Introduction and classification of signals :→

Signal is defined as a function of one or more variables that ... information on the nature of a physical phenomenon

E.g :- Speech signal, ECG signal, Image (for visual communication)

- Electronic mail over the internet.

...al life examples: By listening to the heart beat of a patient and mon... his blood pressure or temp, the doctor is able to diagonise the patient's illness or disease

→ Weather forecast on radio

→ ~~(When the signal depends on a)~~ If the signal is a func^n of one variable the signal is said to be one dimensional

Eg: A speech signal → amplitude varies with time

→ If the signal is a func^n of two or more variables, the signal is said to be multidimensional.

Eg: Image — Horizontal and vertical coordinates represents the 2D.

System :→ "It is defined as an entity that manipulates one or more signals to accomplish a function, there by yielding new signals".

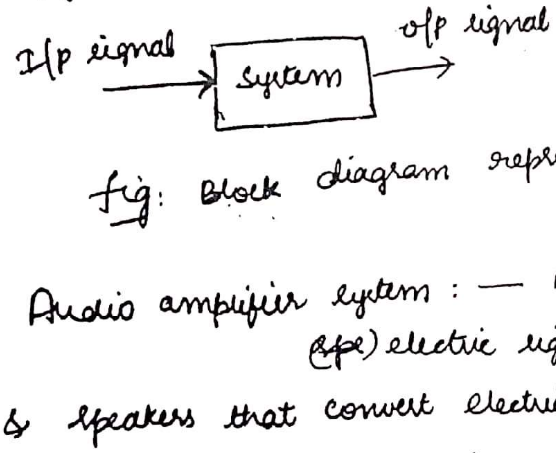
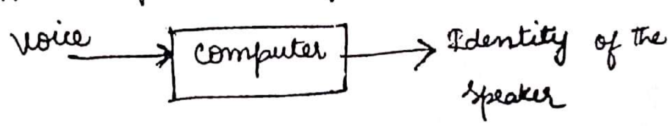


fig: Block diagram representation of a system

Eg. Audio amplifier system :— microphone converts accoustic signals to (ape) electric signals, amplifier amplifies the electric signals & speakers that convert electrical signals to accoustic signals

- Automatic speech recognition system or Speaker recognition system



- communication system
- control system

Communication Systems : →

Message signal → Transmitter →(Transmitted signal)→ channel →(Received signal)→ Receiver → Estimate of message signal

fig: Elements of a Communication system.

- The three basic elements of communication systems are:—
  (i) Transmitter (ii) channel (iii) Receiver.

> The transmitter is placed at one point in space and receiver is placed at one point. channel is the physical medium that connects the transmitter & receiver.

> The purpose of transmitter is to convert the message signal of information source into form suitable for transmission over the channel

- Channel can be optical fiber, coaxial cable, satellite channel or a mobile radio channel.

- As the transmitted signal (travels (propogates over through) the channel, it is distorted due to physical characteristics of channel. Noise and interfering signals contaminate the channel o/p.

- The function of receiver is to operate on received signal and to estimate message signal.

→ (Inorder to recover the transmitted signal at the receiving end exactly). For reliable communication Modulation and demodulation process is done.

→ Analog communication system → Simple
Digital communication system → Quite complex. since have to convert analog signals to digital. by sampling
Quantization
coding

→ Two basic modes of communication:
- Broadcasting
- Point to point communication.

Control Systems :→ 26

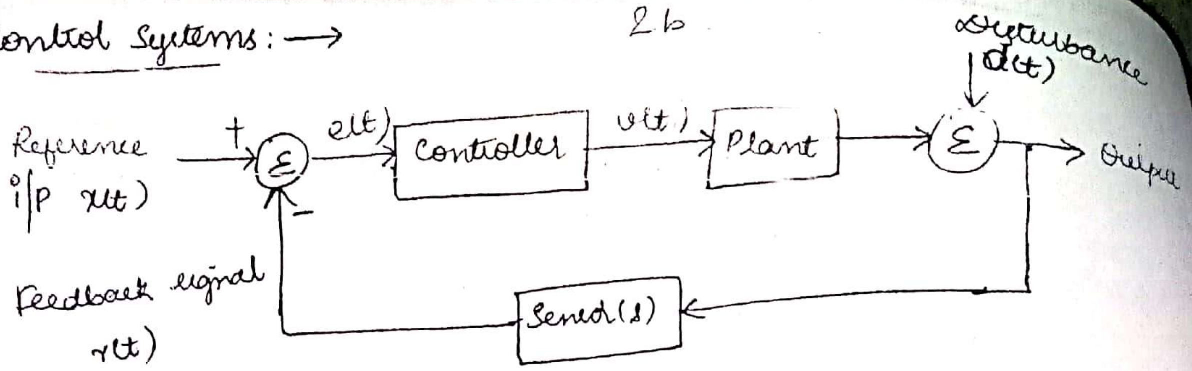


fig: Block diagram of feedback control system.

→ The control of physical s/m is widespread. Paper mills, nuclear reactors, power plants & robots. The object to be controlled is commonly referred to as plant.

→ In any event, the plant is described by mathematical operations that generates the output y(t) in response to plant i/p v(t). and external disturbance d(t).

→ Sensor in feedback loop measures the plant y(t) & converts it to another form usually electrical.

→ sensor o/p r(t) constitutes the feedback signal & is compared with reference i/p x(t) to produce a difference or error s/g. e(t)

→ Error signal e(t) is applied to controller, which in turn generates actuating signal v(t). Actuating signal v(t) performs controlling action on the plant.

→ Depending on number of i/ps to the plants

SISO

MIMO systems

→ controller :→ May be in the form of digital computer or Microprocessor.

(3)

Classification of signals : 5 methods

(i) Continuous and discrete time signals
(ii) Even and odd signals
(iii) Periodic and Non periodic signals
(iv) Deterministic & random signals
(v) Energy and power signals.

Continuous time and discrete time signals :→ A signal x(t) is said to be continuous time signal if it has value of amplitude for all time 't'. Continuous time signal arise naturally when a physical phenomenon (eg. heart beat, accoustic pressure variation) etc is converted into an electrical signal using appropriate transducer.

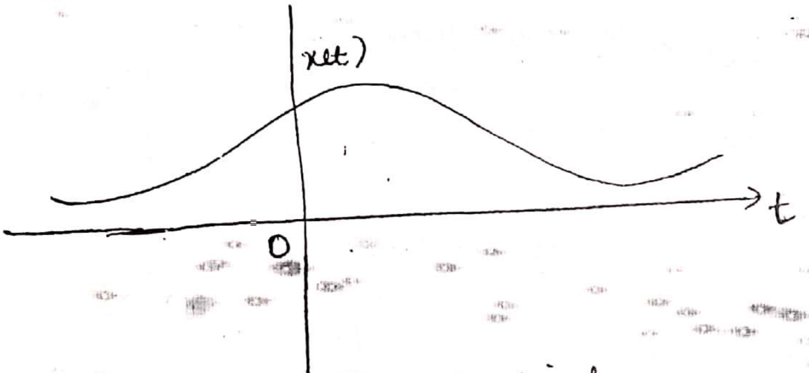


fig: continuous time signal.

Discrete time signal is defined only at discrete instants of time. (ie the independent variable has discrete values only which are usually uniformly spaced).

→ A discrete time signal is derived from a continuous time signal by sampling it at a uniform rate.

→ Let T_s denote the sampling period & n denote an integer that may assume positive & negative values. then sampling a CT sig $x(t)$ at time $t = nT_s$ yields a sample with the value $x(nT_s)$.

$$x[n] = x(nT_s) \quad ; \quad n = 0, \pm 1, \pm 2, \ldots$$

→ Discrete time sigs are represented mathematically as sequence of numbers ; $x[-2], x[-1], x[0], x[1], x[2] \ldots$

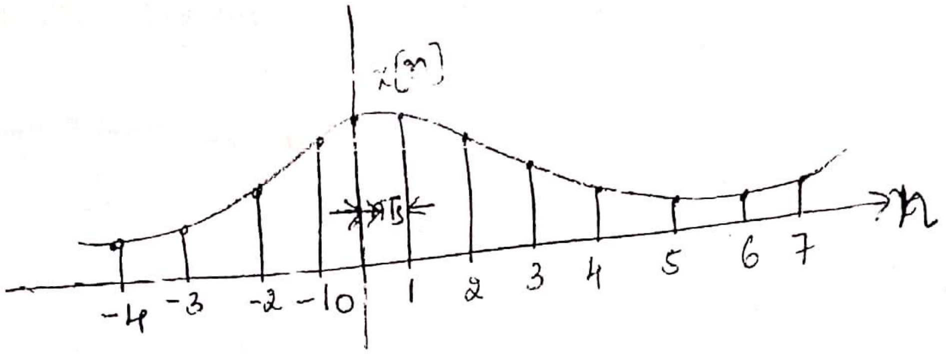


fig: discrete time signal.

- For continuous time signals $\longrightarrow t \rightarrow x(t)$

For discrete time signals $\longrightarrow n \rightarrow x[n]$

Even and odd signals :→ Depending upon the symmetry & time reversing the signals are classified as even and odd signals.

→ A continuous time s/g is said to be even signal if

$$\boxed{x(-t) = x(t)} \text{ for all } t. \; ; \quad \boxed{x[-n] = x[n]} \; \forall n$$

the signal x(t) is said to be an odd signal if

$$\boxed{x(-t) = -x(t)} \text{ for all } t \; ; \quad \boxed{x[-n] = -x[n]} \; \forall n$$

→ Even signals are symmetric about the vertical axis or time origin whereas odd signals are antisymmetric about the time origin.

Eg. Consider the signal $x(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right) & ; \; -T \le t \le T \\ 0 & ; \; \text{otherwise} \end{cases}$

Is the signal x(t) an even or an odd function of time ?

Solun:- Replacing t with -t yields ;

$$x(-t) = \begin{cases} \sin\left(\frac{-\pi t}{T}\right) & ; \; -T \le t \le T \\ 0 & ; \; \text{otherwise} \end{cases}$$

$$= \begin{cases} -\sin\left(\frac{\pi t}{T}\right) & ; \; -T \le t \le T \\ 0 & ; \; \text{otherwise} \end{cases}$$

$\boxed{\{\because \sin(-\theta) = -\sin\theta\}}$

Decomposition of a signal :→ A C.T signal can be decomposed into a sum of two signals, one of which is even $x_e(t)$, the other is $x_o(t)$ such that ;

$$x(t) = x_e(t) + x_o(t) \longrightarrow (1)$$

for $x_e(t)$ to be even ; $x_e(-t) = x_e(t) \longrightarrow (2)$

$x_o(t)$ to be odd ; $x_o(-t) = -x_o(t) \longrightarrow (3)$

Substituting $t = -t$ in equan (1)

$$x(-t) = x_e(-t) + x_o(-t) = x_e(t) - x_o(t) \longrightarrow (4)$$

Solving for $x_e(t)$ and $x_o(t)$ from equan (1) & (4) we get

$$\boxed{x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]}$$ {Adding (1) + (4)}

$$\boxed{x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]}$$ {subtracting (1) − (4)}

For discrete time signals ;

$$\boxed{x_e[n] = \frac{1}{2}\left\{x[n] + x[-n]\right\}}$$

$$\boxed{x_o[n] = \frac{1}{2}\left\{x[n] - x[-n]\right\}}$$

Problems

→ Find the even and odd components of the signal

$$x(t) = \cos t + \sin t + \sin t \cos t$$

Soln:—

$$x(-t) = \cos t + \sin(-t) + \sin(-t)\cos t$$

$$x(-t) = \cos t - \sin t - \sin t \cos t$$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] = \frac{1}{2}\left[2\cos t\right] = \cos t$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right] = \frac{1}{2}\left[2\sin t + 2\sin t\cos t\right] = \sin t\,(1 + \cos t)$$

→ $x(t) = e^{-2t}\cos t$; $x_e(t) = \cosh(2t)\cos t$ $\left\{\because \frac{e^{\theta} + e^{-\theta}}{2} = \cosh\theta\right\}$

$x_o(t) = -\sinh(2t)\cos t$

→ $x(t) = (1+t^3)\cos^3(10t)$ 4c

Solun:— $x(-t) = (1-t^3)(\cos(-10t))^3 = (1-t^3)\cos^3 10t$

$x(t) = \cos^3(10t) + t^3\cos^3(10t)$

$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] = \cos^3(10t)$

$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right] = t^3\cos^3(10t)$

› $x(t) = 1+t+3t^2+5t^3+9t^4$

Solun $x(-t) = 1-t+3t^2-5t^3+9t^4$

$x_e(t) = \frac{1}{2}\left[x(t)+x(-t)\right] = \frac{1}{2}\left[2+6t^2+18t^4\right] = 1+3t^2+9t^4$

$x_o(t) = \frac{1}{2}\left[x(t)-x(-t)\right] = \frac{1}{2}\left[2t+10t^3\right] = t+5t^3$

02-2017

› $x(t) = e^{jt}$

Solun:- $x(-t) = e^{-jt}$

$x_e(t) = \frac{1}{2}\left[x(t)+x(-t)\right] = \frac{1}{2}\left[e^{jt}+e^{-jt}\right] = \cos t$; $\left\{\because \frac{e^{jt}+e^{-jt}}{2} = \cos t\right\}$

$x_o(t) = \frac{1}{2}\left[x(t)-x(-t)\right] = \frac{1}{2}\left[e^{jt}-e^{-jt}\right] = j\sin t$; $\left\{\because \frac{e^{jt}+e^{-jt}}{2j} = \sin t\right\}$

17-02-2017

› ST the product of two even signals or two odd signals is an even signal while the product of an even & odd signal is an odd signal.

Sun:- Let $x(t) = x_1(t)\cdot x_2(t)$

Case (i): If $x_1(t)$ and $x_2(t)$ are even

$x(-t) = x_1(-t)\cdot x_2(-t) = x_1(t)\, x_2(t) = x(t)$ → even signal

Case (ii). If $x_1(t)$ and $x_2(t)$ are odd

$x(-t) = x_1(-t)\cdot x_2(-t) = -x_1(t)\cdot\left[-x_2(t)\right] = +x(t)$ → even ~~odd~~ signal

Case (iii): Let $x_1(t)$ be even & $x_2(t)$ be odd

$x(-t) = x_1(-t)\cdot x_2(-t) = x_1(t)\{-x_2(t)\} = -x_1(t)x_2(t) = -x(t)$

odd signal

2012

(5)

For any arbitrary signal: if $x(t)$ is an [illegible] [illegible] ST $\int_{-\infty}^{\infty} x(t)\,dt = 2\int_{0}^{+\infty} x(t)\,dt$

(ii) if $x(t)$ is odd signal $\int_{-\infty}^{\infty} x(t)\,dt = 0$

Soln:- (i) $x(t)$ is an even ie $x(-t) = x(t)$

$$\int_{-\infty}^{\infty} x(t)\,dt = \int_{-\infty}^{0} x(t)\,dt + \int_{0}^{\infty} x(t)\,dt = -\int_{\infty}^{0} x(t)\,dt + \int_{0}^{\infty} x(t)\,dt$$

Replace t by $-t$ in first term then $\boxed{dt = -dt}$ and limits $0 \le t \le -\infty$

$0 \le -t \le \infty$

$$\int_{-\infty}^{\infty} x(t)\,dt = \int_{0}^{\infty} x(t)\,dt + \int_{0}^{\infty} x(t)\,dt = 2\int_{0}^{\infty} x(t)\,dt$$

) $x(t)$ is odd signal $LHS = \int_{-\infty}^{\infty} x(t)\,dt = \int_{-\infty}^{0} x(t)\,dt + \int_{0}^{\infty} x(t)\,dt$

$x(-t) = -x(t)$

$$= -\int_{0}^{-\infty} x(t)\,dt + \int_{0}^{\infty} x(t)\,dt$$

Replace t by $-t$ in first term then $dt = -dt$

$$\int_{-\infty}^{\infty} x(t)\,dt = \int_{0}^{\infty} -x(t)\,dt + \int_{0}^{\infty} x(t)\,dt = 0$$

ST if $x(n)$ is an odd signal then $\sum_{n=-\infty}^{\infty} x[n] = 0$.

6 Soln:- $LHS = \sum_{n=-\infty}^{\infty} x[n] = \sum_{n=-1}^{-\infty} x[n] + x[0] + \sum_{n=1}^{\infty} x[n]$

Replace n by $-n$ in first term; { For odd signal $x[0] = 0$ Eg $\sin 0 = 0$. }

$$= \sum_{n=1}^{\infty} x(-n) + \cancel{x[0]}^{\,0} + \sum_{n=1}^{\infty} x[n]$$

$$= \sum_{n=1}^{\infty} \{x[n] + x[-n]\} = \sum_{n=1}^{\infty} \{x[n] - x[n]\} = 0$$

5

8-09-2019

→ ST if $x_1(n)$ is an odd signal & $x_2(n)$ is an even signal then $x_1(n)\,x_2(n)$ is an odd signal

→ Let $x(n)$ be an arbitrary signal with even and odd parts by $x_e(n)$ and $x_o(n)$ respectively. ST

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

Solⁿ:-

$$LHS = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} \{x_e[n] + x_o[n]\}^2$$

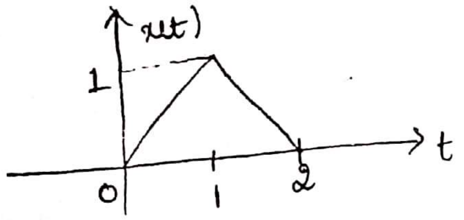
$$= \sum_{n=-\infty}^{\infty} \{x_e^2[n] + x_o^2[n] + 2\,\underbrace{x_e[n]\,x_o[n]}\}$$

→ Product of even & odd signal is odd signal

$$= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] + \cancel{\sum_{n=-\infty}^{\infty} \{2\,x_e[n]\,x_o[n]\}}^{\,0} \rightarrow \sum_{n=-\infty}^{\infty} \text{odd s/g} = 0.$$

$$\therefore \boxed{\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]}$$

→ Determine & sketch the even and odd parts of the signal shown in figure.



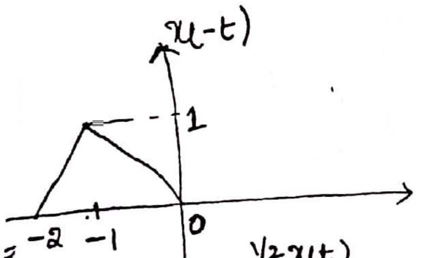
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

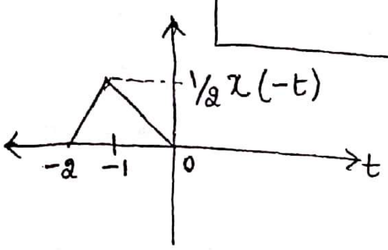
$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

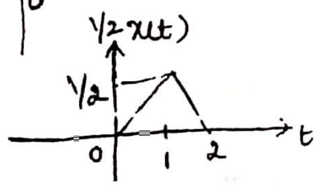
$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

Solⁿ:- Even; $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



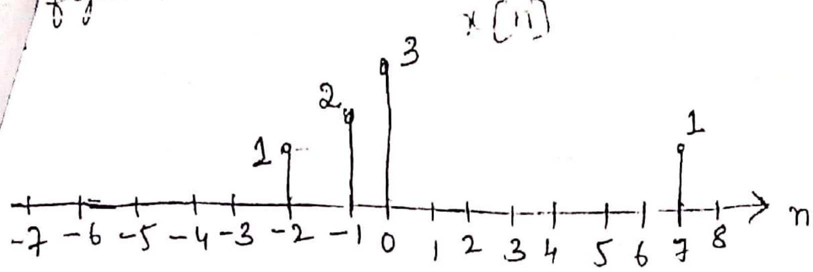
⇒





02-2017

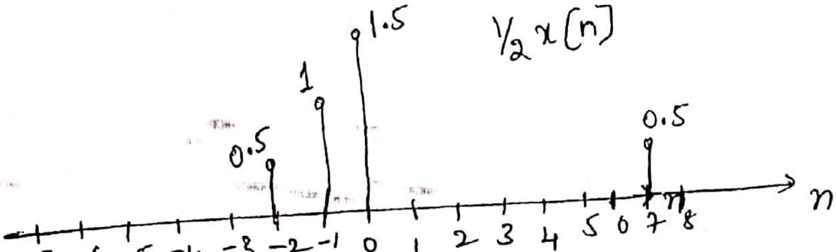
Determine & draw the even & odd parts of the discrete sig shown in figure.

