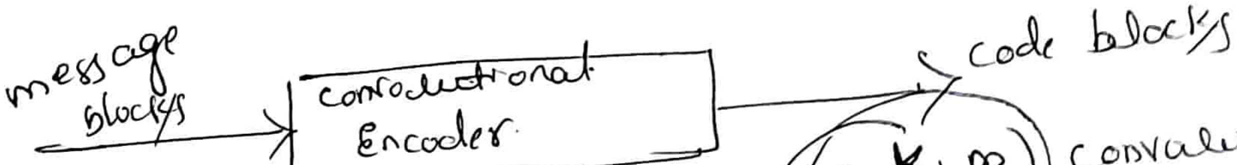
Convolutional Codes

"block codes" : a block of n digits generated by the encoder in a particular time unit depends only on the block of 'K' i/p msg digits within that time unit.

Convolutional codes :- a block of 'n' code digits generated by the encoder in a time unit depends on not only the block of 'K' message digits within that time unit, but also on the preceding (m-1) blocks of msg digits (m > 1). usually the values of k & n will be small.

Encoder for convolutional codes :-



-> 'K' message block -> generate. (n, K, m) convolutional
↓
Convolutional Encoder
code.

~~"nm" digits~~ constrained length "nm" digits

rate efficiency K/n.

-> it depends on previous message block also.

In a general (n, K, m) convolutional encoder, the following notations are used.

n = no. of o/ps = no. of modulo-2 adders (normally)

K ⇒ no. of i/p bits entering at any time.

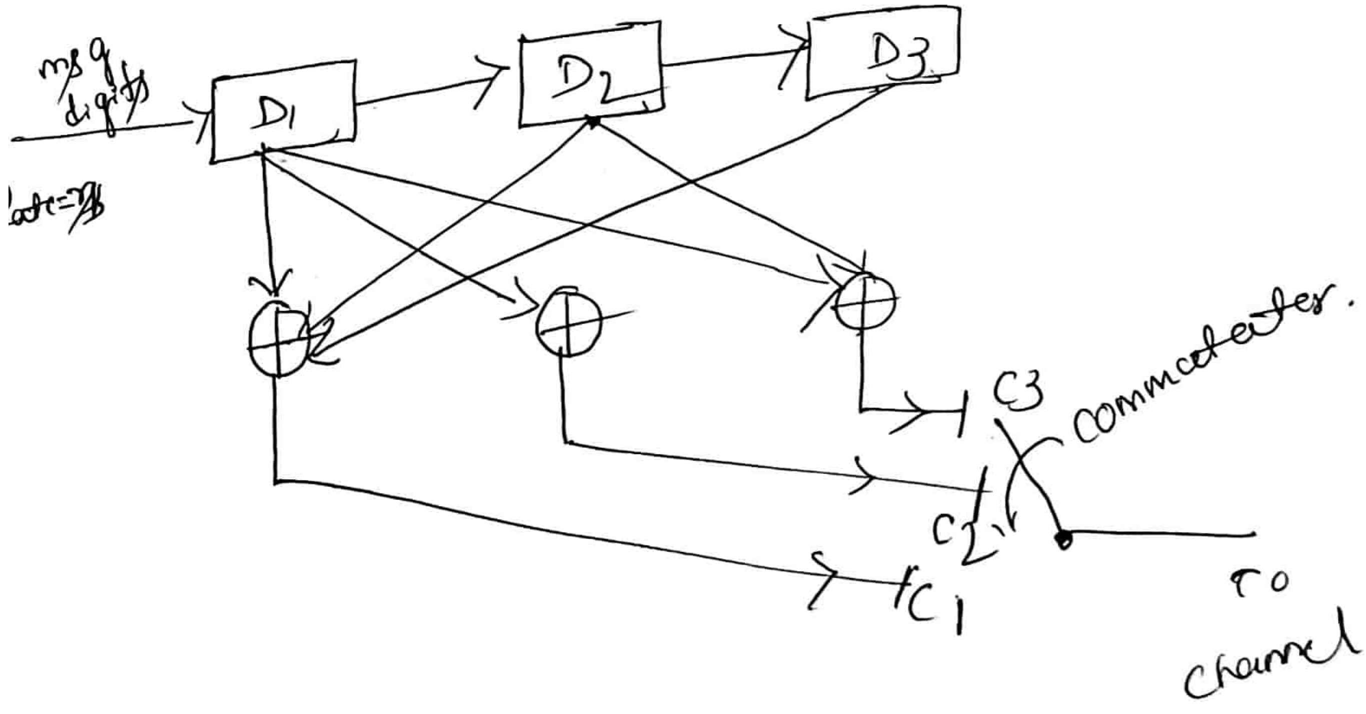
m ⇒ no. of stages of shift register
= no. of flip flops

L = no. of bits in the message sequence.

constraint length = m × n digits

rate efficiency = K/n.

