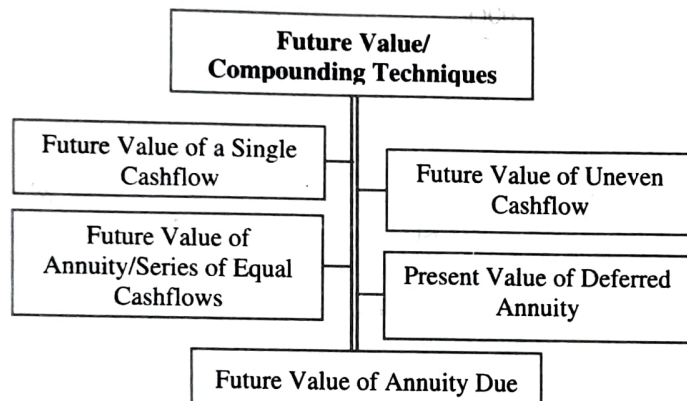


DEPARTMENT OF MASTER OF BUSINESS ADMINISTRATION

The theory of 'Compounding' facilitates ascertaining the Future Value (FV) of today's money. It is similar to the 'Concept of Compound Interest', which anticipates the re-investment of the principal amount along with the interest earned during the previous year at the prevailing rate of interest. The total amount (principal amount + interest accrued thereon) at the end of a period is taken as the principal amount at the beginning of the next period and the interest is calculated accordingly.

The outline of the theory of 'Compounding' to ascertain the 'Future Value' (FV) of the present money is furnished in the following chart:



4.3.1. Future Value of a Single Cashflow

The 'Future Value' of a single cashflow can be computed by applying the following formula:

$$FV = PV(1 + r)^n$$

where,

FV = Future Value

PV = Present Value (given)

r = Rate of interest, and

n = Time gap after which FV is to be ascertained.

From the above equation, it may be seen that there are three variables involved in the calculation of FV, viz., 'Present Value (PV)', 'Rate of Interest (r)', and 'Time Gap (n)'. Changes in any of the above variables would result in the changed FV. The possible combinations of the above three variables are not finite and so is the number of FVs.

Some Examples:

- 1) Calculation of FV of ₹1,000 at 10% after 7 years

$$\begin{aligned} FV &= PV(1+r)^n \\ &= 1,000(1+0.10)^7 = 1,000(1.10)^7 \\ &= 1,000 \times 1.95 = ₹1,950 \end{aligned}$$

- 2) Calculation of FV of ₹5,000 at 11% after 9 years

$$\begin{aligned} FV &= PV(1+r)^n \\ &= 5,000(1+0.11)^9 = 5,000(1.11)^9 \\ &= 5,000 \times 2.56 = ₹12,800 \end{aligned}$$

- 3) Calculation of FV of ₹50,000 at 16% after 3 years

$$\begin{aligned} FV &= PV(1+r)^n = 50,000(1+0.16)^3 = 50,000(1.16)^3 \\ &= 50,000 \times 1.56 = ₹78,000 \end{aligned}$$

Solution: $FV = PV(1 + r)$

When the interest is **compounded yearly**

$$FV = 5,000(1 + 0.085)^6 = 5,000 \times 1.631 = ₹8,155$$

When the interest is **compounded semi-annually**

$$FV = 5,000(1 + 0.085/2)^{6 \times 2} = 5,000 \times 1.648 = ₹8,240$$

WHEN the interest is **compounded quarterly**

$$FV = 5,000(1 + 0.085/4)^{6 \times 4} = 5,000 \times 1.656 = ₹8,280$$

Example 2: You are required to find out the amount to be received by Govind after 8 years from the following data:

- Govind has a deposit of ₹10,000 in a bank.
- The bank pays 8% interest compounded annually for 8 years.

Solution: $FV_n = PV(1 + r)^n$

Where,

Future value (FV) = ?

Present value (PV) = ₹10,000

% Rate of interest (r) = 8%

Time gap after which FV is to be ascertained (n) = 8 years.

$$FV_n = 10,000(1 + .08)^8 = ₹10,000(1.8509) = ₹18,509$$

Example 3: A sum of ₹8,200 is deposited into a time deposit account today that pays 5% per annum. How much will it be in the next 5 years if compounded (i) quarterly, (ii) semi-annually and (iii) annually?

Solution:

- i) **Quarterly Compounding:** $FV = PV(1 + r)^n$

Where, Present value (PV) = ₹8,200

Interest rate (r) = 5% = 0.05/4 = 0.0125

Number of period (n) = 5 years

$$\begin{aligned} FV &= 8,200 \times (1 + 0.0125)^{5 \times 4} \\ &= 8,200 \times (1.0125)^{20} \\ &= 8,200 \times 1.28204 = ₹10,512.73 \text{ or } ₹10,513 \end{aligned}$$

- ii) **Semi-Annually Compounding:** $FV = PV(1 + r)^n$

Where, Present value (PV) = ₹8,200

Interest rate (r) = 5% = 0.05/2 = 0.025

Number of period (n) = 5 years

$$\begin{aligned} FV &= 8,200 \times (1 + 0.025)^{5 \times 2} \\ &= 8,200 \times (1.025)^{10} \\ &= 8,200 \times 1.2801 = ₹10,496.82 \text{ or } ₹10,497 \end{aligned}$$

- iii) **Annually Compounding:** $FV = PV(1 + r)^n$

Where, Present value (PV) = ₹8,200

Interest rate (r) = 5% = 0.05

Number of period (n) = 5 years

$$\begin{aligned} FV &= 8,200 \times (1 + 0.05)^5 \\ &= 8,200 \times (1.25)^5 \\ &= 8,200 \times 3.0517 = ₹25,023.94 \text{ or } ₹25,024 \end{aligned}$$

