MODULE 2

Symmetrical Fault Analysis

Course Objectives:

1. To explain the necessity and conduction of short circuit analysis.
2. To explain analysis of three phase symmetrical faults on synchronous machine and simple power systems.
3. To discuss selection of circuit breaker.

2.1 INTRODUCTION

Transmission lines stretch over large distances and are subject to faults involving one or more phases and ground. Such faults cause momentary power outages but, more important, if a protective action is not taken, can cause permanent damage to transmission equipment such as the transmission line itself and/or the transformer.

2.1.1 FAULTS

A **fault** in a circuit is any failure that interferes with the normal flow of current to the load. In most faults, a short circuit path forms between two or more phases, or between one or more phases and the neutral (ground). Since the impedance of a new path is usually low, an excessive current may flow.
High-voltage transmission lines have strings of **insulators** supporting each phase. The insulators must be large enough to prevent **flashover**–a condition when the voltage difference between the line and the ground is large enough to ionize the air around insulators and thus provide a current path between a phase and a tower.

2.1.2 CAUSES OF FAULTS

- Tree Branches near the right-of-way on transmission lines and shorting them to ground.

- Lightning that represents a current source of thousands of amperes. This current flowing through the tower footing impedance can raise the tower potential above the local ground to such a level that without surge arresters, the insulator strings may flash over.

The reason to analyze faults are:

- ✓ To set the relays so that can detect it.
- ✓ To make sure that the circuit breakers ratings are such that they are capable of interrupting the fault current

2.2 Transient on a Transmission Line

There are two main types of faults:

- **Symmetric faults:** System remains balanced; these faults are relatively rare, but are the easiest to analyze so we'll consider them first.

- **Unsymmetric faults:** System is no longer balanced; very common, but more difficult to analyze.

The most common types of faults on a three phase system are:

- Single line-to-ground (SLG)-70%
- Line-to-line faults (LL)-15%
- Double line-to-ground (DLG) faults -10%
- Balanced three phase faults.-5%

If flashover occurs on a single phase of the line, an arc will be produced. Such faults are called single **line-to-ground faults**. Since the short-circuit path has a low impedance, very high currents flow through the faulted line into the ground and back into the power system. Faults involving ionized current paths are also called transient faults. They usually clear if power is removed from the line for a short time and then restored.

Single line-to-ground faults can also occur if one phase of the line breaks comes into contact with the ground or if insulators break. This fault is called a permanent fault since it will remain after a quick power removing. Approximately 75% of all faults in power systems are either **transient** or **permanent single line-to-ground faults**

Table 2.1 Types of faults

Type of fault	Abbreviation	Type
Single line to ground	SLG	Unsymmetrical
Line to line	LL	Unsymmetrical
Double line to ground	LLG	Unsymmetrical
Symmetrical three phase	3P	Symmetrical

2.2.1 CAUSES OF FAULTS

- Tree Branches near the right-of-way on transmission lines and shorting them to ground.
- Lightning that represents a current source of thousands of amperes. This current flowing through the tower footing impedance can raise the tower potential above the local ground to such a level that without surge arresters, the insulator strings may flash over.
- The reason to analyze faults are
    - ✓ To set the relays so that can detect it.
    - ✓ To make sure that the circuit breakers ratings are such that they are capable of interrupting the fault currents.

2.2.2 LIGHTNING STRIKE EVENT SEQUENCE

1. Lighting hits lines, setting up an ionized path to the ground

   - Millions lightning strikes per year hits every year!

   - A single typical stroke might have 25,000 amps, with a rise time of 10 s, dissipated in 200 s.

   - Multiple strokes can occur in a single flash, causing the lightning to appear to flicker, with the total event lasting up to a second.

2. Conduction path is maintained by ionized air after lightning stroke energy has dissipated, resulting in high fault currents (often > 25,000 amps)

3. Within one to two cycles (16 ms) relays at both ends of line detect high currents, signaling circuit breakers to open the line:
   - Nearby locations see decreased voltages

4. Circuit breakers open to de-energize line in a one to two cycles:
   - Breaking tens of thousands of amps of fault current is no small feat.
   - With line removed voltages usually return to near normal

5. Circuit breakers may reclose after several seconds, trying to restore faulted line to service.

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2.2.3 FAULT ANALYSIS

- Fault currents cause equipment damage due to both thermal and mechanical processes.
- The main goal of fault analysis is to determine the magnitudes of the currents present during the fault:
  - We need to determine the maximum current to ensure devices can survive the fault.
  - We need to determine the maximum current the circuit breakers (CBs) need to interrupt to correctly size the CBs.

To understand fault analysis we need to review the behavior of an *RL* circuit with a switch: transient response and steady state: Circuit Analysis.

2.2.4 NETWORK FAULT ANALYSIS SIMPLIFICATIONS

To simplify the analysis of fault currents in networks we will make several simplifications:

1. Transmission lines are represented by their series reactance
2. Transformers are represented by their leakage reactances.
3. Synchronous machines are modelled as a constant voltage behind direct-axis sub-transient reactance.
4. Induction motors are ignored or treated as synchronous machines
5. Other (non-spinning) loads are ignored.

2.3 Short Circuit of a Synchronous Machine (On No Load), Short Circuit of a Loaded Synchronous Machine

Sub-transient: first cycle or so after the fault – AC current is very large and falls rapidly;

Transient: current falls at a slower rate;

Steady-state: current gets back to normal

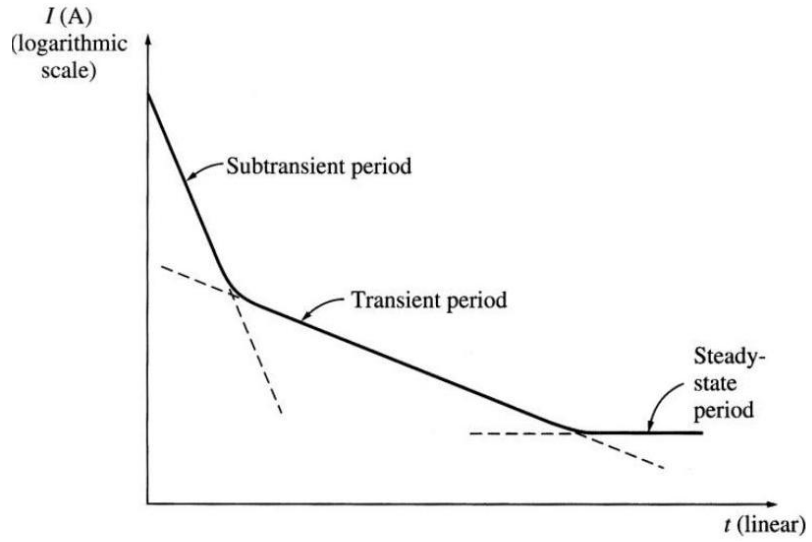


Fig 2.1 Symmetrical AC components of transients

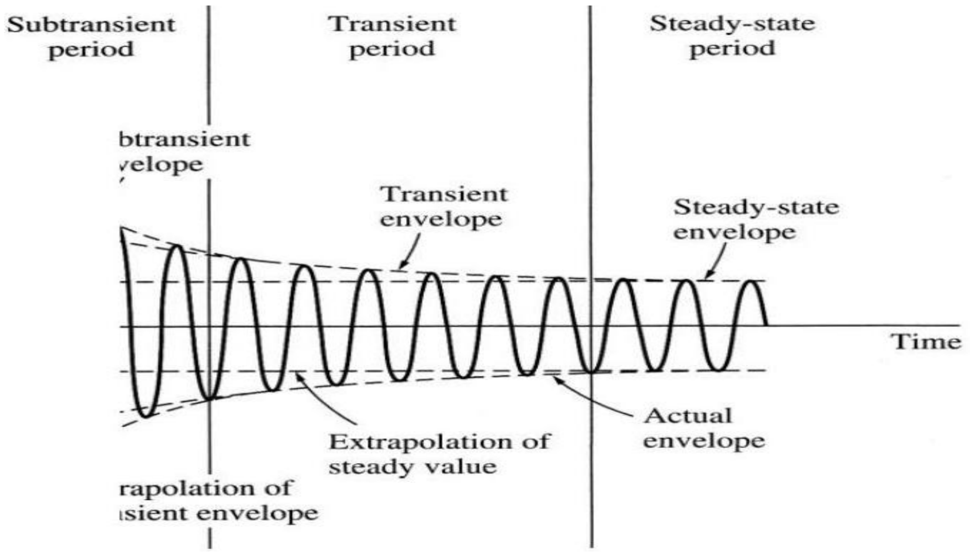


Fig 2.2: Symmetrical short circuit current

- The AC current flowing in the generator during the sub-transient period is called the sub-transient current and is denoted by *I''*. This current is caused by the damper windings of synchronous machines. The time constant of the sub-transient current is denoted by *T''* and it can be determined from the slope. This current may be 10 times the steady-state fault current.

- The AC current flowing in the generator during the transient period is called the transient current and is denoted by *I'*. It is caused by a transient DC component of current induced in the field circuit of a synchronous generator at the time of fault. This transient field current increases the internal generated voltage of a machine and, therefore, an increased fault current.

- The time constant of a field circuit *T'* is much larger than the time constant of the damper winding, therefore, the transient period lasts longer than the sub-transient. This current is often as much as 5 times the steady-state fault current.

After the transient period, the fault current reaches a steady-state condition. The steady-state rms current is denoted by I_{ss} and is approximated by the fundamental frequency component of the internal generated voltage normalized by the synchronous reactance:

$I_{ss}= E_A/X_s$

The rms magnitude of the AC fault current in a synchronous generator varies over time as

I(t)= (I''- I')e –t/T'' + (I'-Iss)e-t/T'' +Iss

The sub-transient reactance is the ratio of the fundamental component of the internal generated voltage to the sub-transient component of current at the beginning of the fault.

$$X''= E_A/I''$$

Similarly, the transient reactance is the ratio of the fundamental component of the internal generated voltage to the transient component of current at the beginning of the fault. This value of current is found by extrapolating the transient region back to time zero.

$$X'=E_A/I'$$
$$I''=E_A/X''$$
$$I'=E_A/X'$$

Course Outcome

At the end of the module, students will be able to:

- Analyze the selection of circuit breaker through short circuit analysis for synchronous machines

SYMMETRICAL THREE PHASE FAULTS

INTRODUCTION:

- A fault is any kind of failure that interferes the normal operation of the system
- Short circuit faults are not common in PS but they do occur
- The result of short circuit (S.C) fault is very high current, because S.C reduces impedance of system considerably thereby increasing the current flowing through the component
- Short circuits are caused mainly because of insulation failure
- The reason for the insulation failures are
  - → Over voltages caused by lightning or switching surges
  - → Insulation contamination (Salt spray, pollution)
    - If equipment or transmission line is in a polluted region or near seashore then the salt spray or pollution creates a thin conducting film on the insulation or insulators on overhead lines
  - → Mechanical causes (Over heating, abrasion)
    - If equipment is over loaded for longer time then insulation deteriorates due to high temperature in the system leading to failure of insulation because of vibration taking place in machine or because of the cycle of contraction or expansion due to heating and cooling in the equipment.

