Course: Transmission and Distribution– BEE402

Module-3: Performance of Transmission Lines

Dr. Shakunthala C
Associate Professor
Dept. of Electrical and Electronics Engineering
ATME College of Engineering, Mysuru

Classification of Overhead Transmission Lines

- A transmission line has three constants R, L and C distributed uniformly along the whole length of the line.
- The resistance and inductance form the series impedance.
- The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line.
- Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as :

  (i) Short transmission lines

  (ii) Medium transmission lines

  (iii) Long transmission lines

Short transmission lines :

- When the length of an overhead transmission line is upto about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line.
- Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected.
- Therefore, while studying the performance of a short transmission line, only resistance and inductance of the line are taken into account.

Medium transmission lines:

- When the length of an overhead transmission line is about 50-150 km and the line voltage is moderately high (>20 kV < 100 kV), it is considered as a medium transmission line.
- Due to sufficient length and voltage of the line, the capacitance effects are taken into account.
- For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

Long transmission lines :

- When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line.
- For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution

Voltage regulation:

- When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line.
- The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S).
- This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called ***Voltage Regulation***.
- The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called *voltage regulation* and is expressed as a percentage of the receiving end voltage.

*** At no load, there is no drop in the line so that at no load, $V_R = V_S$. However, at full load, there is a voltage drop in the line so that receiving end voltage is V_R.*

∴ Difference in voltage at receiving end between no load and full load

$$= V_S - V_R$$

Mathematically, $\% \text{ age Voltage regulation} = \dfrac{V_S - V_R}{V_R} \times 100$

Obviously, it is desirable that the voltage regulation of a transmission line should be low *i.e.*, the increase in load current should make very little difference in the receiving end voltage.

Transmission efficiency

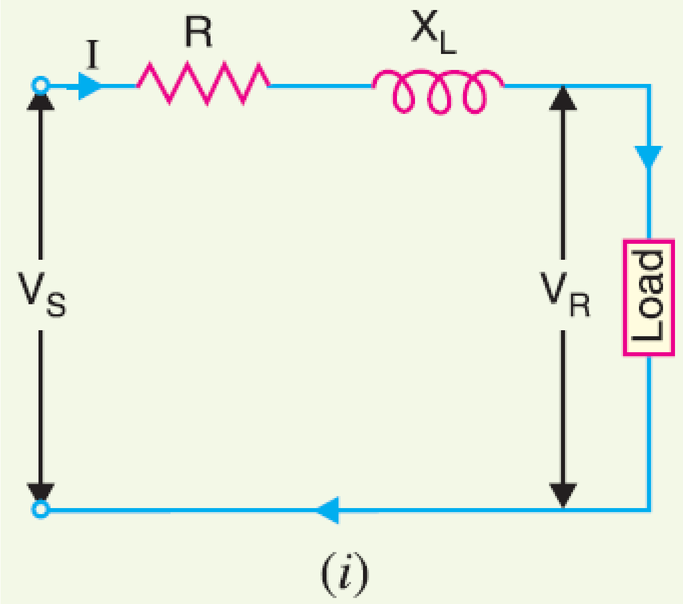
- The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line.
- The ratio of receiving end power to the sending end power of a transmission line is known as the ***transmission efficiency*** of the line i.e.

$$\% \text{ age Transmission efficiency, } \eta_T = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$$= \frac{V_R I_R \cos \phi_R}{V_S I_S \cos \phi_S} \times 100$$

Performance of Single Phase Short Transmission Lines

The equivalent circuit of a single phase short transmission line is shown in Fig 3.1 (i). Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.



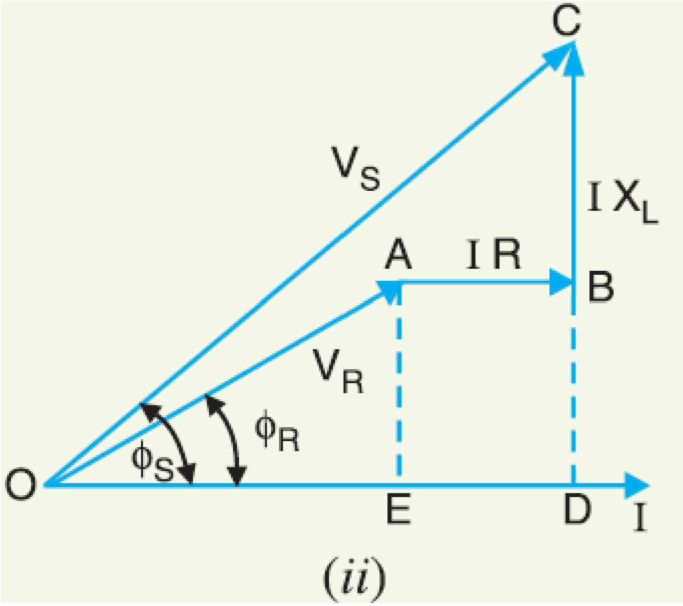


Fig 3.1

The phasor diagram of the line for lagging load power factor is shown in Fig 3.1 (ii).

Let I = load current

R = loop resistance i.e., resistance of both conductors

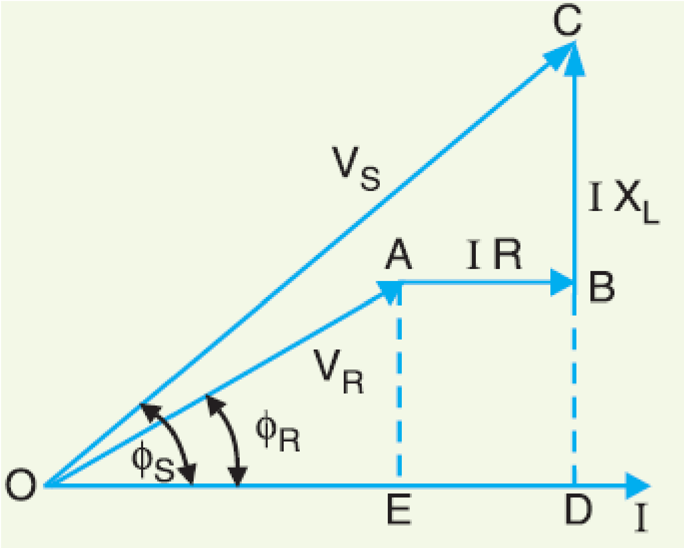
X_L = loop reactance

V_R = receiving end voltage

$\cos \varphi_R$ = receiving end power factor (lagging)

V_S = sending end voltage

$\cos \varphi_S$ = sending end power factor



(ii)

Phasor diagram. Current I is taken as the reference phasor. OA represents the receiving end voltage V_R leading I by ϕ_R. AB represents the drop IR in phase with I. BC represents the inductive drop IX_L and leads I by 90°. OC represents the sending end voltage V_S and leads I by ϕ_S.

