## MODULE - 2

### 2.1 Block Diagram:

A control system may consist of a number of components. In order to show the functions performed by each component in control engineering, we commonly use a diagram called the "Block Diagram".

A block diagram of a system is a pictorial representation of the function performed by each component and of the flow of signals. Such a diagram depicts the inter-relationships which exists between the various components. A block diagram has the advantage of indicating more realistically the signal flows of the actual system.

In a block diagram all system variables are linked to each other through functional blocks. The "Functional Block" or simply "Block" is a symbol for the mathematical operation on the input signal to the block which produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of flow of signals. Note that signal can pass only in the direction of arrows. Thus a block diagram of a control system explicitly shows a unilateral property.

Fig 2.1 shows an element of the block diagram. The arrow head pointing towards the block indicates the input and the arrow head away from the block represents the output. Such arrows are entered as signals.

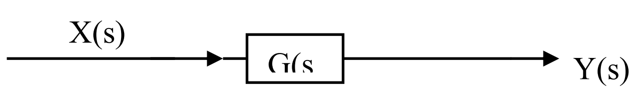



Fig 2.1

The advantages of the block diagram representation of a system lie in the fact that it is easy to form the over all block diagram for the entire system by merely connecting the blocks of the components according to the signal flow and thus it is possible to evaluate the contribution of each component to the overall performance of the system. A block diagram contains information concerning dynamic behavior but does not contain any information concerning the physical construction of the system. Thus many dissimilar and unrelated system can be represented by the same block diagram.

It should be noted that in a block diagram the main source of energy is not explicitly shown and also that a block diagram of a given system is not unique. A number of a different block diagram may be drawn for a system depending upon the view point of analysis.

**Error detector :** The error detector produces a signal which is the difference between the reference input and the feed back signal of the control system. Choice of the error detector is quite important and must be carefully decided. This is because any imperfections in the error detector will affect the performance of the entire system. The block diagram representation of the error detector is shown in fig2.2

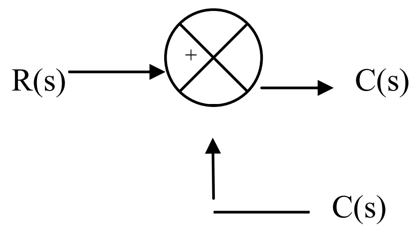



Fig2.2

Note that a circle with a cross is the symbol which indicates a summing operation. The plus or minus sign at each arrow head indicates whether the signal is to be added or subtracted. Note that the quantities to be added or subtracted should have the same dimensions and the same units.

## 2.2 Block diagram of a closed loop system .

**Fig2.3 shows an example of a block diagram of a closed system**

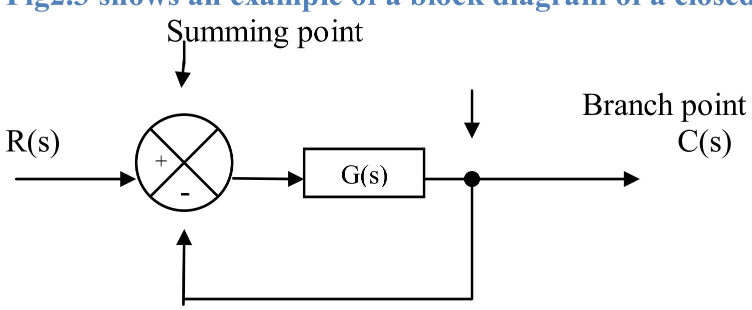



Fig. 2.3

The output C(s) is fed back to the summing point, where it is compared with reference input R(s). The closed loop nature is indicated in fig1.3. Any linear system may be represented by a block diagram consisting of blocks, summing points and branch points. A branch is the point from which the output signal from a block diagram goes concurrently to other blocks or summing points.

When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of output signal to that of he input signal. This conversion is followed by the feed back element whose transfer function is H(s) as shown in fig 1.4. Another important role of the feed back element is to modify the output before it is compared with the input.

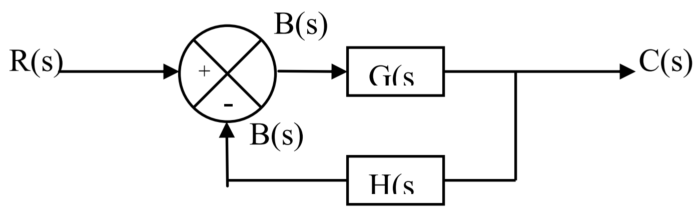



Fig 2.4

The ratio of the feed back signal B(s) to the actuating error signal E(s) is called the open loop transfer function.

open loop transfer function = B(s)/E(s) = G(s)H(s)

The ratio of the output C(s) to the actuating error signal E(s) is called the feed forward transfer function .

Feed forward transfer function = C(s)/E(s) = G(s)

If the feed back transfer function is unity, then the open loop and feed forward transfer function are the same. For the system shown in Fig1.4, the output C(s) and input R(s) are related as follows.

$$C(s) = G(s)\, E(s)$$

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)\, C(s) \qquad \text{but } B(s) = H(s)C(s)$$

Eliminating E(s) from these equations

$$C(s) = G(s)\, [R(s) - H(s)\, C(s)]$$

$$C(s) + G(s)\, [H(s)\, C(s)] = G(s)\, R(s)$$

$$C(s)[1 + G(s)H(s)] = G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

C(s)/R(s) is called the closed loop transfer function.

The output of the closed loop system clearly depends on both the closed loop transfer function and the nature of the input. If the feed back signal is positive, then

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}$$

## 2.3 Closed loop system subjected to a disturbance

Fig2.5 shows a closed loop system subjected to a disturbance. When two inputs are present in a linear system, each input can be treated independently of the other and the outputs corresponding to each input alone can be added to give the complete output. The way in which each input is introduced into the system is shown at the summing point by either a plus or minus sign.

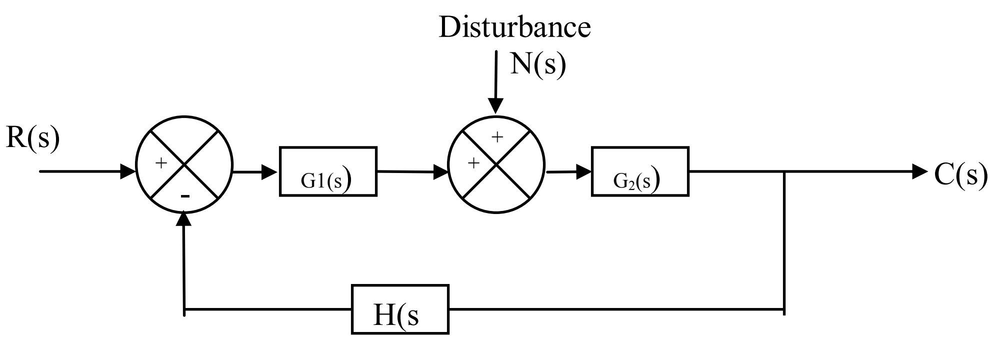



Fig2.5

Fig2.5 closed loop system subjected to a disturbance.