FAULTS ON TRANSMISSION LINE:

- Faults on transmission line (T.L) are most common (60–70%) since the T.L are overhead lines and are exposed to elements of nature
- If lightning strikes the T.L then travelling waves of very high voltages

travel in the line at a speed of light in both direction. When these waves come near towers or insulators they create flashover from insulator to tower (ground) leading to line to ground S.C

NOTE: - If T.L is not loaded then the travelling waves see an open circuit and reflect back doubling their voltage. This effect is called reflection. It is a severe phenomena and should be avoided

- If T.L is loaded then part of the over voltage flows through the load impedance Z_L to ground and the remaining is reflected back. This effect is called refraction

- High winds can topple towers causing lines to fall to ground. This may result in a S.C b/w line to ground (It can be single line to ground, double line to ground or even three lines to ground S.C)

- Trees can fall on T.L causing S.C. Also, since trees are present below T.L they must be trimmed regularly. If the trees are not trimmed regularly, and if T.L carries heavy currents (overloaded) for longer period, then sag of T.L increases due to overheating and the T.L will touch the trees. This results in a S.C on occurance of rain

- Ice loading also creates S.C, ie, the ice coating on T.L increases the line weight and inturn its sag. This may sometime lead to breaking of line near insulators causing mechanical failure of insulator

- S.C can also occur in other elements such as
  - → Under ground Cables (10 - 15%)
  - → Circuit Breakers (10 - 12%)
  - → Generators, motors and transformers (10 - 15%)

  The S.C on these elements are less common and can occur due to over loading for longer periods, deterioration of insulation or mechanical failure

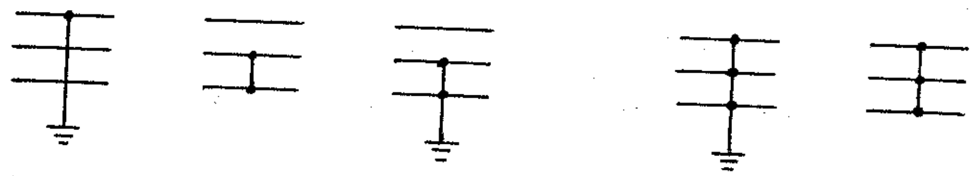
- When fault occurs in a power network, current flowing is determined by internal emf's of machines in the network, by their impedance and by impedances b/w machines and faults

CONSEQUENCES OF SHORT CIRCUIT:

- Current that is several magnitude larger than the normal operating current occurs
- This large current results in thermal damage of the equipment. for example if s.c current is 5 times normal operating current then heating produced being proportional to I^2R will be 25 times the normal operating current. This may cause burning of insulation or equipment itself
- This high current produces heavy magnetic forces b/w the windings of the coil and the bus bar conductors, mechanically damaging them
- Hence faulted section must be removed from service as soon as possible (3 to 5 cycles i.e., b/w 60ms to 100ms in case of extra high voltage or high voltage systems)

TYPES OF SHORT CIRCUIT:

- faults are broadly classified as Asymmetrical and Symmetrical faults.



L-G	L-L	L-L-G	3Ø-G	3Ø
75 - 80 %	5 - 7 %	10 - 12 %	8 - 10 %	
Asymmetrical fault			Symmetrical fault	

L - G — Line to ground fault

L - L — Line to line fault

L - L - G — Double line to ground fault

3Ø - G — Three phase to ground fault

3Ø — Three phase fault

- In case of symmetrical fault, current is same in all the three phases (here current is refers to fault current). Hence system is still balanced even after fault occurrance and analysis is perform

-ed on per phase basis

- In case of Asymmetrical fault, fault current is not same in all the three phases. Hence symmetrical components are used for analysis purpose.

TRANSIENT:

- A transient is nothing but something which is temporary or last only for short time
- An electrical transient is temporary excess of voltage or current in an electrical circuit which only lasts for short duration (millisecond to nano second)
- Transients can occur in all types of electrical, data and communications circuits.
- Lightning is not just the primary cause for transients but 80% of transients are generated inside the facility
- Simple act of turning ON or OFF a light or electrical device (such as circuit breaker) can disturb circuit and create a transient
- Larger the load current of device, greater the disturbance it causes
- Power distribution system and attached load equipment are under constant attacks from various types of power line disturbances
- Transients (also called surges), due to lightning stroke or due to simple switching ON or OFF of light damages and destroys the equipment.

TRANSIENT ON TRANSMISSION LINE (R-L SERIES CIRCUIT TRANSIENTS):

- Consider S.C transient on an unloaded T.L, represented with a lumped series R-L circuit (fig 1)
- Few simplifying assumptions are made

→ Line is fed from constant voltage source

→ S·C takes place when line is unloaded

→ Line capacitance is neglected

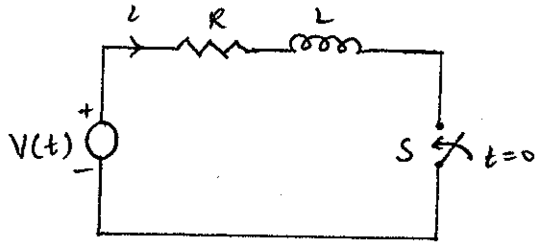


fig 1

$V(t) = \sqrt{2}\, V_{rms} \sin(\omega t + \alpha)$

$= V_m \sin(\omega t + \alpha)$

Where α determines the magnitude of voltage when circuit is closed

i = Current in T·L under S·C condition

Impedance of T·L, $Z = R + jX_L = R + j\omega L$

$$= \sqrt{R^2 + \omega^2 L^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$= |Z| \angle \theta$$

where $|Z| = \sqrt{R^2 + \omega^2 L^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

▷ S·C is assumed to occur at $t = 0$

▷ When switch S is closed at $t=0$, S·C (fault current) flows in the circuit and when KVL is applied we get

$$V(t) - Ri - L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} + Ri = V(t)$$

$$L\frac{di}{dt} + Ri = V_m \sin(\omega t + \alpha)$$

Applying Laplace transform

$$Ls\, i(s) + R\, i(s) = V_m \int_0^{\infty} e^{-st} \sin(\omega t + \alpha)\, dt$$

$$(Ls + R)\, i(s) = V_m \int_0^{\infty} e^{-st} \sin\omega t \cos\alpha\, dt + V_m \int_0^{\infty} e^{-st} \cos\omega t \sin\alpha\, dt$$

$$= V_m \cos\alpha \int_0^{\infty} e^{-st} \sin\omega t\, dt + V_m \sin\alpha \int_0^{\infty} e^{-st} \cos\omega t\, dt$$

$$(Ls + R)\, i(s) = V_m \cos\alpha \left(\frac{\omega}{s^2 + \omega^2}\right) + V_m \sin\alpha \left(\frac{s}{s^2 + \omega^2}\right)$$

$$i(s) = V_m \omega \cos\alpha \left[\frac{1}{(s^2+\omega^2)(Ls+R)}\right] + V_m \sin\alpha \left[\frac{s}{(s^2+\omega^2)(Ls+R)}\right] \to (1)$$

By Using partial fraction for

$$\frac{1}{(s^2+\omega^2)(Ls+R)} = \frac{As+B}{s^2+\omega^2} + \frac{C}{Ls+R} \to (2)$$

$$\frac{1}{(s^2+\omega^2)(Ls+R)} = \frac{(As+B)(Ls+R)+C(s^2+\omega^2)}{(s^2+\omega^2)(Ls+R)}$$

$$(As+B)(Ls+R) + C(\omega^2+s^2) = 1$$

$$LAs^2 + ARs + BLs + BR + C\omega^2 + Cs^2 = 1$$

$$(AL+C)s^2 + (AR+BL)s + (BR+C\omega^2) = 1$$

Equating the like coefficient on b.s we get

$$AL + C = 0$$
$$AR + BL = 0$$
$$BR + C\omega^2 = 1$$

on solving $A = -\frac{C}{L}$

$$AR + BL = 0 \qquad BR + C\omega^2 = 1$$

$$-\frac{C}{L}R + BL = 0 \qquad \left(\frac{CR}{L^2}\right)R + C\omega^2 = 1$$

$$BL = \frac{CR}{L} \qquad \frac{CR^2}{L^2} + C\omega^2 = 1$$

$$B = \frac{CR}{L^2} \qquad C\left(\frac{R^2}{L^2} + \omega^2\right) = 1$$

Using value of C we get $\qquad C\left(\frac{R^2+\omega^2L^2}{L^2}\right) = 1$

$$A = \frac{-L^2}{L(R^2+\omega^2L^2)} = \frac{-L}{R^2+\omega^2L^2} \qquad \therefore C = \frac{L^2}{R^2+\omega^2L^2}$$

$$B = \frac{L^2R}{L^2(R^2+\omega^2L^2)} = \frac{R}{R^2+\omega^2L^2}$$

Putting values of A, B and C in (2)

$$\frac{1}{(s^2+w^2)(Ls+R)} = \frac{1}{R^2+w^2L^2}\left[\frac{-Ls+R}{s^2+w^2} + \frac{L^2}{Ls+R}\right] \rightarrow ③$$

By using partial fraction for

$$\frac{s}{(s^2+w^2)(Ls+R)} = \frac{Ds+E}{s^2+w^2} + \frac{F}{Ls+R} \longrightarrow ④$$

$$\frac{s}{(s^2+w^2)(Ls+R)} = \frac{(Ds+E)(Ls+R)+F(s^2+w^2)}{(s^2+w^2)(Ls+R)}$$

$$(Ds+E)(Ls+R)+F(s^2+w^2) = s$$

$$DLs^2+DRs+ELs+ER+Fs^2+Fw^2 = s$$

$$(DL+F)s^2+(DR+EL)s+(ER+Fw^2) = s$$

Equating the like coefficient on b·s we get

$$DL+F=0 \quad , \quad DR+EL=1 \quad , \quad ER+Fw^2=0$$

On Solving, $D = \frac{-F}{L}$ and $E = \frac{-Fw^2}{R}$

$$DR+EL=1$$

$$-\frac{F}{L}R+EL=1$$

Multiplying $\frac{1}{L}$ on b·s

$$-\frac{F}{L^2}R+E=\frac{1}{L}$$

$$-\frac{F}{L^2}R-\frac{Fw^2}{R}=\frac{1}{L}$$

$$-F\left(\frac{R}{L^2}+\frac{w^2}{R}\right)=\frac{1}{L}$$

$$F\left(\frac{R^2+w^2L^2}{L^2R}\right)=-\frac{1}{L}$$

$$\therefore\ F = -\frac{L^2R}{L(R^2+w^2L^2)} = \frac{-LR}{R^2+w^2L^2}$$

Using value of F in D and E

$$D = \frac{LR}{L(R^2+\omega^2L^2)} = \frac{R}{R^2+\omega^2L^2}$$

$$E = \frac{LR\omega^2}{R(R^2+\omega^2L^2)} = \frac{\omega^2 L}{R^2+\omega^2L^2}$$

