MODULE - 3

Time response analysis of control systems:

3.1 Introduction:

Time is used as an independent variable in most of the control systems. It is important to analyze the response given by the system for the applied excitation, which is function of time. Analysis of response means to see the variation of out put with respect to time. The output behavior with respect to time should be within these specified limits to have satisfactory performance of the systems. The stability analysis lies in the time response analysis that is when the system is stable out put is finite

The system stability, system accuracy and complete evaluation is based on the time response analysis on corresponding results.

DEFINITION AND CLASSIFICATION OF TIME RESPONSE

Time Response:

The response given by the system which is function of the time, to the applied excitation is called time response of a control system.

Practically, output of the system takes some finite time to reach to its final value.
This time varies from system to system and is dependent on different factors.
The factors like friction mass or inertia of moving elements some nonlinearities present etc.
Example: Measuring instruments like Voltmeter, Ammeter.

Classification:

The time response of a control system is divided into two parts.

1 Transient response $c_t(t)$
2 Steady state response $c_{ss}(t)$

$\therefore c(t)=c_t(t)+c_{SS}(t)$

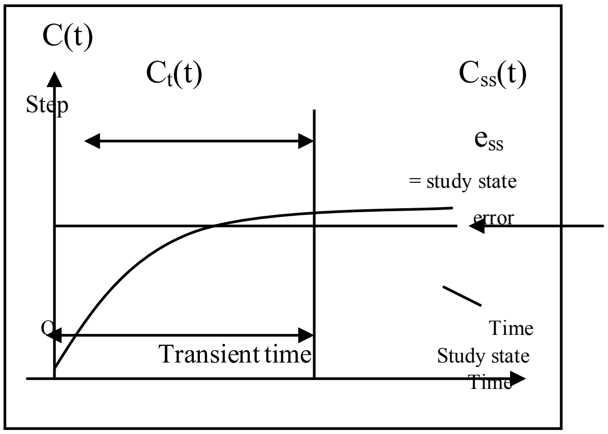
Where c (t) = Time Response
Total Response=Zero State Response +Zero Input Response

Transient Response:

It is defined as the part of the response that goes to zero as time becomes very large. i,e,

$\lim_{t \to \infty} c_t(t) = 0$

A system in which the transient response do not decay as time progresses is an **Unstable system.**



The transient response may be experimental or oscillatory in nature.

3.2 Steady State Response:

It is defined the part of the response which remains after complete transient response vanishes from the system output.

. i,e, $\lim_{t \to \infty} c_t(t)=c_{ss}(t)$

The time domain analysis essentially involves the evaluation of the transient and Steady state response of the control system.

Standard Test Input Signals:

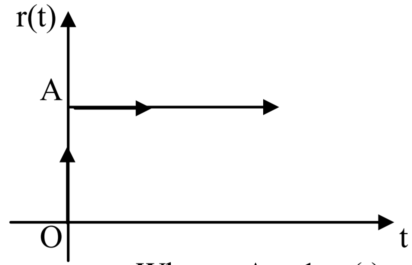
For the analysis point of view, the signals, which are most commonly used as reference inputs, are defined as **standard test inputs.**

- The performance of a system can be evaluated with respect to these test signals.
- Based on the information obtained the design of control system is carried out.
- The commonly used test signals are
  1. Step Input signal.
  2. Ramp Input Signals.
  3. Parabolic Input Signal.
  4. Impulse input signal.

Details of standard test signals

1. Step input signal (position function)
   It is the sudden application of the input at a specified time as usual in the figure or instant any us change in the reference input
   Example :-
   a. If the input is an angular position of a mechanical shaft a step input represent the sudden rotation of a shaft.
   b. Switching on a constant voltage in an electrical circuit.

c. Sudden opening or closing a valve.



When, $A = 1$, $r(t) = u(t) = 1$

The step is a signal who's value changes from 1 value (usually 0) to another level A in Zero time.

In the Laplace Transform form $R(s) = A / S$

Mathematically $r(t) = u(t)$

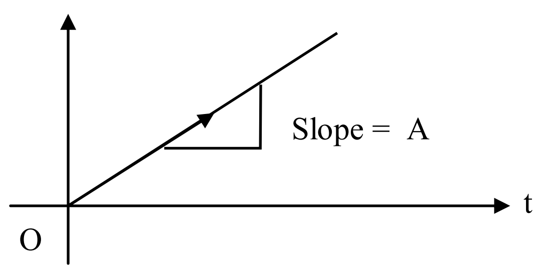
$= 1$ for $t \geq 0$

$= 0$ for $t < 0$

2. <u>Ramp Input Signal (Velocity Functions)</u>:

It is constant rate of change in input that is gradual application of input as shown in fig (2 b).

Ex:- Altitude Control of a Missile



The ramp is a signal, which starts at a value of zero and increases linearly with time.

Mathematically $r(t) = A\,t$ for $t \geq 0$

$= 0$ for $t \leq 0$.

In LT form $R(S) = \dfrac{A}{S^2}$

If A=1, it is called **Unit Ramp Input**

Mathematically

$r(t) = t\,u(t)$

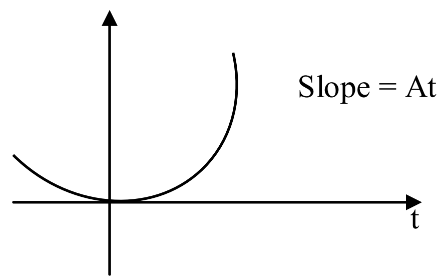
In LT form $R(S) = \dfrac{A}{S^2} = \dfrac{1}{S^2}$ $= \begin{cases} t \text{ for } t \geq 0 \\ 0 \text{ for } t \leq 0 \end{cases}$

3.3 Parabolic Input Signal (Acceleration function):

- The input which is one degree faster than a ramp type of input as shown in fig (2 c) or it is an integral of a ramp.
- Mathematically a parabolic signal of magnitude

A is given by $r(t) = \frac{A\,t^2}{2}\,u(t)$

$$r(t) = \begin{cases} \frac{At^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t \leq 0 \end{cases}$$



In LT form $R(S) = \frac{A}{S^3}$

- If A = 1, a unit parabolic function is defined as $r(t) = \frac{t^2}{2}u(t)$

ie., r(t)

In LT for $R(S) = \frac{1}{S^3} = \begin{cases} \frac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t \leq 0 \end{cases}$

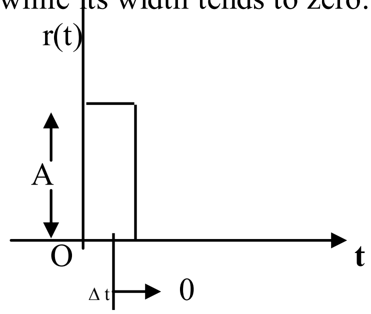
3.4 Impulse Input Signal :

It is the input applied instantaneously (for short duration of time) of very high amplitude as shown in fig 2(d)

Eg: Sudden shocks i e, HV due lightening or short circuit.

It is the pulse whose magnitude is infinite while its width tends to zero.

ie., $t \longrightarrow 0$ (zero) applied momentarily



Area of impulse = its magnitude

If area is unity, it is called **Unit Impulse Input** denoted as $\delta(t)$

Mathematically it can be expressed as

$r(t) = A$ for $t = 0$

$= 0$ for $t \neq 0$

In LT form R(S) = 1 if A = 1

3.1 Standard test Input Signals and its Laplace Transforms.

r(t)	R(S)
Unit Step	1/S
Unit ramp	$1/S^2$
Unit Parabolic	$1/S^3$
Unit Impulse	1

First order system:-

The 1st order system is represent by the differential **Eq:-** $a_1\frac{dc(t)}{dt} + a_o c(t) = b_o r(t)$------ (1)

Where, e (t) = out put , r(t) = input, a0, a1 & b0 are constants.

Dividing **Eq:-** (1) by a0, then $\frac{a1}{a0}.\frac{d\,c(t)}{dt} + c(t) = \frac{bo.r(t)}{ao}$

$$T . \frac{d\,c(t)}{dt} + c(t) = Kr(t) \text{ ---------------------- (2)}$$

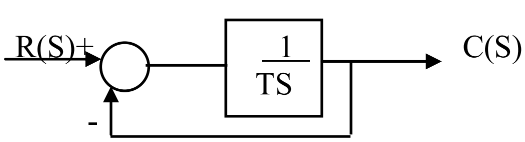
Where, T=time const, has the dimensions of time = $\frac{a1}{a0}$ & K= static sensitivity = $\frac{b0}{a0}$

Taking for L.T. for the above **Eq:-** [TS+1] C(S) = K.R(S)

T.F. of a 1st order system is ; $G(S) = \frac{C(S)}{R(S)} = \frac{K}{1+TS}$

If K=1, Then $G(S) = \boxed{\frac{1}{1+TS}}$ ….I [It‘s a dimensionless T.F.]

This system represent RC ckt. A simplified block diagram is as shown.;



3.1 Unit step response of 1st order system:-

Let a unit step i\p u(t) be applied to a 1st order system,

Then, $r(t) = u(t)$ & $R(S) = \frac{1}{S}$ ---------------(1)

W.K.T. $C(S) = G(S).\ R(S)$

$$C(S) = \frac{1}{1+TS} \cdot \frac{1}{S} = \left[\frac{1}{S} - \frac{T}{TS+1}\right] \text{----------------- (2)}$$

Taking inverse L.T. for the above **Eq:-**

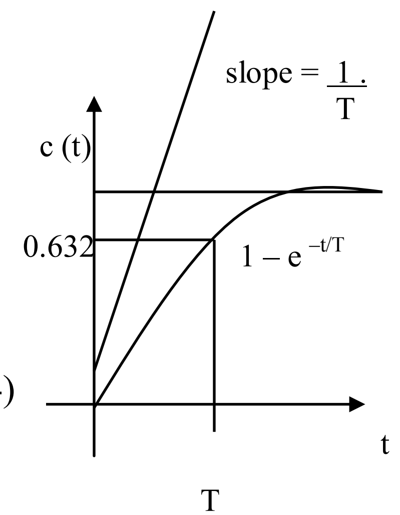
then, $C(t)=u(t) - e^{-t/T}$; $t \geq 0$.------------- (3)

At t=T, then the value of $c(t)= 1- e^{-1} = 0.632$.

The smaller the time const. T. the faster the system response.

The slope of the tangent line at at t= 0 is 1/T.

Since $\frac{dc}{dt} = \frac{1}{T} e^{-t/T} = \frac{1}{T}$ at $t=0$. ------------- (4)



From **Eq:-** (4) , We see that the slope of the response curve c(t) decreases monotonically from $\frac{1}{T}$ at t=0 to zero. At t= ∞

Second order system:-

The 2nd order system is defined as,

$$a_2 \frac{d^2 c(t)}{dt^2} + a_1 \frac{dc(t)}{dt} + a_0\, c(t) = b_0\, .r(t) \text{-----------------(1)}$$

Where c(t) = o/p & r(t) = I/p

-- ing (1) by a_0,

$$\frac{a_2}{a_0} \frac{d^2 c(t)}{dt^2} + \frac{a_1}{a_0} \frac{dc(t)}{dt} + c(t) = \frac{b_0}{a_0} .\ r(t).$$

$$\frac{a_2}{a_0} \frac{d^2 c(t)}{dt^2} + \frac{2a_1}{2\sqrt{a_0}\ \sqrt{a_0}\,.\sqrt{a_2}} \cdot \sqrt{a_2}\ \frac{dc(t)}{dt} + c(t) = \frac{b_0}{a_0} .\ r(t).$$

3) The open loop T.F. of a unity feed back system is given by $G(S) = \frac{K}{S(1+ST)}$ where, T&K are constants having + ve values. By what factor (1) the amplitude gain be reduced so that (a) The peak overshoot of unity step response of the system is reduced from 75% to 25% (b) The damping ratio increases from 0.1 to 0.6.

Solution: $G(S) = \dfrac{K}{S(1+ST)}$

Let the value of damping ratio is, when peak overshoot is 75% & when peak overshoot is 25%

$$Mp = e^{-\left(\frac{\xi \Pi}{\sqrt{1-\xi^2}}\right)}$$

$$\frac{\ln 0.75}{\Pi} = \frac{-\xi}{\sqrt{1-\xi^2}} \Rightarrow -0.0916 = -\frac{\xi}{\sqrt{1-\xi^2}}$$

$\xi_1 = 0.091$

$\xi_2 = 0.4037$

$(0.0084)(1-\xi^2) = \xi^2$

$(1.0084\,\xi^2) = 0.0084$

$\xi = 0.091$

$$\text{w.k.t. T.F.} = \frac{G(S)}{1+G(S)\,.\,H(S)} = \frac{K/(S+S^2T)}{1+\dfrac{K}{(S+S^2T)}} = \frac{K}{S+S^2T+K}$$

$$\text{T.F.} = \frac{K/T}{S^2+\dfrac{S}{T}+\dfrac{K}{T}}$$

Comparing with std Eq :-

$$Wn = \sqrt{\frac{K}{T}}, \quad 2\xi Wn = \frac{1}{T}$$

Let the value of $K = K_1$ When $\xi = \xi_1$ & $K = K_2$ When $\xi = \xi_2$.

Since $2\xi Wn = \dfrac{1}{T}$, $\xi = \dfrac{1}{2TWn} = \dfrac{1}{2\sqrt{KT}}$

$$\therefore \frac{\xi_1}{\xi_2} = \frac{\dfrac{1}{2\sqrt{K_1T}}}{\dfrac{1}{2\sqrt{K_2T}}} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{0.091}{0.4037} = \frac{K_2}{K_1} \Rightarrow \sqrt{\frac{K_2}{K_1}} = 0.0508$$

$$\underline{\underline{K_2 = 0.0508\ K_1}}$$

a) The amplitude K has to be reduced by a factor $= \frac{1}{0.0508} = 20$

b) Let $\xi = 0.1$ Where gain is K_1 and

$\xi = 0.6$ Where gain is K_2

$$\therefore \frac{0.1}{0.6} = \sqrt{\frac{K_2}{K_1}} \qquad \frac{K_2}{K_1} = 0.027 \Rightarrow K_2 = 0.027\ K_1$$

The amplitude gain should be reduced by $\frac{1}{0.027} = 36$

4) Find all the time domain specification for a unity feed back control system whose open loop T.F. is given by

$$G(S) = \frac{25}{S(S+6)}$$

Solution:

$$G(S) = \frac{25}{S(S+6)} \quad \therefore \frac{G(S)}{1 + G(S).H(S)} = \frac{\frac{25}{S(S+6)}}{1 + \frac{25}{S(S+6)}}$$

$$= \frac{25}{S^2 + (6S+25)}$$

$W^2n = 25, \Rightarrow Wn = 5, \quad 2\xi Wn = 6 \Rightarrow \xi = \frac{6}{2 \times 5} = 0.6$

$Wd = Wn\sqrt{1-\xi^2} = 5\sqrt{1-(0.6)^2} = 4$

$tr = \frac{\Pi - \beta}{Wd}, \quad \beta = \tan^{-1}\frac{Wd}{\sigma} \quad \sigma = \xi Wn = 0.6 \times 5 = 3$

$\beta = \tan^{-1}(4/3) = 0.927$ rad.

$tp = \frac{\Pi}{Wd} = \frac{3.14}{4} = 0.785$ sec.

$$MP = \left(e^{-\frac{\xi\Pi}{\sqrt{1-\xi^2}}} \right) = \left(e^{-\frac{0.6}{\sqrt{1-0.6^2}}} \right) \times 3.4 = 9.5\%$$

$$ts = \frac{4}{\xi W_n} \quad \text{for } 2\% = \frac{4}{0.6 \times 5} = 1.3 \ \ldots\ldots\ldots 3\text{sec.}$$

5) The closed loop T.F. of a unity feed back control system is given by

$$\frac{C(S)}{R(S)} = \frac{5}{S^2 + 4S + 5}$$

Determine (1) Damping ratio ξ (2) Natural undamped response frequency Wn. (3) Percent peak over shoot Mp (4) Expression for error resoponse.

