# Module-2
# Three Phase Induction Motors

Review Portion Begins….

## Three Phase Induction Motor:

### Introduction

An electric motor is a device which converts an electrical energy into a mechanical energy. The motors operating on a.c. supply are called a.c. motor. As a.c. supply is commonly available, the a.c. motors are very popularly used in practice. The a.c. motors are classified as three phase induction motors, single phase induction motor, universal motors, synchronous motors etc. The three phase induction motors are widely used for various industrial application. The important features of three phase induction motors are self starting, higher power factor, good speed regulation and robust construction. This chapter explains the construction, working principle and characteristics of three phase induction motors as well as universal motors. The working of three phase induction motors is based on the principle of rotating magnetic field. Let us discuss, the production of rotating magnetic field.

### Rotating Magnetic field (R.M.F.)

The rotating magnetic field can be defined as the field or flux having constant amplitude but whose axis is continuously rotating in a plane with a certain speed. So if the arrangement is made to rotate a permanent magnet, then the resulting field is a rotating magnetic field. But in this method, it is necessary to rotate a magnet physically to produce rotating magnetic field.

But in three phase induction motors such a rotating magnetic field is produced by supplying currents to a set of stationary windings, with the help of three phase a.c. supply. The current carrying windings produce the magnetic field or flux. And due to interaction of three phase fluxes produced due to three phase supply, resultant flux has a constant magnitude and its axis rotating in space, without physically rotating the windings. This type of field is nothing but rotating magnetic field. Let us study how it happens.

### Production of R.M.F.

A three phase induction motor consists of three phase winding as its stationary part called stator. The three phase stator winding is connected in star or delta. The three phase windings are displaced from each other by $120^o$. The windings are supplied by a balanced three phase a.c. supply. This is shown in the Fig. 1. The three phase windings are denoted as R-R' , Y-Y' and B-B'.

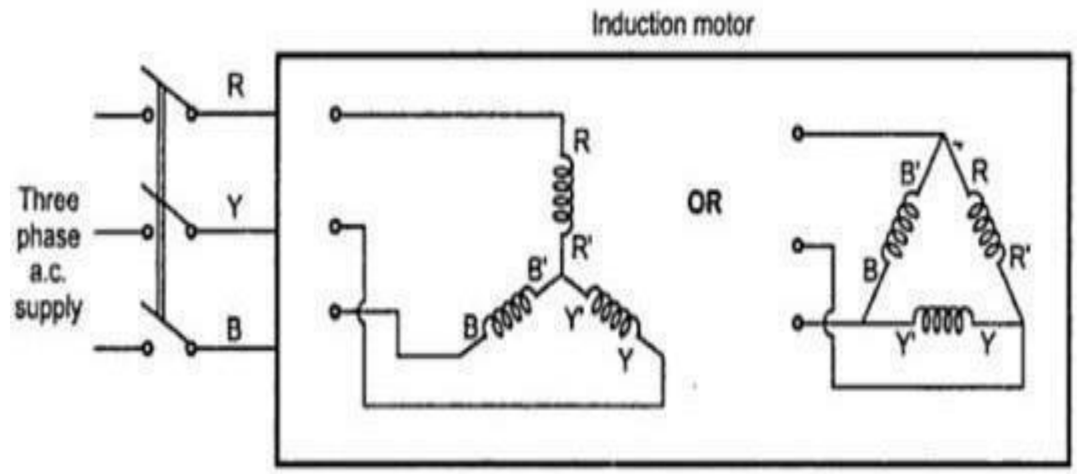


**Fig. 1 Star or delta connected 3phase winding**

The three phase currents flow simultaneously through the windings and are displaced from each other by $120^o$ electrical. Each alternating phase current produces its own flux which is sinusoidal. So all three fluxes are sinusoidal and are separated from each other by $120^o$. If the phase sequence of the windings is R-Y-B, then mathematical equations for the instantaneous values of the three fluxes $\Phi_R$ , $\Phi_Y$ and $\Phi_B$ can be written as,

$\Phi_R = \Phi_m \sin(\omega t) = \Phi_m \sin\theta$................................................(1)
$\Phi_Y = \sin(\omega t - 120^o) = \Phi_m \sin(\theta - 120^o)$...........................(2)
$\Phi_B = \Phi_m \sin(\omega t - 240^o) = \Phi_m \sin(\theta - 240^o)$ ..................... (3)

As winding are identical and supply is balanced, the magnitude of each flux is $\Phi_m$. Due to phase sequence R-Y-B, flux lags behind $\Phi_R$ by $120^o$ and $\Phi_B$ lags $\Phi_Y$ by $120^o$. So $\Phi_B$ ultimately lags $\Phi_R$ by $240^o$. The flux $\Phi_R$ is taken as reference while writing the equations.

The Fig. 2(a) shows the waveforms of three fluxes in space. The Fig.2(b) shows the phasor diagram which clearly shows the assumed positive directions of each flux. Assumed positive direction means whenever the flux is positive it must be represented along the direction shown and whenever the flux is negative it must be represented along the opposite direction to the assumed positive direction.

Let $\Phi_R$, $\Phi_Y$ and $\Phi_B$ be the instantaneous values of the three fluxes. The resultant flux $\Phi_T$ is the phasor addition of $\Phi_R$, $\Phi_Y$ and $\Phi_B$.

$$\therefore \quad \bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_Y + \bar{\phi}_B$$

Let us find $\Phi_T$ at the instants 1, 2, 3 and 4 as shown in the Fig. 2(a) which represents the values of $\theta$ as $0^o$, $60^o$, $120^o$ and $180^o$ respectively. The phasor addition can be performed by obtaining the values of $\Phi_R$, $\Phi_Y$ and $\Phi_B$ by substituting values of $\theta$ in the equation (1), (2) and (3).

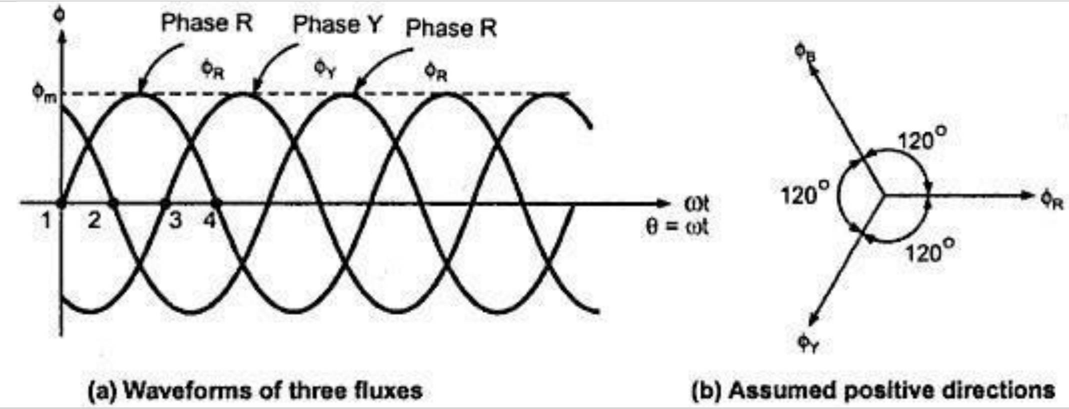


**Fig. 2**

**Case 1** : $\theta = 0^o$

Substituting in the equations (1), (2) and (3) we get,

$\Phi_R = \Phi_m \sin 0^o = 0$

$\Phi_Y = \Phi_m \sin(-120^o) = -0.866\ \Phi_m$

$\Phi_B = \Phi_m \sin(-240^o) = +\ 0.866\ \Phi_m$

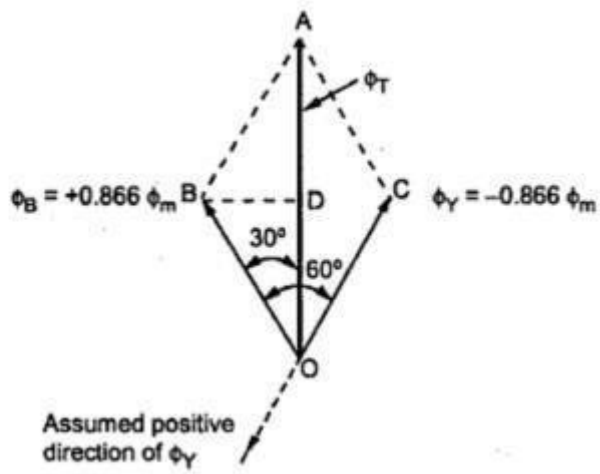


**Fig. 3(a) Vector diagram of θ = 0°**

The phasor addition is shown in the Fig. 3(a). The positive values are shown in assumed positive directions while negative values are shown in opposite direction to the assumed positive directions of the respective fluxes. Refer to assumed positive directions shown in the Fig 3(b). BD is drawn perpendicular from B on $\Phi_T$. It bisects $\Phi_T$.

∴ $OD = DA = \Phi_T/2$

In triangle $\angle OBD = 30^o$

∴ $\cos 30^o = OD/OB = (\Phi_T/2)/(0.866\ \Phi_m)$

∴ $\Phi_T = 2 \times 0.866\ \Phi_m \times \cos 30^o$

$= 1.5\ \Phi_m$

So magnitude of $\Phi_T$ is $1.5\ \Phi_m$ and its position is vertically upwards at $\theta = 0^o$.

**Case 2** $\theta = 60^o$

Equation (1),(2) and (3) give us,

$$\Phi_R = \Phi_m \sin 60^o = +0.866\ \Phi_m$$
$$\Phi_Y = \Phi_m \sin (-60^o) = -0866\ \Phi_m$$
$$\Phi_B = \Phi_m \sin (-180^o) = 0$$

So $\Phi_R$ is positive and $\Phi_Y$ is negative and hence drawing in appropriate directions we get phasor diagram as shown in the Fig. 3(b).

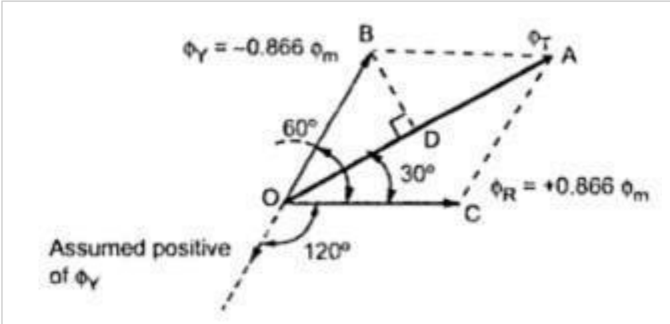


**Fig 3(b) Vector diagram of θ = 60º**

Doing the same construction, drawing perpendicular from B on at D we get the same result as,

$$\Phi_T = 1.5\ \Phi_m$$

But it can be seen that though its magnitude is 1.5 Φm it has rotated through 60ºin space, in clockwise direction, from its previous position.

**Case 3 :** $\theta = 120^o$

Equations (1),(2) and (3) give us,

$$\Phi_R = \Phi m \sin 120^o = +0.866\ \Phi m$$
$$\Phi_Y = \Phi m \sin 0^o = 0$$
$$\Phi_B = \Phi m \sin (-120^o) = -0.866\ \Phi m$$

So $\Phi_R$ is positive and $\Phi_B$ is negative. showing $\Phi_R$ and $\Phi_B$ in the appropriate directions, we get the phasor diagram as shown in the Fig . 3(c).

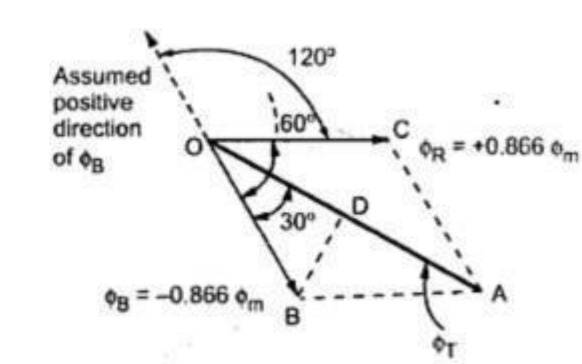


**Fig. 3(c) Vector diagram of θ = 120º**

After doing the construction same as before i.e. drawing perpendicular from B on $\Phi_T$, it can be provided again that,

$$\Phi_T = 1.5\ \Phi m$$

But the position of $\Phi_T$ is such that it has rotated further through $60^o$ from its previous position, in clockwise direction. And from its position at $\theta = 0^o$, it has rotated through $120^o$ in space, in clockwise direction.

**Case 4** : $\theta = 180^o$

From equations (1),(2) and (3),

$$\Phi_R = \Phi m \sin(180^o) = 0$$
$$\Phi_Y = \Phi m \sin(60^o) = +0.866\ \Phi m$$
$$\Phi_B = \Phi m \sin(-60^o)$$
$$= -0.866\ \Phi m$$

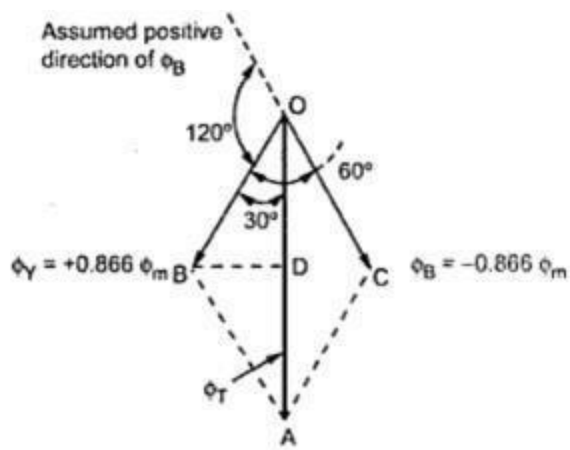


**Fig. 3(d) Vector diagram of θ = 180°**

So $\Phi_R = 0$ , $\Phi_Y$ is positive and $\Phi_B$ is negative. Drawing $\Phi_Y$ and $\Phi_B$ in the appropriate directions, we get the phasor diagram as shown in the Fig. 3(d).

From phasor diagram, it can be easily proved that,

$$\Phi_T = 1.5\ \Phi m$$

Thus the magnitude of $\Phi_T$ once again remains same. But it can be seen that it has further rotated through $60^o$ from its previous position in clockwise direction.

So for an electrical half cycle of $180^o$, the resultant $\Phi_T$ has also rotated through . This is applicable for the windings from the above discussion we have following conclusions :

a) The resultant of the three alternating fluxes, separated from each other by , has a constant amplitude of 1.5 Φm where Φm is maximum amplitude of an individual flux due to any phase.

b) The resultant always keeps on rotating with a certain speed in space.

**Key point** : This shows that when a three phase stationary windings are excited by balanced three phase a.c. supply then the resulting field produced is rotating magnetic field. Though nothing is physically rotating, the field produced is rotating in space having constant amplitude.

**Speed of R.M.F.**

There exists a fixed relation between frequency f of a.c. supply to the windings, the number of poles P for which winding is wound and speed N r.p.m. of rotating magnetic field. For a standard frequency whatever speed of R.M.F. results is called synchronous speed, in case of induction motors. It is denoted as .

$$N_s = \frac{120\,f}{P} = \textbf{Speed of R.M.F.}$$

= (120 f)/P = speed of R.M.F.

Where f = Supply frequency in Hz

p = Number of poles for which winding is wound

This is the speed which R.M.F rotates in space. Let us see how to change direction of rotation of R.M.F.

**Direction of R.M.F.**

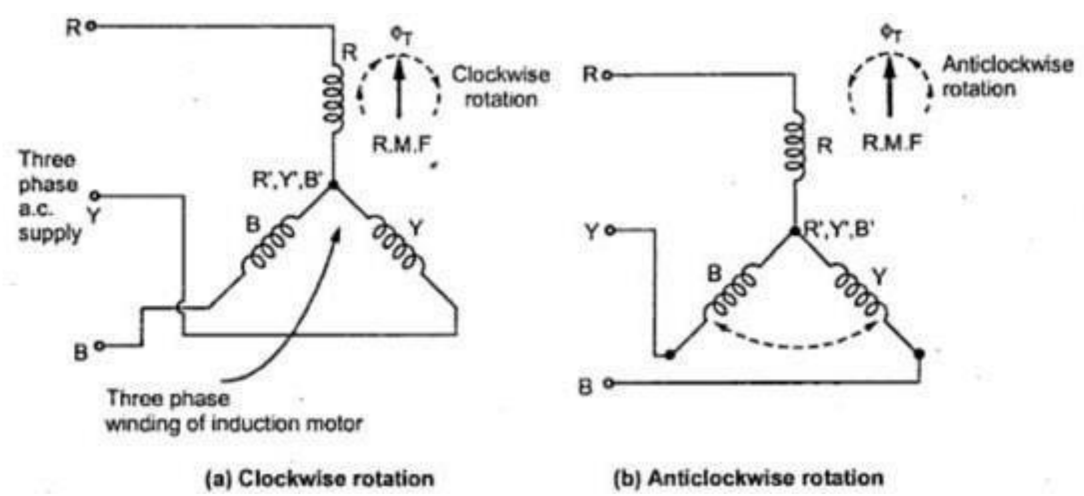


**Fig. 4**

The direction of the R.M.F. is always from the axis of the leading phase of the three phase winding towards the lagging phase of the winding. In a phase sequence of R-Y-B, phase R leads Y by 120$^{o}$ and Y leads B by120$^{o}$. So R.M.F. rotates from axis of R to axis of Y and then to axis of B and so on. So its direction is clockwise as shown in the Fig. 4(a). This direction can be reversed by interchanging any two terminals of the three phase windings while connecting to the three phase supply. The terminals Y and B are shown interchanged in the Fig. 4(b). In such case the direction of R.M.F. will be anticlockwise.

As Y and B of windings are connected to B and Y from winding point of view the phase sequence becomes R-Y-B. Thus R.M.F. axis follows the direction from R to B to Y which is anticlockwise.

**Key point** : Thus by interchanging any two terminals of three phase winding while connecting it to three phase a.c. supply, direction of rotation of R.M.F. gets reversed.

**Working Principle of 3-Phase Induction Motor:**

Induction motor works on the principle of electromagnetic induction.

When a three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced as discussed earlier. The speed of this rotation magnetic field is synchronous speed $N_s$ r.p.m.

$$N_s = \frac{120\,f}{P} = \text{speed of rotating magnetic field}$$

Where f = supply frequency.

p = Number of poles for which stator winding is wound.

This rotating field produces an effect of rotating poles around a rotor. Let direction of rotation of this rotating magnetic field is clockwise as shown in the Fig. 1(a).

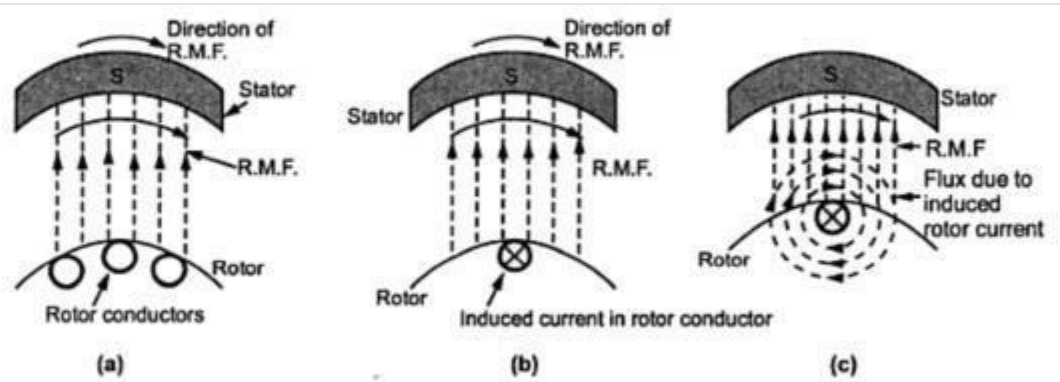


**Fig. 1**

Now at this instant rotor is stationary and stator flux R.M.F. is rotating. So its obvious that there exists a relative motion between the R.M.F. and rotor conductors. Now the R.M.F. gets cut by rotor conductors as R.M.F. sweeps over rotor conductors. Whenever conductors cuts the flux, e.m.f. gets induced in it. So e.m.f. gets induced in the rotor conductors called rotor induced e.m.f. This is electro-magnetic induction. As rotor forms closed circuit, induced e.m.f. circulates current through rotor called rotor current as shown in the Fig.1(b). Let direction of this current is going into the paper denoted by a cross as shown in the Fig. 1(b).

Any current carrying conductor produces its own flux. So rotor produces its flux called rotor flux. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown in the Fig. 1(c). This direction can be easily determined using right hand thumb rule. Now there are two fluxes, one R.M.F. and other rotor flux. Both the fluxes interact with each as shown in the Fig. 1(d). On left of rotor conductor, two fluxes cancel each other to produce low flux area. As flux lines act as stretched rubber band, high flux density area exerts a push on rotor conductor towards low flux density area. So rotor conductor experiences a force from left to right in this case, as shown in the Fig. 1(d), due to interaction of the two fluxes.

As all the rotor conductors experience a force, the overall rotor experiences a torque and starts rotating. So interaction of the two fluxes is very essential for a motoring action. As seen from the Fig. 1(d), the direction of force experienced is same as that of rotating magnetic field. Hence rotor starts rotating in the same direction as that of rotating magnetic field.

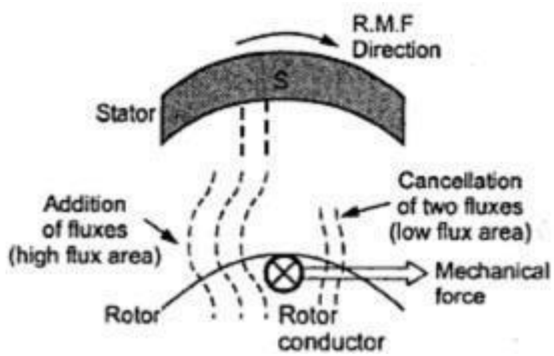


**Fig 1.d**

Alternatively this can be explained as :

According to Lenz's law the direction of induced current in the rotor is so as to oppose the cause producing it. The cause of rotor current is the induced e.m.f. which is induced because of relative motion present between the rotating magnetic field and the rotor conductors. Hence to oppose the relative motion i.e. to reduce the relative speed, the rotor experiences a torque in the same direction as that of R.M.F. and tries to catch up the speed of the rotating magnetic field.

So, $N_s$ = Speed of rotating magnetic field in r.p.m.

$N$ = Speed of rotor i.e. motor in r.p.m.

$N_s - N$ = Relative speed between the two, rotating magnetic field and the rotor conductors.

Thus rotor always rotates in same direction as that of R.M.F.

**Can N = Ns ?**

When rotor starts rotating, it tries to catch the speed of rotating magnetic field.

If it catches the speed of the rotating magnetic field, the relative motion between rotor and the rotating magnetic field will vanish ( Ns - N = 0). In fact the relative motion is the main cause for the induced e.m.f. in the rotor. So induced e.m.f. will vanish and hence there cannot be rotor current and the rotor flux which is essential to produce the torque on the rotor. Eventually motor will stop. But immediately there will exist a relative motion between rotor and rotating magnetic field and it will start. But due to inertia of rotor, this does not happen in practice and motor continues to rotate with a speed slightly less than the synchronous speed of the rotating magnetic field in the steady state. The induction motor never rotates at synchronous speed. The speed at which it rotates is hence called sub synchronous speed and motor sometimes called synchronous motor.s

$\therefore \qquad N < Ns$

So it can be said that rotor slips behind the rotating magnetic field produced by stator. The difference between the two is called slip speed of the motor.

$$Ns - N = \text{Slip speed of the motor in r.p.m.}$$

This speed decides the magnitude of the induction e.m.f. and the rotor current, which in turn decides the torque produced. The torque produced is as per the requirements of overcoming the friction and iron losses of the motor along with the torque demanded by the load on the rotor.

**Construction of Three Phase Induction motor:**

Basically the induction motor consists of two main parts, namely

1. The part i.e. three phase windings, which is stationary called stator.
2. The part which rotates and is connected to the mechanical load through shaft called rotor.

The conversion of electrical power to mechanical power takes place in a rotor. Hence rotor develops a driving torque and rotates.

**Stator**

The stator has a laminated type of construction made up of stampings which are 0.4 to 0.5 mm thick. The stampings are slotted in its periphery to carry the stator winding. The stampings are insulated from each other. Such a construction essentially keeps the iron losses to a minimum value. The number of stampings are stamped together to build the stator core. The built up core is then fitted in a casted or fabricated steel frame. The choice of material for the stampings is generally silicon steel, which minimizes the hysteresis loss. The slots in the periphery of the stator core carries a three phase winding, connected either in star or delta. This three phase winding is called stator winding. It is wound for definite number of poles. This winding when excited by a three phase supply produces a magnetic rotating field as discussed earlier. The choice of number of poles depends on the speed of the rotating magnetic field required. The radial ducts are provided for the cooling purpose. In some cases, all the six terminals of three phase stator winding are brought out which gives flexibility to the user to connect them either in star or delta. The Fig. 1 shows a stator lamination.

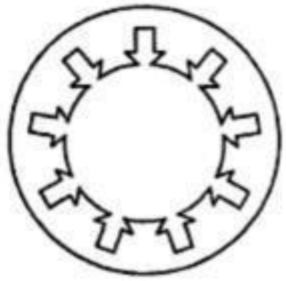

**Fig. 1 Stator lamination**

**Rotor**

The rotor is placed inside the stator. The rotor core is also laminated in construction and uses cast iron. It is cylindrical, with slots on its periphery. The rotor conductors or winding is placed in the rotor slots. The two typed of rotor constructions which are used for induction motors are,

1. Squirrel cage rotor and
2. Slip ring wound rotor

**Squirrel Cage Rotor**

The rotor core is cylindrical and slotted on its periphery. The rotor consists of uninsulated copper or aluminium bars called rotor conductors. The bars are placed in the slots. These bars are permanently shorted at each end with the help of conducting copper ring called end ring. The bars are usually brazed to the end rings to provide good mechanical strength. The entire structure looks like a cage, forming a closed electrical circuit. So the rotor is called squirrel cage rotor. The construction is shown in the Fig. 1.