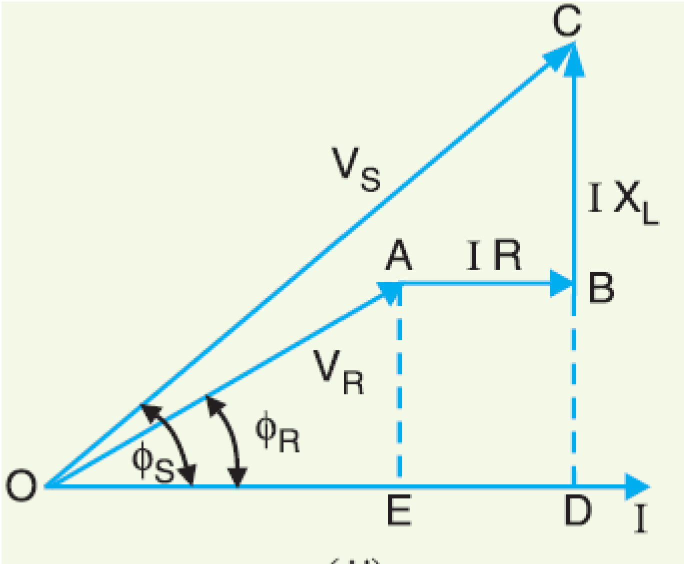
From the right angled triangle *ODC*, we get,

$$(OC)^2 = (OD)^2 + (DC)^2$$

$$V_S^2 = (OE + ED)^2 + (DB + BC)^2$$

$$= (V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2$$

$$V_S = \sqrt{(V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2}$$



(*ii*)

(*i*) %age Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100$

(*ii*) Sending end *p.f.*, $\cos\phi_S = \dfrac{OD}{OC} = \dfrac{V_R \cos\phi_R + IR}{V_S}$

(*iii*) Power delivered $= V_R I_R \cos\phi_R$

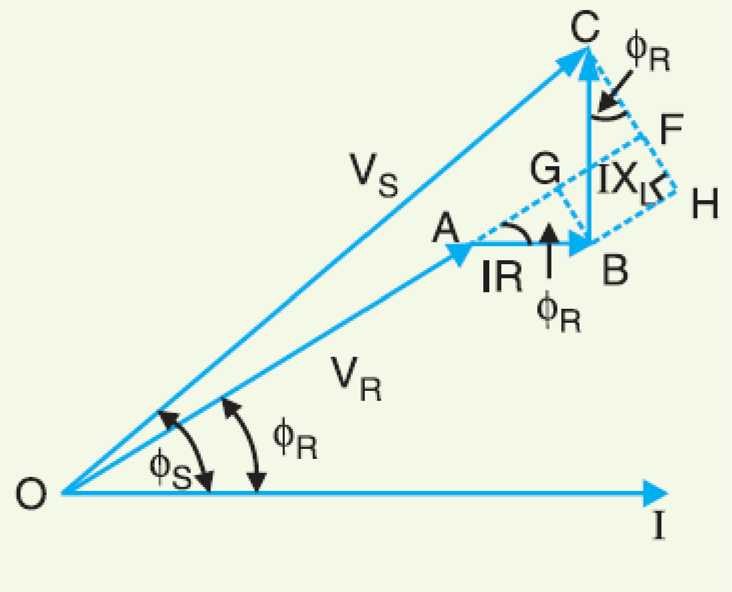
Line losses $= I^2 R$

Power sent out $= V_R I_R \cos\phi_R + I^2 R$

$$\%\text{age Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent out}} \times 100$$

$$= \frac{V_R\, I_R \cos\phi_R}{V_R\, I_R \cos\phi_R + I^2 R} \times 100$$

An approximate expression for the sending end voltage *VS* can be obtained as follows. Draw perpendicular from *B* and *C* on *OA* produced as shown in Fig. 10.2. Then *OC* is *nearly* equal to *OF*



$$\text{i.e.,}\quad OC = OF = OA + AF = OA + AG + GF$$

$$= OA + AG + BH$$

$$V_S = V_R + IR\cos\phi_R + IX_L \sin\phi_R$$

Formula for Voltage Regulation in Short Transmission Line

$$\%\text{age Voltage regulation} = \frac{I R \cos \phi_R + I X_L \sin \phi_R}{V_R} \times 100 \quad (\textit{for lagging p.f.})$$

$$\%\text{age Voltage regulation} = \frac{I R \cos \phi_R - I X_L \sin \phi_R}{V_R} \times 100 \quad (\textit{for leading p.f.})$$

Example 5.5.2 *A 3-phase short transmission line delivers 3 MW at a p.f. of 0.8 lagging to a load. If the sending end voltage is 33 kV, determine : (i) Receiving end voltage; (ii) Line current; (iii) Transmission efficiency; (iv) Regulation. The resistance and reactance of each conductor are 5 Ω and 8 Ω respectively.* **VTU : Jan.-15,18, July-19, Marks 10**

Solution : Resistance, R = 5 Ω, Reactance, X_L = 8 Ω

$$\cos\phi_R = 0.8 \text{ lagging}; \quad \phi_R = \cos^{-1} 0.8 = 36.86°, \quad \sin\phi_R = 0.6$$

Let V_R be receiving end phase voltage.

Sending end line voltage, $V_{S_L} = 33$ kV $\quad V_S = \dfrac{V_{S_L}}{\sqrt{3}} = \dfrac{33\times 10^3}{\sqrt{3}} = 19052.55$ V

$$\text{Line current,} \quad I_L = \frac{\text{Power delivered / Phase}}{V_R \times \cos\phi_R} = \frac{(3\times 10^6)/3}{V_R \times 0.8} = \frac{1250000}{V_R}$$

The approximate expression for V_S is given by,

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R = V_R + I\left[R\cos\phi_R + X_L\sin\phi_R\right]$$

$$19052.55 = V_R + \frac{1250000}{V_R}\left[(5)\ (0.8) + (8)\ (0.6)\right]$$

$$19052.55\,V_R = V_R^2 + 1250000\ [4 + 4.8] = V_R^2 + 11000000$$

$$V_R^2 - 19052.55\ V_R + 11 \times 10^6 = 0$$

$$V_R = \frac{19052.55 \pm \sqrt{(19052.55)^2 - (4)\ (11 \times 10^6)}}{2}$$

$$V_R = \frac{19052.55 \pm 17860.56}{2} = 595.99\text{ V}, 18.45 \times 10^3\text{ V}$$

$V_R = 18.45 \times 10^3$ V $=$ **18.45 kV** (Neglecting other root)

Line value of receiving voltage, $V_{R_L} = \sqrt{3}\ V_R$ $=$ **31.95 kV**

$$\% \text{ regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{19052.55 - 18.45 \times 10^3}{18.45 \times 10^3} \times 100 = \mathbf{3.266}$$

$$\text{Line current} = \frac{1250000}{V_R} = \frac{1250000}{18.45 \times 10^3} = \mathbf{67.75\ A}$$

$$\text{Line losses} = 3\, I^2 R = 3(67.75)^2\ (5) = 68.85 \times 10^3\ W$$

$$\% \text{ Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power delivered + Losses}} \times 100$$

$$= \frac{3 \times 10^6}{3 \times 10^6 + 68.85 \times 10^3} \times 100 = \mathbf{97.75\ \%}$$

Example 5.7.2 *A 3 phase line delivers 5000 kW at 22 kV and at a p.f. of 0.8 lagging to a load. Determine i) Sending end voltage ii) % Regulation iii) Transmission efficiency. The resistance and reactance of each conductor is 4 Ω and 6 Ω respectively.*

VTU : Aug.-11, July-17, Marks 8

Solution : Resistance of each conductor, R = 4 Ω ,Reactance of each conductor ,X_L = 6 Ω

$$\cos\phi_R = 0.8 \text{ lagging} \quad V_R \text{ per phase} = \frac{22000}{\sqrt{3}} = 12701.70 \text{ V}$$

Let V_R be the receiving end voltage

$$\text{Line current,} \quad I = \frac{\text{Power delivered / phase}}{V_R \times \cos\phi_R} = \frac{5000/3\times10^3}{V_R \times 0.8} = 164 \text{ A}$$