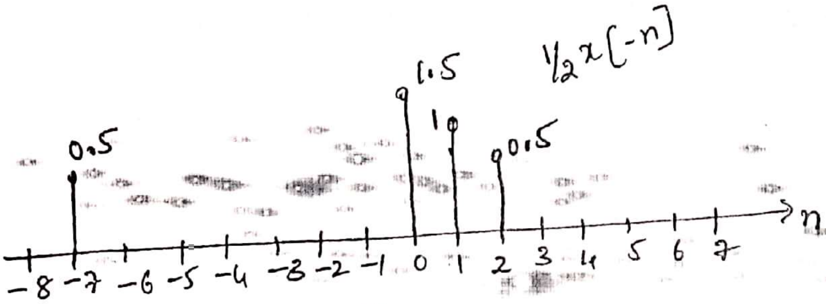


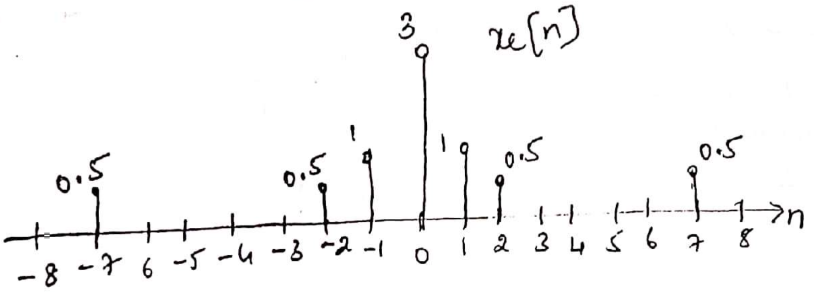
Solun :-

$$x_e[n] = \tfrac{1}{2}x[n] + \tfrac{1}{2}x[-n]$$

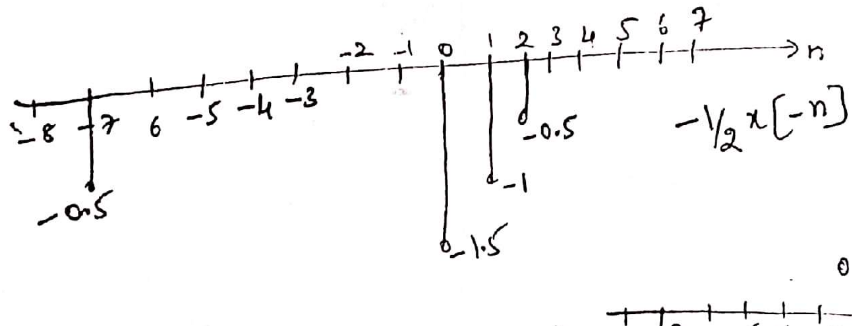
$$x_o[n] = \tfrac{1}{2}x[n] + \{-\tfrac{1}{2}x[-n]\}$$

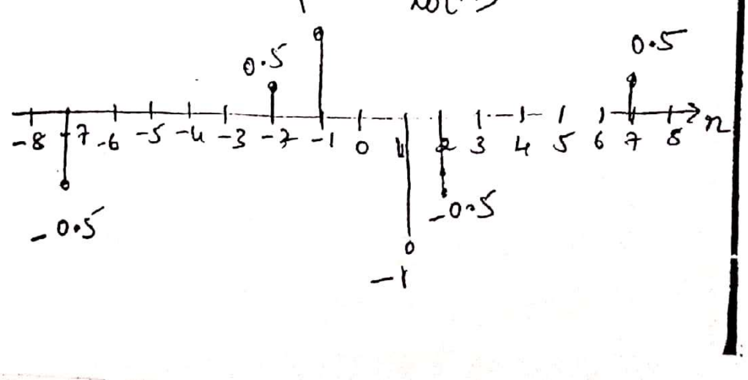




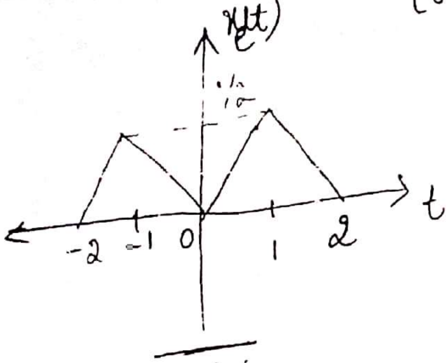


⇒ Even component.

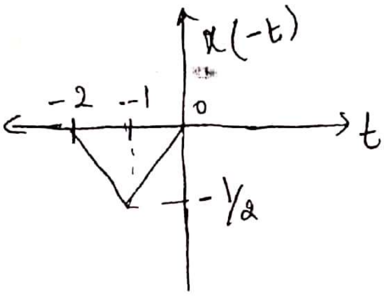


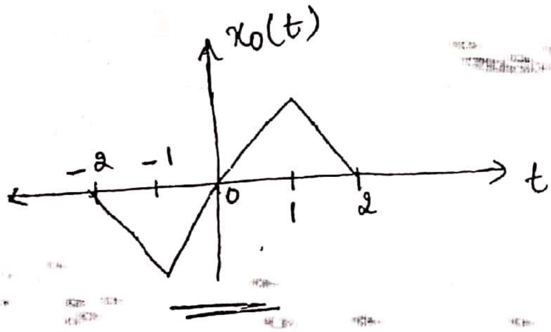


Odd component

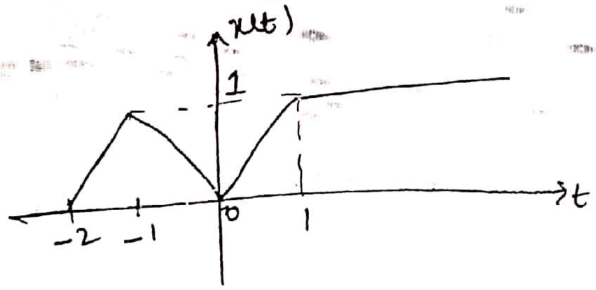


or odd component $x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$



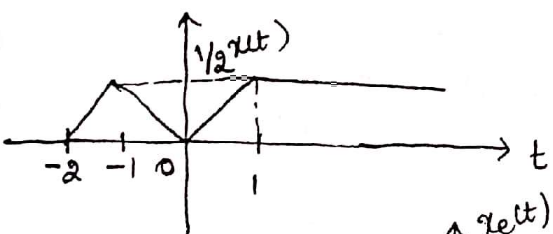


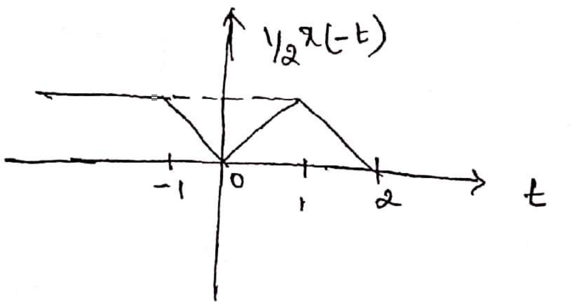
→ Determine and sketch even & odd components of $x(t)$

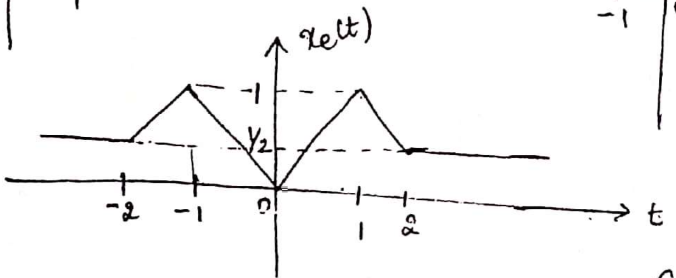


Soln:– $x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$

$x_o(t) = \frac{1}{2}x(t) + \{-\frac{1}{2}x(-t)\} = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$



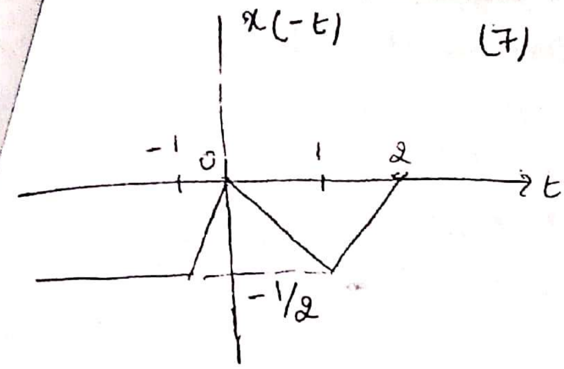


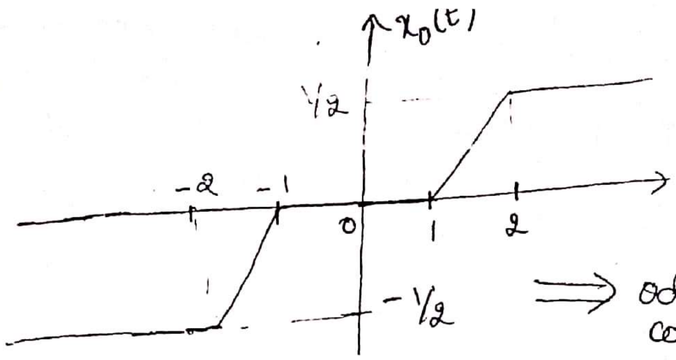


⟹ Even Component

(continued in (7))

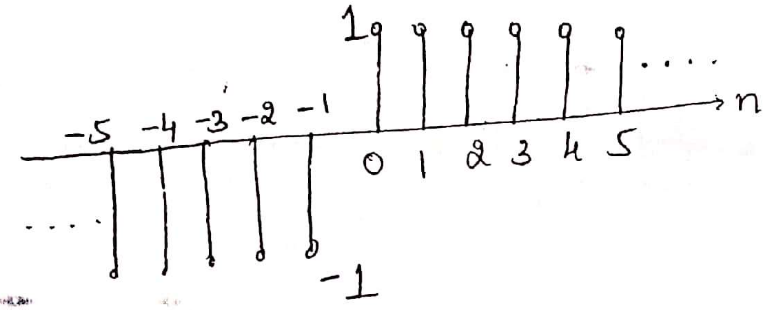
(7)





$\Rightarrow$ odd component

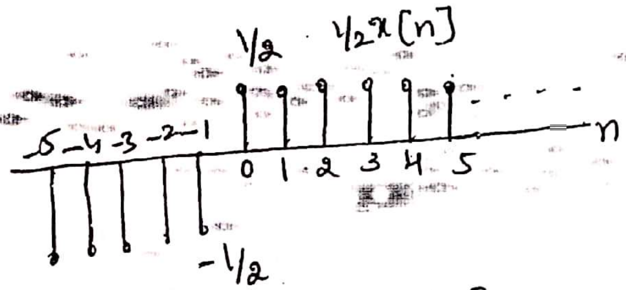
→ Determine & sketch even & odd parts of the signal shown in fig.

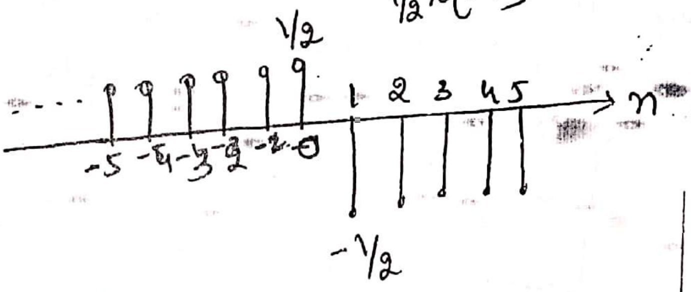


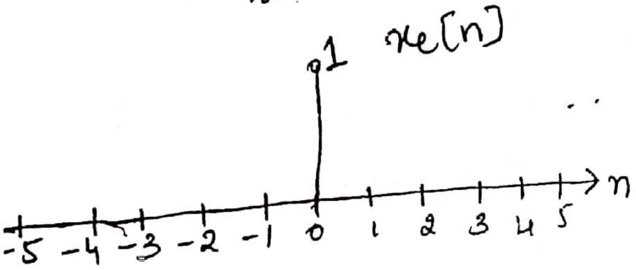
Soln :-

$$x_e[n] = \tfrac{1}{2}x[n] + \tfrac{1}{2}x[-n]$$

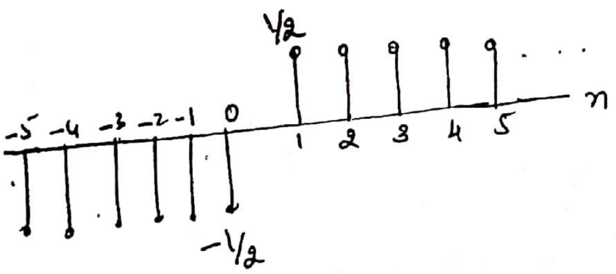
$$x_o[n] = \tfrac{1}{2}x[n] + \{-\tfrac{1}{2}x[-n]\}$$

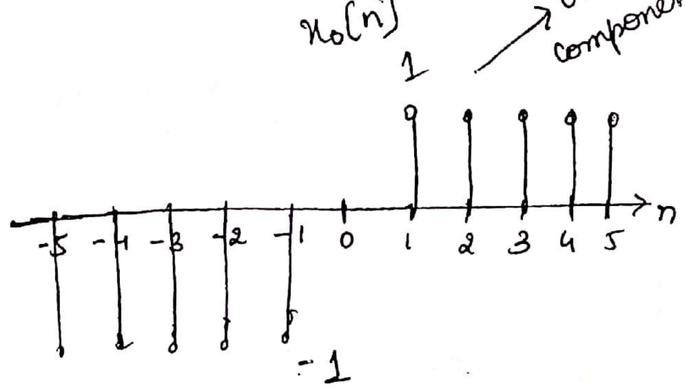






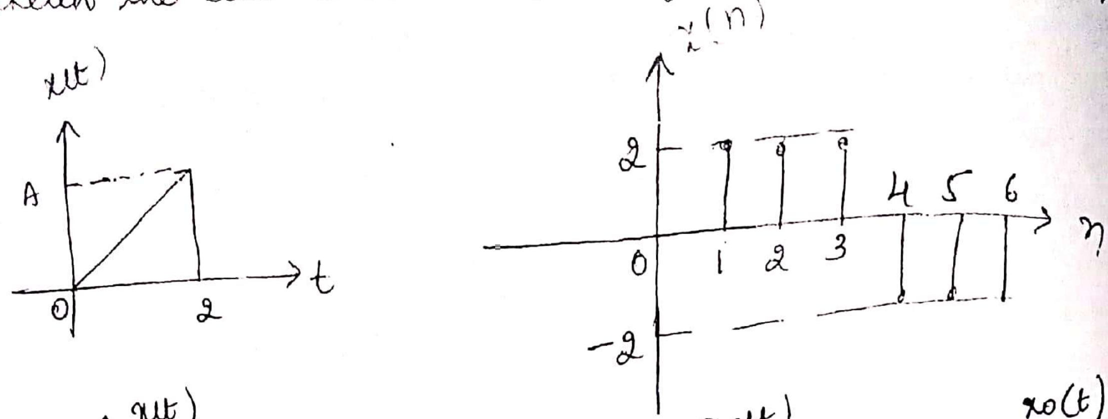
→ even component

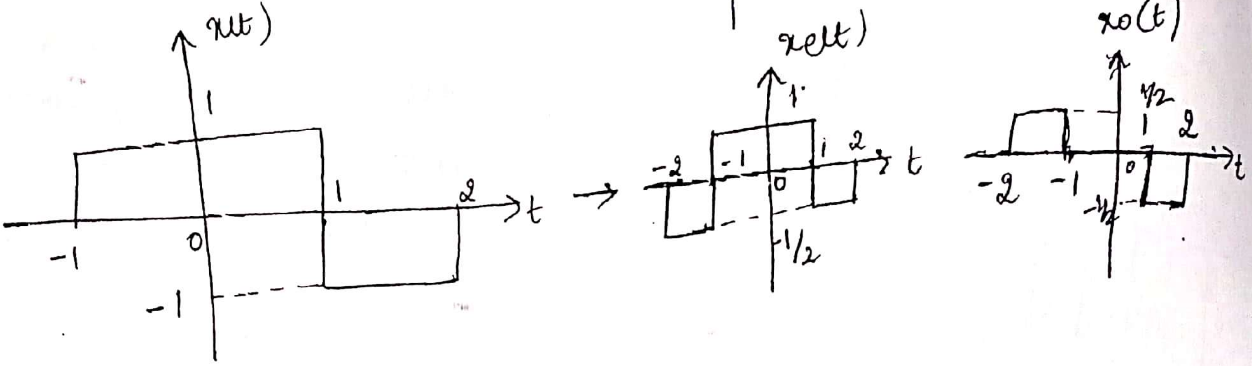




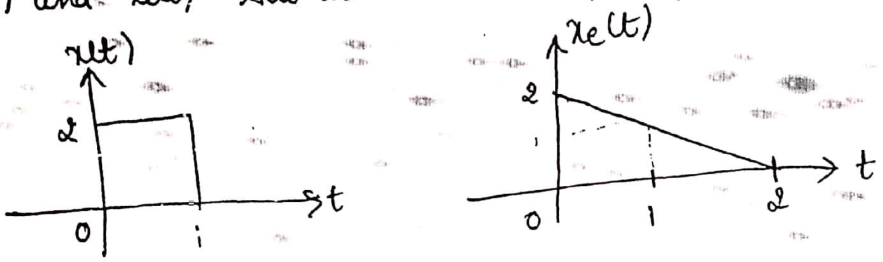
→ odd component

→ Sketch the even and odd parts of the signal shown in fig

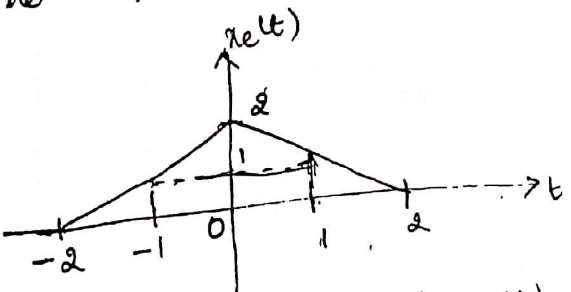




→ fig shows part of the signal x(t) & its even part xe(t) respectively for t≥0 only. x(t) and even part xe(t) for t<0 is not shown complete the plots of x(t) and xe(t). Also draw the odd part of x(t) [i.e xo(t)]



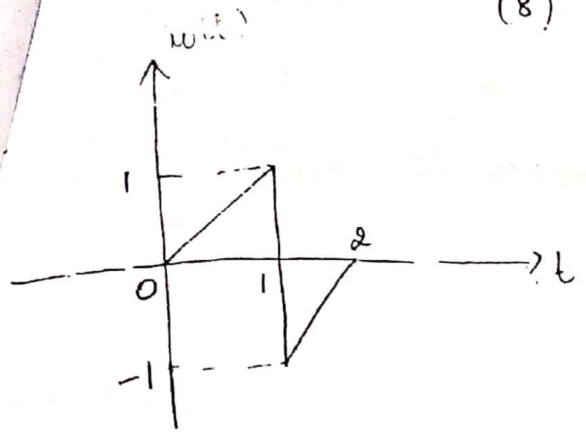
Even :— step 1: WKT the even part is symmetric about $t=0$. Thus we have the complete even part xe(t)



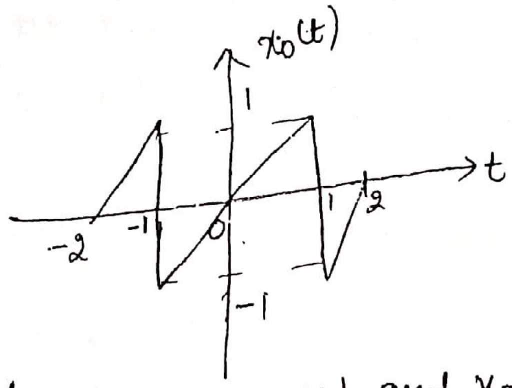
Step 2:— $x(t) = x_e(t) + x_o(t)$

Now $x_o(t)$ for $t \geq 0$ is obtained in such a way that

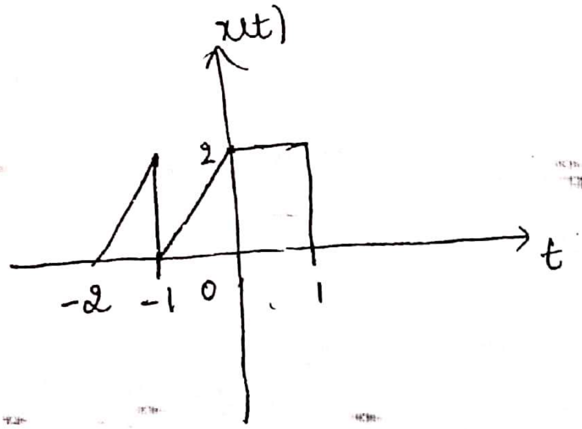
$x_o(t) = x(t) - x_e(t)$.