problem: An encoder for a (n, K, m) = (3, 1, 3) convolutional code shown in below. Explain the operation of the encoder & hence obtain the o/p of the encoder.



assignment

10011

i/p :- 1 0 1 1 0 (d_1 d_2 d_3 d_4 d_5)

i/p	Contents of Registers D1	D2	D3	o/p
	0	0	0	111
	0	0	0	101
1	1	0	0	011
0	0	1	1	010
1	1	0	0	001
1	1	1	1	100
0	0	0	1	000
0	0	0	0	
0	0			

$C_3 = D_1 \oplus D_2$

$C_2 = D_1$

$C_1 = D_1 + D_2 + D_3$

impulse response from top adder.

$$g^{(1)} = 1011 - g_1^{(1)}\, g_2^{(1)}\, g_3^{(1)}\, g_4^{(1)}$$

using eqn (3) ~~$d_{l-i} = 0$ for al~~

$l = 1$ $\quad c_1^{(1)} = d_1 g_1^{(1)} + 0 + 0 + 0 = 1(1) = 1$ //

$l = 2$ $\quad c_2^{(1)} = d_2 g_1^{(1)} + d_1 g_2^{(1)} + 0 + 0 =$

$= 0(1) + 1(0) = 0$

$l = 3$ $\quad c_3^{(1)} = d_3 g_1^{(1)} + d_2 g_2^{(1)} + d_1 g_3^{(1)} + 0$

$= (1)(1) + (0)(0) + (1)(1) = 1 + 1 = 0$

$l = 4$ $\quad c_4^{(1)} = d_4 g_1^{(1)} + d_3 g_2^{(1)} + d_2 g_3^{(1)} + d_1 g_4^{(1)}$

$= 1 + 1 = 0$

$l = 5$ $\quad c_5^{(1)} = d_5 g_1^{(1)} + d_4 g_2^{(1)} + d_3 g_3^{(1)} + d_2 g_4^{(1)}$

$= 1 + 1 = 0$ //

$l = 6$ $\quad c_6^{(1)} = d_6 g_1^{(1)} + d_5 g_2^{(1)} + d_4 g_3^{(1)} + d_3 g_4^{(1)}$

$= 1 + 1 = 0$

$l = 7$ $\quad c_7^{(1)} = d_7 g_1^{(1)} + d_6 g_2^{(1)} + d_5 g_3^{(1)} + d_4 g_4^{(1)}$

$l = 8$ $\quad c_8^{(1)} = d_8 g_1^{(1)} + g_7 g_2^{(1)} + d_6 g_3^{(1)} + d_5 g_4^{(1)}$

$= 0 + 0 + 0 + 1(1) = 1$ //

o/p of top adder is

$$c^{(1)} = c_1^{(1)}\, c_2^{(1)}\, c_3^{(1)}\, c_4^{(1)}\, c_5^{(1)}\, c_6^{(1)}\, c_7^{(1)}\, c_8^{(1)} = 10000001$$

<u>For $j = 2$</u>

$$c_l^{(2)} = \sum_{i=0}^{3} d_{l-i}\, g_{i+1}$$

$$c_l^{(2)} = d_l g_1^{(2)} + d_{l-1} g_2^{(2)} + d_{l-2} g_3^{(2)} + d_{l-3} g_4^{(2)} \quad -②$$

$$g^{(2)} = g_1^{(2)} g_2^{(2)} g_3^{(2)} g_4^{(2)} = 1111$$

For $l = 1$ $\quad c_1^{(2)} = 1$ //

$$c^{(2)} = 11011101$$

∴ encoder o/p is

$$c = [11, 01, 00, 01, 01, 01, 00, 11]$$

IInd Method :-

no. of rows equal to = no. of digits in the msg Sequence. (L)

coloumn equal to = $n(L+m)$

$$G = \begin{bmatrix} g_1^{(1)} g_1^{(2)} & g_2^{(1)} g_2^{(2)} & g_3^{(1)} g_3^{(2)} & \cdots & g_{m+1}^{(1)} g_{m+2}^{(2)} & 0\,0\,0 \ldots 00 \\ 0 \cdot 0 & g_1^{(1)} g_2^{(1)} & g_2^{(1)} g_2^{(1)} & \cdots & & g_{m+1}^{(1)} g_{m+2}^{(1)} 0 \ldots 00 \\ \vdots & & & & & \\ & & & & & g_{m+1}^{(1)} g_{m+2}^{(2)} \end{bmatrix}$$

$$c = dG$$

$$d = 10111$$

$g^{(1)} = 1011$

$g^{(2)} = 1111$

$L = 5$, $n(L+m) = 2(5+3) = 16$ column.