The approximate expression for V_S is given by

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$

If $\cos\phi_R = 0.8$ then we have $\sin\phi_R = 0.6$

$$V_S = \frac{12701.70}{(164)} \times (0.8) \times 4 + (164) \times 6 \times (0.6) = 13816.9 \text{ V}$$

Line voltge at sending end is given as,

$$V_{S\ line} = \sqrt{3} \cdot 13.82 = \mathbf{23.93\ kV}$$

$$\%\text{ Regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{13.82 - 12.70}{12.70} \times 100 = \mathbf{8.82\ \%}$$

$$\text{Line losses} = 3\ I^2R = 3\ (164)^2\ (4) = 322.752 \text{ kW}$$

$$\%\text{ Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power delivered} + \text{Losses}} \times 100$$

$$= \frac{5000}{5000 + 322.752} \times 100 = \mathbf{93.94\ \%}$$

Medium Transmission Lines

- In medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected.
- Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.
- The capacitance is uniformly distributed over the entire length of the line.
- However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points.
- The most commonly used methods (known as localised capacitance methods) for the solution of medium transmissions lines are :

  (i) End condenser method **(ii) Nominal T method** **(iii) Nominal π method.**

End Condenser Method

- In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Figure.
- This method of localising the line capacitance at the load end overestimates the effects of capacitance.
- In Figure one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.

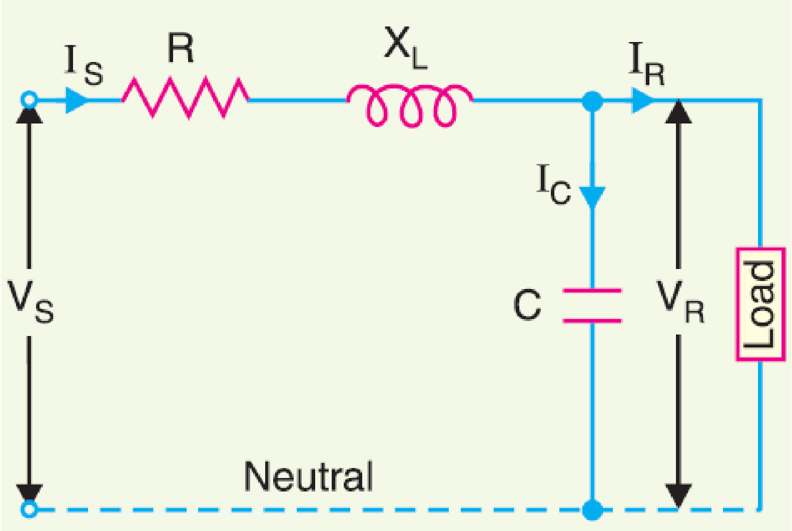
Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

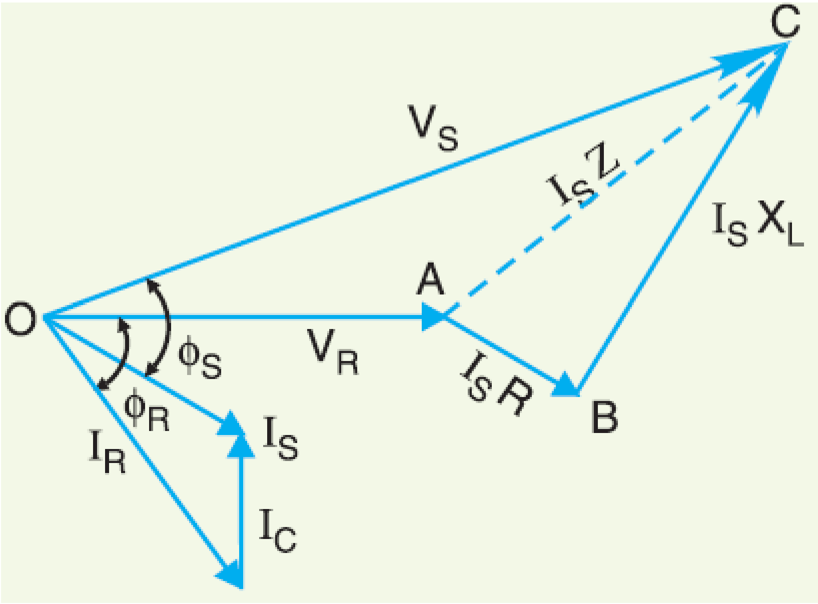
C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*)



V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig



Taking the receiving end voltage $\vec{V_R}$ as the reference phasor,

we have, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\vec{I_C} = j\,\vec{V_R}\,\omega C = j\,2\pi f C\,\vec{V_R}$

The sending end current $\vec{I_S}$ is the phasor sum of load current $\vec{I_R}$ and capacitive current $\vec{I_C}$ *i.e.*,

$$\vec{I_S} = \vec{I_R} + \vec{I_C}$$

$$= I_R(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C\,V_R$$

$$= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi f C V_R)$$

Voltage drop/phase $= \vec{I}_S \vec{Z} = \vec{I}_S (R + jX_L)$

Sending end voltage, $\vec{V}_S = \vec{V}_R + \vec{I}_S \vec{Z} = \vec{V}_R + \vec{I}_S (R + jX_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\% \text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\% \text{ Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase} + \text{losses / phase}} \times 100$$

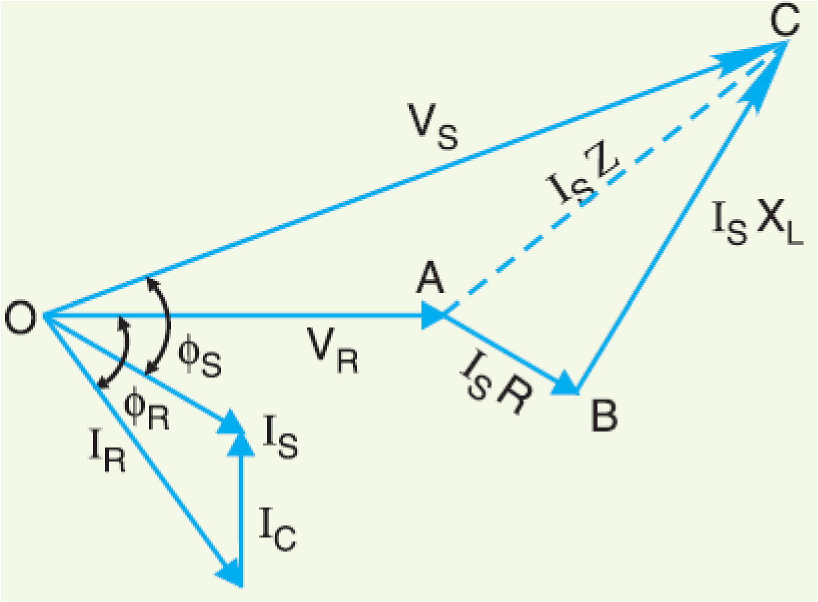
$$= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I_S^2 R} \times 100$$

Limitations.

Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

(*i*) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(*ii*) This method overestimates the effects of line capacitance.