Solution:

$$\frac{C(S)}{R(S)} = \frac{5}{S^2 + 4S + 5}, \quad W_n^2 = 5 \Rightarrow W_n = \sqrt{5} = 2.236$$

$$2\xi W_n = 4 \Rightarrow \xi = \frac{4}{2 \times 2.236} = 0.894. \quad W_d = 1.0018$$

$$MP = e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\Pi\right)} = e^{-\left(\frac{0.894}{\sqrt{1-(0.894)^2}}\right) X\ 3.14} = 0.19\%$$

$$\text{W. K.T. } C(t) = e^{-\xi W_n t}\left[\text{Cos } W_d t_r + \frac{\boldsymbol{\xi}}{\sqrt{1-\boldsymbol{\xi}^2}} \sin w_d t_r\right]$$

$$= e^{-0.894 \times 2.236t}\left(\text{Cos } 1.0018t + \frac{\mathbf{0.894}}{\sqrt{1-\mathbf{(0.894)}^2}} \sin 1.0018t\right)$$

6) A servo mechanism is represent by the **Eq:-**

$$\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 150E,$$

$E = R-\theta$ is the actuating signal calculate the value of damping ratio, undamped and damped frequency of ascillation.

Soutions:- $$\boldsymbol{\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 15\,(r - \theta), \quad = 150r - 150\theta.}$$

Taking L.T., $[S^2 + 10S + 150]\,\theta(S) = 150\ R(S)$.

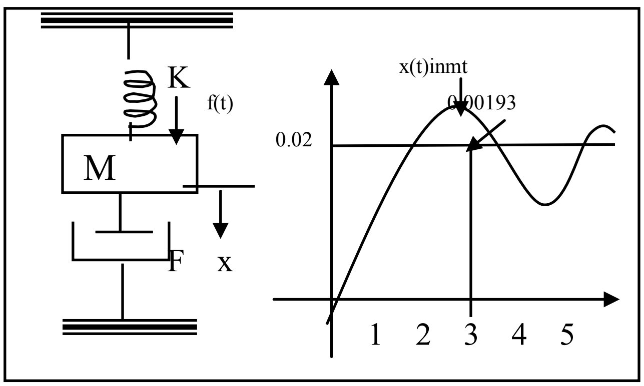
$$\frac{\theta(S)}{R(S)} = \frac{150}{S^2 + 10S + 15O}$$

$Wn^2 = 150 \Rightarrow Wn = \underline{\underline{12.25.}}$ ……………………………rad sec .1

$2\xi Wn = 10 \Rightarrow \xi = \frac{10}{2 \times 12.25} = 0.408.$

$Wd = Wn\sqrt{1 - \xi 2} = 12.25\sqrt{1 - (0.408)2} = \underline{\underline{11.18.}}$ rad 1sec.

7) Fig shows a mechanical system and the response when 10N of force is applied to the system. Determine the values of M, F, K,.



The T.F. of the mechanical system is ,

$$\frac{X(S)}{F(S)} = \frac{1}{MS^2 + FS = K}$$

$$f(t) = M\frac{d^2X}{dt^2} + F\frac{dX}{dt} + KX$$

$$F(S) = (MS^2 + FS + K)\, x\,(S)$$

Given :- $F(S) = \frac{10}{S.}$

$$\therefore\ X(S) = \frac{10}{S(MS^2 + FS + K)}$$

$$SX(S) = \frac{10}{MS^2 + FS + K}$$

The steady state value of X is By applying final value theorem,

$$\lim_{S \to O} SX(S) = \frac{10}{M(0) + F(0) + K} = \frac{10}{K.} = 0.02 \text{ (Given from Fig.)} \qquad (K = 500.)$$

$$MP = \frac{0.00193}{0.02} = 0.0965 = 9.62\%$$

$$Mp = e^{-\left(\frac{\xi\Pi}{\sqrt{1-\xi^2}}\right)}$$

$$0.744 = \frac{\xi}{\sqrt{1-\xi^2}} \Rightarrow 0.5539 = \frac{\xi^2}{\sqrt{1-\xi^2}}$$

$$0.5539 - 0.5539\,\xi^2 = \xi^2$$

$$\xi = \underline{0.597} = \underline{\underline{0.6}}$$

$$t_p = \frac{\Pi}{Wd} = \frac{\Pi}{Wn\sqrt{1-\xi^2}}$$

$$3 = \frac{\Pi}{Wn\sqrt{(1-(0.6)^2)}} \Rightarrow Wn = 1.31\ldots\ldots \text{ rad / Sec.}$$

$$S\,x(S) = \frac{10/M}{\left(S^2 + \frac{F}{M}S + \frac{K}{M}\right)}$$

Comparing with the std. 2nd order **Eq :-**, then,

$$Wn^2 = \frac{K}{\mathbf{M}} \Rightarrow Wn = \sqrt{\frac{K}{\mathbf{M}}} \qquad (1.31)^2 = \frac{500}{\mathbf{M}} \qquad M = 291.36 \text{ kg.}$$

$$\frac{F}{\mathbf{M}} = 2\xi Wn \qquad F = 2 \times 0.6 \times 291 \times 1.31$$

$$\mathbf{F = \underline{458.7\ N/M/\ Sec.}}$$

8) Measurements conducted on sever me mechanism show the system response to be $c(t) = 1+0.2e^{-60t} - 1.2e^{-10t}$, When subjected to a unit step i/p. Obtain the expression for closed loop T.F the damping ratio and undamped natural frequency of oscillation .

Solution:

$$C(t) = 1+0.2e^{-60t} - 1.2e^{-10t}$$

Taking L.T., $C(S) = \frac{1}{S} + \frac{0.2}{S+60} - \frac{1.2}{S+10}$

$$C(S) = \frac{600/S}{S^2 + 70S + + 600}$$

Given that :- Unit step i/p r(t) = 1 ∴ R(S) = 1 .

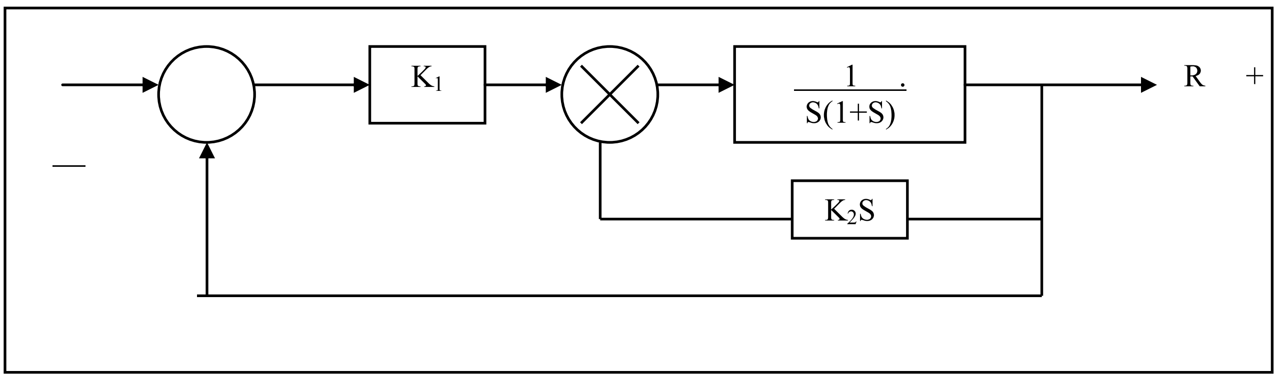
$$\frac{C(S)}{R(S)} = \frac{600/S}{S^2 + 70S + + 600}$$

Comparing, $W_n^2 = 600$, 24.4 …..rad / Sec

$2\ \xi W_n = 70, \Rightarrow \xi = \frac{70}{2 \times 24.4} = 1.428$

10) A feed back system employing o/p damping is as shown in fig.

1) Find the value of K_1 & K_2 so that closed loop system resembles a 2[nd] order system with $\xi = 0.5$ & frequency of damped oscillation 9.5 rad / Sec.
2) With the above value of K_1 & K_2 find the % overshoot when i/p is step i/p
3) What is the % overshoot when i/p is step i/p, the settling time for 2% tolerance?



$$\frac{C}{R} = \frac{K_1}{S^2 + (1 + K_2)S + K_1}$$

$W_n^2 = K_1 \Rightarrow W_n = \sqrt{K_1}$

$2\xi W_n = 1 + K_2 \Rightarrow \xi = \frac{1 + K_2}{2\sqrt{K_1}}$

$W_d = W_n\sqrt{1 - \xi^2} \Rightarrow \therefore W_n = \frac{9.5}{\sqrt{1 - 0.5^2}}$ 10.96 rad/Sec

$K_1 = (10.96)^2 = \underline{\underline{120.34}}$

$2\xi W_n = 1 + K_2$, $K_2 = \underline{\underline{9.97}}$

$$M_p = e^{-\left(\frac{\xi \Pi}{\sqrt{1-\xi^2}}\right)}$$

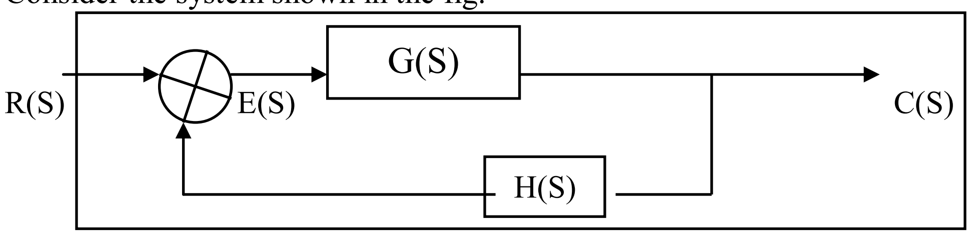
$$M_p = 16.3\%$$

$$T_s = \frac{4}{\xi W_n} = \frac{4}{0.5 \times 10.97} = 0.729 \text{ sec}$$

Steady state Error :-

Steady state errors constitute an extremely important aspect of system performance. The state error is a measure of system accuracy. These errors arise from the nature of i/p's type of system and from non-linearties of the system components. The steady state performance of a stable control system is generally judged by its steady state error to step, ramp and parabolic i/p.

Consider the system shown in the fig.



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S) \cdot H(S)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The closed loop T.F is given by (1). The T.F. b/w the actuating error signal e(t) and the i/p signal r(t) is,

$$\frac{E(S)}{R(S)} = \frac{R(S) - C(S)\,H(S)}{R(S)} = 1 - \frac{C(S) \cdot H(S)}{R(S)}$$

$$= 1 - \frac{G(S) \cdot H(S)}{1 + G(S) \cdot H(S)} = 1 + \frac{G(S) \cdot H(S) - G(S)H(S)}{1+G(S) \cdot H(S)}$$

$$= \frac{1}{1 + G(S) \cdot H(S)}$$

Where e(t) = Difference b/w the i/p signal and the feed back signal

$$\therefore E(S) = \frac{1}{1 + G(S) . H(S)} .R(S) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The steady state error e_{ss} may be found by the use of final value theorem and is as follows;

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{S \to O} SE(S)$$

Substituting (1), $$e_{ss} = \lim_{S \to O} \frac{S.R(S)}{1+G(S) . H(S)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Eq :- (2) Shows that the steady state error depends upon the i/p R(S) and the forward T.F. G(S) and loop T.F G(S) . H(S).

The expression for steady state errors for various types of standard test signals are derived below;

1) Steady state error due to step i/p or position error constant (K_p):-

The steady state error for the step i/p is
I/P r(t) = u(t). Taking L.T., R(S) = 1/S.

From Eq:- (2), $$e_{ss} = \lim_{S \to O} \frac{S. R(s)}{1 +G(S). H.S} = \frac{1}{1 + \lim_{S \to O} G(S). H(S)}$$

$$\lim_{(S \to O)} G(S) . H(S) = Kp$$

Where Kp = proportional error constant or position error const.

$$\therefore e_{ss} = \frac{1}{1 + Kp}$$

$$(1 + Kp)\, e_{ss} = 1 \quad \Rightarrow \quad Kp = \frac{1 - e_{ss}}{e_{ss}}$$

<u>Note</u> :- $e_{ss} = \frac{R}{1 + Kp}$ for non-unit step i/p

2) Steady state error due to ramp i/p or static velocity error co-efficient (Kv) :-

The e_{ss} of the system with a unit ramp i/p or unit velocity i/p is given by,

r (t) = t. u(t) , Taking L -T, R(S) = $1/S^2$

Substituting this to e_{ss} Eq:-

$$\therefore e_{ss} = \lim_{S \to O} \frac{S}{1 + G(S) . H(s)} . \frac{1}{S^2} = \lim_{S \to O} \frac{1}{S +S\, G(S)\, H(s)S}$$

$$\lim = SG(S) . H(S) = Kv = \text{velocity co-efficient then}$$

$S \to O$

$$e_{ss} = \lim_{S \to O} \frac{1}{(S + K_v)} \qquad \therefore e_{ss} = \frac{1}{K_v}$$

Velocity error is not an error in velocity , but it is an error in position error due to a ramp i/p

3) Steady state error due to parabolic i/p or static acceleration co-efficient (K_a) :-

The steady state actuating error of the system with a unit parabolic i/p (acceleration i/p) which is defined by $r(t) + \frac{1}{2} t^2$ Taking L.T. $R(S)=\frac{1}{S^3}$

$$e_{ss} = \lim_{S \to O} \frac{S}{1 + G(S) . H(S)} . \frac{1}{S^3} \qquad \lim_{S \to O} \frac{1}{S^2 + S^2 G(S) . H(S)}.$$

$$\lim_{S \to O} S^2 G(S) . H(S) = K_a.$$

$$\therefore e_{ss} = \lim_{S \to O} \frac{1}{S^2 + K_a} = \frac{1}{K_a}$$

Note :- $e_{ss} = \frac{R}{K_a}$ for non unit parabolic.

Types of feed back control system :-

The open loop T.F. of a unity feed back system can be written in two std, forms;

1) Time constant form and **2) Pole Zero form,**

$$\therefore G(S) = \frac{K(T_aS + 1)\,(T_bS + 1)\ldots\ldots}{S^n(T_1 S+1)\;(T_2S + 1)\ldots\ldots}$$

Where K = open loop gain.

Above Eq:- involves the term S^n in denominator which corresponds to no, of integrations in the system. A system is called Type O, Type1, Type2,……….. if n = 0, 1, 2, ………….. Respectively. The Type no., determines the value of error co-efficients. As the type no., is increased, accuracy is improved; however increasing the type no., aggregates the stability error. A term in the denominator represents the poles at the origin in complex S plane. Hence Index n denotes the multiplicity of the poles at the origin. The steady state errors co-efficient for a given type have definite values. This is illustration as follows.