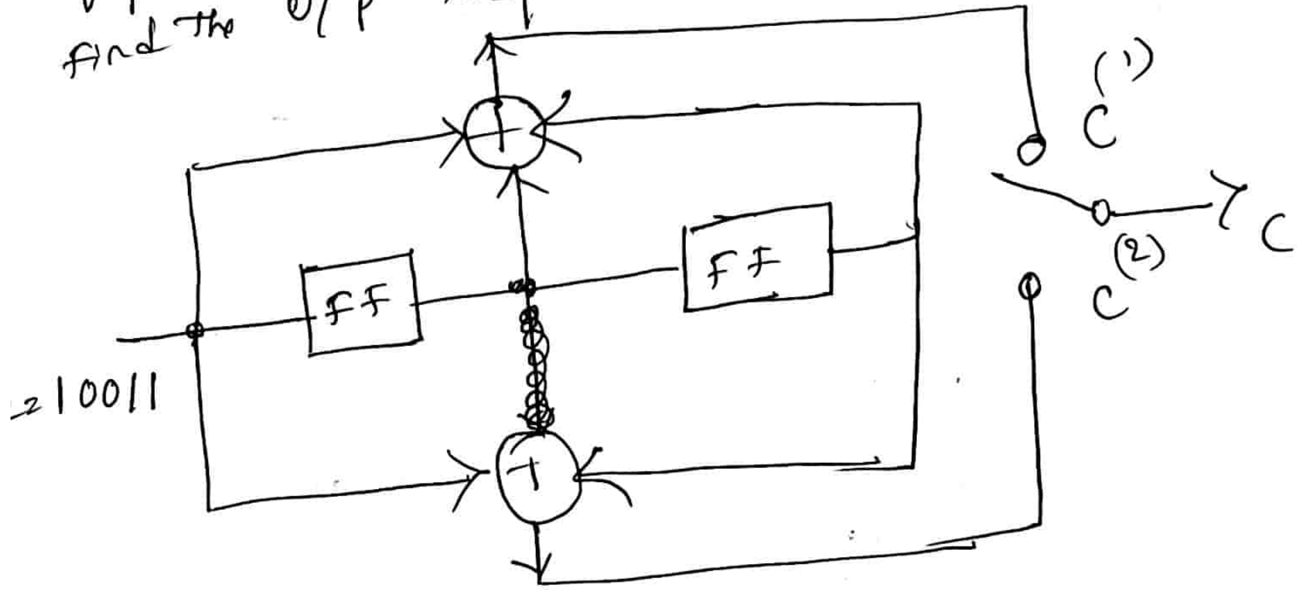
$$G = \begin{bmatrix} 11 & 01 & 11 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 01 & 11 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 01 & 11 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 01 & 11 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 01 & 11 & 11 \end{bmatrix}$$

$c = DG$

$c = [11, 01, 00, 01, 01, 01, 00, 11]$

Assignment

For the convolutional encoder shown in below figure. The information sequence is $d = 10011$. Find the o/p sequence.



$n = 2$, $K = 1$, $m = 2$

$(2,1,2)$ convolutional encoder.

Ans: $[11 \; 10 \; 11 \; 11 \; 01 \; 01 \; 11]$

top adder

$g^{(1)} = 111$, $g^{(2)} = 101$

$d = 10011$, $k = 5$, $C = n(L+m)$
$= 2(5+2) = 14$

$$G = \begin{bmatrix} 11 & 10 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 10 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 10 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 10 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 10 & 11 \end{bmatrix}$$

$[c] = [1\ 0\ 0\ 1\ 1]$

$= [11\ \ 10\ \ 11\ \ 11\ \ 01\ \ 01\ \ 11]$

Transform Domain Approach :

~~$g^{i}(x) = g_1^{(i)} + g_2^{(i)} x + g_3^{(i)} x^2 + \dots + g_{m+1}^{(i)} x^m$~~

$g^{(1)} = 111$, $d = 10011$

$d(x) = 1 + x^3 + x^4$

$g^{(2)} = 101$

$g^{(1)}(x) = 1 + x + x^2$

$g^{2}(x) = 1 + x^2$

$\rightarrow c^{(i)}(x) = d(x)\, g^{(i)}(x)$

$c(x) = d(x)\, g(x)$

$c'(x) = d(x)\, g^{1}(x)$

$= (1 + x^3 + x^4)(1 + x + x^2)$

~~$= 1 + x + x^2 + x^4 + x^5 + x + x^2 + x^5 + x^6$~~

~~$= 1 + x^4 + x^6$~~

$= 1 + x + x^2 + x^3 + x^4 + x^6 + x^4 + x^5 + x^6$

$= 1 + x + x^2 + x^3 + x^6$

$c^{(2)}(x) = d(x) \cdot g^{(2)}(x)$

$= (1+x^3+x^4)(1+x^2)$

$= 1+x^2+x^3+x^5+x^4+x^6$

~~$c(x) = c^{(1)}(x) + c^{(2)}(x)$~~

~~$= 1+x^2+x^4+x^6+x$~~

$c(x) = c^{(1)}(x^n) + x\, c^{(2)}(x^n) + x^2 c^{(3)}(x^n) + \dots + x^{n-1} c^{(n)}(x^n)$