Note the construction of phasor diagram. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The capacitive current $\vec{I_C}$ leads $\vec{V_R}$ by 90° as shown. The phasor sum of $\vec{I_C}$ and $\vec{I_R}$ is the sending end current $\vec{I_S}$. The drop in the line resistance is $\vec{I_S}R$ (AB) in phase with $\vec{I_S}$ whereas inductive drop $\vec{I_S}X_L$ (BC) leads I_S by 90°. Therefore, OC represents the sending end voltage $\vec{V_S}$. The angle ϕ_S between the sending end voltage $\vec{V_S}$ and sending end current $\vec{I_S}$ determines the sending end power factor $\cos\phi_S$.

Example 5.8.1 *A medium single phase transmission line 100 km long has the following constants*

Resistance / km /phase = 0.15 Ω

Inductive reactance / km / phase = 0.377 Ω

Capacitive reactance / km / phase = 31.87 Ω

Receiving end line voltage = 132 kV

Assuming that the total capacitance of the line is localised at the receiving end alone determine

i) Sending end current   ii) Line value of sending end voltage

iii) Regulation   iv) Sending end power factor.

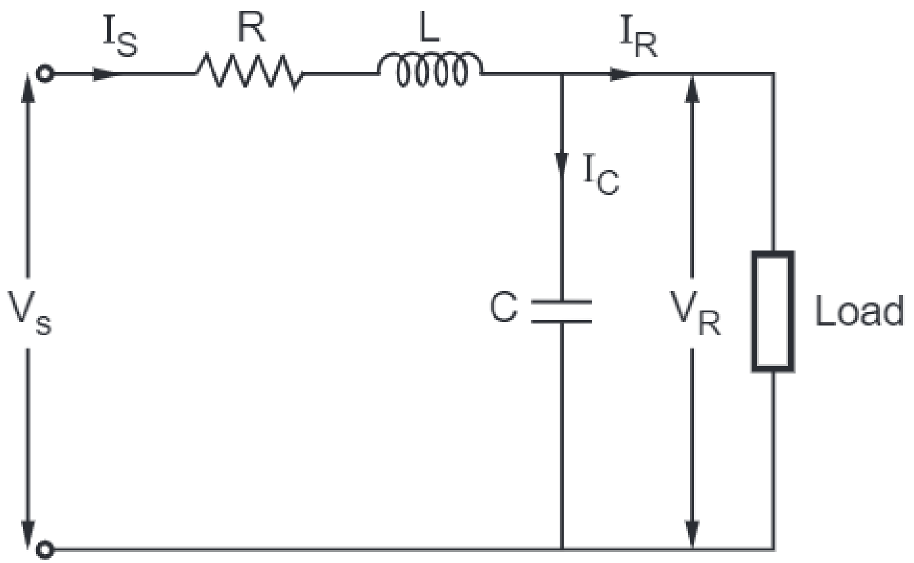
The line is delivering 72 MW at 0.8 p.f. lagging.

Solution : Total resistance $= 0.15 \times 100 = 15\ \Omega$

Total inductive reactance $= 0.377 \times 100 = 37.7\ \Omega$

Total capacitive reactance $= 31.87 \times 100 = 3187\ \Omega$

V_R line $= 132$ kV



Capacitive current I_C is given by,

$$\bar{I}_C = j\,\omega\,C\,V_R$$

$$= j\frac{V_R}{X_C} = j\frac{132\times 10^3}{3187}$$

$$= j\,41.41\ \text{A}$$

Receiving end power $= V_R\,I_R \cos\phi_R$

$$I_R = \frac{72\times 10^6}{132\times 10^3\times 0.8} = 681.81\ \text{A}$$

$$\bar{I}_R = 681.81 \angle -\cos^{-1} 0.8 = 681.81 \angle -36.86° \text{ A}$$

$$\bar{I}_S = \bar{I}_R + \bar{I}_C = [681.81 \angle -36.86°] + [j\, 41.41] = \mathbf{657.79 \angle -34°}$$

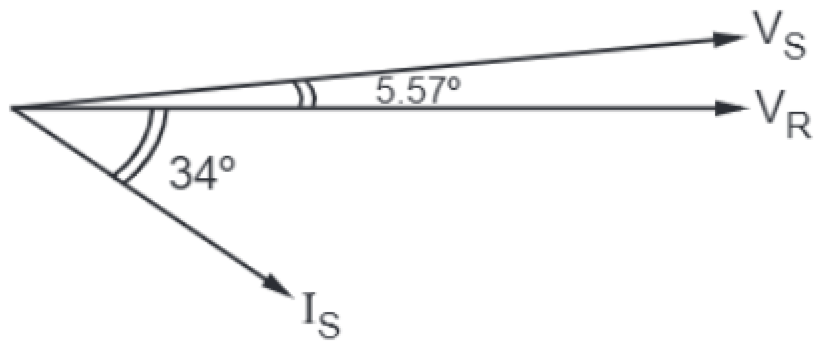
$$\text{Sending end voltage, } \bar{V}_S = \bar{V}_R + \bar{I}_S \bar{Z}$$

$$= [132000 + j0] + [657.79 \angle -34°]\,[15 + j\, 37.7]$$

$$= 154778.03 \text{ volts} \angle 5.57° = \mathbf{154.778 \text{ kV}}$$

$$\text{Sending end power factor} = \cos[34° + 5.57°] = \mathbf{0.77 \text{ lag}}$$

$$\therefore \quad \% \text{ Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

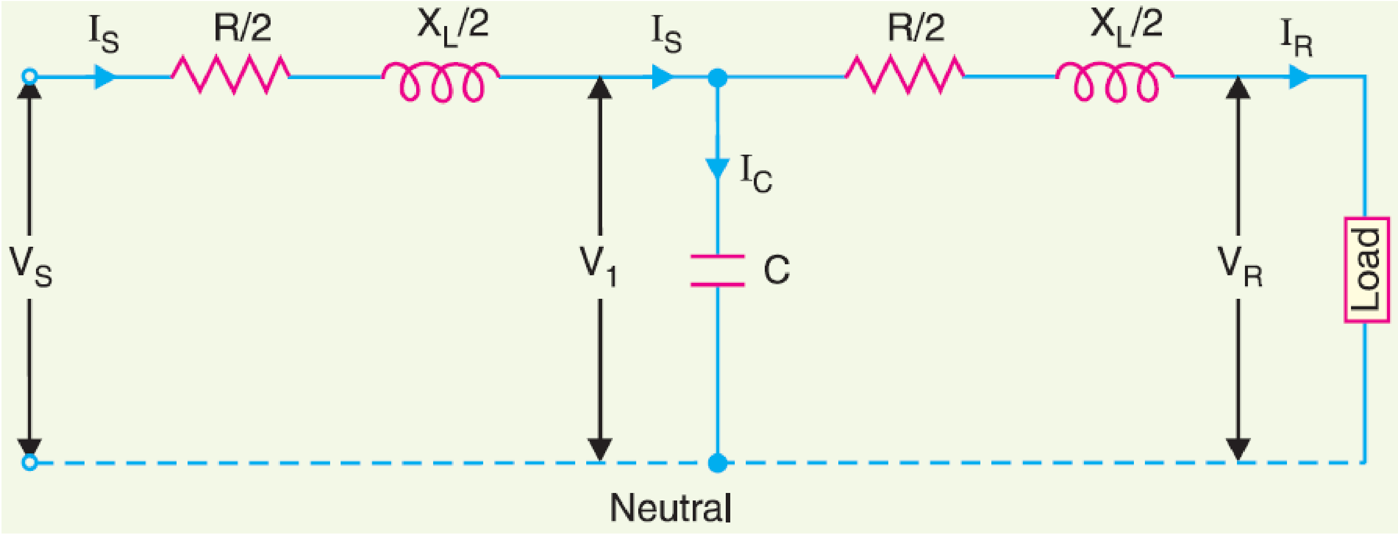


$$= \frac{154.77 - 132}{132} \times 100$$

$$= \mathbf{17.25 \, \%}$$

Nominal T Method

- In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig.
- Therefore, in this arrangement, full charging current flows over half the line.