Step3 :— $x_o(t)$ is an antisymmetric about $t=0$. (passes it through origin)



Step4 :— Complete $x(t)$ is obtained by adding $x_e(t)$ and $x_o(t)$



Periodic and Non periodic signals :→

A continuous time signal $x(t)$ is said to be periodic if it satisfies the condition :

$$x(t) = x(t+T) \text{ ; for all } 't' \longrightarrow (1)$$

where T is a positive constant

If the condition is satisfied for $T=T_0$ then it is also satisfied for any

$T = nT_0$ where $n = 1, 2, 3, \ldots$

ie $T = 2T_0, 3T_0, 4T_0 \ldots$

The smallest value of 'T' that satisfies the equation is called the fundamental period of $x(t)$. This fundamental period is the time taken by the signal $x(t)$ to complete its one cycle.

The reciprocal of the fundamental period 'T' is known as the fundamental frequency of the signal.

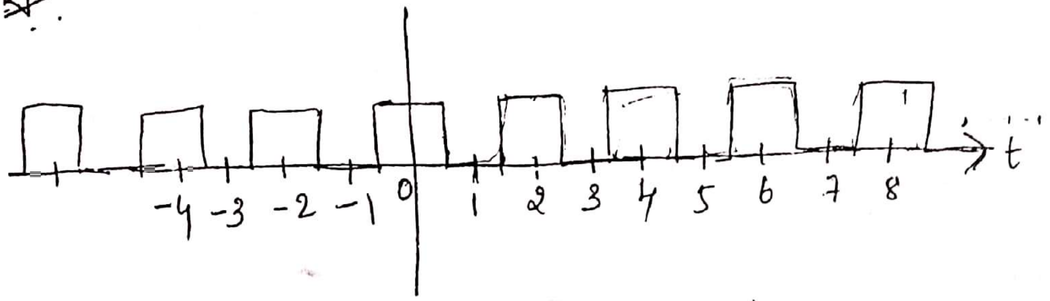
$$f = \frac{1}{T} \quad \text{(Hertz)}$$

The fundamental angular frequency

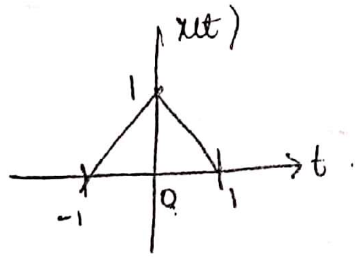
$$\omega = 2\pi f = \frac{2\pi}{T} \quad (rad/sec)$$

ny continuous time signal x(t) which doesnot satisfy equation(1) is ca[lled] non periodic or Aperiodic signal.

Eg. Periodic signal : T = 2



Non periodic signal



For discrete time signal:

If x(n) is said to be periodic then it has to satisfy the condition

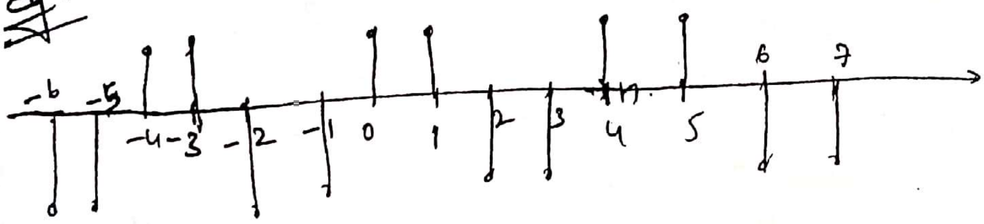
$x(n) = x(n+N)$: for all 'n' ; N is a positive integer →(2)

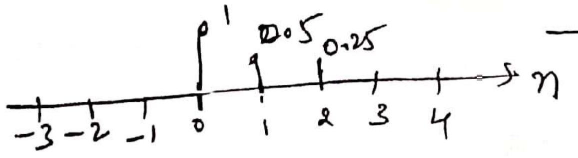
he smallest value of N which satisfies equa^n(1) is called the fundamental eriod of the signal x(n)

The fundamental angular frequency $\Omega = \frac{2\pi}{N}$ (radians)

Any DTS which doesnot satisfies equa^n(2) is called non periodic signal or Aperiodic signal.

Eg. periodic signal with N = 4





→ Non periodic signal.

(9)

Q: For the following signals, determine whether it is periodic or not. If it is periodic find the fundamental period.

(i) $x(t) = \cos^2(2\pi t)$ (ii) $x(t) = \sin^3(2t)$ (iii) $x(t) = e^{-2t}\cos(2\pi t)$

(iv) $x[n] = (-1)^n$ (v) $x[n] = (-1)^{n^2}$ (vi) $x[n] = \cos(2n)$

(vii) $x[n] = \cos(2\pi n)$.

Soln:- $x(t) = \cos^2(2\pi t)$

$\{\because \cos 2\theta = 1 + 2\cos^2\theta$ (2cos²θ − 1)

$\cos^2\theta = \frac{1}{2}[1 + \cos 2\theta]\}$

$\theta = 2\pi t$

$$x(t) = \frac{1}{2}[1 + \cos 4\pi t] = \underbrace{1/2}_{\text{constant}} + 1/2 \cos 4\pi t$$

↳ cosine signal with amplitude of 1/2

comparing $\cos 4\pi t$ with $\cos \omega_0 t$; $\omega_0 = 4\pi$

$$\frac{2\pi}{T} = 4\pi$$

$$\boxed{T = \frac{2\pi}{4\pi} = 0.5 \text{ sec}}$$

Verification: $x(t+T) = x(t)$: $T = 0.5$ sec

$$x(t+0.5) = \cos^2(2\pi(t+0.5))$$

$$= \frac{1}{2}[1 + \cos 4\pi(t+0.5)] = \frac{1}{2} + \frac{1}{2}\underbrace{\cos(4\pi t + 2\pi)}_{\cos(A+B)}$$

WKT

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$x(t+T) = \frac{1}{2} + \frac{1}{2}[\cos 4\pi t \overbrace{\cos 2\pi}^{1} - \sin 4\pi t \overbrace{\sin 2\pi}^{0}]$$

$$x(t+T) = \frac{1}{2} + \frac{1}{2}\cos 4\pi t = \frac{1}{2}[1 + \cos 2(2\pi t)] = \cos^2(2\pi t) = x(t)$$

Signal $x(t)$ is periodic with fundamental period $T = 0.5$ secs.

i) $x(t) = \sin^3(2t)$

$\theta = 2t$

$\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

$$x(t) = \frac{1}{4}\left[3\sin 2t - \sin 6t\right]$$

$$= \frac{3}{4}\sin 2t - \frac{1}{4}\sin 6t = \frac{3}{4}\sin 2t - \frac{1}{4}\sin 3(2t)$$

comparing with $\sin\omega_0 t$

$\omega_0 = 2$; $\omega_0 = 2\pi f = \frac{2\pi}{T}$

$T = \frac{2\pi}{2} = \pi$ secs

$\sin(2\pi + \theta) = \sin\theta$

$$x(t+T) = \frac{1}{4}\left[3\sin(2t + 2\pi) - \sin(6t + 6\pi)\right]$$

$$= \frac{1}{4}\left[3\sin 2t - \sin 6t\right] = x(t)$$

$x(t)$ is periodic with $T = \pi$ secs.

i) $x(t) = e^{-2t}\cos 2\pi t$

Comparing e^{-2t} with $e^{j\omega t}$ there is no 'j' term it is an Aperiodic signal.

comparing with $\cos\omega_0 t$ ~~with~~

$\omega_0 = 2\pi$

$T = \frac{2\pi}{\omega_0} = 1$ secs.

Product of periodic & Non periodic signals is a Non periodic signal

Hence $x(t)$ is Non periodic.

(10)

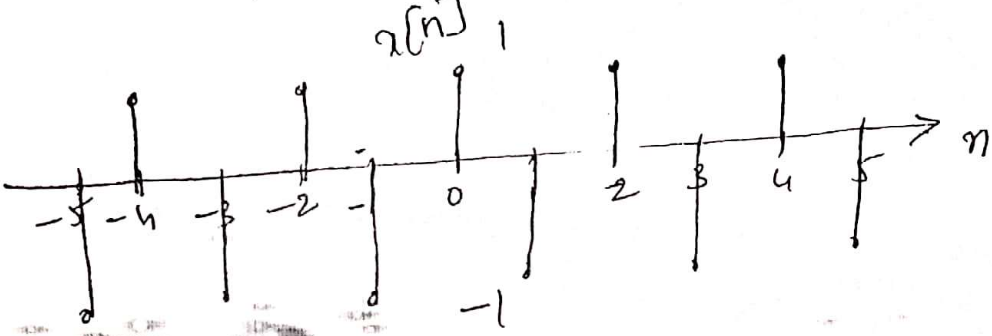
5 a) $x[n] = (-1)^n$; $\because \cos n\pi = (-1)^n$

$\Omega_0 = \pi$ $\quad \cos \Omega_0 n$

$x[n] = \begin{cases} 1 & ; n = \text{even} \\ -1 & ; n = \text{odd} \end{cases}$ $\quad \Omega_0 = \frac{2\pi}{N}$

$\boxed{N = 2}$

The signal $x[n]$ is plotted as below



$x[n]$ is periodic with 2 samples.

i) $x[n] = (-1)^{n^2}$;

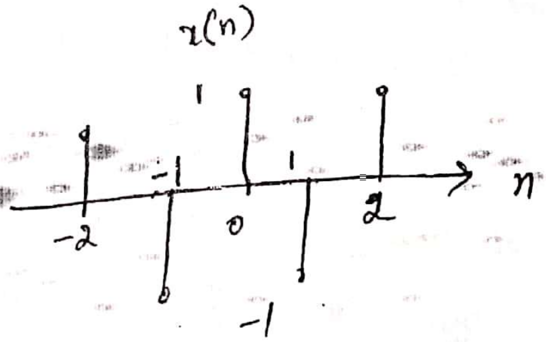
$n=0$; $x[n] = 1$

$n=1$; $x[n] = -1$

$n=2$; $x[n] = 1$

$n=-1$; $x[n] = -1$

$n=-2$; $x[n] = 1$



$\underline{N = 2 \text{ samples}}$.

$x[n]$ is periodic.

2-02-2017

i) $x[n] = \cos(2n)$

$\Omega_0 = 2$. $\longrightarrow$ We cannot express Ω_0 (ratio of integers) is rational multiple of 2π $\quad \Omega_0 = 2\pi \frac{m}{N}$.

$N = \frac{2\pi}{\Omega_0} = \pi$.

Hence $x[n]$ is non periodic.

ii) $x[n] = \cos(2\pi n)$ $\quad \Omega_0 = 2\pi \cdot \underbrace{\frac{1}{1}}_{\text{rational}}$; $\boxed{N = \frac{2\pi}{2\pi} = 1}$

Comparing with $\cos \Omega_0 n$ $\Rightarrow$

$x[n]$ is periodic

check for the periodicity

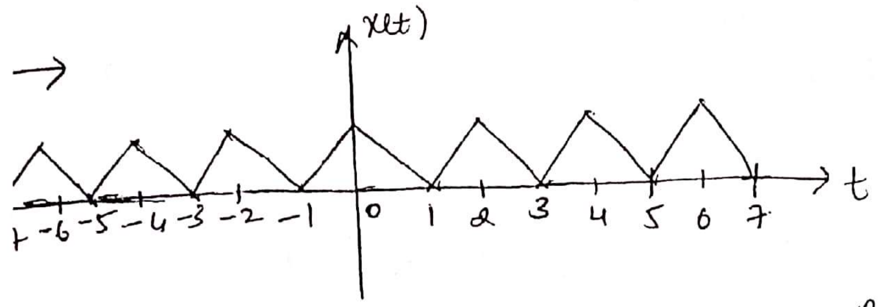
→ $x(n) = \cos\left(\frac{2\pi n}{7} + 2\right)$ → periodic $N = 7$ samples

~~$x(n) = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{5}\pi n)$~~

→ $x(t) = 2\cos(3t + \pi/4)$ — periodic with $T = \frac{2\pi}{3}$ sec.

→ $x(t) = e^{j\pi t}$ → periodic with $T = 2$ sec

→



Non periodic since t has the values from -7 to 7
to be periodic t must be $t = -\infty$ to $t = \infty$.

, Determine whether the CTS $x(t) = x_1(t) + x_2(t) + x_3(t)$ is periodic
where $x_1(t)$, $x_2(t)$ and $x_3(t)$ have periods of $8/3$, 1.26 & $\sqrt{2}$ sec resp.

soln :- $x_1(t)$, $x_2(t)$ & $x_3(t)$ are periodic signals. To find whether
summation of these signals are periodic or not

i) obtain the ratio T_1/T_i & convert it into a ratio of integers (rational)

where T_1: fundamental period of $x_1(t)$
T_i: fundamental period of $x_i(t)$ where $2 \leq i \leq M$

ii) If this conversion is not possible, then the sum signal is not periodic.

ii) If possible, then find gcd (greatest common divisor) from the numerator & denominator of each individual ratio

) Then find LCM of the denominators of the resulting ratios say it is 'l'

i) Then the period of the sum signal $T = T_1 \cdot l$

given : $x(t) = x_1(t) + x_2(t) + x_3(t)$
$T_1 = 8/3$ sec $T_2 = 1.26$ sec $T_3 = \sqrt{2}$ sec.

$$= \frac{8/3}{1.26} = \frac{8}{3.78} = \frac{800}{378} = \frac{400}{189} \Rightarrow \text{rational}$$

$$\frac{T_1}{T_3} = \frac{8/3}{\sqrt{2}} = \frac{8}{3\sqrt{2}} \rightarrow \text{not rational}$$

Hence $x(t)$ is not periodic.

→ $y(t) = y_1(t) + y_2(t) + y_3(t)$ with $T_1 = 1.08$ secs $T_2 = 3.6$ secs &

$T_3 = 2.025$ secs

step1

$$\frac{T_1}{T_2} = \frac{1.08}{3.6} = \frac{108}{360} = \frac{3}{10} \rightarrow \text{rational}$$

$$\frac{T_1}{T_3} = \frac{1.08}{2.025} = \frac{1080}{2025} = \frac{8}{15} \rightarrow \text{rational}$$

step2: $y(t)$ is periodic

step3: $\frac{T_1}{T_2} = \frac{3}{10}$; $\frac{T_1}{T_3} = \frac{8}{15}$

gcd of $(3, 8) = 1$

gcd of $(10, 15) = 5$

step 4: lcm of denominators $10, 15 = 30$

step5: Period of the sum signal $y(t) \Rightarrow T = T_1 \cdot l$

$= 1.08 \times 30$

$\therefore \boxed{T = 32.4 \text{ sec}}$

imp/ July 2013

→ $x(n) = \cos\left(\frac{1}{5}\pi n\right) \sin\left(\frac{1}{3}\pi n\right)$.

$N_1 = 15$ $N_2 = 15$ $\sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$

$\therefore \boxed{N = 15}$

→ Determine whether the following signal is periodic or not:

$x(n) = \cos\left(\frac{1}{3}\pi n\right)\sin(3n)$ → Non periodic since Ω_2 cannot be expressed as rational multiple of 2π

$\downarrow$

periodic with period $N=6$ samples

Hence $x(n)$ is a non-periodic signal

→ Find the periodicity of the signal.

$$x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$$

Soln:- $N_1 = 5 \qquad N_2 = 7$

$\frac{N_1}{N_2} = \frac{5}{7}$ (rational)

Lcm of the denominator $l = 7$

Fundamental period $N = N_1 \cdot l = 5 \times 7 = 35$

Dec 2013.

→ (i) $x(n) = \cos(20\pi n) + \sin(50\pi n)$ $\qquad$ (ii) $x(t) = [\cos(2\pi t)]^2$

$\downarrow \qquad\qquad \downarrow$

$\Omega_1 = 20\pi \qquad \Omega_2 = 50\pi n$

$2\pi \frac{m_1}{N_1} = 2\pi \frac{10}{1} \qquad 2\pi \frac{m_2}{N_2} = 2\pi \frac{25}{1}$

$\boxed{N_1 = 1} \qquad \boxed{N_2 = 1}$

$\frac{N_1}{N_2} = 1$

Lcm $= 1$

$\boxed{N = 1}$

Dec 2015

→ $x(n) = \cos\left(\frac{\pi n}{8}\right)\sin\left(\frac{\pi n}{4}\right)$ $\qquad$ (ii) $x(t) = x_1(t) + x_2(t) + x_3(t)$ for x_1, x_2 & x_3 with fundamental periods of 3.2, 9.6 & 12.8 secs respectively

June 2014.

→ $x(n) = \cos\left(\pi/8 \; n^2\right)$ $\longrightarrow$ $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}$

→ continuous time sig.

-2017

Determinitic & Random signals :→ A Deterministic signal is a signal about which there is no uncertainity wrt its value at anytime. The deterministic signal behaves in a fixed known way wrt time.

Ex: square wave, rectangular wave etc

Random signal takes on one of several possible value at each time for which signal value is defined. The signal has uncertainity wrt its value at any time. The existence of random signal in a system is a random process which requires probabilistic models.

Eg. ECG signal, noise generated in the amplifier of radio receiver.