1) **<u>Type – O system</u> :-** If, n = 0, the system is called type – 0, system. The steady state error are as follows;

Let, $G(S) = \dfrac{K}{S+1}$ $[\because H(s) = 1]$

$$e_{ss}\text{ (Position)} = \frac{1}{1 + G(O)\;.\;H(O)} = \frac{1}{1+K} = \frac{1}{1+K_p}$$

$$\left(\because K_p = \lim_{S\to 0} G(S)\,.\,H(S) = \lim_{S\to 0}\left(\frac{K}{S+1}\right) = K\right)$$

$$e_{ss}\text{ (Velocity)} = \frac{1}{Kv} = \frac{1}{0} = \infty$$

$$Kv = \lim_{S\to 0} G(S)\,.\,H(S) = \lim_{S\to 0} S\left(\frac{K}{S+1}\right) = 0.$$

$$e_{ss}\text{ (acceleration)} = \frac{1}{Ka} = \frac{1}{0} = \infty$$

$$Ka = \lim_{S\to 0} S^2\, G(S)\,.\,H(S) = \lim_{S\to 0} S^2\left(\frac{K}{S+1}\right) = 0$$

2) **<u>Type 1 –System</u> :-** If, n = 1, the e_{ss} to various std, i/p, $G(S) = \dfrac{K}{S\,(S+1)}$

$$e_{ss}\text{ (Position)} = \frac{1}{1+\infty} = O$$

$$\left(Kp = \lim_{S\to 0} G(S)\,.\,H(S) = \lim_{S\to O}\left(\frac{K}{S(S+1)}\right) = \infty\right)$$

$$Kv = \lim_{S\to 0} S\,\frac{K}{S(S+1)} = K$$

$$e_{ss}\text{ (Velocity)} = \frac{1}{K}$$

$$e_{ss}\text{ (acceleration)} = \frac{1}{0} = \infty$$

$$\left(K_a = \lim_{S \to 0} S^2 \frac{K}{S(S+1)} = 0. \right)$$

3) <u>Type 2 –System</u> :- If, n = 2, the e_{ss} to various std, i/p, are , $G(S) = \frac{K}{S^2(S+1)}$.

$$K_p = \lim_{S \to 0} \frac{K}{S^2(S+1)} = \infty$$

$$\therefore \quad e_{ss}\text{ (Position)} = \frac{1}{\infty} = 0$$

$$K_v = \lim_{S \to 0} S \frac{K}{S^2(S+1)} = \infty$$

$$\therefore \quad e_{ss}\text{ (Velocity)} = \frac{1}{\infty} = 0$$

$$K_a = \lim_{S \to 0} S^2 \frac{K}{S^2(S+1)} = K.$$

$$\therefore \quad e_{ss}\text{ (acceleration)} = \frac{1}{K}$$

3) <u>Type 3 –System</u> :- Gives $K_p = K_v = K_a = \infty$ & $e_{ss} = 0$.
(Onwards)

The error co-efficient Kp, Kv, & Ka describes the ability of the system to eliminate the steady state error therefore they are indicative of steady state performance. It is generally described to increase the error co-efficient while maintaining the transient response within an acceptable limit.

PROBLEMS;

1. The unit step response of a system is given by

$C(t) = 5/2 + 5t - 5/2\, e^{-2t}$. Find the T. F of the system.

T/P = r(t) = U (t). Taking L.T, R(s) = 1/S.

Response $C(t) = 5/2+5t-5/2\, e^{-2t}$

$$\left(\frac{1}{S} + \frac{2}{S^2} - \frac{1}{S+2} \right)$$

Taking L.T, C(s) = 5 1 +5 1 5 1 = 5
2 S S^2 2 (S+2) 2

S^2 = 5 (S^2+2S+2S+4) C(s) = 5 S(S+2)+2(S+2)-
2 2 S^2(S+2)

= 10 (S+1)
S2 (S+2)

T.F = C (S) = 10 (S+1) S = 10 (S+1)
R (S) S^2(S+2) S(S+2)

2. The open loop T F of a unity food back system is G(s) = 100
S (S+10)

Find the static error constant and the steady state error of the system when subjected to an i/p given by the polynomial

R(t) = Po + p1t + P2 t^2
2

G(s) = 100 position error co-efficient
S (S+10)

KP = lt G(s) = lt 100 = :
S→ 0 →S 0 S (S+10)

Similarly KV = lt SG(s) = lt 100 x s = 100 = 10
S→ 0 →S 0 S (S+10)

lt 100 x s^2 = 0
S→ 0 →S 0 S (S+10)

Ka = lt S2 G(S)

Given :- r(t) = Po+P1t +P2 $\frac{t2}{2}$

Therefore steady state error ess $\frac{R_1}{1+Kp} + \frac{R_2}{Kv} + \frac{R_3}{Ka}$

ess R_1 R_2 R_3+ + = P0 P1 P2 + +

Ess = 0+0.1 P1 + ∞ = ∞

3. Determine the error co-efficeint and static error for G(s) = $\frac{1}{S(S+1)(S+10)}$
And H(s) = (S+2)

The error constants for a non unity feed back system is as follows

Kp = lt G(S) H(S) = $\frac{(0+2)}{0(0+1)(0+10)}$ = ∞

G(S).H(S) = $\frac{(S+2)}{S(S+1)(S+10)}$

Kv = lt G(S) H(S) = $\frac{(0+2)}{0(0+1)(0+10)}$ = 1/5 = 0.2

Ka = 0

Static Error:-

Steady state error for unit step i/p = 0

Unit ramp i/p $\frac{1}{Kv} = \frac{1}{0.2}$ = 5

Unit parabolic i/p = 1/0 = ∞

4. A feed back C.S is described as $G(S) = \frac{50}{S\ (S+2)\ (S+5)}$ H(S)=1/s.

For unit step i/p,cal steady state error constant and errors.

$$K_p = \lim_{S \to 0} G(S)\ H(S) = \lim_{S \to 0} \frac{50}{S^2\ (S+2)\ (S+5)} = \infty$$

$$K_v = \lim_{S \to 0} G(S)\ H(S) = \lim_{S \to 0} \frac{50 \times S}{S^2\ (S+2)\ (S+5)} = \infty$$

$$K_a = \lim_{S \to 0} G(S)\ H(S) = \lim_{S \to 0} \frac{S^2 \times 50}{S^2\ (S+2)\ (S+5)} = \frac{50}{10} = 5$$

The steady state error $E_{ss} = \lim_{S \to 0} \frac{S.\ 1/S}{1+50}$ $\lim_{S \to 0} \frac{S^2\ (S+2)\ (S+5)}{S^2\ (S+2)\ (S+5) + 50}$

= 0/50 = 0

5. A certain feed back C.S is described by following C.S $G(S) = \frac{K}{S^2\ (S+20)\ (S+30)}$ H(S) = 1

Determine steady state error co-efficient and also determine the value of K to limit the steady to 10 units due to i/p r(t) = 1 + 10 + t 20/2 t2.

$$K_p = \lim_{S \to 0} G(S)\ H(S) = \lim_{S \to 0} \frac{50}{S^2\ (S+20)\ (S+30)} = \infty$$

$$K_v = \lim_{S \to 0} S \frac{K}{S^2 (S+20)(S+30)} = \infty$$

$$K_a = \lim_{S \to 0} S^2 \frac{K}{S^2 (S+20)(S+30)} \quad \frac{K}{600}$$

Steady state error:-

Error due to unit step i/p $\frac{1}{1+K_p} + \frac{1}{1+\infty} = 0$

Error due o r(t) ramp i/p

$$\frac{10}{K_v} + \frac{10}{\infty} = 0$$

Error due to para i/p, $\frac{20}{K_a} = \frac{40}{2K_a} = \frac{20 \times 600}{K} = \frac{12000}{K}$

,

r (t) = (0+0 12000)/K= 10 = K = 1200

<u>First order system:-</u>

The 1st order system is represent by the differential **Eq:-** $a_1 \frac{dc(t)}{dt} + a_o c(t) = b_o r(t)$------ (1)

Where, e (t) = out put , r(t) = input, a0, a1 & b0 are constants.

Dividing **Eq:-** (1) by a0, then $\frac{a_1}{a_0} \cdot \frac{d\,c(t)}{dt} + c(t) = \frac{b_o \cdot r(t)}{a_o}$

$$T \cdot \frac{d\,c(t)}{dt} + c(t) = K r(t) \text{ ---------------------- (2)}$$

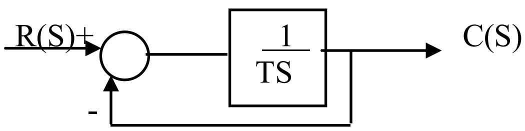
Where, T=time const, has the dimensions of time = $\frac{a1}{a0}$ & K= static sensitivity = $\frac{b0}{a0}$

Taking for L.T. for the above **Eq:-** [TS+1] C(S) = K.R(S)

T.F. of a 1st order system is ; $G(S) = \frac{C(S)}{R(S)} = \frac{K}{1+TS}$

If K=1, Then $G(S) = \boxed{\frac{1}{1+TS}}$ ….I [It's a dimensionless T.F.]

This system represent RC ckt. A simplified bloc diagram is as shown.;



Unit step response of 1st order system:-

Let a unit step i\p u(t) be applied to a 1st order system,

Then, r (t)=u (t) & $R(S) = \frac{1}{S}$ ---------------(1)

W.K.T. C(S) = G(S). R(S)

$$C(S) = \frac{1}{1+TS} \cdot \frac{1}{S} = \left[\frac{1}{S} - \frac{T}{TS+1} \right]$$ ----------------- (2)

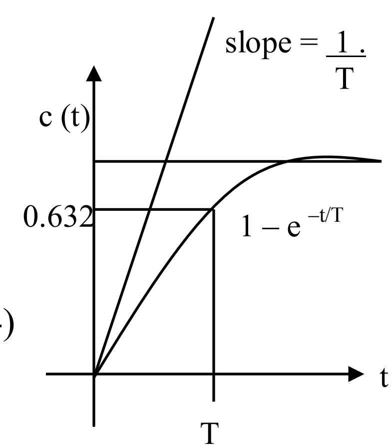
Taking inverse L.T. for the above **Eq:-**

then, $C(t)=u(t) - e^{-t/T}$; $t \geq 0$.------------- (3)

At t=T, then the value of c(t)= $1- e^{-1}$ = 0.632.

The smaller the time const. T. the faster the system response.
The slope of the tangent line at at t= 0 is 1/T.

Since $\frac{dc}{dt} = \frac{1}{T} e^{-t/T} = \frac{1}{T}$ at t .=0. ------------- (4)



From **Eq:-** (4) , We see that the slope of the response curve c(t) decreases monotonically from 1 . at t=0 to zero. At t= ∞

T

Second order system:-

The 2nd order system is defined as,

$$a_2 \frac{d^2 c(t)}{dt^2} + a_1 \frac{dc(t)}{dt} + a_0 c(t) = b_0 .r(t) \text{-----------------}(1)$$

Where c(t) = o/p & r(t) = I/p

-- ing (1) by a_0,

$$\frac{a_2}{a_0} \frac{d^2 c(t)}{dt^2} + \frac{a_1}{a_0} . \frac{dc\,(t)}{dt} + c(t) = \frac{b_0}{a_0} . r(t).$$

$$\frac{a_2}{a_0} \frac{d^2 c(t)}{dt^2} + \frac{2a_1}{2\sqrt{a_0}\ \sqrt{a_0} . \sqrt{a_2}} . \frac{\sqrt{a_2}}{} . \frac{dc\,(t)}{dt} + c(t) = \frac{b_0}{a_0} . r(t).$$

3.3 Step response of 2nd order system:

The T.F. = $\frac{C(s)}{R(s)} = \frac{W_n^2}{3^2+2\xi W_n S+ W_n^2}$ Based on value

The system may be,

2) Under damped system ($0< \xi<1$)
3) Critically damped system ($\xi=1$)
4) Over damped system ($\xi>1$)

1) Under damped system :- ($0< \xi<1$)

In this case $\frac{C(s)}{R(s)}$ can be written as

$$\frac{C(s)}{R(s)} = \frac{W_n^2}{(S+ \xi w_n + jw_d)\ (S+ \xi w_n - jw_d)}$$

Where $w_d = w_n \sqrt{1-\xi^2}$ The Freq. w_d is called damped natural frequency

For a unit step i/p :- [R(t)= 1 ⇒ R(S) = 1/S]

$$C(S) = \frac{W_n^2}{(S^2+2\xi W_n S+W_n^2)} \times R(S) = \frac{W_n^2}{(S^2+2\xi W_n S+W_n^2)} = \frac{1}{S}$$

$$C(S) = \frac{1}{S} - \frac{S+2\xi W_n}{S^2+2\xi W_n S+W_n^2}$$

$$= \frac{1}{S} - \frac{S+\xi W_n}{(S+\xi W_n)^2+W_d^2} - \frac{\xi W_n}{(S+\xi W_n^2+W_d^2)} \text{ --------------- (5)}$$

$$C(S) = \frac{1}{S} - \frac{S+\xi W_n}{(S+\xi W_n)^2+W_d^2} - \frac{\xi}{\sqrt{1-\xi^2}} \cdot \frac{W_d}{(S+\xi W_n)^2+W_d^2}$$

Taking ILT, $C(t) = 1-e^{-\xi W_n t}\left[\cos W_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin W_d t\right]$ ----------- (6)

The error signal for this system is the difference b/w the I/p & o/p.

$\therefore\ e(t) = r(t) - c(t)$.

$= 1 - c(t)$

$= e^{-\xi W_n t}\left[\cos W_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin W_d t\right]$ --------------------- (7)

$t \geq 0$.

At $t = \infty$, error exists b/w the i/p & o/p.

If the damping ratio $\xi = 0$, the response becomes undamped & oscillations continues indefinitely.

The response C(t) for the zero damping case is ,

$c(t) = 1-1(\cos w_n t) = 1-\cos w_n t\ ;\ t \geq 0$ --------------------- (8)

From Eq:- (8) , we see that the W_n represents the undamped natural frequency of the system. If the linear system has any amount of damping the undamped natural frequency cannot be observed experimentally. The frequency, which may be observed, is the damped natural frequency.

$W_d = w_n\sqrt{1-\xi^2}$ This frequency is always lower than the undamped natural frequency. An increase in ξ would reduce the damped natural frequency W_d . If ξ is increased beyond unity, the response over damped & will not oscillate.

<u>Critically damped case</u>:- (ξ=1).

If the two poles of $\frac{C(S)}{R(S)}$ are nearly equal, the system may be approximated by a Critically damped one.

For a step I/p R(S) = 1/S

$$\therefore\ C(S) = \frac{W_n^2}{S^2+2\xi W_n S+W_n^2} \bullet \frac{1}{S}$$

$$= \frac{1}{S} - \frac{1}{(S+W_n)} - \frac{W_n}{(S+W_n)^2}$$

$$= \frac{1}{S} - \frac{W_n^2}{(S+W_n)^2 S}$$

Taking I.L.T.,

$$C(t) = 1 - e^{-W_n t}(1+w_n t)$$

<u>Over damped system</u> :- ($\xi > 1$)

If this case, the two poles of $\frac{C(S)}{R(S)}$ are negative, real and unequal.

For a unit step I/p R(S) = 1/S , then,

$$C(S) = \frac{W_n^2}{(S+\xi W_n + W_n\sqrt{\xi^2-1})\,(S+\xi W_n - W_n\sqrt{\xi^2-1})}$$

Taking ILT, $C(t) = 1+\frac{1}{S\sqrt{\xi^2-1}\,(\xi+\sqrt{\xi^2-1})}\ \bar{e}\ (\xi+\sqrt{\xi^2-1})\,W_n t.$

$$\frac{1}{S\sqrt{(\xi^2-1)}[(\xi \mp \sqrt{(\xi^2-1)}]}\ \bar{e}\ (\xi+\sqrt{(\xi^2-1)})\,W_n t.$$

$$C(t) = 1+\frac{W_n}{S\sqrt{\xi^2-1}}\left[\frac{e^{-S_1 t}}{S_1} - \frac{e^{-S_2 t}}{S_2}\right];\ t \geq O$$

Where $S_1 = (\xi + \sqrt{\xi^2-1})\,W_n$

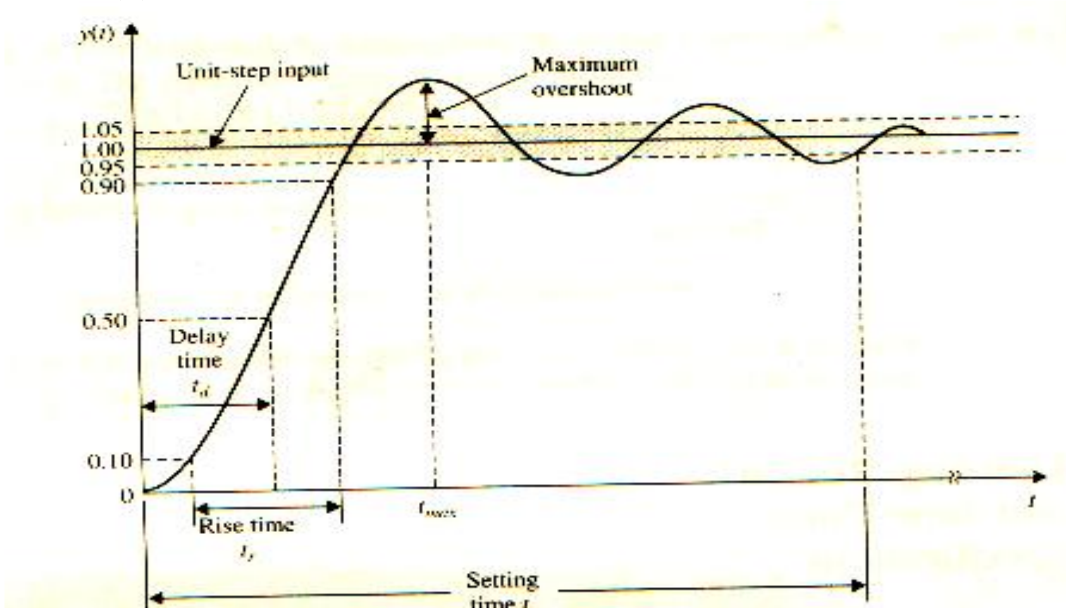
$S_2 = (\xi - \sqrt{\xi^2-1})\,W_n$

3.4 Time response (Transient) Specification (Time domain) Performance :-

The performance characteristics of a controlled system are specified in terms of the transient response to a unit step i/p since it is easy to generate & is sufficiently drastic.