Let I_R = load current per phase ; R = resistance per phase

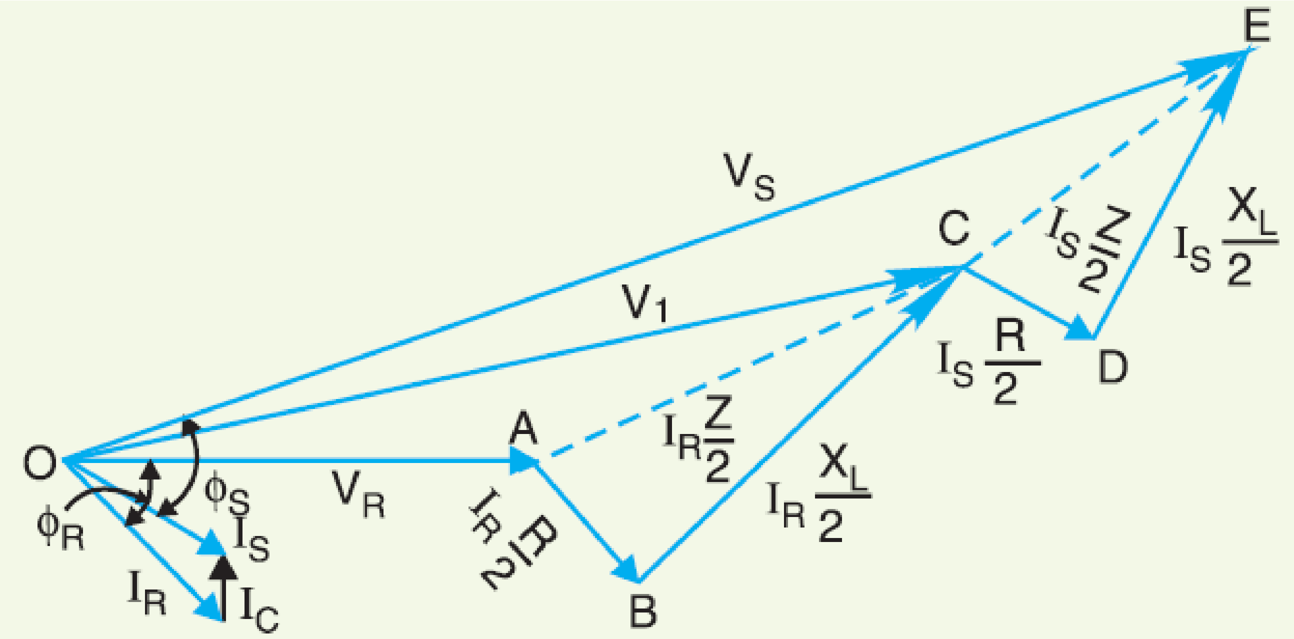
X_L = inductive reactance per phase ; C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase

V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig.

Taking the receiving end voltage $\vec{V}_R$ as the reference phasor, we have,



* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by OA. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The drop $AB = I_R R/2$ is in phase with $\vec{I_R}$ and $BC = I_R \cdot X_L/2$ leads $\vec{I_R}$ by 90º. The phasor OC represents the voltage $\vec{V_1}$ across condenser C. The capacitor current $\vec{I_C}$ leads $\vec{V_1}$ by 90º as shown. The phasor sum of $\vec{I_R}$ and $\vec{I_C}$ gives $\vec{I_S}$. Now $CD = I_S R/2$ is in phase with $\vec{I_S}$ while $DE = I_S X_L/2$ leads $\vec{I_S}$ by 90º. Then, OE represents the sending end voltage $\vec{V_S}$.

Receiving end voltage, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current, $\vec{I_C} = j\,\omega\,C\,\vec{V_1} = j\,2\pi f\,C\,\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\frac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\frac{R}{2} + j\frac{X_L}{2}\right)$

Example 5.8.3 *A 3 phase, 50 Hz overhead transmission line 100 km long has the following constants.*

Resistance/km/phase = 0.1 Ω

Inductance reactance/km/phase = 0.2 Ω

Capacitive susceptance/km/phase = 0.4×10^{-14} siemen

Determine i) Sending end current ii) Sending end voltage

iii) Sending end power factor

iv) Transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV with p.f. of 0.8 lagging. Use nominal T method.

VTU : Aug.-01,06,07, Jan.-05,06, Feb.-09, Marks 10

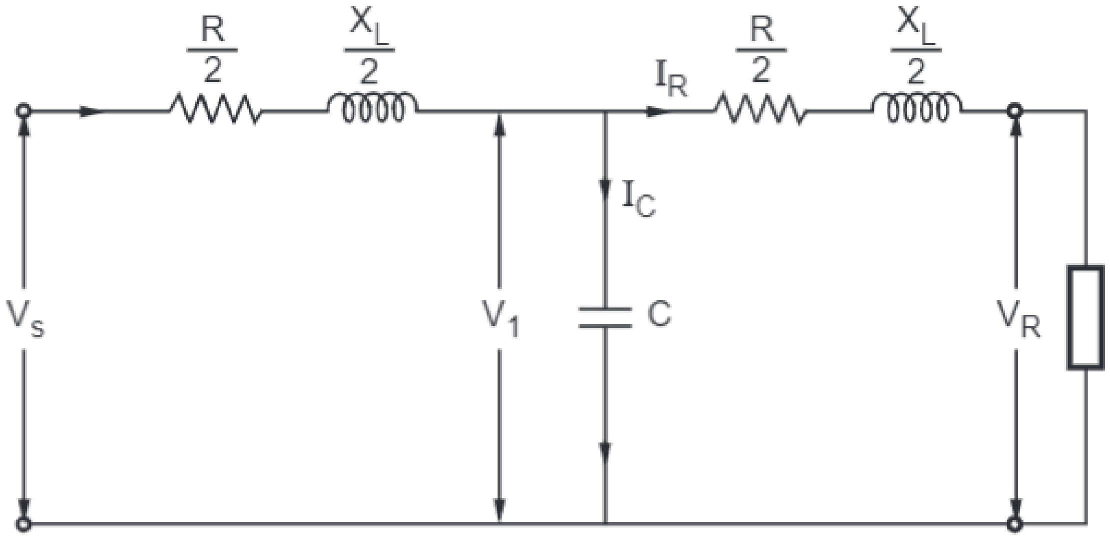
Solution :

$$R = 0.1 \times 100 = 10\ \Omega$$

$$\text{Total } X_L = 0.2 \times 100 = 20\ \Omega$$

$$\text{Total } Y_C = 0.4 \times 10^{-14} \times 100 = 4 \times 10^{-13} \text{ mho}$$

$$\bar{V}_R = V_R \angle 0^\circ = \frac{66000}{\sqrt{3}} \angle 0^\circ = 38105.118 \angle 0^\circ \text{ volts}$$



$$\text{Power delivered} = \sqrt{3}\, V_R I_R \cos \phi_R$$

$$\text{i.e.} \quad I_R = \frac{10{,}000 \times 10^3}{\sqrt{3} \times 66000 \times 0.8} = 109.34 \text{ A}$$