Consider the system shown in fig 2.5. We assume that the system is at rest initially with zero error. Calculate the response $C_N(s)$ to the disturbance only. Response is

$$\frac{C_N(s)}{R(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

On the other hand, in considering the response to the reference input R(s), we may assume that the disturbance is zero. Then the response CR(s) to the reference input R(s)is

$$\frac{CR(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s).}$$

The response C(s) due to the simultaneous application of the reference input R(s) and the disturbance N(s) is given by

$$C(s) = C_R(s) + C_N(s)$$

$$C(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} [G_1(s)R(s) + N(s)]$$

## 2.4 **Procedure for drawing block diagram :**

To draw the block diagram for a system, first write the equation which describes the dynamic behaviour of each components. Take the laplace transform of these equations, assuming zero initial conditions and represent each laplace transformed equation individually in the form of block. Finally assemble the elements into a complete block diagram.

As an example consider the Rc circuit shown in fig2.6 (a). The equations for the circuit shown are

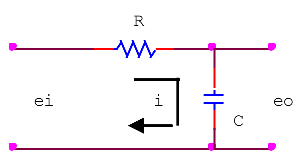



Fig. 2.6a

$e_i = iR + 1/c\int idt$ -----------(1)

And

$e_o = 1/c\int idt$ ---------(2)

*Equation (1) becomes*

$e_i = iR + e_o$

$$\frac{e_i - e_o}{R} = i$$ --------------(3)

*Laplace transforms of equations (2) & (3) are*

$$Eo(s) = 1/CsI(s) \quad \text{-----------}(4)$$

$$\frac{Ei(s) - Eo(s)}{R} = I(s) \quad \text{--------} (5)$$

Equation (5) represents a summing operation and the corresponding diagram is shown in fig1.6 (b). Equation (4) represents the block as shown in fig2.6(c). Assembling these two elements, the overall block diagram for the system shown in fig2.6(d) is obtained.

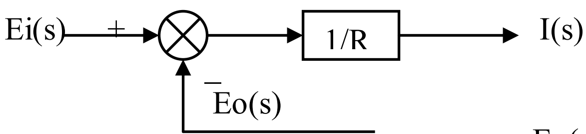



Fig2.6(b)

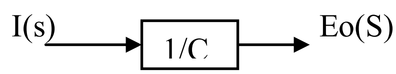



Fig2.6(c)

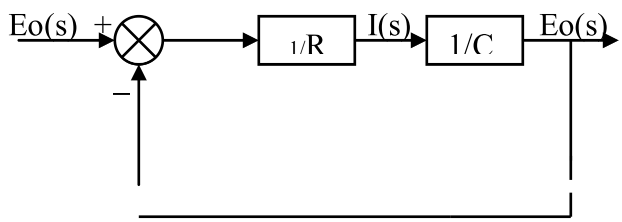



Fig2.6(d)

## 2. SIGNAL FLOW GRAPHS

An alternate to block diagram is the signal flow graph due to S. J. Mason. A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. Each signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable, and each branch acts as a signal multiplier. The signal flows in the direction indicated by the arrow.

**Definitions:**

**Node:** A node is a point representing a variable or signal.

**Branch:** A branch is a directed line segment joining two nodes.

**Transmittance:** It is the gain between two nodes.

**Input node:** A node that has only outgoing branche(s). It is also, called as source and corresponds to independent variable.

**Output node:** A node that has only incoming branches. This is also called as sink and corresponds to dependent variable.

**Mixed node:** A node that has incoming and out going branches.

**Path:** A path is a traversal of connected branches in the direction of branch arrow.

**Loop:** A loop is a closed path.

**Self loop:** It is a feedback loop consisting of single branch.

**Loop gain:** The loop gain is the product of branch transmittances of the loop.

**Nontouching loops:** Loops that do not posses a common node.

**Forward path:** A path from source to sink without traversing an node more than once.

**Feedback path:** A path which originates and terminates at the same node.

**Forward path gain:** Product of branch transmittances of a forward path.

**Properties of Signal Flow Graphs:**

1) Signal flow applies only to linear systems.
2) The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as a function of causes. Nodes are used to represent variables. Normally the nodes are arranged left to right, following a succession of causes and effects through the system.
3) Signals travel along the branches only in the direction described by the arrows of the branches.
4) The branch directing from node $X_k$ to $X_j$ represents dependence of the variable $X_j$ on $X_k$ but not the reverse.
5) The signal traveling along the branch $X_k$ and $X_j$ is multiplied by branch gain $a_{kj}$ and signal $a_{kj}X_k$ is delivered at node $X_j$.

### 2.1.1 Guidelines to Construct the Signal Flow Graphs:

The signal flow graph of a system is constructed from its describing equations, or by direct reference to block diagram of the system. Each variable of the block diagram becomes a node and each block becomes a branch. The general procedure is

1) Arrange the input to output nodes from left to right.
2) Connect the nodes by appropriate branches.
3) If the desired output node has outgoing branches, add a dummy node and a unity gain branch.
4) Rearrange the nodes and/or loops in the graph to achieve pictorial clarity.

## Signal Flow Graph Algebra

**Addtion rule**
The value of the variable designated by a node is equal to the sum of all signals entering the node.
**Transmission rule**
The value of the variable designated by a node is transmitted on every branch leaving the node.
**Multiplication rule**
A cascaded connection of n-1 branches with transmission functions can be replaced by a single branch with new transmission function equal to the product of the old ones.

## Mason's Gain Formula

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

Gain P is given by

$$P = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

Where, $P_k$ is gain of $k^{th}$ forward path,
$\Delta$ is determinant of graph

$\Delta$=1-(sum of all individual loop gains)+(sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + $\cdots$

$\Delta_k$ is cofactor of $k^{th}$ forward path determinant of graph with loops touching $k^{th}$ forward path. It is obtained from $\Delta$ by removing the loops touching the path $P_k$.