<u>n = 2</u>

$\therefore c(x) = c^{(1)}(x^2) + x\, c^{(2)}(x^2)$

$= 1+x^2+x^4+x^5+x^{12} + x(1+x^4+x^6+x^{10}+x^8+x^{12})$

$= 1+x^2+x^4+x^6+x^{12} + x + x^5 + x^7 + x^{11} + x^9 + x^{13}$

$= 1+x+x^2+x^4+x^5+x^6+x^7+x^9+x^{11}+x^{12}+x^{13}$

$= [11\ 10\ 11\ 11\ 01\ 01\ 11]$

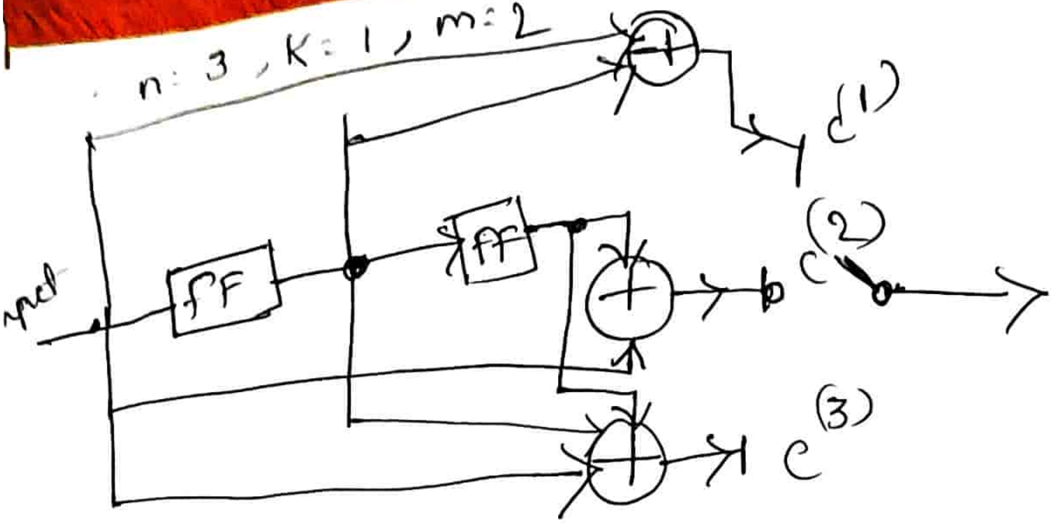
<u>Assignment Question</u> 6:

Consider the (3, 1, 2) convolutional code with $g^{(1)} = 110$, $g^{(2)} = (101)$ & $g^{(3)} = (111)$.