$$\bar{I}_R = 109.34 \angle -\cos^{-1} 0.8 = 109.34 \angle -36.86^\circ \text{ A}$$

$$\bar{V}_1 = \bar{V}_R + \bar{I}_R\left(\frac{\bar{Z}}{2}\right) = \bar{V}_R + \bar{I}_R\left[\frac{R}{2} + j\frac{X_L}{2}\right] = [38105.118 \angle 0^\circ] + [109.34 \angle -36.86^\circ]\left[\frac{10}{2} + j\frac{20}{2}\right]$$

$$= 39202.241 \angle 0.7991^\circ \text{ volts}$$

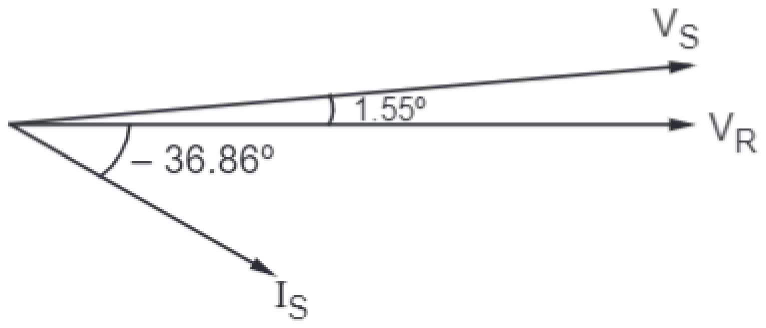
$$V_1 = 39.202 \text{ kV}$$

$$\bar{I}_C = j Y_C \bar{V}_1 = j\,[4 \times 10^{-13}]\,[39202.241 \angle 0.7991^\circ] = 1.5680 \times 10^{-8} \angle 90.7991^\circ$$

$$\bar{I}_S = \bar{I}_C + \bar{I}_R = [1.5680 \times 10^{-8} \angle 90.7991^\circ] + [109.34 \angle -36.86^\circ] = 109.34 \angle -36.86^\circ \text{ A}$$

$$\bar{V}_S = \bar{V}_1 + \bar{I}_S\left(\frac{\bar{Z}}{2}\right) = \bar{V}_1 + \bar{I}_S\left[\frac{R}{2} + j\frac{X_L}{2}\right] = 40306.56 \angle 1.55^\circ \text{ vol}$$

$$V_S = \mathbf{40.306\ kV}$$



Line value of sending end voltage $= \sqrt{3} \times 40.306 = \mathbf{69.81\ kV}$

p.f. angle between V_S and I_S $= 1.55^\circ + 36.86^\circ = 38.41^\circ$

p.f. at sending end $= \cos(38.41^\circ) = \mathbf{0.7835\ lagging}$

$$\text{Total line losses} = 3\,I_S^2\frac{R}{2} + 3\,I_R^2\frac{R}{2} = 3\,\frac{R}{2}\,[I_S^2 + I_R^2] = 3 \times \frac{10}{2}\,[(109.34)^2 + (109.34)^2] = 388.657 \text{ kW}$$

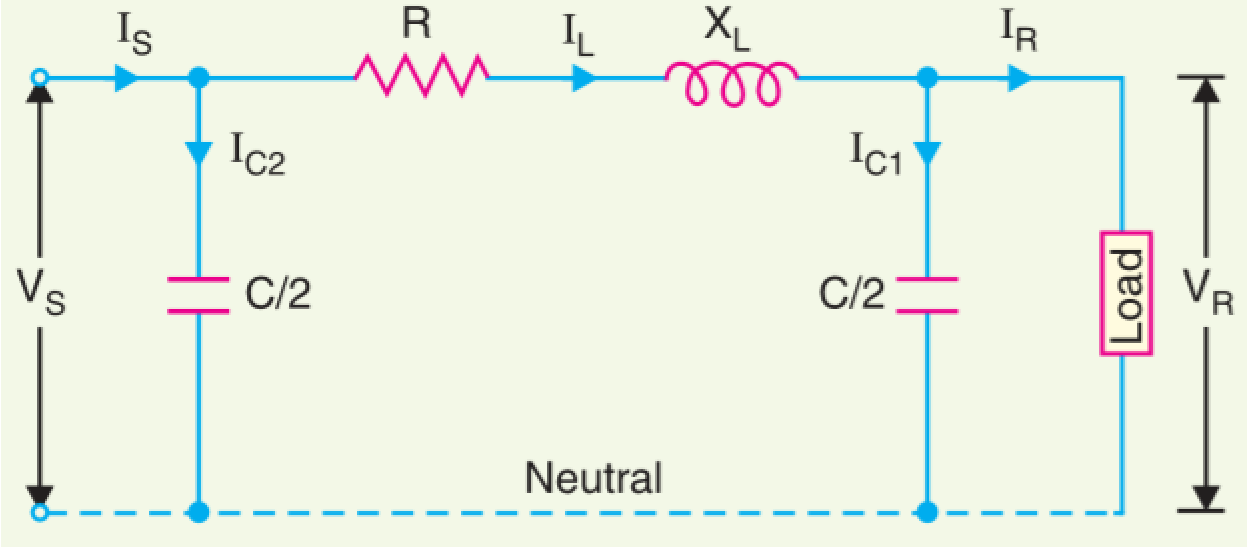
$$\%\ \text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power delivered} + \text{Losses}} \times 100$$

$$= \frac{10000}{10000 + 388.657} \times 100 = \mathbf{96.25\ \%}$$

Nominal π Method

- In this method, capacitance of each conductor (i.e., line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig.
- It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.

I_R = load current per phase
R = resistance per phase
X_L = inductive reactance per phase
C = capacitance per phase
$\cos \phi_R$ = receiving end power factor (*lagging*)
V_S = sending end voltage per phase



The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage as the reference phasor, we have,

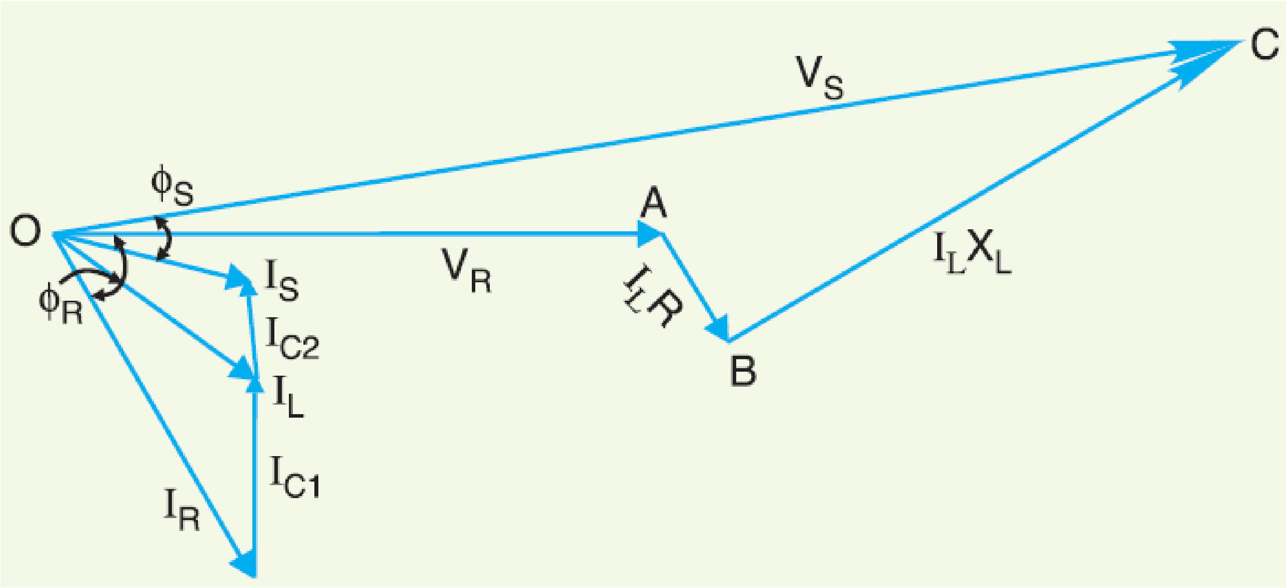
$$\vec{V_R} = V_R + j\,0$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Charging current at load end is

$$\vec{I_{C1}} = j\,\omega\,(C/2)\,\vec{V_R} = j\,\pi\,f\,C\,\vec{V_R}$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}}$