Example 4: Find the future value at the end of one year if the present value is ₹20,000 and the interest rate is 16% and the frequencies are compounded:

- Monthly
- Daily

Solution:1) **Monthly Compounding:** $FV = PV(1 + r)^n$

Given, Present value of money (PV) = ₹20,000

Interest rate (r) = 16% or 0.16 = 0.16/12 = 0.0133

$$\begin{aligned} FV &= 20,000(1 + 0.0133)^{12} \\ &= 20,000(1.0133)^{12} \\ &= 20,000 \times 1.1718 = ₹23,436 \end{aligned}$$

2) **Daily Compounding:** $FV = PV(1 + r)^n$

Given, Present value of money (PV) = ₹20,000

Interest rate (r) = 16% or 0.16 = 0.16/365 = 0.00044

$$\begin{aligned} FV &= 20,000(1 + 0.00044)^{365} \\ &= 20,000(1.00044)^{365} \\ &= 20,000 \times 1.1742 = ₹23,484 \end{aligned}$$

4.3.2. Future Value of Uneven Cashflow

The formula to be applied to calculate future value of uneven cashflow streams is:

$$FV = R_1(1 + r)^{n-1} + R_2(1 + r)^{n-2} + R_3(1 + r)^{n-3} \dots + R_n$$

Where,

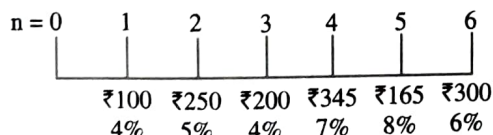
FV = Future Value

R = Payment per compounding period

r = Interest rate per compounding periods

The above formula of 'Uneven Cashflow Stream' gives some relaxation with regard to the basic three presumptions; viz., cashflows are equal in (i) timing, (ii) amount and (iii) interest rate. With the help of this formula, it is possible to calculate cashflows, which vary in timing, amount and interest rates.

Uneven cashflow streams have different payments over different time periods with different interest rates.



In order to calculate the uneven payments across varying time intervals with varying interest rates, the Future Value (FV) compounding interest formula for each 'Cashflow Stream' may be used for each period and their totals may be taken.

The future value compounding interest formula for 'Single Cashflow' is:

$$FV = PV(1 + r)^n$$

In the above formula, Present Value (PV), is replaced with R (payment per compounding period).

Example 5: Calculate the future value of the income stream from the following data:

- i) Mr. A is planning to save some money to purchase a car and plans to deposit the following cashflow stream each year:

Year	1	2	3	4
₹	1,500	3,000	2,200	3,000

- ii) Interest rate is 9% for 4 years compounded semi-annually.

Solution: $FV = R_1(1 + r)^{n-1} + R_2(1 + r)^{n-2} + R_3(1 + r)^{n-3} + R_4(1 + r)^{n-4}$

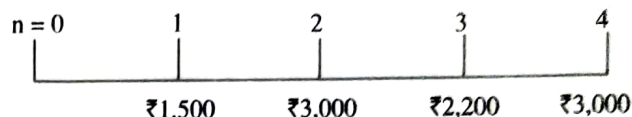
Where,

Future Value (FV) = ?

Payment per compounding period (R_1) = ₹1,500Payment per compounding period (R_2) = ₹3,000Payment per compounding period (R_3) = ₹2,200Payment per compounding period (R_4) = ₹3,000

Interest rate per compounding periods (r) = 0.09/2 = 0.045

Time gap after which FV is to be ascertained (n) = 4 years



$$\begin{aligned} FV &= ₹1,500(1 + 0.045)^{4-1 \times 2} + ₹3,000(1 + 0.045)^{4-2 \times 2} + \\ &\quad ₹2,200(1 + 0.045)^{4-3 \times 2} + ₹3,000(1 + 0.045)^{4-4 \times 2} \\ &= ₹1,500(1.3022) + ₹3,000(1.1925) + ₹2,200(1.0920) \\ &\quad + ₹3,000(1.045) \\ &= ₹1,953.39 + ₹3,577.5 + ₹2,402.4 + ₹3,000 \\ &= ₹10,933.29 \end{aligned}$$

Example 6: Calculate the future value of the investment from the following data:

- 1) Mr. A is planning to paint his house in Diwali which would incur cost of ₹12,000. He decided to deposit ₹2,500 during each period.

- 2) Interest rates for each period are expected to be as follows:

Period	1	2	3	4
%	3	4	5.5	5

- 3) Interest rates are compounded annually.

Solution: $FV = R_1(1 + r)^{n-1} + R_2(1 + r)^{n-2} + R_3(1 + r)^{n-3} + R_4(1 + r)^{n-4}$

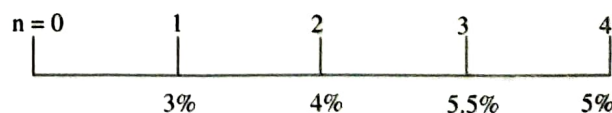
Where,

Future Value (FV) = ?