MP The transient response of a practical C.S often exhibits damped oscillations before reaching steady state. In specifying the transient response characteristic of a C.S to unit step i/p, it is common to specify the following terms.

1) Delay time (td)
2) Rise time (tr)



Response curve

3) Peak time (tp)
4) Max over shoot (Mp)
5) Settling time (ts)

1) <u>Delay time</u> :- (t_d)

It is the time required for the response to reach 50% of its final value for the 1st time.

2) <u>Rise time</u> :- (t_r)

It is the time required for the response to rise from 10% and 90% or 0% to 100% of its final value. For under damped system, second order system the 0 to 100% rise time is commonly used. For over damped system, the 10 to 90% rise time is commonly used.

3) <u>Peak time</u> :- (t_p)

It is the time required for the response to reach the 1st of peak of the overshoot.

4) Maximum over shoot :- (MP)

It is the maximum peak value of the response curve measured from unity. The amount of max over shoot directly indicates the relative stability of the system.

5) Settling time :- (t_s)

It is the time required for the response curve to reach & stay with in a range about the final value of size specified by absolute percentage of the final value (usually 5% to 2%). The settling time is related to the largest time const., of C.S.

Transient response specifications of second order system :-

W. K.T. for the second order system,

$$T.F. = \frac{C(S)}{R(S)} = \frac{W_n^2}{S^2+2\xi W_n S + W_n^2} \quad \text{------------------------------}(1)$$

Assuming the system is to be underdamped (**ξ< 1)**

Rise time tr

$$\text{W. K.T. } C(t_r) = 1 - e^{-\xi W nt}\left[\text{Cos } W d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin w d t_r\right]$$

Let $C(t_r) = 1$, i.e., substituting t_r for t in the above Eq:

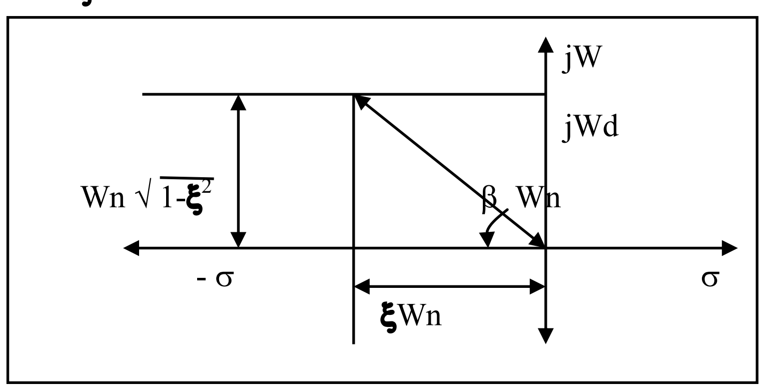
$$\text{Then, } C(t_r) = 1 = 1 - e^{-\xi W ntr}\left[\text{Cos } w d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin w d t_r\right]$$

$$\text{Cos } w d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin w d t_r = \tan w d t_r = -\frac{\sqrt{1-\xi^2}}{\xi} = -\frac{wd}{\sigma}$$

Thus, the rise time t_r is ,

$$t_r = \frac{1}{Wd}\tan^{-1}\left(\frac{-wd}{\sigma}\right) = \frac{\Pi - \beta}{wd} \text{ secs}$$

When β must be in radians.



S- Plane

Peak time :- (tp)

Peak time can be obtained by differentiating C(t) W.r.t. t and equating that derivative to zero.

$$\left.\frac{dc}{dt}\right/_{t = t_p} = O = \text{Sin } Wdtp \; \frac{Wn}{\sqrt{1-\xi^2}} . e^{-\xi Wntp}$$

Since the peak time corresponds to the 1st peak over shoot.

$\therefore$ Wdtp = Π = tp = $\dfrac{\Pi}{Wd}$.

The peak time tp corresponds to one half cycle of the frequency of damped oscillation.

Maximum overshoot :- (MP)

The max over shoot occurs at the peak time.

i.e. At t = tp = $\dfrac{\Pi}{Wd}$.

Mp = $e^{-(\sigma/ Wd)\,\Pi}$ or $e^{-(\xi/\sqrt{1-\xi^2})\,\Pi}$

Settling time :- (ts)

An approximate value of ts can be obtained for the system O < ξ <1 by using the envelope of the damped sinusoidal waveform.

Time constant of a system = T = $\dfrac{1}{\xi Wn}$.

Setting time ts = 4x Time constant.

= 4x $\dfrac{1}{\xi Wn}$. for a tolerance band of +/- 2% steady state.

Delay time :- (td)

The easier way to find the delay time is to plot Wn td VS ξ. Then approximate the curve for the range O<ξ< 1 , then the Eq. becomes,

$$Wn\,td = 1+0.7\,\xi$$

$$\therefore td = \frac{1+0.7\,\xi}{Wn}$$

PROBLEMS:

(1) Consider the 2nd order control system, where ξ = 0.6 & Wn = 5 rad / sec, obtain the rise time tr, peak time tp, max overshoot Mp and settling time ts When the system is subject to a unit step i/p.,

Given :- ξ = 0.6, Wn = 5rad /sec, tr = ?, tp = ?, Mp = ?, ts = ?

$$W_d = W_n\sqrt{1-\xi^2} = 5\sqrt{1-(0.6)^2} = 4$$

$$\sigma = \xi W_n = 0.6 \times 5 = 3.$$

$$t_r = \frac{\pi-\beta}{W_d}, \quad \beta = \tan^{-1}\left(\frac{W_d}{\sigma}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 0.927 \text{ rad}$$

$$t_r = \frac{3.14 - 0.927}{4} = \underline{0.55\text{sec.}}$$

$$t_p = \frac{\pi}{W_d} = \frac{3.14}{4} = \underline{0.785 \text{ sec.}}$$

$$MP = e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\pi} = e^{-(\sigma/W_d)\pi}$$

$$MP = e^{-(3/4)\times 3.14} = 0.094 \times 100 = \underline{9.4\%}$$

ts :- For the 2% criteria.,

$$t_s = \frac{4}{\xi W_n} = \frac{4}{0.6\times 5} = \underline{1.33 \text{ sec.}}$$

For the 5% criteria.,

$$t_s = \frac{3}{\sigma} = \frac{3}{3} = \underline{1 \text{ sec}}$$

EXERCISE:

(2) A unity feed back system has on open loop T.F. $G(S) = \frac{K}{S(S+10)}$

Determine the value of K so that the system has a damping factors of 0.5 For this value of K determine settling time, peak over shoot & time for peak over shoot for unit step i/p

LCS

The error co-efficient Kp, Kv, & Ka describes the ability of the system to eliminate the steady state error therefore they are indicative of steady state performance. It is generally described to increase the error co-efficient while maintaining the transient response within an acceptable limit.

PROBLEMS;

2. The unit step response of a system is given by

C (t) = 5/2 +5t – 5/2 e^{-2t}. Find the T. F of the system.

T/P = r(t) = U (t). Taking L.T, R(s) = 1/S.

Response C(t) = 5/2+5t-5/2 e^{-2t}

Taking L.T, C(s)

$$= \frac{5}{2}\frac{1}{S} + 5\frac{1}{S^2} - \frac{5}{2}\frac{1}{(S+2)} = \frac{5}{2}\left[\frac{1}{S} + \frac{2}{S^2} - \frac{1}{S+2}\right]$$

$$C(s) = \frac{5}{2}\left[\frac{S(S+2)+2(S+2)-S^2}{S^2(S+2)}\right] = \frac{5}{2}\left[\frac{S^2+2S+2S+4-S^2}{S^2(S+2)}\right]$$

$$= \frac{10\,(S+1)}{S^2\,(S+2)}$$

$$\text{T.F} = \frac{C(S)}{R(S)} = \frac{10\,(S+1)\,S}{S^2(S+2)} = \frac{10\,(S+1)}{S(S+2)}$$

2. The open loop T F of a unity food back system is $G(s) = \frac{100}{S\,(S+10)}$

Find the static error constant and the steady state error of the system when subjected to an i/p given by the polynomial

$$R(t) = Po + p1t + \frac{P2}{2}\, t^2$$

$G(s) = \frac{100}{S\,(S+10)}$ position error co-efficient

$$KP = \lim_{S \to 0} G(s) = \lim_{S \to 0} \frac{100}{S\,(S+10)} = :$$

Similarly $K_V = \lim_{S \to 0} SG(s) = \lim_{S \to 0} \frac{100 \times s}{S(S+10)} = \frac{100}{} = 10$

$$\lim_{S \to 0} \frac{100 \times s^2}{S(S+10)} = 0$$

$K_a = \lim S2\ G(S)$

Given :- $r(t) = \frac{}{2}$ Po+P1t +P2 t2

Therefore steady state error ess $\frac{R_1}{1+K_p} + \frac{R_2}{K_v} + \frac{R_3}{K_a}$

$\frac{R_1}{1+\infty} + \frac{R_2}{10} + \frac{R_3}{0}$ ess $\frac{P0}{1+\infty} + \frac{P1}{10} = \frac{P2}{0}$

Ess = 0+0.1 P1 + ∞ = ∞

3. Determine the error co-efficeint and static error for $G(s) = \frac{1}{S(S+1)(S+10)}$ And H(s) = (S+2)

The error constants for a non unity feed back system is as follows

$$G(S).H(S) = \frac{(S+2)}{S(S+1)(S+10)}$$

$$K_p = \lim_{S \to 0} G(S)\ H(S) = \lim_{S \to 0} \frac{(0+2)}{0(0+1)(0+10)} = \infty$$

$$K_v = \lim_{S \to 0} G(S)\,H(S) = \frac{(0+2)}{0(0+1)(0+10)} = 1/5 = 0.2$$

Ka = 0

Static Error:-

Steady state error for unit step i/p = 0

Unit ramp i/p $\frac{1}{K_v} = \frac{1}{0.2} = 5$

Unit parabolic i/p = 1/0 = ∞

4. A feed back C.S is described as $G(S) = \frac{50}{S\,(S+2)(S+5)}$ H(S)=1/s.

For unit step i/p,calculate steady state error constant and errors.

$$K_p = \lim_{S \to 0} G(S)\,H(S) = \frac{50}{S^2\,(S+2)(S+5)} = \infty$$

$$K_v = \lim_{S \to 0} G(S)\,H(S) = \frac{50 \times S}{S^2\,(S+2)(S+5)} = \infty$$

$$K_a = \lim_{S \to 0} G(S)\,H(S) = \lim_{S \to 0} \frac{S^2 \times 50}{S^2\,(S+2)(S+5)} = \frac{50}{10} = 5$$

The steady state error $E_{ss} = \underset{S \to 0}{lt} \dfrac{S.\ 1/S}{1+50} \quad \underset{S \to 0}{Lt} \dfrac{S^2 (S+2)\ (S+5)}{S^2 (S+2)\ (S+5)\ + 50}$

$= 0/50 = 0$

5. A certain feed back C.S is described by following C.S $G(S) = \dfrac{K}{S^2\ (S+20)\ (S+30)}$ $H(S) = 1$

Determine steady state error co-efficient and also determine the value of K to limit the steady to 10 units due to i/p r(t) = 1 + 10 + t 20/2 t2.

$$K_p = \underset{S \to 0}{lt}\ G(S)\ H(S) = \underset{S \to 0}{lt} \frac{50}{S^2\ (S+20)\ (S+30)} = \infty$$

$$K_v = \underset{S \to 0}{lt} \frac{S\ K}{S^2\ (S+20)\ (S+30)} = \infty$$

$$K_a = \underset{S \to 0}{lt} \frac{S^2\ K}{S^2\ (S+20)\ (S+30)} \quad \frac{K}{600}$$

Steady state error:-

Error due to unit step i/p $\dfrac{1}{1+K_p} + \dfrac{1}{1+\infty} = 0$

Error due o r(t) ramp i/p $\dfrac{10}{K_v} + \dfrac{10}{\infty} = 0$

Error due to para i/p, $\dfrac{20}{K_a} = \dfrac{40}{2K_a} = \dfrac{20 \times 600}{K} = \dfrac{12000}{K}$

$$r(t) = \frac{0+0\ 12000}{K} = 10 \ = K = 1200$$

3) The open loop T.F. of a unity feed back system is given by $G(S) = \frac{K}{S(1+ST)}$ where,

T&K are constants having + Ve values.By what factor (1) the amplitude gain be reduced so that (a) The peak overshoot of unity step response of the system is reduced from 75% to 25% (b) The damping ratio increases from 0.1 to 0.6.

Solution: $G(S) = \frac{K}{S(1+ST)}$

Let the value of damping ratio is, when peak overshoot is 75% & when peak overshoot is 25%

$$Mp = e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\Pi}$$

$$\frac{\ln 0.75}{\Pi} = \frac{-\xi}{\sqrt{1-\xi^2}} \Rightarrow -0.0916 = -\frac{\xi}{\sqrt{1-\xi^2}}$$

$\xi_1 = 0.091$

$\xi_2 = 0.4037$

$$(0.0084)(1-\xi^2) = \xi^2$$

$$(1.0084\ \xi^2) = 0.0084$$

$$\xi = 0.091$$

$$\text{w.k.t. T.F.} = \frac{G(S)}{1+G(S)\ .\ H(S)} = \frac{\frac{k}{S+S^2T}}{1+\frac{K}{S+S^2T}} = \frac{K}{S+S^2T+K}$$

$$\text{T.F.} = \frac{K/T}{S^2+\frac{S}{T}+\frac{K}{T}}$$

Comparing with std Eq :-

$$Wn = \sqrt{\frac{K}{}} \quad , \ 2\,\xi Wn = \frac{1}{}$$

T T

Let the value of $K = K_1$ When $\xi = \xi_1$ & $K = K_2$ When $\xi = \xi_2$.