i) Draw the encoder block diagram

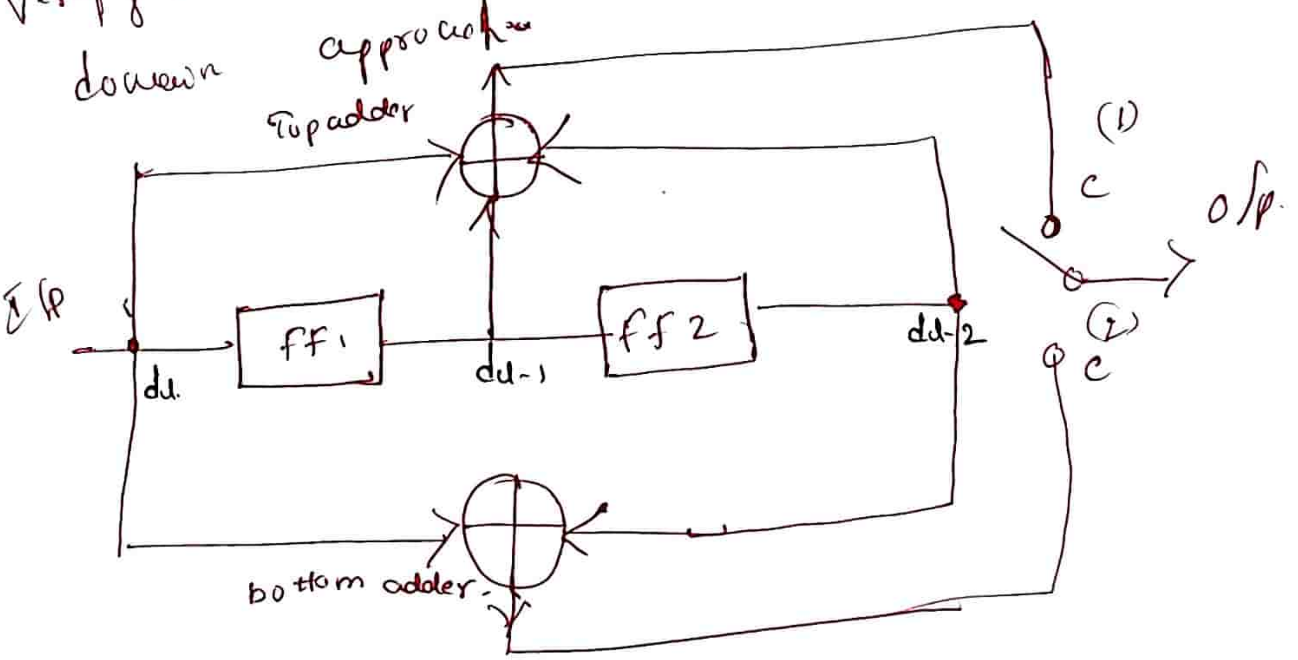
ii) Find the generator Matrix

iii) Find the code-word corresponding to the information Sequence (11101) using Time-domain & Transform-domain approach.



Convolutional code State graph, code tree

1) consider the binary convolutional encoder shown in figure below. Draw the state table, state transition table, state Diagram & the corresponding code tree. Using the code tree, find the encoded sequence for the message (10111). Verify the o/p sequence so obtained using transform domain approach.



2 flip flops $2^2 = 4$ states

State diagram

State	Binary description
S_0	00
S_1	10
S_2	01
S_3	11

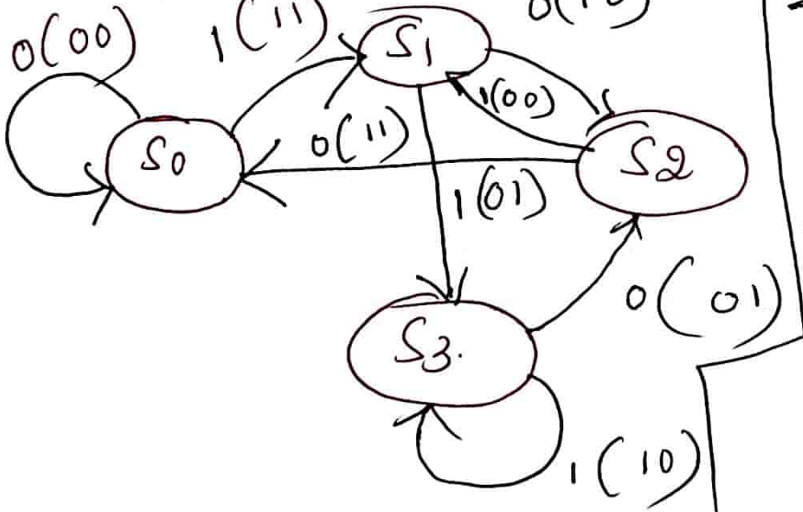
$c^{(1)} = d_l + d_{l-1} + d_{l-2}$

$c^{(2)} = d_l + d_{l-2}$.

State Transition Table

Present state	Binary Description	Input	Next state	Binary Description	d_l	d_{l-1}	d_{l-2}	o/p $c^{(1)}$ $c^{(2)}$
S_0	00	0	S_0	00	0	0	0	00
		1	S_1	10	1	0	0	11
S_1	10	0	S_2	01	0	1	0	10
		1	S_3	11	1	1	0	01
S_2	01	0	S_0	00	0	0	1	11
		1	S_1	10	1	0	1	00
S_3	11	0	S_2	01	0	1	1	01
		1	S_3	11	1	1	1	10

State diagram.