Putting values of D, E and F in ④

$$\frac{s}{(s^2+\omega^2)(Ls+R)} = \frac{1}{R^2+\omega^2L^2}\left[\frac{Rs+\omega^2L}{s^2+\omega^2} - \frac{LR}{Ls+R}\right] \rightarrow ⑤$$

Putting ③ and ⑤ in ①

$$(R^2+\omega^2L^2)\, i(s) = V_m\omega\cos\alpha\left(\frac{-Ls+R}{s^2+\omega^2} + \frac{L^2}{Ls+R}\right) + V_m\sin\alpha\left(\frac{Rs+\omega^2L}{s^2+\omega^2} - \frac{LR}{Ls+R}\right)$$

$$= -V_m\omega L\cos\alpha\left(\frac{s}{s^2+\omega^2}\right) + V_m R\cos\alpha\left(\frac{\omega}{s^2+\omega^2}\right) + V_m\omega\cos\alpha\, L\left(\frac{1}{s+\frac{R}{L}}\right)$$

$$+ V_m\sin\alpha\, R\left(\frac{s}{s^2+\omega^2}\right) + V_m\omega L\sin\alpha\left(\frac{\omega}{s^2+\omega^2}\right) - V_m R\sin\alpha\left(\frac{1}{s+\frac{R}{L}}\right)$$

$$\left[\ \frac{L^2}{Ls+R} = \frac{L^2}{L(s+\frac{R}{L})} = L\left(\frac{1}{s+\frac{R}{L}}\right)\right.$$

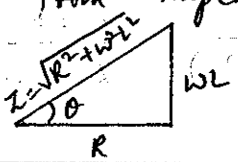
$$\left. |||^{ly}\quad \frac{LR}{Ls+R} = \frac{LR}{L(s+\frac{R}{L})} = R\left(\frac{1}{s+\frac{R}{L}}\right)\ \right]$$

Taking inverse laplace transform

$$(R^2+\omega^2L^2)\, i(t) = -V_m\omega L\cos\alpha\cos\omega t + V_m R\cos\alpha\sin\omega t + V_m\omega L\cos\alpha\, e^{-\frac{R}{L}t}$$
$$+ V_m R\sin\alpha\cos\omega t + V_m\omega L\sin\alpha\sin\omega t - V_m R\sin\alpha\, e^{-\frac{R}{L}t}$$

$$= V_m R\,(\cos\alpha\sin\omega t + \sin\alpha\cos\omega t) + V_m\omega L\,(\sin\alpha\sin\omega t - \cos\alpha\cos\omega t)$$
$$- V_m e^{-\frac{R}{L}t}\,(R\sin\alpha - \omega L\cos\alpha)$$

[from Impedance triangle



$|Z^2| = R^2+\omega^2L^2$

$\sin\theta = \frac{\omega L}{Z}$ and $\cos\theta = \frac{R}{Z}$]

$$i(t) = \frac{V_m}{|Z^2|} R \sin(\omega t + \alpha) - \frac{V_m}{|Z^2|} \omega L \cos(\omega t + \alpha) - \frac{V_m}{|Z^2|} e^{-\frac{R}{L}t} (R \sin\alpha - \omega L \cos\alpha)$$

$$= \frac{V_m}{|Z|} \cdot \frac{R}{|Z|} \sin(\omega t + \alpha) - \frac{V_m}{|Z|} \cdot \frac{\omega L}{|Z|} \cos(\omega t + \alpha) - \frac{V_m}{|Z|} e^{-R/L\, t} \left(\frac{R}{|Z|} \sin\alpha - \frac{\omega L}{|Z|} \cos\alpha \right)$$

$$= \frac{V_m}{|Z|} \cos\theta \sin(\omega t + \alpha) - \frac{V_m}{|Z|} \sin\theta \cos(\omega t + \alpha) - \frac{V_m}{|Z|} e^{-\frac{R}{L}t} (\cos\theta \sin\alpha - \sin\theta \cos\alpha)$$

$$= \frac{V_m}{|Z|} \sin(\omega t + \alpha - \theta) - \frac{V_m}{|Z|} e^{-\frac{R}{L}t} \sin(\alpha - \theta)$$

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \alpha - \theta) + \frac{V_m}{|Z|} e^{-\frac{R}{L}t} \sin(\theta - \alpha) \rightarrow (6)$$

- As seen from the above equation $i(t)$ consists of two terms
  - → first term is an a.c current and varies sinusoidally with time. It is represented as $i_{ac}(t)$
  - → Second term is the d.c offset current that decays exponentially with time constant L/R. It is represented as $i_{dc}(t)$

$$\therefore\ i(t) = i_{ac}(t) + i_{dc}(t)$$

$i_{ac}(t)$ = Symmetrical s.c current / Steady State Current (Varies sinusoidally with time)

$i_{dc}(t)$ = Dc offset current (Decays exponentially with time constant L/R)

$i(t)$ = Asymmetrical fault current

- $i_{dc}(t)$ causes $i(t)$ to be unsymmetrical till transient decays.

▷ wave form for $i_{ac}(t)$, $i_{dc}(t)$ and $i(t)$ is shown separately in fig 2 (a),(b) and (c) respectively.

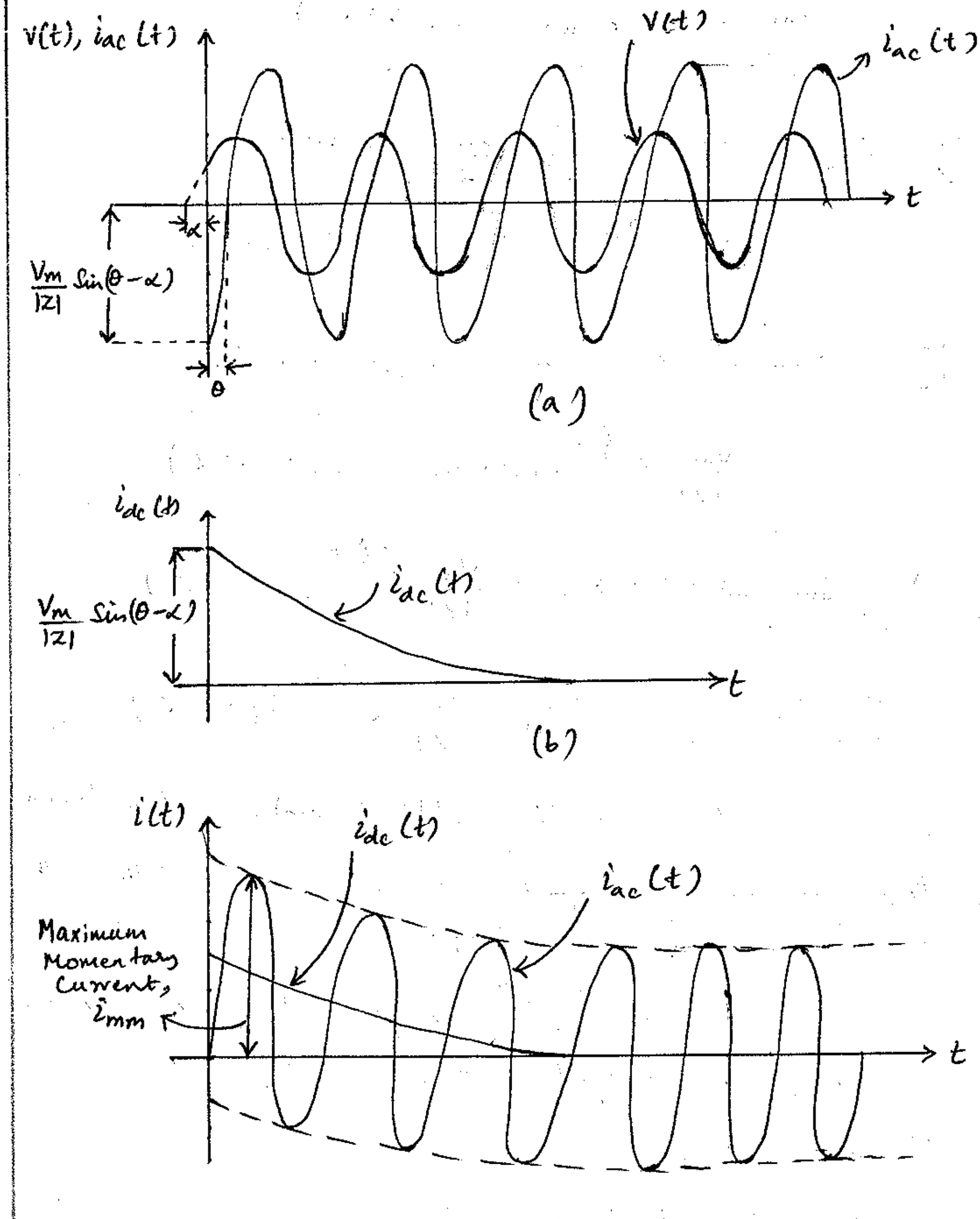


fig 2

▷ As seen from fig 2 (c), i(t) is not symmetrical about x-axis. As $t \to \infty$, $i_{dc}(t)$ decays to zero and i(t) becomes symmetrical.

▷ <u>MAXIMUM MOMENTARY CURRENT</u> (i_{mm}):

→ i_{mm} is maximum momentary current and corresponds to first peak of i(t)

→ first peak value is obtained when $\sin(\omega t + \alpha - \theta) = 1$ in equation (6)

$$\therefore i_{mm} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} e^{-\frac{R}{L}t} \sin(\theta - \alpha)$$

→ If decay of transient current b/w $t=0$ and time at which first peak occurs is neglected, then

$$i_{mm} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} e^{-\frac{R}{L}(0)} \sin(\theta - \alpha) \quad (\because t=0)$$

$$= \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \sin(\theta - \alpha)$$

→ In T.L resistance is very small than the reactance, so neglecting resistance (i.e., $R=0$)

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}(\infty) = 90^\circ$$

$$i_{mm} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \sin(90 - \alpha)$$

$$= \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \cos\alpha$$

→ i_{mm} will have maximum possible value at $\alpha = 0$ (i.e., when voltage wave is at 0)

$$\therefore i_{mm\,(max\ possible)} = \frac{V_m}{|Z|} + \frac{V_m}{|Z|} \cos(0)$$

$$i_{mm\,(max\ possible)} = 2\frac{V_m}{|Z|} \rightarrow (7)$$

→ from (7) it is clear that max possible value of i_{mm} is twice the max of symmetrical s.c current when voltage wave is going through zero. This effect is called <u>doubling effect</u>

→ The safe choice for selection of circuit breaker is to take i_{mm} corresponding to its max possible value.