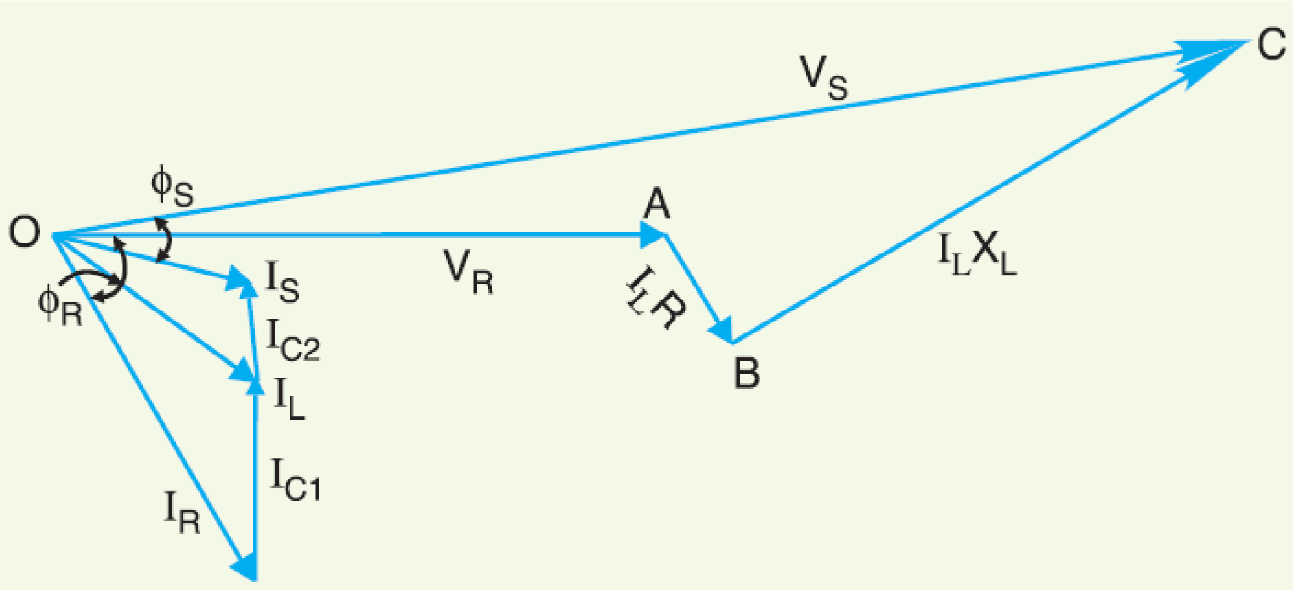


Sending end voltage,

$$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + jX_L)$$

Charging current at the sending end is $\vec{I_{C2}} = j\,\omega\,(C/2)\,\vec{V_S} = j\,\pi\,f\,C\,\vec{V_S}$

$\therefore$ Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$



Note the construction of phasor diagram. $\overrightarrow{V_R}$ is taken as the reference phasor represented by OA. The current $\overrightarrow{I_R}$ lags behind $\overrightarrow{V_R}$ by ϕ_R. The charging current $\overrightarrow{I_{C1}}$ leads $\overrightarrow{V_R}$ by 90°. The line current $\overrightarrow{I_L}$ is the phasor sum of $\overrightarrow{I_R}$ and $\overrightarrow{I_{C1}}$. The drop $AB = I_L R$ is in phase with $\overrightarrow{I_L}$ whereas drop $BC = I_L X_L$ leads $\overrightarrow{I_L}$ by 90°. Then OC represents the sending end voltage $\overrightarrow{V_S}$. The charging current $\overrightarrow{I_{C2}}$ leads $\overrightarrow{V_S}$ by 90°. Therefore, sending end current $\overrightarrow{I_S}$ is the phasor sum of the $\overrightarrow{I_{C2}}$ and $\overrightarrow{I_L}$. The angle ϕ_S between sending end voltage V_S and sending end current I_S determines the sending end p.f. $\cos \phi_S$.

Example 5.8.2 A 3 phase, 50 Hz, 150 km transmission line has the following constants.

Resistance / phase / km = 0.1 Ω

Reactance / phase / km = 0.5 Ω

Capacitive shunt admittance / phase / km = 3×10^{-6} mho

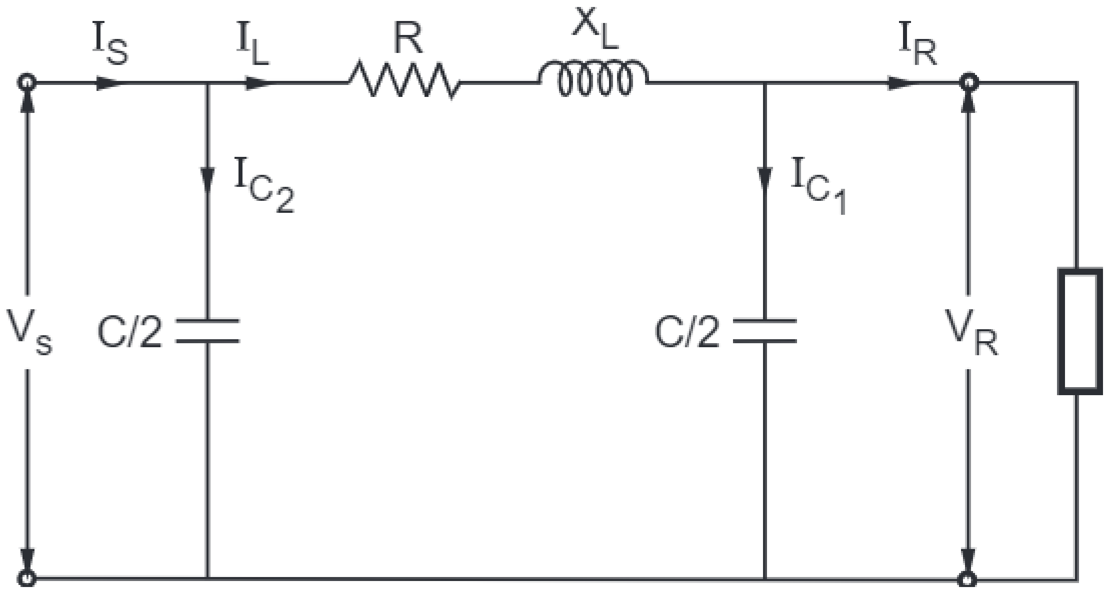
If the line supplies a load of 50 MW at 0.8 p.f. lagging at 110 kV at the receiving end calculate by using nominal π method.