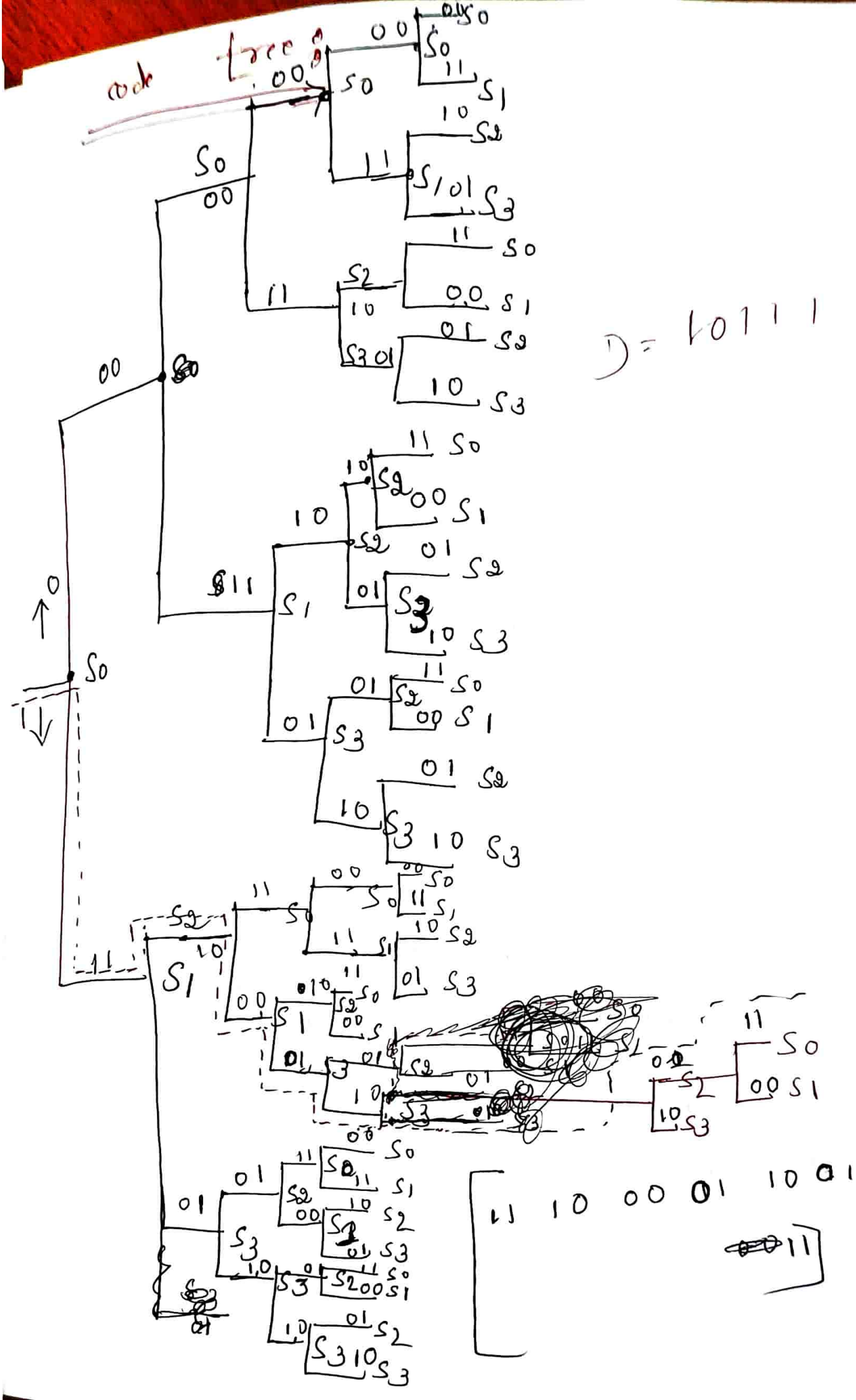


$d = 10111$

$S_0 \xrightarrow{1} S_1 \xrightarrow{0} S_2 \xrightarrow{1} S_1$
$\xrightarrow{1} S_3 \xrightarrow{1} S_3 \xrightarrow{0}$
$S_2 \xrightarrow{0} S_0$

o/p

[11 10 00 01 10 01 11]


code tree
D = 10111
11 10 00 01 10 01
11

$g^{(1)} = 1\ 1\ 1 \quad = 1 + x + x^2$

$g^{(2)} = 1\ 0\ 1 \quad + \quad 1 + x^2$

$c^{(1)}(x) = d(x)\, g^{(1)}(x)$

$= 1 + x^2 + x^3 + x^4 \, (1 + x + x^2)$

$= 1 + x + x^2 + x^2 + x + x^3 + x^4 + x^3 + x^4 + x^5 +$

$x^4 + x^5 + x^6$

$= 1 + x + x^4 + x^6$ //

$c^2(x) = (1 + x^2 + x^3 + x^4)\,(1 + x^2)$

$= 1 + x^2 + x^2 + x^4 + x^3 + x^5 + x^4 + x^6$

$= 1 + x^3 + x^5 + x^6$

no. of modulo - 2 adder $n = 2$.