Payment per compounding period (R) = ₹2,500

Interest rate per compounding periods (r_1) = 0.03Interest rate per compounding periods (r_2) = 0.04Interest rate per compounding periods (r_3) = 0.055Interest rate per compounding periods (r_4) = 0.05

Time gap after which FV is to be ascertained (n) = 4 years



$$\begin{aligned} FV &= ₹2,500(1 + 0.030)^{4-1} + ₹2,500(1 + 0.040)^{4-2} + \\ &\quad ₹2,500(1 + 0.055)^{4-3} + ₹2,500(1 + 0.050)^{4-4} \\ &= ₹2,500(1.0927) + ₹2,500(1.0816) + ₹2,500(1.1130) \\ &\quad + ₹2,500(1) \\ &= ₹2,731.75 + ₹2,704 + ₹2,637.5 + ₹2,500 \\ &= ₹10,573.25 \end{aligned}$$

4.3.3. Future Value of Annuity/Series of Equal Cashflows

Many a times an investment decision may involve regular cashflows of equal amount during successive years in future for a number of years, in lieu of a single cashflow. Such

regular cashflows are referred to as 'Annuity'. For example, if ₹1,000 is deposited at the end of every year starting from the current year for a period of three years, it will be termed as an 'Annuity deposit of ₹1,000 for three years'.

An 'Annuity' may, therefore, be defined as 'any regular and equal cashflows at fixed intervals for a specified period of time'. An annuity that has no definite end is called 'Perpetuity'.

If the rate of interest is 10% compounded annually, computation of the Future Value (FV) of an annuity (₹1,000) may be represented in a graphical manner as under (figure 4.1):

Year 0	Year 1	Year 2	Year 3
	₹1,000	₹1,000	₹1,000
			₹1,100
			₹1,210
Total			₹3,310

Figure 4.1: Computation of Future Value (FV) of an Annuity (₹1,000) of 3 Years (at $r = 10\%$)

In order to ascertain the Future Value (FV) of each cashflow, it is subject to 'Compounding'. The total of 'FVs' of individual cashflows would be the total FV of the 'Annuity'. The FV of an 'Annuity' thus depends upon three variables, viz.:

- 1) The annual amount,
- 2) The rate of interest, and
- 3) The specific period of time.

Following is the general formula, which may be applied to calculate the 'Future Value' of an 'Annuity':

$$FVA_n = A \left[\frac{(1+r)^n - 1}{r} \right]$$

where,

FVA_n = Future Value of an Annuity with a time period of 'n' number of years.

A = Constant periodic flow.

r = Rate of Interest (ROI) for each period.

n = Time period of the annuity.

The 'Future Value' (FV) of an 'Annuity' may also be found out with the help of a 'Pre-calculated Mathematical Table', which is available for various combinations of all the three variables, i.e.:

- 1) The annual amount (A),
- 2) The rate of interest (r), and
- 3) The specific period of time (n).

From the above, it may be concluded that the above three variables are the determinants of the 'Future Value' (FV) of any 'Annuity'. A change in any of them would result in a changed FV.

Example 7: Calculate the future value of annuity at the end of five years from the following data:

- i) Deposition of ₹10,000 in five equal annual payments in the FD.
- ii) Interest rate per year is 10%.

Solution: $FVA_n = A \left[\frac{(1+r)^n - 1}{r} \right]$

Where,

Future value of an annuity which has time period of n years (FVA_n) = ?

Constant periodic flow (A) = ₹10,000

Interest rate per period (r) = 10%

Time period of the annuity (n) = 5 years

$$\begin{aligned} FVA_n &= 10,000 \left[\frac{(1+.10)^5 - 1}{.10} \right] \\ &= 10,000 \left[\frac{(1.10)^5 - 1}{.10} \right] = 10,000 \left[\frac{1.611 - 1}{.10} \right] \\ &= 10,000 [6.11] = ₹61,100 \end{aligned}$$

Example 8: A student plans to deposit ₹50 in savings account at the end of each quarter for the next 6 years. Interest is earned at a rate of 8% per year compounded quarterly. What should her account balance be 6 years from now? How much interest will she earn?

Solution: $FVA_n = A \left[\frac{(1+r)^n - 1}{r} \right]$

Given, A = ₹50, $r = 8\% = 0.08/4 = 0.02$, $n = (6 \text{ years} \times 4 \text{ quarters per year}) = 24$

$$\begin{aligned} FVA_n &= 50 \left[\frac{(1+0.02)^{24} - 1}{0.02} \right] = 50 \left[\frac{1.6084 - 1}{0.02} \right] \\ &= 50 \left[\frac{0.6084}{0.02} \right] = 50 \times 30.42 = ₹1,521 \end{aligned}$$

Example 9: How much amount is required to be invested every year so as to accumulate ₹3,00,000 at the end of 10 years if the interest is compounded annually at 10%?

Solution: $FVA_n = A \left[\frac{(1+r)^n - 1}{r} \right]$

$$3,00,000 = A \left[\frac{(1+0.1)^{10} - 1}{0.1} \right]$$

$$3,00,000 = A \times 15.9374248$$

$$A = \frac{3,00,000}{15.9374248} = ₹18,823.62$$

Growing Annuity

A finite series of regular cashflow growing at fixed rate every year is referred as growing annuity. Following is the general formula which may be applied to calculate 'Growing Annuity':

Where $i < \text{or} > g$:

$$FV(A) = A \cdot \frac{(1+i)^n - (1+g)^n}{i - g}$$

Where $i = g$:

$$FV(A) = A \cdot n(1+i)^{n-1}$$

where,

$FV(A)$ = The value of the annuity at time

A = The value of the individual payments in each compounding period,

worth tomorrow, given its capacity to earn interest. Discounting is the method, which is used to ascertain the worth of future payments or receipts today. This concept is just the opposite of what is the concept of 'Compounding Technique' and is known as the 'Present Value'. While with the help of 'Compounding Technique', the 'Future Value' (FV) of the present investments is calculated, with the help of 'Discounting Technique', the 'Present Value' of the future cashflows is calculated by using following formula:

$$PV = FV / (1 + r)^n$$

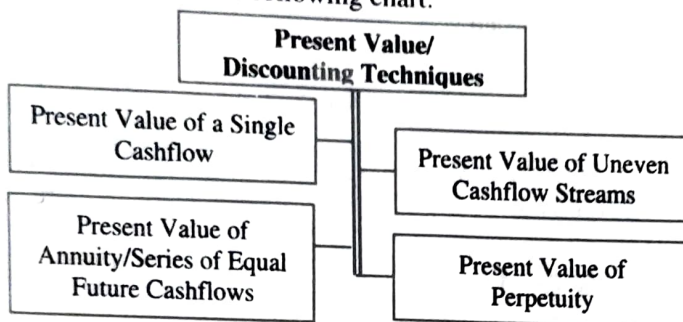
where, PV = Present Value

FV = Future Value (given)

r = % Rate of interest, and

n = No. of years for which discounting is done.