→ Modern circuit breakers interrupts current in first few cycles

(5 cycles or less) so $i_{dc}(t)$ also contributes for interrup-tion

→ Instead of computing DC offset value at time of interrup-tion (this would be highly complex even for moderately large network) $i_{ac}(t)$ alone is calculated

→ The result is then multiplied with a multiplying factor to account for $i_{dc}(t)$

<u>NOTE:</u> Refer the topic "Selection of circuit breaker" for multiplying factor values.

<u>SHORT CIRCUIT CURRENTS AND THE REACTANCES OF SYNCHRONOUS MACHINES (ON NO LOAD):</u>

▷ Alternator consists of armature and field winding wound on 3 phases of stator and on rotor respectively

▷ The damper windings (copper bars) are placed on rotor and are shorted on both end by end rings

▷ Field winding is excited by dc and when rotor rotates armature winding cuts the flux inducing alternating emf in all the 3 phases

▷ This inturn induces alternating current in these three windings producing a rotating magnetic field which rotates at synchronous speed (N_s) in air gap

▷ Rotor rotates at synchronous speed (N_s) along with rotating magnetic field produced by stator conductors

▷ Under normal operation rotating magnetic field of stator currents will be relatively stationary with respect to field and damper winding. Hence there will be no induced voltage or current in them

▷ But when symmetrical short circuit occurs under constant excitation, then S-C armature current changes from 0 to very high value in all the three phases

▷ This S-C armature current will have a.c and d.c offset current in all the 3 phases.

▷ Offset d-c component will be different in each phase and is accounted separately on empirical basis

▷ To analyse effect of 3ϕ faults, Oscillogram of symmetrical s.c armature current in one of the phase is shown in fig 3. Here offset d-c current is neglected

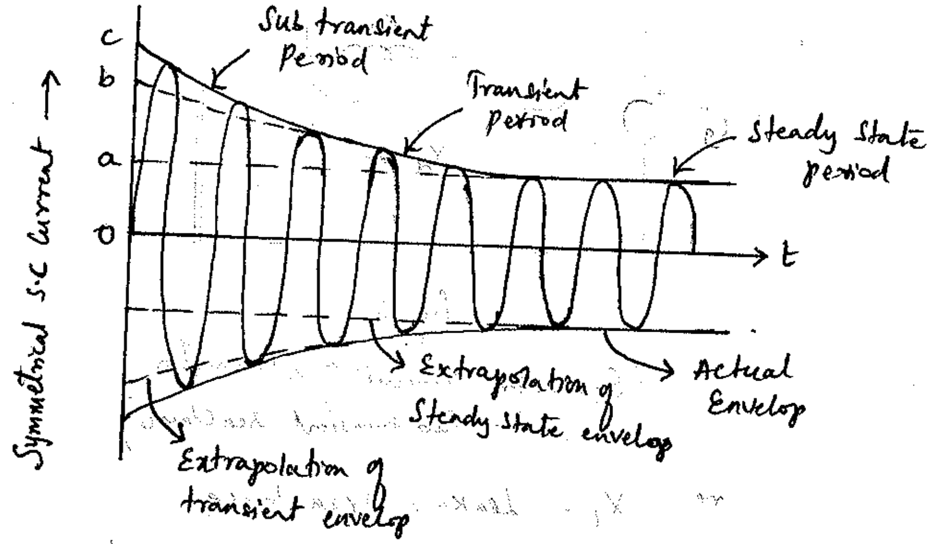


fig 3

▷ Armature current during symmetrical s.c can be divided into three regions

(i) <u>SUB-TRANSIENT REGION</u> (APPROXIMATELY 0 to 0.1 SECOND):

→ The first peaks of a.c waveforms are in this time frame when passive circuit elements are affected but electromechanical devices are not capable of responding

→ At instant of s.c d.c offset current appears in all the three phases of stator and induces current in rotor field winding and damper winding by transformer action

→ Since synchronous machine shows very low impedance under s.c, the sub-transient current would be much larger. This is because decrease in air gap flux of machine, caused by demagnetization of armature s.c current, cannot take place immediately. This result in induction of current in field & damper windings in a direction to help the main flux

→ This effect is represented by two reactances (field & damper winding reactance) in parallel with armature reactance, in fig 4

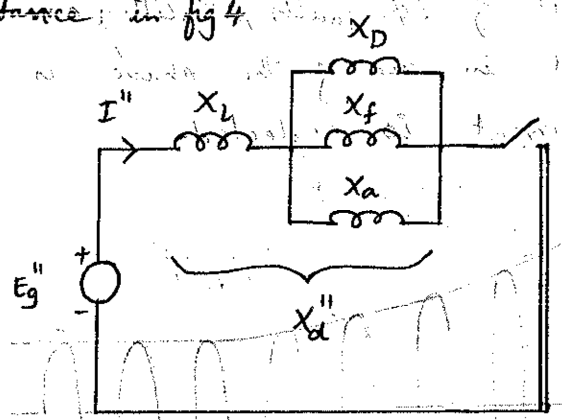


fig 4

(Equivalent Circuit of alternator under Sub-transient reactance)

Where X_L = Leakage reactance

X_a = Armature reaction reactance

X_f = field winding reactance

X_D = Damper winding reactance

X_d'' = Direct axis Sub-transient reactance

$|I''|$ = Sub-transient Current / Initial Symmetrical rms Current, rms value excluding d-c component

→ Combined effect of X_D, X_f and X_a is to reduce the total X_d'' so that S.C Current is very high

→ Total X_d'' is given by

$$X_d'' = X_L + (X_D \,||\, X_f \,||\, X_a) \rightarrow ①$$

→ From the waveform of fig 3

$$|I''| = \frac{OC}{\sqrt{2}}$$

Since $|I''|$ is rms current, OC being the peak value is divided by $\sqrt{2}$

→ from fig 4

$$X_d'' = \frac{|E_g''|}{|I''|} = \frac{|E_g''|}{oc/\sqrt{2}}$$

(ii) TRANSIENT REGION (APPROXIMATELY 0.1 to 6 SECOND):

→ This is usually the operating time interval for electromechanical protective devices such as circuit breakers and interrupters, which senses abnormal voltage or currents.

→ The induced current in damper and field winding decreases exponentially depending on their time constant ($\tau = L/R$)

→ Time constant of damper winding is much less than time constant of field winding. Hence induced current of damper winding dies out very fast (within 2 to 3 cycles)

→ Since current through X_D has died out, it acts as open circuit and the resulting reactance is called transient reactance

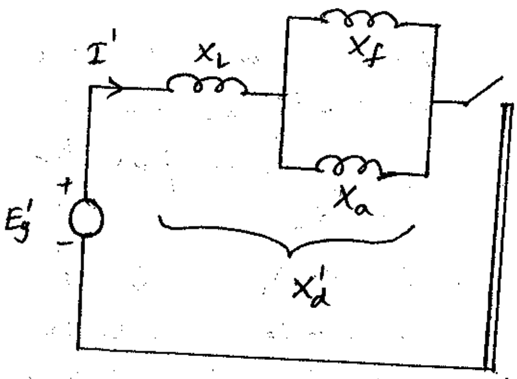


fig 5
(Equivalent circuit of alternator under transient reactance)

X_d' = direct axis transient reactance

$$X_d' = X_L + (X_f \parallel X_a) \rightarrow ②$$

→ from waveform of fig 3

$$|I'| = \frac{ob}{\sqrt{2}}$$

Where $|I'|$ = transient current, rms value excluding d-c

Component

→ from fig 5

$$X_d' = \frac{|E_g'|}{|I'|} = \frac{|E_g'|}{Ob/\sqrt{2}}$$

(iii) STEADY STATE REGION (APPROXIMATELY MORE THAN 6 SECOND):

→ As determined by machine time constant all transients have decayed to zero

→ The induced current of field winding dies out within 5 to 10 cycles

→ This open circuits X_f and the resulting reactance is called direct axis synchronous reactance

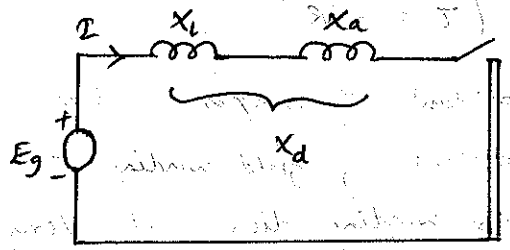


Fig 6

(Equivalent circuit of alternator under steady-state reactance)

Where X_d = direct axis synchronous reactance

$|I|$ = Steady state current, rms value

$$X_d = X_l + X_a \longrightarrow ③$$

NOTE: - Under steady state S.C condition armature reaction of synchronous generator produces demagnetising flux. This effect is represented by X_a

- Sum of X_l and X_a is called synchronous reactance X_s. In case of salient pole machine it is called direct axis synchronous reactance X_d

- Since power factor is very low during S.C, small resistance of armature is neglected

→ from waveform of fig 3

$$|I| = \frac{Oa}{\sqrt{2}}$$

→ From fig 6

$$X_d = \frac{|E_g|}{|I|} = \frac{|E_g|}{oa/\sqrt{2}}$$

▷ from ①, ② and ③, X_d'' is smallest and X_d is largest

$$\therefore \quad X_d'' < X_d' < X_d$$

▷ Practically circuit breaker will open the generator circuit before steady state fault condition is reached

▷ Maximum momentary current (i_{mm}) for generators and motors are determined by using X_d''

▷ Interrupting capacity of circuit breaker is determined by using X_d'' for generators and X_d' for motors

<u>NOTE:</u> for S.C analysis mostly we consider sub transient reactance only. So maximum current in sub transient period is

$$i_{ac}(t) = \sqrt{2}\, E_g'' \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/\tau_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/\tau_d'} + \frac{1}{X_d} \right] \sin\left(\omega t + \alpha - \pi/2\right)$$

$$i_{dc\,max}(t) = \frac{\sqrt{2}\, E_g''}{X_d''}\, e^{-t/\tau_A} = \sqrt{2}\, I''\, e^{-t/\tau_A}$$

$\therefore$ Total S.C current $i(t)$ is

$$i(t) = i_{ac}(t) + i_{dc\,max}(t)$$

Where τ_d'' = Direct axis sub-transient time constant

τ_d' = Direct axis transient time constant

τ_A = Armature time constant

▷ Values of machine constants for different types of machines is shown in table 1