Since $2\xi W_n = \frac{1}{T}$, $\xi = \frac{1}{2TW_n} = \frac{1}{2\sqrt{KT}}$

$$\therefore \frac{\xi_1}{\xi_2} = \frac{\frac{1}{2\sqrt{K_1 T}}}{\frac{1}{2\sqrt{K_2 T}}} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{0.091}{0.4037} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \frac{K_2}{K_1} = 0.0508$$

$$\underline{\underline{K_2 = 0.0508\ K_1}}$$

a) The amplitude K has to be reduced by a factor $= \frac{1}{0.0508} = 20$

b) Let $\xi = 0.1$ Where gain is K_1 and

$\xi = 0.6$ Where gain is K_2

$\therefore \frac{0.1}{0.6} = \sqrt{\frac{K_2}{K_1}}$ $\frac{K_2}{K_1} = 0.027 \Rightarrow K_2 = 0.027\ K_1$

The amplitude gain should be reduced by $\frac{1}{0.027} = 36$

4) Find all the time domain specification for a unity feed back control system whose open loop T.F. is given by

$$G(S) = \frac{25}{S(S+6)}$$

Solution:

$$G(S) = \frac{25}{S(S+6)} \quad \therefore \frac{G(S)}{1 + G(S).H(S)} = \frac{\frac{25}{S(S+6)}}{1 + \frac{25}{S(S+6)}}$$

$$= \frac{25}{S^2 + (6S+25)}$$

$W^2n = 25$, $\Rightarrow W_n = 5$, $2\xi W_n = 6 \Rightarrow \xi = \frac{6}{2 \times 5} = 0.6$

$W_d = W_n\sqrt{1-\xi^2} = 5\sqrt{1-(0.6)^2} = 4$

$t_r = \frac{\Pi - \beta}{W_d}$, $\beta = \tan^{-1}\frac{W_d}{\sigma}$ $\sigma = \xi W_n = 0.6 \times 5 = 3$

$\beta = \tan^{-1}(4/3) = 0.927$ rad.

$$tp = \frac{\Pi}{Wd} = \frac{3.14}{4} = 0.785 \text{ sec.}$$

$$MP = e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\Pi} = e^{-\left(\frac{0.6}{\sqrt{1-0.6^2}}\right)} \text{x}3.4 = 9.5\%$$

$$ts = \frac{4}{\xi Wn} \text{ for } 2\% = \frac{4}{0.6 \times 5} = 1.3 \;\ldots\ldots\ldots 3\text{sec.}$$

5) The closed loop T.F. of a unity feed back control system is given by

$$\frac{C(S)}{R(S)} = \frac{5}{S^2 + 4S + 5}$$

Determine (1) Damping ratio ξ (2) Natural undamped response frequency Wn. (3) Percent peak over shoot Mp (4) Expression for error resoponse.

Solution:

$$\frac{C(S)}{R(S)} = \frac{5}{S^2 + 4S + 5}, \quad Wn^2 = 5 \Rightarrow Wn = \sqrt{5} = 2.236$$

$$2\xi Wn = 4 \Rightarrow \xi = \frac{4}{2 \times 2.236} = 0.894. \quad Wd = 1.0018$$

$$MP = e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\Pi} = e^{-\left(\frac{0.894}{\sqrt{1-(0.894)^2}}\right)} \text{X } 3.14 = 0.19\%$$

$$\text{W. K.T. } C(t) = e^{-\xi Wnt}\left[\text{Cos } Wdt_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin wdt_r\right]$$

$$= e^{-0.894 \times 2.236t}\left(\text{Cos } 1.0018t + \frac{0.894}{\sqrt{1-(0.894)^2}} \sin 1.0018t\right)$$

6) A servo mechanism is represent by the **Eq:-**

$\frac{d^2\theta}{} + 10 \frac{d\theta}{} = 150E$, $E = R-\theta$ is the actuating signal calculate the

$\frac{d^2}{dt^2}$ $\frac{}{dt}$ value of damping ratio, undamped and damped frequency of ascillation.

Soutions:- $\mathbf{\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 15(r - \theta)}$, $\mathbf{= 150r - 150\theta.}$

Taking L.T., $[S^2 + 10S + 150]\,\theta(S) = 150\,R(S)$.

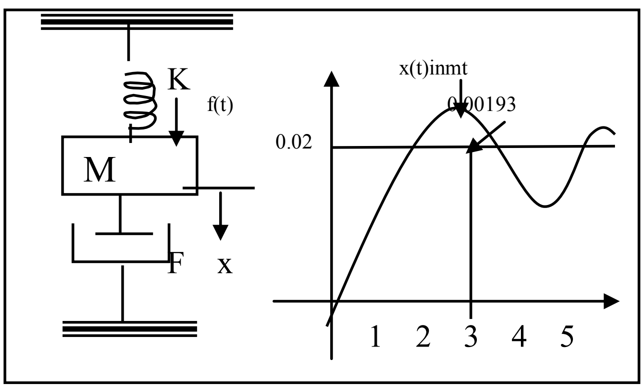
$$\frac{\theta(S)}{R(S)} = \frac{150}{S^2 + 10S + 15O}.$$

$Wn^2 = 150 \Rightarrow Wn = \underline{\underline{12.25.}}$ ………………………….rad sec .1

$2\xi Wn = 10 \Rightarrow \xi = \frac{10}{2 \times 12.25}. = 0.408.$

$$\mathbf{Wd = Wn\sqrt{1 - \xi 2} = 12.25\sqrt{1 - (0.408)2} = \underline{\underline{11.18.}}\ rad\ 1sec.}$$

7) Fig shows a mechanical system and the response when 10N of force is applied to the system. Determine the values of M, F, K,.



The T.F. of the mechanical system is ,

$$\frac{X(S)}{F(S)} = \frac{1}{MS^2 + FS = K}.$$

$$f(t) = M\frac{d^2X}{dt^2} + F\frac{dX}{dt} + KX$$

$$F(S) = (MS^2 + FS + K)\,x(S)$$

Given :- $F(S) = \frac{10}{S.}$

$$\therefore X(S) = \frac{10}{S(MS^2 + FS + K)}.$$

$$SX(S) = \frac{10}{MS^2 + FS + K}.$$

The steady state value of X is By applying final value theorem,

lt. $SX(S) = \frac{10}{}. = \frac{10}{} = 0.02$ (Given from Fig.)

$\longrightarrow$ S O M(0) + F (0) + K K. (K = 500.)

$$MP = \frac{0.00193}{0.02} = 0.0965 = 9.62\%$$

$$0.744 = \frac{\xi}{\sqrt{1-\xi^2}} \Rightarrow 0.5539 = \frac{\xi^2}{\sqrt{1-\xi^2}}$$

$$0.5539 - 0.5539\,\xi^2 = \xi^2$$

$$\xi = \underline{0.597} = \underline{0.6}$$

$$tp = \frac{\Pi}{Wd} = \frac{\Pi}{Wn\sqrt{(1-\xi^2)}}$$

$$3 = \frac{\Pi}{Wn\sqrt{1-(0.6)^2}} \Rightarrow Wn = 1.31\ldots\ \text{rad / Sec.}$$

$$Sx(S) = \frac{10/M}{\left(S^2 + \frac{F}{M}S + \frac{K}{M}\right)}$$

Comparing with the std. 2nd order **Eq :-**, then,

$$Wn^2 = \frac{K}{\mathbf{M}} \Rightarrow Wn = \sqrt{\frac{K}{\mathbf{M}}} \qquad (1.31)^2 = \frac{500}{\mathbf{M}} \qquad M = 291.36\ \text{kg.}$$

$$\frac{F}{\mathbf{M}} = 2\xi Wn \qquad F = 2 \times 0.6 \times 291 \times 1.31$$

F = <u>458.7 N/M/ Sec.</u>

9) Measurements conducted on sever me mechanism show the system response to be $c(t) = 1+0.2e^{-60t} - 1.2e^{-10t}$, When subjected to a unit step i/p. Obtain the expression for closed loop T.F the damping ratio and undamped natural frequency of oscillation .

<u>Solution:</u>

$$C(t) = 1+0.2e^{-60t} - 1.2e^{-10t}$$

Taking L.T., $C(S) = \frac{1}{S} + \frac{0.2}{S+60} - \frac{1.2}{S+10}$

$$C(S) = \frac{600/S}{S^2 + 70S + + 600}$$

Given that :- Unit step i/p r(t) = 1 $\therefore$ $R(S) = \frac{1}{S}$

$$\frac{C(S)}{R(S)} = \frac{600 / S}{S^2 + 70S + + 600}$$

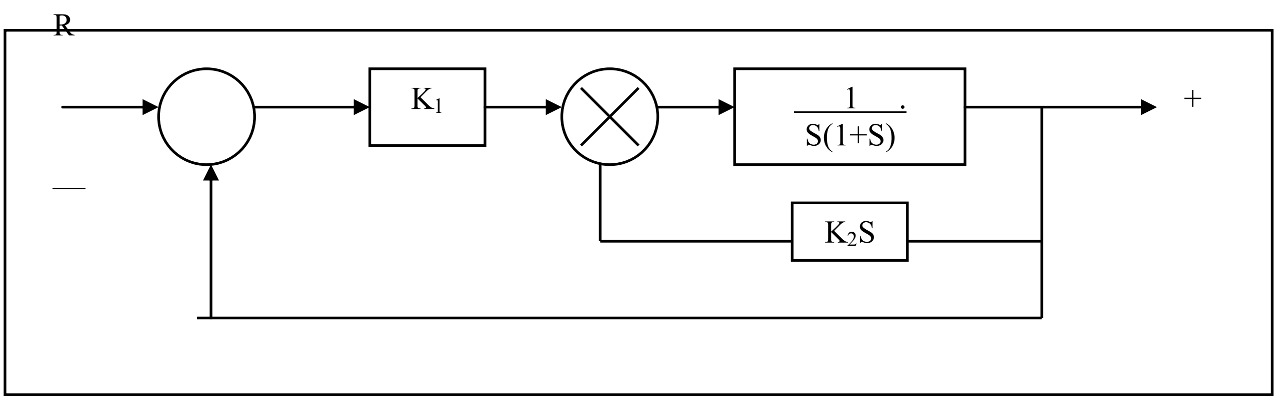
Comparing, $Wn^2 = 600$, 24.4 …..rad / Sec

$$2\,\xi Wn = 70, \Rightarrow \xi = \frac{70}{2 \times 24.4} = 1.428$$

10) The C.S. shown in the fig employs proportional plus error rate control. Determine the value of error rate const. Ke, so the damping ratio is 0.6 . Determine the value of settling time, max overshoot and steady state error, if the i/p is unit ramp, what will be the value of steady state

10) A feed back system employing o/p damping is as shown in fig.

4) Find the value of K_1 & K_2 so that closed loop system resembles a 2nd order system with ξ = 0.5 & frequency of damped oscillation 9.5 rad / Sec.
5) With the above value of K_1 & K_2 find the % overshoot when i/p is step i/p
6) What is the % overshoot when i/p is step i/p, the settling time for 2% tolerance?



$$\frac{C}{R} = \frac{K_1}{S^2 + (1 + K_2)S + K_1}$$

$$Wn^2 = K_1 \Rightarrow Wn = \sqrt{K_1}$$

$$2\xi Wn = 1 + K_2 \Rightarrow \xi = \underline{1 + K_2}$$

$$Wd = Wn\sqrt{(1-\xi^2)} \Rightarrow \quad \therefore Wn = \frac{9.5}{\sqrt{(1-0.5^2)}} = 10.96 \text{ rad/Sec}$$

$2\sqrt{K_1}$

$K_1 = (10.96)^2 = \underline{\underline{120.34}}$

$2\xi Wn = 1 + K_2, \quad K_2 = \underline{\underline{9.97}}$

$$MP = e^{\left(\frac{-\xi \Pi}{\sqrt{(1-\xi^2)}}\right)} = 16.3\%$$

$$ts = \frac{4}{\xi Wn} = \frac{4}{0.5 \times 10.97} = 0.729 \text{ sec}$$

3.5 Types of feed back control system :-

The open loop T.F. of a unity feed back system can be written in two std, forms;

1) Time constant form and **2) Pole Zero form,**

$$\therefore G(S) = \frac{K(TaS+1)(TbS+1)\ldots\ldots}{S^n(T_1 S+1)(T_2S+1)\ldots\ldots}$$

Where K = open loop gain.

Above Eq:- involves the term S^n in denominator which corresponds to no, of integrations in the system. A system is called Type O, Type1, Type2,……….. if n = 0, 1, 2, ………….. Respectively. The Type no., determines the value of error co-efficients. As the type no., is increased, accuracy is improved; however increasing the type no., aggregates the stability error. A term in the denominator represents the poles at the origin in complex S plane. Hence Index n denotes the multiplicity of the poles at the origin. The steady state errors co-efficient for a given type have definite values. This is illustration as follows.

2) <u>Type – O system</u> :- If, n = 0, the system is called type – 0, system. The steady state error are as follows;

Let, $G(S) = \frac{K}{S+1}$ $[\because H(s) = 1]$

$$e_{ss}\,(\text{Position}) = \frac{1}{1 + G(O)\,.\,H(O)} = \frac{1}{1+K} = \frac{1}{1+K_p}$$

$$\left(\because K_p = \lim_{S \to 0} G(S)\,.\,H(S) = \lim_{S \to 0} \left(\frac{K}{S+1}\right) = K\right)$$

$$e_{ss}\text{ (Velocity)} = \frac{1}{Kv} = \frac{1}{0} = \infty$$

$$Kv = \lim_{S \to 0} G(S)\,.\,H(S) = \lim_{S\to 0} S\left(\frac{K}{S+1}\right) = 0$$

$$e_{ss}\text{ (acceleration)} = \frac{1}{Ka} = \frac{1}{0} = \infty$$

$$Ka = \lim_{S \to 0} S^2\, G(S)\,.\,H(S) = \lim_{S\to 0} S^2\left(\frac{K}{S+1}\right) = 0.$$

2) <u>Type 1 –System</u> :- If, n = 1, the e_{ss} to various std, i/p, $G(S) = \frac{K}{S\,(S+1)}$

$$e_{ss}\text{ (Position)} = \frac{1}{1+\infty} = 0$$

$$\left(Kp = \lim_{S\to 0} G(S)\,.\,H(S) = \lim_{S \to 0}\left(\frac{K}{S(S+1)}\right) = \infty\right)$$

$$Kv = \lim_{S\to 0} S\,\frac{K}{S(S+1)} = K$$

$$e_{ss}\text{ (Velocity)} = \frac{1}{K}$$

$$e_{ss}\text{ (acceleration)} = \frac{1}{0} = \left(\frac{1}{0}\right) = \infty$$

$$\left(Ka = \lim_{S \to 0} S^2\,\frac{K}{S\,(S+1)} = O.\right)$$

3) <u>Type 2 –System</u> :- If, n = 2, the e_{ss} to various std, i/p, are , $G(S) = \frac{K}{S^2\,(S+1)}$

$$Kp = \lim \frac{K}{\quad} = \infty$$

$$S \to O \qquad S^2 (S+1)$$

$$\therefore \quad e_{ss} \text{ (Position)} = \frac{1}{\infty} = O$$

$$K_v = \lim_{S \to O} S \frac{K}{S^2 (S+1)} = \infty$$

$$\therefore \quad e_{ss} \text{ (Velocity)} = \frac{1}{\infty} = O$$

$$K_a = \lim_{S \to O} S^2 \frac{K}{S^2 (S+1)} = K.$$

$$\therefore \quad e_{ss} \text{ (acceleration)} = \frac{1}{K}$$

3) <u>Type 3 –System</u> :- Gives $K_p = K_v = K_a = \infty$ & $e_{ss} = O$. (Onwards)

<u>Recommended Questions</u>

1. Define and classify time response of a system.

2. Mention the Standard Test Input Signals and its Laplace transform

3. The open loop T.F. of a unity feed back system is given by $G(S) = \frac{K}{S(1+ST)}$. Where, T&K are constants having + Ve values.By what factor (1) the amplitude gain be reduced so that (a) The peak overshoot of unity step response of the system is reduced from 75% to 25% (b) The damping ratio increases from 0.1 to 0.6.

4. Find all the time domain specification for a unity feed back control system whose open loop T.F. is given by

$$G(S) = \frac{25}{S(S+6)}$$

5. The closed loop T.F. of a unity feed back control system is given by

$$\frac{C(S)}{R(S)} = \frac{5}{S^2 + 4S + 5}$$

Determine (1) Damping ratio ξ (2) Natural undamped response frequency Wn. (3) Percent peak over shoot Mp (4) Expression for error resoponse.