Energy & Power signals :→ In electrical system, a signal may represent voltage or current. Consider a voltage $V(t)$ developed across resistor R producing a current $i(t)$. The instantaneous power dissipated in resistor is

$$P(t) = \frac{V^2(t)}{R} \quad \text{or} \quad i^2(t)R$$

In both the case, the instantaneous power is proportional to square of amplitude of signal. In signal analysis, power is defined for 1Ω resistor regardless of whether a given signal $x(t)$ represents a voltage or a current. The instantaneous power is expressed as

$$P(t) = x^2(t)$$

The total energy of continuous time signal $x(t)$:

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt$$

If $x(t)$ is complex, then $E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$

The average power of continuous time signal $x(t)$ is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt$$

The average power of a periodic CTS $x(t)$ of fundamental period T

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

lly
or discrete time signals :

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The average Power of DTS $x(n)$ is given by

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

The average power of periodic DTS $x(n)$ is given by

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n).$$

The signal is refered to as an energy signal if the total energy E of the signal satisfies the condition:

$0 < E < \infty$ [ie E must be finite]

It is referred as power signal if the average power P of the signal satisfies the condition.

$0 < P < \infty$ [ie P must be finite]

Examples for power s/gs:

- all periodic signals
- Random signals

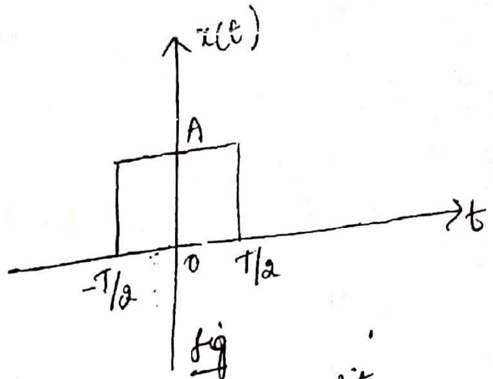
amples for Energy S/g: • Both deterministic & non periodic.

...u is the total energy of the rectangular pulse shown in fig.

...:- we have

$$x(t) = A \quad ; \quad -T/2 < t < T/2 .$$

$$= 0 \quad ; \quad \text{elsewhere}$$



fig

...he total energy of a CTS is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

∴ x(t) has non zero value in the range $-T/2 < t < T/2$ we can write

$$E = \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \int_{-T/2}^{T/2} |A|^2 \, dt = A^2 \, t \Big|_{-T/2}^{T/2} = A^2 \left[T/2 + T/2 \right]$$

$$\boxed{E = A^2 . T}$$

What is the average power of the square wave shown in figure.

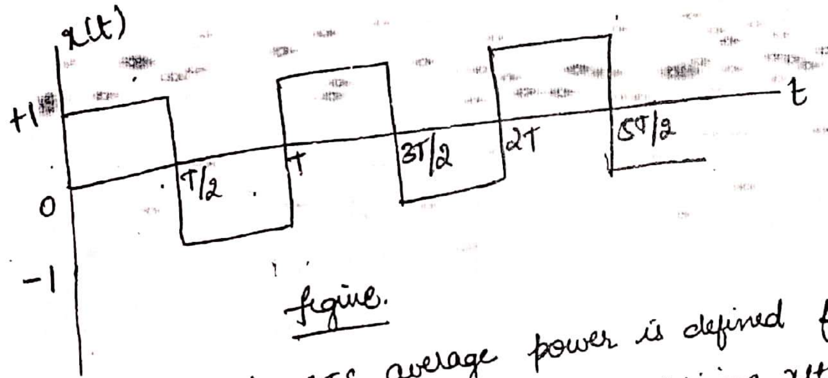


figure.

...ln: In a periodic CTS average power is defined for one cycle. The period of the signal is T. Mathematically defining x(t) for one cycle:

$$x(t) = +1 \quad ; \quad 0 < t < T/2$$

$$= -1 \quad ; \quad T/2 < t < T$$

Average power $P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt$

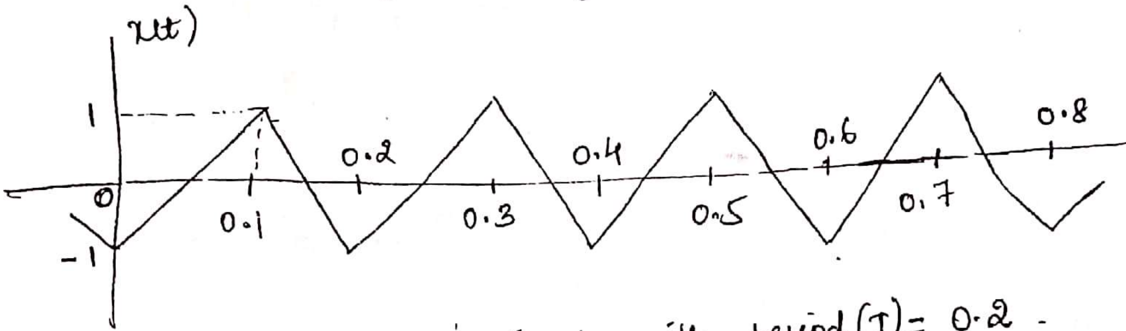
Integration must be carried out for one complete cycle

$$P = \frac{1}{T} \int_0^T x^2(t) . dt = \frac{1}{T} \left[\int_0^{T/2} x^2(t) \, dt + \int_{T/2}^{T} x^2(t) . dt \right]$$

$$P = \frac{1}{T}\left[\int_0^{T/2} 1^2\,dt + \int_{T/2}^{T} (-1)^2\,dt\right] = \frac{1}{T}\left[\frac{T}{2} + \frac{T}{2}\right] = \frac{T}{T}$$ 13B

$$\boxed{P = 1}$$

what is the average power of the rectangular wave shown in figure



Sun:- It is a periodic signal with period $(T) = 0.2$.

Average Power $P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$

integration must be carried out for one complete cycle.

$x(t) = 20t - 1$; $0 < t < 0.1$

$= -20t + 3$; $0.1 < t < 0.2$

on points (2, 3)

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\frac{x(t) - 1}{1 + 1} = \frac{t - 0.1}{-0.1} \Rightarrow x(t) - 1 = \frac{2t - 0.2}{-0.1}$$

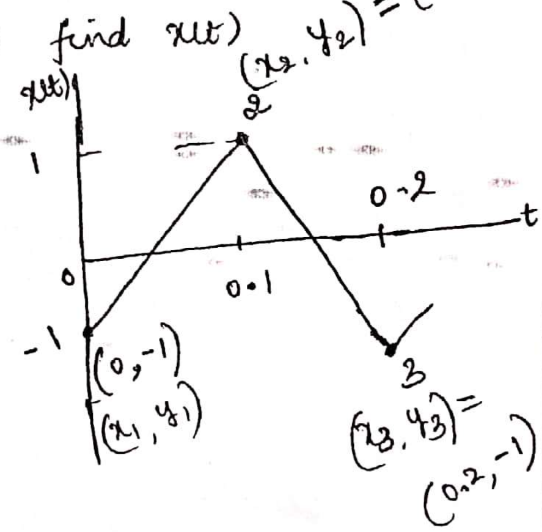
$x(t) - 1 = -20t + 2$

$\boxed{x(t) = -20t + 3}$; $0.1 < t < 0.2$

verage power

$$= \frac{1}{0.2}\left[\int_0^{0.1} (20t - 1)^2\,dt + \int_{0.1}^{0.2} (-20t + 3)^2\,dt\right.$$

To find x(t)



From points (1, 2). the straight line equation is

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\frac{x(t) + 1}{-1 - 1} = \frac{t - 0}{-0.1}$$

$$x(t) + 1 = \frac{-2t}{-0.1} = 20t$$

$\boxed{x(t) = 20t - 1}$; $0 < t < 0.1$

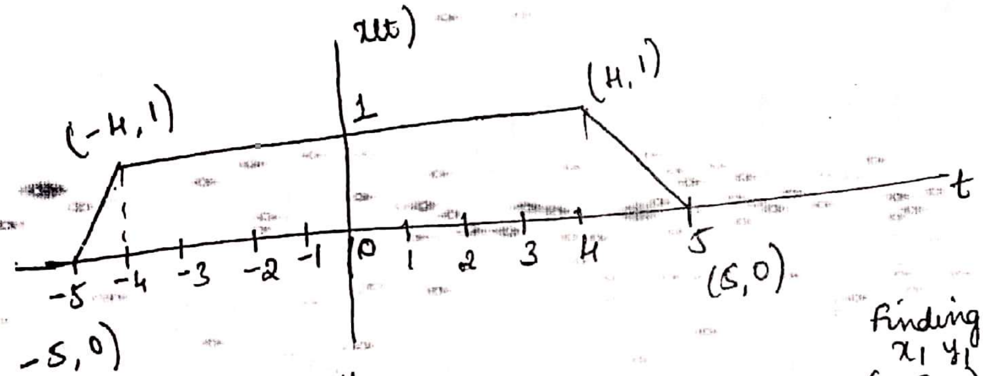
$$= \frac{1}{0.2}\left[\int_0^{0.1}(400t^2+1-40t)\,dt + \int_{0.1}^{0.2}(400t^2+9-120t)\,dt\right]$$

$$= \frac{1}{0.2}\left[\underbrace{\frac{400\,t^3}{3}\Big|_0^{0.1} + t\Big|_0^{0.1} - \frac{40\,t^2}{2}\Big|_0^{0.1}}_{} + \frac{400}{3}\,t^3\Big|_{0.1}^{0.2} + 9t\Big|_{0.1}^{0.2} - \frac{120\,t^2}{2}\Big|_{0.1}^{0.2}\right]$$

$$\frac{1}{0.2}\left[0.0333 + 0.0333\right] \quad ; \quad \cancel{P = 2.33}$$

$$\boxed{P = 0.333 \text{ or } 1/3} \quad ;$$

? For the trapezoidal pulse $x(t)$ find the total energy:



, $\overset{x_1\,y_1}{(-4,1)}$ & $\overset{x_2\,y_2}{(4,1)}$

$$\frac{x(t)-1}{0} = \frac{t+4}{-8}$$

$$\boxed{x(t) = 1} \quad -4 \le t \le 4$$

, $(4,1)$ $(5,0)$

$$\frac{x(t)-1}{1} = \frac{t-4}{-1}$$

$$x(t) - 1 = -t + 4$$

$$\boxed{x(t) = 5-t} \; ; \; 4 \le t \le 5$$

finding equations:

$\rightarrow$ $\overset{x_1\,y_1}{(-5,0)}$ & $\overset{x_2\,y_2}{(-4,1)}$

$$\frac{x(t)}{\cancel{1}} = \frac{t+5}{\cancel{1}}$$

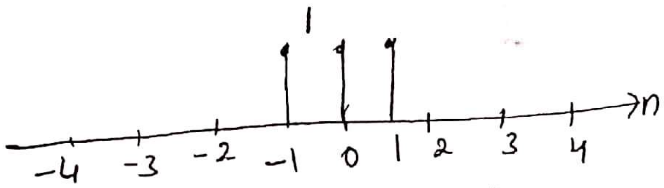
$$\boxed{x(t) = t+5} \; ; \; -5 \le t \le -4$$

$$x(t) = \begin{cases} 5-t & ; \; 4 \le t \le 5 \\ 1 & ; \; -4 \le t \le 4 \\ t+5 & ; \; -5 \le t \le -4 \\ 0 & ; \; \text{otherwise} \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x^2(t)|^2 dt = \int_{-5}^{5} |x(t)|^2 dt = \int_{-5}^{-4} (t+5)^2 dt + \int_{-4}^{4} dt + \int_{4}^{5} (5-t)^2 dt$$

$$\boxed{= 26/3}$$

- What is the total energy of the discrete time signal $x(n)$ 14B

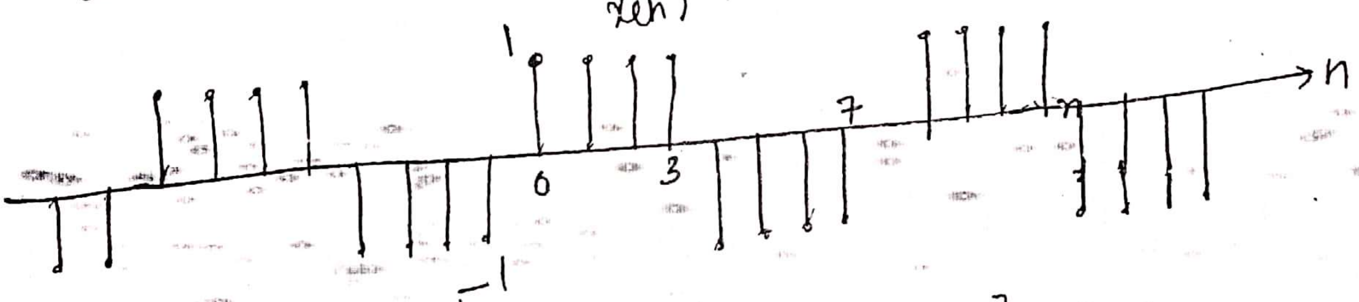


Solun -

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-1}^{1} |x(n)|^2 = 1^2 + 1^2 + 1^2 = 3.$$

8-02-2017

- What is the average power of the periodic DTS shown in figure.



Solun:-

$N = 8$

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n) = \frac{1}{8} \sum_{n=0}^{7} x^2(n)$$

$$= \frac{1}{8} \left[x^2(0) + x^2(1) + x^2(2) + x^2(3) + x^2(4) + x^2(5) + x^2(6) + x^2(7) \right]$$

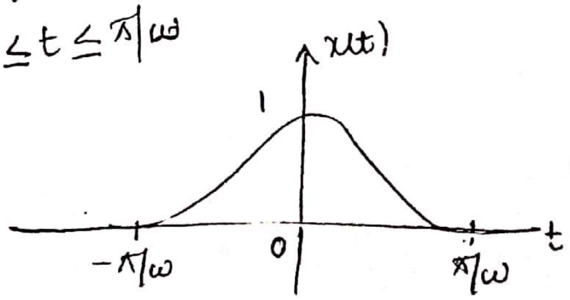
$$P = \frac{1}{8} \left[1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 \right]$$

$$\boxed{P = 1}$$

- The raised cosine pulse $x(t)$ shown in fig. is given by

$$x(t) = \tfrac{1}{2}\left[\cos(\omega t) + 1\right] \quad ; \; -\pi/\omega \le t \le \pi/\omega$$

$$= 0 \quad ; \; \text{otherwise}$$



Find the total energy of $x(t)$

(1.5)

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\pi/\omega}^{\pi/\omega} \left|\frac{1}{2}[\cos(\omega t)+1]\right|^2 dt = \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega} (\cos^2\omega t + 1 + 2\cos\omega t)\, dt$$

$$= \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega} \left(\frac{1}{2} + \frac{1}{2}\cos 2\omega t + 1 + 2\cos\omega t\right) dt \; ; \quad \cos^2\theta = \frac{1}{2}[1+\cos 2\theta]$$

$$\boxed{= \frac{3\pi}{4\omega}}$$

(Dec 2012)

Check whether the following signal $x(n)$ is energy or power signal & find the corresponding value:

$$x(n) = \begin{cases} n & ; \; 0 \le n \le 5 \\ 10-n & ; \; 5 < n \le 10 \\ 0 & ; \; \text{otherwise.} \end{cases}$$

$$E = \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2$$

$$= [(0+1^2+2^2+3^2+4^2+5^2) + \{(10-6)^2 + (10-7)^2 + (10-8)^2 + (10-9)^2\}]$$

$$\boxed{E = 85} < \infty$$

$\therefore$ $x(n)$ is energy signal.

Check whether the following are energy or power signals? Also find the corresponding value.

(i) $x_1(n) = \cos(\pi n)$; $-4 \le n \le 4$
$= 0$; otherwise

(ii) $x_2(n) = \cos \pi n$; $n \ge 0$
$= 0$; otherwise

Soln:- $x_1(n) = \cos(\pi n)$; $-4 \le n \le 4$
$= 0$; otherwise

Since the given is a non periodic signal it is an energy signal

$$E_1 = \sum_{n=-\infty}^{\infty} |x_1(n)|^2 = \sum_{n=-4}^{4} \cos^2 \pi n = \sum_{n=-4}^{4} (-1)^{2n}$$

Put $m = n+4$; $E_1 = \sum_{m=0}^{8} (-1)^{2(m-4)} = (-1)^{-8} \sum_{m=0}^{8} (-1)^{2m} = 1 \cdot \sum_{m=0}^{8} 1$

$\boxed{E_1 = 9}$; $\left(\sum_{n=0}^{N-1} 1 = N \right)$

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i) $x_2(n) = \cos(\pi n)$; $n \geq 0$

$= 0$; otherwise

The given signal is of infinite length, so if we calculate energy it is also infinite. Therefore it is a power signal. It repeats for every 2 samples

$N = 2$; $P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{2}\sum_{n=0}^{1} (\cos \pi n)^2 = \frac{1}{2}\sum_{n=0}^{1} (-1)^{2n}$

$$P = \frac{1}{2}\sum_{n=0}^{1} 1 = \frac{1}{2}(2) = 1$$

The trapezoidal pulse $x(t) = 5 - t$; $4 \leq t \leq 5$
$= 1$; $-4 \leq t \leq 4$
$= t+5$; $-5 \leq t \leq -4$
$= 0$; otherwise

is applied to a differentiator defined by $y(t) = \frac{d}{dt}x(t)$

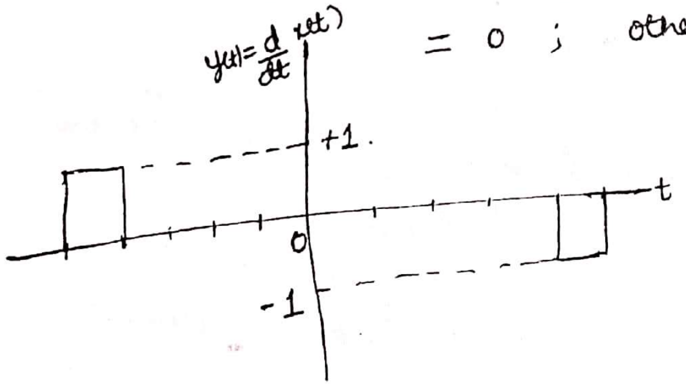
1) Find the resulting output $y(t)$ of the differentiator.

ii) Find the total energy of $y(t)$.

soln:- $y(t) = \frac{d}{dt}x(t) = -1$; $4 \leq t \leq 5$

$= 1$; $-5 \leq t \leq -4$

$= 0$; otherwise



Energy $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$; It is having non zero value only in the ranges $4 \leq t \leq 5$ & $-5 \leq t \leq -4$

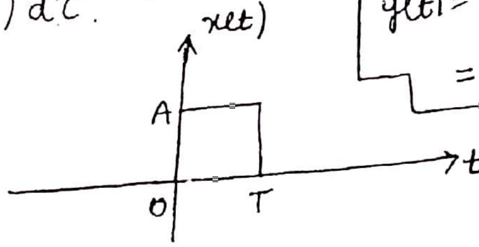
$$= \int_{4}^{5} (-1)^2 \, dt + \int_{-5}^{-4} (1)^2 \, dt = t\Big|_4^5 + t\Big|_{-5}^{-4} = 1+1$$

$\boxed{E = 2}$

...gular pulse $x(t)$ is defined by $x(t) = \begin{cases} A ; & 0 \le t \le T \\ 0 ; & \text{otherwise} \end{cases}$ is applied

... integrator defined by $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$.