The concept of 'Discounting Technique' to calculate the PV is depicted in the following chart:



4.4.1. Present Value of a Single Cashflow

The 'Present Value' of a future 'Cashflow' would be logically less than the 'Future Value' (FV), because of the 'Opportunity Cost', which may be defined as "the cost of an alternative that must be forgone in order to pursue a certain action". In other words, the benefits that could have been received by taking an alternative action. One foregoes the opportunity to invest elsewhere and to earn interest during that period. This interest foregone is the cost the investor has to pay and the money expected in future needs to be adjusted to take care of this cost. If the waiting period for the future money is more, the cost involved in such delay also increases, which shows the compounded value of the lost opportunities.

With a view to ascertain the 'PV' of future money in an accurate manner, the 'Opportunity Cost' needs to be taken into account.

The 'Present Value' (PV) of a single 'Cashflow' may be calculated by applying following formula:

$$PV = FV_n \left(\frac{1}{1+r} \right)^n \text{ or } PV = \frac{FV}{(1+r)^n}$$

where,

FV_n = Future value n years hence.

r = Rate of interest per annum.

n = No. of years for which discounting is done.

From the above formula, it may be seen that the 'Present Value' (PV) of future cashflow depends upon three variables, viz., 'FV_n', 'r' and 'n'.

Example 13: Calculate the present value from the following data:

- A person will receive ₹3,000 after 8 years.
- The discount rate is 10%.

$$\text{Solution: } PV = FV_n \left(\frac{1}{1+r} \right)^n$$

where,

Present value (PV) = ?

Future value n years (FV_n) = ₹3,000

Rate of interest per annum (r) = 10%

No. of years for which discounting is done (n) = 8 years

$$PV = 3,000 \left(\frac{1}{1+0.10} \right)^8 = 3,000 (.46651) = ₹1,399.53 \text{ or } ₹1,400$$

Example 14: Seema wants to have ₹1 million when she retires in 20 years. If she can earn a 10% annual return, compounded annually, on her investments, then what is the lump-sum amount she would need to invest today to reach her goal?

$$\text{Solution: } PV = FV \left(\frac{1}{1+r} \right)^n$$

Given,

Future value of money (FV) = 1 Million = ₹10,00,000

Interest rate (r) = 10%

Yearly time periods (n) = 20,

Present value (PV) = ?

$$PV = 10,00,000 \left(\frac{1}{1+0.1} \right)^{20} = 10,00,000 \left(\frac{1}{1.1} \right)^{20} \\ = 10,00,000 \times 0.148644 = ₹1,48,644$$

Example 15: A sum of ₹10,500 is needed in 5 years from now. What will be the single sum of money need to be deposited today in an account that pays 5% per annum compounded (i) quarterly, (ii) semi-annually and (iii) annually?

Solution:

$$\text{i) Quarterly Compounding: } PV = FV \left(\frac{1}{1+r} \right)^n$$

Where, Future value (FV) = ₹10,500

Interest rate (r) = 5% = 0.05/4 = 0.0125

Number of period (n) = 5 years

$$PV = 10,500 \left(\frac{1}{1+0.0125} \right)^{5 \times 4} = 10,500 \left(\frac{1}{1.0125} \right)^{20} \\ = 10,500 \times 0.780 = ₹8,190$$

$$\text{ii) Semi-annual Compounding: } PV = FV \left(\frac{1}{1+r} \right)^n$$

Where, Future value (FV) = ₹10,500

Interest rate (r) = 5% = 0.05/2 = 0.025

Number of period (n) = 5 years

$$PV = 10,500 \left(\frac{1}{1+0.025} \right)^{5 \times 2} = 10,500 \left(\frac{1}{1.025} \right)^{10}$$

$$= 10,500 \times 0.7812 = ₹8,202.6 \text{ or } ₹8,203$$

iii) **Annual Compounding:** $PV = FV \left(\frac{1}{1+r} \right)^n$

Where, Future value (FV) = ₹10,500

Interest rate (r) = 5% = 0.05

Number of period (n) = 5 years

$$PV = 10,500 \left(\frac{1}{1+0.05} \right)^5 = 10,500 \left(\frac{1}{1.05} \right)^5$$

$$= 10,500 \times 0.7835 = ₹8,226.75 \text{ or } ₹8,227$$

4.4.2. Present Value of Uneven Cashflow Streams

The 'Present Value' (PV) in the case of uneven 'Cashflows' may be calculated by applying the following formula:

$$PV = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + \dots + \frac{R_n}{(1+r)^n}$$

where,

PV = Present Value

R = Payment per compounding period

r = Interest rate per compounding periods

To find out the 'Present Value' (PV) of irregular payments across varying time intervals with varying interest rates, the 'Present Value Compounding Interest' formula (given below) may be used for each period and their summation may be taken thereafter.

$$PV = \frac{FV}{(1+r)^n}$$

'Future Value' (FV) in the above equation is replaced with R (Payment per Compounding Period).