**Example1**
Draw the signal flow graph of the block diagram shown in Fig.2.7

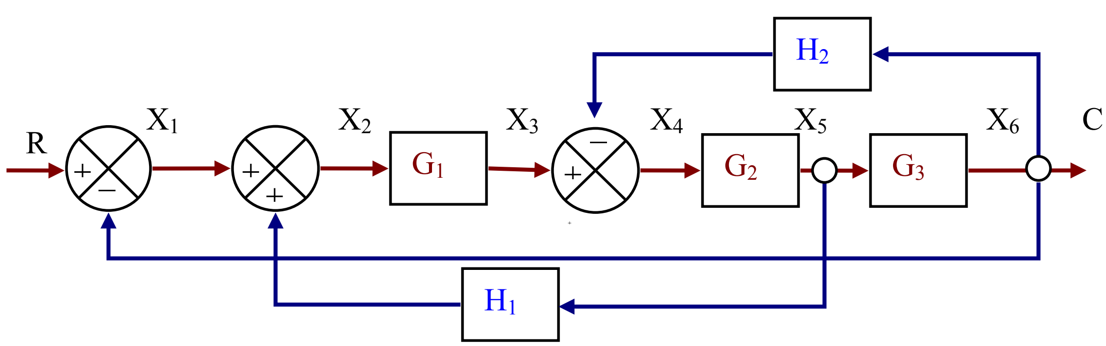



**Figure 2.7 Multiple loop system**

Choose the nodes to represent the variables say $X_1$ .. $X_6$ as shown in the block diagram. Connect the nodes with appropriate gain along the branch. The signal flow graph is shown in Fig. 2.7

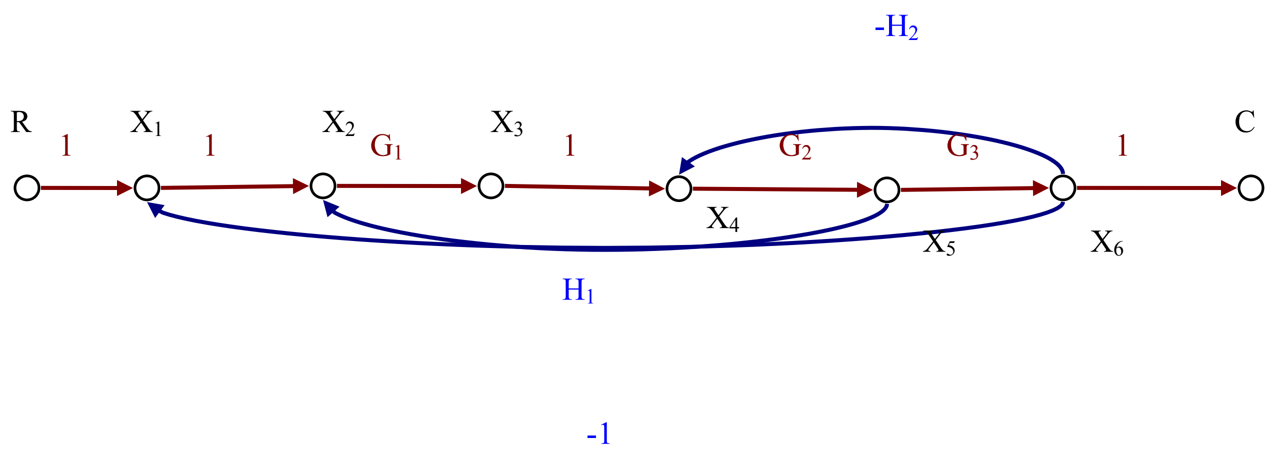



**Figure 1.8 Signal flow graph of the system shown in Fig. 2.7**

**Example 2.9**

Draw the signal flow graph of the block diagram shown in Fig.2.9.

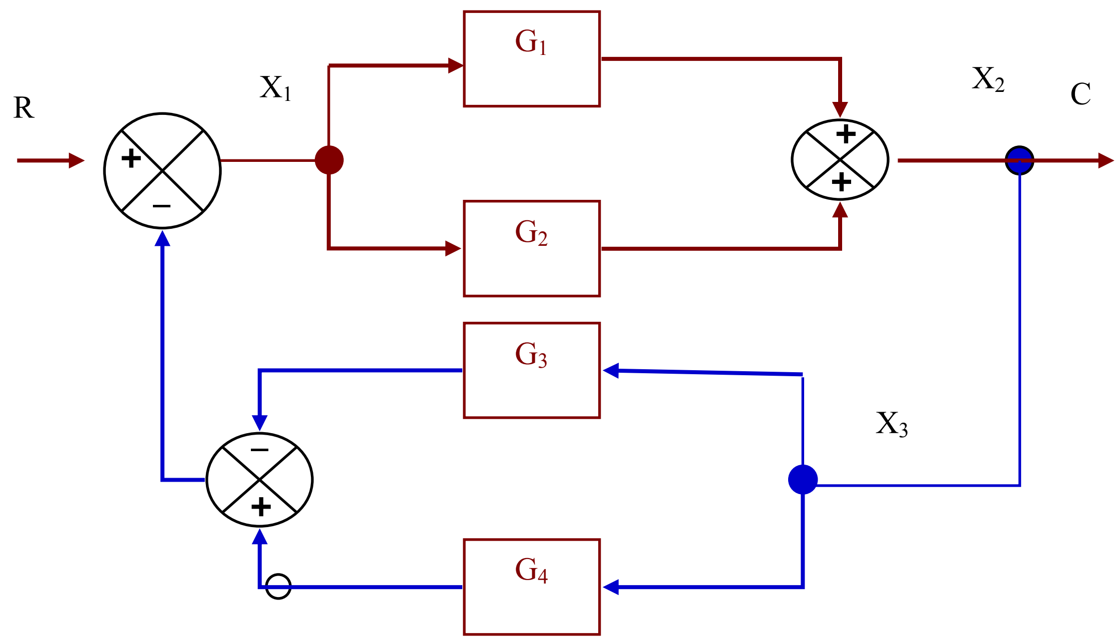



**Figure 2.9 Block diagram feedback system**

The nodal variables are $X_1$, $X_2$, $X_3$.
The signal flow graph is shown in Fig. 2.10.