...d the total energy of the o/p $y(t)$



$$y(t) = \int_0^t A \cdot dt = A \cdot t$$

$y(t) = A \cdot t$

$$\boxed{E = \frac{A^2 T^3}{3}}$$

Basic or Elementary signals :→ Basic continuous time signals

Some of the important elementary basic signals are

(i) Exponential signal

(ii) Sinusoidal signals

(iii) Exponentially damped sinusoidal signals

(iv) unit step function

(v) unit Impulse function

(vi) unit ramp function.

Exponential signals :→ A real exponential CTS is given by $x(t) = Ce^{at}$ → (1)

C & a are real constant

C is called amplitude of exponential signal at $t=0$

If $a<0$ then $x(t)$ is called decaying exponential signal

$a>0$ then $x(t)$ is called growing exponential signal.

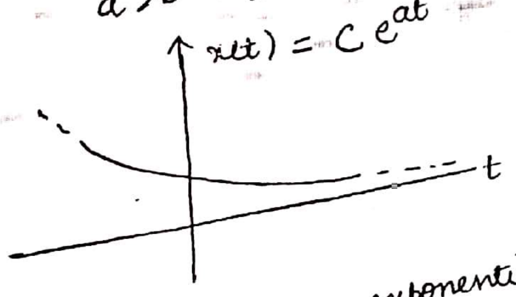


fig: decaying exponential ($a<0$)

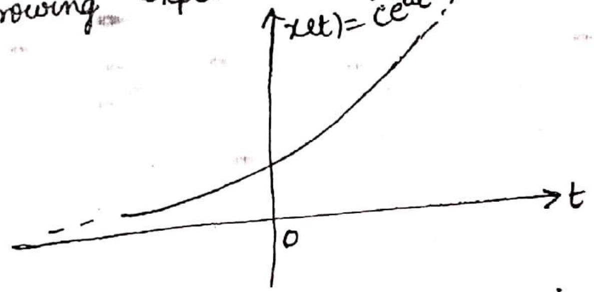


fig: Growing exponential ($a>0$)

If 'c' or 'a' or both are complex numbers then $x(t)$ is known as CT complex exponential signal.

$C=1$ & 'a' is imaginary

ie $x(t) = e^{j\omega_0 t}$ → (2)

Equation (2) is periodic with fundamental period $T = \frac{2\pi}{\omega_0}$

• $e^{j\omega_0 t}$ & $e^{-j\omega_0 t}$ are periodic with fundamental period $T = \frac{2\pi}{\omega_0}$

Sinusoidal signals :→ It is given by $\boxed{x(t) = A\cos(\omega_0 t + \phi)}$

$\omega_0 = 2\pi f_0$ = angular frequency (rad/sec)

f_0 = linear frequency (Hz)

ϕ = phase shifts (radians)

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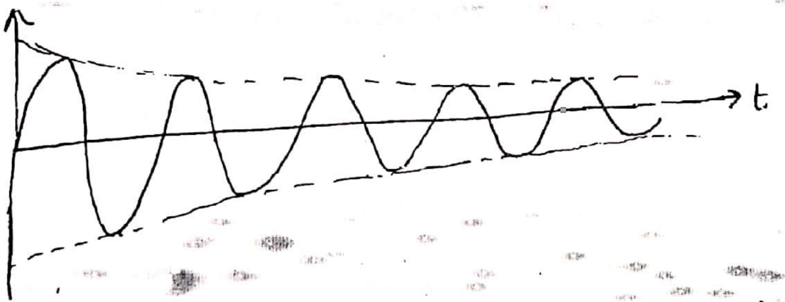
fundamental period $T = \frac{2\pi}{\omega_0}$

Exponentially damped sinusoidal signal :→ It is given by

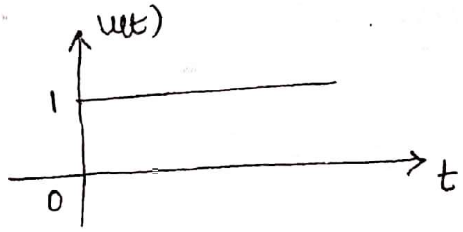
$$\boxed{x(t) = e^{-at}\sin\omega t} \; ; \; a > 0$$

As t increases the amplitude of sinusoidal oscillation decreases exponentially & approaches zero at $t \to \infty$. An exponentially damped sinusoidal signal is shown below



unit step function :→ A CT unit step function is defined as

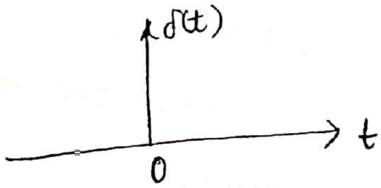
$$u(t) = \begin{cases} 1 ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}$$



Unit Impulse function :→ The CT unit impulse function $\delta(t)$ is defined as

$$\delta(t) = 0 \quad ; \; t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$



$\delta(t)$ is called dirac delta function

The impulse function $\delta(t)$ is derivative of step function wrt time

The area covered by an unit impulse function is unity.

01-03-2017

Basic Discrete time signal :→ (i) Exponential signal
(ii) Sinusoidal signal
(iii) Exponentially damped sinusoidal signal
(iv) unit step sequence
(v) unit impulse sequence
(vi) unit ramp sequence.

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Exponential signal :→ In discrete time, a real exponential signal is written as

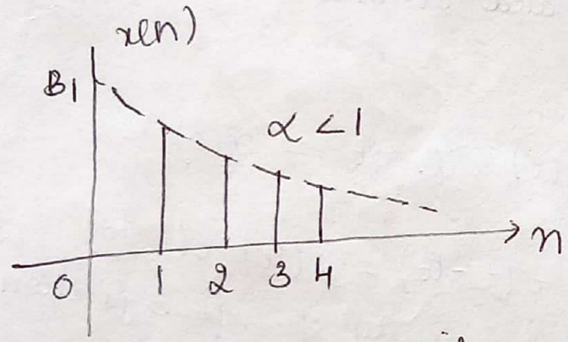
$$x(n) = c\,\alpha^{n} \quad \text{where} \quad \alpha = e^{\beta}$$

c, α & β are real constants

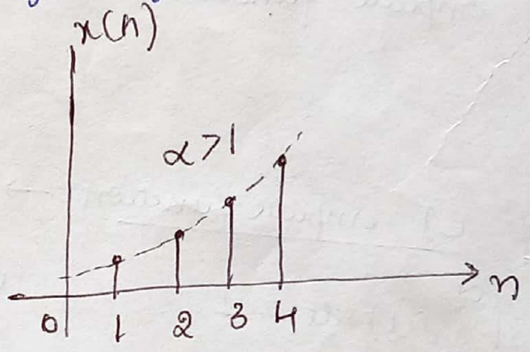
c is known as amplitude of sequence at $n = 0$

if $|\alpha| < 1$ then signal decays exponentially

if $|\alpha| > 1$ then signal is called growing exponential sequence



Decaying exponential



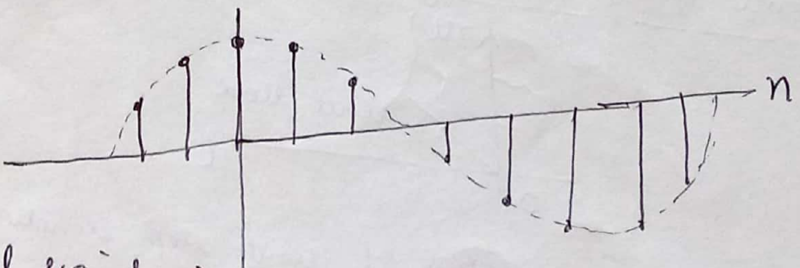
Growing exponential

Sinusoidal signal :→ $x(n) = A\cos[\Omega_0 n + \phi]$

Ω_0 is angular frequency ; ϕ is Phase angle

For discrete time sinusoidal signal to be periodic, angular frequency Ω_0 & ϕ must be rational multiple of 2π

ie $\Omega_0 = 2\pi \dfrac{m}{N}$; m & N are integers



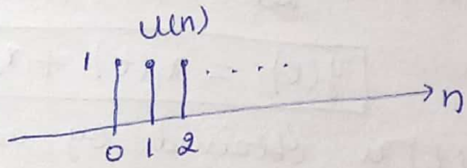
Exponentially damped sinusoidal signal :→

It is given by $x(n) = c\,\alpha^{n}\sin(\Omega n + \phi)$; $0 < \alpha < 1$

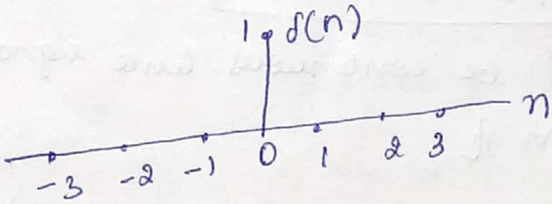
$x(n)$ decreases as 'n' increases

03-03-2017

unit step sequence :→ $u(n) = 1 \; ; \; n \geq 0$
$= 0 \; ; \; n < 0$



Discrete time unit impulse function :→ $\delta(n) = 1; \; n = 0$
$= 0; \; n \neq 0$



<u>Properties</u>: (i) $x(n)\,\delta(n) = x(0)\,\delta(n)$

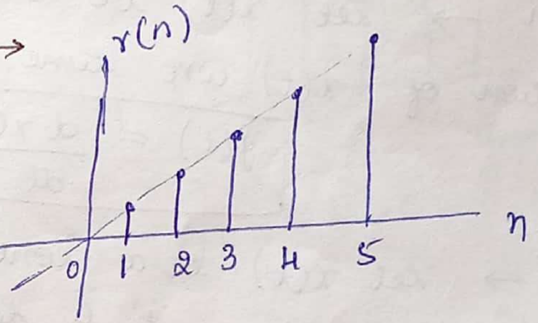
(ii) $x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$

(iii) $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n) = x(0)$

(iv) $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$; shifting property of impulse

Discrete time unit ramp sequence :→

$r(n) = n \; ; \; n \geq 0$
$0 \; ; \; 0$



<u>Basic operations on signals</u> :→ One dimensional signal can be defined using 2 variables (i) Dependent variable
(ii) Independent variable.

Dependent variables corresponds to amplitude or value of signal
Independent variable corresponds to time 't' or 'n'

<u>Operations Performed on dependent variable</u> :→

(i) Amplitude Scaling :→ Let $x(t)$ be a continuous signal then

$$\boxed{y(t) = c\,x(t)}$$

c is called scaling factor
The signal $y(t)$ is obtained by multiplying amplitude of $x(t)$ by scalar 'c' for all values of 't'
Illy for discrete time signal $\boxed{y(n) = c\,x(n)}$

Addition :→ Let $x_1(t)$ & $x_2(t)$ are continuous time signals then,

$$\boxed{y(t) = x_1(t) + x_2(t)}$$

$y(t)$ is obtained by adding the amplitudes of $x_1(t)$ & $x_2(t)$ for all t.

Ily for discrete time signal ; $\boxed{y(n) = x_1(n) + x_2(n)}$

Multiplication :→ Let $x_1(t)$ & $x_2(t)$ be continuous time signal then

$$\boxed{y(t) = x_1(t) \cdot x_2(t)} \quad \forall t$$

$y(t)$ is obtained by multiplying amplitude of $x_1(t)$ & $x_2(t)$ for all t.

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Ily $\boxed{y(n) = x_1(n) \cdot x_2(n)} \quad \forall n$

Differentiation :→ Let $x(t)$ be continuous time signal then differentiation of $x(t)$ wrt time is defined as

$$\boxed{y(t) = \frac{d\,x(t)}{dt}}$$

Integration :→ Let $x(t)$ be a continuous time signal. Then the integration of $x(t)$ wrt to time 't' is defined as

$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau}$$

Operations performed on the independent variables :→

(i) Time scaling :→ Let $x(t)$ be a continuous time signal. The signal $y(t)$ obtained by scaling the independent variable 't' by a factor 'a' is given by

$$\boxed{y(t) = x(at)}$$

If $a > 1$, the signal $y(t)$ is a compressed version of $x(t)$ and if $0 < a < 1$, the signal $y(t)$ is an expanded version of $x(t)$.

Eg: Let $x(t)$ be as shown in fig (a)

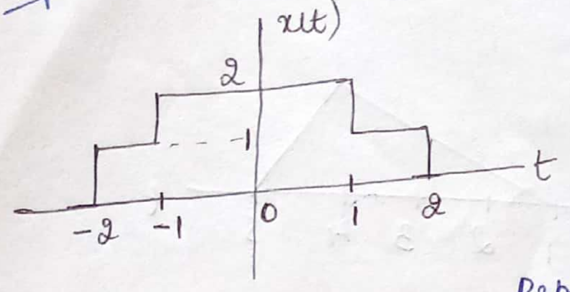


fig (a)

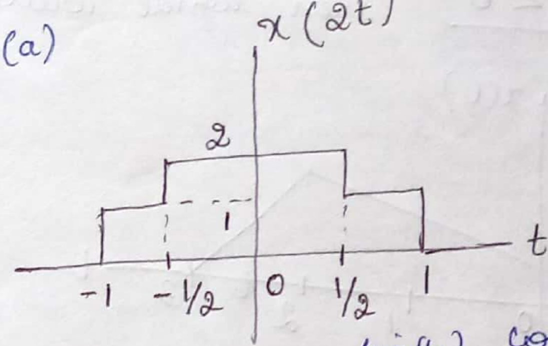


fig (b) compressed version of $x(t)$ by a factor 2.

Replacing t by $2t$

i.e. $2t = 1$; $2t = 2$; $2t = -1$; $2t = -2$

$t = 1/2$; $t = 1$; $t = -1/2$; $t = -1$

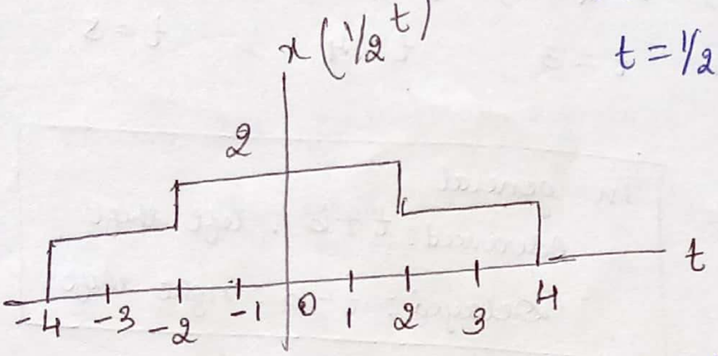
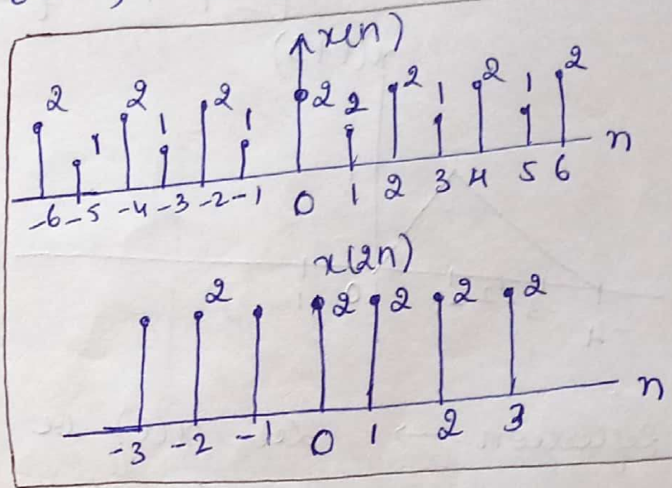


fig (c)

$\frac{1}{2}t = 1$;

$t = 2$;

Expanded version of $x(t)$ by a factor 2



For discrete time sequence $\boxed{y(n) = x(kn)}$; $k > 0$

where 'k' is an integer. If $k > 1$ then some samples of $x(n)$ would be lost

06-03-2017

Eg Let $x(n)$ is as shown in fig (a)

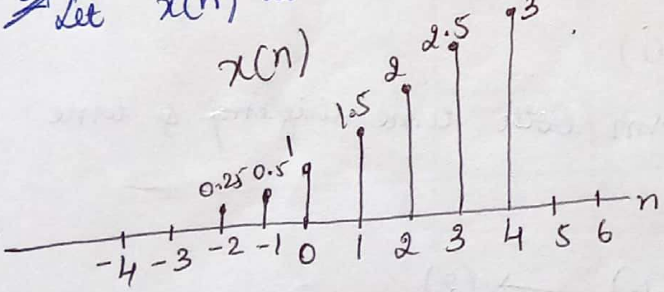


fig (a)

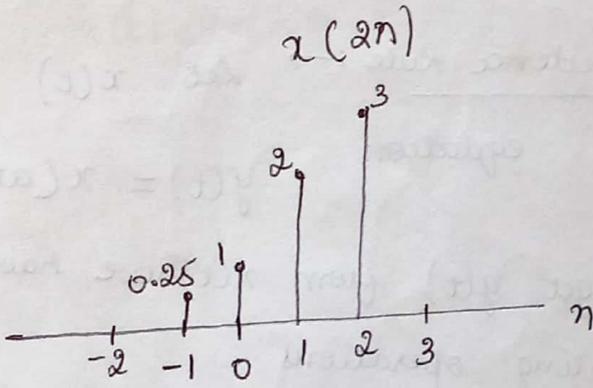


fig (b): compressed version of $x(n)$ by a factor 2.

Time shifting: $\rightarrow$ Let $x(t)$ be a continuous time signal, then

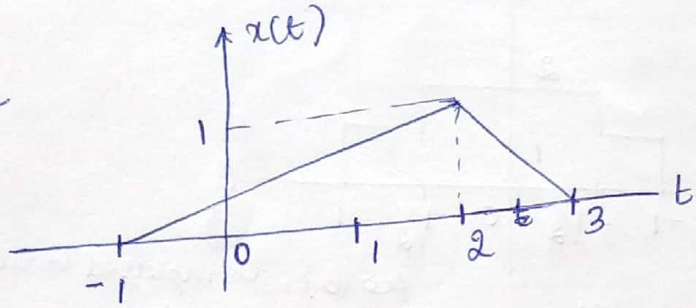
$\boxed{y(t) = x(t - t_0)}$ is called time shifted version of $x(t)$

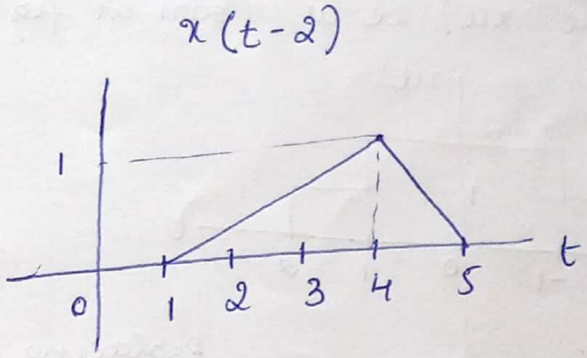
t_0 is the time shift.

If $t > 0$ then signal waveform is shifted to the right

If $t_0 < 0$ then signal waveform is shifted to left

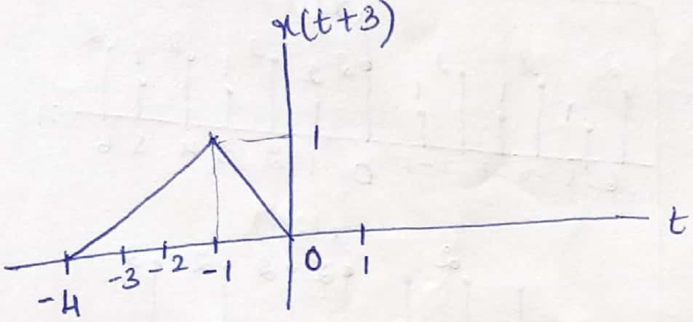
Eg.