$c(x) = c^{(1)}(x^2) + x\, c^{(2)}(x^2)$

$= 1 + x^2 + x^8 + x^{12} + x(1 + x^6 + x^{10} + x^{12})$

$= 1 + x + x^2 + x^8 + x^{12} + x + x^7 + x^{11} + x^{13}$

$= 1 + x + x^2 + x^8 + x^{12} + x + x^7 + x^{11} + x^{12} + x^{13}$

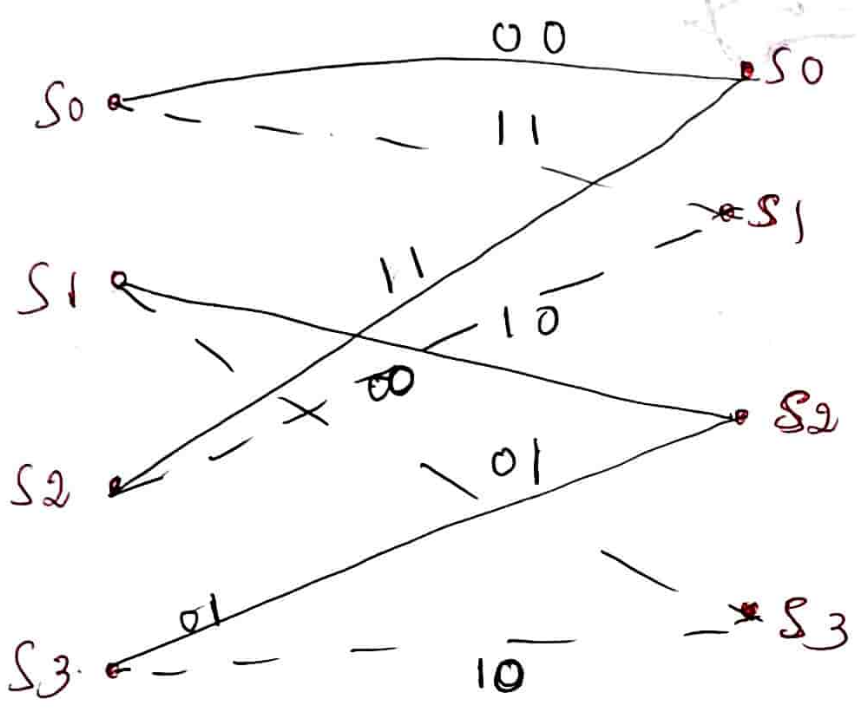
$c(x) = 1 + x + x^2 + x^7 + x^8 + x^{11} + x^{12} + x^{13}$

$= [11\ \ 10\ \ 00\ \ 01\ \ 10\ \ 01\ \ 11]$

Trellis Diagram :

take example as previous one

Solid lines - 0
Dot lines - 1



D = 10111

S0 —1→ S1 —0→ S2 —10→ S1 —1→ S3 —1→ S3 —0→

S2 —0→ S0.

[11 10 00 01 10 01 11]

Viterbi algorithm

Basis of VA

* It is a method of decoding convolution codes.
* Here we use trellis decoder to decode received data.

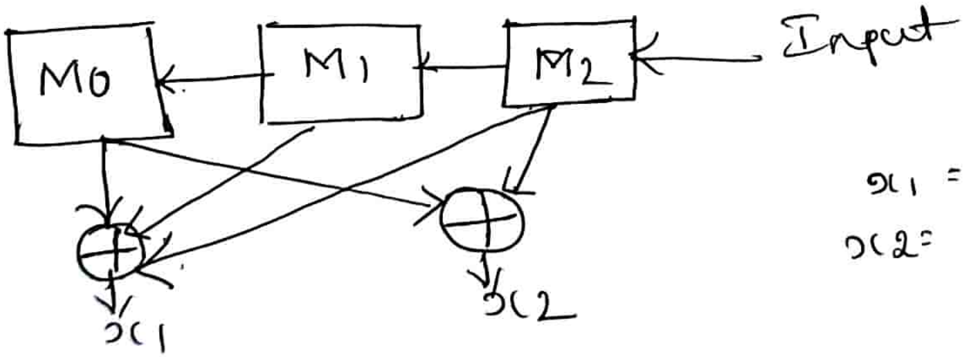
objective of Viterbi algorithm :-

To find the best path through the trellis that is closest to the received data bit sequence.

process of Viterbi Algorithm

* following steps are to be followed to solve viterbi algorithm.
* Make trellis encoder
* from Trellis encoder make trellis diagram
* Match weightage with respect to trellis diagram
* find minimum weightage path on trellis.