Machine Constant	Turbo Generator	Hydro Generator	Synchronous Condensor	Synchronous Motor
X_d	1.1	1.15	1.8	1.2
X_d'	0.23	0.37	0.4	0.35
X_d''	0.12	0.24	0.25	0.3
τ_d'	1.1	1.8	2	1.4
τ_d''	0.035	0.035	0.035	0.035
τ_A	0.16	0.15	0.17	0.15

Table 1

SHORT CIRCUIT OF A LOADED SYNCHRONOUS GENERATOR:

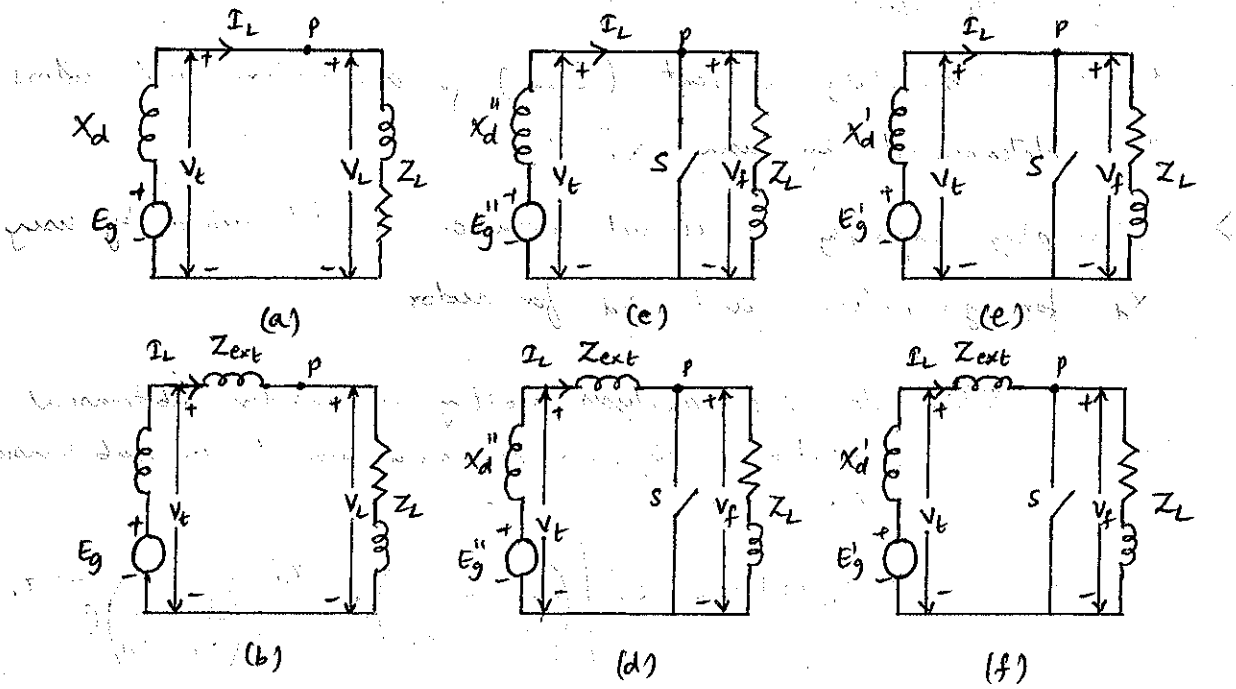


Fig 7

- (a) and (b) of fig 7 shows equivalent circuit of generator under balanced 3ϕ load, supplying load current I_L at terminal voltage of V_t
- External impedance (Z_{ext}) is shown b/w generator terminals and the point P at which fault occurs
- Z_{ext} represents total impedance of all the components till the fault point P. (Components may be step up & step down transformers and T.L)
- Voltage at the fault is V_f, and voltage across load is V_L
- E_g is induced emf under loaded condition and X_d is direct axis

Synchronous reactance of the machine

▷ Applying KVL to (a)

$$E_g - jX_d I_L - V_L = 0$$

$$E_g = V_L + jX_d I_L$$

or

$$E_g = V_t + jX_d I_L \quad (\because V_t = V_L)$$

Applying KVL to (b)

$$E_g - jX_d I_L - Z_{ext} I_L - V_L = 0$$

$$E_g = V_L + (Z_{ext} + jX_d) I_L$$

▷ When a 3ϕ fault occurs at point P, There is a s.c from P to neutral

▷ To calculate sub-transient current (I'') reactance of generator must be X_d'' and to calculate transient current (I') reactance of generator must be X_d'

▷ The equivalent circuits for studying the sub-transient and transient condition is shown in (c), (d), (e) and (f) of fig 7

▷ If switch S is open then E_g'' in series with X_d'' (in (c) and (d)) or E_g' in series with X_d' (in (e) and (f)) supplies steady state current I_L

▷ When S is closed E_g'' and E_g' supplies current to s.c through X_d'' or X_d' and Z_{ext}

▷ Applying KVL to (c) and (d) when S is open

For (c) $E_g'' - jX_d'' I_L - V_L = 0$ (Fault voltage V_f is considered only on occurance of fault. Since S is open i.e., there is no s.c voltage across the load V_L is considered)

$$E_g'' = V_L + jX_d'' I_L$$

or

$$E_g'' = V_t + jX_d'' I_L$$

$$(\because V_t = V_L)$$

For (d) $E_g'' - jX_d'' I_L - Z_{ext} I_L - V_L = 0$

$$E_g'' = V_L + (Z_{ext} + jX_d'') I_L$$

▷ Applying KVL to (c) and (d) when S is closed

For (c) $E_g'' - jX_d'' I_L - V_f = 0$

$$E_g'' = V_f + jX_d'' I_L$$

for (d) $E_g'' - jX_d'' I_L - Z_{ext} I_L - V_f = 0$

$$E_g'' = V_f + (Z_{ext} + jX_d'') I_L$$

▷ Similarly by applying KVL to (e) and (f) we get

when S is open

$$E_g' = V_t + jX_d' I_L \quad (\text{for (e)})$$

$$E_g' = V_L + (Z_{ext} + jX_d') I_L \quad (\text{for (f)})$$

when S is closed

$$E_g' = V_f + jX_d' I_L \quad (\text{for (e)})$$

$$E_g' = V_f + (Z_{ext} + jX_d') I_L \quad (\text{for (f)})$$

Where E_g'' = Sub-transient internal voltage

E_g' = Transient internal voltage

▷ Synchronous motors have reactances of same type as generators

▷ If motor is short circuited it will no longer recieve electrical energy from the power line, but its field remains energised and the inertia of its rotor and connected load keeps it rotating for indefinite period

▷ The internal voltage of synchronous motor causes it to contribute current to the system, since it is acting as generator.

▷ By comparing with formula's of generators, the sub-transient internal voltage and transient internal voltage of synchronous motor is given as

when S is open

$$E_m'' = V_t - jX_d'' I_L$$

$$E_m'' = V_L - (Z_{ext} + jX_d'') I_L$$

$$E_m' = V_t - jX_d' I_L$$

$$E_m' = V_L - (Z_{ext} + jX_d') I_L$$

When S is closed

$$E_m'' = V_f - jX_d'' I_L$$

$$E_m'' = V_f - (Z_{ext} + jX_d'') I_L$$

$$E_m' = V_f - jX_d' I_L$$

$$E_m' = V_f - (Z_{ext} + jX_d') I_L$$

▷ Motor expressions under 3ϕ balanced load would be

$$E_m = V_t - jX_d I_L$$

$$E_m = V_L - (Z_{ext} + jX_d) I_L$$

ANALYSIS OF SYMMETRICAL 3ϕ FAULTS:

▷ System that contains generators and motors under load can be solved by using
→ Thevenin's theorem
→ Kirchoff's laws

▷ Thevenin's theorem method is the fastest and easier method for computing s.c currents of large networks

▷ A s.c changes the structure of the network leading to addition of a fault impedance (Z_f) at the point of fault ($Z_f = 0$ for a solid short circuit)

▷ The change in Voltage and Current due to the structural change in network is analysed by using Thevenin's theorem

▷ THEVENIN'S THEOREM:

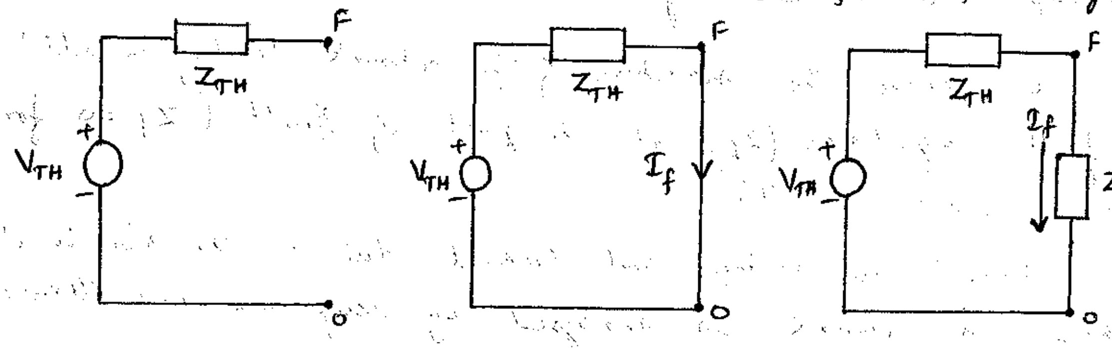
The changes in network voltages and currents due to the addition of an impedance b/w two points of a network are identical with those voltages and currents that would be caused by placing an emf, having a magnitude and polarity equal to the prefault voltage b/w the nodes, in series with the impedance all other voltage source being zeroed

PROCEDURE TO CALCULATE SHORT CIRCUIT CURRENT USING THEVENIN'S THEOREM:

- ▷ Choose appropriate base values and determine prefault condition reactance diagram of the given power system (prefault condition reactance diagram is seperately formed for sub-transient, transient and steady state condition of fault)

- ▷ Calculate prefault voltage at fault point using prefault current (load current). (If system is unloaded then the prefault voltage is 1 p.u. Also internal emf for subtransient and transient state are same as steady state induced emf)

- ▷ Prefault voltage at fault point is thevenin's voltage
- ▷ To determine thevenin's impedance of the system at fault point replace all the sources by zero value source and then reduce the resultant network to single equivalent impedance
- ▷ Draw thevenin's equivalent at fault point. Fault can be represented by a solid S.C or by Z_f i.e, fault impedance



- ▷ Per unit value of fault current I_f is given by

$$I_f = \frac{V_{TH}}{Z_{TH} + Z_f} \quad (Z_f = 0 \text{ for solid S.C})$$

- ▷ Multiplying p.u. value of I_f by base value given the actual value of fault current I_f.