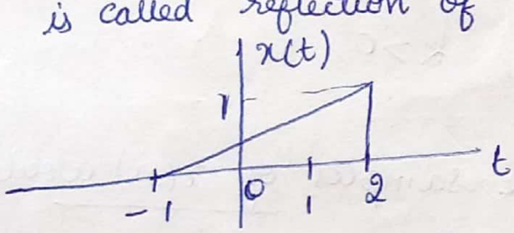
$t-2=-1$; $t-2=0$; $t-2=1$; $t-2=2$; $t-2=3$

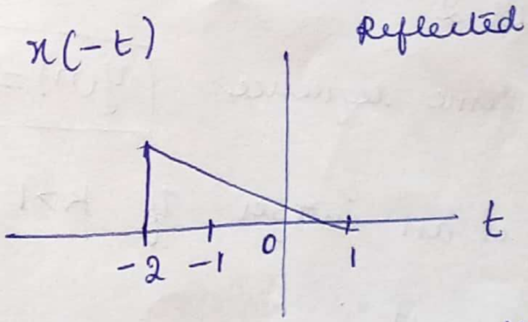
$t=1$; $t=2$; $t=3$; $t=4$; $t=5$



In general
Advanced: $t+b$: left shift
Delayed: $t-b$: right shift.

Reflection :→ Let $x(t)$ be a continuous time signal then $\boxed{y(t)=x(}$ is called reflection of $x(t)$





Reflected version of $x(t)$

Precedence Rule :→ Let $x(t)$ & $y(t)$ are related by the following equation

$$y(t) = x(at-b) \longrightarrow (1)$$

To get $y(t)$ from $x(t)$ we have to perform both time shifting & time scaling operations.

Put $t=0$ in equa^n (1) $\quad y(0) = x(-b) \rightarrow (2)$

Put $t = b/a$ in equa^n (1) $\quad y(b/a) = x(0) \rightarrow (3)$

Once we obtain $y(t)$ from $x(t)$ by performing time shifting & time scaling operations it must satisfy the equation (2) & (3)

This is possible only if the time shifting operations is performed first on $x(t)$ which yields an intermediate signal signal $v(t)$ given by

$$v(t) = x(t-b)$$

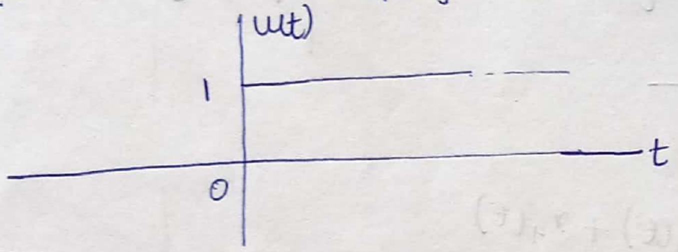
Next, the time scaling operation is performed on $v(t)$ to obtain $y(t)$

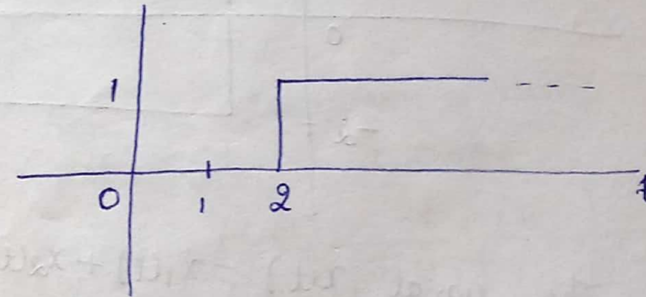
ie $y(t) = v(at) = x(at-b)$

Problems

→ Sketch the following signal : $x(t) = u(t) - u(t-2)$

Soln :- The unit step function $u(t)$ $u(t-2)$





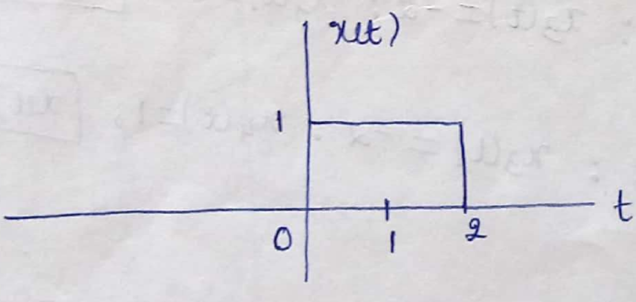
By using time shifting propert

$$x(t) = u(t) - u(t-2)$$

for $t < 0$; $u(t) = u(t-2) = 0$; $x(t) = 0$

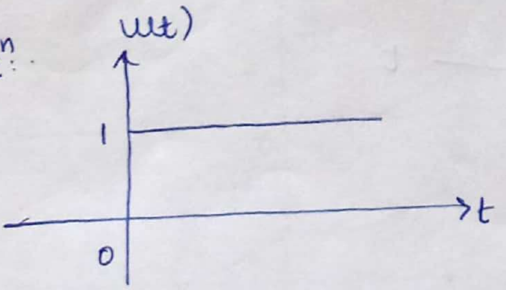
for $0 < t < 2$; $u(t) = 1$; $u(t-2) = 0$; $x(t) = 1 - 0 = 1$

for $t > 2$; $u(t) = 1$; $u(t-2) = 1$; $x(t) = 1 - 1 = 0$



→ Sketch the signal $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3$

Soln.



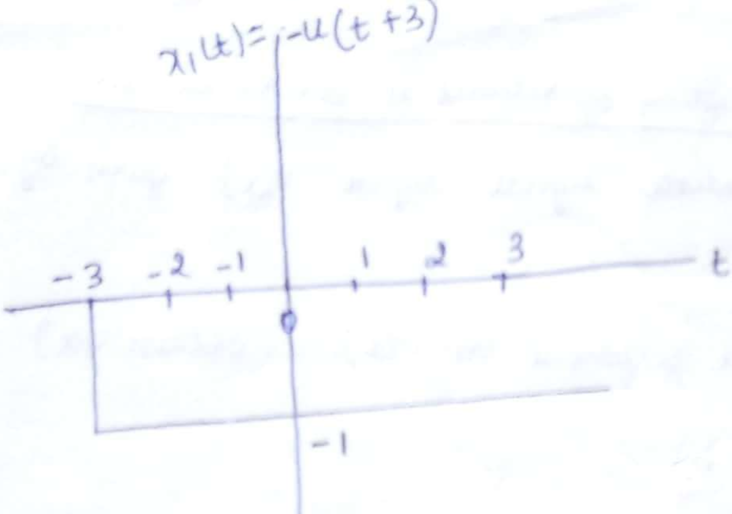
$x_1(t) = -u(t+3)$

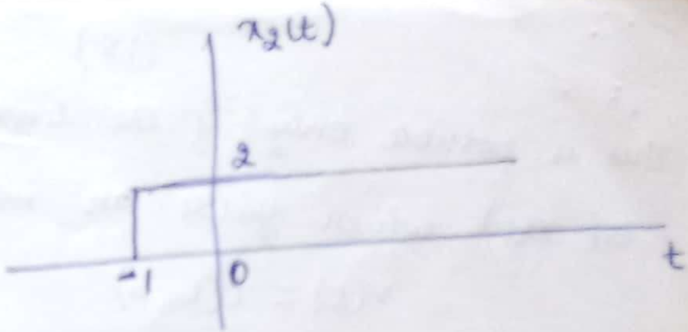
$x_2(t) = 2u(t+1)$

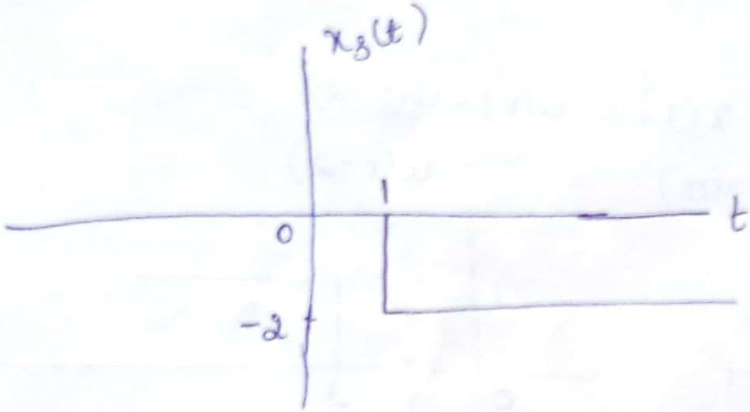
$x_3(t) = -2u(t-1)$

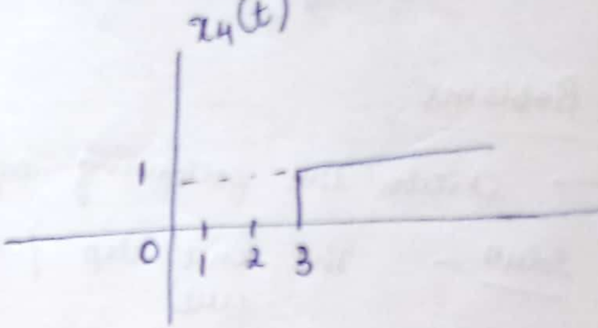
$x_4(t) = u(t-3)$

$x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$









The signal $x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$

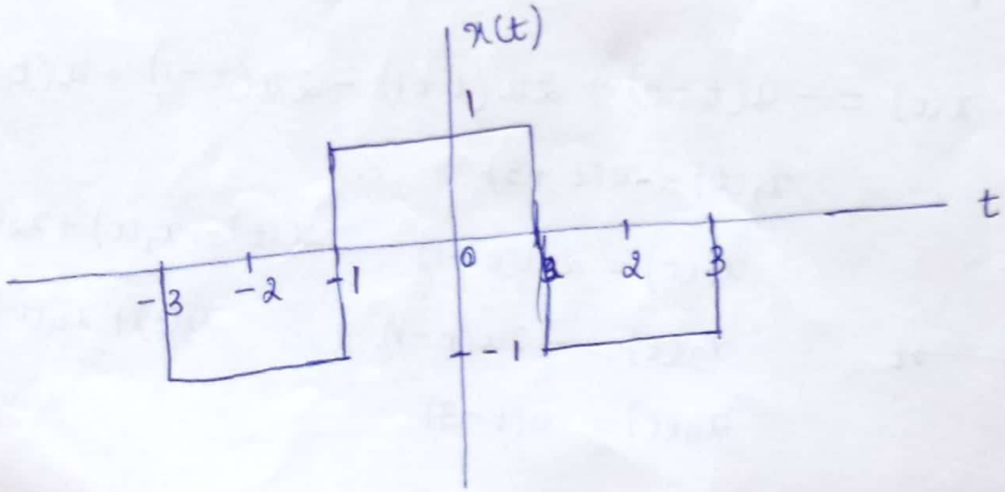
For $t < -3$; $x_1(t) = 0 : x_2(t) = 0 : x_3(t) = 0 : x_4(t) = 0$; $\boxed{x(t) = 0}$

For $-3 < t < -1$; $x_1(t) = -1 ; x_2(t) = 0 : x_3(t) = 0 : x_4(t) = 0$; $\boxed{x(t) = -1}$

For $-1 < t < 1$; $x_1(t) = -1 : x_2(t) = 2 : x_3(t) = 0 : x_4(t) = 0$; $\boxed{x(t) = 1}$

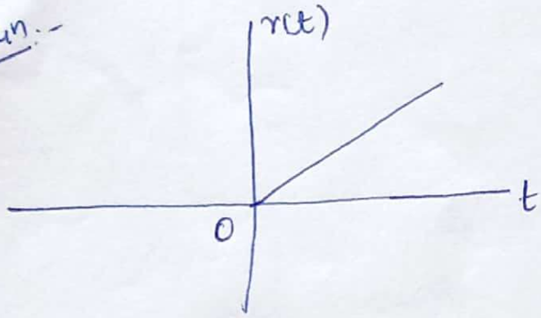
For $1 < t < 3$; $x_1(t) = -1 : x_2(t) = 2 : x_3(t) = -2 ; x_4(t) = 0$; $\boxed{x(t) = -1}$

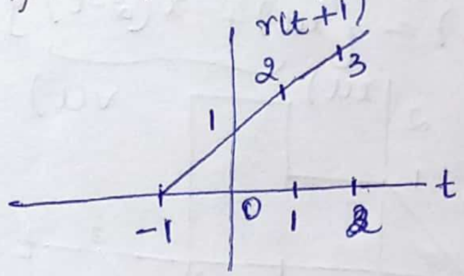
For $t > 3$; $x_1(t) = -1 : x_2(t) = 2 : x_3(t) = -2 : x_4(t) = 1$; $\boxed{x(t) = 0}$

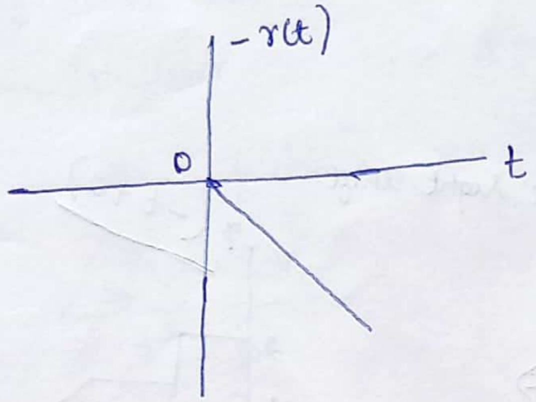


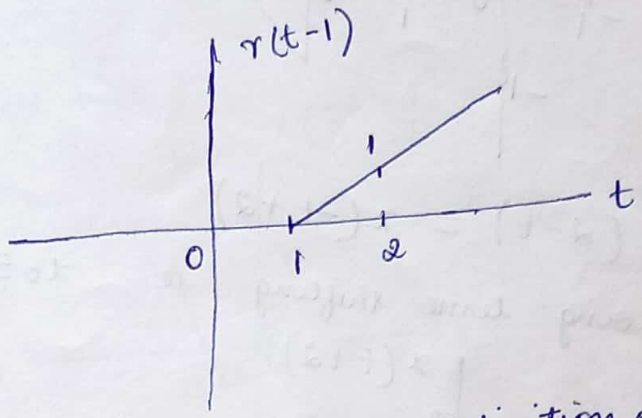
→ Sketch the signal ; $x(t) = r(t+1) - r(t) + r(t-1)$

Soln:-





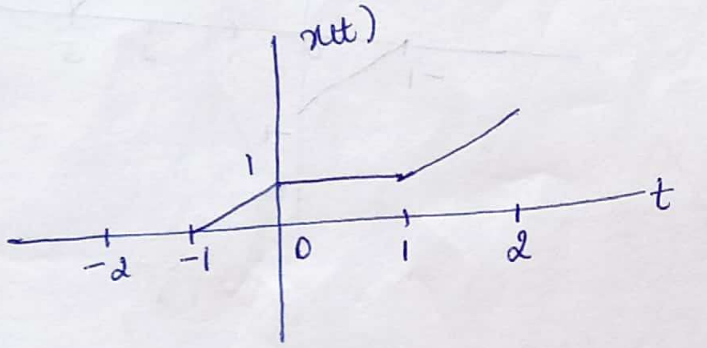




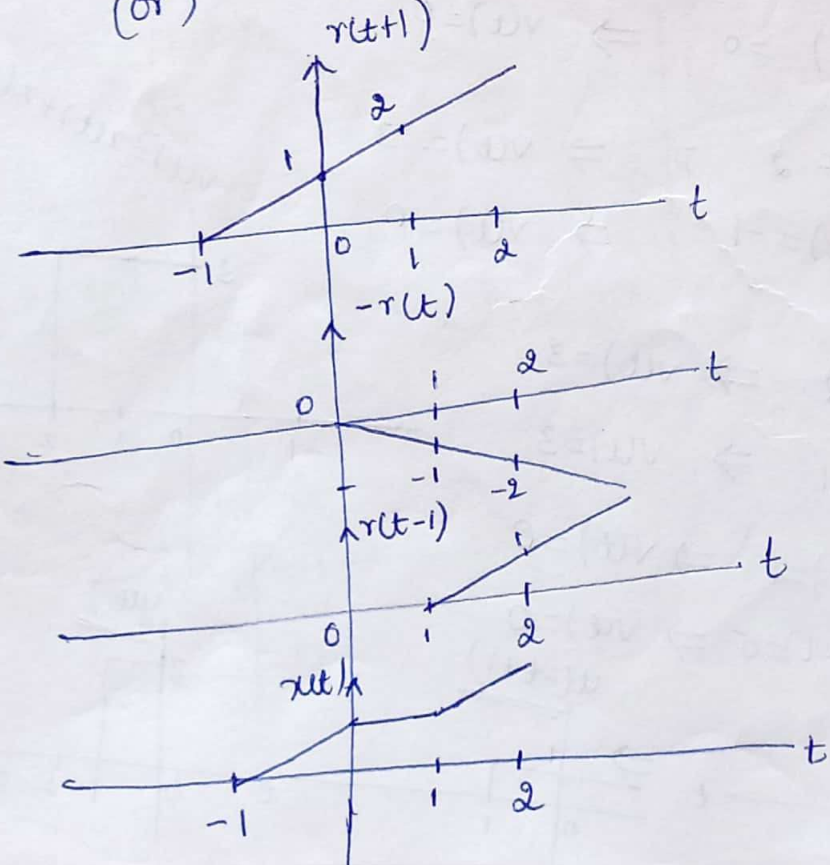
From the definition of ramp

$r(t) = t \quad ; \quad t \geq 0$

t	$r(t+1)$ $(t+1)$	$-r(t)$ $-t$	$r(t-1)$ $(t-1)$
-1	0	0	0
0	1	0	0
1	2	-1	0
2	3	-2	1

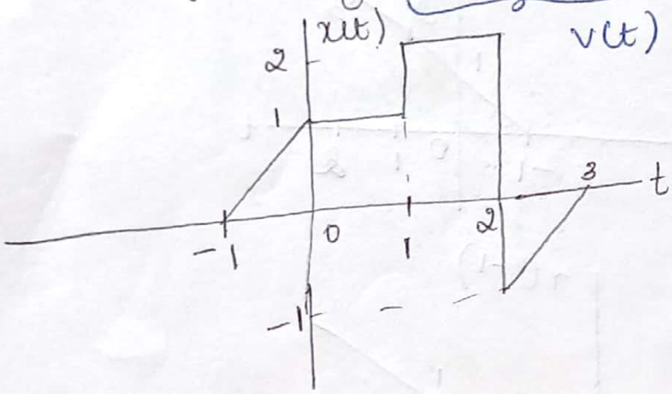


(or)