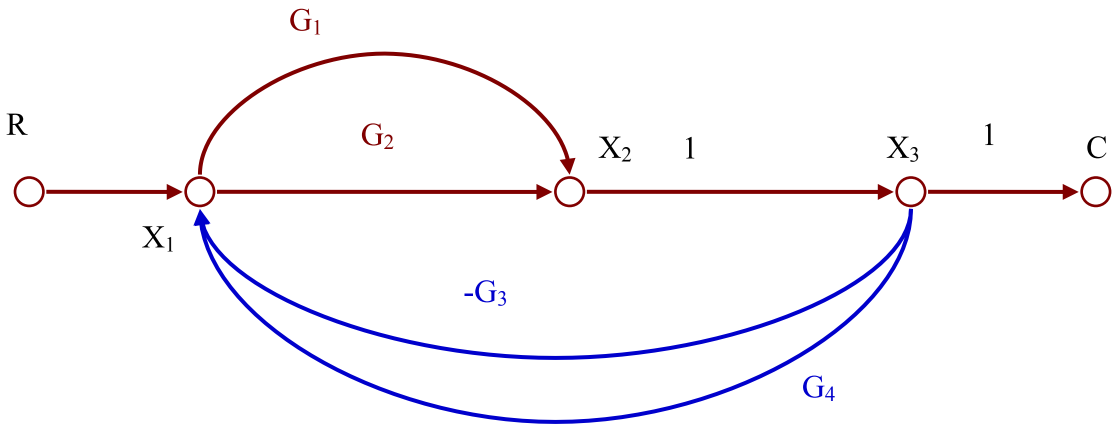



**Figure 2.10 Signal flow graph of example 2**

**Example 3**
Draw the signal flow graph of the system of equations.

$$X_1 = a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + b_1u_1$$
$$X_2 = a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + b_2u_2$$
$$X_3 = a_{31}X_1 + a_{32}X_2 + a_{33}X_3$$

The variables are $X_1$, $X_2$, $X_3$, $u_1$ and $u_2$ choose five nodes representing the variables.
Connect the various nodes choosing appropriate branch gain in accordance with the equations.
The signal flow graph is shown in Fig. 2.11.

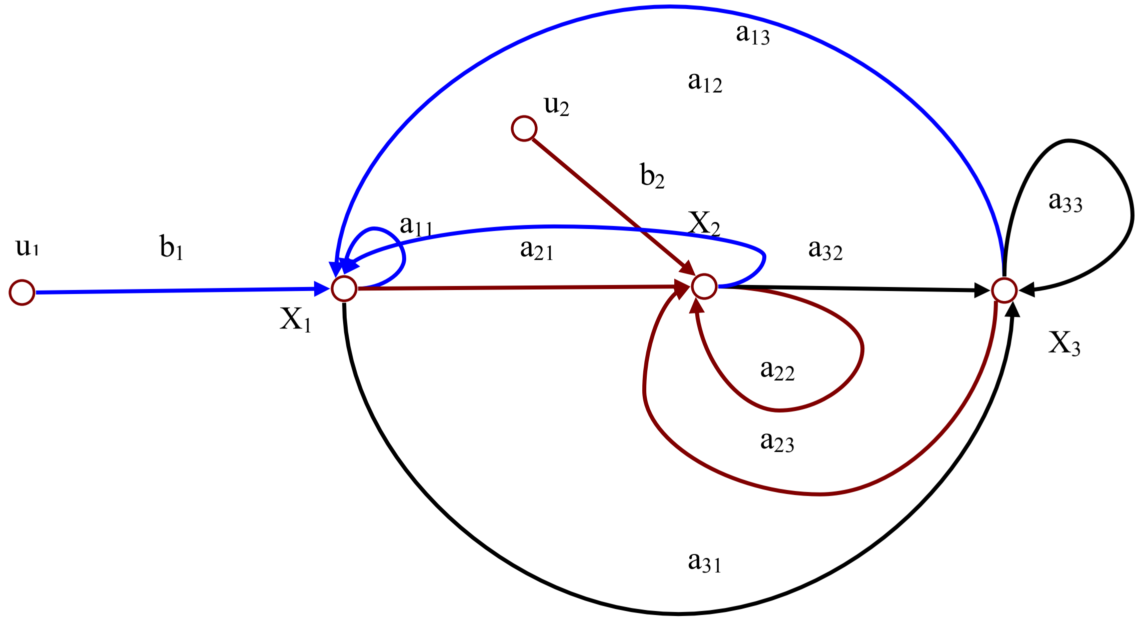



**Figure 2.11 Signal flow graph of example 2**

**Example 4**

LRC net work is shown in Fig. 2.12. Draw its signal flow graph.

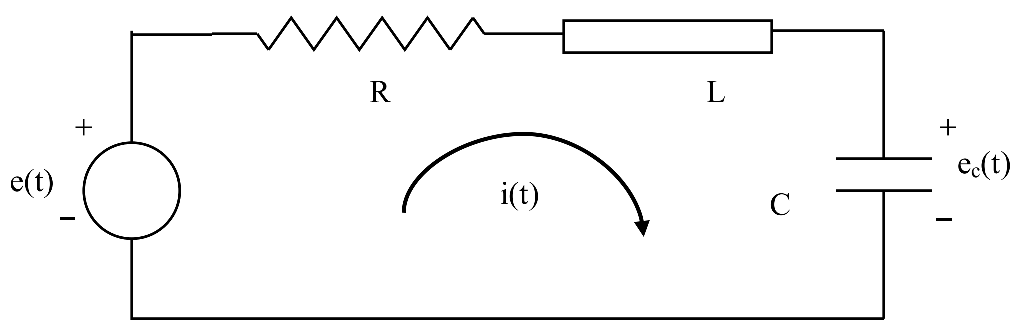



**Figure 2.12 LRC network**

The governing differential equations are

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e(t)\cdots(1)$$

*or*

$$L\frac{di}{dt} + Ri + e_c = e(t)\cdots(2)$$

$$C\frac{de_c}{dt} = i(t)\cdots(3)$$

Taking Laplace transform of Eqn.1 and Eqn.2 and dividing Eqn.2 by L and Eqn.3 by C

$$sI(s)-i(0^{+})+\frac{R}{L}I(S)+\frac{1}{L}E_{c}(s)=\frac{1}{L}E(s)\cdots(4)$$

$$sE_{c}(s)-e_{c}(0^{+})=\frac{1}{C}I(s)\cdots(5)$$

Eqn.4 and Eqn.5 are used to draw the signal flow graph shown in Fig.7.

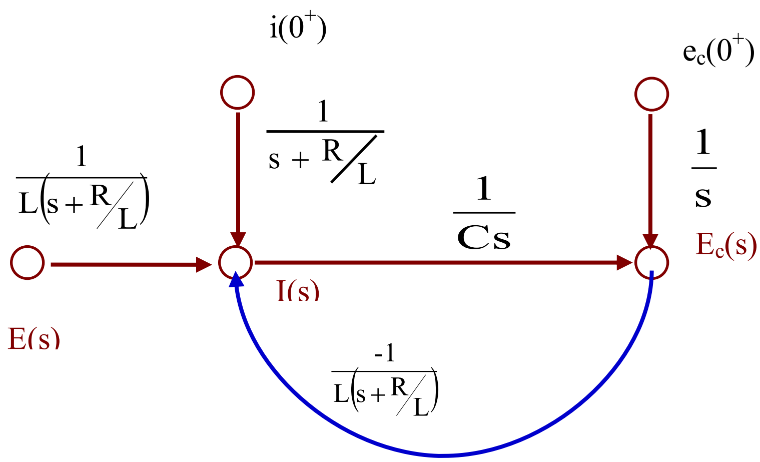



**Figure 2.12 Signal flow graph of LRC system**

## 2.3 SIGNAL FLOW GRAPHS

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

### Mason's Gain Formula

Gain P is given by

$$P = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

Where, $P_k$ is gain of $k^{th}$ forward path,
$\Delta$ is determinant of graph

$\Delta$=1-(sum of all individual loop gains) + (sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + ···

$\Delta_k$ is cofactor of $k^{th}$ forward path determinant of graph with loops touching $k^{th}$ forward path. It is obtained from $\Delta$ by removing the loops touching the path $P_k$.