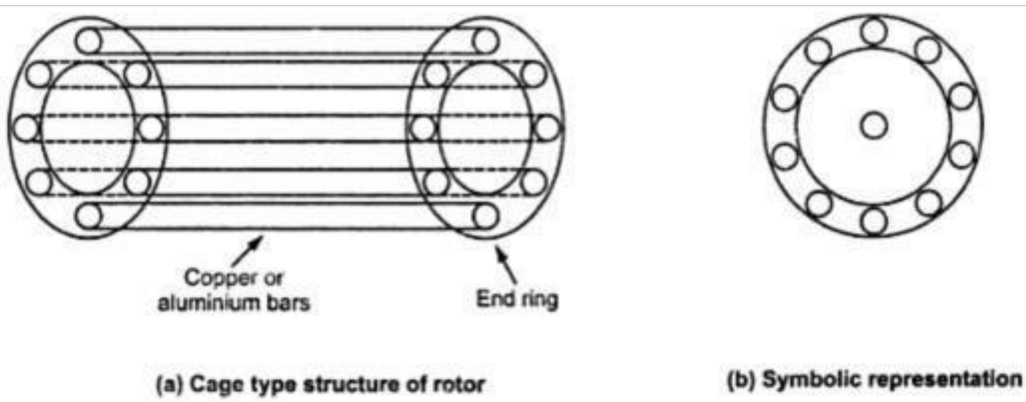


**Fig. 1 Squirrel cage rotor**

As the bars are permanently shorted to each other through end ring, the entire rotor resistance is very very small. Hence this rotor is also called short circuited rotor. As rotor itself is short circuited, no external resistance can have any effect on the rotor resistance. Hence no external resistance can be introduced in the rotor circuit. So slip ring and brush assembly is not required for this rotor. Hence the construction of this rotor is very simple.

Fan blades are generally provided at the ends of the rotor core. This circulates the air through the machine while operation, providing the necessary cooling. The air gap between stator and rotor is kept uniform and as small as possible.

In this type of rotor, the slots are not arranged parallel to the shaft axis but are skewed as shown in the Fig. 2.

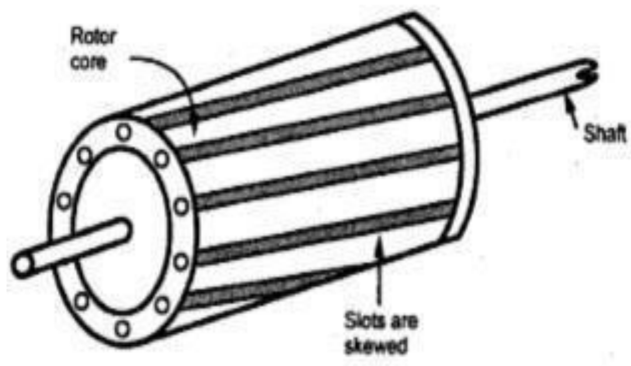


**Fig. 2 Skewing in rotor construction**

**The advantages of skewing are,**

1. A magnetic hum i.e. noise gets reduced due to skewing hence skewing makes the motor operation quite.
2. It makes the rotor operation smooth.
3. The stator and rotor teeth may get magnetically locked. Such a tendency of magnetic locking gets reduced due to skewing.
4. It increases the effective transformation ratio between stator and rotor.

**Slip Ring Rotor or Wound Rotor**

In this type of construction, rotor winding is exactly similar to the stator. The rotor carries a three phase star or delta connected, distributed winding, wound for same number of poles as that of stator. The rotor construction is laminated and slotted. The slots contain the rotor winding. The three ends of three phase winding, available after connecting the winding in star or delta, are permanently connected to the slip rings. The slip rings are mounted on the same shaft. We have seen that slip rings are used to connect external stationary circuit to the internal rotating circuit. So in this type of rotor, the external resistances can be added with the help of brushes and slip ring arrangement, in series with each phase of the rotor winding. This arrangement is shown in the Fig. 1.

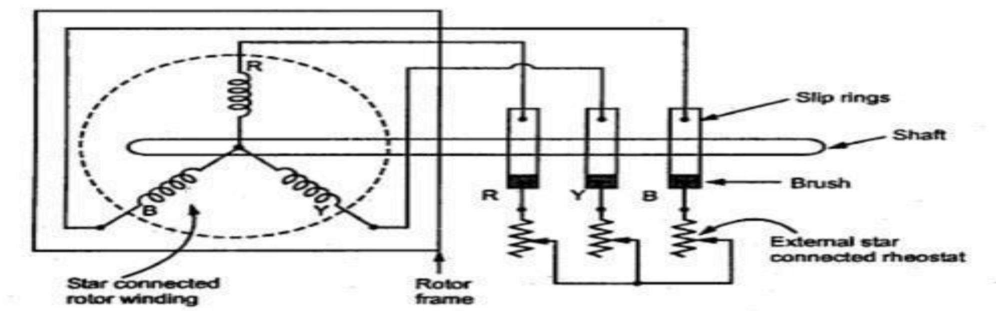


**Fig. 1 Slip rings or wound rotor**

**Key point** : This way the value of rotor resistance per phase can be controlled. This helps us to control some of the important characteristics of the motor like starting torque, speed etc.

In the running condition, the slip rings are shorted. This is possible by connecting a metal collar which gets pushed and connects all the slip rings together, shorting them. At the same time brushes are also lifted from the slip rings. This avoids wear and tear of the brushes due to friction. The possibility of addition of an external resistance in series with the rotor, with the help of slip sings is the main feature of this type of rotor.

**Comparison of Squirrel Cage and Wound Rotor**

| Sr. No. | Wound or slip ring rotor | Squirrel cage rotor |
|---|---|---|
| 1 | Rotor consists of a three phase winding similar to the stator winding. | Rotor consists of bars which are shorted at the ends with the help of end rings. |
| 2 | Construction is complicated. | Construction is very simple. |
| 3 | Resistance can be added externally. | As permanently shorted, external resistance cannot be added. |
| 4 | Slip rings and brushes are present to add external resistance. | Slip rings and brushes are absent. |
| 5 | The construction is delicate and due to brushes, frequent maintenance is necessary. | The construction is robust and maintenance free. |
| 6 | The rotors are very costly. | Due to simple construction, the rotors are cheap. |
| 7 | Only 5% of induction motors in industry use slip ring rotor. | Very common and almost 95% induction motors use this type of rotor. |
| 8 | High starting torque can be obtained. | Moderate starting torque which cannot be controlled. |
| 9 | Rotor resistance starter can be used. | Rotor resistance starter cannot be used. |
| 10 | Rotor must be wound for the same number of poles as that of stator. | The rotor automatically adjusts itself for the same number of poles as that of stator. |
| 11 | Speed control by rotor resistance is possible. | Speed control by rotor resistance is not possible. |
| 12 | Rotor copper losses are high hence efficiency is less. | Rotor copper losses are less hence have higher efficiency. |
| 13 | Used for lifts, hoists, cranes, elevators, compressors etc. | Used for lathes, drilling machines, fans, blowers, water pumps, grinders, printing machines etc. |

**Concept of Slip Rings and Brush Assembly:**

Whenever there is a need of connecting the rotating member of the machine to the stationary external circuit, then slip rings and brush assembly is used.

Consider a three phase rotating star connected winding as shown in the Fig. 1. It is required to connect external three stationary star connected resistances to this windings. The winding must keep on rotating and external resistance must remain stationary and still there should be contact between the two. This is possible by slip rings and brushes.

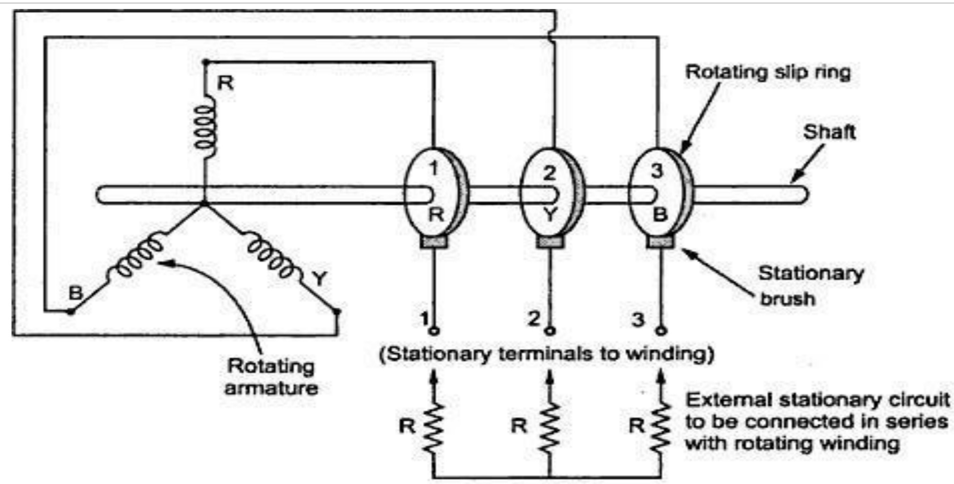


**Fig. 1 Concept of slip rings and brush**

The three rings made up of conducting material called slip rings are mounted on the same shaft with which winding is rotating. Each terminal of winding is connected to an individual slip ring, permanently. Thus three ends R-Y-B of winding are available at the three rotating slip rings. The three brushes are then used. Each brush is resting on the corresponding slip ring, making contact with the slip ring but the brushes are stationary. So rotating three ends R-Y-B are now available at the brushes which are stationary as shown in the Fig. 1. Now stationary external circuit can be connected to the brushes which are nothing but the three ends of the winding.

Thus the external stationary circuit can be connected to the rotating internal part of the machine with the help of slip rings and brush assembly. Not only the external circuit can be connected but the voltage also can be injected to the rotating winding, by connecting stationary supply to the brushes externally.

**Key point** : Such slip rings and brush assembly plays an important role in the working of slip ring induction motor.

**Review Portion Ends….**
**Note: (No Question shall be set from the review potion)**
**Actual Syllabus Begins….**

**Slip of Induction Motor:**

We have seen that rotor rotates in the same direction as that of R.M.F. but in steady state attains a speed less than the synchronous speed. The difference between the two speeds i.e. synchronous speed of R.M.F. ( Ns ) and rotor speed (N) is called slip speed. This slip speed is generally expressed as the percentage of the synchronous speed.

So slip of the induction motor is defined as the difference between the synchronous speed (Ns) and actual speed of rotor i.e. motor (N) expressed as a friction of the synchronous speed(Ns). This is also called absolute slip or fractional slip and is denoted as 's'.

Thus

$$\boxed{s = \frac{N_s - N}{N_s}} \qquad \text{... (absolute slip)}$$

The percentage slip is expressed as,

$$\boxed{\%\ s = \frac{N_s - N}{N_s} \times 100} \qquad \text{... (percentage slip)}$$

In terms of slip, the actual speed of motor (N) can be expressed as,

$$\boxed{N = N_s\,(1 - s)} \qquad \text{... (from the expression of slip)}$$

At start, motor is at rest and hence its speed N is zero.

$$\boxed{s = 1 \text{ at start}}$$

This is maximum value of slip s possible for induction motor which occurs at start. While s = 0 given us N = Ns which is not possible for an induction motor. So slip of induction motor cannot be zero under any circumstances.

Practically motor operates in the slip range of 0.01 to 0.05 i.e. 1% to 5%. The slip corresponding to full load speed of the motor is called full load slip.

**Example 1** : A 4 pole, 3 phase induction motor is supplied from Hz supply. Determine its synchronous speed. On full load, its speed is observed to be 1410 r.p.m. calculate its full load slip.

**Solution** : Given values are,

P = 4, f = 50 Hz , N = 1410 r.p.m.

Ns = 120f / P = 120 x 50 / 4 = 1500 r.p.m.

Full load absolute slip is given by,

$s = (Ns - N)/ V_2 = (1500\text{-}1410)/ 1500 = 0.06$

$\therefore \quad \%s = 0.06 \times 100 = 6\ \%$

**Example 2** : A 4 pole, 3 phase, 50 Hz, star connected induction motor has a full load slip of 4 %. Calculate full load speed of the motor.

**Solution** : Given values are,

$P = 4, \quad f = 50\ Hz, \quad \%\ s_{fl} = 4\%$

$s_{fl}$ = Full load absolute slip = 0.04

$Ns = 120f / P = 120 \times 50 / 4 = 1500$ r.p.m.

$s_{fl} = (Ns - N_{fl}) / Ns$ = where = full load speed of motor

$\therefore \quad 0.04 = (1500 - N_{fl})/ 1500$

$\therefore \quad N_{fl} = 1440$ r.p.m.

**Effect of Slip on Rotor Parameters**

In case of a transformer, frequency of the induced e.m.f. in the secondary is same as the voltage applied to primary. Now in case of induction motor at start N = 0 and slip s = 1. Under this condition as long as s = 1, the frequency of induced e.m.f. in rotor is same as the voltage applied to the stator. But as motor gathers speed, induction motor has some slip corresponding to speed N. In such case, the frequency of induced e.m.f. in rotor is no longer same as that of stator voltage. Slip affects the frequency of rotor induced e.m.f. Due to this some other rotor parameters also get affected. Let us study the effect of slip on the following rotor parameters.

1. Rotor frequency 2. Magnitude of rotor induced e.m.f. 3. Rotor reactance
4. Rotor power factor and 5. Rotor current

**1. Effect on rotor frequency**

In case of induction motor, the speed of rotating magnetic field is,

$N_s = (120\,f)/P$........................................................ (1)

Where f = Frequency of supply in Hz

At start when N = 0, s = 1 and stationary rotor has maximum relative motion with respect to R.M.F. Hence maximum e.m.f. gets induced in the rotor at start. The frequency of this induced e.m.f. at start is same as that of supply frequency.

As motor actually rotates with speed N, the relative speed of rotor with respect R.M.F. decreases and becomes equal to slip speed of $N_s$ - N. The induced e.m.f. in rotor depends on rate of cutting flux i.e. relative speed $N_s$ - N. Hence in running condition magnitude of induced e.m.f. decreases so as to its frequency. The rotor is wound for same number of poles as that of stator i.e. P. If $f_r$ is the frequency of rotor induced e.m.f. in running condition at slip speed $N_s$ - N then there exists a fixed relation between ($N_s$ - N), $f_r$ and P similar to equation (1). So we can write for rotor in running condition,

$(N_s - N) = (120\,f_r)/P$ , rotor poles = stator poles = P..........................(2)

Dividing (2) by (1) we get,

$(N_s - N)/N_s = (120\ f_r / P)/(120\ f / P)$ but $(N_s - N)/N_s$ = slip s

$s = f_r/f$

$f_r = s\ f$

Thus frequency of rotor induced e.m.f. in running condition ($f_r$) is slip times the supply frequency (f).

At start we have s = 1 hence rotor frequency is same as supply frequency. As slip of the induction motor is in the range 0.01 to 0.05, rotor frequency is very small in the running condition.

**Example** : A 4 pole, 3 phase, 50 Hz induction motor runs at a speed of 1470 r.p.m. speed. Find the frequency of the induced e.m.f in the rotor under this condition.

**Solution** : The given values are,

P = 4, f = 50 Hz, N = 1470 r.p.m.

$N_s = (120\ f)/ P = (120 \times 50)/4 = 1500$ r.p.m.

$s = (N_s - N)/N_s = (1500-1470)/1500 = 0.02$

$f_r = s\ f = 0.02 \times 50 = 1$ Hz

It can be seen that in running condition, frequency of rotor induced e.m.f. is very small.

**2. Effect of Slip on Magnitude of Rotor Induced E.M.F**

We have seen that when rotor is standstill, s = 1, relative speed is maximum and maximum e.m.f. gets induced in the rotor. Let this e.m.f. be,

$E_2$ = Rotor induced e.m.f. per phase on standstill condition

As rotor gains speed, the relative speed between rotor and rotating magnetic field decreases and hence induced e.m.f. in rotor also decreases as it is proportional to the relative speed $N_s - N$. Let this e.m.f. be,

$E_{2r}$ = Rotor induced e.m.f. per phase in running condition

Now $E_{2r} \alpha N_s$ while $E_{2r} \alpha N_s - N$

Dividing the two proportionality equations,

$E_{2r}/E_2 = ( N_s - N)/N_s$ but $(N_s - N)/N$ = slip s

$E_{2r}/E_2 = s$

$E_{2r} = s\ E_2$

The magnitude of the induced e.m.f in the rotor also reduces by slip times the magnitude of induced e.m.f. at standstill condition.

**3. Effect on Rotor Resistance and Reactance**

The rotor winding has its own resistance and the inductance. In case of squirrel cage rotor, the rotor resistance is very very small and generally neglected but slip ring rotor has its own resistance which can be controlled by adding external resistance through slip rings. In general let,

$R_2$ = Rotor resistance per phase on standstill

$X_2$ = Rotor reactance per phase on standstill

Now at standstill, fr = f hence if $L_2$ is the inductance of rotor per phase,

$X_2 = 2\pi fr\ L_2 = 2\pi f\ L_2\ \Omega$/ph

While $R_2$ = Rotor resistance in $\Omega$/ph

Now in running condition, $f_r = s\ f$ hence,

$X_{2r} = 2\pi f_r\ L_2 = 2\pi fs\ L_2 = s\ .(2\pi f\ L_2)$

$X_{2r} = s\ X_2$

where $X_{2r}$ = Rotor reactance in running condition

Thus resistance as independent of frequency remains same at standstill and in running condition. While the rotor reactance decreases by slip times the rotor reactance at standstill.

Hence we can write rotor impedance per phase as :

$Z_2$ = Rotor impedance on standstill (N = 0) condition

$= R_2 + j\ X_2\ \Omega$/ph

$Z_2 = \sqrt{( R_2^2+ X_2)^2)}\ \Omega$/ph..................magnitude

While $Z_{2r}$ = Rotor impedance in running condition

$= R_2 + j\ X_{2r} = R_2 + j\ (s\ X_2)\ \Omega$/ph

$Z_{2r} = \sqrt{(R_2^2+ (s\ X_2)^2)}$ $\Omega$/ph ............ magnitude

**4. Effect on Rotor Power Factor**

From rotor impedance, we can write the expression for the power factor of rotor at standstill and also in running condition.

The impedance triangle on standstill condition is shown in the Fig1. From it we can write,

$\cos \Phi_2$ = Rotor power factor on standstill

$= R_2/Z_2 = R_2/\sqrt{(R_2^2+ X_2^2)}$

The impedance in running condition becomes $Z_{2r}$ and the corresponding impedance triangle is shown in the Fig.2. From Fig. 2 we can write,

$\cos \Phi_2$r = Rotor power factor in running condition

$= R_2/Z_{2r} = R_2/\sqrt{(R_2^2+ (s\ X_2)^2)}$

**Key point** : As rotor winding is inductive, the rotor p.f. is always lagging in nature.

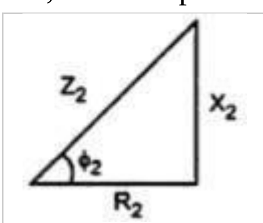


**Fig. 1**

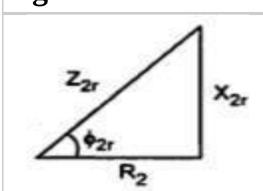


**Fig. 2**

**5. Effect on Rotor Current**

Let $I_2$ = Rotor current per phase on standstill condition

The magnitude of $I_2$ depends on magnitude of $E_2$ and impedance $Z_2$ per phase.

$$I_2 = (E_2 \text{ per phase})/(Z_2 \text{ per phase}) \text{ A}$$

Substituting expression of $Z_2$ we get,

$$I_2 = E_2 / \sqrt{(R_2^2 + X_2^2)} \quad \text{A}$$

The equivalent rotor circuit on standstill is shown in the Fig.3. The $\Phi_2$ is the angle between $E_2$ and $I_2$ which determines rotor p.f. on standstill.

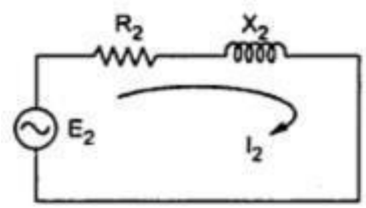


**Fig. 3**

In the running condition, $Z_2$ changes to $Z_{2r}$ while the induced e.m.f. changes to $E_{2r}$. Hence the magnitude of current in the running condition is also different than on standstill. The equivalent circuit on running condition is shown in the Fig. 4.

$I_{2r}$ = Rotor current per phase in running condition

The value of slip depends on speed which inturn depends on load on motor hence $X_{2r}$ is shown variable in the equivalent circuit. From the equivalent we can write,

$$I_{2r} = E_{2r}/Z_{2r} = (s\,E_2)/\sqrt{(R_2^2 + (s\,X_2)^2)}$$

$\Phi_{2r}$ is the angle between $E_{2r}$ and $I_{2r}$ which decides p.f. in running condition.

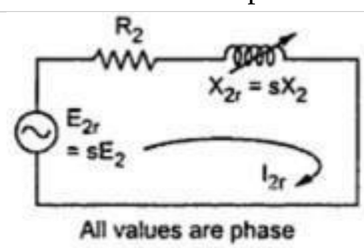


**Fig. 4**

**Key point** : Putting s = 1 in the expression obtained in running condition, the values at standstill can be obtained.

**Induction Motor as a Transformer**

We know that, transformer is a device in which two windings are magnetically coupled and when one winding is excited by a.c. supply of certain frequency, the e.m.f. gets induced in the second winding having same frequency as that of supply given to the first winding. The winding to which supply is given is called primary winding while winding in which e.m.f. gets induced is called secondary winding. The induction motor can be regarded as the transformer.

The difference is that the normal transformer is an alternating flux transformer while induction motor is rotating flux transformer. The normal transformer has no air gap as against this an induction motor has distinct air gap between its stator and rotor.

In an alternating flux transformer the frequency of induced e.m.f. and current in primary and secondary is always same. However in the induction motor frequency of e.m.f. and current on the stator side remains same but frequency of rotor e.m.f. and current depends on the slip and slip depends on load on the motor. So we have a variable frequency on the rotor side. But it is important to remember that at start when N = 0 the value of slip is unity (s = 1), then frequency of supply to the stator and of induced e.m.f. in the rotor is same. The effect of slip on the rotor parameters is already discussed in the previous section.

And last difference is that in case of the alternating flux transformer the entire energy present in the secondary circuit, is in the electrical form. As against this, in an induction motor part of its energy in the rotor circuit is in electrical form and the remaining part is converted into mechanical form.

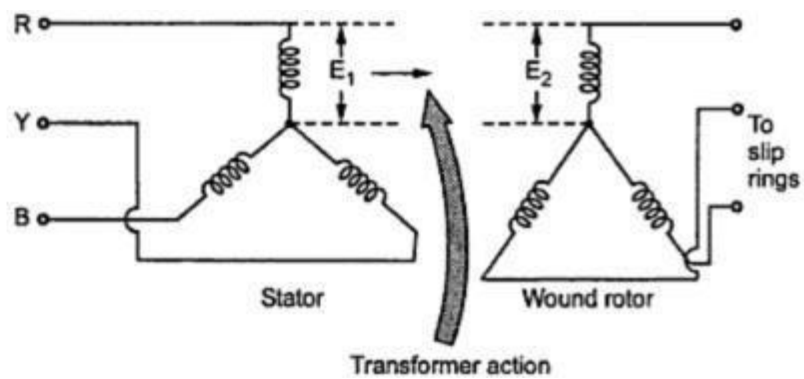


**Fig. 1 Induction motor as a transformer**

In general, an induction motor can be treated as a generalized transformer as shown in the Fig. 1. In this, the slip ring induction motor with star connected stator and rotor is shown.

So if $E_1$ = Stator e.m.f. per phase in volts.

$E_2$ = Rotor induced e.m.f. per phase in volts at start when motor is at standstill.

Then according to general transformer there exists a fixed relation between $E_1$ and $E_2$ called transformer ratio.

$\therefore$ At start when N =0 , s =1

and we get,

$$\boxed{\frac{E_2}{E_1} = K = \frac{\text{Rotor turns / phase}}{\text{Stator turns / phase}}}$$

**Key Point** : So if stator supply voltage is known and ratio of stator to rotor turns per phase is known then the rotor induced e.m.f. on standstill can be obtained.

**Torque Equation**

The torque produced in the induction motor depends on the following factors :

1. The part of rotating magnetic field which reacts with rotor and is responsible to produce induced e.m.f. in rotor.
2. The magnitude of rotor current in running condition.
3. The power factor of the rotor circuit in running condition.

Mathematically the relationship cab be expressed as,

$T \alpha \Phi I_{2r} \cos \Phi_{2r}$ ..........(1)

where $\Phi$ = Flux responsible to produce induced e.m.f.

$I_{2r}$ = Rotor running condition

$\cos \Phi_{2r}$ = Running p.f. of motor

The flux $\Phi$ produced by stator is proportional to i.e. stator voltage.

$\therefore$ $\Phi \alpha E_1$..........(2)

while $E_1$ and $E_2$ are related to each other through ratio of stator turns to rotor turns i.e. k.

$\therefore$ $E_2/E_1 = K$..........(3)

Using (3) in (2) we can write,

Thus in equation (1), $\Phi$ can be replaced by $E_2$.

While $I_{2r} = E_{2r}/Z_{2r} = (s E_2)/\sqrt{(R_2^2 + (s X_2)^2)}$.......... (5)

and $\cos \Phi 2r = R_2/Z_{2r} = R_2/\sqrt{(R_2^2 + (s X_2)^2)}$..........(6)

Using (4), (5), (6) in equation (1),

$$T \propto E_2 \cdot \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\therefore \quad \boxed{T \propto \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}}$$

$\therefore$ $T = (k\, s\, E_2^2\, R_2)/(R_2^2 + (s X_2)^2)$..........(7)

where k = Constant of proportionality

The constant k is provided to be 3/2 for three phase induction motor.

$\therefore$ $k = 3/(2 \pi n_s)$..........(8)

**Key Point** : $n_s$ = synchronous speed in r.p.s. = $N_s/60$

Using (8) in (7) we get the torque equation as,

$$\boxed{T = \frac{3}{2\pi n_s} \cdot \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \text{ N - m}} \quad \text{... (9)}$$

So torque developed at any load condition can be obtained if slip at that load is known and all standstill rotor parameters are known.