6. A servo mechanism is represent by the Eq:-

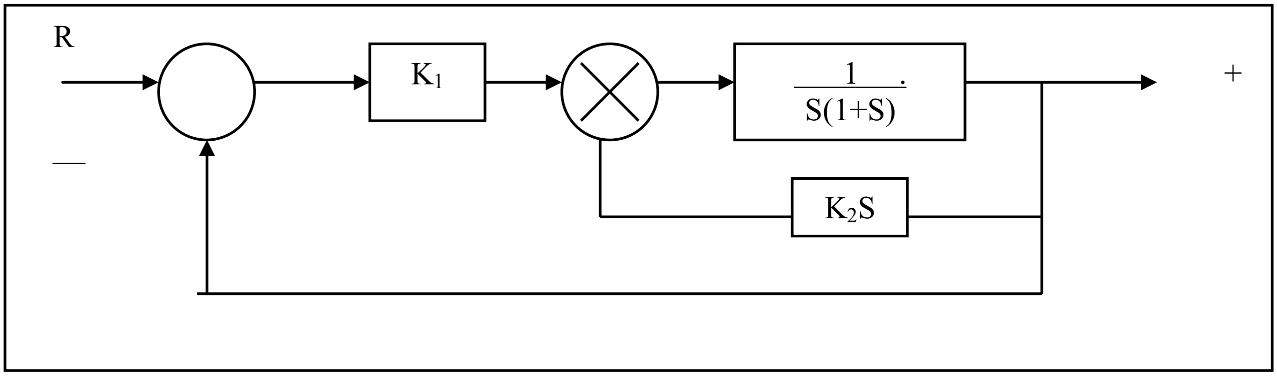
$$\frac{d^2\theta}{dt^2} + 10 \frac{d\theta}{dt} = 150E$$

, $E = R - \theta$ is the actuating signal calculate the value of damping ratio, undamped and damped frequency of ascillation.

7. Measurements conducted on sever me mechanism show the system response to be c (t) = $1+0.2e^{-60t} - 1.2e^{-10t}$, When subjected to a unit step i/p. Obtain the expression for closed loop T.F the damping ratio and undamped natural frequency of oscillation .

8. A feed back system employing o/p damping is as shown in fig.

1) Find the value of K_1 & K_2 so that closed loop system resembles a 2nd order system with ξ = 0.5 & frequency of damped oscillation 9.5 rad / Sec.
   2) With the above value of K_1 & K_2 find the % overshoot when i/p is step i/p
   3) What is the % overshoot when i/p is step i/p, the settling time for 2% tolerance?



4. Stability Analysis

Every System, for small amount of time has to pass through a transient period. Whether system will reach its steady state after passing through transients or not. The answer to this question is whether the system is stable or unstable. This is stability analysis.

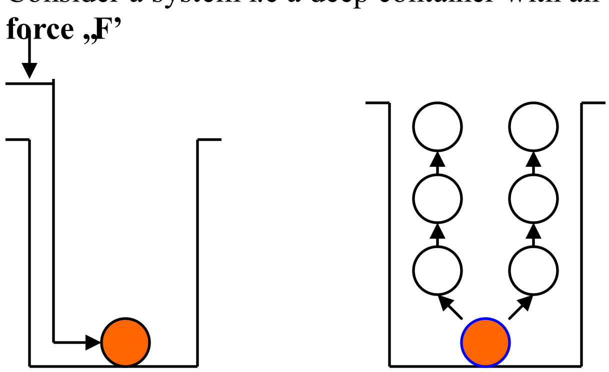
For example, we want to go from one station to other. The station we want to reach is our final steady state. The traveling period is the transient period. Now any thing
may happen during the traveling period due to bad weather, road accident etc, there is a chance that we may not reach the next station in time. The analysis of wheather the given system can reach steady state after passing through the transients successfully is called the stability analysis of the system.

In this chapter, we will steady

1. The stability & the factor on which system stability depends.
2. Stability analysis & location of closed loop poles.
3. Stability analysis using Hurwitz method.
4. Stability analysis using Routh-Hurwitz method.
5. Special cases of Routh's array.
6. Applications of Routh-Hurwitz method.

4.1.1 Concept of stability:

Consider a system i.e a deep container with an object placed inside it as shown in fig(1)



(a) fig(1) (b)

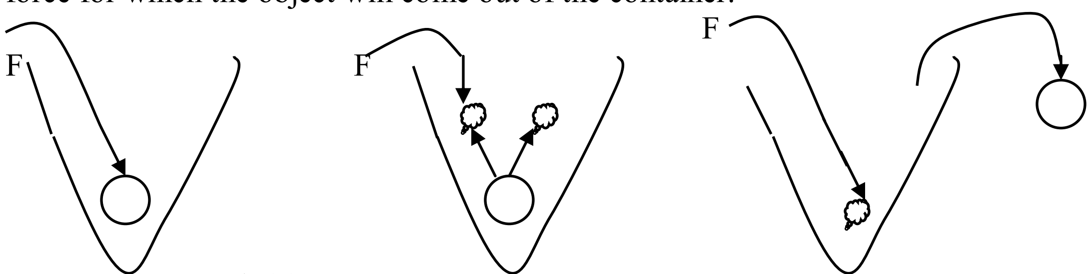
Now, if we apply a force to take out the object, as the depth of container is more, it will oscillate & settle down again at original position.

Assume that force required to take out the object tends to infinity i.e always object will oscillate when force is applied & will settle down but will not come out such a system is called absolutely stable system. No change in parameters, disturbances, changes the output.

Now consider a container which is pointed one, on which we try to keep a circular object. In this object will fall down without any external application of force. Such system is called Unstable system.

(a) fig(2) (b)

While in certain cases the container is shallow then there exsists a critical value of force for which the object will come out of the container.



Fig(3) $F<F_{critical}$ $F>F_{critical}$

As long as $F<F_{critical}$ object regains its original position but if $F>F_{critical}$ object will come out. Stability depends on certain conditions of the system, hence system is called conditionally stable system.

Pendulum where system keeps on oscillating when certain force is applied. Such systems are neither stable nor unstable & hence called critically stable or marginally stable systems

Stability of control systems:

The stability of a linear closed loop system can be determined from the locations of closed loop poles in the S-plane.

If the system has closed loop T.F.

$$\frac{C(s)}{R(s)} = \frac{10}{(S+2)\,(S+4)}$$

Output response for unit step input $R(s) = \dfrac{1}{S}$

$$C(s) = \frac{10}{S(S+2)(S+4)} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S+4}$$

Find out partial fractions

$$C(s) = \left(\frac{\frac{1}{8}}{S} - \frac{\frac{1}{4}}{S+2} + \frac{\frac{1}{8}}{S+4} \right) 10$$

$$= \frac{1.25}{S} - \frac{2.5}{S+2} + \frac{1.25}{S+4}$$

$$C(s) = 1.25 - 2.5e^{-2t} + 1.25e^{-4t}$$

$$= Css + C_t(t)$$

If the closed loop poles are located in left half of s-plane, Output response contains exponential terms with negative indices will approach zero & output will be the steady state output.

i.e.

$$C_t(t) = 0$$
$$t \rightarrow \infty$$

Transient output = 0

Such system are called absolutely stable systems.

Now let us have a system with one closed loop pole located in right half of s- plane

$$\frac{C(s)}{R(s)} = \frac{10}{S(S-2)(s+4)}$$

$$= \left[\frac{A}{S} + \frac{B}{S-2} + \frac{C}{S+4} \right] 10$$

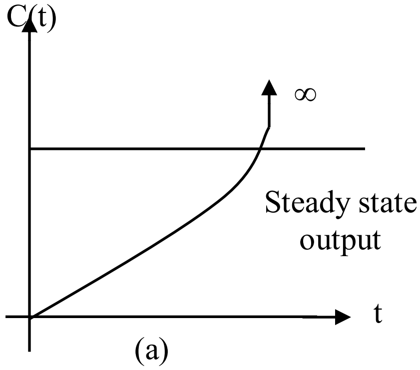
$$C(t) = -1.25 + 0.833e^{2t} + 0.416e^{-4t}$$

Here there is one exponential term with positive in transient output

Therefore Css = - 1.25

t	C(t)
0	0
1	+ 4.91
2	+ 44.23
4	+2481.88
∞	∞

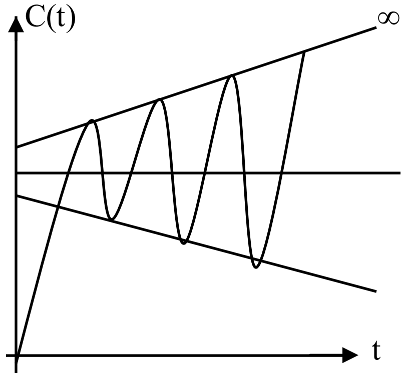
From the above table, it is clear that output response instead of approaching to steady state value as $t \to \infty$ due to exponential term with positive index, transients go on increasing in amplitude. So such system is said to be unstable. In such system output is uncontrollable & unbounded one. Output response of such system is as shown in fig(4).



(a)

OR

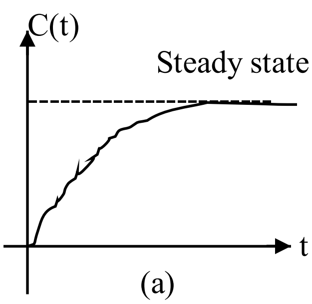
fig(4)



(b)

For such unstable systems, if input is removed output may not return to zero. And if the input power is turned on, output tends to ∞ . If no saturation takes place in system & no mechanical stop is provided then system may get damaged.

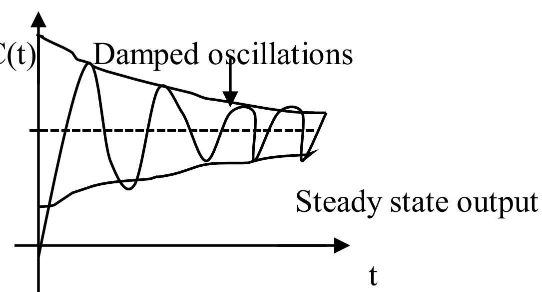
If all the closed loop poles or roots of the characteristic equation lies in left of s-plane, then in the output response contains steady state terms & transient terms. Such transient terms approach to zero as time advances eventually output reaches to equilibrium & attains steady state value. Transient terms in such system may give oscillation but the amplitude of such oscillation will be decreasing with time & finally will vanish. So output response of such system is shown in fig5 (a) & (b).



(a)

OR

fig 5



(b)

BIBO Stability : This is bounded input bounded output stability.

4.1.2 Definition of stable system:

A linear time invariant system is said to be stable if following conditions are satisfied.

1. When system is excited by a bounded input, output is also bounded & controllable.
2. In the absence of input, output must tend to zero irrespective of the initial conditions.

Unstable system:

A linear time invariant system is said to be unstable if,

1. for a bounded input it produces unbounded output.
2. In the absence of input, output may not be returning to zero. It shows certain output without input.

Besides these two cases, if one or more pairs simple non repeated roots are located on the imaginary axis of the s-plane, but there are no roots in the right half of s-plane, the output response will be undamped sinusoidal oscillations of constant frequency & amplitude. Such systems are said to be critically or marginally stable systems.

4.1.3 Critically or Marginally stable systems:

A linear time invariant system is said to be critically or marginally stable if for a bounded input its output oscillates with constant frequency & Amplitude. Such oscillation of output are called Undamped or Sustained oscillations.

For such system one or more pairs of non repeated roots are located on the imaginary axis as shown in fig6(a). Output response of such systems is as shown in fig6(b).

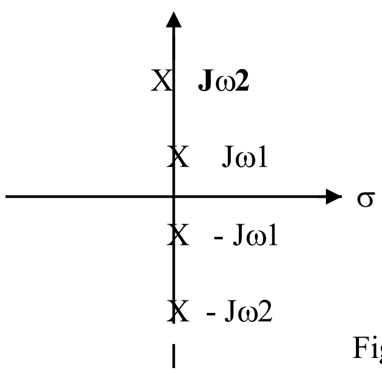
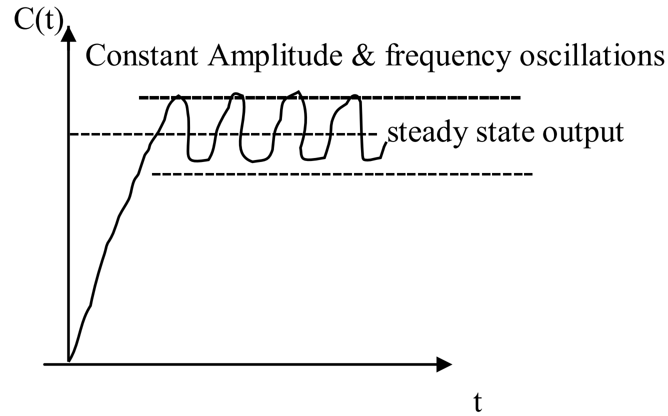
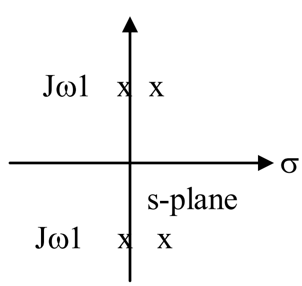


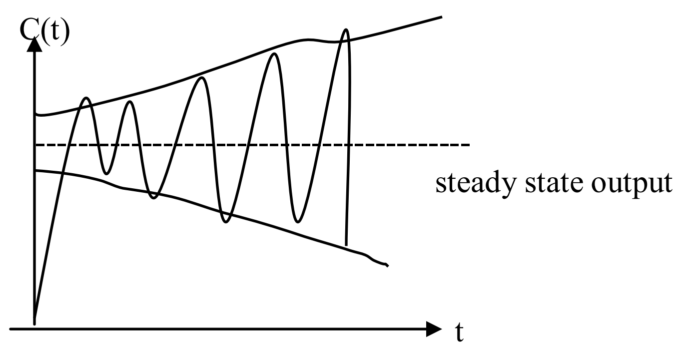
Fig 6(a)

non repeated poles on Jω axis.



If there are repeated poles located purely on imaginary axis system is said to be unstable.

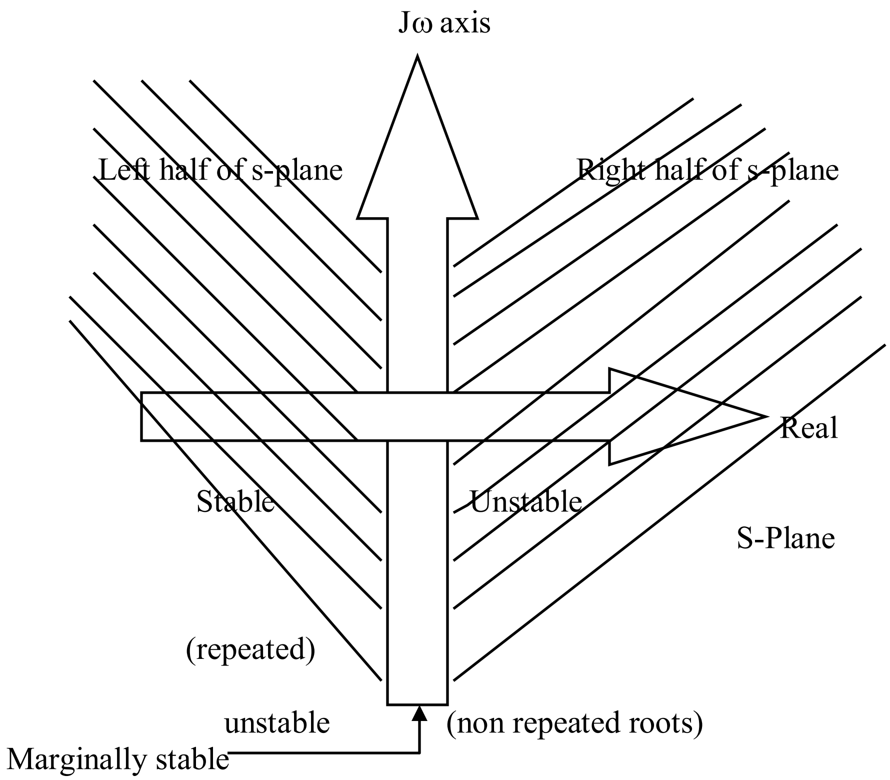


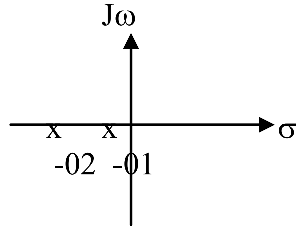
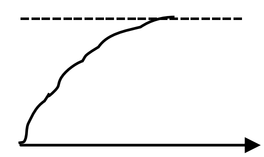
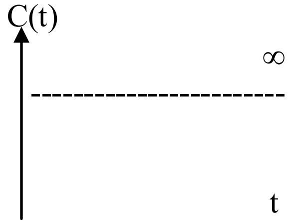
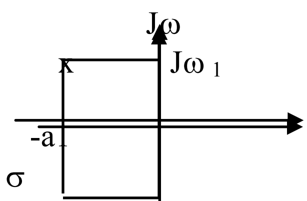
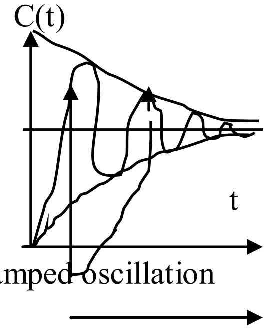
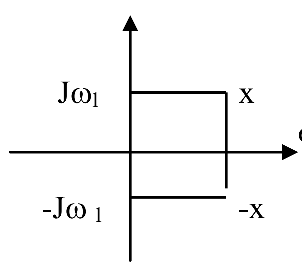
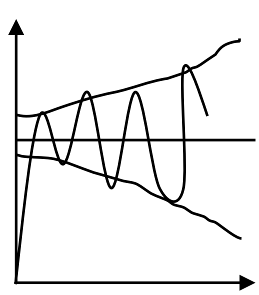