**Example 1**
Obtain the transfer function of C/R of the system whose signal flow graph is shown in Fig.2.13

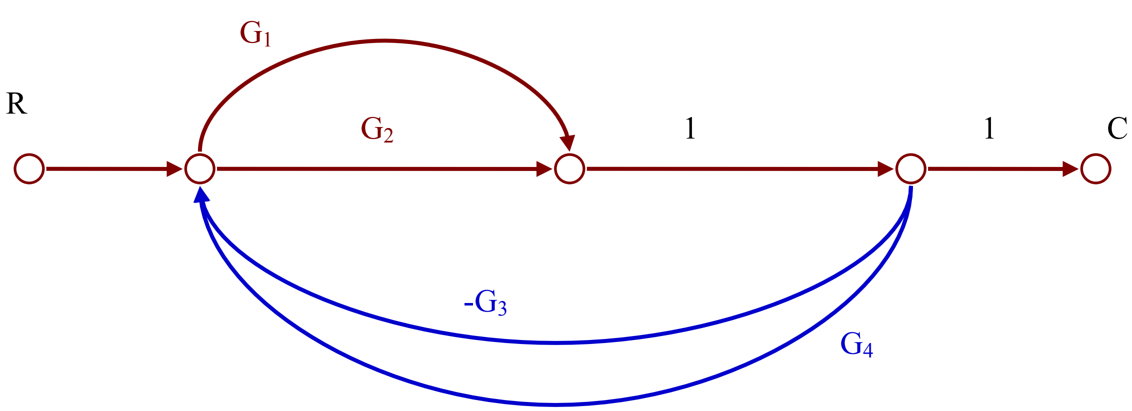



**Figure 2.13 Signal flow graph of example 1**

There are two forward paths:
Gain of path 1 : $P_1=G_1$
Gain of path 2 : $P_2=G_2$

There are four loops with loop gains:
$L_1=-G_1G_3$, $L_2=G_1G_4$, $L_3= -G_2G_3$, $L_4= G_2G_4$
There are no non-touching loops.
$\Delta = 1+G_1G_3-G_1G_4+G_2G_3-G_2G_4$
Forward paths 1 and 2 touch all the loops. Therefore, $\Delta_1= 1$, $\Delta_2= 1$

The transfer function $T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1G_3 - G_1G_4 + G_2G_3 - G_2G_4}$

**Example 2**
Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in Fig.2.14.

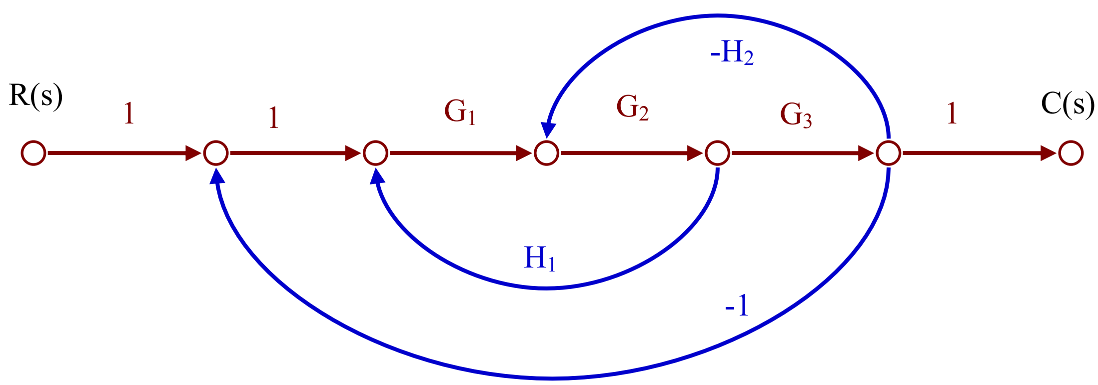



**Figure 2.14 Signal flow graph of example 2**

There is one forward path, whose gain is: $P_1=G_1G_2G_3$
There are three loops with loop gains:
$L_1=-G_1G_2H_1$, $L2=G_2G_3H_2$, $L3= -G_1G_2G_3$
There are no non-touching loops.
$\Delta = 1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3$
Forward path 1 touches all the loops. Therefore, $\Delta_1= 1$.
The transfer function $T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_1G_3H_2 + G_1G_2G_3}$

**Example 3**
Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in Fig.2.15.

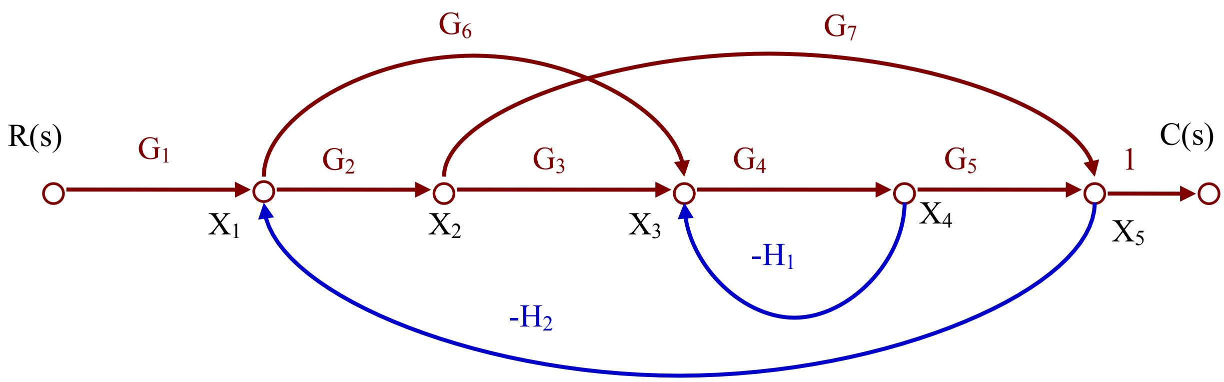



**Figure 2.15 Signal flow graph of example 3**

There are three forward paths.