**Starting Torque**

Starting torque is nothing but the torque produced by an induction motor as start. At start, N= 0 and slip s = 1. So putting s = 1 in the torque equation we can write expression for the starting torque $T_{st}$ as,

$$\boxed{T_{st} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2 R_2}{(R_2^2 + X_2^2)}} \quad \ldots (10)$$

**Key Point** : From the equation (10), it is clear that by changing the starting torque can be controlled.

The change in $R_2$ at start is possible in case of slip ring induction motor only. This is the principle used in case of slip induction motor to control the starting torque $T_{st}$.

**Example 1** : A 3 phase, 400 V, 50 Hz, 4 pole induction motor has star connected stator winding. The rotor resistance and reactance are 0.1 Ω and 1 Ω respectively. The full load speed is 1440 r.p.m. Calculate the torque developed on full load by the motor.
Assume stator to rotor ratio as 2 :1.

**Solution** : The given values are,

$P = 4$, $f = 50$ Hz, $R_2 = 0.1\ \Omega$, $X_2 = 1\ \Omega$, $N = 1440$ r.p.m.

Stator turns/Rotor turns = 2/1

∴ $K = E_2 / E_1$ = Rotor turns/Stator turns = 1/2 = 0.5

$N_s = 120f/P = 120 \times 50 / 4 = 1500$ r.p.m.

$E_{1line} = 400$ V.................................................. Stator line voltage given

∴ $E_{1ph} = E_{1line} / \sqrt{3} = 400/\sqrt{3} = 230.94$ V

But $E_{2ph} / E_{1ph} = 0.5 = K$

∴ $E_{2ph} = 0.5 \times 230.94 = 115.47$ V

Full load slip, $s = (N_s - N)/N_s = (1500-1400)/1500 = 0.04$

$n_s$ = Synchronous speed in r.p.s.

$= N_s/60 = 1500/60 = 25$ r.p.s.

$$T = \frac{3}{2\pi n_s} \cdot \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi \times 25} \times \frac{0.04 \times (115.47)^2 \times 0.1}{[(0.1)^2 + (0.04 \times 1)^2]}$$

$= 87.81$ N-m

**Condition of Maximum Torque:**

From the torque equation, it is clear that torque depends on slip at which motor is running. The supply voltage to the motor is usually rated and constant and there exists a fixed ratio between $E_1$ and $E_2$. Hence $E_2$ is also constant. Similarly $R_2$, $X_2$ and $n_s$ are constants for the induction motor.

Hence while finding the condition for maximum torque, remember that the only parameter which controls the torque is slip s.

Mathematically for the maximum torque we can write,

$$dT/ds = 0$$

where $T = (k\, s\, E_2^2 R_2)/(R_2^2 + (s X_2)^2)$

While carrying out differential remember that $E_2$, $R_2$, $X_2$ and k are constants. The only variable is slip s. As load on motor changes, its speed changes and hence slip changes. This slip decides the torque produced corresponding to the load demand.

$$T = (k\, s\, E_2^2 R_2)/(R_2^2 + s^2 X_2^2) \text{ .................. Writing } (s X_2)^2 = s^2 X_2^2$$

As both numerator and denominator contains s terms, differential T with respect to s using the rule of differentiation for u/v.

$$\therefore \frac{dT}{ds} = \frac{(k\, s\, E_2^2 R_2)\frac{d}{ds}(R_2^2 + s^2 X_2^2) - (R_2^2 + s^2 X_2^2)\frac{d}{ds}(k\, s\, E_2^2 R_2)}{(R_2^2 + s^2 X_2^2)^2} = 0$$

$\therefore \quad k\, s\, E_2^2 R_2 (2s X_2^2) - (R_2^2 + s^2 X_2^2)(k E_2^2 R_2) = 0$

$\therefore \quad 2 s^2 k X_2^2 E_2^2 R_2 - R_2^2 k E_2^2 R_2 - k s^2 X_2^2 E_2^2 R_2 = 0$

$\therefore \quad k s^2 X_2^2 E_2^2 R_2 - R_2^2 k E_2^2 R_2 = 0$

$\therefore \quad s^2 X_2^2 - R_2^2 = 0 \qquad$ Taking $k E_2^2 R_2$ common.

$\therefore \quad s^2 = R_2^2/X_2^2$

$\therefore \quad s = R_2/X_2 \qquad$ Neglecting negative slip

This is the slip at which the torque is maximum and is denoted as $s_m$.

$\therefore \quad s_m = R_2/X_2$

It is the ratio of standstill per values of resistance and reactance of rotor, when the torque produced by the induction motor is at its maximum.

**Magnitude of Maximum Torque**

This can be obtained by substituting $s_m = R_2/X_2$ in the torque equation. It is denoted by $T_m$.

$$T_m = (k\, s_m E_2^2 R_2)/(R_2^2 + (s_m X_2)^2)$$

$$\therefore \quad T_m = \frac{k\left(\frac{R_2}{X_2}\right) E_2^2 R_2}{R_2^2 + \left(\frac{R_2}{X_2} X_2\right)^2}$$

$$\therefore \quad \boxed{T_m = \frac{k E_2^2}{2X_2} \text{ N-m.}}$$

From the expression of $T_m$, it can be observed that

1. It is inversely proportional to the rotor reactance.
2. It is directly proportional to the square of the rotor induced e.m.f. at standstill.
3. The most interesting observation is, the maximum torque is not dependent on the rotor resistance $R_2$. But the slip at which it occurs i.e. speed at which it occurs depends on the value of rotor resistance $R_2$.

**Example 1** : A 400 V, 4 pole, 3 phase, 50 Hz star connected induction motor has a rotor resistance and reactance per phase equal to 0.01 Ω and 0.1 Ω respectively. Determine i) Starting torque ii) slip at which maximum torque will occur iii) speed at which maximum torque will occur iv) maximum torque v) full load torque if full load slip is 4 %. Assume ratio of stator to rotor turns as 4.

**Solution** : The given values are,

P = 4, f = 50 Hz, stator turns/ rotor turns = 4, $R_2$ = 0.01 Ω, $X_2$ = 0.1 Ω

$E_{1line}$ = stator line voltage = 400 V

$E_{1ph} = E_{1line}/\sqrt{3} = 400/\sqrt{3} = 230.94$ V............................ star connection

$K = E_{2ph}/E_{1ph}$ = Rotor turns/ Stator turns = 1/4

$\therefore \quad E_2 = (1/4) \times E_{1ph} = 230.94/4 = 57.735$ V

$N_s = 120f/P = 120 \times 50 / 4 = 1500$ r.p.m.

i) At start, s =1

$\therefore \quad T_{st} = (k\ E_2^2\ R_2)/(R_2^2 + (X_2)^2)$ where $k = 3/(2\ \pi\ n_s)$

$n_s = N_s/60 = 1500/60 = 25$ r.p.s.

$\therefore \quad k = 3/(2\pi \times 25) = 0.01909$

$\therefore \quad T_{st} = (0.01909 \times 57.735^2 \times 0.01)/(0.01^2 + 0.1^2) = 63.031$ N-m

ii) Slip at which maximum torque occurs is,

$s_m = R_2/X_2 = 0.01/0.1 = 0.1$

$\%s_m = 0.1 \times 100 = 10\%$

iii) Speed at which maximum torque occurs is speed corresponding to,

$N = N_s(1 - s_m) = 1500\ (1 - 0.1) = 1350$ r.p.m.

iv) The maximum torque is,

$T_m = (k\ E_2^2)/(2\ X_2) = (0.01909 \times 57.735^2)/(2 \times 0.1) = 318.16$ N-m

v) Full load slip, $s_f = 0.04$ as % $s_f$ = 4 %

$\therefore T_{f.l.} = (k\ s_f\ E_2^2\ R_2)/(R_2^2 + (s_f\ X_2)^2) = (0.01909 \times 0.04 \times 57.735^2 \times 0.01)/(0.01^2 + (0.04 \times 0.1)^2)$

$= 219.52$ N-m

**Torque-Slip Characteristics**

As the induction motor is located from no load to full load, its speed decreases hence slip increases. Due to the increased load, motor has to produce more torque to satisfy load demand. The torque ultimately depends on slip as explained earlier. The behaviour of motor can be easily judged by sketching a curve obtained by plotting torque produced against slip of induction motor. The curve obtained by plotting torque against slip from s = 1 (at start) to s = 0 (at synchronous speed) is called torque-slip characteristics of the induction motor. It is very interesting to study the nature of torque-slip characteristics.

We have seen that for a constant supply voltage, $E_2$ is also constant. So we can write torque equations as,

$$\boxed{T \propto \frac{s R_2}{R_2^2 + (sX_2)^2}}$$

Now to judge the nature of torque-slip characteristics let us divide the slip range (s = 0 to s = 1) into two parts and analyze them independently.

**i) Low slip region :**

In low slip region, 's' is very very small. Due to this, the term $(s X_2)^2$ is so small as compared to $R_2^2$ that it can be neglected.

$$\therefore \quad \boxed{T \propto \frac{s R_2}{R_2^2} \propto s} \quad \text{As } R_2 \text{ is constant.}$$

Hence in low slip region torque is directly proportional to slip. So as load increases, speed decreases, increasing the slip. This increases the torque which satisfies the load demand.

Hence the graph is straight line in nature.

At $N = N_s$, s = 0 hence T = 0. As no torque is generated at $N = N_s$, motor stops if it tries to achieve the synchronous speed. Torque increases linearly in this region, of low slip values.

**ii) High slip region :**

In this region, slip is high i.e. slip value is approaching to 1. Here it can be assumed that the term $R_2^2$ is very very small as compared to $(s X_2)^2$. Hence neglecting from the denominator, we get

$$\boxed{T \propto \frac{s R_2}{(sX_2)^2} \propto \frac{1}{s}} \quad \text{where } R_2 \text{ and } X_2 \text{ are constants}$$

So in high slip region torque is inversely proportional to the slip. Hence its nature is like rectangular hyperbola.

Now when load increases, load demand increases but speed decreases. As speed decreases, slip increases. In high slip region as T $\alpha 1/s$, torque decreases as slip increases.

But torque must increases to satisfy the load demand. As torque decreases, due to extra loading effect, speed further decreases and slip further increases. Again torque decreases as T $\alpha 1/s$ hence same load acts as an extra load due to reduction in torque produced. Hence speed further drops. Eventually motor comes to standstill condition. The motor cannot continue to rotate at any point in this high slip region. Hence this region is called unstable region of operation.

So torque - slip characteristics has two parts,

**1.** Straight line called **stable region of operation**

**2.** Rectangular hyperbola called **unstable region of operation.**

Now the obvious question is upto which value of slip, torque - slip characteristics represents stable operation ?

In low slip region, as load increases, slip increases and torque also increases linearly. Every motor has its own limit to produce a torque. The maximum torque, the motor can produces as load increases is $T_m$ which occurs at $s = s_m$. So linear behaviour continues till $s = s_m$.

If load is increased beyond this limit, motor slip acts dominantly pushing motor into high slip region. Due to unstable conditions, motor comes to standstill condition at such a load. Hence i.e. maximum torque which motor can produce is also called **breakdown torque** or **pull out torque**. So range $s = 0$ to $s = s_m$ is called low slip region, known as stable region of operation. Motor always operates at a point in this region. And range $s = s_m$ to $s = 1$ is called high slip region which is rectangular hyperbola, called unstable region of operation. Motor can not continue to rotate at any point in this region.

At $s = 1$, $N = 0$ i.e. start, motor produces a torque called starting torque denoted as $T_{st}$. The entire torque - slip characteristics is shown in the Fig. 1.

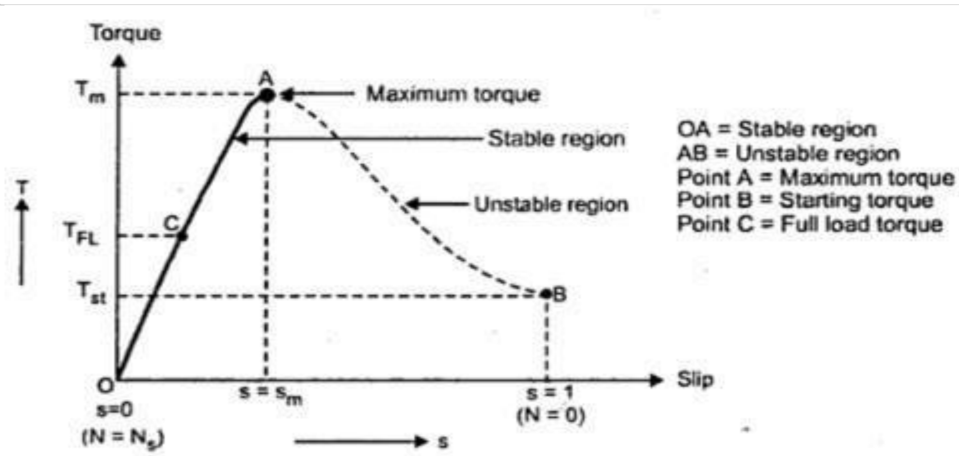


**Fig. 1 Torque speed characteristics**

**Full load torque**

When the load on the motor increases, the torque produced increases as speed decreases and slip increases. The increases torque demand is satisfied by drawing motor current from the supply.

The load which motor can drive safely while operating continuously and due to such load, the current drawn is also within safe limits is called full load condition of motor. When current increases, due to heat produced the temperature rises. The safe limit of current is that which when drawn for continuous operation of motor, produces a temperature rise well within the limits. Such a full load point is shown on the torque-slip characteristics torque as $T_{F.L.}$.

The interesting thing is that the load on the motor can be increased beyond point C till maximum torque condition. But due to high current and hence high temperature rise there is possibility of damage of winding insulation, if motor is operated for longer time duration in this region i.e. from point C to B. But motor can be used to drive loads more than full load, producing torque upto maximum torque for short duration of time. Generally full load torque is less than the maximum torque.

So region OC upto full load condition allows motor operation continuously and safely from the temperature point pf view. While region CB is possible to achieve in practice but only for short duration of time and not for continuous operation of motor. This is the difference between full load torque and the maximum or breakdown torque. The breakdown torque is also called stalling torque.

$$\therefore \qquad \boxed{T_{\text{Full load}} < T_m}$$

**Generating and Braking Region:**

When the slip lies in the region 0 and 1 i.e. when $0 \leq s \leq 1$, the machine runs as a motor which is the normal operation. The rotation of rotor is in the direction of rotating field which is developed by stator currents. In this region it takes electrical power from supply lines and supplies mechanical power output. The rotor speed and corresponding torque are in same direction.

When the slip is greater than 1, the machine works in the braking mode. The motor is rotated in opposite direction to that of rotating field. In practice two of the stator terminals are interchanged which changes the phase sequence which in turn reverses the direction of rotation of magnetic field. The motor comes to quick stop under the influence of counter torque which produces braking action. This method by which the motor comes to rest is known as plugging. Only care is taken that the stator must be disconnected from the supply to avoid the rotor to rotate in other direction

To run the induction machine as a generator, its slip must be less than zero i.e. negative. The negative slip indicates that the rotor is running at a speed above the synchronous speed. When running as a generator it takes mechanical energy and supplies electrical energy from the stator.

Thus the negative slip, generation action takes place and nature of torque - slip characteristics reverses in this generating region.

The Fig.2 shows the complete torque - slip characteristics showing motoring, generating and the braking region.

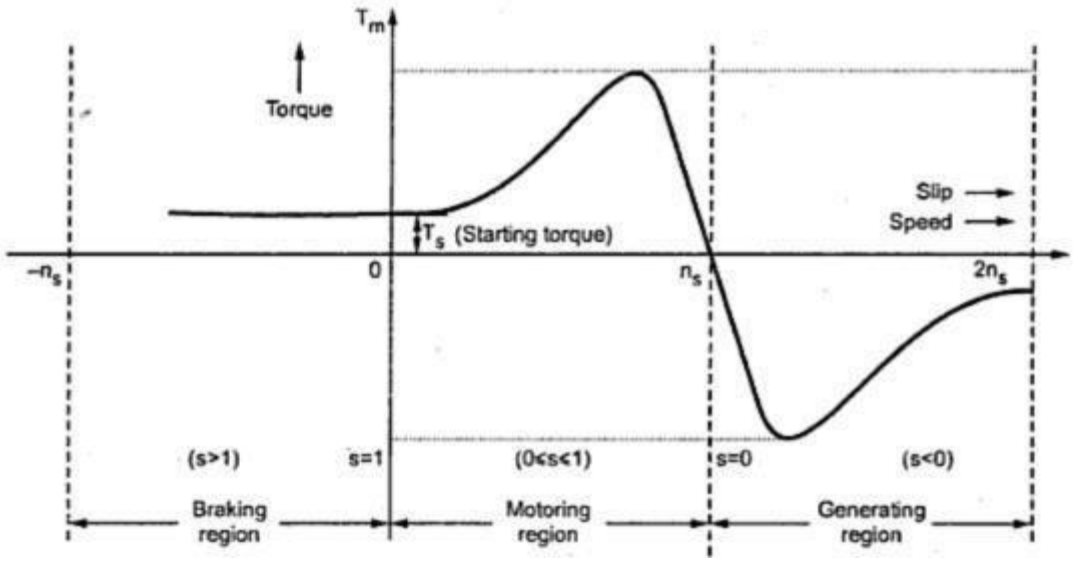


**Fig. 2 Regions of torque - slip characteristics**

**Torque Ratios**

The performance of the motor is sometimes expressed in terms of comparison of various torques such as full load torque, starting torque and maximum torque. The comparison is obtained by finding out ratios of these torques.

**Full load and Maximum Torque Ratio**

In general, $T \alpha \, (s E_2^2 R_2)/(R_2^2 + (s X_2)^2)$

Let $s_f$ = Full load slip

∴ $T_{F.L.} \, \alpha \, (s_f E_2^2 R_2)/(R_2^2 + (s_f X_2)^2)$

and $s_m$ = Slip for maximum torque $T_m$

∴ $T_m \, \alpha \, (s_m E_2^2 R_2)/(R_2^2 + (s_f X_2)^2)$

$$\therefore \quad \frac{T_{F.L.}}{T_m} = \frac{s_f E_2^2 R_2}{[R_2^2 + (s_f X_2)^2]} \times \frac{[R_2^2 + (s_m X_2)^2]}{s_m E_2^2 R_2}$$

$$\therefore \quad \frac{T_{F.L.}}{T_m} = \frac{s_f}{s_m} \times \frac{[R_2^2 + (s_m X_2)^2]}{[R_2^2 + (s_f X_2)^2]}$$

Dividing both numerator and denominator by $X_2^2$ we get,

$$\therefore \qquad \frac{T_{F.L.}}{T_m} = \frac{s_f}{s_m} \times \frac{\left[\frac{R_2^2}{X_2^2} + s_m^2\right]}{\left[\frac{R_2^2}{X_2^2} + s_f^2\right]}$$

But $R_2/X_2 = s_m$

$T_{F.L.}/T_m = (s_f \times 2 s_m^2)/(s_m \times (s_m^2 + s_f^2))$

$T_{F.L.}/T_m = (2 s_f s_m)/(s_m^2 + s_f^2)$

**Starting Torque and Maximum Torque Ratio:**

Against starting with torque equation as,

$T \alpha (s E_2^2 R_2)/(R_2^2 + (s X_2)^2)$

Now for $T_{st}$, $s = 1$

$T_{st} \alpha (E_2^2 R_2)/(R_2^2 + (X_2)^2)$

While for $T_m$, $s = s_m$

$$\therefore \qquad T_m \propto \frac{s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2}$$

$$\therefore \qquad \frac{T_{st}}{T_m} = \frac{E_2^2 R_2}{[R_2^2 + X_2^2]} \times \frac{[R_2^2 + (s_m X_2)^2]}{s_m E_2^2 R_2}$$

$$\therefore \qquad \frac{T_{st}}{T_m} = \frac{[R_2^2 + (s_m X_2)^2]}{s_m [R_2^2 + X_2^2]}$$

Dividing both numerator and denominator by $X_2^2$ we get,

$$\therefore \qquad \frac{T_{st}}{T_m} = \frac{\left[\frac{R_2^2}{X_2^2} + s_m^2\right]}{s_m \left[\frac{R_2^2}{X_2^2} + 1\right]}$$

Substituting $R_2/X_2 = s_m$

$$\therefore \qquad \boxed{\frac{T_{st}}{T_m} = \frac{2 s_m^2}{s_m (1 + s_m^2)} = \frac{2 s_m}{1 + s_m^2}}$$

Infact using the same method, ratio of any two torques at two different slip values can be obtained.

Sometimes using the relation, $R_2 = a X_2$ the torque ratios are expressed interms of constant a as,

$T_{F.L.}/T_m = (a s_f)/(a^2 + s_f^2)$

and $T_{st}/T_m = 2a/(1+a^2)$
where $a = R_2/X_2 = s_m$

**Example 1** : A 24 pole, 50 Hz, star connected induction motor has rotor resistance of 0.016 Ω per phase and rotor reactance of 0.265 Ω per phase at standstill. It is achieving its full load torque at a speed of 247 r.p.m. Calculate the ratio of
i) Full load torque to maximum torque ii) starting torque to maximum torque
**Solution** : Given values are,

$P = 24$, $f = 50$ Hz, $R_2 = 0.016\ \Omega$, $X_2 = 0.265\ \Omega$, $N = 247$ r.p.m.

$N_s = 120f / P = (120 \times 50)/24 = 250$ r.p.m.

$s_f = (N_s - N)/N_s = (250-247)/250 = 0.012$ = Full load slip

$s_m = R_2/X_2 = 0.016/0.265 = 0.06037$

i) $T_{F.L.}/T_m = (2\, s_m\, s_f)/(s_m^2 + s_f^2) = (2 \times 0.06037 \times 0.012)/(0.06037^2 + 0.012^2)$

ii) $T_{st}/T_m = (2\, s_m)/(1 + s_m^2) = (2 \times 0.06037)/(1 + 0.06037^2) = 0.1203$

**Speed Torque Characteristics:**

Uptill now, we have seen torque - slip characteristics of an induction motor. To compare the performance of induction motor with d.c. shunt and series motors, it is possible to plot speed-torque curve of an induction motor.

At $N = T_s$, the motor stops as it cannot produce any torque, as induction motor cannot rotate at synchronous motor.

At N = 0, the starting condition, motor produces a torque called starting torque.

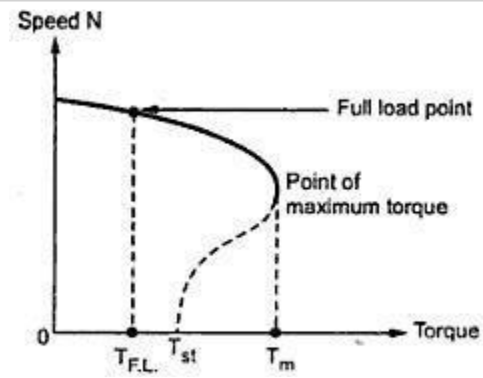


**Fig. 1 Speed Torque characteristics**

For low slip region, i.e. speed near the region is stable and the characteristics is straight in nature. Fall in speed from no load to full load is about 4 to 6 %. The characteristics is shown in the Fig.1. It can be seen from that the figure that for the stable region of operation, the characteristics is similar to that of d.c. shunt motor. Due to this, three phase induction motor is practically said to be 'constant speed' motor as drop in speed from no load to full load is not significant. The unstable region of operation is shown dotted in the Fig.1.

**Effect of Change in Rotor Resistance on Torque:**

It is shown that in slip ring induction motor, externally resistance can be added in the rotor. Let us see the effect of change in rotor resistance on the torque produced.

Let $R_2$ = Rotor resistance per phase

Corresponding torque, $T \alpha (s E_2^2 R_2)/\sqrt{}(R_2^2 + (s X_2)^2)$

Now externally resistance is added in each phase of rotor through slip rings.

Let $R_2'$ = New rotor resistance per phase

Corresponding torque $T' \alpha (s E_2^2 R_2')/\sqrt{}(R_2'^2 + (s X_2)^2)$

Similarly the starting torque at s = 1 for $R_2$ and $R_2'$ can be written as

$T_{st} \alpha (E_2^2 R_2)/\sqrt{}(R_2^2 + (X_2)^2)$

and $T'st \alpha (E_2^2 R'_2)/\sqrt{}(R'_2{}^2 + (X_2)^2)$

Maximum torque $T_m \alpha (E_2^2)/(2X_2)$

**Key Point** : It can be observed that $T_m$ is independent of $R_2$ hence whatever may be the rotor resistance, maximum torque produced never change but the slip and speed at which it occurs depends on $R_2$.

For $R_2$, $s_m = R_2/X_2$ where $T_m$ occurs

For $R_2'$, $s_m' = R_2'/X_2'$ where same $T_m$ occurs

As $R_2' > R_2$, the slip $s_m' > s_m$. Due to this, we get a new torque-slip characteristics for rotor resistance . This new characteristics is parallel to the characteristics for with same but $T_m$ occurring at $s_m'$. The effect of change in rotor resistance on torque-slip characteristics shown in the Fig. 1.

It can be seen that the starting torque T'st for $R_2'$ is more than Tst for $R_2$. Thus by changing rotor resistance the starting torque can be controlled.

If now resistance is further added to rotor to get resistance as $R_2'$ and so on, it can be seen that $T_m$ remains same but slip at which it occurs increases to $s_m'$ and so on. Similarly starting torque also increases to T'st and so on.