Example 16: Calculate the present value of the following cashflow stream from the following data:

Period	1	2	3	4
%	3	4	5.5	5
₹	1,500	3,000	2,200	3,000

Solution: $PV = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + \frac{R_4}{(1+r)^4}$

where,

Present value (PV) = ?

Payment per compounding period (R_1) = ₹1,500

Payment per compounding period (R_2) = ₹3,000

Payment per compounding period (R_3) = ₹2,200

Payment per compounding period (R_4) = ₹3,000

Interest rate per compounding periods (r_1) = 0.03

Interest rate per compounding periods (r_2) = 0.04

Interest rate per compounding periods (r_3) = 0.055

Interest rate per compounding periods (r_4) = 0.05

n = 0	1	2	3	4
	₹1,500	₹3,000	₹2,200	₹3,000
	3%	4%	5.5%	5%

$$PV = \frac{₹1,500}{(1+0.03)^1} + \frac{₹3,000}{(1+0.04)^2} + \frac{₹2,200}{(1+0.055)^3} + \frac{₹3,000}{(1+0.05)^4}$$

$$= ₹1,456.31 + ₹2,773.67 + ₹1,873.55 + ₹2,468.11$$

$$= ₹8,571.64$$

4.4.3. Present Value of Annuity/ Series of Equal Future Cashflows

An investment decision taken today may yield a chain of 'cashflows' of the similar amount in future. To understand the above statement in a better manner an example may be taken. Suppose there is an offer from an agency for a three year contract with following two options:

- 1) **Option I:** Payment of ₹2,500 only at present with no provision of payment for the next three years.
- 2) **Option II:** Payment of ₹900 each at the end of first year, second year and third year from now.

With the rate of interest @ 10% p.a., analysis on the basis of the 'Present Values' (PVs) of both options may be carried out as under:

Option I: This option involves payment of ₹2,500 now, which is in terms of the 'Present Value' and therefore, no modification is required.

Option II: This option involves payment of ₹900 as an annuity for 3 years. This can be expressed graphically as follows (**figure 4.2**):

Year 0	Year 1	Year 2	Year 3
	₹900	₹900	₹900
PV(818) ←			
+ PV(744) ←			
+ PV(676) ←			
₹2,238	Total		

Figure 4.2: Calculation of Present Value of an Annuity (at r = 10%)

With a view to ascertain the PV of a chain of payments, the PVs of various amounts accruing at different times need to be computed and then their total needs to be obtained. In the above example, as depicted graphically (**figure 4.2**), the total PV is ₹2,238 under option II, as against the PV of ₹2,500 under the option I. Obviously, option number II is a preferred one, as it involves outgo of a lower amount (₹2,238) in real terms.

The present value of an annuity may be expressed as follows:

$$PVA_n = \frac{A}{(1+r)^1} + \frac{A}{(1+r)^2} + \dots + \frac{A}{(1+r)^{n-1}} + \frac{A}{(1+r)^n}$$

$$= A \left[\frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^{n-1}} + \frac{1}{(1+r)^n} \right]$$

$$PVA_n = A \left[\frac{1 - \frac{1}{(1+r)^n}}{r} \right] = A \left[\frac{(1+r)^n - 1}{r(1+r)^n} \right]$$

where,

Cashflow₁ = The cashflow at the end of the first period

r = Rate of interest

g = Growth rate in perpetuity amount

It is important to note that the above formula can be applied successfully only if the rate of interest (r) is more than the rate of growth (g), i.e., $r > g$.

Example 24: Calculate the present value of perpetuity from the following data:

- An investment gives an expected return of ₹2,500 p.a.
- The rate of interest is 12% p.a.

Solution: $PV_P = \text{Annual Cash flow}/r$

Where,

Present value of perpetuity (PV_P) = ?

Annual Cash flow = ₹2,500

Rate of interest (r) = 12%

$$PV_P = ₹2,500 / .12 = ₹2,083.33$$

Example 25: Calculate the present value of repair expenses of ₹20,000 p.a. which are assumed to be incurred continuously at 8% p.a.

Solution: $PV_P = \text{Annual Cash flow}/r$

where,

Present value of perpetuity (PV_P) = ?

Annual Cash flow = ₹20,000

Rate of interest (r) = 8%

$$PV_P = \frac{₹20,000}{0.08} = ₹2,50,000$$

Example 26: Calculate the present value of perpetuity from the following data:

- A University has established an endowment fund that will generate ₹50,000 per year in scholarships.
- It earns at a rate of return of 5.5%.

Solution: $PV_P = \text{Annual Cash flow}/r$

where,

Present value of perpetuity (PV_P) = ?

Annual Cash flow = ₹50,000

Rate of interest (r) = 5.5%

$$PV_P = \frac{₹50,000}{5.5} = ₹9,090.91$$

4.5. DOUBLING PERIOD

Investors are generally concerned regarding the time likely to take place to double the value of their investment. Two fundamental rules, which govern the time taken for the investment value to double, are:

- Rule of 72:** The 'Rule of 72' is a simple way to determine how long an investment will take to double itself, at a fixed rate of interest. By dividing 72 by the annual 'Rate of Return', a rough estimate of how many years it will take for the initial investment to double itself may be obtained.

For example, in order to ascertain the number of years it will take to double any amount of the money at 12% rate of interest, 72 may be divided by 12. The result would be six years.

2020 (10) b-

Example 27: Mr. X deposited ₹5,000/- in a bank which pays 12% interest rate. How long Mr. X has to wait for the amount as per "rule 72".