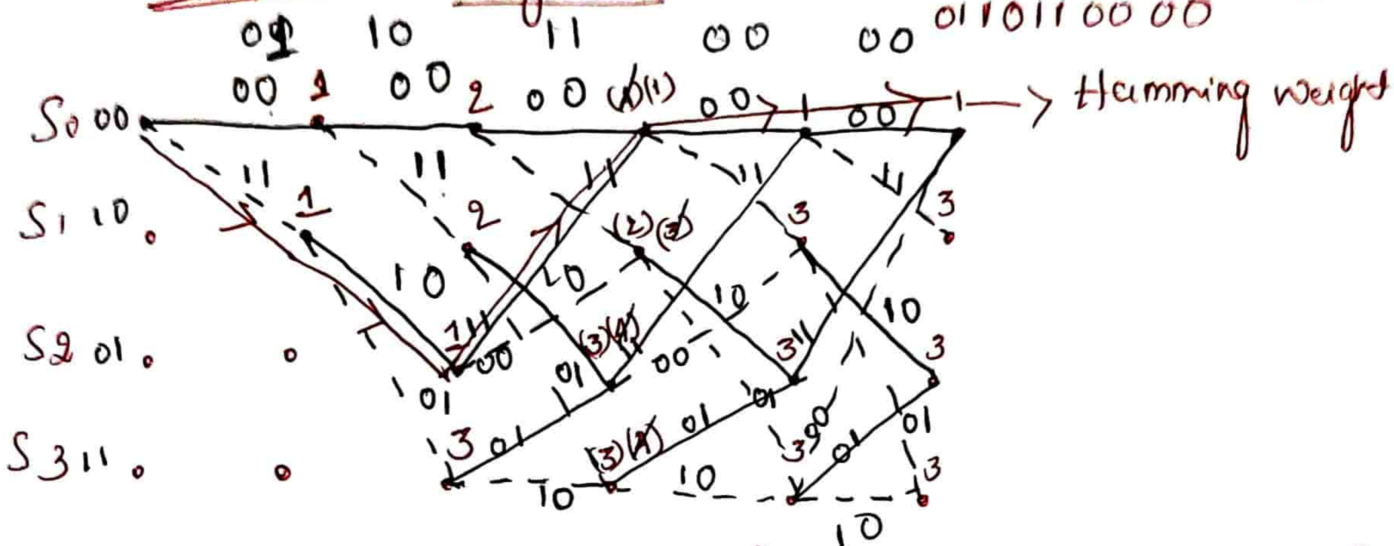
problem :-



$x_1 = M_0 \oplus M_1 \oplus M_2$

$x_2 = M_0 \oplus M_2$

Viterbi Algorithm :- Received Data bit Sequence 01 10 11 00 00



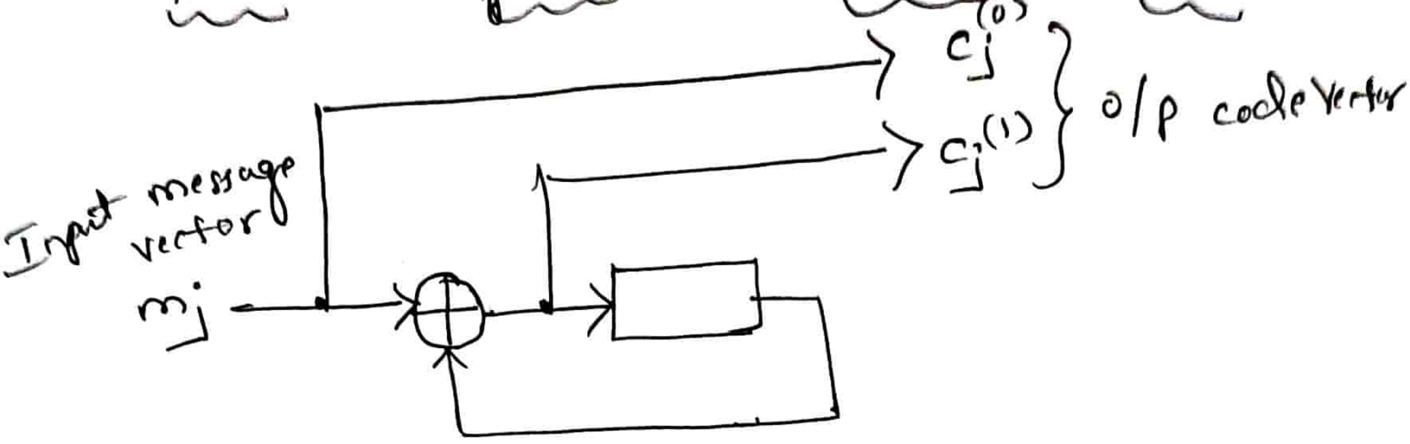
Decoded data Sequence is 11 10 11 00 00

* Received Data bit Sequence is given by
  0|1 10 11 00 00

* Decoded data bit Sequence is given by
  1|1 10 11 0000

If we compare both code will get only single bit Error.

Recursive Systematic convolutional codes



There are 2 distinguishing features.

1. The code is Systematic, in that the incoming message vector m_j at time-unit j defines the Systematic part of the code vector c_j at the output of the encoder.

2. The code is recursive by virtue of the fact that the other constituent of the code vector, namely the parity-check vector b_j, is related to the message vector m_j by the modulo-2 recursive equation.

$$m_j + b_{j-1} = b_j \quad -(1)$$

where $b_{j-1} \to$ past value

$b_j \to$ present value.

It is more convenient to work in the transform D-domain.

$$b_{j-1} = D[b_j] \quad -(2)$$

equivalent eq of eqn (1)

$$b_j = \frac{1}{1+D}[m_j] \quad -(3)$$

where the transfer fun. $\frac{1}{(1+D)}$ operates on m_j to produce b_j. with the code vector c_j consisting of the message vector m_j followed by the parity-check vector b_j, we may express the code vector c_j produced in response to the message vector m_j as follows.

$$c_j = (m_j, b_j)$$

$$= \left(1, \frac{1}{1+D}\right) m_j$$

$$G(D) = \left(1, \frac{1}{1+D}\right)$$

* For recursive systematic convolutional codes, the transform domain matrix $G(D)$ is easier to use as the code generator than the corresponding time-domain matrix G whose entries contain sequences of infinite length.

* A single error in the systematic bits of an RSC code produces an infinite no. of parity-check errors due to the use of feedback in the encoder.