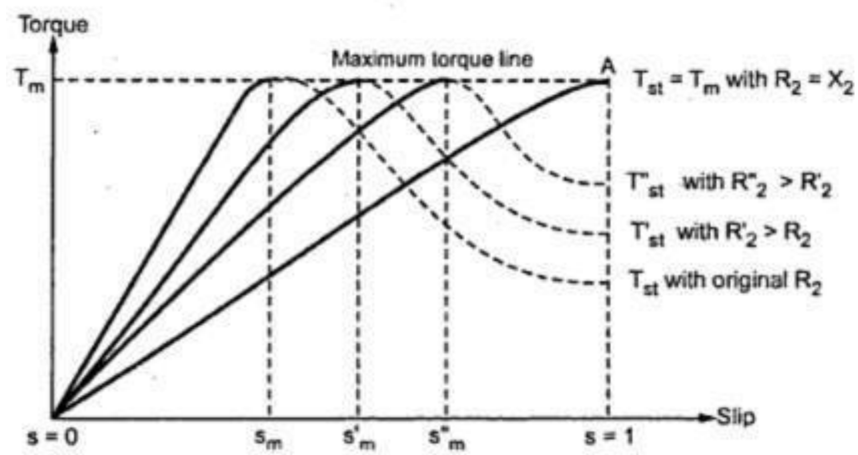


**Fig. 1 Effect of rotor resistance on torque-slip curves**

If maximum torque Tm is required at start then $s_m = 1$ as at start slip is always unity, so

$$s_m = R_2/X_2 = 1$$

$$R_2 = X_2 \qquad \text{Condition for getting } T_{st} = T_m$$

**Key Point** : Thus by adding external resistance to rotor till it becomes equal to $X_2$, the maximum torque can be achieved at start.

It is represented by point A in the Fig. 1.If such high resistance is kept permanently in the circuit, there will be large copper losses ($I^2$ R) and hence efficiency of the motor will be very poor. Hence such added resistance is cut-off gradually and finally removed from the rotor circuit, in the normal running condition of the motor. So this method is used in practice to achieve higher starting torque hence resistance in rotor is added only at start. Thus good performance at start and in the running condition is ensured.

**Key Point** : This is possible only in case of slip type of induction motor as in squirrel cage due to short circuited rotor, extra rotor resistance cannot be added.

**Example** : Rotor resistance and standstill reactance per phase of a 3 phase induction motor are 0.04 Ω and 0.2 Ω respectively. What should be the external resistance required at start in rotor circuit to obtain.
i) maximum torque at start ii) 50% of maximum torque at start.

**Solution** :

$R_2 = 0.04\ \Omega, \quad X_2 = 0.2\ \Omega$

i) For $T_m = T_{st}$, $\quad s_m = R_2'/X_2 = 1$

$\therefore \quad R_2' = X_2 = 0.2$

Let $R_{ex}$ = external resistance required in rotor.

$R_2' = R_2 + R_{ex}$

$\therefore \quad R_{ex} = R_2' - R_2 = 0.2 - 0.04 = 0.16\ \Omega$ per phase

ii) For $T_{st} = 0.5\,T_m$,

Now $T_m = (k\,E_2^2)/(2\,X_2)$ and

$T_{st} = (k\,E_2^2\,R_2)/(R_2^2 + X_2^2)$

But at start, external resistance $R_{ex}$ is added. So new value of rotor resistance is say $R_2'$.

$R_2' = R_2 + R_{ex}$

$\therefore \quad T_{st} = (k\,E_2^2\,R_2')/(R_2'^2 + X_2^2)$ with added resistance

but $T_{st} = 0.5T_m$ required.

Substituting expressions of $T_{st}$ and $T_m$, we get

$(k\,E_2^2\,R_2')/(R_2'^2 + X_2^2) = 0.5\,(k\,E_2^2)/\,(2X_2)$

$\therefore \quad 4\,R_2'\,X_2 = (R_2'^2 + X_2^2)$

$\therefore \quad (R_2'^2) - 4 \times 0.2 \times R_2' + 0.2^2 = 0$

$\therefore \quad (R_2'^2) - 0.8\,R_2' + 0.04 = 0$

$\therefore \quad R_2' = \{0.8 + \sqrt{(0.8^2 - 4 \times 0.04)}\}/2$

$\therefore \quad R_2' = 0.0535\ ,\ 0.7464\ \Omega$

But $R_2'$ can not greater than $X_2$ hence,

$R_2' = 0.0535 = R_2 + R_{ex}$

$\therefore \quad 0.0535 = 0.04 + R_{ex}$

$\therefore \quad R_{ex} = 0.0135\ \Omega$ per phase

This is much resistance is required in the rotor externally to obtain $T_{st} = 0.5\ T_m$.

# Module-3
## Performance of three phase induction motors

**Phasor Diagram of Induction Motor:**

The phasor diagram of loaded induction motor is similar to the loaded transformer. The only difference is the secondary of induction motor is rotating and short circuited while transformer secondary is stationary and connected to load. The load on induction motor is mechanical while load on transformer is electrical. Still by finding electrical equivalent of mechanical load on the motor, the phasor diagram of induction motor can be developed.

Let $\Phi$ = Magnetic flux links with both primary and secondary.

There is self induced e.m.f. $E_1$ in the stator while a mutually induced e.m.f. $E_{2r}$ in the rotor.

Let $R_1$ = Stator resistance per phase.

$X_1$ = Stator reactance per phase

The stator voltage per phase $V_1$ has to counter balance self induced e.m.f. $E_1$and has to supply voltage drops $I_1 R_1$ and $I_1 X_1$. So on stator side we can write,

$$\overline{V_1} = -\overline{E_1} + \overline{I_1 R_1} + j\,\overline{I_1 X_1} = \overline{E_1} + \overline{I_1}\,(R_1 + j\,X_1) = -\overline{E}_1 + \overline{I}_1\overline{Z}_1$$

The rotor induced e.m.f. in the running condition has to supply the drop across impedances as rotor short circuited.

$$\therefore \qquad \overline{E}_{2r} = \overline{I_{2r} R_2} + j\overline{I_{2r} X_2} = \bar{I}_{2r}\,(R_2 + jX_2) = \overline{I_{2r}}\;\overline{Z_{2r}}$$

The value of $E_{2r}$ depends on the ratio of rotor turns to stator turns.

The rotor current in the running condition is $I_{2r}$ which lags $E_{2r}$ by rotor p.f. angle $\Phi_{2r}$.

The reflected rotor current $I_{2r}'$ on stator side is the effect of load and is given by,

$$I_{2r}' = K\,I_{2r}$$

The induction motor draws no load current $I_o$ which is phasor sum of $I_c$ and $I_m$. The total stator current drawn from supply is,

$$\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}'$$

The $\Phi_1$ is angle between $V_1$ and $I_1$ and $\cos\Phi_1$ gives the power factor of the induction motor.

Thus using all above relations the phasor diagram of induction motor on load can be obtained.

The steps to draw phasor diagram are,

1. Takes $\Phi$ as reference phasor.
2. The induced voltage $E_1$ lags $\Phi$ by $90^o$.
3. Show - $E_1$ by reversing voltage phasor.
4. The phasor $E_{2r}$ is in phase with $E_1$. So $I_{2r}$ show lagging $E_{2r}$ i.e. $E_1$ direction by $\Phi_{2r}$.
5. Show $I_{2r} R_2$ in phase with $I_{2r}$ and $I_{2r} X_{2r}$ leading the resistive drop by $90^o$, to get exact location of.

6. Reverse $I_{2r}$ to get $I_{2r}'$.
7. $I_m$ is in phase with Φ while Ic is at leading with. Add $I_m$ and Ic to get $I_o$.
8. Add $I_o$ and $I_{2r}'$ to get $I_1$.
9. From tip of - $E_1$ phasor, add $I_1 R_1$ in phase with $I_1$ and $I_1X_1$ at $90^o$ leading to $I_1$ to $V_1$ get phasor.
10. Angle between $V_1$ and $I_1$ is $\Phi_1$.

The phasor diagram is shown in the Fig. 1.

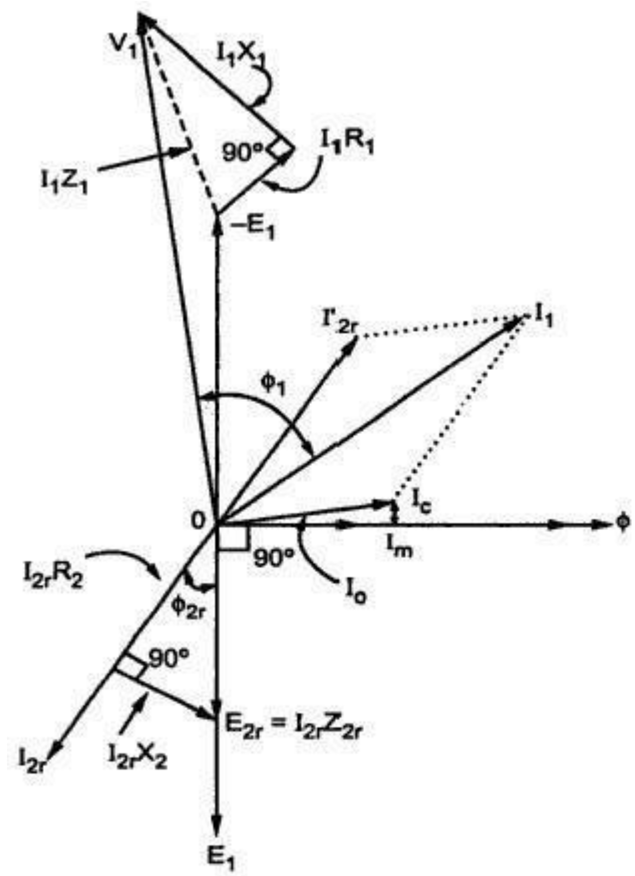


**Fig. 1 On load phasor diagram of induction motor**

**Equivalent Circuit of Induction Motor :**

We have already seen that the induction motor can be treated as generalized transformer. Transformer works on the principle of electromagnetic induction. The induction motor also works on the same principle. The energy transfer from stator to rotor of the induction motor takes place entirely with the help of a flux mutually linking the two. Thus stator acts as a primary while the rotor acts as a rotating secondary when induction motor is treated as a transformer.

If $E_1$ = Induced voltage in stator per phase

$E_2$ = Rotor induced e.m.f. per phase on standstill

k = Rotor turns / Stator turns

then $k = E_2 / E_1$

Thus if $V_1$ is the supply voltage per phase to stator, it produces the flux which links with both stator and rotor. Due to self induction $E_1$, is the induced e.m.f. in stator per phase while $E_2$ is the induced e.m.f. in rotor due to mutual induction, at standstill. In running condition the induced e.m.f. in rotor becomes $E_{2r}$ which is s $E_2$.

Now $E_{2r}$ = Rotor induced e.m.f. in running condition per phase

$R_2$ = Rotor resistance per phase

$X_{2r}$ = Rotor reactance per phase in running condition

$R_1$ = Stator resistance per phase

$X_1$ = Stator reactance per phase

So induction motor can be represented as a transformer as shown in the Fig. 1.

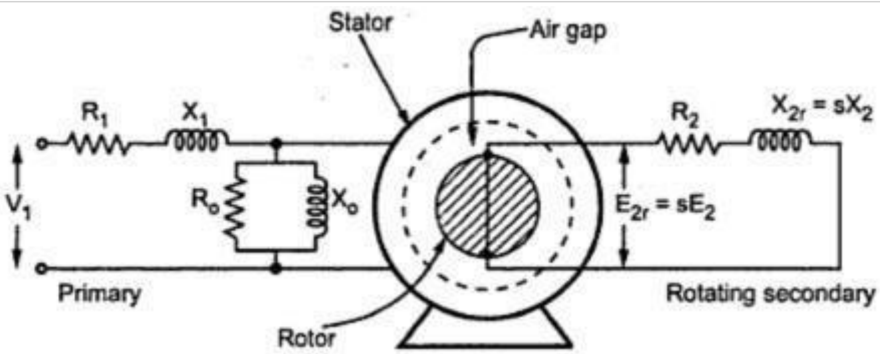


**Fig. 1 Induction motor as a transformer**

When induction motor is on no load, it draws a current from the supply to produce the flux in air gap and to supply iron losses.

1. $I_c$ = Active component which supplies no load losses
2. $I_m$ = Magnetizing component which sets up flux in core and air gap

These two currents give us the elements of an exciting branch as,

$R_o$ = Representing no load losses = $V_1 / I_c$

and $X_o$ = Representing flux set up = $V_1 / I_m$

Thus, $\bar{I}_o = \bar{I}_c + \bar{I}_m$

The equivalent circuit of induction motor thus can be represented as shown in the Fig. 2.

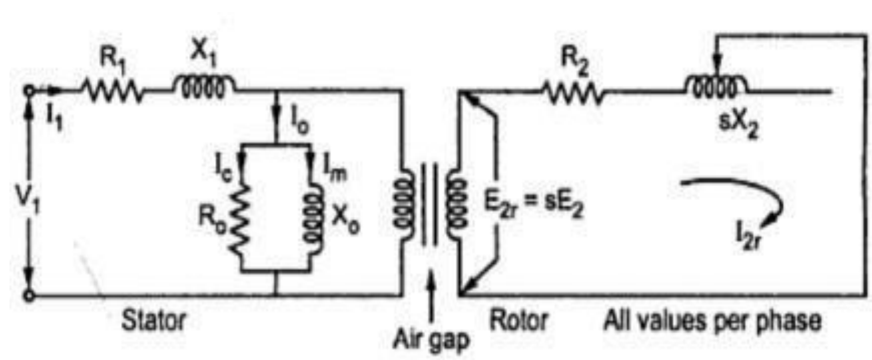


**Fig. 2 Basic equivalent circuit**

The stator and rotor sides are shown separated by an air gap.

$I_{2r}$ = Rotor current in running condition

$= E_{2r}/Z_{2r} = (s\,E_2)/\sqrt{(R_2^2 + (s\,X_2)^2)}$

It is important to note that as load on the motor changes, the motor speed changes. Thus slip changes. As slip changes the reactance $X_{2r}$ changes. Hence $X_{2r} = sX_2$ is shown variable.

**Representing of rotor impedance :**

It is shown that, $I_{2r} = (sE_2)/\sqrt{(R_2^2 + (s\,X_2)^2)} = E_2/\sqrt{((R_2/s)^2 + X_2^2)}$

So it can be assumed that equivalent rotor circuit in the running condition has fixed reactance $X_2$, fixed voltage $E_2$ but a variable resistance $R_2/s$, as indicated in the above equation.

Now $R_2/s = R_2 + (R_2/s) - R_2$

$$\therefore \quad \boxed{\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right) = R_2 + R_2\left(\frac{1-s}{s}\right)}$$

So the variable rotor resistance $R_2/s$ has two parts.

1. Rotor resistance $R_2$ itself which represents copper loss.
2. $R_2(1 - s)/s$ which represents load resistance $R_L$. So it is electrical equivalent of mechanical load on the motor.

**Key Point** : Thus the mechanical load on the motor is represented by the pure resistance of value $R_2(1 - s)/s$.

So rotor equivalent circuit can be shown as,

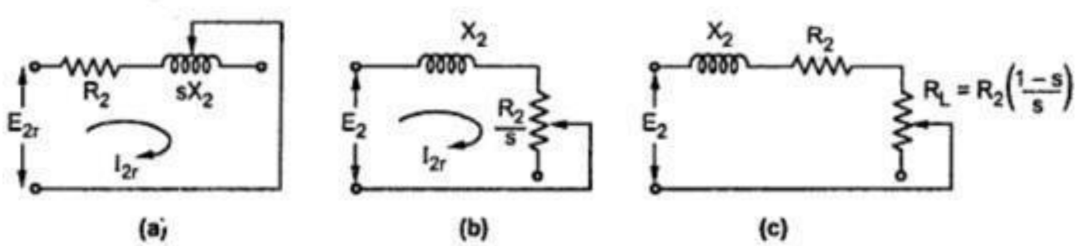


**Fig. 3 Rotor equivalent circuit**

Now let us obtain equivalent circuit referred to stator side.

**Equivalent circuit referred to stator :**

Transfer all the rotor parameters to stator,

$k = E_2/E_1$ = Transformation ratio

$E_2' = E_2/k$

The rotor current has its reflected component on the stator side which is $I_{2r}'$.

$I_{2r}' = k\,I_{2r} = (k\,s\,E_2)/\sqrt{(R_2^2 + (s\,X_2)^2)}$

$X_2' = X_2/K^2$ = Reflected rotor reactance

$R_2' = R_2/K^2$ = Reflected rotor resistance

$R_L' = R_L/K^2 = (R_2/K^2)(1\text{-}s / s)$
$= R_2' (1\text{-}s / s)$

Thus $R_L'$ is reflected mechanical load on stator.

So equivalent circuit referred to stator can be shown as in the Fig. 4

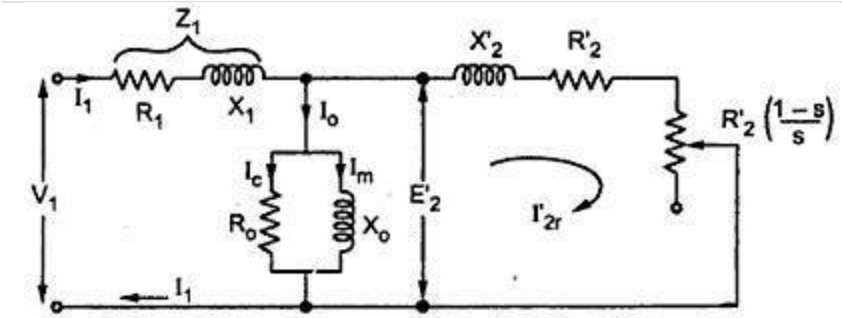


**Fig. 4 Equivalent circuit referred to stator**

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The resistance $R_2' (1 \text{-}s)/ s = R_L'$ is fictitious resistance representing the mechanical load on the motor.

**Approximate Equivalent Circuit**

Similar to the transformer the equivalent circuit can be modified by shifting the exciting current ($R_o$ and $X_o$) purely across the supply, to the left of $R_1$ and $X_1$. Due to this, we are neglecting the drop across $R_1$ and $X_1$ due to $I_o$, which is very small. Hence the circuit is called approximate equivalent circuit. The circuit is shown in the Fig.5.

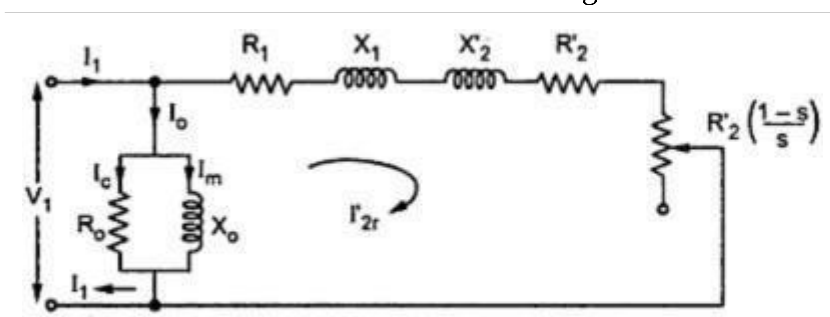


**Fig. 5 Approximate equivalent circuit**

Now the resistance $R_1$ and $R_2'$ while reactance $X_1$ and $X_2'$ can be combined. So we get,

$R_{1e}$ = Equivalent resistance referred to stator = $R_1 + R_2'$

$X_{1e}$ = Equivalent reactance referred to stator = $X_1 + X_2'$

$R_{1e} = R_1 + (R_2/K^2)$

and $X_{1e} = X_1 + (X_2/K^2)$

While $\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}'$**......................** phasor diagram

and $\bar{I}_o = \bar{I}_c + \bar{I}_m$

Thus the equivalent circuit can be shown in the Fig.6.

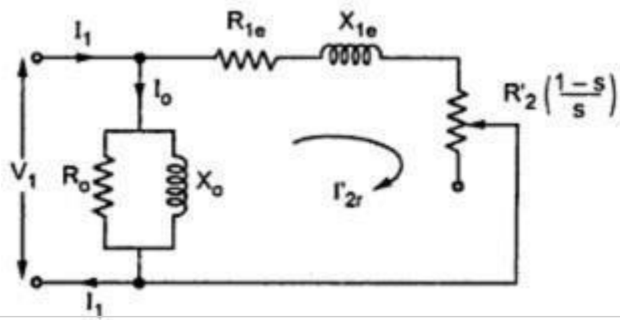


**Fig. 6**

## Power Equations from Equivalent Circuit

With reference to approximate equivalent circuit shown in the Fig. 6, we can write various power equations as,

$P_{in}$ = input power = 3 $V_1 I_1 \cos \Phi$

where $V_1$ = Stator voltage per phase

$I_1$ = Current drawn by stator per phase

$\cos \Phi$ = Power factor of stator

Stator core loss = $I_m^2 R_o$

Stator copper loss = 3 $I_1^2 R_o$

where $R_1$ = Stator resistance per phase

$P_2$ = Rotor input = $(3 I_{2r}'^2 R_2')/s$

$P_c$ = Rotor copper loss = 3 $I_{2r}'^2 R_2'$

Thus $P_c = s P_2$

$P_m$ = Gross mechanical power developed

$$\therefore \quad \boxed{P_m = P_2 - P_c = \frac{3 (I_{2r}')^2 R_2'}{s} - 3 (I_{2r}')^2 R_2' = 3 (I_{2r}')^2 R_2' \left(\frac{1-s}{s}\right)}$$

T = Torque developed

$$\therefore \quad \boxed{T = \frac{P_m}{\omega} = \frac{3 (I_{2r}')^2 R_2' \left(\frac{1-s}{s}\right)}{\frac{2\pi N}{60}}}$$

where N = Speed of motor

But N = $N_s$(1-s) =, so substituting in above

$$T = \frac{\frac{3(I'_{2r})^2 R'_2}{s}}{\frac{2\pi N_s}{60}} = 9.55 \times \frac{\frac{3(I'_{2r})^2 R'_2}{s}}{N_s} \quad \text{N-m}$$

and $I_{2r}' = V_1 /(( R_{1e} + R_L') + j X_{1e} )$

where $R_L' = R_2' (1-s)/s$

$I_{2r}' = V_1/\sqrt{(( R_{1e} + R_L')^2 + X_{1e}^2 )}$

**Key Point** : Remember that in all the above formula all the values per phase values.

**Maximum Power Output**

Consider the approximate equivalent circuit as shown in the Fig.7

In this circuit, the exciting current $I_o$ is neglected hence the exciting no load branch is not shown.

$\therefore$ $I_1 = I_{2r}'$

The total impedance is given by,

$Z_T = (R_{1e} + R_L')+$ where $R_L' = R_2' (1-s)/s$

$I_1 = V_1 /\sqrt{((R_{1e} + R_L')^2 +(X_{1e})^2)}$

The power supplied to the load i.e. $P_{out}$ per phase is,

Per phase $P_{out} = I_1^2 R_L'$ watts per phase

$\therefore$ Total = $3 I_1^2 R_L'$

$$\therefore \quad P_{out} = 3 \frac{V_1^2}{[(R_{1e} + R_L')^2 + (X_{1e})^2]} (R_L')$$

To obtain maximum output power, differentiate the equation of total $P_{out}$ with respect to variable $R_L'$ and equal to zero.

$$\therefore \quad \frac{d}{dR_L'}\left[\frac{3 V_1^2 (R_L')}{[(R_{1e} + R_L')^2 + (X_{1e})^2]}\right] = 0$$

$$\therefore \quad [(R_{1e} + R_L')^2 + (X_{1e})^2] [3 V_1^2] - 3 V_1^2 (R_L') [2 (R_{1e} + R_L')] = 0$$

$$\therefore \quad (R_{1e} + R_L')^2 + (X_{1e})^2 - 2 (R_L') (R_{1e} + R_L') = 0 \qquad \text{... Taking } 3 V_1^2 \text{ common}$$

$$\therefore \quad R_{1e}^2 + (R_L')^2 + 2 R_{1e} R_L' + X_{1e}^2 - 2 R_{1e} R_L' - 2(R_L')^2 = 0$$

$$\therefore \quad R_{1e}^2 + X_{1e}^2 = (R_L')^2$$

But $Z_{1e} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$ = Leakage impedance referred to stator

$\therefore$ $Z_{1e}^2 = R_L'^2$

Thus the mechanical load on the induction motor should be such that the equivalent load resistance referred to stator is equal to the total leakage impedance of motor referred to stator.

**Slip at maximum $P_{out}$** : This can be obtained as,

$R_L' = Z_{1e} = R_2'(1-s)/s$ where $R_L' = R_2/K^2$

$\therefore$ $s Z_{1e} = R_2' - sR_2'$

$\therefore$ $s(Z_{1e} + R_2') = R_2'$

$$\boxed{s = \frac{R_2'}{(R_2' + Z_{1e})}}$$

This is slip at maximum output.

**Expression for maximum $P_{out}$** : Using the condition obtained in expression of total $P_{out}$ , we can get maximum $P_{out}$.

$\therefore$ $(P_{out})_{max} = 3 I_1^2 Z_{1e}$ as $R_L' = Z_{1e}$

$$= 3 \frac{V_1^2}{(R_{1e}+Z_{1e})^2 + (X_{1e})^2} \cdot Z_{1e} \quad \text{as} \quad R_L' = Z_{1e}$$

$$= 3 \frac{V_1^2}{(R_{1e}^2 + 2R_{1e} Z_{1e} + Z_{1e}^2 + X_{1e}^2)} \cdot Z_{1e}$$

But $R_{1e}^2 + X_{1e}^2 = Z_{1e}$

$$\therefore \quad (P_{out})_{max} = 3 \frac{V_1^2}{2Z_{1e}^2 + 2R_{1e} Z_{1e}} \cdot Z_{1e} = 3 \frac{V_1^2}{2Z_{1e} + (R_{1e} + Z_{1e})} \cdot Z_{1e}$$

$$\therefore \quad \boxed{(P_{out})_{max} = \frac{3V_1^2}{2(R_{1e} + Z_{1e})} \text{ watts}}$$

**Maximum Torque**

In case of induction motor, the speed of the motor decreases with increase in load. Thus the maximum power output is not obtained at a slip which corresponds to maximum torque. In the previous section we have seen the condition for maximum power output. In this section we will find the condition which gives maximum torque.