Solution: Doubling Period under Rule 72

$$\text{Period} = \frac{72}{\text{Interest Rate}} = \frac{72}{12} = 6 \text{ years}$$

- Rule of 69:** A general rule estimating how long it will take for an investment to double, assuming continuously compounding interest. This may be calculated by dividing 69 by the 'Rate of Return'. According to this, the doubling period is equal to $0.35 + (69/\text{Interest rate})$. The formula is:

$$= 0.35 + (69/\text{Interest rate})$$

For example, if interest rate is 10%, then doubling period will be

$$= 0.35 + (69/10)$$

$$= 0.35 + 6.90 = 7.25 \text{ years}$$

4.6. SIMPLE INTEREST & COMPOUND INTEREST

4.6.1. Meaning of Simple Interest

Simple interest is the most basic type of interest. Simple interest is just the amount of money paid on the money borrowed. It is the easiest type of interest to calculate and understand. Simple interest is calculated only on the principal amount, or on that portion of the principal amount that remains unpaid. The amount of interest due is based on three factors – the principal, the rate of interest, and the time period during which the principal is used. One year is generally used as the time period. Simple interest is calculated on original principal only.

4.6.2. Calculation of Simple Interest

The amount of simple interest is calculated according to the following formula:

$$S.I. = \frac{P \times r \times n}{100} \text{ or } S.I. = P \times r \times n$$

Here,

S.I. = Simple Interest

P = Principal

r = Rate of Interest

n = Time Period

Therefore, the amount (A) is given as: **Amount = P + S.I.**

Although the time span of a loan may be given in days, months, or years, the rate of interest is an annual rate. Thus, when the duration of a loan is given in months or days, the time must be converted to years.

Example 28: If a bank accepts deposit of ₹1,00,000 for the period of 1 year under the interest rate 20%. What will be the simple interest?

$$\text{Solution: S.I.} = \frac{P \times r \times n}{100}$$

Given, $P = ₹1,00,000$, $r = 20\%$ or 0.20 , $n = 1$ year

$$\text{S.I.} = \frac{1,00,000 \times 20 \times 1}{100} = \frac{20,00,000}{200} = ₹20,000$$

Example 29: Find the simple interest and amount when:

- 1) Principal = ₹1,050, Rate = 7% p.a. and Time = $4\frac{1}{2}$ years.
- 2) Principal = ₹1,560, Rate = 10% p.a. and Time = 3 years 4 months.
- 3) Principal = ₹6,250, Rate = 1% per month and Time = 73 days.

Solution:

- 1) Given, $P = ₹1,050$, Rate = 7% p.a., Time = $4\frac{1}{2} = \frac{9}{2}$ years

$$\therefore \text{S.I.} = \frac{P \times r \times n}{100} = \frac{1,050 \times 7 \times 9}{100 \times 2} = \frac{66150}{200} = ₹330.75$$

$$\therefore \text{Amount} = P + \text{S.I.} = ₹1,050 + ₹330.75 = ₹1,380.75$$

- 2) Given, $P = ₹1,560$, $r = 10\%$ p.a., $n = 3$ years + $\frac{4}{12}$ years = $\frac{10}{3}$ years

$$\text{S.I.} = \frac{P \times r \times n}{100} = \frac{1,560 \times 10 \times 10}{3 \times 100} = \frac{156,000}{300} = ₹520$$

$$\therefore \text{Amount} = P + \text{S.I.} = ₹1,560 + ₹520 = ₹2,080$$

- 3) Given, $P = ₹6,250$, $r = 1\%$ per month = $(1 \times 12)\%$ p.a. = 12% p.a.,

$$n = 73 \text{ days} = \frac{73}{365} \text{ year} = \frac{1}{5} \text{ year}$$

$$\text{S.I.} = \frac{P \times r \times n}{100} = \frac{6,250 \times 12 \times 1}{100 \times 5} = \frac{75,000}{500} = ₹150$$

$$\therefore \text{Amount} = P + \text{S.I.} = ₹6,250 + ₹150 = ₹6,400$$

4.6.3. Meaning of Compound Interest

Compound interest is calculated each period on the original principal and all interest accumulated during past periods. This concept of adding accumulated interest back to the principal is called **compounding**. In these cases, the interest, as it falls due over a period of time, is again invested to earn further interest.

Thus, the interest is added to the principal to form a new principal which may be reinvested for the next time period. This process can be repeated for several time periods. Compound interest, rather than simple interest, must be used to properly evaluate long-term investment proposals.

The difference between the original principal and the amount at the end of the last time period is known as the **Compound Interest** on the original principal for the period. The time period after which interest is added each time to form a new principal is called the **conversion period**. It may be one year, six months, three months or one month and in these cases, the interest is said to be **compounded annually, semi-annually, quarterly or monthly**, respectively.

4.6.4. Calculation of Compound Interest

The compound interest formula for the amount 'A' accrued on a principal 'P' at an annual interest rate 'r' compounded annually for the period of 'n' years is given as:

$$A = P \left(1 + \frac{r}{100} \right)^n$$

The formula gives the amount (A), now compound interest can be determined as follows:

Compound Interest (C.I.) = A - P

$$= P \left(1 + \frac{r}{100} \right)^n - P \quad [\text{Using above equation}]$$

$$= P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right]$$

When interest is compounded annually, semi-annually, quarterly, monthly and daily; the **table 4.1** show the compounding period, frequency of conversion and related interest rate for different compounding assumptions,

Let, annual interest rate (r) is 12%.