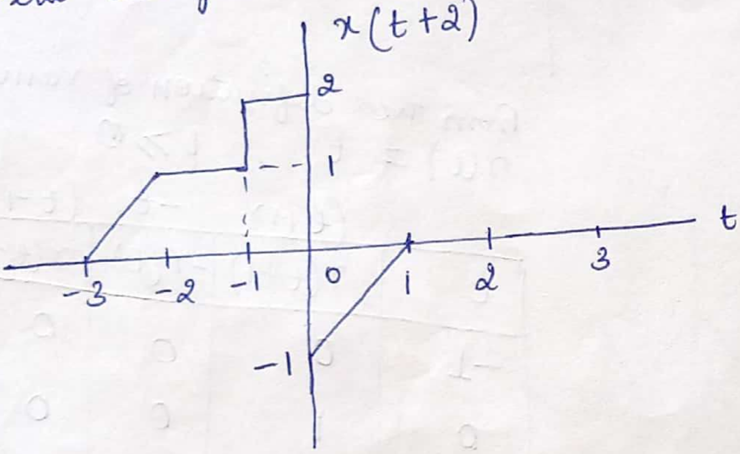
→ A CTS $x(t)$ shown in figure. Draw the signal

$$y(t) = \{\underbrace{x(t) + x(2-t)}_{v(t)}\} u(1-t)$$

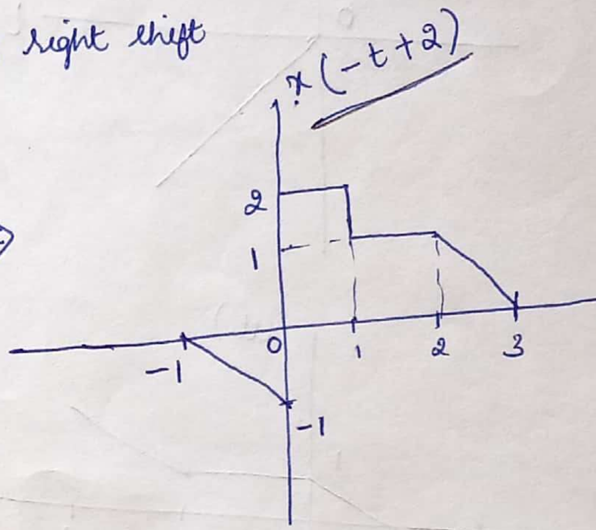


Soln:- $x(2-t) = x(-t+2)$

First doing time shifting ie $t_0 = -2$: right shift



⇒



For $t < -1$; $x(t)=0$; $x(2-t)=0$ ⇒ $v(t)=0$

$t=-1$; $x(t)=0$; $x(2-t)=0$ ⇒ $v(t)=0$

$t=0$; $x(t)=1$; $x(2-t)=2$; ⇒ $v(t)=3$

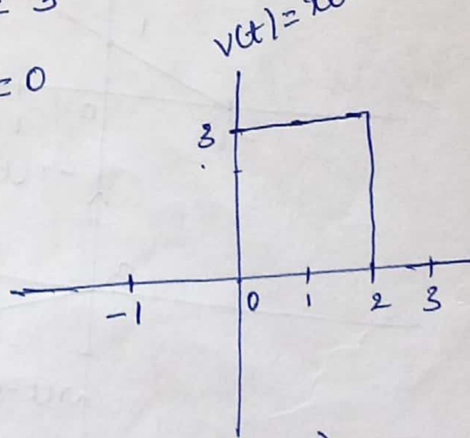
$x(2-t)=-1$; ⇒ $v(t)=0$

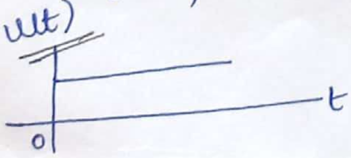
$0<t<1$; $x(t)=1$; $x(2-t)=2$ ⇒ $v(t)=3$

$1<t<2$; $x(t)=2$; $x(2-t)=1$ ⇒ $v(t)=3$

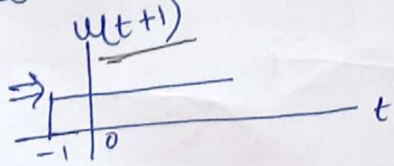
$t=2$; $x(t)=-1$; $x(2-t)=1$ ⇒ $v(t)=0$

$t=3$; $x(t)=0$; $x(2-t)=0$ ⇒ $v(t)=0$

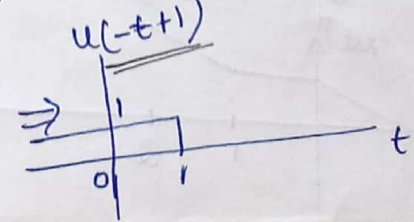


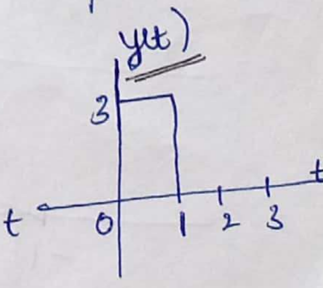


⇒



⇒

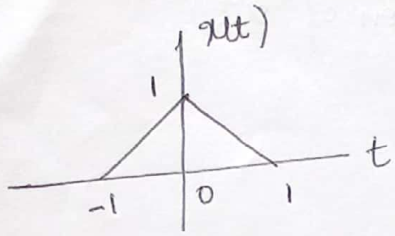




(17)

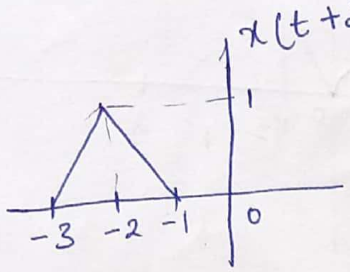
→ For the CTS $x(t)$ shown in fig. Sketch the signal $y(t) = x(3t+2)$

$x(3t)$

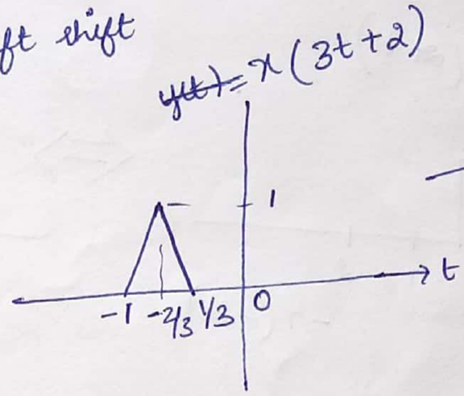


Soln:- According to precedence rule, first applying time shifting property & then time scaling

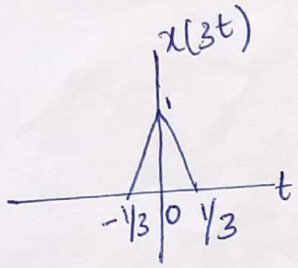
Time shifting ; $t_0 = -2$; left shift

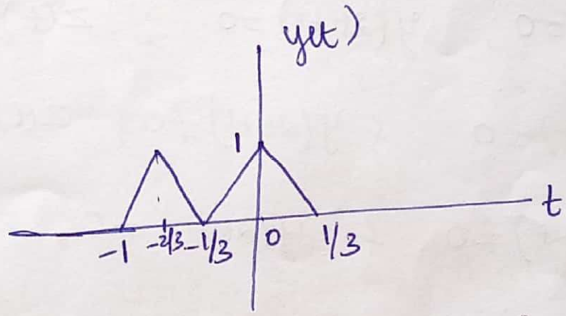


⟹

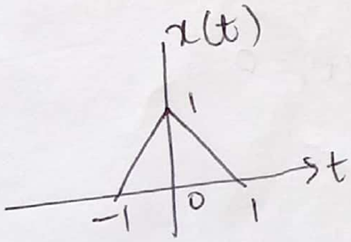


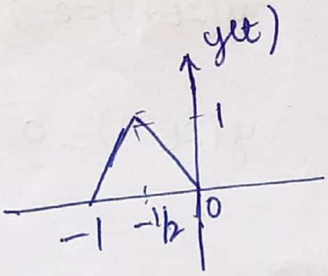
→ Time scaled + time shifted





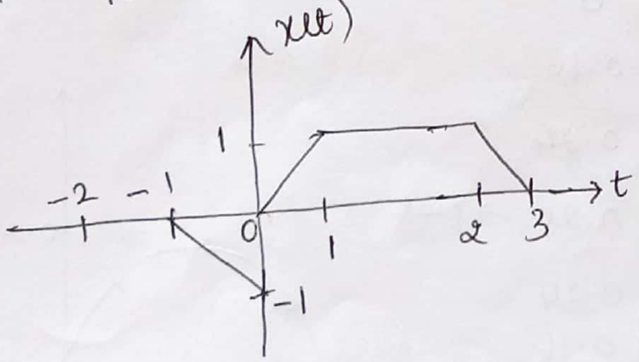
→ Draw $y(t) = x(-2t-1)$ for $x(t)$ given in the figure

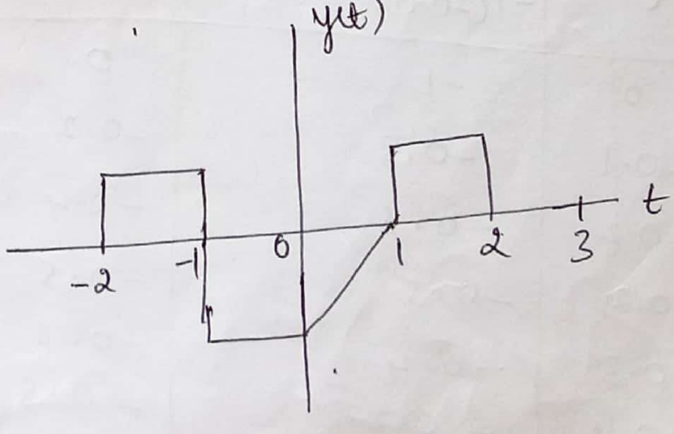




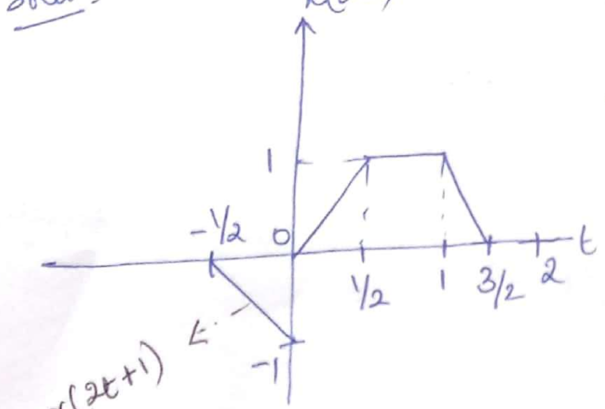
08-03-2017

→ Two CTS $x(t)$ & $y(t)$ are given in figure. Draw $z(t) = x(2t).y(2$

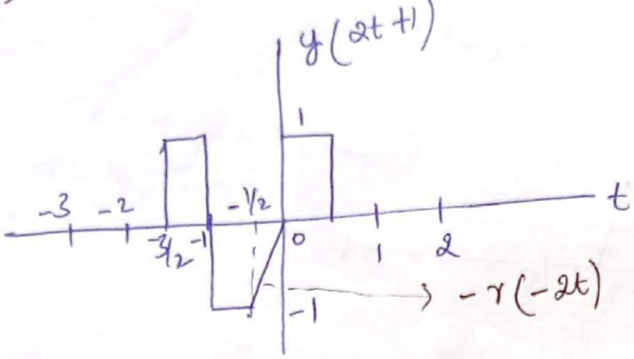




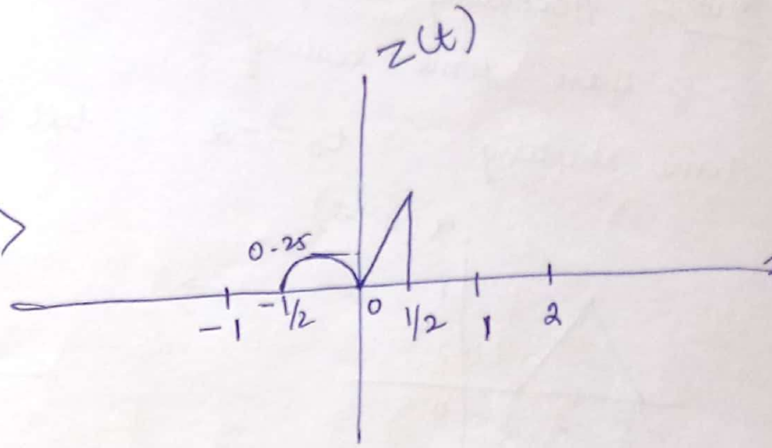
Soln:-



$\Rightarrow$ the compressed version of x(t)



$\Leftrightarrow$



For $t < -3/2$; $x(2t) = 0$; $y(2t+1) = 0$; $z(t) = 0$

$-3/2 < t < -1/2$; $x(2t) = 0$ & $y(2t+1) \neq 0$; $z(t) = 0$

$-1/2 < t < 0$; $x(2t) \neq 0$ & $y(2t+1) \neq 0$; $z(t) \neq 0$

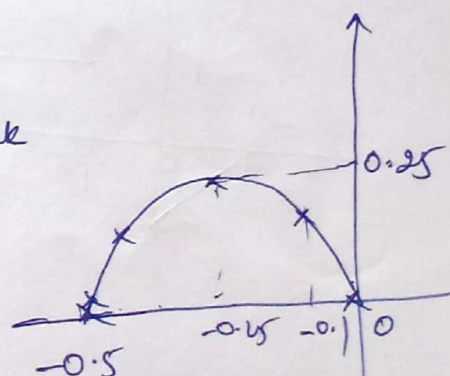
$0 < t < 1/2$; $x(2t) \neq 0$ & $y(2t+1) = 1$; $z(t) = x(2t)$

$1/2 < t < 3/2$; $x(2t) \neq 0$ $y(2t+1) = 0$; $z(t) = 0$

$t > 3/2$; $z(2t) = 0$ $y(2t+1) = 0$; $z(t) = 0$

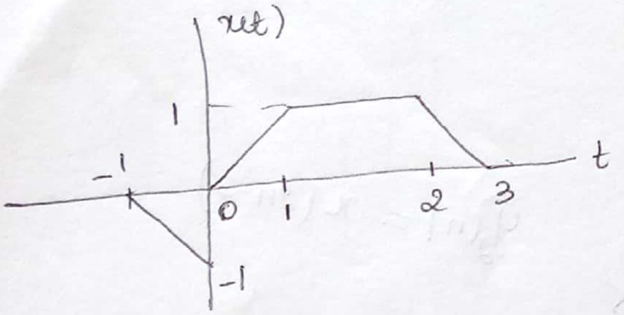
For $-1/2 < t < 0$.

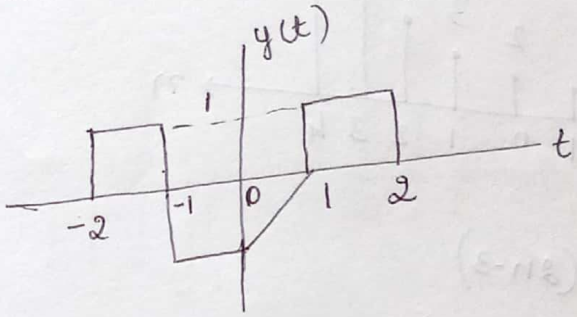
t	$x(2t)$ $-r(2t+1) = -(2t+1)$	$y(2t+1)$ $-r(-2t) = +2t$	$z(t) = x(2t) \cdot y(2t+1)$
0	-1	0	0
-0.1	-0.8	-0.2	0.16
-0.2	-0.6	-0.4	0.24
-0.25	-0.5	-0.5	0.25 → peak
-0.3	-0.4	-0.6	0.24
-0.4	-0.2	-0.8	0.16
-0.5	0	1	0



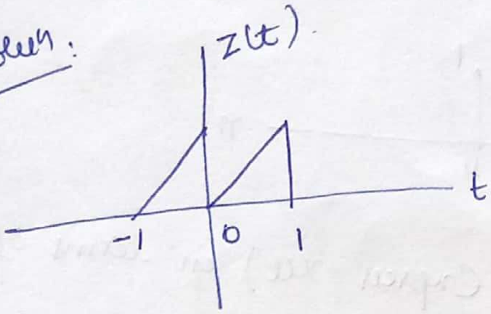
(18)

→ For $x(t)$ & $y(t)$ shown in figure draw $z(t) = x(t)\, y(-1-t)$





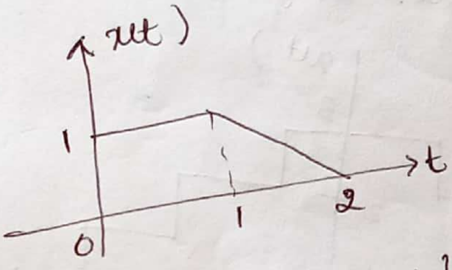
Soln:



Jan 2015

→ Given the signal $x(t)$ as shown in fig Sketch the following

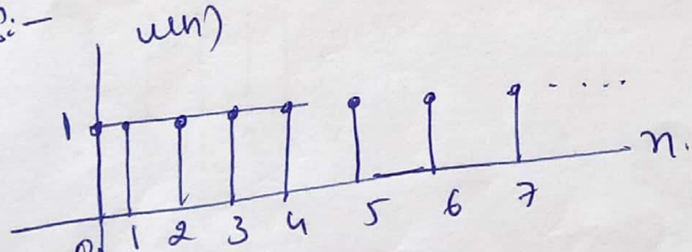
(i) $x(-2t+3)$ (ii) $x(t/2 - 2)$



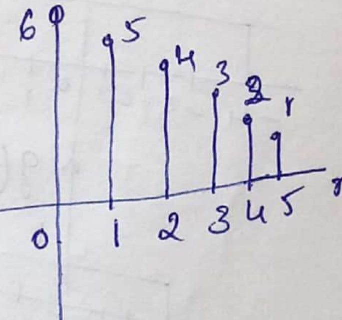
→ Given the signal ~~(Two signals x(t) & g(t))~~ $x(n) = (6-n)\{\underbrace{u(n) - u(n-6)}_{v(n)}\}$ make a sketch of $x(n)$, $y_1(n) = x(4-n)$ & $y_2(n) = x(2n-3)$

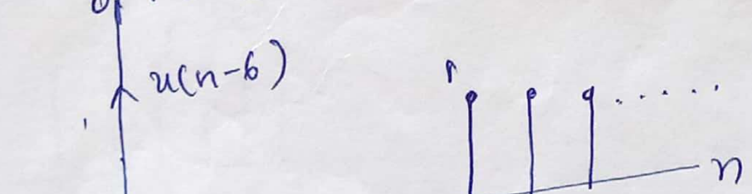
$x(n) = (6-n)\, v(n)$

Soln:–



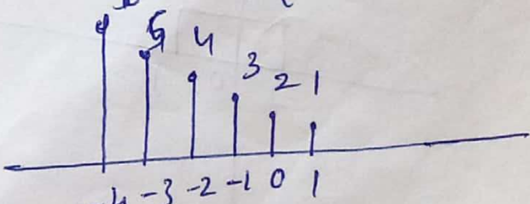
for $n=0$.
$n=1$
$n=2$
$n=3$
$n=4$
$n=5$