The gains of the forward path are: $P_1=G_1G_2G_3G_4G_5$

$P_2=G_1G_6G_4G_5$

$P_3= G_1G_2G_7$

There are four loops with loop gains:

$L_1=-G_4H_1$, $L_2=-G_2G_7H_2$, $L3= -G_6G_4G_5H_2$, $L_4=-G_2G_3G_4G_5H_2$

There is one combination of Loops $L_1$ and $L_2$ which are nontouching with loop gain product $L_1L_2=G_2G_7H_2G_4H_1$

$\Delta = 1+G_4H_1+G_2G_7H_2+G_6G_4G_5H_2+G_2G_3G_4G_5H_2+ G_2G_7H_2G_4H_1$

Forward path 1 and 2 touch all the four loops. Therefore $\Delta_1= 1$, $\Delta_2= 1$.

Forward path 3 is not in touch with loop1. Hence, $\Delta_3= 1+G_4H_1$.

The transfer function

T= C(s) / R(s)

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta} = \frac{G_1G_2G_3G_4G_5 + G_1G_4G_5G_6 + G_1G_2G_7(1+G_4H_1)}{1+G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_2G_4G_7H_1H_2}$$

**Example 4**

Find the gains $\frac{X_6}{X_1}, \frac{X_5}{X_2}, \frac{X_3}{X_1}$ for the signal flow graph shown in Fig.2.16.

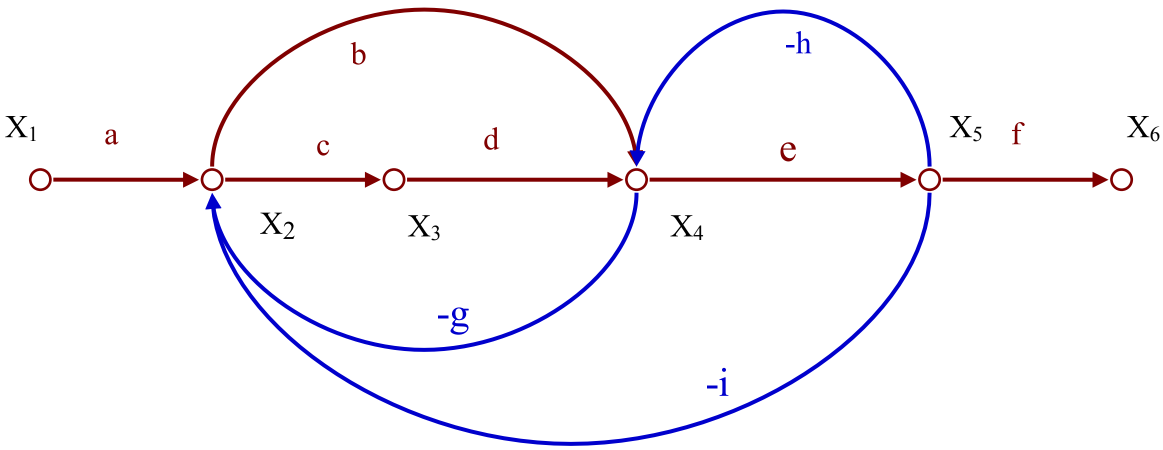



**Figure 2.16 Signal flow graph of MIMO system**

**Case 1:** $\frac{X_6}{X_1}$

There are two forward paths.
The gain of the forward path are: $P_1$=acdef
$P_2$=abef
There are four loops with loop gains:
$L_1$=-cg, $L_2$=-eh, $L_3$= -cdei, $L_4$=-bei
There is one combination of Loops $L_1$ and $L_2$ which are nontouching with loop gain product $L_1L_2$=cgeh
$\Delta$ = 1+cg+eh+cdei+bei+cgeh
Forward path 1 and 2 touch all the four loops. Therefore $\Delta_1$= 1, $\Delta_2$= 1.

The transfer function T = $\frac{X_6}{X_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{cdef + abef}{1 + cg + eh + cdei + bei + cgeh}$

**Case 2:** $\frac{X_5}{X_2}$

The modified signal flow graph for case 2 is shown in Fig.2.17.

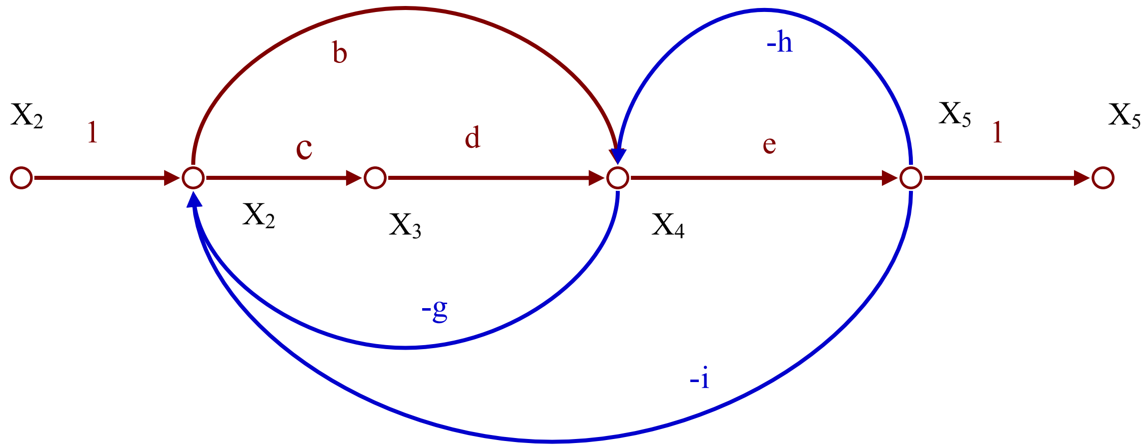



**Figure 2.17 Signal flow graph of example 4 case 2**

The transfer function can directly manipulated from case 1 as branches a and f are removed which do not form the loops. Hence,

The transfer function T= $\frac{X_5}{X_2} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{cde + be}{1 + cg + eh + cdei + bei + cgeh}$

**Case 3:** $\frac{X_3}{X_1}$

The signal flow graph is redrawn to obtain the clarity of the functional relation as shown in Fig.2.18.

