchronous machines and not on the rotor speed.

Induction generator is most commonly used in wind turbines because of low cost, ruggedness, operates with slip (Synchronous not required), availability in many sizes and advance technology available.

Induction generators have outstanding operation as either motor or generator. They have robust construction features. It provides natural protection against short circuits. The abrupt changes in speed are easily absorbed by its solid rotor. Also any surge in the current is damped by the magnetization path of the core, avoiding the possibility of demagnetization which is possible incase of permanent magnet generators.