The expression for torque is given by,

$$T = \frac{3(I'_{2r})^2 R'_2}{s\,\omega_s} = \frac{3(I'_{2r})^2 R'_2}{s\left(\frac{2\pi N_s}{60}\right)}$$

The condition for maximum torque can be obtained from maximum power transfer theorem. When $I_{2r}'^2\, R_2'/s$ is maximum consider the approximate equivalent circuit of induction motor as shown in The Fig. 8.

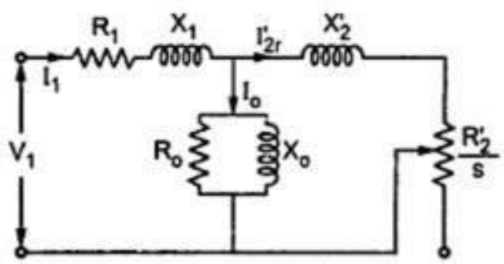


**Fig. 8**

The value of $R_o$ is assumed to be negligible. Hence the circuit will be reduced as shown below.

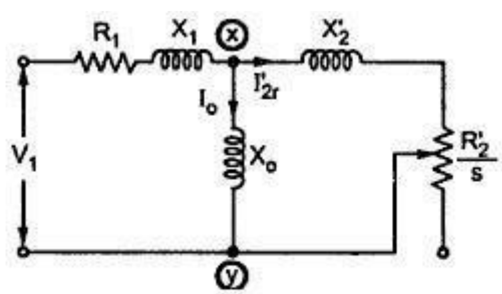


**Fig. 9**

The thevenin's equivalent circuit for the above network is shown in the Fig.10 across the terminals x and y.

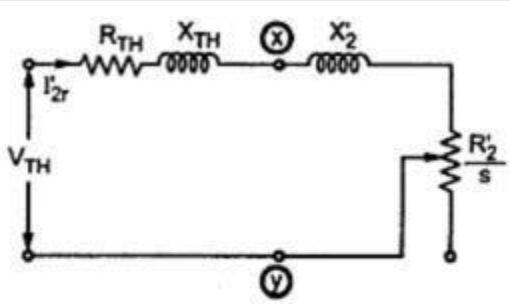


**Fig. 10**

Now, $$Z_{TH} = R_{TH} + jX_{TH} = (R_1 + j\,X_1) \parallel j\,X_o$$

$$V_{TH} = \frac{V_1(jX_o)}{R_1 + j(X_{TH} + X_o)}$$

The mechanical torque developed by rotor is maximum if there is maximum power transfer to the resistor $R_2'/s$. This takes place when $R_2'/s$ equals to impedance looking back into the supply source.

$$\frac{R_2'}{s} = R_{TH} + j\left(X_{TH} + X_2'\right)$$

$$\frac{R_2'}{s} = \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}$$

$$\therefore \quad s = s_m = \frac{R_2'}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}}$$

This is the slip corresponding to the maximum torque. The maximum torque is given by,

$$T_m = \frac{3}{\omega_s} \cdot \frac{R_2'}{s_m} \cdot (I_{2r}')^2$$

$$\frac{R_2'}{s_m} = \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}$$

$$\therefore \quad I_{2r}' = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2'}{s_m}\right)^2 + \left(X_{TH} + X_2'\right)^2}}$$

$$I_{2r}'^2 = \frac{V_{TH}^2}{\left(R_{TH} + \frac{R_2'}{s_m}\right)^2 + \left(X_{TH} + X_2'\right)^2}$$

Substituting,

$$T_m = \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2} \cdot \frac{V_{TH}^2}{\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}\right]^2 + \left(X_{TH} + X_2'\right)^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2} \cdot \frac{V_{TH}^2}{2R_{TH}^2 + 2R_{TH}\sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2} + 2\left(X_{TH} + X_2'\right)^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2} \cdot \frac{R_{TH}^2}{2\left[R_{TH}^2 + \left(X_{TH} + X_2'\right)^2 + R_{TH}\sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}\right]}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2} \cdot \frac{0.5\,V_{TH}^2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2'\right)^2}\right]}$$

$$\therefore \quad T_m = \frac{3}{\omega_s} \cdot \frac{0.5\, V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}}$$

From the above expression, it can be seen that the maximum torque is independent of rotor resistance.

**Synchronous Watt:**

The torque produced in the induction motor is given by,

$$T = \frac{\dfrac{3(I_{2r}')^2\, R_2'}{s}}{\dfrac{2\pi N_s}{60}} = \frac{P_2}{\dfrac{2\pi N_s}{60}} \text{ N-m}$$

Thus torque is directly proportional to the rotor input. By defining new unit of torque which is synchronous watt we can write,

$T = P_2$ synchronous-watts

If torque is given in synchronous-watts then it can be obtained in N-m as,

$$1 \text{ syn-watt} = \frac{60}{2\pi N_s} \text{ N-m}$$

$$\text{i.e.} \quad 1 \text{ N-m} = \frac{2\pi N_s}{60} \text{ syn-watt}$$

**Key Point** : Unit synchronous watt can be defined as the torque developed by the motor such that the power input to the rotor across the air gap is 1 W while running at synchronous speed.

**Losses in Induction Motor:**

The various power losses in an induction motor can be classified as,

i) Constant losses

ii) Variable losses

**i) Constant losses :**

These can be further classified as core losses and mechanical losses.

Core losses occur in stator core and rotor core. These are also called iron losses. These losses include eddy current losses and hysteresis losses. The eddy current losses are minimizsed by using laminated construction while hysteresis losses are minimized by selecting high grade silicon steel as the material for stator and rotor.

The iron losses depends on the frequency. The stator frequency is always supply frequency hence stator iron losses are dominate. As against this in rotor circuit, the frequency is very small which is slip times the supply frequency. Hence rotor iron losses are very small and hence generally neglected, in the running condition.

The mechanical losses include frictional losses at the bearings and windings losses. The friction changes with speed but practically the drop in speed is very small hence these losses are assumed to be the part of constant losses.

**ii) Variable losses :**

This include the copper losses in stator and rotor winding due to current flowing in the winding. As current changes as load changes as load changes, these losses are said to be variable losses.

Generally stator iron losses are combined with stator copper losses at a particular load to specify total stator losses at particular load condition.

Rotor copper loss = $3 I_{2r}^2 R_2$ .............................Analysed separately

where $I_{2r}$ = Rotor current per phase at a particular load

$R_2$ = Rotor resistance per phase

**Power Flow in an Induction Motor**

Induction motor converts an electrical power supplies to it into mechanical power. The various stages in this conversion is called power flow in an inductor motor.

The three phase supply given to the stator is the net electrical input to the motor. If motor power factor is cos Φ and $V_L$, $I_L$ are line values of supply voltage and current drawn, then net electrical supplied to the motor can be calculated as,

$$\boxed{P_{in} = \sqrt{3}\, V_L\, I_L \cos\phi}$$

Where $\boxed{P_{in} = \text{Net input electrical power.}}$

This is nothing but the stator input.

The part of this power is utilized to supply the losses in the stator which are stator core as well as copper losses.

The remaining power is delivered to the rotor magnetically through the air gap with the help of rotating magnetic field. This is called rotor input denoted as $P_2$.

So $$\boxed{P_2 = P_{in} - \text{stator losses (core + copper)}}$$

The rotor is not able to convert its entire input to the mechanical as it has to supply rotor losses. The rotor losses are dominantly copper losses as rotor iron losses are very small and hence generally neglected. So rotor losses are rotor copper losses denoted as $P_c$.

So $$\boxed{P_c = 3 \times I_{2r}^2 \times R_2}$$

where $I_{2r}$ = Rotor current per phase in running condition

$R_2$ = Rotor resistance per phase.

After supplying these losses, the remaining part of $P_2$ is converted into mechanical which is called gross mechanical power developed by the motor denoted as $P_m$.

$$\therefore \quad \boxed{P_m = P_2 - P_c}$$

Now this power, motor tries to deliver to the load connected to the shaft. But during this mechanical transmission, part of $P_m$ is utilized to provide mechanical losses like friction and windage.

And finally the power is available to the load at the shaft. This is called net output of the motor denoted as $P_{out}$. This is also called shaft power.

$$\therefore \quad \boxed{P_{out} = P_m - \text{Mechanical losses.}}$$

The rating of the motor is specified in terms of value of $P_{out}$ when load condition is full load condition.

The above stages can be shown diagrammatically called power flow diagram of an induction motor.

This is shown in the Fig.1.

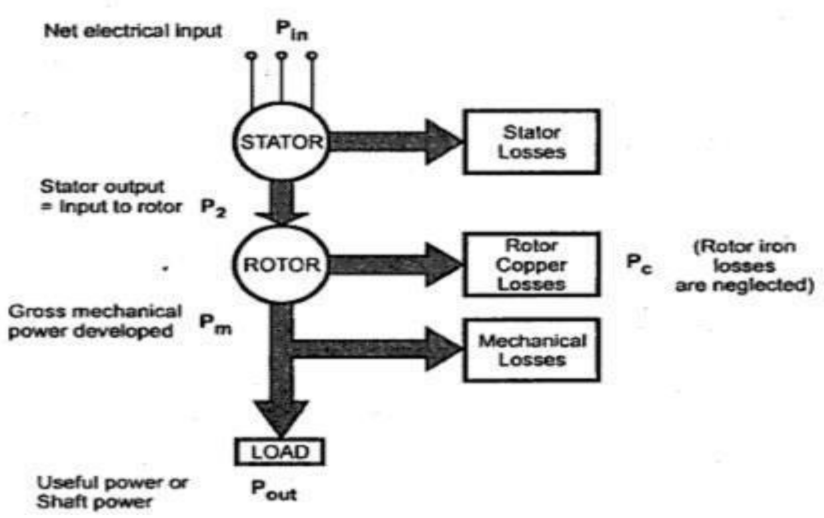


**Fig. 1 Power flow diagram**

From the power flow diagram we can define,

$$\text{Rotor efficiency} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{gross mechanical power developed}}{\text{rotor input}}$$

$$= \mathbf{P_m / P_2}$$

$$\text{Net motor efficiency} = \frac{\text{net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_{in}}$$

**Relation between P2, Pc, and Pm:**

The rotor input $P_2$, rotor copper loss $P_c$ and gross mechanical power developed $P_m$ are related through the slip s. Let us derive this relationship.

Let $T$ = Gross torque developed by motor in N-m.

We know that the torque and power are related by the relation,

$$P = T \times \omega$$

where P = Power

and $\omega$ = angular speed

$= (2\pi N)/60$ , N = speed in r.p.m.

Now input to the rotor $P_2$ is from stator side through rotating magnetic field which is rotating at synchronous speed $N_s$.

So torque developed by the rotor can be expressed interms of power input and angular speed at which power is inputted i.e. $\omega_s$ as,

$P_2 = T \times \omega_s$ where $\omega_s = (2\pi N_s)/60$ rad/sec

$P_2 = T \times (2\pi N_s)/60$ where $N_s$ is in r.p.m ..................... (1)

The rotor tries to deliver this torque to the load. So rotor output is gross mechanical power developed $P_m$ and torque T. But rotor gives output at speed N and not $N_s$. So from output side $P_m$ and T can be related through angular speed $\omega$ and not $\omega_s$.

$P_m = T \times \omega$ where $\omega = (2\pi N)/60$

$P_m = T \times (2\pi N)/60$ ..........................................................(2)

The difference between $P_2$ and $P_m$ is rotor copper loss $P_c$.

$P_c = P_2 - P_m = T \times (2\pi N_s/60) - T \times (2\pi N/60)$

$P_c = T \times (2\pi/60)(N_s - N)$ = rotor copper loss ....................... (3)

Dividing (3) by (1),s

$$\frac{P_c}{P_2} = \frac{T\times\frac{2\pi}{60}(N_s - N)}{T\times\frac{2\pi}{60}\times N_s} = \frac{N_s - N}{N_s}$$

$P_c/P_2 = s$ as $(N_s - N)/N_s =$ slip s

Rotor copper loss $P_c = s$ x Rotor input $P_2$

Thus total rotor copper loss is slip times the rotor input.

Now $P_2 - P_c = P_m$

$P_2 - sP_2 = P_m$

$(1 - s)P_2 = P_m$

Thus gross mechanical power developed is (1 - s) times the rotor input

The relationship can be expressed in the ratio from as,

$$\boxed{P_2 : P_c : P_m \quad \text{is} \quad 1 : s : 1 - s}$$

The ratio of any two quantities on left hand side is same as the ratio of corresponding two sides on the right hand side.

For example, $\frac{P_c}{P_m} = \frac{s}{1-s}$, $\frac{P_2}{P_c} = \frac{1}{s}$ and so on.

This relationship is very important and very frequently required to solve the problems on the power flow diagram.

**Key Point** : The torque produced by rotor is gross mechanical torque and due to mechanical losses entire torque cannot be available to drive load. The load torque is net output torque called shaft torque or useful torque and is denoted as $T_{sh}$. It is related to $P_{out}$ as,

$$\boxed{T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)}}$$

and $T_{sh} < T$ due to mechanical losses.

**Derivation of k in Torque Equation**

We have seen earlier that

$$T = (k\, s\, E_2^2\, R_2)/(R_2^2 + (s\, X_2)^2)$$

and it mentioned that $k = 3/(2\pi\, n_s)$ . Let us see its proof.

The rotor copper losses can be expressed as,

$$P_c = 3 \times I_{2r}^2 \times R_2$$

but $I_{2r} = (s\, E_2)/\sqrt{(R_2^2 + (s\, X_2)^2)}$, hence substituting above

$$P_c = 3\times\left[\frac{s\,E_2}{\sqrt{R_2^2 + (sX_2)^2}}\right]^2 \times R_2$$

$$\therefore \quad P_c = \frac{3 s^2 E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Now as per $P_2 : P_c : P_m$ is $1 : s : 1\text{-}s$ ,

$$P_c/P_m = s/(1-s)$$

Now $P_m = T \times \omega$

$= T \times (2\pi N/60)$

$$\therefore \quad T\times\frac{2\pi N}{60} = \frac{(1-s)\,3\, s\, E_2^2\, R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \quad T = \frac{60}{2\pi N}\times\frac{(1-s)\,3\, s\, E_2^2\, R_2}{R_2^2 + (sX_2)^2}$$

Now $N = N_s(1\text{-}s)$ from definition of slip, substituting in above,

$$\therefore \quad T = \frac{60}{2\pi N_s\,(1-s)}\times\frac{(1-s)\,3\, s\, E_2^2\, R_2}{R_2^2 + (sX_2)^2}$$

$$= \frac{3}{2\pi\left(\frac{N_s}{60}\right)}\times\frac{s\,E_2^2\,R_2}{R_2^2 + (sX_2)^2}$$

but $N_s/60 = n_s$ in r.p.m.

So substituting in the above equation,

$$T = \frac{3}{2\pi n_s} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Comparing the two torque equations we can write,

$$\boxed{k = \frac{3}{2\pi n_s}} \text{ where } n_s \text{ is in r.p.s.}$$

**Efficiency of an Induction Motor:**

The ratio of net power available at the shaft ($P_{out}$) and the net electrical power input ($P_{in}$) to the motor is called as overall efficiency of an induction motor.

$$\therefore \quad \% \eta = \frac{P_{out}}{P_{in}} \times 100$$

The maximum efficiency occurs when variable losses become equal to constant losses. When motor is on no load, current drawn by the motor is small. Hence efficiency is low. As load increases, current increases so copper losses also increases. When such variable losses achieve the same value as that of constant losses, efficiency attains its maximum value. If load is increased further, variable losses becomes greater than constant losses hence deviating from condition for maximum, efficiency starts decreasing. Hence the nature of the curve of efficiency against output power of the motor is shown in the Fig. 1.

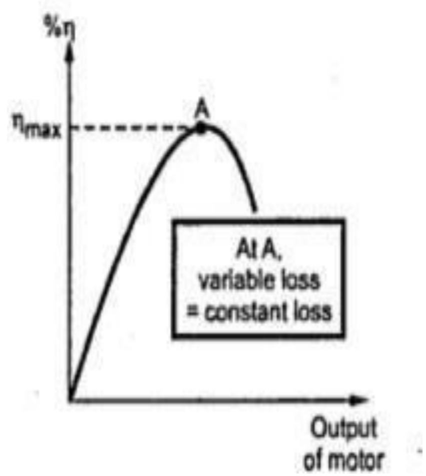


**Fig. 1 Efficiency curve for an induction motor**

**No Load Test:**

In this test, the motor is made to run without any load i.e. no load condition. The speed of the motor is very close to the synchronous speed but less than the synchronous speed. The rated voltage is applied to the stator. The input line current and total in put power is measured. The two wattmeter method is used to measure the total input power. The circuit diagram for the test is shown in the Fig. 1.

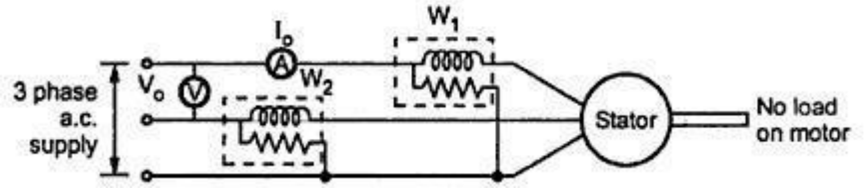


**Fig. 1 No load test**

As the motor is on no load, the power factor is very low which is less than 0.5 and one of the two wattmeters read negative. It is necessary to reverse the current coil or pressure coil connections of such a wattmeter to get the positive reading. This reading must be taken negative for the further calculations.

The total power input $W_o$ is the algebraic sum of the two wattmeter readings. The observation table is,

| $V_o$ volts Rated line voltage | $I_o$ Amp No load current | $W_o = W_1 + W_2$ (Algebraic sum) in watts |
|---|---|---|
| | | |

The calculations are,

$$W_o = \sqrt{3} V_o I_o \cos\Phi_o$$

$$\therefore \quad \cos\phi_o = \frac{W_o}{\sqrt{3}\, V_o I_o} \quad \text{where } V_o, I_o \text{ are line values}$$

**This is no load power factor.**

Thus we are now in a position to obtain magnitude and phase angle of no load current $I_o$, which is required for the circle diagram.

From the knowledge of $I_o$ and $\Phi_o$, the parameters of the equivalent circuit can be obtained as,

$I_c = I_o \cos\Phi_o$ = Active component of no load current

$I_m = I_o \sin\Phi_o$ = Magnetising component of no load current

$R_o = V_o$ (per phase) / $I_c$ (per phase) = No load branch resistance

$X_o = V_o$ (per phase)/ $I_m$ (per phase) = No load branch resistance

The power input $W_o$ consists of following losses,

1. Stator copper loss i.e. $3\, I_o^2 R_1^2$ where $I_o$ is no load per phase current and $R_1$ is stator resistance per phase.
2. Stator core loss i.e. iron loss.
3. Friction and windage loss.

The no load rotor current is very small and hence rotor copper loss is negligibly small. The rotor frequency is s times supply frequency and on no load it is very small. Rotor iron losses are proportional to this frequency and hence are negligibly small.

**Key Point** : Under no load condition, $I_o$ is also very small and in many practical cases it is also neglected.
Thus $W_o$ consists of stator iron loss and friction and windage loss which are consists for all load condictions. Hence $W_o$ is said to give fixed losses of the motor.

∴ $W_o$ = No load power input

∴ $\boxed{W_o = \text{Fixed Loss}}$ ... Neglecting stator copper loss

**Separating No Load Losses:**

The no load losses are the constant losses which include core loss and friction and windage loss. The separation between the two can be carried out by the no load test conducted from variable voltage, rated frequency supply.

When the voltage is decreased below the rated value, The core loss reduces as nearly square of voltage. The slip does not increase significantly the friction and windage loss almost remains constant.

The voltage is continuously decreased till the machine slip suddenly begins to increase and the motor tends to stall. At no load, this takes place at a sufficiently reduced voltage. The graph showing no load losses $P_{N.L.}$ versus as shown in the Fig. is extrapolated to V = 0 which gives friction and windage loss as iron or core loss is zero at zero voltage.

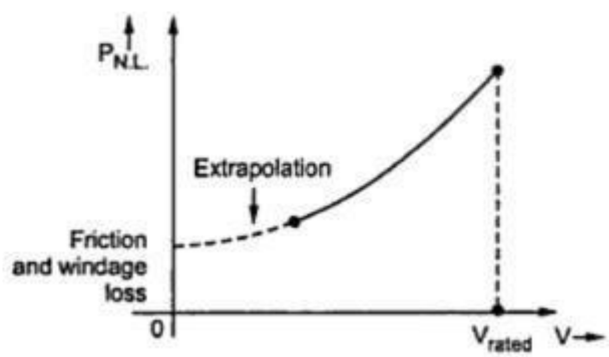


**Fig.3**

**Blocked Rotor Test**

In this test, the rotor is locked and it is not allowed to rotate. Thus the slip s = 1 and $R_L'$ = $R_2'$ (1-s)/s is zero. If the motor is slip ring induction motor then the windings are short circuited at the slip rings.

The situation is exactly similar to the short circuit test on transformer. If under short circuit condition, if primary is excited with rated voltage, a large shot circuit current can flow which is dangerous from the windings point of view. So similar to the transformer short circuit test, the reduced voltage (about 10 to 15 % of rated voltage) just enough such that stator carries rated current is applied. Now the applied voltage $V_{sc}$, the input power $W_{sc}$ and a short circuit current $I_{sc}$ are measured.

As $R_L' = 0$, the equivalent circuit is exactly similar to that of a transformer and hence the calculations are similar to that of short circuit test on a transformer.

$V_{sc}$ = Short circuit reduced voltage (line value)

$I_{sc}$ = Short circuit current (line value)

$W_{sc}$ = Short circuit input power

Now $W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos\Phi_{sc}$.......................................................... Line values

$$\therefore \quad \cos\phi_{sc} = \frac{W_{sc}}{\sqrt{3}\, V_{sc}\, I_{sc}}$$

This gives us short circuit power factor of a motor.

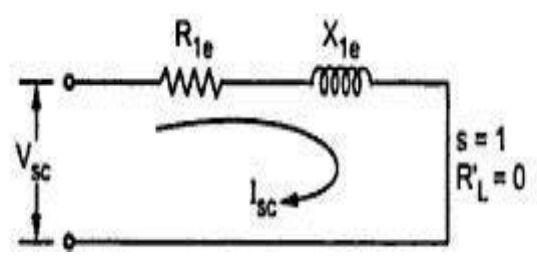


**Fig. 1**

Now the equivalent circuit is as shown in the Fig. 1.

$$\therefore \quad W_{sc} = 3\,(I_{sc})^2\, R_{1e}$$

where $I_{sc}$ = Per phase value

$$\therefore \quad R_{1e} = \frac{W_{sc}}{3\,(I_{sc})^2}$$

This is equivalent resistance referred to stator.

$Z_{1e} = V_{sc}$ (per phase)/ $I_{sc}$ (per phase) = Equivalent impedance referred to stator.

$$\therefore \quad R_{1e} = \frac{W_{sc}}{3\,(I_{sc})^2}$$

$$\therefore \quad X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$$

$$= \text{Equivalent reactance referred to stator}$$

During this test, the stator carries rated current hence the stator copper loss is also dominant. Similarly the rotor also carries short circuit current to produce dominant rotor copper loss. As the voltage is reduced, the iron loss which is proportional to voltage is negligibly small. The motor is at standstill hence mechanical loss i.e. friction and windage loss is absent. Hence we can write,

$$W_{sc} = \text{Stator copper loss} + \text{Rotor copper loss}$$

But it is necessary to obtain short circuit current when normal voltage is applied to the motor. This is practically not possible. But the reduced voltage test results can be used to find current $I_{SN}$ which is short circuit current if normal voltage is applied.

If $V_L$ = Normal rated voltage (line value)

$V_{sc}$ = Reduced short circuit voltage (line voltage)

then $$I_{SN} = \left(\frac{V_L}{V_{sc}}\right) \times I_{sc}$$

where $I_{sc}$ = Short circuit current at reduced voltage

Thus, $I_{SN}$ = Short circuit current at normal voltage

Now power input is proportional to square of the current.

So $W_{SN}$ = Short circuit input power at normal voltage

This can be obtained as,

$$W_{SN} = \left(\frac{I_{SN}}{I_{sc}}\right)^2 W_{sc}$$

But at normal voltage core loss cannot be negligible hence,

$W_{SN}$ = Core loss + Stator and rotor copper loss

**Circle Diagram :**

**Introduction**

In a particular circuit, if one of the circuit elements is variable, then depending upon its value, the circuit characteristics varies. As the value of the variable element is changed, the circuit parameters like current, power factor, power losses etc. also change. The locus of the extremity of the current phasor, obtained for various values of a variable element is called a locus diagram.

From the equivalent circuit of an induction motor, the motor can be treated as series R-L circuit where the element resistance of the circuit is variable which varies as slip s. Thus for variable load conditions, the resistance changes and hence the current drawn by the motor also changes. The locus diagram of such a current phasor is circular in nature and hence called circle diagram of a three phase induction motor. Using this diagram, all the performance characteristics of an induction motor like power factor, efficiency, stator losses, rotor losses, maximum output, maximum torque etc. can be predicted. Thus, a circle diagram is a graphical approach of predetermining the operation characteristics of an induction motor.

Let us prove that the locus diagram obtained for a current phasor is a circle, for a series R-L circuit with an element R as variable.

**Circle Diagram for a Series R-L Circuit:**

Consider a series R-L circuit with a variable R as shown in the Fig. 1. It is excited by an alternating source of V volts. The frequency of the source is f Hz.

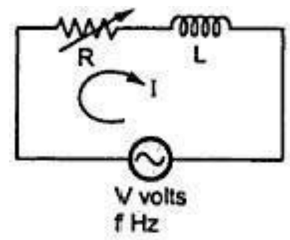


**Fig. 1**

Let $I$ = Current flowing through the circuit

$Z$ = Impedance of the circuit

$Z = R + j X_L$ where $X_L = 2\prod fL$

Now R is variable while $X_L$ is fixed.

$$\therefore \quad I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$= \frac{V}{X_L} \cdot \frac{X_L}{\sqrt{R^2 + X_L^2}} \quad \text{... Multiply and divide by } X_L$$

The phasor diagram is shown in the Fig. 2(a). The current I lags voltage V by angle as the circuit is inductive. The impedance triangle is shown in the Fig. 2(b).