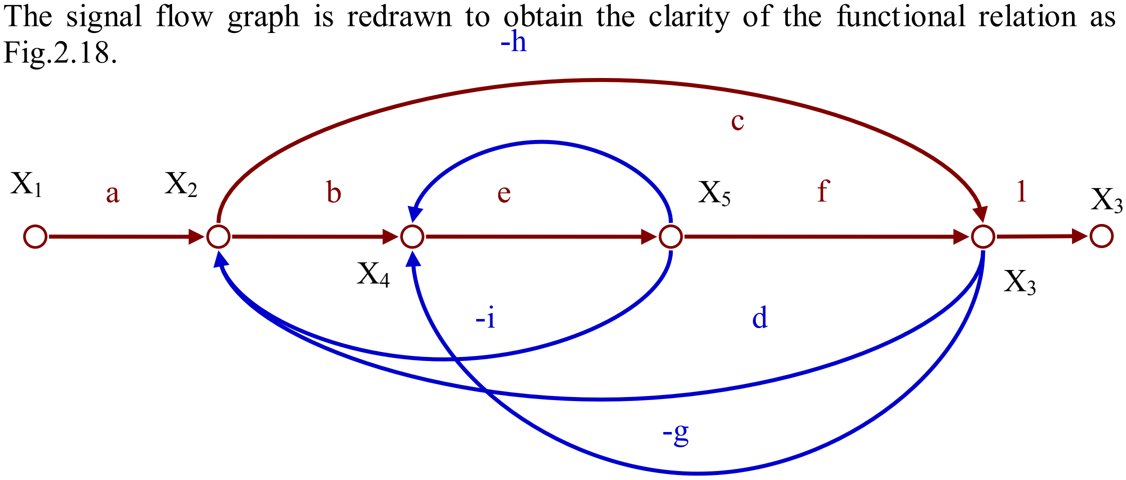



**Figure 2.18 Signal flow graph of example 4 case 3**

There are two forward paths.
The gain of the forward path are: $P_1$=abcd
$P_2$=ac
There are five loops with loop gains:
$L_1$=-eh, $L_2$=-cg, $L_3$= -bei, $L_4$=edf, $L_5$=-befg
There is one combination of Loops $L_1$ and $L_2$ which are nontouching with loop gain product $L_1L_2$=ehcg
$\Delta$ = 1+eh+cg+bei+efd+befg+ehcg
Forward path 1 touches all the five loops. Therefore $\Delta_1$= 1.
Forward path 2 does not touch loop $L_1$. Hence, $\Delta_2$= 1+ eh

The transfer function T = $\dfrac{X_3}{X_1} = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \dfrac{abef + ac(1+eh)}{1+eh+cg+bei+efd+befg+ehcg}$

**Example 5**

For the system represented by the following equations find the transfer function X(s)/U(s) using signal flow graph technique.

$$X = X_1 + \beta_3 u$$
$$\dot{X}_1 = -a_1 X_1 + X_2 + \beta_2 u$$
$$\dot{X}_2 = -a_2 X_1 + \beta_1 u$$

Taking Laplace transform with zero initial conditions

$$X(s) = X_1(s) + \beta_3 U(s)$$
$$sX_1(s) = -a_1 X_1(s) + X_2(s) + \beta_2 U(s)$$
$$sX_2(s) = -a_2 X_1(s) + \beta_1 U(s)$$

Rearrange the above equation

$$X(s) = X_1(s) + \beta_3 U(s)$$
$$X_1(s) = \frac{-a_1}{s} X_1(s) + \frac{1}{s} X_2(s) + \frac{\beta_2}{s} U(s)$$
$$X_2(s) = \frac{-a_2}{s} X_1(s) + \frac{\beta_1}{s} U(s)$$

The signal flow graph is shown in Fig.2.19.

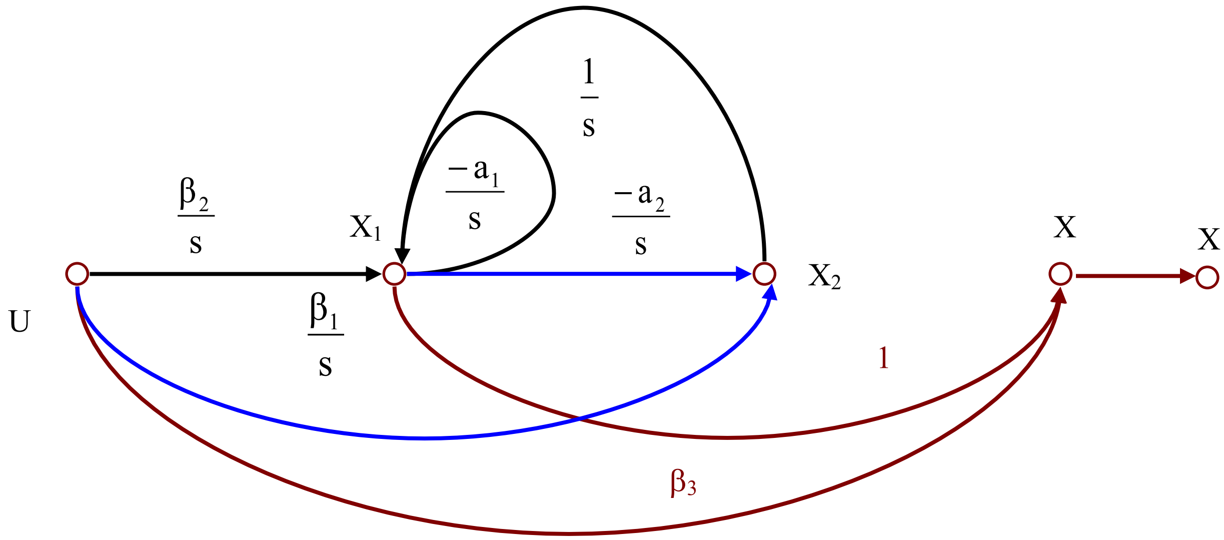



**Figure 2.19 Signal flow grapgh of example 5**

There are three forward paths.

The gain of the forward path are: $P_1=\beta_3$

$P_2=\beta_1/ s^2$

$P_3=\beta_2/ s$

There are two loops with loop gains:

$$L_1 = \frac{-a_1}{s}$$

$$L_2 = \frac{-a_2}{s^2}$$

$L_1$=-eh, $L_2$=-cg, $L_3$= -bei, $L_4$=edf, $L_5$=-befg

There are no combination two Loops which are nontouching.

$$\Delta = 1 + \frac{a_1}{s} + \frac{a_2}{s^2}$$

Forward path 1 does not touch loops $L_1$ and $L_2$. Therefore

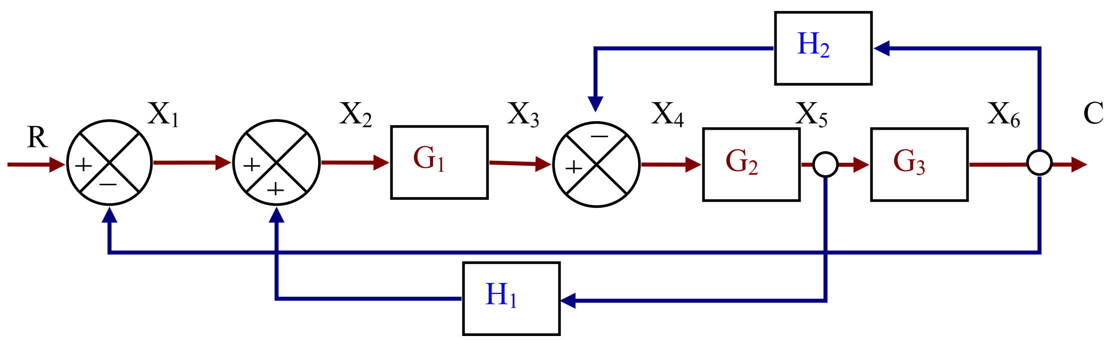
$$\Delta_1 = 1 + \frac{a_1}{s} + \frac{a_2}{s^2}$$

Forward path 2 path 3 touch the two loops. Hence, $\Delta_2$= 1, $\Delta_2$= 1.