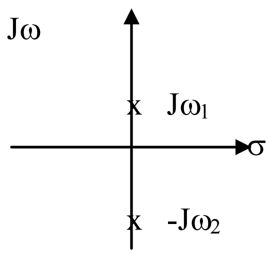
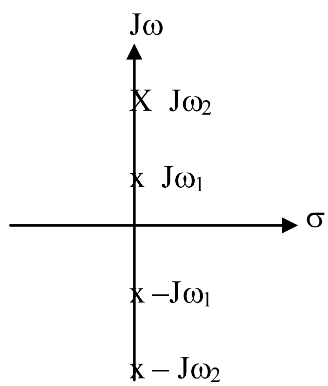
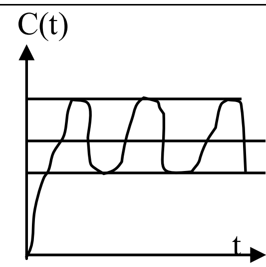
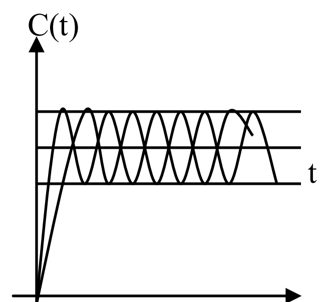
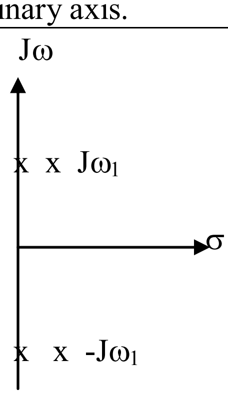
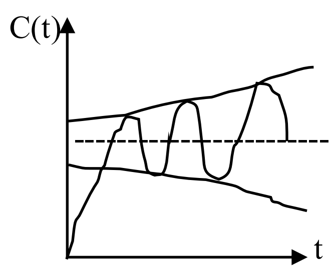
Conditionally Stable:

A linear time invariant system is said to be conditionally stable, if for a certain condition if a particular parameter of the system, its output is bounded one. Otherwise if that condition is violated output becomes unbounded system becomes unstable. i.e. Stability of the system depends the on condition of the parameter of the system. Such system is called conditionally stable system.

S-plane can be divided into three zones from stability point of view.



Sl. No	Nature of closed loop poles.	Location of closed loop poles in s-plane	Step response	Stability condition
1.	Real negative i.e in LHS of splane			Absolutely stable
2.	Real positive in RHS of s-plane	Jω x σ a_1	 increasing towards ∞	Unstable
3.	Complex conjugate with negative real part		 Damped oscillation	Absolutely stable.
4.	Complex conjugate with positive real part		 oscillations with increasing amplitude	Unstable.

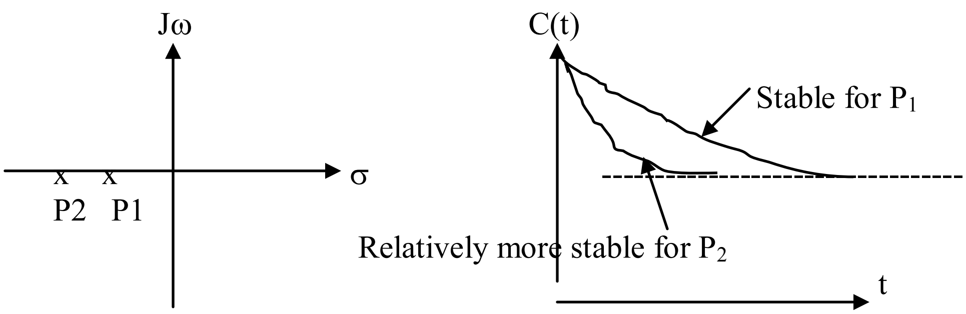
5.	Non repeated pair on imaginary axis	 OR  Two non repeated pairs on imaginary axis.	  Sustained oscillations with two frequencies ω_1 & ω_2	Marginally or critically stable Marginally or critically stable
6.	Repeated pair on imaginary axis		 oscillation of increasing amplitude	Unstable

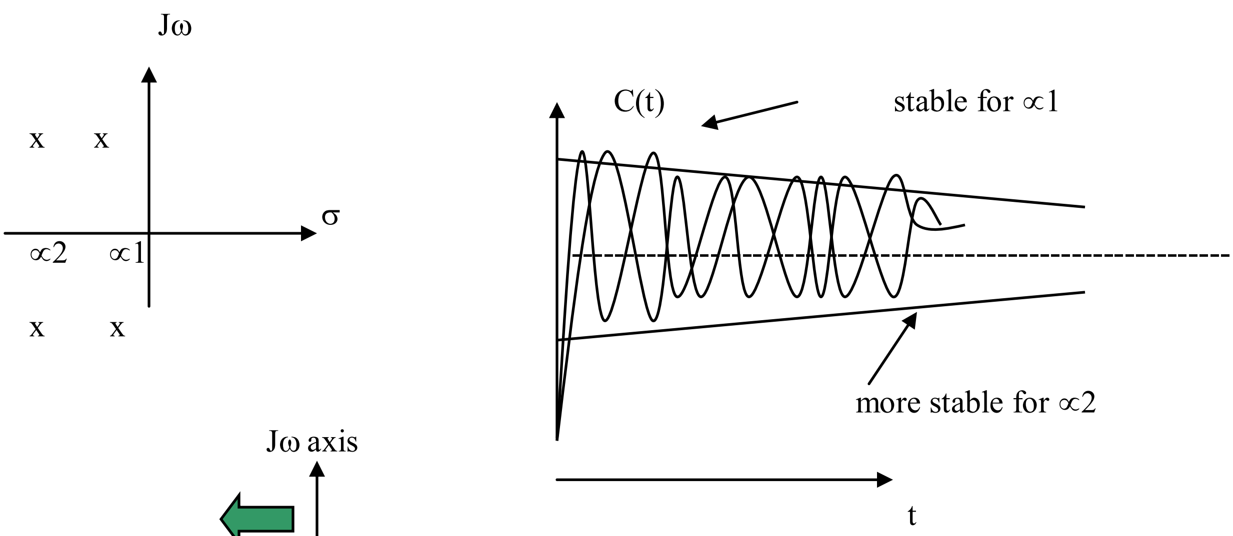
4.1.4 Relative Stability:

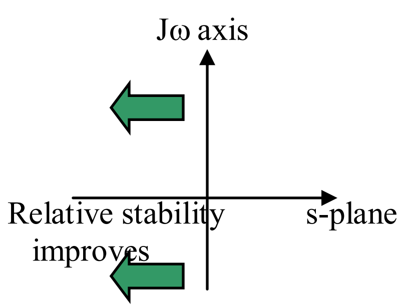
The system is said to be relatively more stable or unstable on the basis of settling time. System is said to be more stable if settling time for that system is less than that of other system.

The settling time of the root or pair of complex conjugate roots is inversely proportional to the real part of the roots.

Sofar the roots located near the Jω axis, settling time will be large. As the roots move away from Jω axis i.e towards left half of the s-plane settling time becomes lesser or smaller & system becomes more & more stable. So the relative stability improves.







4.2 Routh – Hurwitz Criterion :

This represents a method of determining the location of poles of a characteristics equation with the respect to the left half & right half of the s-plane without actually solving the equation.

The T.F.of any linear closed loop system can be represented as,

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_m}{a_0 s^n + a_1 s^{n-1} + \ldots + a_n}$$

Where ‗a' & ‗b' are constants.

To find the closed loop poles we equate F(s) =0. This equation is called as **Characteristic Equation** of the system.

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots.. + a_n = 0.$$

Thus the roots of the characteristic equation are the closed loop poles of the system which decide the stability of the system.

Necessary Condition to have all closed loop poles in L.H.S. of s-plane.

In order that the above characteristic equation has no root in right of s-plane, it is necessary but not sufficient that,

1. All the coefficients off the polynomial have the same sign.
2. Non of the coefficient vanishes i.e. all powers of ‗s' must be present in descending order from ‗n' to zero.

These conditions are not sufficient.

Hurwitz's Criterion :

The sufficient condition for having all roots of characteristics equation in left half of s-plane is given by Hurwitz. It is referred as Hurwitz criterion. It states that:

The necessary & sufficient condition to have all roots of characteristic equation in left half of s-plane is that the sub-determinants D_K, K = 1, 2,………n obtained from Hurwitz determinant ‗H' must all be positive.

Method of forming Hurwitz determinant:

$$H = \begin{vmatrix} a_1 & a_3 & a_5 & \dots\dots\dots & a_{2n-1} \\ a_0 & a_2 & a_4 & \dots\dots\dots & a_{2n-2} \\ 0 & a_1 & a_3 & \dots\dots\dots & a_{2n-3} \\ 0 & a_0 & a_2 & \dots\dots\dots & a_{2n-4} \\ 0 & 0 & a_1 & \dots\dots\dots & a_{2n-5} \\ - & - & & \dots\dots\dots & - \\ - & - & - & \dots\dots\dots & - \\ 0 & - & - & \dots\dots & a_n \end{vmatrix}$$

The order is n*n where n = order of characteristic equation. In Hurwitz determinant all coefficients with suffices greater than ‗n‘ or negative suffices must all be replaced by zeros. From Hurwitz determinant subdeterminants, D_K, K= 1, 2, ….n must be formed as follows:

$$D_1 = \begin{vmatrix} a_1 \end{vmatrix} \quad D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} \quad D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} \quad D_K = \begin{vmatrix} H \end{vmatrix}$$

For the system to be stable, all above determinants must be positive.

Determine the stability of the given characteristics equation by Hurwitz,s method.

Ex 1: $F(s)= s^3 + s^2 + s^1 + 4 = 0$ is characteristic equation.

$a_0 = 1, \ a_1 = 1, \ a_2 = 1, \ a_3 = 4, \ n = 3$

$$H = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} 1 \end{vmatrix} = 1$$

$$D_2 = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -3$$

$$D_3 = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 4 - 16 = -12.$$

As D_2 & D_3 are negative, given system is unstable.

Disadvantages of Hurwitz's method :

1. For higher order system, to solve the determinants of higher order is very complicated & time consuming.
2. Number of roots located in right half of s-plane for unstable system cannot be judged by this method.
3. Difficult to predict marginal stability of the system.

Due to these limitations, a new method is suggested by the scientist Routh called Routh's method. It is also called Routh-Hurwitz method.

Routh's Stability Criterion:

It is also called Routh's array method or Routh-Hurwitz's method
Routh suggested a method of tabulating the coefficients of characteristic equation in a particular way. Tabulation of coefficients gives an array called Routh's array.
Consider the general characteristic equation as,

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots.. + a_n = 0.$$

Method of forming an array :

$$\begin{array}{c|ccccc}
S^n & a_0 & a_2 & a_4 & a_6 & \ldots\ldots\ldots \\
S^{n-1} & a_1 & a_3 & a_5 & a_7 & \\
S^{n-2} & b_1 & b_2 & b_3 & & \\
S^{n-3} & c_1 & c_2 & c_3 & & \\
- & - & - & - & & \\
- & - & - & - & & \\
S^0 & a_n & & & &
\end{array}$$

Coefficients of first two rows are written directly from characteristics equation. From these two rows next rows can be obtained as follows.

$$b_1 = \frac{a_1\, a_2 - a_0\, a_3}{a_1}, \qquad b_2 = \frac{a_1\, a_4 - a_0\, a_5}{a_1}, \qquad b_3 = \frac{a_1\, a_6 - a_0\, a_7}{a_1}$$

From 2^{nd} & 3^{rd} row , 4^{th} row can be obtained as

$$C_1 = \frac{b_1\, a_3 - a_1\, b_2}{b_1}, \qquad C_2 = \frac{b_1\, a_5 - a_1\, b_3}{b_1}$$

This process is to be continued till the coefficient for s^0 is obtained which will be a_n. From this array stability of system can be predicted.

Routh‘s criterion :

The necessary & sufficient condition for system to be stable is ―All the terms in the first column of Routh‘s array must have same sign. There should not be any sign change in first column of Routh‘s array”.

If there are sign changes existing then,

1. System is unstable.
2. The number of sign changes equals the number of roots lying in the right half of the s-plane.

Examine the stability of given equation using Routh‘s method :

Ex.2: $s^3+6s^2 + 11s + 6 = 0$

Sol: $\mathbf{a_0 = 1, \quad a_1 = 6, \quad a_2 = 11, \quad a_3 = 6, \quad n = 3}$

$$\begin{array}{c|cc}
S^3 & 1 & 11 \\
S^2 & 6 & 6 \\
S^1 & \dfrac{11 * 6 - 6}{6} = 10 & 0 \\
S^0 & 6 &
\end{array}$$

As there is no sign change in the first column, system is stable.

Ex. 3 $s^3 + 4s^2 + s + 16 = 0$

Sol: $a_0 = 1$, $a_1 = 4$, $a_2 = 1$, $a_3 = 16$

S^3	1	1
S^2	+4	16
S^1	$\frac{4 - 16}{4} = -3$	0
S^0	+16	

As there are two sign changes, system is unstable.

Number of roots located in the right half of s-plane = number of sign changes = 2.

4.3 Special Cases of Routh's criterion :

Special case 1 :

First element of any of the rows of Routh's array is zero & same remaining rows contains at least one non-zero element.

Effect : The terms in the new row become infinite & Routh's test fails.

e.g. : $s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$

S^5	1	3	2
S^4	2	6	1
S^3	$\boxed{0}$	1.5	0
S^2	∞	….	…

Special case 1 Routh's array failed

Following two methods are used to remove above said difficulty.

First method : Substitute a small positive number ‗ε‘ in place of a zero occurred as a first element in the row. Complete the array with this number ‗ε‘. Then examine

Sign change by taking $\lim_{\varepsilon \to 0}$. Consider above Example.

$$
\begin{array}{c|ccc}
S^5 & 1 & 3 & 2 \\
S^4 & 2 & 6 & 1 \\
S^3 & \varepsilon & 1.5 & 0 \\
S^2 & \dfrac{6\varepsilon - 3}{\varepsilon} & 1 & 0 \\
S^1 & \dfrac{\dfrac{1.5(6\varepsilon - 3)}{\varepsilon} - \varepsilon}{\dfrac{(6\varepsilon - 3)}{\varepsilon}} & 0 & \\
S^0 & 1 & &
\end{array}
$$

To examine sign change,

$$\lim_{\varepsilon \to 0} = \frac{6\varepsilon - 3}{\varepsilon} = \lim_{\varepsilon \to 0} \; 6 - \frac{3}{\varepsilon}$$

$$= 6 - \infty$$

$$= -\infty \quad \text{sign is negative.}$$

$$\lim_{\varepsilon \to 0} \frac{1.5(6\varepsilon - 3) - \varepsilon^2}{6\varepsilon - 3} = \lim_{\varepsilon \to 0} \frac{9\varepsilon - 4.5 - \varepsilon^2}{6\varepsilon - 3}$$

$$= \frac{0 - 4.5 - 0}{0 - 3}$$

$$= +1.5 \quad \text{sign is positive}$$

Routh‘s array is,

S^5	1	3	2
S^4	2	6	1
S^3	$+\varepsilon$	1.5	0
S^2	$-\infty$	1	0
S^1	+1.5	0	0
S^0	1	0	0

As there are two sign changes, system is unstable.