Table 4.1: Interest Rate for Different Compounding Assumptions

Compounding Period	Frequency of Conversion	Related Interest Rate
Annually	1	12% = 0.12
Semi-annually	2	12% / 2 = 0.12 / 2 = 0.06
Quarterly	4	12% / 4 = 0.12 / 4 = 0.03
Monthly	12	12% / 12 = 0.12 / 12 = 0.01
Daily	365	12% / 365 = 0.12 / 365 = 0.00032

So, compound interest can be written as, $A = P(1+r)^n$

If the interest is compounded **k times in a year**, then the equation becomes:

$$A = P \left(1 + \frac{r}{k} \right)^{n \times k}$$

So, when the interest is **compounded semi-annually**, the amount after n years will be given as:

$$A = P \left(1 + \frac{r}{2} \right)^{2n}$$

When the interest is **compounded quarterly**, the amount after n years will be given as:

$$A = P \left(1 + \frac{r}{4} \right)^{4n}$$

Example 45: Mr. S purchased a flat for the amount of ₹5,00,000. For this purpose, he has taken a loan from HDFC at 8% p.a. You are required to compute EMI for the next 10 years and compare the EMIs computed under the reducing balance interest rate method and corresponding flat interest rate method.

Solution: Calculation of EMI

1) Reducing Balance Interest Rate

$$EMI = P \times \left(r + \frac{r}{(1+r)^n - 1} \right)$$

Where, P = Principle amount

r = Monthly interest rate

n = Time period in months

$$\begin{aligned} EMI &= 5,00,000 \left(\frac{0.08}{12} + \frac{\frac{0.08}{12}}{\left(1 + \frac{0.08}{12}\right)^{120} - 1} \right) \\ &= 5,00,000 \left(0.00667 + \frac{0.00667}{(1.00667)^{120} - 1} \right) \\ &= 5,00,000 \left(0.00667 + \frac{0.00667}{1.2205} \right) \\ &= 5,00,000 \times 0.01217 = 6,0085 \end{aligned}$$

Hence, EMI is ₹6,085

2) Flat Interest Rate

$$\begin{aligned} EMI &= \frac{P + \left(\frac{P \times r \times n}{100} \right)}{\text{Number of Months}} \\ &= \frac{5,00,000 + \left(\frac{5,00,000 \times 8 \times 10}{100} \right)}{10 \times 12} = ₹7,500 \end{aligned}$$

Hence, EMI is ₹7,500

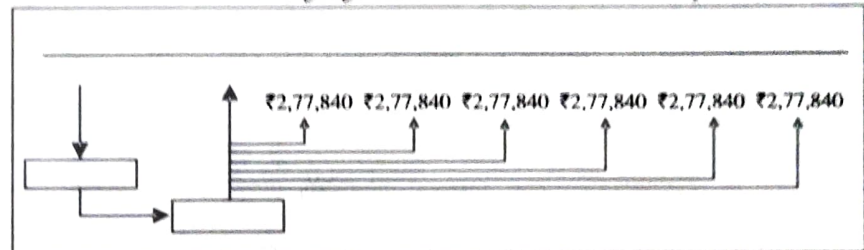
4.8.5. Loan Amortization Schedule

The term '**amortization**' can simply be used to refer to the act of settling or repaying debt over a specified period in the form of installments. A borrower taking loan is required to repay the amount borrowed (i.e., principal amount) together with the interest accrued thereon, over a specified tenure. This amount is repaid in the form of installments. In this way, '**amortization table or schedule**' or '**repayment schedule**' refers to a table reflecting the division between the principal and interest component of each EMI paid to settle or repay a loan. The amount of each EMI remains same throughout the loan tenure but when represented through amortization schedule, it can be observed that in the beginning of the tenure the amount of interest component is higher whereas at the end of the tenure the amount of principal component is higher.

Let's assume that a firm or a person takes out a loan of a specific amount at a given rate of interest. The borrower is

willing to repay the money in equal annual amount. The first installment is required to begin a couple of years after taking the loan. In such a case, the interest payable between the period of initial borrowing and the time of repayment is delayed. This amount of interest should be considered while calculating the annual amount to be paid.

For example, on a loan of ₹10,00,000, the interest is charged at the rate of 10%. The repayment of the loan is scheduled to start at the end of the third year from now. Following figure shows the amount to be repaid.



Following method are used for calculating the amount of annual payments after the end of the year 3 from now.

Step 1: In the first step, calculate the total amount which is due at the end of the 2nd year which corresponds with the start of the third year from now, using 10% rate. Following formula needs to be used for this purpose:

$$FV = PV (1+r)^n = ₹10,00,000 (1+0.10)^2 = ₹12,10,000$$

Step 2: The PV of the 6 year annuity at 10% interest rate is ₹12,10,000. Following equation may be used for calculating the annuity amount:

$$\begin{aligned} PV &= \text{Annuity Amount} \times PVAF_{(r, n)} \\ ₹12,10,000 &= \text{Annuity Amount} \times PVAF_{(10\%, 6)} \\ &= \text{Annuity Amount} (4.355) \end{aligned}$$

$$\begin{aligned} \text{Therefore, Annuity Amount} &= ₹12,10,000 / 4.355 \\ &= ₹2,77,840 \end{aligned}$$

The annuity amount is ₹2,77,840, which needs to be paid every year with effect from 3rd year for 6 years. The repayment is enough to cover the principle amount as well as the interest for eight year i.e. 6 years of actual annuity period and delayed time period of three years.