- ▷ The fault current I_f is sum of prefault current and change in current due to fault

- ▷ This change in current due to fault is estimated by connecting thevenin's source with reverse polarity (i.e., negative of thevenin's voltage source) at the fault. Replace all other sources by zero value sources. Now currents in various parts of systems are the change in currents due to fault. Calculate these currents by any conventional technique

NOTE: → In practical PS normal working voltage is close to 1 p.u. So prefault voltage is assumed as 1 p.u.

→ S.C current is 10 to 20 times load current (I_L) and is purely imaginary. I_L is almost real and negligible. So prefault current is neglected

PROCEDURE TO CALCULATE SHORT CIRCUIT CURRENT USING KIRCHOFF'S LAWS:

- Select appropriate base values and determine the prefault condition reactance diagram of the given PS
- Calculate the internal emf's of synchronous machines and prefault voltages at fault point using prefault current (load current). If PS is unloaded prefault voltage is 1 p.u.
- Draw fault condition reactance diagram of the system. This diagram is same as prefault reactance diagram except that fault is represented by a solid s.c or by specified Z_f. The currents in this reactance diagram is fault condition currents
- Calculate p.u. value of fault current in various parts of the system and at fault point
- Actual value of I_f is obtained by multiplying p.u. values by respective base values.

SELECTION OF CIRCUIT BREAKERS:

- A circuit breaker (C.B) performs two functions
  - → Acts as switch under normal load condition
  - → Automatically isolates faulty part of system in event of a fault
- It is used in PS in places where power level is very high like high voltage TL, substation, generating station, heavy loads in industries etc

- Selection of C.B depends not only on the current the breaker has to carry under normal operating condition but also on the maximum current it has to carry momentarily and the current it has to interrupt at the voltage of line in which it is placed

- Choice of C.B for any particular application depends on the following rating of C.B
  - → Normal working power level specified as rated interrupting current or rated interrupting MVA
  - → Fault level specified as rated symmetrical s.c interrupting current or rated s.c interrupting MVA
  - → Rated momentary current
  - → Normal working voltage
  - → Speed of C.B

- Speed of C.B is measure of time from occurrance of fault to extinction of arc (i.e., when C.B contacts opens). It is specified in power frequency (1 cycle for 50Hz power frequency is 1/50 = 0.02ms). Standard speed of C.B are 8, 5, 3 or $1\frac{1}{2}$ cycles.

- Symmetrical momentary current (current that flows during subtransient period) is obtained by using X_d'' for synchronous machine

- This current is then multiplied by a factor of 1.8 (to account for d.c offset current during subtransient period) to get maximum momentary current during fault in rms.
  * Multiply 1.8 with $\sqrt{2}$ to obtain peak value of i_{mm}, since i_{mm} is actually a peak value

- Usually C.B opens its contacts in transient period, so the s.c interrupting current rating depends on I'

- To obtain maximum s.c interrupting current a.c component of I' is multiplied with factor 1 to 1.5 to account for d.c offset current during transient period

* Refer to NOTE in page 13

- C.B is chosen such that its s.c interrupting current rating is less than the calculated value
- Multiplying factor to find interrupting current depends on speed of C.B. For different speed of C.B the values of multiplying factor is given in table 2.

Speed of CB	Multiplying factor
8 cycles or more	1
5 cycles	1.1
3 cycles	1.2
2 cycles	1.4
1 cycle	1.5

Table 2

- If s.c MVA is more than 500 then above multiplying factors are increased by 0.1 each
- Multiplying factor for air breakers rated 600V or below is 1.25

RATING OF CIRCUIT BREAKERS:

- Current that C.B can interrupt is inversely proportional to operating voltage over a certain range

$$\text{Amperes at operating voltage} = \frac{\text{Amperes at rated voltage} \times \text{Rated voltage}}{\text{Operating voltage}}$$

- Operating voltage cannot exceed maximum design value
- No matter how low the voltage is rated interrupting current cannot exceed rated maximum interrupting current
- Over this range of voltages, the product of operating voltage and interrupting is constant
- Therefore it is logical and convinient to express C.B rating interms of S.C MVA that can be interrupted. It is defined as

Rated Interrupting MVA (3ϕ) Capacity =

$$\sqrt{3} \times |V_L|_{rated} \times |I_L|_{rated\ interrupting\ current}$$

$|V_L|$ is in KV and $|I_L|$ is in KA

▷ Instead of computing S.C current to be interrupted, we Compute 3ϕ S.C MVA to be interrupted

$$SC\ MVA\ (3\phi) = \sqrt{3} \times \text{Prefault voltage in KV} \times \text{S.C current in KA}$$

$$= \sqrt{3} \times |V_{pf}| \times |I_f|$$

▷ If voltage and current are in p.u. then

$$SC\ MVA\ (3\phi) = |V_{pf,pu}| \times |I_{f,pu}| \times MVA_b$$

▷ Obviously rated interrupting MVA is greater or equal to SC MVA

▷ To select CB for particular location, maximum possible SC MVA to be interrupted is found with respect to the type and location of fault, generating capacity and synchronous motor load connected to system

▷ 3ϕ fault, although very rare, are generally the one to give highest SC MVA and C.B must be capable of interrupting it.

*** NOTE:** $2 \times 0.9 \times \sqrt{2} = 2.55$ (i_{mm} is obtained by multiplying I'' by 2.55. 2.55 also accounts for d.c offset current)

→ Factor 2 represents the doubling effect i.e., when $t=0$ i_{mm} is 2 times symmetrical s.c current

→ As seen from fig 2 (c) i_{mm} occurs after first quarter cycle, till then the s.c current would have decayed by 10% – 20%. Remaining s.c. current (90% – 80%) is accounted by the factor 0.9 or 0.8 (Usually current decays by 10% so we take factor 0.9 into consideration)

→ Obtained current is the rms current so factor $\sqrt{2}$ is multiplied to obtain i_{mm} in peak form.

SYMMETRICAL THREE PHASE FAULTS

PROBLEMS:

1. A 25MVA, 13.2KV synchronous generator is connected to a synchronous motor of same rating. Both have a sub-transient reactance of 15%. The line connecting them has reactance of 10% on the machine base. The motor is drawing a power of 18MW at 0.8 pf lead, at 12.9KV, when a short circuit occurs at its terminals. find the sub-transient currents in the motor, generator and at fault points

(JUNE-JULY 2009 - 12M)

Sol

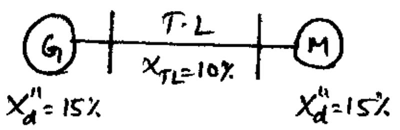


Fig 1: SLD
(SINGLE LINE DIAGRAM)

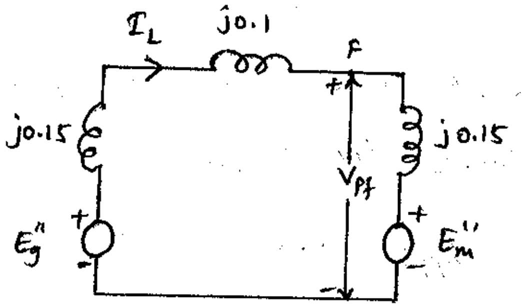


Fig 2: Circuit Model to Compute Sub-transient fault current (I_f'')

In fig 2 value of voltage sources are sub-transient internal voltages

$$MVA_b = 25\,MVA, \quad KV_b = 13.2\,KV$$

Prefault voltage at fault point, $V_{pf} = 12.9\,KV$

Per unit value of $V_{pf} = \dfrac{12.9}{13.2} = 0.977\angle 0^\circ$ pu

V_{pf} is taken as reference

$$\text{Base Current}, \; I_b = \frac{MVA_b \times 1000}{\sqrt{3} \times KV_b} = \frac{25 \times 1000}{\sqrt{3} \times 13.2}$$

$$= 1093.47 \text{ A}$$

$$\text{Load Current}, \; |I_L| = \frac{P}{\sqrt{3} \times |V_L| \times \cos\phi} = \frac{18 \times 10^6}{\sqrt{3} \times 12.9 \times 10^3 \times 0.8}$$

$|I_L| = 1007 \text{ A}$

$\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8) = 36.87^\circ$

$\therefore I_L = |I_L| \angle\phi = 1007 \angle 36.87^\circ \text{ A}$

Per Unit value of load Current $I_{L\,pu} = \dfrac{1007 \angle 36.87}{1093.47}$

$= 0.736 + j0.552$

$= 0.921 \angle 36.87 \text{ pu}$

[NOTE: $I_{L\,pu} = \dfrac{I_L}{I_b}$]

METHOD - 1: USING KIRCHOFF'S LAWS

PREFAULT CONDITION:

Apply KVL to fig 2

$E_g'' - j0.15 I_L - j0.1 I_L - V_{pf} = 0$

$E_g'' = V_{pf} + j0.25 I_L$

$= 0.977 \angle 0^\circ + 0.25 \angle 90^\circ \times 0.921 \angle 36.87$

$= 0.8388 + j0.1842$

$= 0.8588 \angle 12.38 \text{ pu}$

[NOTE: V_{pf} and I_L are in p.u. and $j0.25$ is written in polar form as $0.25 \angle 90^\circ$]

Apply KVL to fig 2

$E_m'' + j0.15 I_L - V_{pf} = 0$

$E_m'' = V_{pf} - j0.15 I_L$

$= 0.977 \angle 0^\circ - 0.15 \angle 90^\circ \times 0.921 \angle 36.87^\circ$

$= 1.059 - j0.1105$

$= 1.0656 \angle -5.95 \text{ pu}$

FAULT CONDITION:

Equivalent circuit of the system on occurrance of a 3ϕ fault is shown in fig 3

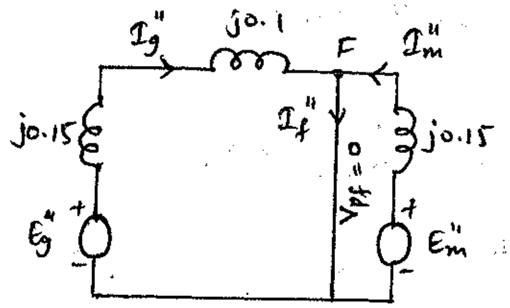


Fig 3

I_g'' = Sub-transient fault current in the generator

I_m'' = Sub-transient fault current in the Motor

I_f'' = Sub-transient fault current at fault point
$= I_g'' + I_m''$

Apply KVL to fig 3

$$E_g'' - j0.15\, I_g'' - j0.1\, I_g'' = 0$$

$$E_g'' = j0.25\, I_g''$$

$$I_g'' = \frac{E_g''}{j0.25} = \frac{0.8588\angle 12.38}{0.25\angle 90} = 0.736 - j\,3.355$$

$$= 3.435\angle -77.62 \text{ pu}$$

$$\therefore I_{g\,actual}'' = I_g'' \text{ in pu} \times I_b$$

$$= 3.435\angle -77.62 \times 1093.47$$

$$= 805.28 - j\,3668.73$$

$$= 3756.07\angle -77.62 \text{ A}$$

Apply KVL to fig 3

$$E_m'' - j0.15\, I_m'' = 0$$

$$E_m'' = j0.15\, I_m''$$

$$I_m'' = \frac{E_m''}{j0.15} = \frac{1.0656\angle -5.95^\circ}{0.15\angle 90^\circ} = -0.736 - j7.065$$

$$= 7.104\angle -95.95 \text{ pu}$$

$$\therefore I_{m\,actual}'' = I_m'' \text{ in pu} \times I_b$$

$$= 7.104\angle -95.95 \times 1093.47$$

$= -805.236 - j7726.162$

$= 7768.01 \angle -95.95 \;\; A$

Sub-transient fault current at fault point

$I_f'' = I_g'' + I_m''$

$= 3.435 \angle -77.62 + 7.104 \angle -95.95$

$= 4.026 \times 10^{-5} - j10.421$

$= 10.421 \angle -90^\circ \;\; pu$

$I''_{f\,actual} = I_f'' \text{ in } pu \times I_b$

$= 10.421 \angle -90^\circ \times 1093.47$

$= 0.044 - j11394.89$

$= 11394.89 \angle -90^\circ \;\; A$

[NOTE: Here I_g'' and I_m'' are in pu. Instead directly the actual values of I_g'' and I_m'' can be added together to obtain $I''_{f\,actual}$]

METHOD - 2: USING THEVENINS THEOREM

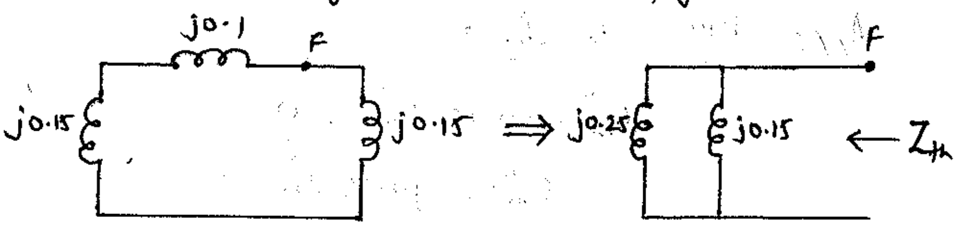
TO FIND FAULT CURRENT:

V_{pf} is taken as the thevenin's voltage at fault point

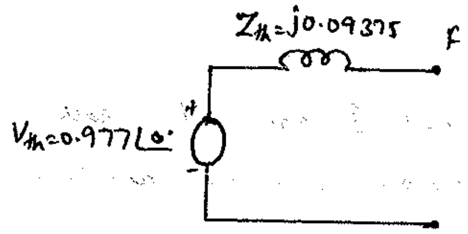
i.e., $V_{th} = V_{pf} = 0.977 \angle 0^\circ$ pu

To COMPUTE Z_{th}:

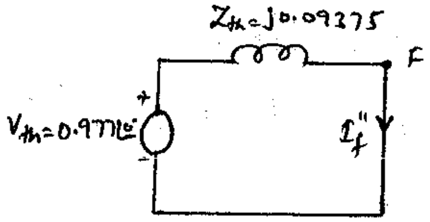
S.C all voltage sources in fig 2, we get



$$Z_{th} = \frac{j0.25 \times j0.15}{j0.25 + j0.15} = j0.09375$$



Prefault thevenin's Equivalent at fault point



Thevenin's equivalent under fault Condition

$\therefore$ Current at fault point, $I_f'' = \dfrac{V_{th}}{Z_{th}} = \dfrac{0.977\angle 0^\circ}{0.09375\angle 90^\circ}$

$= -j10.421$

$= 10.421\angle -90$ pu

$\therefore I''_{f\,actual} = I_f''$ in pu $\times I_b$

$= 10.421\angle -90^\circ \times 1093.47$

$= -j11395.41$

$= 11395.41\angle -90^\circ$ A

<u>To find Change in Current Due To Fault:</u>

The change in current due to fault is calculated by Connecting thevenins voltage, V_{th}, with reverse polarity at fault point as shown in fig 4. Here all voltage sources are replaced by s.c

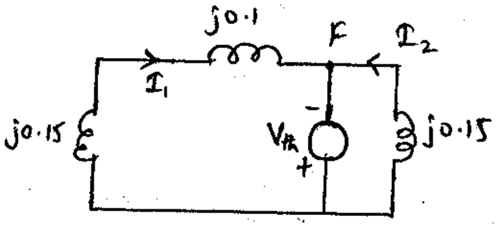


fig 4

Apply KVL to fig 4

$-j0.15\,I_1 - j0.1\,I_1 + V_{th} = 0$

$V_{th} = j0.25\,I_1$

$I_1 = \dfrac{V_{th}}{j0.25} = \dfrac{0.977\angle 0^\circ}{0.25\angle 90^\circ}$

$= -j3.908 = 3.908\angle -90^\circ$ pu

Apply KVL to fig 4

$-j0.15 I_2 + V_{th} = 0$

$V_{th} = j0.15\,I_2$

$I_2 = \dfrac{V_{th}}{j0.15} = \dfrac{0.977\angle 0^\circ}{0.15\angle 90^\circ} = -j6.513$

$= 6.513\angle -90^\circ$ pu

TO FIND I_g'' AND I_m'':

$I_g'' = I_L + I_1$ (Since I_L and I_1 are aiding each other as seen from fig 2 and fig 4)

$= 0.921\angle 36.87 + 3.908\angle -90^\circ$

$= 0.7368 - j3.355$

$= 3.435\angle -77.62$ pu

$I_{g\,actual}'' = I_g''$ in pu $\times I_b$

$= 3.435\angle -77.62 \times 1093.47$

$= 805.28 - j3668.73$

$= 3756.07\angle -77.62$ A

$I_m'' = I_2 - I_L$ (Since I_L and I_2 are opposing each other as seen from fig 2 and fig 4)

$= 6.513\angle -90^\circ - 0.921\angle 36.87$

$= -0.7368 - j7.0656$

$= 7.104\angle -95.95$ pu

$I_{m\,actual}'' = I_m''$ in pu $\times I_b$

$= 7.104\angle -95.95 \times 1093.47$

$= -805.236 - j7726.162$

$= 7768.01\angle -95.95$ A

∴ As seen both the methods yield the same values of I_g'', I_m'' and I_f''

2. A synchronous generator and motor are rated 30 MVA, 13.2 KV and both have $X_d'' = 20\%$. The line connecting them has a reactance of 10% on the base of the machine ratings. The motor is drawing 20 MW at 0.8 pf leading at a T.V. (terminal voltage) of 12.8 KV, when a symmetrical 3ϕ fault occurs at the motor terminals. Find the sub-transient current in the generator, motor and fault using the internal voltages of the machines. Verify the value of sub-transient current in the fault. (DEC.09/JAN 10 – 12M)

Sol

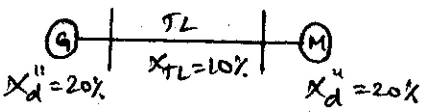


Fig 1 : SLD

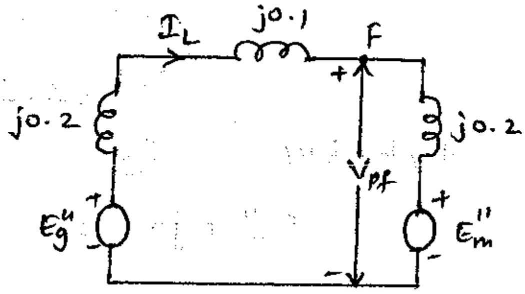


Fig 2 : Circuit Model to Compute I_f''

$MVA_b = 30\,MVA$, $KV_b = 13.2\,KV$

Prefault voltage at fault point, $V_{pf} = 12.8\,KV$

Per Unit value of $V_{pf} = \dfrac{12.8}{13.2} = 0.97\angle 0^\circ$ pu

Base current, $I_b = \dfrac{MVA_b \times 1000}{\sqrt{3} \times KV_b} = \dfrac{30 \times 1000}{\sqrt{3} \times 13.2} = 1312.16\ A$

Load Current, $|I_L| = \dfrac{P}{\sqrt{3} \times |V_L| \times \cos\phi} = \dfrac{20 \times 10^6}{\sqrt{3} \times 12.8 \times 10^3 \times 0.8}$

$= 1127.64\ A$

$\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8) = 36.87$

$\therefore\ I_L = |I_L|\angle\phi = 1127.64\angle 36.87\ A$

Per Unit value of load current, $I_{L_{pu}} = \dfrac{I_L}{I_b}$

$= \dfrac{1127.64\angle 36.87}{1312.16}$

$= 0.6875 + j0.5156$

$= 0.8594\angle 36.87$ pu

METHOD - 1 : USING KIRCHOFFS LAWS

PREFAULT CONDITION:

Apply KVL to fig 2

$E_g'' - j0.2 I_L - j0.1 I_L - V_{pf} = 0$

$E_g'' = V_{pf} + j0.3 I_L$

$= 0.97\angle 0^\circ + 0.3\angle 90^\circ \times 0.8594\angle 36.87$

$$= 0.8153 + j0.2063$$
$$= 0.841 \angle 14.2 \text{ pu}$$

Apply KVL to fig 2

$$E_m'' + j0.2 I_L - V_{pf} = 0$$
$$E_m'' = V_{pf} - j0.2 I_L$$
$$= 0.97 - 0.2\angle 90 \times 0.8594 \angle 36.87$$
$$= 1.0731 - j0.1375$$
$$= 1.082 \angle -7.3 \text{ pu}$$

FAULT CONDITION:

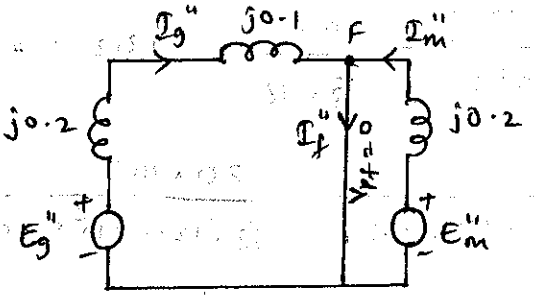


fig 3: Equivalent circuit on occurance of 3ϕ fault

Apply KVL to fig 3

$$E_g'' - j0.2 I_g'' - j0.1 I_g'' = 0$$
$$E_g'' = j0.3 I_g''$$
$$I_g'' = \frac{E_g''}{j0.3} = \frac{0.841 \angle 14.2}{0.3 \angle 90}$$
$$= 0.6876 - j2.7176$$
$$= 2.803 \angle -75.8 \text{ pu}$$

$$I_{g\,actual}'' = I_g'' \text{ in pu} \times I_b$$
$$= 2.803 \angle -75.8 \times 1312.16$$
$$= 902.34 - j3566.02$$
$$= 3678.42 \angle -75.8 \text{ A}$$

Apply KVL to fig 3

$$E_m'' - j0.2 I_m'' = 0$$
$$E_m'' = j0.2 I_m''$$
$$I_m'' = \frac{E_m''}{j0.2} = \frac{1.082 \angle -7.3}{0.2 \angle 90} = -0.687 - j5.366$$
$$= 5.41 \angle -97.3 \text{ pu}$$

$$I_{m\,actual}'' = I_m'' \text{ in pu} \times I_b$$
$$= 5.41 \angle -97.3 \times 1312.16$$
$$= -902 - j7041.24$$
$$= 7098.78 \angle -97.3 \text{ A}$$

Sub-transient fault current at fault point

$$I_f'' = I_g'' + I_m''$$

$$= 2.803 \angle -75.8 + 5.41 \angle -97.1$$

$$= 1.77 \times 10^{-4} - j8.083$$

$$= 8.083 \angle -90 \ pu$$

$$I_{f\,actual}'' = I_f'' \text{ in pu} \times I_b$$

$$= 8.083 \angle -90^\circ \times 1312.16$$

$$= 0.232 - j10606.85$$

$$= 10606.85 \angle -90^\circ \ A$$

METHOD - 2: USING THEVENINS THEOREM.