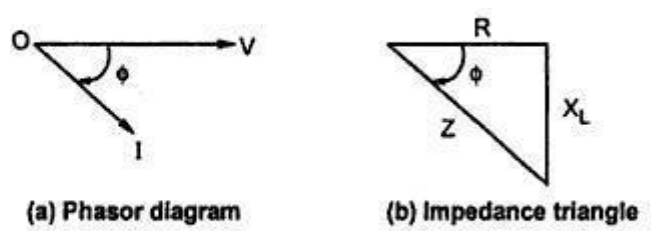


**Fig. 2**

From the impedance triangle we can write,

$\sin \Phi = X_L/Z$

Substituting in the expression for I,

$I = (V/X_L) \sin\Phi$ ........................................................ (1)

This is the equation of a circle in polar co-ordinates with a diameter equal to $(V/X_L)$.

When the resistance R =0, then $\Phi = 90^o$ hence $\sin \Phi = 1$.

$\therefore \quad I = I_m = (V/X_L)$

This is the maximum value of current.

As R resistance, the phase angle decreases thus decreasing sin. Effectively current I also decrease. When $R \rightarrow \infty$ the $\Phi \rightarrow 0^o$ and current becomes zero.

The locus obtained of extremities of a current phasor plotted for various values of R is a semicircle. The semicircle is shown in the Fig. 3. The voltage axis is taken as vertical axis as a reference, with respect to which the various current phasors are plotted.

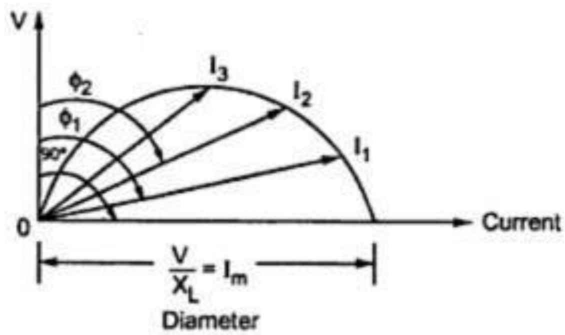


**Fig. 3 Circle diagram**

The power factors at various conditions are $\cos\Phi_1$, $\cos\Phi_2$ etc. As $\Phi$ varies only from $0^o$to $90^o$, the diagram is semicircle, infact it is a half part of a circle hence it is known as circle diagram.

This theory of series R-L circuit can be easily extended to a three phase induction motor.

Circle Diagram of a 3 Phase Induction Motor

The equivalent circuit of a 3 phase induction motor is shown in the Fig.1.

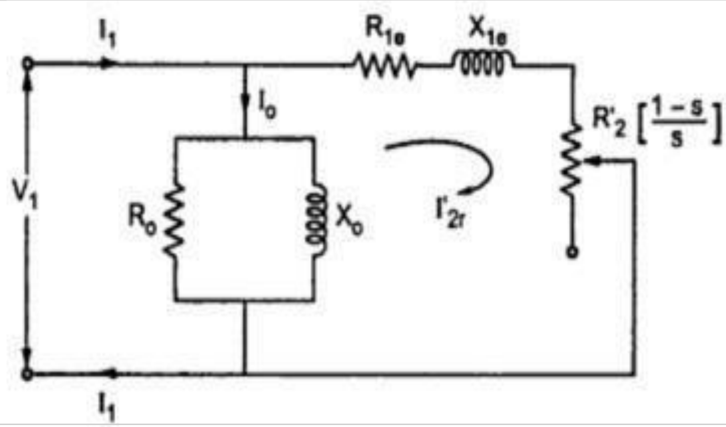


**Fig. 1 Equivalent circuit of a 3 phase induction motor**

All the values shown are per phase values. The circuit is similar to series R-L circuit. The reactance $X_{1e}$ is fixed while the total resistance $R_{1e}$ + ($R_2$'(1-s)/s) is variable. This is because the slip s varies as load varies. The voltage across the parallel exciting branch is $V_1$. Hence we can write the expression for the rotor current referred to stator as,

$$I_{2r}' = V_1/\sqrt{(R_{1e} + R_L')^2 + X_{1e}^2}$$

Where $R_L' = R_2'(1-s)/s$ = Variable equivalent load resistance

$R_{1e} = R_1 + R_2'$ = Equivalent resistance of motor referred to stator

$X_{1e} = X_1 + X_2'$ = Equivalent reactance of motor referred to stator

Dividing and multiplying by,

$$I'_{2r} = \frac{V_1}{X_{1e}} \times \frac{X_{1e}}{\sqrt{(R_{1e} + R'_L)^2 + (X_{1e})^2}}$$

∴ $I_{2r}' = I_{max}$ sin ....................................................(1)

where $\sin\Phi = X/Z = X_{1e}/\sqrt{((R_{1e} + R_L')^2 + X_{1e}^2)}$

and $I_{max} = V_1/X_{1e}$

The $I_{2r}'$ will be at its maximum when $R_{1e} + R_L' = 0$ i.e., there exists an ideal short circuit. Hence current $I_{max}$ is called ideal short circuit current of an induction motor.

The equation (1) represents equation of a circle with as its diameter. Thus locus of extremely of $I_{2r}'$ is a circle, as shown in the Fig.2.

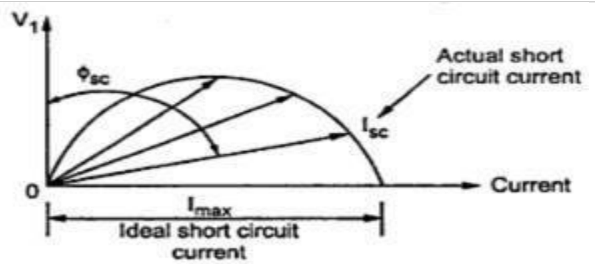


**Fig. 2 Circle diagram of $I_{2r}'$**

But the total stator current $I_1$ per phase is a vector addition of current $I_o$ and $I_{2r}'$.

$I_1 = I_o + I_{2r}'$ **................................**Vector addition

For and induction motor, $I_o$ has a fixed value and phase angle $\Phi_o$ which is decided by its active component and magnetising component $I_m$.

$$\bar{I}_o = \bar{I}_c + \bar{I}_m$$

As $I_o$ has fixed magnitude and phase, the locus of extremities of $I_1$, which is $I_o + I_{2r}'$ is also a circle with a diameter still as $V_1/X_{1e}$. The only charge will be that the diameter $V_1/X_{1e}$ will no longer be along X-axis i.e. current axis but will get shifted at the tip of the $I_o$ phasor. All the $I_{2r}'$ phasors are to be drawn from $I_o$ phasor to get $I_1$, as has fixed magnitude and phase angle $\Phi_o$.

**Key Point** : Thus the current locus for a stator current is also a semicircle which is truly called circle diagram of a three phase induction motor. This diagram once obtained can be used to predict the performance of an induction motor under variable load conditions.

The circle diagram is shown in the Fig. 3.

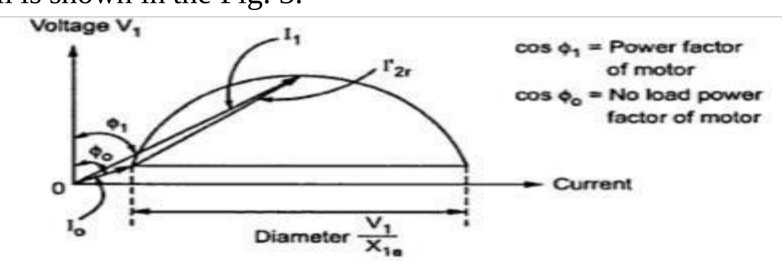


**Fig. 3 Circle diagram of a three phase induction motor**

Let us see, how to obtain the data for plotting the circle diagram.

**Obtaining Data to Plot Circle Diagram**

The data required to draw the circle diagram is obtained by conducting two testes which are,

1. No load test or open circuit test
2. Blocked rotor test or short circuit test

**Construction of Circle Diagram:**

By using the data obtained from the no load test and the blocked rotor test, the circle diagram can be drawn using the following steps :

**Step 1** : Take reference phasor V as vertical (Y-axis).

**Step 2** : Select suitable current scale such that diameter of circle is about 20 to 30 cm.

**Step3** : From no load test, $I_o$ and are $\Phi_o$ obtained. Draw vector $I_o$, lagging V by angle $\Phi_o$. This is the line OO' as shown in the Fig. 1.

**Step 4** : Draw horizontal line through extremity of $I_o$ i.e. O', parallel to horizontal axis.

**Step 5** : Draw the current $I_{SN}$ calculated from $I_{sc}$ with the same scale, lagging V by angle $\Phi_{sc}$, from the origin O. This is phasor OA as shown in the Fig. 1.

**Step 6** : Join O'A is called output line.

**Step 7** : Draw a perpendicular bisector of O'A. Extend it to meet line O'B at point C. This is the centre of the circle.

**Step 8** : Draw the circle, with C as a center and radius equal to O'C. This meets the horizontal line drawn from O' at B as shown in the Fig. 1.

**Step 9** : Draw the perpendicular from point A on the horizontal axis, to meet O'B line at F and meet horizontal axis at D.

**Step 10** : Torque line.

The torque line separates stator and rotor copper losses.

Note that as voltage axis is vertical, all the vertical distances are proportional to active components of currents or power inputs, if measured at appropriate scale.

Thus the vertical distance AD represents power input at short circuit i.e. $W_{SN}$, now which consists of core loss and stator, rotor copper losses.

Now FD = O'G

= Fixed loss

Where O'G is drawn perpendicular from O' on horizontal axis. This represents power input on no load i.e. fixed loss.

Hence AF $\alpha$ Sum of stator and rotor copper losses

Then point E can be located as,

AE/EF = Rotor copper loss / Stator copper loss

The line O'E under this condition is called torque line.

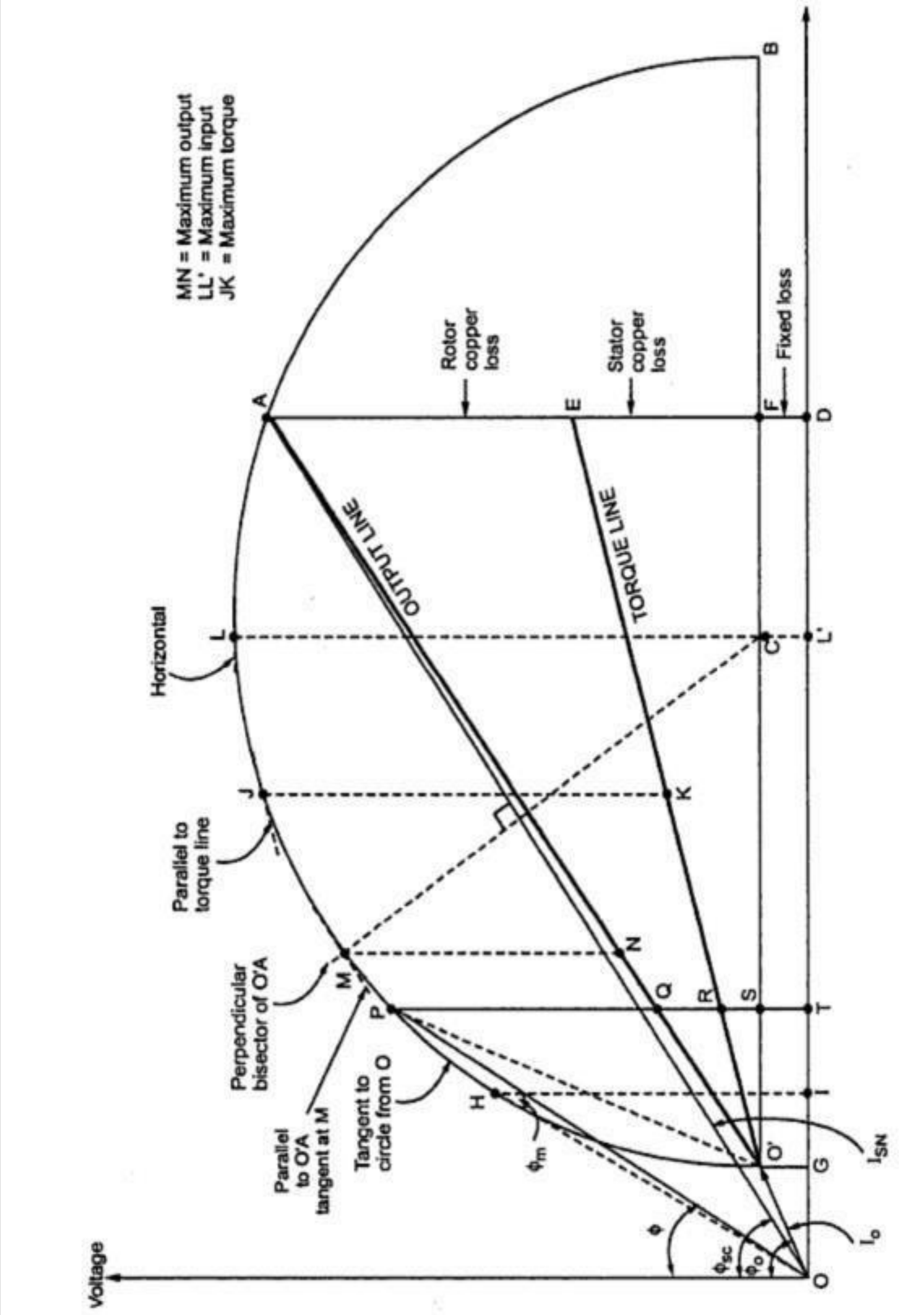
MN = Maximum output
LL' = Maximum input
JK = Maximum torque
Voltage
Horizontal
Parallel to torque line
Perpendicular bisector of O'A
Parallel to O'A tangent at M
Tangent to circle from O
Rotor copper loss
Stator copper loss
Fixed loss
OUTPUT LINE
TORQUE LINE
A
B
C
D
E
F
G
H
I
J
K
L
L'
M
N
O
O'
P
Q
R
S
T
$\phi$
$\phi_{sc}$
$\phi_o$
$\phi_m$
$I_o$
$I_{SN}$

**Fig. 1**

**Power scale** : As AD represents $W_{SN}$ i.e. power input on short circuit at normal voltage, the power scale can be obtained as,

Power scale = $W_{SN}/l(AD)$ W/cm

where $l(AD)$ = Distance AD in cm

**Location of Point E** : In a slip ring induction motor, the stator resistance per phase $R_1$ and rotor resistance per phase $R_2$ can be easily measured. Similarly by introducing ammeters in stator and rotor circuit, the currents $I_1$ and $I_2$ also can be measured.

∴ $K = I_1/I_2$ = Transformation ratio

Now AF/EF = Rotor copper loss / Stator copper loss = $(I_2^2R_2)/(I_1^2R_1) = (R_2/R_2)(I_2^2/I_1^2) = (R_2/R_2).(1/K^2)$

But $R_2' = R_2/K^2$ = Rotor resistance referred to stator

∴ $AE/EF = R_2'/R_1$

Thus point E can be obtained by dividing line AF in the ratio $R_2'$ to $R_1$.

In **a squirrel cage motor**, the stator resistance can be measured by conducting resistance tset.

∴ Stator copper loss = $3I_{SN}^2 R_1$ where $I_{SN}$ is phase value.

Neglecting core loss, $W_{SN}$ = Stator Cu loss + Rotor Cu loss

∴ Rotor copper loss = $W_{SN} - 3I_{SN}^2 R_1$

∴ $AE/EF = (W_{SN} - 3I_{SN}^2 R_1)/(3I_{SN}^2 R_1)$

Dividing line AF in this ratio, the point E can be obtained and hence O'E represents torque line.

**Predicting Performance Form Circle Diagram:**

Let motor is running by taking a current OP as shown in the Fig. 1. The various performance parameters can be obtained from the circle diagram at that load condition.

Draw perpendicular from point P to meet output line at Q, torque line at R, the base line at S and horizontal axis at T.

We know the power scale as obtained earlier.

Using the power scale and various distances, the values of the performance parameters can be obtained as,

Total motor input = PT x Power scale

Fixed loss = ST x power scale

Stator copper loss = SR x power scale

Rotor copper loss = QR x power scale

Total loss = QT x power scale

Rotor output = PQ x power scale

Rotor input = PQ + QR = PR x power scale

Slip s = Rotor Cu loss = QR/PR

Power factor cos = PT/OP

Motor efficiency = Output / Input = PQ/PT

Rotor efficiency = Rotor output / Rotor input = PQ/PR

Rotor output / Rotor input = 1 - s = $N/N_s$ = PQ/PR

The torque is the rotor input in synchronous watts.

**Maximum Quantities:**

The maximum values of various parameters can also be obtained by using circle diagram.

**1. Maximum Output** : Draw a line parallel to O'A and is also tangent to the circle at point M. The point M can also be obtained by extending the perpendicular drawn from C on O'A to meet the circle at M. Then the maximum output is given by l(MN) at the power scale. This is shown in the Fig. 1.

**2. Maximum Input** : It occurs at the highest point on the circle i.e. at point L. At this point, tangent to the circle is horizontal. The maximum input given l(LL') at the power scale.

**3. Maximum Torque** : Draw a line parallel to the torque line and is also tangent to the circle at point J. The point J can also be obtained by drawing perpendicular from C on torque line and extending it to meet circle at point J. The l(JK) represents maximum torque in synchronous watts at the power scale. This torque is also called stalling torque or pull out torque.

**4. Maximum Power Factor** : Draw a line tangent to the circle from the origin O, meeting circle at point H. Draw a perpendicular from H on horizontal axis till it meets it at point I. Then angle OHI gives angle corresponding to maximum power factor angle.

∴ Maximum p.f. = cos ∟{OHI}

= HI/OH

**5. Starting Torque** : The torque is proportional to the rotor input. At s = 1, rotor input is equal to rotor copper loss i.e. l(AE).

∴ $T_{start}$ = l(AE) x Power scale.............................in synchronous watts

**Full load Condition:**

The full load motor output is given on the name plates in watts or h.p. Calculates the distance corresponding to the full load output using the power scale.

Then extend AD upwards from A onwards, equal to the distance corresponding to full load output, say A'. Draw parallel to the output line O'A from A' to meet the circle at point P'. This is the point corresponding to the full load condition, as shown in the Fig. 2.

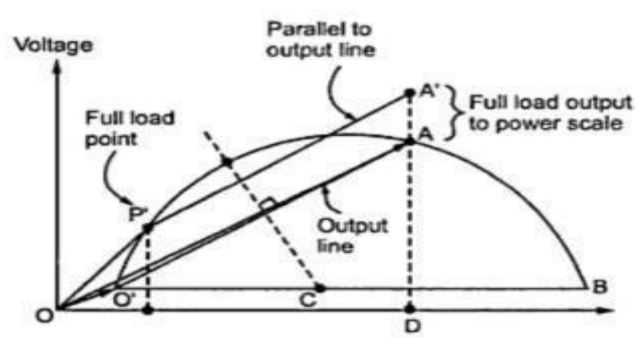


**Fig. 2 Locating full load point**

Once point P' is known, the other performance parameters can be obtained easily as discussed above.

**Load Test on Three Phase Induction Motor:**

By conducting the load test on three phase induction motor, the performance of the motor viz. slip, power factor, input, efficiency etc. at various loads can be studied.

The induction motor is located by any of the following methods :

1. Brake test
2. By connecting a d.c. generator

In case of loading by connecting a d.c. generator, the induction motor is connected to a d.c. generator. The generator is loaded by a lamp bank. Thus inturn an induction motor is loaded. The Fig. 1 shows the experimental set up for conducting load test on three phase induction motor using a d.c. generator.

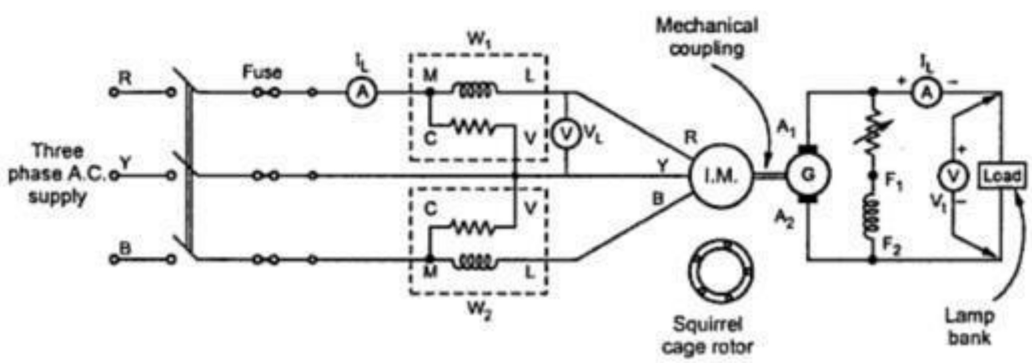


**Fig. 1 Load test on three phase induction motor**

On induction motor side, ammeter reads line current  and voltmeter reads line voltage $V_L$. The two wattmeters are connected as per the two wattmeter method hence,

$$P_{in} = \text{Power input} = W_1 + W_2$$

On generator side, the ammeter reads load current and voltmeter reads terminal voltage $V_t$.

By varying the lamp bank, load on generator i.e. load on induction motor can be varied. The induction motor can be star or delta connected and can be squirrel cage or slip ring type. The speed readings are taken using tachometer. The load is  increased till induction motor carries rated line current. The following observation table is prepared,

| Induction Motor | | | | | Generator | | |
|---|---|---|---|---|---|---|---|
| Sr No. | $I_L$ A | $V_L$ V | $W_1$ W | $W_2$ W | $I_L$ A | $V_t$ V | N r.p.m. |
| 1<br>2<br>⋮ | | | | | | | |

**Calculations** : The output of induction motor is input to a d.c. generator.

Output of d.c. generator = $V_t \times I_L$ W

Assume $\eta_{gen} = 80\,\%$

∴ $P_{out}$ of induction motor = $P_{in}$ of d.c. generator

= $P_{out}$ of d.c. generator / $\eta_{gen}$ = $(V_t\ I_L)/\eta_{gen}$ W

$P_{in}$ of induction motor = $W_1 + W_2$ W

$\cos\Phi = P_{in}/(\sqrt{3}V_L I_L) = (W_1 + W_2)/(\sqrt{3}V_L I_L)$ = power factor

$$\%\ \eta_{motor} = \frac{P_{out}\text{ of motor}}{P_{in}\text{ of motor}} \times 100$$

$$\%\ \text{slip} = \frac{N_s - N}{N_s} \times 100$$

where $N_s = 120f / P$ for a given motor

For various loads above parameters are obtained.

As the load on the induction motor increases,

1. The output of motor increases.
2. The power factor increases.
3. The efficiency increase upto certain load and then decreases.
4. The speed decreases marginally.
5. The slip increases.
6. The input current increases.

The various performance characteristics can be obtained as shown in the Fig. 2.

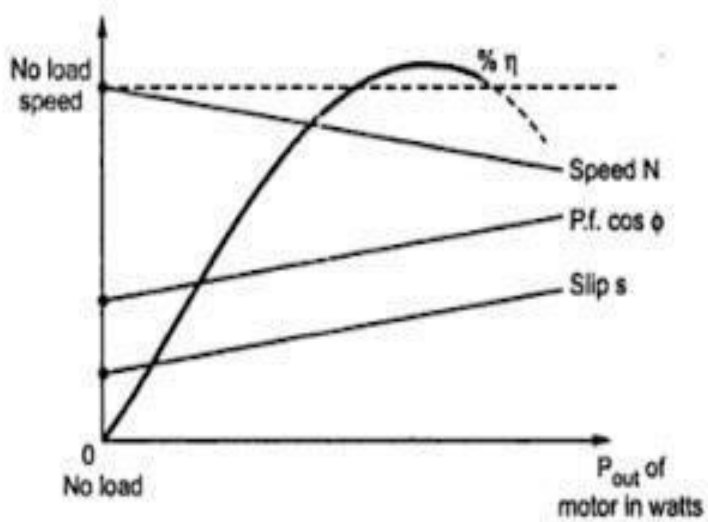


**Fig. 2**

The graphs indicate the behaviour of various performance parameters against output of the induction motor and not shown to the scale.

**Effect of harmonics on Performance of 3-ph Induction Motor**

The induction motor performance is affected by the harmonics in the time variation of the impressed voltage. But its effect on the performance of the motor is not predominant hence it is not considered here.

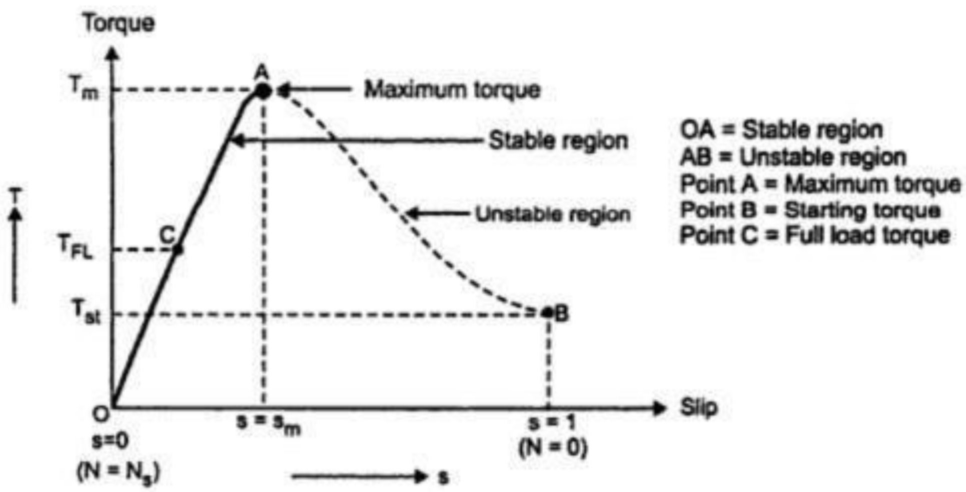


**Fig. 1 Torque speed characteristics**

The torque-slip characteristics as shown in Fig.1 is obtained when the space distribution of flux wave along the air gap periphery is sinusoidal. But the air gap flux is not purely sinusoidal as it contains odd harmonics ($5^{th}$, $7^{th}$, $11^{th}$ etc). Hence at low speeds, the torque-slip characteristic is not smooth. The distribution of stator winding and variation of air gap reluctance due to stator and rotor slots are main causes of air gap flux harmonics.