Second method : To solve the above difficulty one more method can be used. In this, replace ‗s‘ by ‗1/Z‘ in original equation. Taking L.C.M. rearrange characteristic equation in descending powers of ‗Z‘. Then complete the Routh‘s array with this new equation in ‗Z‘ & examine the stability with this array.

Consider $F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$

Put $s = 1 / Z$

$$\therefore \frac{1}{Z^5} + \frac{2}{Z^4} + \frac{3}{Z^3} + \frac{6}{Z^2} + \frac{2}{Z} + 1 = 0$$

$$Z^5 + 2Z^4 + 6Z^3 + 3Z^2 + 2Z + 1 = 0$$

Z^5	1	6	2
Z^4	2	3	1
Z^3	4.5	1.5	0
Z^2	2.33	1	0
Z^1	- 0.429	0	
Z^0	1		

As there are two sign changes, system is unstable.

Special case 2 :

All the elements of a row in a Routh's array are zero.
Effect : The terms of the next row can not be determined & Routh's test fails.

$$\begin{array}{c|ccc} S^5 & a & b & c \\ S^4 & d & e & f \\ S^3 & \boxed{0 \quad 0 \quad 0} \end{array} \longleftarrow \text{Row of zeros, special case 2}$$

This indicates no availability of coefficient in that row.

4.3.1 Procedure to eliminate this difficulty :

1. Form an equation by using the coefficients of row which is just above the row of zeros. Such an equation is called an **Auxillary equation** denoted as A(s). For above case such an equation is,

$$A(s) = ds^4 + es^2 + f$$

Note that the coefficients of any row are corresponding to alternate powers of 's' starting from the power indicated against it.

So 'd' is coefficient corresponding to s^4 so first term is ds^4 of A(s).

Next coefficient 'e' is corresponding to alternate power of 's' from 4 i.e. s^2 Hence the term es^2 & so on.

2. Take the derivative of an auxillary equation with respect to 's'.

i.e. $$\frac{dA(s)}{ds} = 4d\,s^3 + 2e\,s$$

3. Replace row of zeros by the coefficients of $\frac{dA(s)}{ds}$

$$\begin{array}{c|ccc} S^5 & a & b & c \\ S^4 & d & e & f \\ S^3 & 4d & 2e & 0 \end{array}$$

4. Complete the array of zeros by the coefficients.

Importance of auxillary equation :

Auxillary equation is always the part of original characteristic equation. This means the roots of the auxillary equation are some of the roots of original characteristics equation. Not only this but roots of auxillary equation are the most dominant roots of the original characteristic equation, from the stability point of view. The stability can be predicted from the roots of A(s)=0 rather than the roots of characteristic equation as the roots of A(s) = 0 are the most dominant from the stability point of view. The remaining roots of the characteristic equation are always in the left half & they do not play any significant role in the stability analysis.

e.g. Let F(s) = 0 is the original characteristic equation of say order n = 5.
Let A(s) = 0 be the auxillary equation for the system due to occurrence of special case 2 of the order m = 2.

Then out of 5 roots of F(s) = 0, the 2 roots which are most dominant (dominant means very close to imaginary axis or on the imaginary axis or in the right half of s-plane) from the stability point of view are the 2roots of A(s) = 0. The remaining 5 –2 = 3 roots are not significant from stability point of view as they will be far away from the imaginary axis in the left half of s-plane.

The roots of auxillary equation may be,

1. A pair of real roots of opposite sign i.e.as shown in the fig. 8.10 (a).

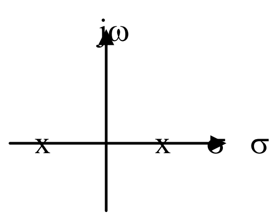


Fig 8. 10(a)

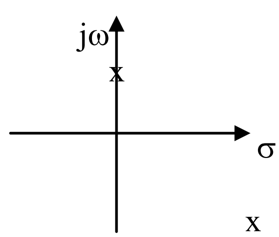
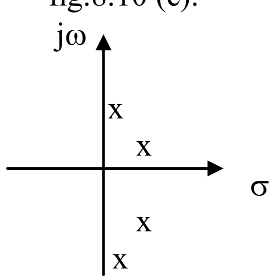


Fig. 8. 10 (b)

2. A pair of roots located on the imaginary axis as shown in the fig. 8.10(b).
3. The non-repeated pairs of roots located on the imaginary axis as shown in the fig.8.10 (c).



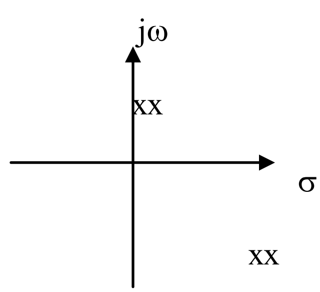


Fig. 8.10(c) Fig. 8.10(d).

4. The repeated pairs of roots located on the imaginary axis as shown in the Fig.8.10 (d).

   Hence total stability can be determined from the roots of A(s) = 0, which can be out of four types shown above.

Change in criterion of stability in special case 2 :

After replacing a row of zeros by the coefficients of $\frac{dA(s)}{ds}$, complete the Routh's array.

But now, the criterion that, no sign in 1st column of array for stability, no longer remains sufficient but becomes a necessary. This is because though A(s) is a part of original characteristic equation, $\frac{dA(s)}{ds}$ is not, which is in fact used to complete the array.

So if sign change occurs in first column, system is unstable with number of sign changes equal to number of roots of characteristics equation located in right half of s-plane.

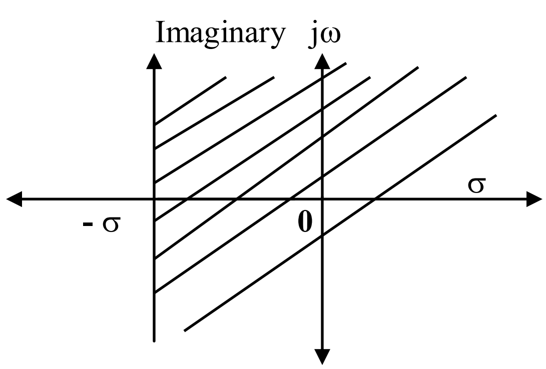
But there is no sign changes, system cannot be predicted as stable . And in such case stability is to be determined by actually solving A(s) = 0 for its roots. And from the location of roots of A(s) = 0 in the s-plane the system stability must be determined. Because roots A(s) = 0 are always dominant roots of characteristic equation.

Application of Routh's of criterion :

Relative stability analysis :

If it is required to find relative stability of system about a line $s = -\sigma$. i.e. how many roots are located in right half of this line $s = -\sigma$, the Routh's method can be used effectively.

To determine this from Routh's array, shift the axis of s – plane & then apply Routh's array i.e. substitute $s = s^1 - \sigma$, (σ = constant) in characteristic equation. Write polynomial in terms of s^1. Complete array from this new equation. The number of sign changes in first column is equal to number of roots those are located to right of the vertical line $s = -\sigma$.



Determining range of values of K :

In practical system, an amplifier of variable gain K is introduced .
The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{KG(s)}{1+ KG(s)\, H(s)}$$

Hence the characteristic equation is

$$F(s) = 1+ KG(s)\, H(s) = 0$$

So the roots of above equation are dependent on the proper selection of value of ‗K‘.

So unknown ‗K‘ appears in the characteristic equation. In such case Routh‘s array is to be constructed in terms of K & then the range of values of K can be obtained in such away that it will not produce any sign change in first column of the Routh‘s array. Hence it is possible to obtain the range of values of K for absolute stability of the system using Routh‘s criterion. Such a system where stability depends on the condition of parameter K, is called conditionally stable system.

Advantages of Routh‘s criterion :

Advantages of routh‘s array method are :

1. Stability of the system can be judged without actually solving the characteristic equation.
2. No evaluation of determinants, which saves calculation time.
3. For unstable system it gives number of roots of characteristic equation having positive real part.
4. Relative stability of the system can be easily judged.
5. By using the criterion, critical value of system gain can be determined hence frequency of sustained oscillations can be determined.
6. It helps in finding out range of values of K for system stability.
7. It helps in finding out intersection points of roots locus with imaginary axis.

Limitation of Routh‘s criterion :

1. It is valid only for real coefficients of the characteristic equation.
2. It does not provide exact locations of the closed loop poles in left or right half of s-plane.
3. It does not suggest methods of stabilizing an unstable system.
4. Applicable only to linear system.

Ex.1. $s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$ Find the number of roots of this equation with positive real part, zero real part & negative real part

Sol:

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	0	0	0	

$$A(s) = 3S^4 - 48 = 0 \qquad \frac{dA}{ds} = 12s^3$$

S^6	1	3	-16	-48
S^5	4	0	-64	0
S^4	3	0	-48	0
S^3	12	0	0	0
S^2	$(\varepsilon)0$	-48	0	0
S^1	$\frac{576}{\varepsilon}$	0	0	0
S^0	-48			

$$\lim_{\varepsilon \to 0} \frac{576}{\varepsilon} = +\infty$$

Therefore One sign change & system is unstable. Thus there is one root in R.H.S of the s – plane i.e. with positive real part. Now
solve A(s) = 0 for the dominant roots

$$A(s) = 3s^4 - 48 = 0$$

Put $S^2 = Y$

$\therefore \quad 3Y^2 = 48 \qquad \therefore \quad Y^2 = 16, \qquad \therefore \quad Y = \pm\sqrt{16} = \pm 4$

$S^2 = +4 \qquad S^2 = -4$

$S = \pm 2 \qquad S = \pm 2j$

So $S = \pm 2j$ are the two parts on imaginary axis i.e. with zero real part. Root in R.H.S. indicated by a sign change is $S = \pm 2$ as obtained by solving $A(s) = 0$. Total there are 6 roots as $n = 6$.

Roots with Positive real part = 1
Roots with zero real part = 2
Roots with negative real part = 6 –2 – 1 = 3

Ex.2 : For unity feed back system,

$$G(s) = \frac{k}{S(1 + 0.4s)(1 + 0.25s)}$$

, Find range of values of K, marginal value of K & frequency of sustained oscillations.

Sol : Characteristic equation, $1 + G(s)H(s) = 0$ & $H(s) = 1$

$$\therefore 1 + \frac{K}{s(1 + 0.4s)(1 + 0.25s)} = 0$$

$$s[1 + 0.65s + 0.1s^2\} + K = 0$$

$$\therefore 0.1s^3 + 0.65s^2 + s + K = 0$$

S^3	0.1	1	From s^0, $K > 0$
S^2	0.65	K	from s^1,
S^1	$\frac{0.65 - 0.1K}{0.65}$	0	$0.65 - 0.1K > 0$ $\therefore 0.65 > 0.1K$
S^0	K		$\therefore 6.5 > K$

$\therefore$ Range of values of K, $0 < K < 6.5$

Now marginal value of ‗K‘ is that value of ‗K‘ for which system becomes marginally stable. For a marginal stable system there must be row of zeros occurring in Routh‘s array. So value of ‗K‘ which makes any row of Routh array as row of zeros is called marginal value of ‗K‘. Now K = 0 makes row of s^0 as row of zeros but K = 0 can not be marginal value because for K = 0, constant term in characteristic equation becomes zeros ie one coefficient for s^0 vanishes which makes system unstable instead of marginally stable.

Hence marginal value of ‗K' is a value which makes any row other than s^0 as row of zeros.

$\therefore \quad 0.65 - 0.1\, K_{mar} = 0$

$\therefore \quad K_{mar} = 6.5$

To find frequency, find out roots of auxiliary equation at marginal value of ‗K'

$A(s) = 0.65\, s^2 + K = 0$;

$\therefore \quad 0.65\, s^2 + 6.5 = 0$ Because K = 6.5

$s^2 = -10$

$s = \pm j\, 3.162$

comparing with $s = \pm j\omega$

ω = frequency of oscillations = 3.162 rad/ sec.

Ex : 3 For a system with characteristic equation

$F(s) = s^5 + s4 + 2s^3 + 2s^2 + 3s + 15 = \infty$, examine the stability

Solution :

S^5	1	2	3
S^4	1	2	15
S^3	0	-12	0
S^2			
S^1			
S^0			

S^5	1	2	3
S^4	1	2	15
S^3	ε	-12	0
S^2	$\frac{(2\varepsilon + 12)}{\varepsilon}$	15	0
S^1	$\dfrac{\frac{(2\varepsilon + 12)}{\varepsilon}(-12) - 15\varepsilon}{\frac{2\varepsilon + 12}{\varepsilon}}$	0	

$$S^0 \quad \lim_{\varepsilon \to 0} \frac{2\varepsilon + 12}{\varepsilon} = 2 + \frac{12}{\varepsilon} = 2 + \infty = +\infty$$

(the S^0 row entry is 15)

$$\lim_{\varepsilon \to 0} \frac{\frac{(2\varepsilon + 12)(-12) - 15\varepsilon}{\varepsilon}}{\frac{2\varepsilon + 12}{\varepsilon}} = \lim_{\varepsilon \to 0} \frac{-24\varepsilon - 144 - 15\varepsilon^2}{2\varepsilon + 12}$$

$$= \frac{0 - 144 - 0}{0 + 12} = -12$$

S^5	1	2	3
S^4	1	2	15
S^3	ε	-12	0
S^2	$+\infty$	15	0
S^1	- 12	0	
S^0	15		

There are two sign changes, so system is unstable.

Ex : 4 Using Routh Criterion, investigate the stability of a unity feedback system whose open loop transfer function is

$$G(s) = \frac{e^{-sT}}{s(s+1)}$$

Sol : The characteristic equation is

$$1 + G(s)\ H(s) = 0$$

$$\therefore \quad 1 + \frac{e^{-sT}}{s(s+1)} = 0$$

$$\therefore \quad s^2 + s + e^{-sT} = 0$$

Now e^{-sT} can be Expressed in the series form as

$$e^{-sT} = \left[1 - sT + \frac{s^2 T^2}{2!} + \ldots\ldots\right]$$

Trancating the series & considering only first two terms we get

$$e^{sT} = 1 - sT$$

$$\therefore \quad s^2 + s + 1 - sT = 0$$
$$\therefore \quad s^2 + s\,(1 - T) + 1 = 0$$

So routh's array is

S^2	1	1
S	1-T	0
S^0	1	

$$\therefore \; 1 - T > 0 \quad \text{for stability}$$
$$\therefore \quad T < 1$$

This is the required condition for stability of the system.

Ex : 5 Determine the location of roots with respect to s = -2 given that

$$F(s) = s^4 + 10\,s^3 + 36s^2 + 70s + 75$$

Sol : shift the origin with respect to s = -2

$$s = s^1 - 2$$

$$(s' - 2)^4 + 10\,(s' - 2)^3 + 36(s' - 2)^2 + 70\,(s' - 2) + 75 = 0$$

$$s'^4 + 2s'^3 + 0s'^2 + 14s' + 15 = 0$$

S'^4	1	0	15
S'^3	2	14	0
S'^2	-7	15	0
S'^1	18.28	0	0
S'^0	15		

Two sign change, there are two roots to the right of s = -2 & remaining ‗2‘ are to the left of the line s = -2. Hence the system is unstable.

Recommended Questions:

1. Explain briefly how system depends on poles and zeros.

2. Mention the necessary condition to have all closed loop poles in LHS of S-Plane

3. Explain briefly the Hurwitz‘s Criterion.

4. Explain briefly the Routh‘s Stability Criterion.

5..Examine the stability of given equation using Routh‘s method

$$s^3+6s^2 + 11s + 6 =0$$

6. Examine the stability of given equation using Routh‘s method

$$s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$$

7. Using Routh Criterion, investigate the stability of a unity feedback system whose open loop transfer function is

$$G(s) = \frac{e^{-sT}}{s(s+1)}$$