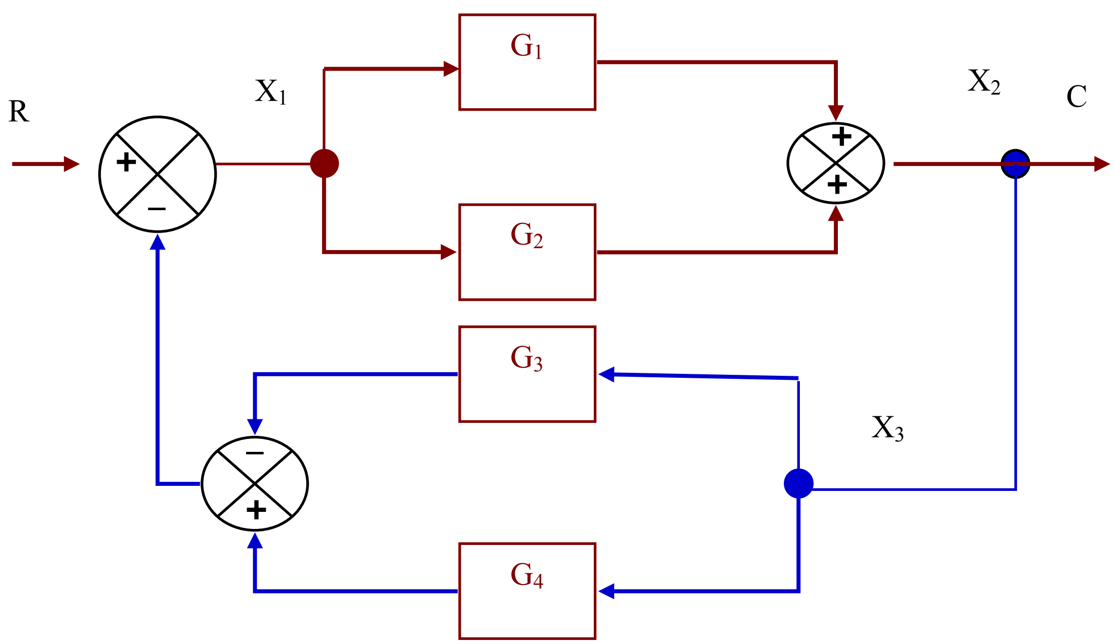
The transfer function T = $\dfrac{X_3}{X_1} = \dfrac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta} = \dfrac{\beta_3\left(s^2 + a_1 s + a_2\right) + \beta_2 s + \beta_1}{s^2 + a_1 s + a_2}$

**<u>Recommended Questions:</u>**

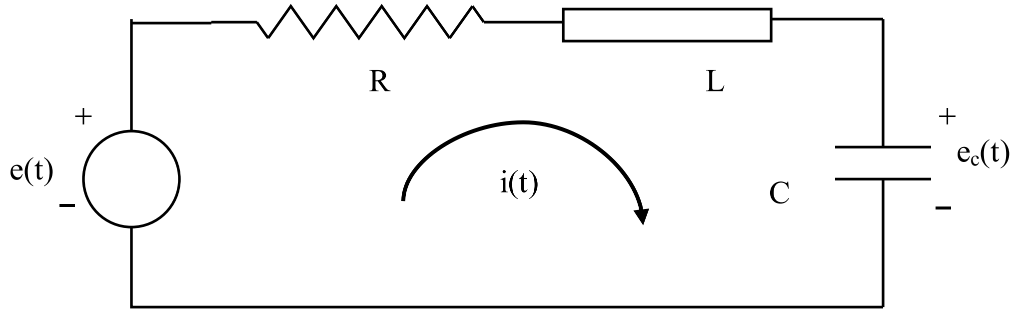
1. Define block diagram & depict the block diagram of closed loop system.
2. Write the procedure to draw the block diagram.
3. Define signal flow graph and its parameters
4. Explain briefly Mason's Gain formula
5. Draw the signal flow graph of the block diagram shown in Fig below.




6. Draw the signal flow graph of the block diagram shown in Fig below

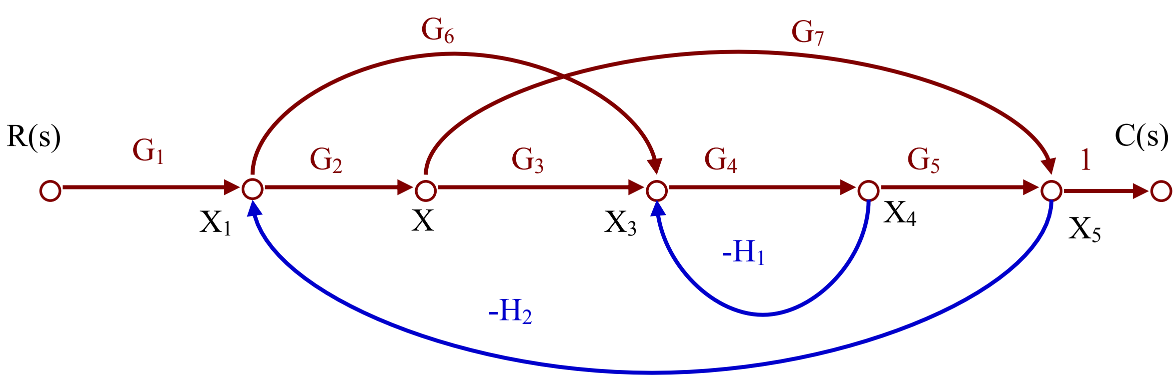


7. For the LRC net work is shown in Fig Draw its signal flow graph.




**Figure**

**8.** Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in Fig.




**Q.9** For the system represented by the following equations find the transfer function X(s)/U(s) using signal flow graph technique.

$$X = X_1 + \beta_3 u$$

$$\dot{X}_1 = -a_1 X_1 + X_2 + \beta_2 u$$

$$\dot{X}_2 = -a_2 X_1 + \beta_1 u$$