Example 46: A company named Vimal took a loan for purchasing a van costing ₹10,00,000 @ 14% interest rate. The loan has to be repaid over the next 5 years. Estimate the monthly instalment and prepare the loan amortisation schedule. 2020(10)

Solution: Annual payment may be calculated as follows:

$$₹10,00,000 = R[PVIFA]_{14\%, 5}$$

where,

R = Annual Installment Paid

PVIFA = Present Value Interest Factor of an Annuity at 14% and 5 years is 3.433

$$10,00,000 = R \times 3.433$$

$$R = \frac{10,00,000}{3.433} = ₹2,91,290 \text{ per annum to be paid over the course of 5 years}$$

Following is the calculation of monthly installment:

$$EMI = 2,91,290 / 12 = ₹24,274 \text{ per month.}$$

Table 4.2: Loan Amortisation Schedule

Year End (1)	Installment Payment (2)	Annual Interest [(5) _{t-1} × 0.14] (3)	Principal Payment [2] – [3] (4)	Principal Amount Outstanding [(5) _{t-1} – (3)] (5)
0	—	—	—	10,00,000
1	2,91,290	1,40,000	1,51,290	8,48,710
2	2,91,290	1,18,820	1,72,470	6,76,240
3	2,91,290	94,670	1,96,620	4,79,620
4	2,91,290	67,150	2,24,140	2,55,480
5	2,91,290	35,770	2,55,520	—

The above **table 4.2** shows that the calculated amount of ₹2,91,290 will finish off the entire loan and interest in five year. The installment amount calculated includes both principal and interest rate. The annual interest payable is calculated by charging 14% to the total outstanding amount of ₹10,00,000. The interest element may be deducted from instalment to calculate the contribution towards the payment of principal. The interest is charged only on the outstanding principal amount.

Example 47: Firm X took, ₹1,00,000 loan at the annual interest rate of 7%. Calculate the amount of EMI if the loan is required to be paid in 12 months equal annual instalments.

$$\begin{aligned}
 \text{Solution: EMI} &= P \times \left(r + \frac{r}{(1+r)^n - 1} \right) \\
 &= 1,00,000 \left(\frac{0.07}{12} + \frac{\frac{0.07}{12}}{\left(1 + \frac{0.07}{12}\right)^{12} - 1} \right) \\
 &= 1,00,000 \left(.005833 + \frac{.005833}{(1.005833)^{12} - 1} \right) \\
 &= 1,00,000 \left(.005833 + \frac{.005833}{1.07229 - 1} \right) \\
 &= 1,00,000 (.005833 + .0807) \\
 &= 1,00,000 \times .08653 = ₹8,653
 \end{aligned}$$

Principal payment for 1st month = 8,652 – (1,00,000 × 7%/12) = ₹8,069 and so on for other months.

Loan Amortisation Schedule

Months	Beginning Amount (₹) (1)	Annual Instalment (₹) (2)	Interest (₹) (3) = (1) × 7%/12	Principal Repayment (₹) (4) = (2) – (3)	Balance Outstanding (₹) (5) = (1) – (4)
1	1,00,000	8,653	583	8,069	91,931
2	91,931	8,653	536	8,116	83,814
3	83,814	8,653	489	8,164	75,650
4	75,650	8,653	441	8,211	67,439
5	67,439	8,653	393	8,259	59,180
6	59,180	8,653	345	8,307	50,872
7	50,872	8,653	297	8,356	42,517
8	42,517	8,653	248	8,405	34,112
9	34,112	8,653	199	8,454	25,658
10	25,658	8,653	150	8,503	17,155
11	17,155	8,653	100	8,553	8,602
12	8,602	8,653	50	8,602	0

Example 48: Y bought a T.V. costing ₹10,000 and agreeing to make equal annual payment for 5 years. How much would be each payment if the interest paid is 10% annually. Also draw loan amortisation schedule for the same.

Solution: In the present case we have present value of the annuity, i.e., ₹10,000 and we have to calculate equal annual payment over the period of five years.

Here, $n = 5$ and $r = 0.10$

$PV = \text{Annuity Amount} \times PVA_{F(r, n)}$

₹10,000 = Annuity Amount $\times PVA_{F(10\%, 5)}$

$$= \frac{10,000}{3.791} = ₹2,637.83$$

(Note: 3.791 is calculated by using the Present Value of Annuity Table)

Therefore, each payment would be ₹3,432.05

Loan Amortisation Schedule

Year	Beginning Amount (₹) (1)	Annual Instalment (₹) (2)	Interest (₹) (3) = (1) × 10%	Principal Repayment (₹) (4) = (2) – (3)	Balance Outstanding (₹) (5) = (1) – (4)
1	10,000	2,637.83	1,000	1,637.83	8,362.17
2	8,362.17	2,637.83	836.22	1,801.61	6,560.56
3	6,560.56	2,637.83	656.06	1,981.77	4,578.79
4	4,578.79	2,637.83	457.88	2,179.95	2,398.84
5	2,398.84	2,637.83	239.88	2,398	0.84 (approx to 0)

4.9. EXERCISE

4.9.1. Very Short Answer Type Questions

- 1) What is Time Value of Money?
- 2) What is simple interest?
- 3) What is compound interest?
- 4) How the 'future value' of an 'annuity' is calculated?
- 5) What are sinking funds?

4.9.2. Short Answer Type Questions

- 1) What are the reasons for time value of money?
- 2) Explain the concept of annuity due with an example.
- 3) Write a note on annuity due.
- 4) Explain the two discounting techniques.
- 5) Discuss the concept of sinking funds.

4.9.3. Long Answer Type Questions

- 1) Explain the present and future of annuity with illusionary figure.
- 2) Elaborate the techniques of time value of money.
- 3) Discuss the concept of capital recovery and loan amortization with an example.

4.9.4. Practical Questions

Time Value of Money

- 1) Calculate the future value of an investment of ₹500 for a period of 3 years that pays an interest rate of 6% compounded semi-annually.

[Ans: ₹597.03]