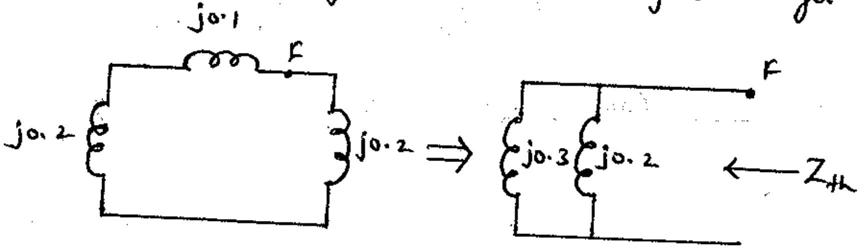
TO FIND FAULT CURRENT:

V_{pf} is taken as thevenins voltage at fault point

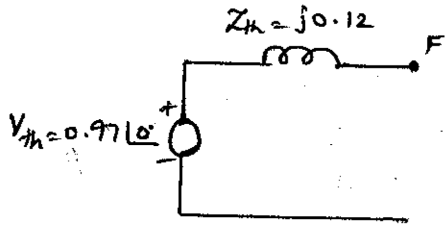
$$\therefore V_{th} = V_{pf} = 0.97 \angle 0^\circ \ pu$$

TO COMPUTE Z_{th}:

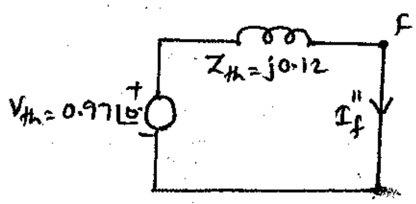
S.C all voltage sources in fig 2, we get



$$Z_{th} = \frac{j0.3 \times j0.2}{j0.3 + j0.2} = j0.12$$



Prefault thevenins equivalent at fault point



Thevenins equivalent under fault condition

Current at fault point, $I_f'' = \frac{V_{th}}{Z_{th}} = \frac{0.97\angle 0^\circ}{0.12\angle 90^\circ} = -j8.083$

$$= 8.083 \angle -90^\circ \ pu$$

$$I''_{f\,actual} = I''_f \text{ in pu} \times I_b$$
$$= 8.083\angle{-90^\circ} \times 1312.16$$
$$= -j\,10606.2$$
$$= 10606.2\angle{-90^\circ}\ A$$

To find Change in Current due to fault:

The change in current due to fault is calculated by connecting V_{th} with reverse polarity at fault point as shown in fig 4. Here all voltage sources are replaced by s.c

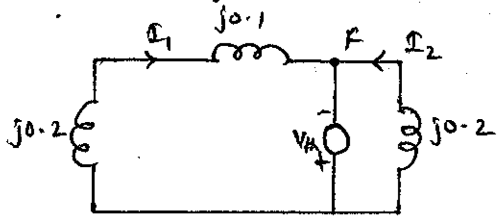


fig 4

Apply KVL to fig 4

$$-j0.2\,I_1 - j0.1\,I_1 + V_{th} = 0$$
$$V_{th} = j0.3\,I_1$$
$$I_1 = \frac{V_{th}}{j0.3} = \frac{0.97\angle{0^\circ}}{0.3\angle{90}} = -j\,3.234$$
$$= 3.234\angle{-90^\circ}\ pu$$

Apply KVL to fig 4

$$-j0.2\,I_2 + V_{th} = 0$$
$$V_{th} = j0.2\,I_2$$
$$I_2 = \frac{V_{th}}{j0.2} = \frac{0.97\angle{0^\circ}}{0.2\angle{90}} = -j\,4.85 = 4.85\angle{-90^\circ}\ pu$$

To find I_g'' and I_m'':

$$I_g'' = I_L + I_1$$
$$= 0.8594\angle{36.87} + 3.234\angle{-90}$$
$$= 0.6875 - j2.718$$
$$= 2.804\angle{-75.8}\ pu$$

$$I''_{g\,actual} = I_g'' \text{ in pu} \times I_b$$
$$= 2.804\angle{-75.8} \times 1312.16$$
$$= 902.135 - j\,3566.92$$
$$= 3679.23\angle{-75.8}\ A$$

$$I_m'' = I_2 - I_L$$
$$= 4.85\angle{-90^\circ} - 0.8594\angle{36.87}$$
$$= -0.6875 - j\,5.355 = 5.4\angle{-97.3}\ pu$$

$$I''_{m\,actual} = I_m'' \text{ in pu} \times I_b$$
$$= 5.4\angle{-97.3} \times 1312.16$$
$$= -902.135 - j\,7027.46$$
$$= 7085.12\angle{-97.3}\ A$$

3. A generator is connected to a synchronous motor through transformer. Reduced to a common base, the per unit sub-transient reactances of generator and motor are 0.15 and 0.35 pu respectively. The leakage reactance of the transformer is 0.1 pu. A 3ϕ S.C fault occurs at terminals of the motor when terminal voltage of generator is 0.9 pu and output current of the generator is 1 pu at 0.8 pf leading. Find the sub-transient current in the fault, generator and motor

(MAY/JUNE 2010 – 8M)

Sol:

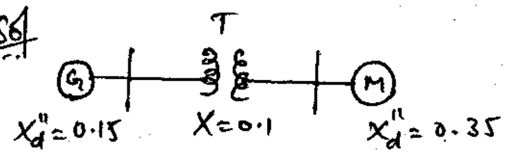


fig 1: SLD

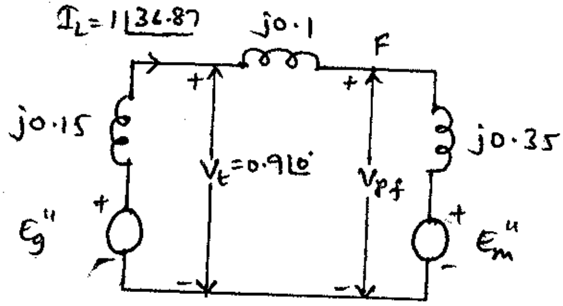


fig 2: Circuit Model to Compute I_f''

In fig 2, V_t = Terminal voltage of generator, and it is used as a reference vector

[NOTE: – Since MVA and KV rating are not specified for any component (G, T and M) the reactance diagram is directly draw from the given values of their reactances.
– Also base current cannot be calculated, so final values of I_g'', I_f'' and I_m'' are retained in per unit]

Load current, $|I_L| = 1$ pu

$$\cos\phi = 0.8$$
$$\phi = \cos^{-1}(0.8) = 36.87°$$

$$\therefore\ I_L = |I_L|\angle\phi = 1\angle 36.87 \text{ pu}$$

PREFAULT CONDITION:

Apply KVL to fig 2

$$E_g'' - j0.15\, I_L - V_t = 0$$

$$\begin{aligned} E_g'' &= V_t + j0.15\, I_L \\ &= 0.9\angle 0° + 0.15\angle 90 \times 1\angle 36.87 \\ &= 0.8099 + j0.12 \\ &= 0.8188\angle 8.42 \text{ pu} \end{aligned}$$

Apply KVL to fig 2

$$V_t - j0.1 I_L - V_{pf} = 0$$

$$V_{pf} = V_t - j0.1 I_L$$
$$= 0.9\angle 0^\circ - 0.1\angle 90^\circ \times 1\angle 36.87^\circ$$
$$= 0.96 - j0.08$$
$$= 0.9633\angle -4.76 \text{ pu}$$

Apply KVL to fig 2

$$E_m'' + j0.35 I_L - V_{pf} = 0$$

$$E_m'' = V_{pf} - j0.35 I_L$$
$$= 0.9633\angle -4.76 - 0.35\angle 90^\circ \times 1\angle 36.87$$
$$= 1.17 - j0.36$$
$$= 1.224\angle -17.1 \text{ pu}$$

FAULT CONDITION:

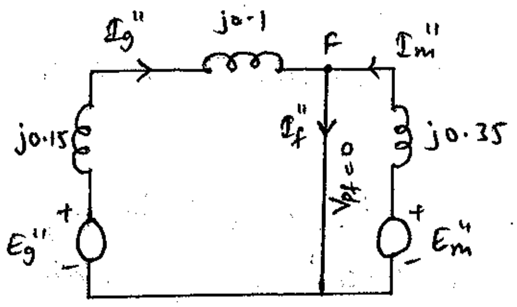


fig 3: Equivalent circuit on occurance of 3ϕ fault

Apply KVL to fig 3

$$E_g'' - j0.15 I_g'' - j0.1 I_g'' = 0$$

$$E_g'' = j0.25 I_g''$$

$$I_g'' = \frac{E_g''}{j0.25} = \frac{0.8188\angle 8.42}{0.25\angle 90^\circ}$$
$$= 0.48 - j3.24$$
$$= 3.275\angle -81.58 \text{ pu}$$

Apply KVL to fig 3

$$E_m'' - j0.35 I_m'' = 0$$

$$E_m'' = j0.35 I_m''$$

$$I_m'' = \frac{E_m''}{j0.35} = \frac{1.224\angle -17.1}{0.35\angle 90^\circ} = -1.028 - j3.342$$
$$= 3.497\angle -107.1 \text{ pu}$$

Sub-transient fault current at fault point

$$I_f'' = I_g'' + I_m''$$
$$= 3.275\angle -81.58 + 3.497\angle -107.1$$
$$= -0.548 - j6.582$$
$$= 6.605\angle -94.76 \text{ pu}$$