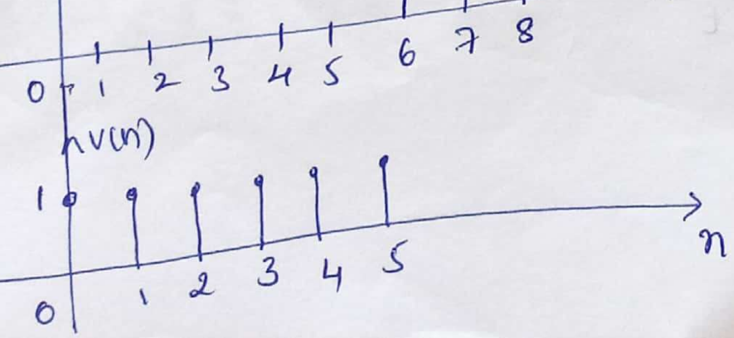




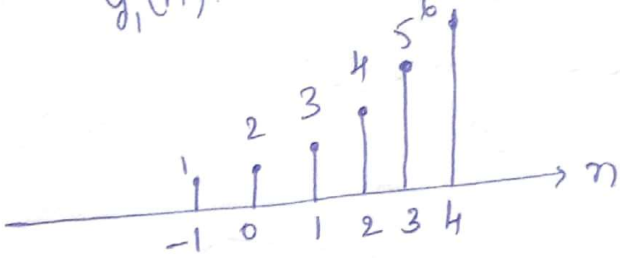
$y_1(n) = x(-n+4)$

$x(n+4)$



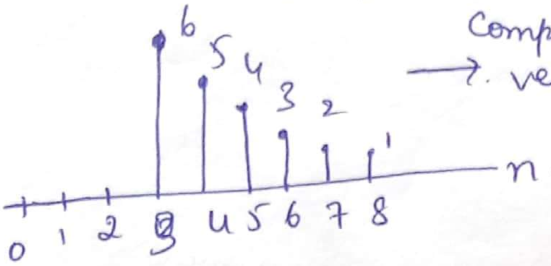


$y_1(n)$



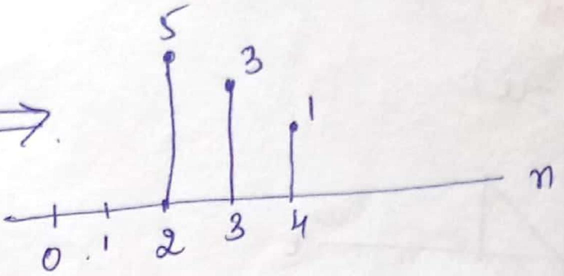
$y_2(n) = x(2n-3)$

$x(n-3)$

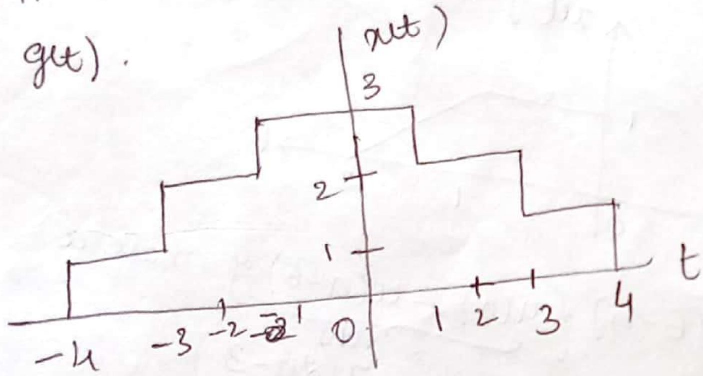


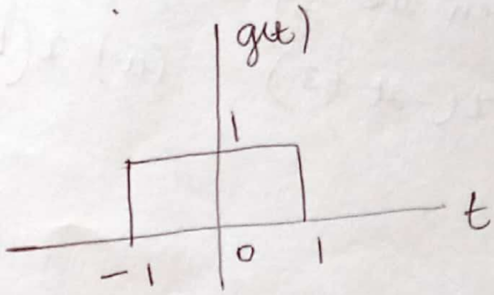
Compressed → version ⇒

$y_2(n) = x(2n-3)$

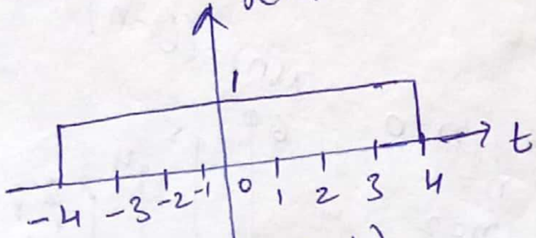


→ A CTS $x(t)$ & $g(t)$ is shown in fig. Express $x(t)$ in terms of $g(t)$.



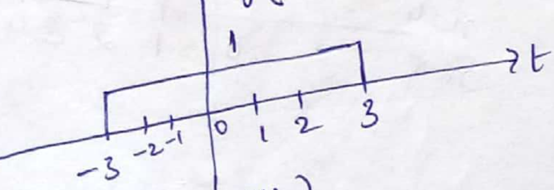


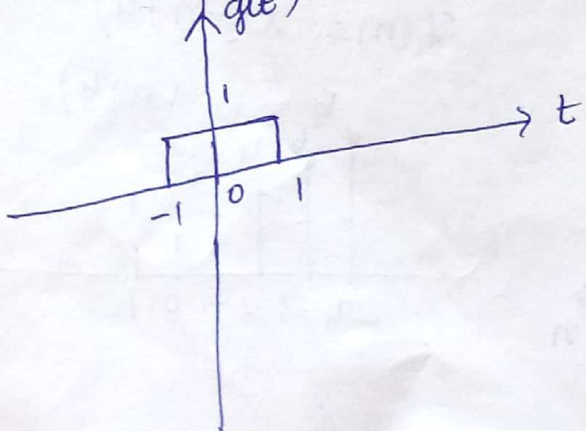
$g(\frac{1}{4}t)$



$$x(t) = g(t) + g(\tfrac{1}{3}t) + g(\tfrac{1}{4}t)$$

$g(\frac{1}{3}t)$

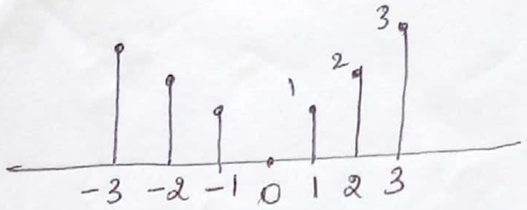




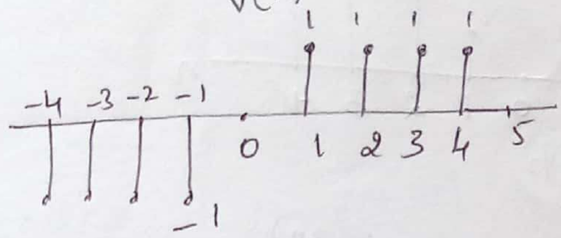
(19)

→ Sketch the following signals $z(n) = x(2n)\, v(n-4)$

$x(n)$



$v(n)$



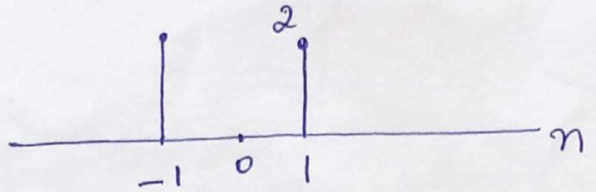
(or)

$$x(n) = [\,3\ \ 2\ \ 1\ \ \underset{\uparrow}{0}\ \ 1\ \ 2\ \ 3\,]$$

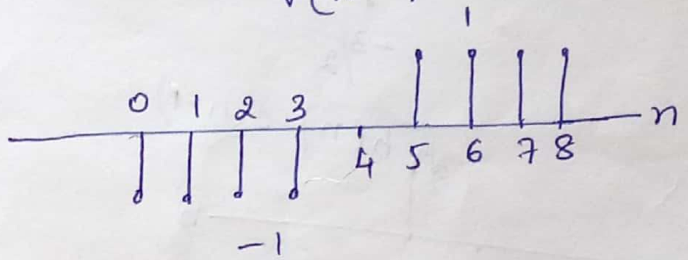
$$v(n) = [-1, -1, -1, -1, \underset{\uparrow}{0}, 1, 1, 1$$

Soln:-

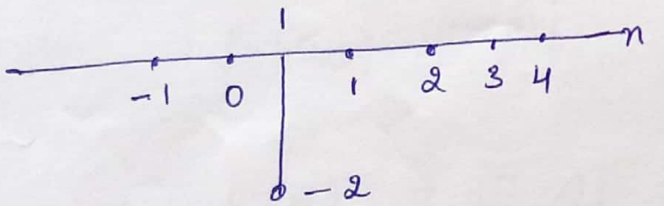
$x(2n)$



$v(n-4)$

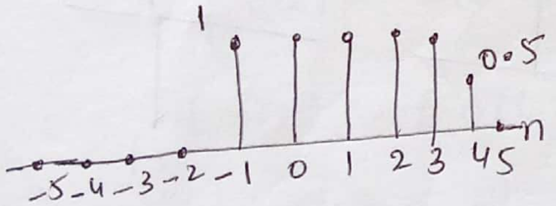


$z(n)$



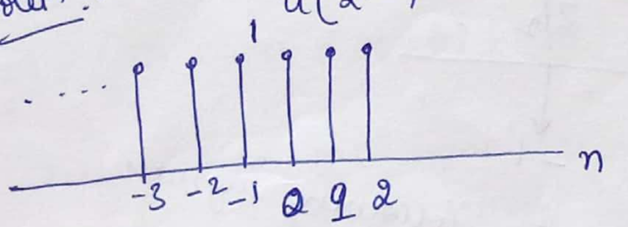
→ A DTS $x(n)$ is shown in figure Draw $y(n) = x(n)\, u(2-n)$

$x(n)$

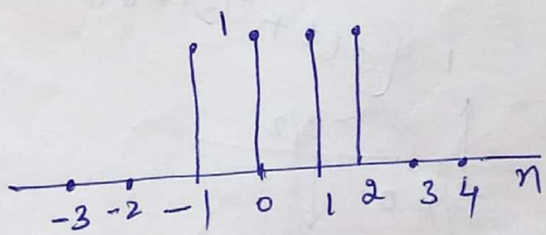


Soln:-

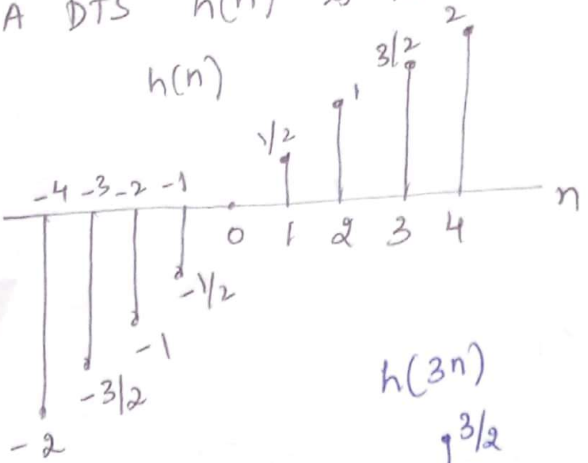
$u(2-n)$



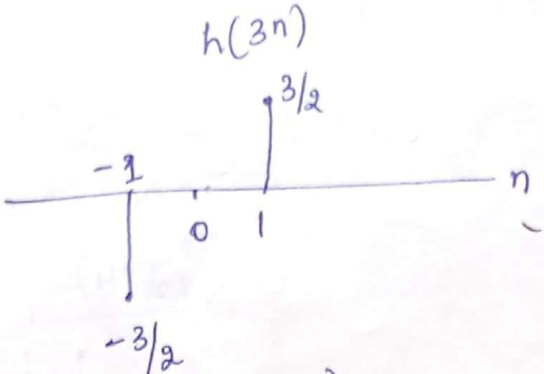
$y(n)$

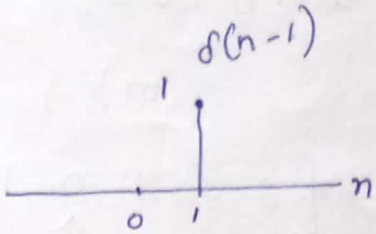


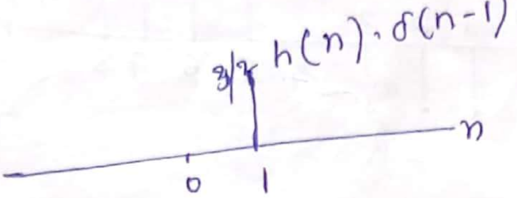
→ A DTS $h(n)$ is shown in figure. Draw $h(3n) . \delta(n-1)$



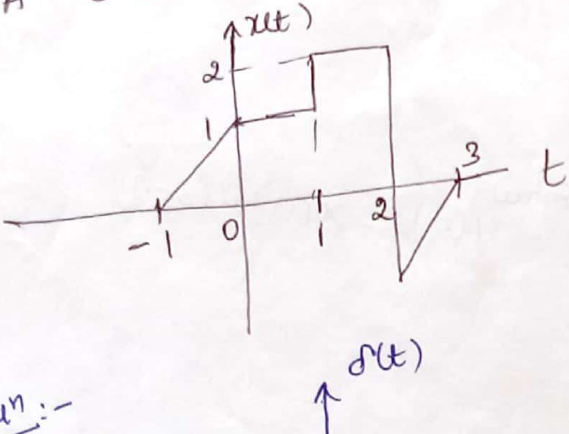
Soln:-



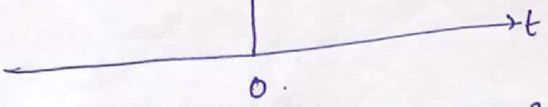


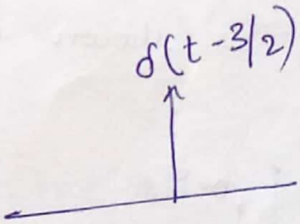


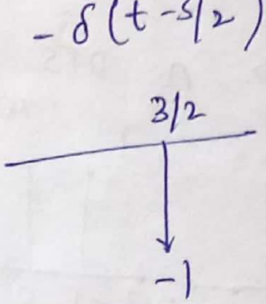
→ A CTS shown in fig find $y(t) = x(t)\left[\delta(t+3/2) - \delta(t-3/2)\right]$

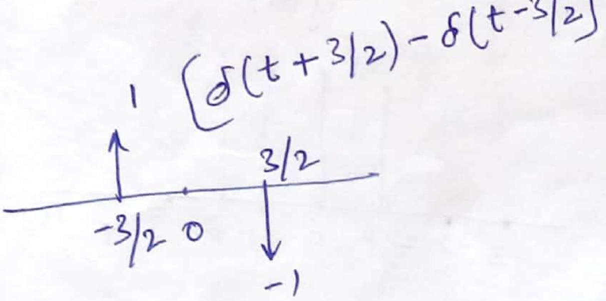


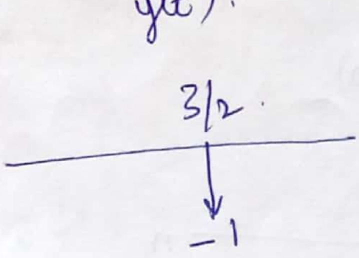
Soln:-











→ find & sketch the first derivatives of the following signals

(i) $x(t) = u(t) - u(t-a)$; $a>0$

(ii) $y(t) = t\left[u(t) - u(t-a)\right]$; $a>0$

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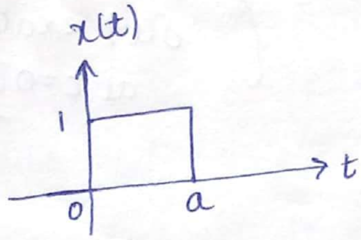
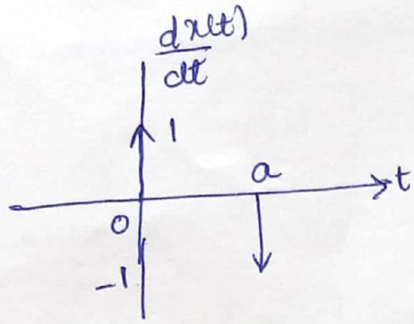
(20)

(i) $x(t) = u(t) - u(t-a)$

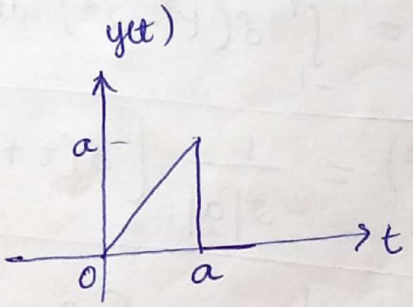


$$\frac{dx(t)}{dt} = \frac{d}{dt}u(t) - \frac{d}{dt}u(t-a)$$

$$\frac{dx(t)}{dt} = \delta(t) - \delta(t-a)$$



(ii) $y(t) = t\left[u(t) - u(t-a)\right]$



$$\frac{dy(t)}{dt} = \frac{d}{dt}\left\{t\left[u(t) - u(t-a)\right]\right.$$

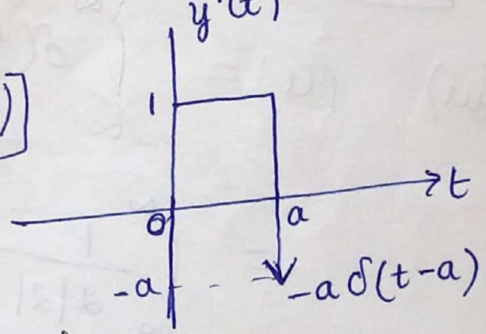
$$= t\left[\frac{d}{dt}u(t) - \frac{d}{dt}u(t-a)\right] + \left[u(t) - u(t-a)\right] \qquad \left\{\because \frac{d(t)}{dt} = 1\right\}$$

$$= t\left[\delta(t) - \delta(t-a)\right] + \left[u(t) - u(t-a)\right]$$

$$= t\,\delta(t) - t\,\delta(t-a) + \left[u(t) - u(t-a)\right]$$

$$= t\Big|_{t=0}\delta(t) - t\Big|_{t=a}\delta(t-a) + \left[u(t) - u(t-a)\right]$$

$$y'(t) = 0 - a\,\delta(t-a) + u(t) - u(t-a).$$



→ check whether the signal is energy or power $j^n + j^{-n}$

$$j^n + j^{-n} = e^{jn\pi/2} + e^{-jn\pi/2} = 2\cos\left(\frac{n\pi}{2}\right) \; ; \quad \Omega_0 = \pi/2 = \frac{2\pi}{4} \; ; \; \boxed{N=4}$$

Hence $x(n)$ is periodic with $N=4$

$x(n) = (2, 0, -2, 0)$ for one period $\quad ; \; P = \frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2$

$$P = \frac{1}{4}\sum_{n=0}^{3}|x(n)|^2 = \frac{1}{4}(4+4) = 2W$$

→ Determine each of the following

(i) $f(t) = e^{-3(t-1)}\delta(t)$

$= e^{-3t}\, e^{3} \cdot \delta(t)$

$= [e^{-3t}\,\delta(t)]\, e^{3} = e^{3}\left[e^{-3t}\Big|_{t=0}\delta(t)\right]$ {∵ $\delta(t)$ exits only at $t=0$}

$= e^{3}[e^{0}\,\delta(t)]$

$\boxed{f(t) = e^{3}\cdot\delta(t)}$

(ii) $f(t) = \int_{-1}^{1}\delta(t^2-4)\,dt$

$= \int_{-1}^{1}\delta(t^2-2^2)\,dt$

$\boxed{\delta(t^2-a^2) = \frac{1}{2|a|}\left[\delta(t+a)+\delta(t-a)\right]}$

$f(t) = \int_{-1}^{1}\frac{1}{2(2)}\left[\delta(t+2)+\delta(t-2)\right] = \frac{1}{4}\left[\int_{-1}^{1}\delta(t+2)\,dt + \int_{-1}^{1}\delta(t-2)\,dt\right]$

$= \frac{1}{4}[0+0]$

$\boxed{f(t)=0}$

(iii) $f(t) = \int_{-\infty}^{\infty}\delta(t^2-4)\,dt$

$= \int_{-\infty}^{\infty}\frac{1}{2|2|}\left[\delta(t+2)+\delta(t-2)\right].dt = \frac{1}{4}\left[\int_{-\infty}^{\infty}\delta(t+2)\,dt + \int_{-\infty}^{\infty}\delta(t-2)\,dt\right]$

$= \frac{1}{4}[1+1]$

$f(t) = 2/4$

∴ $\boxed{f(t) = 1/2}$