The harmonics caused due to variation of air gap reluctance are called tooth or slot harmonics . Due to these harmonics produced in air gap flux, unwanted torque is developed along with vibration and noise.

Now eventhough stator currents are sinusoidal, the stator m.m.f. is not sinusoidal as stator winding has the number of slots not more than 3 to 4 per phase. If carry out analysis of stator m.m.f. with the help of Fourier series it can be seen that in addition to fundamental wave it contains odd harmonics m.m.f. waves.

The third harmonic flux waves produced by each of the three phases neutralize each other as it differs in time phase by $120^o$. Thus air gap flux does not contain third harmonics and its multiplies. The fundamental mmf wave produces flux which rotates at synchronous speed which given as $n_s = 2f_1/P$ rps where $f_1$ is supply frequency and P is number of poles. Similarly fifth harmonic mmf wave produces flux which rotates at $2f_1/5P = n_s/5$ rps and in direction opposite to the fundamental mmf wave. The seven harmonic mmf produces flux which rotates at $n_s/7$ rps and in the direction of fundamental m.m.f. wave.

Thus it can be seen that harmonic m.m.f. wave produces flux which rotates at 1/K times the fundamental speed and in the direction of fundamental wave if K = 6m + 1 and in the reversed direction if K = 6m - 1 where m is any integer. The most important and predominant harmonics whose effects must be studied are 5th and 7th harmonics.

The electromagnetic torque that is developed in the induction motor is because of zero relative speed between stator and rotor fields. This fact can be explained as follows :

When rotor is revolving in the same direction of rotation as the stator field, the frequency of rotor currents is $sf_1$ and the rotor field produced will have speed of $sn_s$ rpm with respect to rotor in the forward direction. But there is mechanical rotation of rotor at n rpm which is superimposed on this. The speed of rotor field in space is thus given by sum of these speeds

$$sn_s + n = sn_s + n_s(1\text{-}s) = n_s$$

The stator and rotor fields are thus stationary with respect to each other which produces a steady torque maintaining the rotation. This torque existing at any mechanical speed n other than synchronous speed is called synchronous torque.

The fifth harmonic field rotates at $n_s/5$ rps and in a direction opposite to direction of rotor. Therefore slip of rotor with respect to fifth harmonic field speed is

$$s_5 = \frac{n_{s\ \text{fifth harmonic}} - n_r}{n_{s\ \text{fifth harmonic}}} \quad \text{where } n_r \text{ is rotor speed.}$$

$$= \frac{-\frac{n_s}{5} - n_r}{-\frac{n_s}{5}} = \frac{-\frac{n_s}{5} - n_s(1-s)}{-\frac{n_s}{5}} = 1 + 5\,(1 - s) = 6 - 5\,s$$

Here $-n_s/5$ represents fifth harmonic field rotating opposite to the rotor. The frequency of rotor currents induced by fifth harmonic rotating field is

$$f_{2\ \text{fifth harmonic}} = s_5 \times \text{Stator frequency}$$

$$= (6\ \text{-}5s) \times f_1$$

Now speed of fifth harmonic rotor field with respect to rotor is given by

$$\frac{2\,(f_{2\ \text{fifth harmonic}})}{5\,p} = \frac{2}{5p} f_1\,(6 - 5s) = \frac{n_s}{5}\,(6 - 5s)$$

Now, speed of fifth harmonic rotor field with respect to stator

$$= \begin{array}{c}\text{Speed of fifth harmonic rotor}\\ \text{field with respect to rotor}\end{array} + \text{Rotor speed}$$

$$= -\frac{n_s}{5}\,(6 - 5s) + n_r = -\frac{6}{5}\,n_s + sn_s + n_s\,(1 - s) = -\frac{n_s}{5}\ \text{rps}$$

Negative sign is used before $n_s/5$ (6 - 5s) which indicates 5th harmonic field rotates opposite to rotor movement. Thus it can be seen that speed of fifth harmonic stator field and rotor field is equal and relative speed between the two is zero. Thus it produces 5th harmonic induction motor torque similar to torque produced by fundamental component.

Similar analysis can be made on 7th harmonic to show 7th harmonic torque produced similar to fundamental one. Thus each space harmonic can be considered to produces its own asynchronous torque. The induction motor can be considered as equivalent to number of induction motors in series having poles equal to number of harmonics multiplied by number of poles. The torque produced by fundamental component and the harmonic are shown in the Fig. 2.

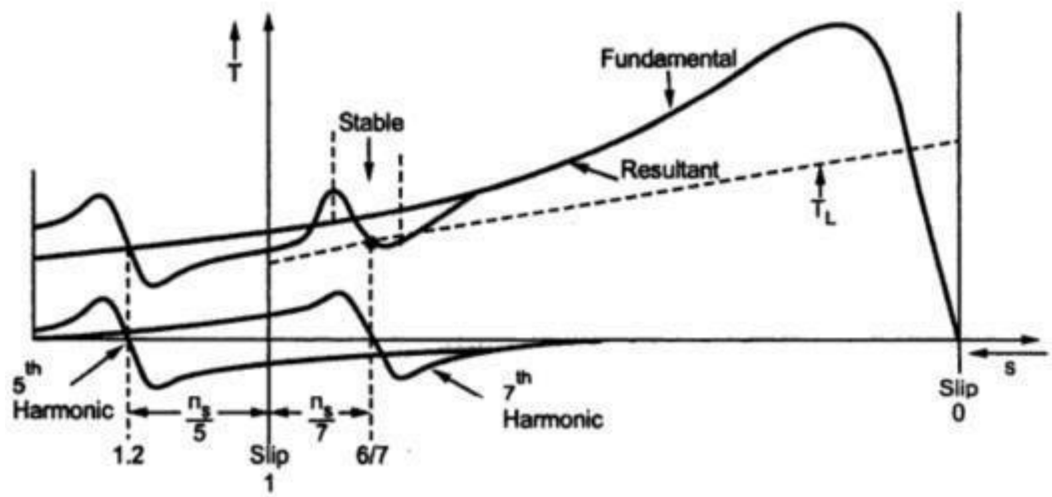


**Fig. 2 Presence of harmonics**

**Crawling:**

As fifth harmonic field rotates opposite to the rotor rotation, the torque produced by fifth harmonic opposes fundamental torque and it acts as braking torque on motor. The seventh harmonic field rotates in the direction of rotor rotation, the torque produced by seventh harmonic aids the fundamental torque. The resultant torque is shown in the Fig. 2 which shows the addition of fundamental, fifth harmonic and seventh harmonic torque. The fifth harmonic torque is zero at $-n_s/5$ rps while seventh harmonic torque is zero at $+n_s/7$.

There are two dips which can be seen in the resultant torque, one is near the slip 1.2 and other near slip 6/7. The slip near s =6/7 is more important as torque here decreases with increase in speed. The load torque is shown in figure. The rotor will run at $n_s/7$ with X as the operating point. Thus stable operation is obtained near sub-synchronous speed $n_s/7$. This is called crawling or synchronous crawling. Due to crawling there is much higher stator current accompanied by noise and vibration. The torque obtained from induction motor here is called synchronous called.

When two harmonic fluxes of same order one because of stator and the rotor because of rotor interact with each other at one particular speed and produces harmonic synchronous torque just like that produced in synchronous motor. These torques are caused by tooth harmonics. The stable operation at synchronous speed caused by slot harmonics is called synchronous crawling which is associated with vibration and noise.

**Cogging:**

A special behaviour is shown by squirrel cage induction motor during starting for certain combination of number of stator and rotor slots. If number of stator slots $S_1$are equal to number of rotor slots $S_2$ or integral multiple of rotor slots $S_2$ then variation of reluctance as a function of space will have pronounced effect producing strong forces than the accelerating torque. Due to this motor fails to start. This phenomenon is called cogging. Such combination of stator and rotor slots should be avoided while designing the motor.

Let the slots of stator and rotor be 24. The stator-slotting produces its tooth harmonics of order $2S_1/P \pm 1$ whereas the rotor-slotting produces its tooth harmonics of order $2S_1/P \pm 1$ where $S_1$ and $S_2$ are number of stator and rotor slots. The plus sign refers to the harmonic field rotation in the direction of rotor.

Here $S_1 = S_2$ so stator and rotor slot harmonics are same and given by,

Let $P = 4$

$$(2x24/ 4) \pm 1 = 11 \text{ or } 23$$

The harmonics of order 11 produce backward rotating field for both stator and rotor. The harmonics of order 13 produces forward rotating field.

The two harmonics fields of same order say 11$^{th}$ harmonic would be stationary with respect to each other only when

$$n_r - (n_s-n_r / 11) = -n_s/11$$

$$n_r = 0$$

As the harmonic field due to 11$^{th}$ harmonic rotates backward with respect to stator hence negative sign is used for $n_s/11$.

Similarly, for 13$^{th}$ harmonic produced by stator and rotor would be stationary with respect to each other when

$$= (n_s-n_r / 13) + n_r = n_s/13$$

$$n_r = 0$$

Hence it can be seen that harmonic synchronous torque is produced at zero rotor speed. The 11$^{th}$ and 13$^{th}$ harmonic fields produced by stator and rotor and stationary with respect to each other. The harmonic synchronous torque is produced at zero rotor speed and the motor will remain at rest. This is called cogging. The torque speed characteristic with harmonic synchronous torque as $n_s/7$ is shown in the Fig.3.

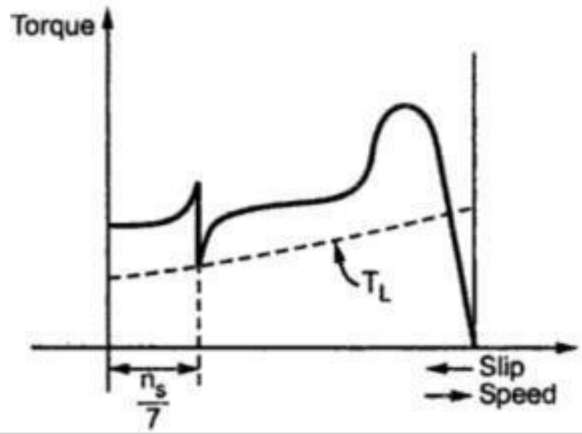


**Fig. 2 Cogging**

The stator slot harmonics of order $2S_1/P \pm 1$ may interact with rotor slot harmonics of order $2S_2/P \pm 1$ to develop the harmonic synchronous torques.

$$2S_1/P + 1 = 2S_2/P + 1$$

$$S_1 = S_2$$

And $$2S_1/P - 1 = 2S_2/P + 1$$

$$S_1 - S_1 = P$$

It can be thus seen that if $S_1 = S_2$ or $S_1 - S_2 = P$ then cogging will be definitely observed in the induction motor.

The cogging and crawling is not predominately in slip ring induction motor as these motors are started with higher starting torques with external resistance in rotor circuit.

The crawling effect can be reduced by taking proper car during the design. Still if crawling is observed then it can be overcome by applying a sudden external torque to the driven load in the direction of rotor. If there is reduction in supply voltage then torque also decreases ($T \alpha V_1^2$). Hence asynchronous crawling may be observed which is absent under rated voltage conditions. Thus asynchronous torques cannot be avoided but can be reduced by proper choice of coil span and by skewing the stator or rotor slots.

**Key Point** : The synchronous harmonics torques can be totally eliminated by proper combination of stator and rotor slots.

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## High Torque Rotors:

### Special Rotor Constructions and Applications

In case of slip ring induction motor an external resistance can be added in the rotor circuit during starting which gives higher starting torque and lower starting line current at an improved power factor. This resistance is then gradually cut from the rotor circuit which would otherwise result in decrease of full load speed, poor speed regulation, more rotor losses and hence reduced efficiency. With lower rotor resistance it gives constant speed, low slip, less losses and high efficiency. This is the major advantage of slip ring induction motor that it gives high rotor resistance at starting and low rotor resistance at normal operating speed.

In case of squirrel cage induction motor there is no provision made for adding external resistance. If the resistance is designed in such a way that it gives better running performance then it has high starting current and consequently low starting torque. This is major disadvantage of squirrel cage induction motor although it is having the other qualities of low cost, ruggedness and maintenance free operation. Thus the designer had found different ways of improving the starting performance of the motor without affecting the running performance of the motor.

In squirrel cage induction motor high starting torque can be obtained by the use of deep bar or double cafe rotors. Both these types of rotors make use of skin effect in which distribution of current is not uniform but the alternating current has the tendency to concentrate near the surface of the conductor. Due to this effect, effective area of cross section of the conductor is reduced and hence resistance of the conductor is increased when carrying alternating current.

The solid conductor can be considered to be makeup of large number of strands each carrying a small part of current. The inductance of each strand will vary according to the position. The strands in proximity of the centre are surrounded by greater magnetic flux and has greater inductance than near the surface. Due to high reactance at the centre, the alternating current flows near the surface of the conductor. The skin effect depends upon nature of material, diameter of wire, shape of wire and frequency.

Thus the current in the rotor during starting is having the frequency of supply. While under running condition the frequency of rotor current reduces to slip frequency. This variation in frequency changes the rotor resistance as it depends on skin effect. During starting it gives high resistance whereas it gives low resistance during running condition which is desirable. Thus the variation in rotor resistance can be achieved by deep bar or double cage construction of rotor and induction motor. Both these types of construction make use of skin effect phenomenon.

**Deep Bar Rotor Construction:**

There is no constructional difference between stator of deep bar motor and that of ordinary induction motor. The rotor consists of deep bars, short circuited by two end rings one on each side. The deep and narrow rotor bar of rectangular cross section is shown in the Fig. 1(a). The other rotor bar shapes are shown in the Fig. 1(b). The magnetic leakage flux lines are shown by dotted lines, Now consider that the bar consists of many number of layers of different depths. The top and bottom layers are shown in the Fig. 1.

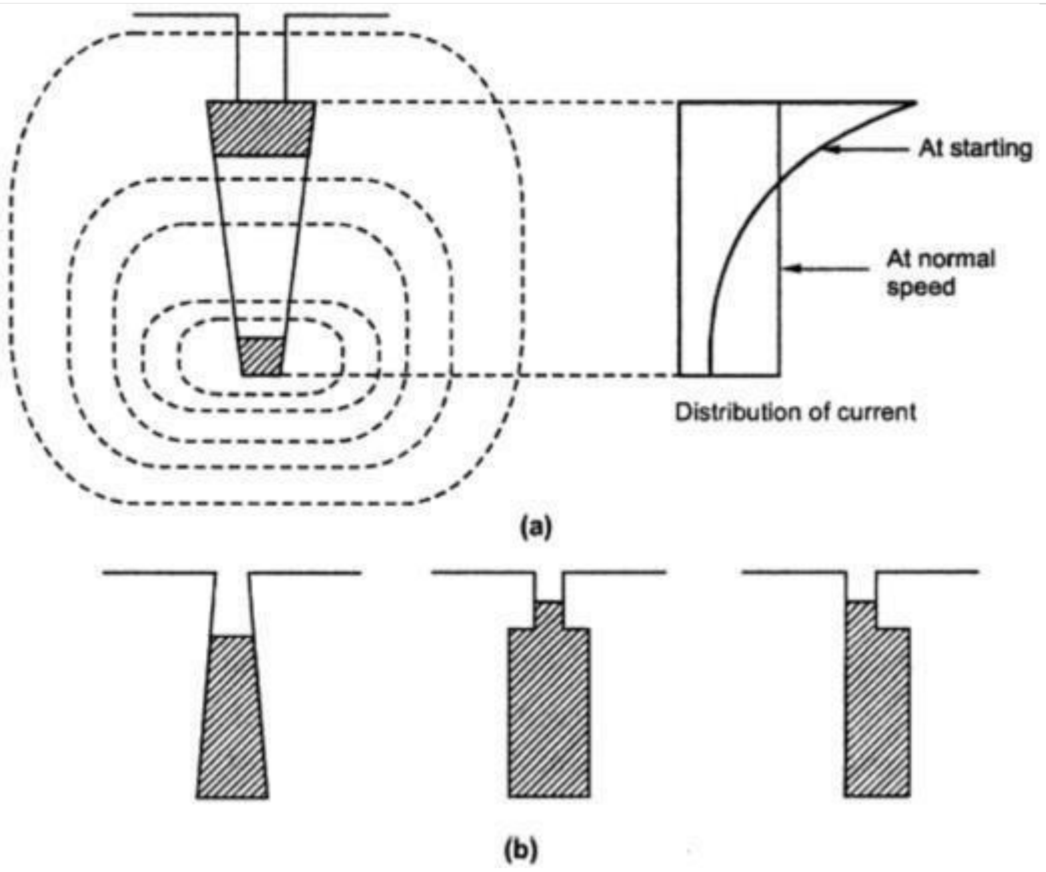



**Fig. 1 Deep bar rotor**

The leakage inductance of the bottom strips is greater than that of top strips as more flux links with bottom strip compared to top strip. All the strips are in parallel electrically. The bottom strip has greater leakage inductance than the top strip. During starting the rotor frequency is equal to the stator frequency and hence leakage reactance of bottom strip is largest and current in it is least. The top strip has low leakage reactance and current in it is large. Thus the current in low reactance top strip will be greater than that in high reactance lower strip and the current will be forced towards the top of the slot and phase of current in upper strip will lead that of the current in lower one. Thus there is non-uniform distribution of current which is shown in the Fig. 1. Due to this non-uniform distribution of current, and use to skin effect, effective are of cross section decreases. Hence rotor resistance increases resulting i high starting torque.

As leakage reactance is proportional to frequency, the non-uniform distribution of current depends upon the rotor frequency. The Fig. 2 shows a curve indicating a.c. effective resistance to d.c. resistance with change in frequency for a copper bar of 2.5 cm deep. The skin effect is maximum when rotor is at standstill.

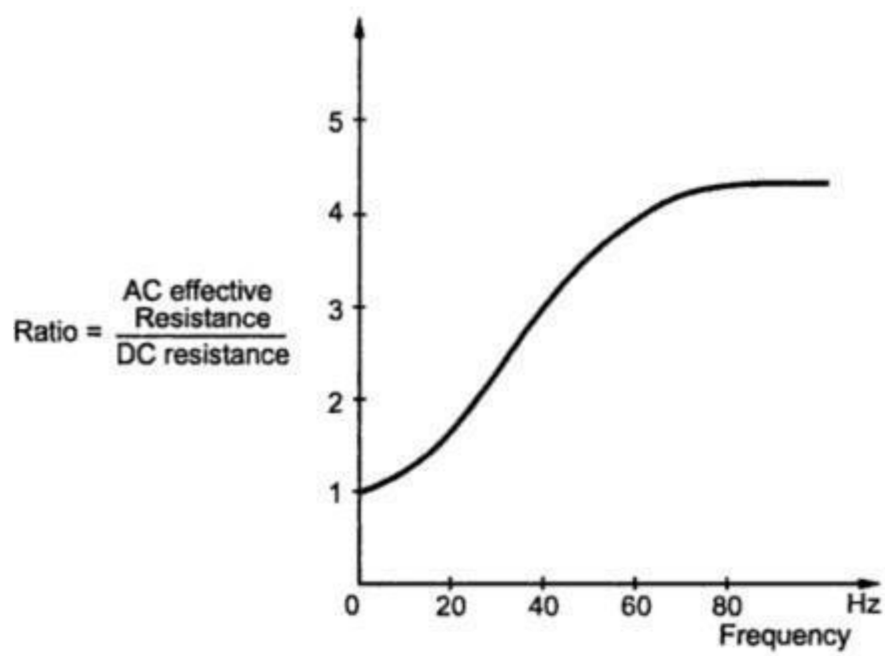



**Fig. 2**

With the increase in rotor speed, the rotor frequency decreases and skin effect also decreases. The reactances of different strips at this low frequency become almost equal and the current density over the conductor cross section becomes uniform so its a.c. resistance is equal to d.c. resistance. Thus with deep bar rotor has a low starting current with high starting torque without affecting running performance of motor. The net reactance of deep bar rotor at standstill is higher than that in a normal bar design, the breakdown or pull out torque in deep bar rotor is lower. The torque-slip characteristics of deep bar motor and normal induction motor is shown in the Fig.3.

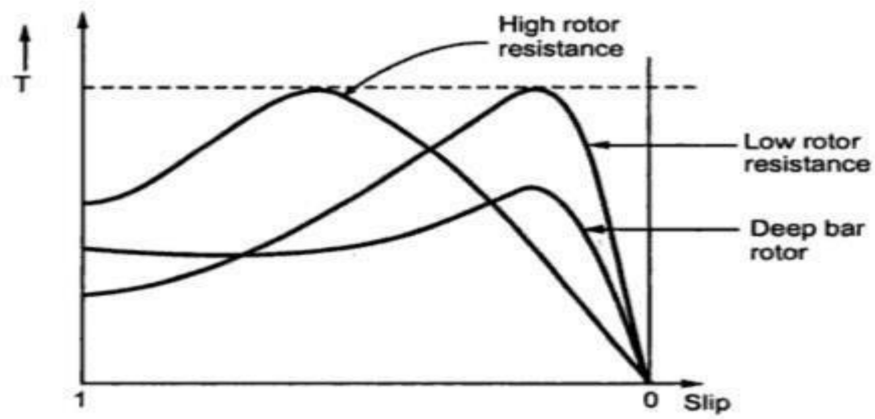



**Fig. 3 Torque slip characteristics of deep bar rotor**

The equivalent circuit of induction motor is applicable to deep bar rotor also wherein proper value of $r_2'$ and $x_2'$ must be determined for satisfactory running performance. During starting their values should correspond to effective value at stator frequency. During running their values should correspond to their effective values at low rotor frequency.

**Double Cage Rotor Construction:**

This is another way of obtaining improved starting performance without affecting its running performance. Though it is more expensive it gives better performance than deep bar rotor construction.

The stator of double cage rotor induction motor is same as that of ordinary induction motor whereas its rotor consists of two cages or two layers of bars short circuited by end rings since the upper cage is having smaller cross-sectional area than the lower cage, the upper cage is having higher resistance than that of lower cage. With equal cross sectional areas of two cages the upper cage is made up of high resistance material like brass, aluminium, bronze etc. and the lower cage is made up of low resistance material like copper. The upper cage and lower cage are separated by a narrow slit or constriction. This is shown in the Fig. 4.

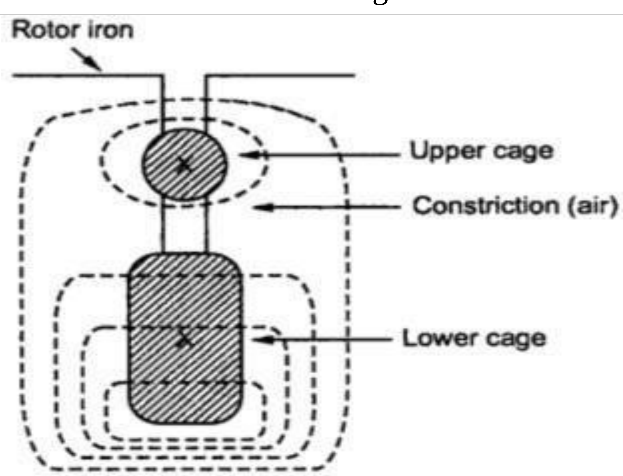



**Fig. 4 Double cage rotor construction**

The slot leakage flux pattern is also shown in the Fig. 4 for the double cage rotor. Similar to deep bar rotor construction the rotor bars in the upper cage have less leakage flux linkage and therefore has lower reactance. The dimension of air construction controls the self leakage flux linking upper and lower bars. If air constriction would have been absent then the main flux would return via iron path between the two slots and thereby missing the bars in the lower cages which will not contribute to production of torque in that case. Hence it can be seen that the upper cage has high resistance and low reactance whereas the lower cage has low resistance and high reactance.

During starting the rotor frequency is same as stator frequency or supply frequency. The division of rotor current in upper and lower cage is inversely proportional to their leakage impedances. At the time of starting the leakage reactance of lower cage is very high and consequently its leakage impedance is several times greater than that of upper cage whose leakage reactance is small. Hence most rotor current flows in upper cage having lower leakage impedance. The upper cage having high resistance sharing the rotor current results in low starting current at improved power factor giving high starting torque.

When rotor speeds up, the rotor frequency decreases which decreases the leakage reactance of lower cage. At normal operating speed the reactance difference between the two cages is negligibly small. Hence the division of rotor current in this case is mainly decided by the resistances of the two cages. As resistance of upper cage is very high most of the current flows through the lower cage giving excellent operating characteristics under running condition. It can be noted that starting current is confined mainly with upper cage so if there is frequent starting of motor then it would cause overheating and burning of upper cage.

The torque-slip characteristics of double cage induction motor are shown in the Fig. 5.

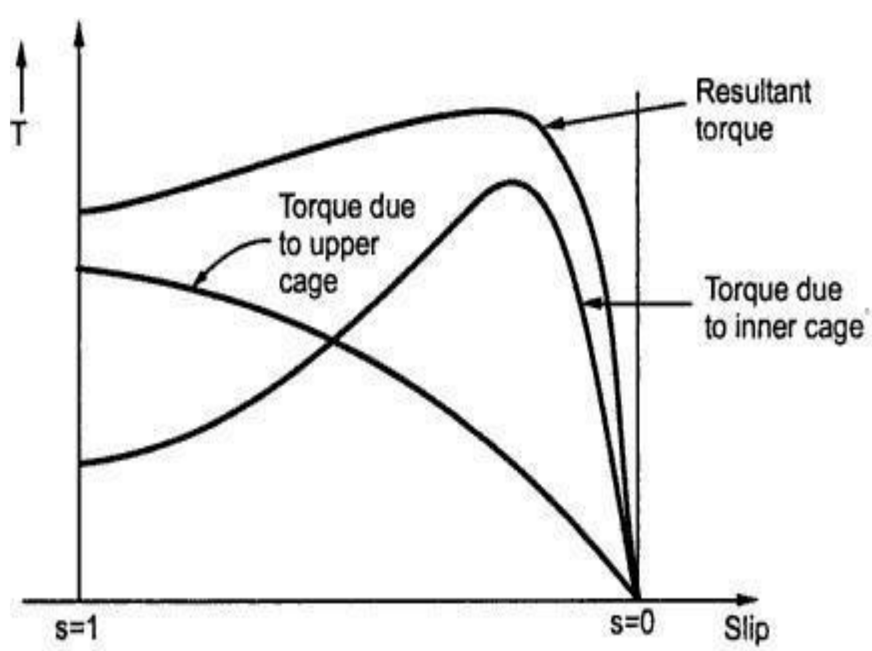



**Fig. 5 Torque slip characteristics of double cage induction motor**

Another type of double cage rotor construction is also possible which is shown in the Fig. 6. The slot-leakage flux pattern for this type of construction is also shown.

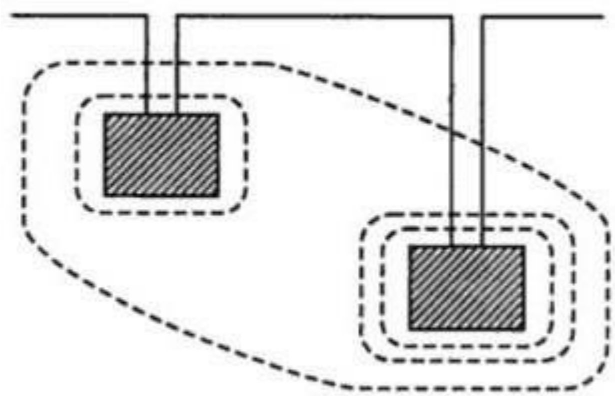


**Fig. 6**

The approximate equivalent circuit of double cage rotor induction motor is shown in the Fig. 7. Though the two cages are somewhat coupled magnetically, they can be treated as independent for simplicity and it gives approximately same results. The two cages are assumed to be parallel while drawing the equivalent circuit.

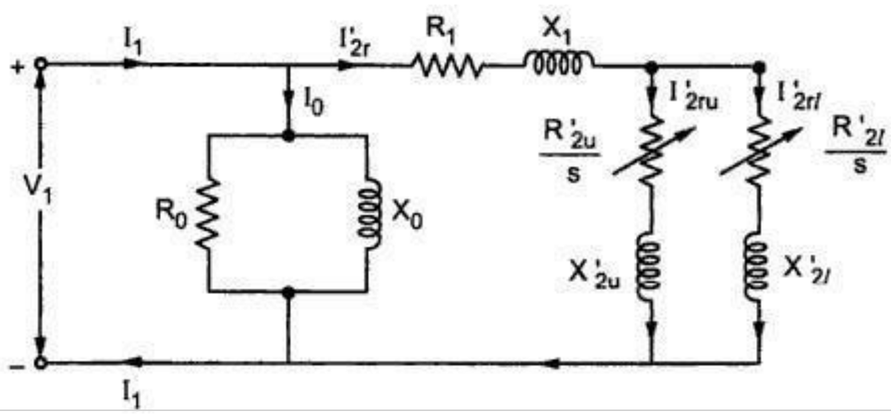



**Fig. 8 Equivalent circuit of double cage induction motor**

$I_{2ru}'$ and $I_{2r}'$ are the currents in the upper and lower cages respectively referred to the stator $R_{2u}'$and $R_{2e}'$ are the resistance of upper and lower cages referred to the stator whereas and are leakage reactances of the two cages referred to the stator of the motor.

**Comparison of Single Cage and Double Cage Motors**

| | Single cage | Double cage |
|---|---|---|
| 1 | Starting current is high hence not suitable for direct on line starting. | Starting current is low hence suitable for direct on line starting. |
| 2 | Starting torque is low. | Starting torque is high. |
| 3 | Effective rotor resistance is low hence at start rotor heating is not severe. | Effective rotor resistance is high hence at start rotor heating is large. |
| 4 | As rotor resistance is low rotor copper losses are less and efficiency is more. | The rotor copper losses are high due to high rotor resistance and efficiency is less. |
| 5 | The breakdown torque or maximum torque is more. | The breakdown torque or maximum torque is smaller as two cages produce maximum torques at different speeds. |
| 6 | The leakage reactance is low. | The effective leakage reactance is high. |
| 7 | The power factor is high. | The power factor is low. |
| 8 | The torque-slip characteristics are fixed and constant. | With proper choice of resistances and reactances of inner and outer cages, wide range of torque-slip characteristics can be obtained. |
| 9 | For same rating, cost is low. | For same rating, cost is high due to double cages. |

**Application**

i) Squirrel cage type of motors having moderate starting torque and constant speed characteristics preferred for driving fans, blowers, water pumps, grinders, lathe machines, printing machines, drilling machines.

ii) Slip ring induction motors can have high starting torque as high as maximum torque. Hence they are preferred for lifts, hoists, elevators, cranes, compressor.

**Induction Generator :**

The torque-slip or the torque-speed characteristics of the induction motor are shown in the Fig. 1. The operating mode of induction machine as a generator or motor or braking depends on value of slip s.

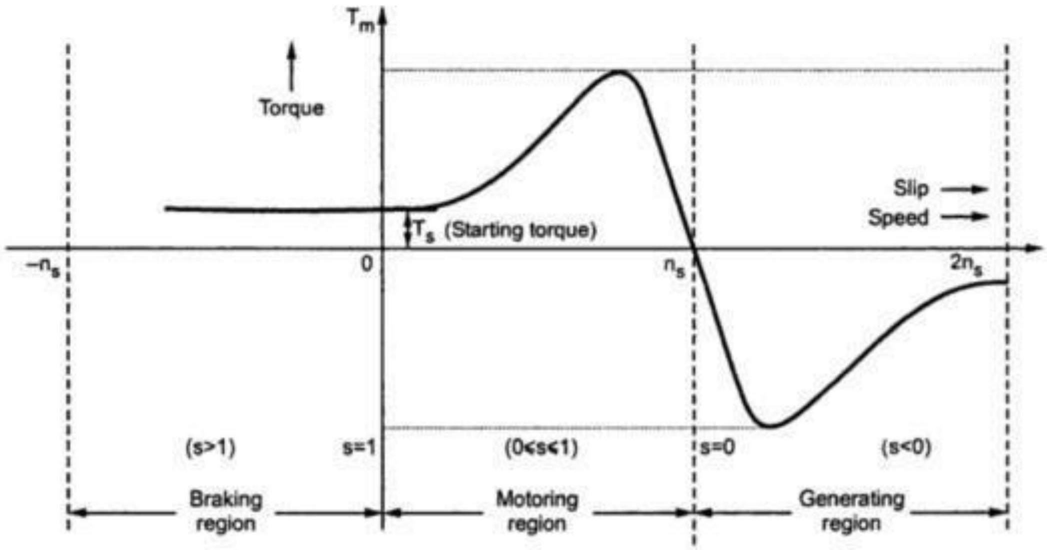



**Fig. 1**

When the slip lies in the region 0 and 1 i.e. when $0 \leq s \leq 1$, the machines runs as a motor which is the normal operation. The rotation of rotor is in the direction of rotating field which is developed by stator currents. In this region it takes electrical power from supply lines and supplies mechanical power output. The rotor speed and corresponding torque are in same direction.

When the slip is greater than 1, the machines works in braking mode. The motor is rotated in opposite direction to that of rotating field. In practice two of the stator terminals are interchanged which changes the phase sequence which in turn reverses the direction of rotation of magnetic field. The motor comes to quick stop under the influence of counter torque which produces braking action. This method by which the motor comes to rest is known as plugging. Only care is taken that the stator must be disconnected from the supply to avoid the rotor in other direction.

To run the induction machines as a generator, its slip must be less than zero i.e. negative. The negative slip indicates that the rotor is running at a speed above the synchronous speed. When running as a generator it takes mechanical energy and supplies electrical energy from the stator. As the speed of induction generator is not in synchronism with the line frequency, it is often called asynchronous generator.

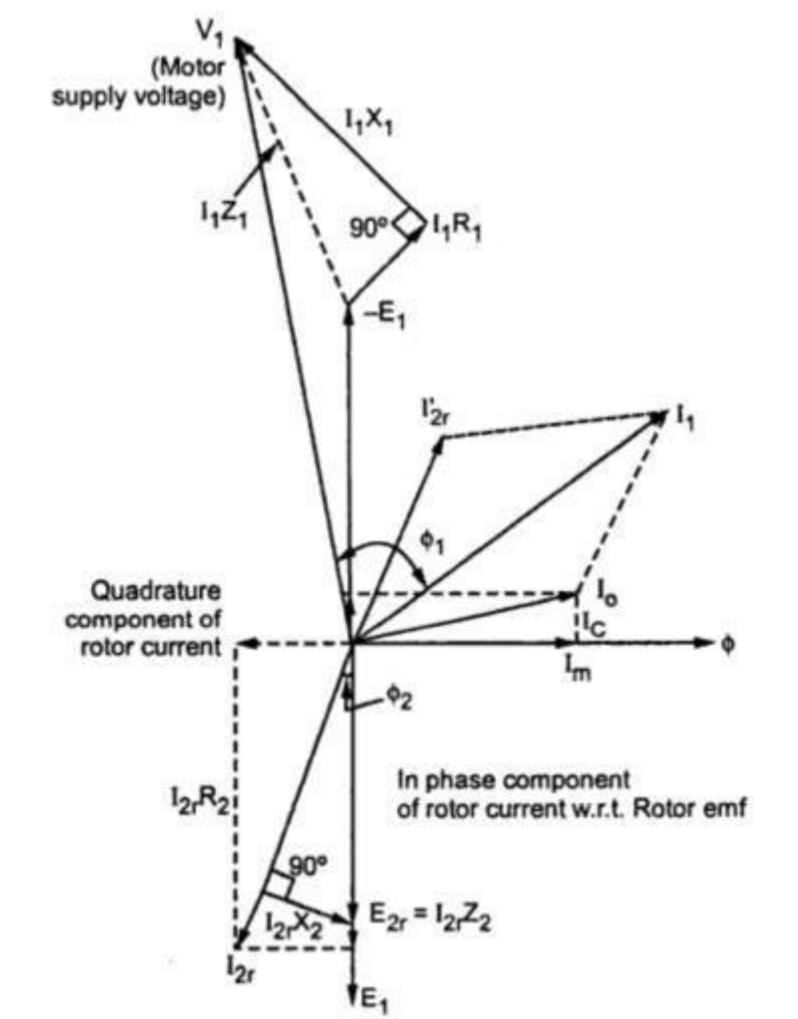
Thus when the slip of the induction motor is negative i.e. when the induction motor runs faster than synchronous speed, the induction motor runs as a generator called induction generator.

The stator of induction generator must be connected to a voltage source to produce the necessary rotating magnetic field revolving at synchronous speed. When rotor is rotated above synchronous speed and the stator is disconnected from the supply, the generating action will not take place.

When rotor of induction machine is driven above synchronous speed, the rotor conductors cut the flux of rotating field in opposite direction to that when it is operating as a motor. The rotor currents are also reversed. Due to the transformer action, currents are induced in the stator and the induction motor can be runs as a generator.

The construction of induction generator is same as that of motor with the difference that the direction of rotation of the motor and a generator is opposite for the same current direction.

The action of induction machine as a generator can be explained from the phasor diagram.




S

**Fig. 2**

Consider the phasor diagram of the induction motor on load.

Let us consider the speed of the induction machine is less than synchronous speed so that machine takes current $I_1$ from supply. This current $I_1$ is phasor sum of no load current $I_o$ and $I_{2r}'$ which is opposite of $I_{2r}$ and referred as reflected rotor current in stator. The rotor current can be resolved into two components, one in phase with rotor emf and the other one is quadrature component.

The rotor current $I_{2r}$ is given by,

$$I_{2r} = \frac{sE_2}{R_2 + jsX_2} = \frac{E_2}{\frac{R_2}{s} + jX_2}$$

Rationalizing the denominator we get,

$$I_{2r} = \frac{E_2\left(\frac{R_2}{s} - jX_2\right)}{\left(\frac{R_2}{s} + jX_2\right)\left(\frac{R_2}{s} - jX_2\right)} = \frac{E_2\left(\frac{R_2}{s}\right)}{\frac{R_2}{s^2} + X_2^2} - j\frac{E_2\,X_2}{\frac{R_2}{s^2} + X_2^2}$$

$$= \frac{sER_2}{R_2^2 + s^2X_2^2} - j\frac{E_2 s^2 X_2}{R_2^2 + s^2X_2^2}$$

Let the real part of above current be denoted by A while the imaginary part of the current be denoted by B. Thus the total rotor current $I_{2r}$ be assumed as A- jB.

Now let the speed of the induction machine is increased. With increase in speed of prime mover i.e. of rotor of induction motor slip goes on reducing and hence the rotor current also as it depends on it. Thus $I_{2r}$ decreases. At synchronous speed, it completely vanishes. Hence its opposite current $I_{2r}'$ also vanished and the resultant stator current is nothing but the no load current . The core losses are supplied from line whereas friction and windage losses are supplied mechanically.

When the speed is increased further the machines enters in generating region. At zero power factor no power is interchanged between machine and supply lines, But the machine generates power to meet its core losses. When the speed is increased, the current $I_{2r}$ increases in magnitude but it changes the phase. The current supplied by the generator will be then vector sum of $I_o$ and $I_{2r}'$ which is reversed in phase as indicated in the phasor diagram.

The rotor current is now given by

$$I_{2r} = \frac{E_2R_2(-s)}{R_2^2 + s^2X_2^2} - j\frac{X_2\,(-s)^2 E_2}{R_2^2 + s^2X_2^2}$$

$\therefore \quad I_{2r} = -A - jB$

It can be seen that the in phase component reverses while the quadrature component remains in the same direction.

The phasor diagram of induction machine as generator is shown in Fig. 2.

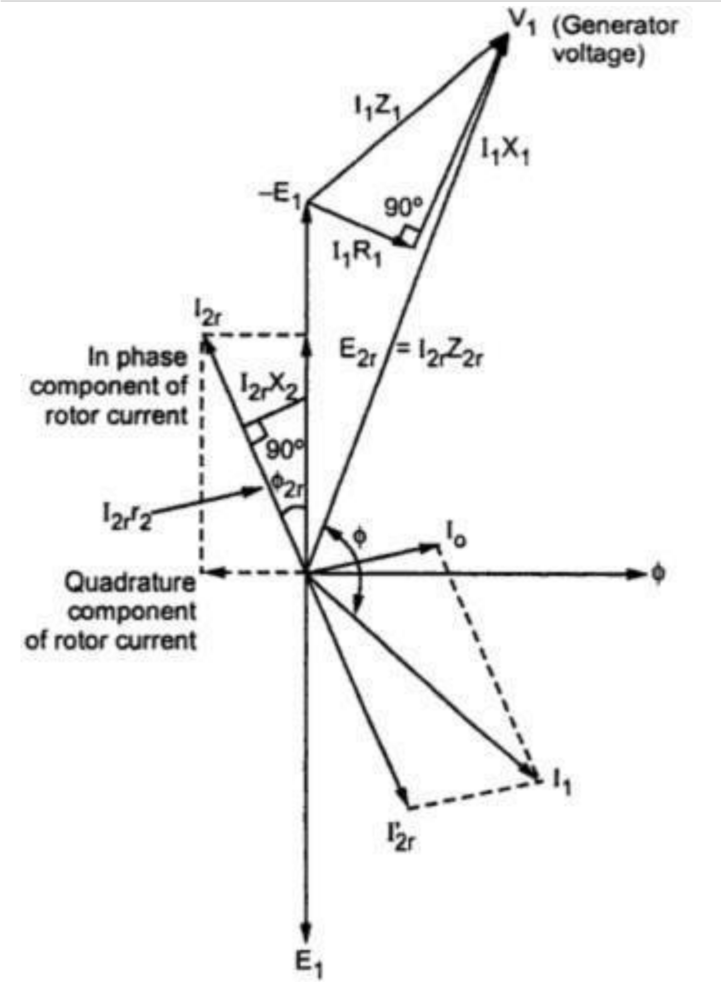



**Fig. 2**

The current $I_{2r}$ leads the voltage - $E_{2r}$ which is opposite of $E_{2r}$. The angle between $V_1$ and $I_1$ is more than which shows that electrical power of the machine is negative i.e. it is supplying the power. Thus when the rotor is rotated above synchronous speed with the rotating field remaining in the same direction, then the direction of cutting of rotor is in opposite direction which results in reversal of rotor emf, current and torque. The machine is said to be operating in generating mode.

The induction generator is not self excited as it cannot generates its own exciting current. Thus it must be always connected to an a.c. supply. Generally it is operated in parallel with synchronous machines. It is shown in the Fig.3.

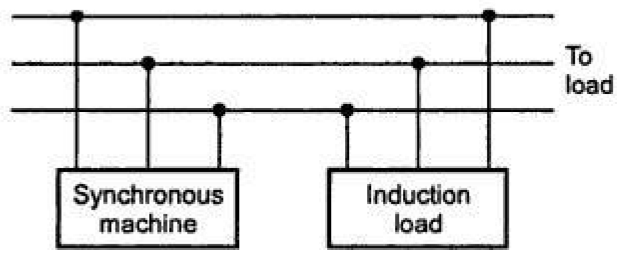



**Fig. 3**

Consider an example of a load which requires a lagging current which can not be supplied by induction generator alone as it supplies leading current.

But this current requirement is fulfilled with the help of synchronous generators operating in parallel with induction generator. Consider the following phasor diagram.

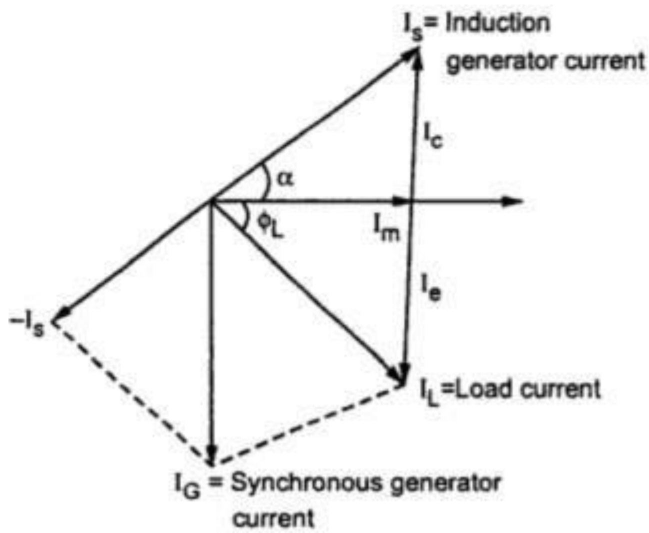



**Fig. 4**

The load current $I_L$ can be resolved into two components one in phase component $I_m$ and the other quadrature component $I_e$. The speed of the induction generator is adjusted in such a way that it supplies current $I_c$ which is leading one. The induction generator current $I_s$ is nothing but vector sum of $I_c$ and $I_m$.

The synchronous generator which is in parallel with the induction generator must supply the remaining part of load current. For this the induction generator current $I_s$ is subtracted vectorially from $I_L$ (subtracting vectorially means reversing $I_s$ and adding it with $I_L$ ). This current is nothing but algebraic sum of currents $I_c$ and $I_e$. The synchronous generator supplies no power. The total current supplied by synchronous generator is lagging quadrature current.

If the load requires a leading current then theoretically the quadrature component of current can be supplied entirely by the induction generator. But for satisfactory operation it should be run in parallel with synchronous generator.

If the bank of delta connected capacitors is operated in parallel with induction generator then the reactive power requirement of induction generator is met by capacitors. This arrangements is shown in Fig. 5.

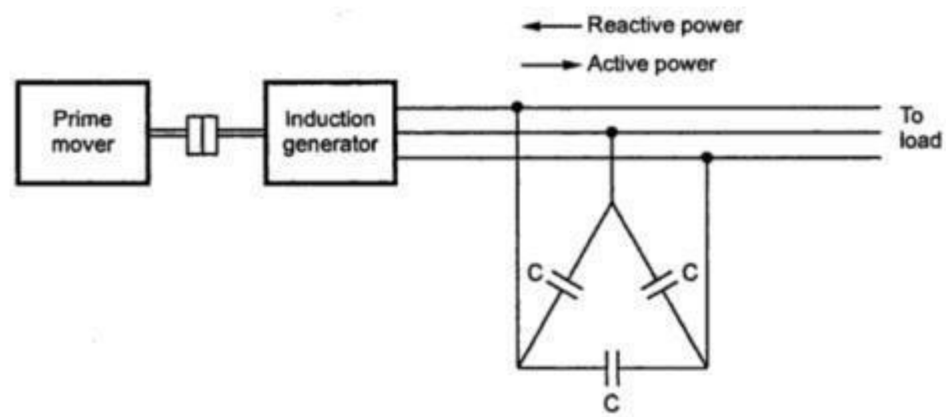



**Fig. 5**

The induction generator in this case is said to be isolated induction generator supplying a load. The external voltage source is not required in this case.

Unlike in synchronous generators, induction generators are not rotating at a definite speed at a given frequency. The speed varies with load as the load is proportional to slip. The frequency of the induction generator is same as the frequency of the line to which it is connected.

**Circle Diagram of Induction Generator**

Using circle diagram, the induction generators can also be analyzed.

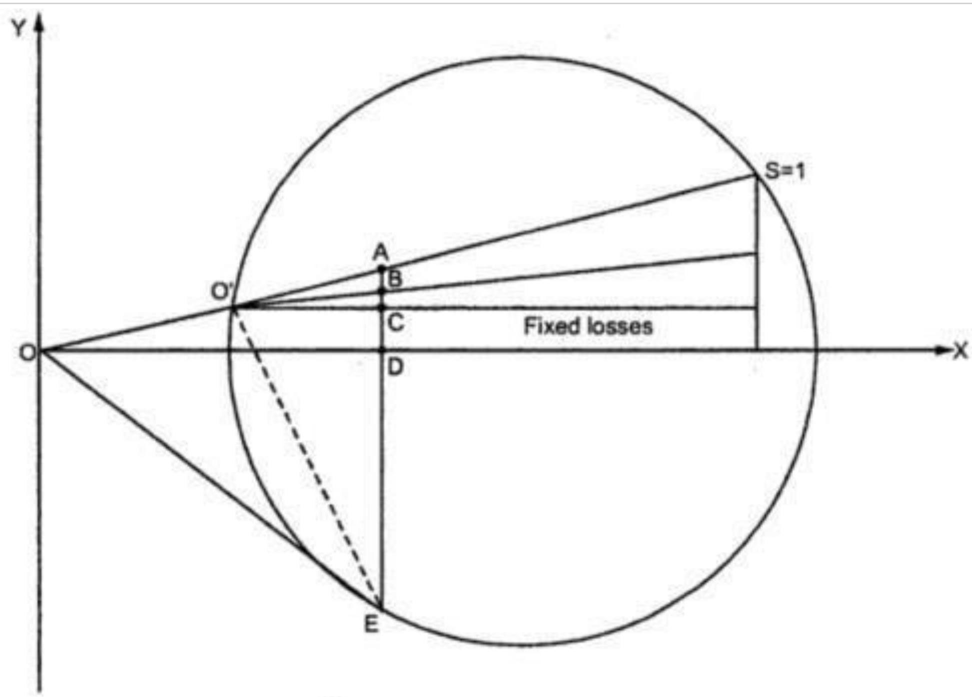



**Fig. 1**

As the antiphase component of current reverses, direction of current also changes. It will be below horizontal shown by OE. E is the operating point. As seen from circle diagram.

AB = Rotor Cu loss.
BC = Stator Cu loss
CD = Constant losses.
DE = Generator output.
BE = Rotor input.
The slip is given by,

$$s = \frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{AB}{BE}$$

Similarly other required quantities can be obtained from the circle diagram.

**Comparison of Induction Generator and Synchronous Generator:**

The distinct features of induction generator compared to synchronous generators are as follows :

i) It will not require d.c. excitation.
ii) It is not self excited but external a.c. supply of fixed frequency is required.
iii) The frequency of induction generator is decided by the frequency of the excitation voltage which is supplying current to it.
iv) Synchronization of generator is not required as no emf is generated until it is connected to the line.

**Advantages:**

The following are the advantages of induction generator.

i) Synchronization for induction generator is required.
ii) The construction is rugged for rotating parts.
iii) Unlike in synchronous machine, there is no danger of hunting or drop out of synchronism for induction generators.
iv) When it short circuited, it delivers small power as the excitation quickly reduces to zero.
v) Induction generators are more suitable for high speeds.
vi) With the help of excitation supply and frequency, the voltage and frequency of induction generator are controlled.

**Disadvantages:**

Although induction generators are having above mentioned advantages, it has following advantages.

i) It must be run in parallel with the synchronous machine.
ii) The load is not deciding the power factor of induction generator but the power factor depends on slip.s

**Applications:**

Because of distinct superiority of the synchronous generator, induction generators are rarely used to supply commercial power.

One application of induction generator is in railway for braking purposes. When the train is moving down a gradient, the induction generators runs above synchronism. As the torque in this region is negative, the braking action is achieved in the train. In addition to this the energy generated by induction generator is given to the line so that the load on main generating station is somewhat relieved. In this case no complicated control apparatus is required.

**Importance of Induction Generators in Wind Mill:**

The induction generator is extremely important in wind power electricity generation system. It is suitable because the stator frequency depends on that of the paralleled synchronous machines and not on the rotor speed.

Induction generator is most commonly used in wind turbines because of low cost, ruggedness, operates with slip (Synchronous not required), availability in many sizes and advance technology available.

Induction generators have outstanding operation as either motor or generator. They have robust construction features. It provides natural protection against short circuits. The abrupt changes in speed are easily absorbed by its solid rotor. Also any surge in the current is damped by the magnetization path of the core, avoiding the possibility of demagnetization which is possible incase of permanent magnet generators.