i) Sending end current   ii) Sending end voltage

iii) Sending end power factor

VTU : Feb.-10, July-16, Jan.-19, Marks 10

Solution : The equivalent circuit using nominal π method is shown in the Fig.



$$\text{Total } R = 0.1 \times 150 = 15\ \Omega$$

$$\text{Total } X_L = 0.5 \times 150 = 75\ \Omega$$

$$\text{Total } Y_C = 3 \times 10^{-6} \times 150 = 4.5 \times 10^{-4} \text{ mho}$$

$$\therefore \quad \frac{Y_C}{2} = \frac{4.5 \times 10^{-4}}{2}$$

$$V_R = \frac{110 \times 10^3}{\sqrt{3}} = 63508.52 \text{ V}$$

Receiving end load current, $I_R = \dfrac{\text{Power delivered}}{\sqrt{3} \times V_R \times \cos\phi_R} = \dfrac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0.8} = 328 \text{ A}$

$$\bar{I}_R = 328 \angle -\cos^{-1} 0.8 = 328 \angle -36.86^\circ \text{ A}$$

$$\bar{I}_{C_1} = j\,(2\pi\ f)\,\frac{C}{2}\,\bar{V}_R \quad = j\,\omega\,\frac{C}{2}\,\bar{V}_R$$

Now, $$\bar{I}_{C_1} = j\,\frac{Y_C}{2}\,\bar{V}_R \quad = j\left(\frac{4.5\times10^{-4}}{2}\right)(63508.52\ \angle\ 0^\circ) \quad = j\ 14.29\ \text{A}$$

$$\bar{I}_L = \bar{I}_R + \bar{I}_{C_1} \quad = [328\ \angle -36.86^\circ] + [j\ 14.29] \quad = 319.62\ \angle -34.80^\circ\ \text{A}$$

i.e. $I_L = 319.62$ A

$$\bar{V}_S = \bar{V}_R + \bar{I}_L\ \bar{Z} \quad = \bar{V}_R + \bar{I}_L\ [R + j\,X_L]$$

$$= [63508.52 + j0] + [319.62\ \angle -34.80^\circ]\ [15 + j\ 75]$$

$$= 82876.17\ \angle\ 11.8^\circ\ \text{volts}$$

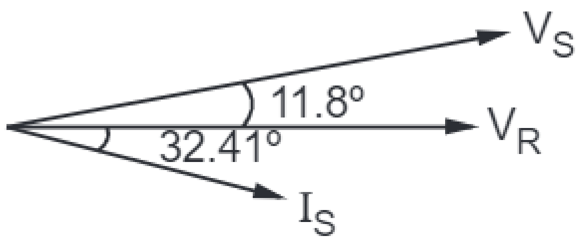
i.e. $V_S =$ **82.87 kV**

Line value of sending end voltage $= \sqrt{3}\times 82.87 \quad =$ **143.53 kV**

$$\bar{I}_{C_2} = j\frac{Y_C}{2}\bar{V}_S = j\left(\frac{4.5\times10^{-4}}{2}\right)(81125 + j\ 16946.8) = 18.64 \angle 101.79\ °A$$

$$\bar{I}_S = \bar{I}_L + \bar{I}_{C_2} = [319.62 \angle -34.80°] + [18.64 \angle 101.79°] = 306.34 \angle -32.41°A$$

i.e. $I_S =$ **306.34 A**



Angle between V_S and $I_S = 11.8° + 32.41°$

$$\phi_S = 44.21°$$

Sending end p.f. $= \cos\phi_S = \cos 44.21°$

$=$ **0.7167 lagging**

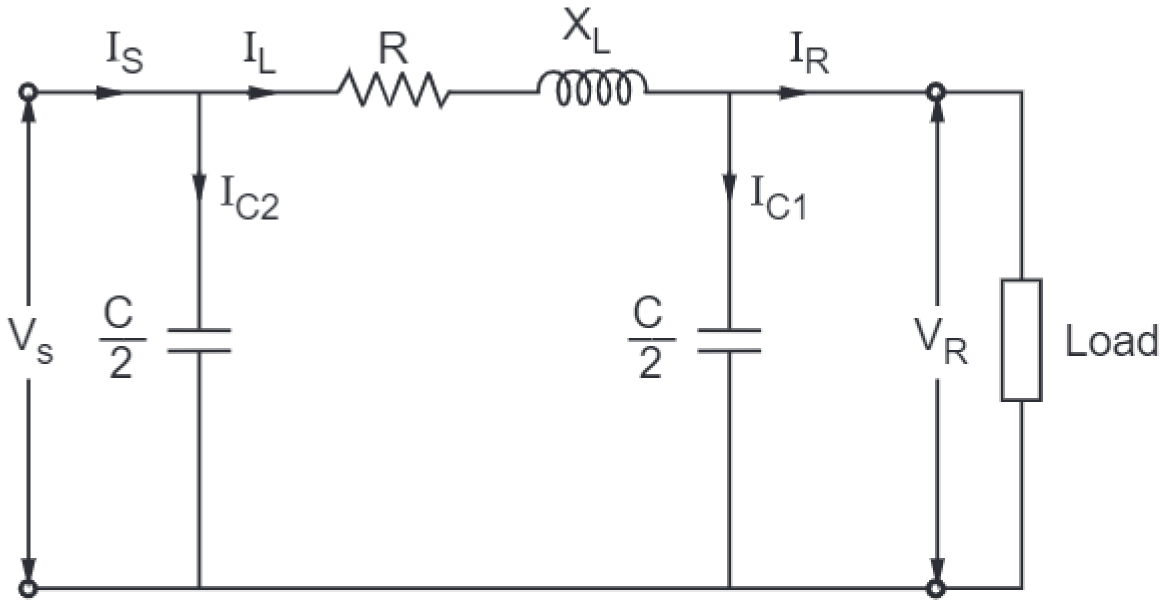
Example 5.8.7 *A 3 phase 50 Hz overhead transmission line has the following constants per phase $R = 28\ \Omega$, $X = 63\ \Omega$, $Y = 4 \times 10^{-4}$ (℧). If the load at the receiving end is 75 MVA at 0.8 pf lag with 132 kV between lines, calculate the voltage, current and pf at the sending end. Use nominal π method.* **VTU : July-12,16, Jan.-14,16, Marks 10**

Solution : $R = 28\ \Omega, \quad X_L = 63\ \Omega$

$$Y_C = 4 \times 10^{-4} \text{ mho}$$

$$\frac{Y_C}{2} = \frac{4 \times 10^{-4}}{2} = 2 \times 10^{-4}$$

$$V_R = \frac{132 \times 10^3}{\sqrt{3}} = 76.21 \text{ kV}$$



Receiving end load current, $I_R = \dfrac{\text{Power delivered}}{\sqrt{3} \times V_R \times \cos\phi_R} = \dfrac{75 \times 10^6 \times 0.8}{\sqrt{3} \times 132 \times 10^3 \times 0.8}$

$$\bar{I}_R = 328 \angle -\cos^{-1} 0.8 = 328 \angle -36.86° \text{ A}$$

$$\bar{I}_{C_1} = j\pi f C \bar{V}_R = j \frac{\omega C}{2} \bar{V}_R$$

Now, $X_C = \dfrac{1}{\omega C}$; $\omega C = \dfrac{1}{X_C}$ $\quad \therefore \quad \dfrac{\omega C}{2} = \dfrac{1}{2 X_C} = \dfrac{Y_C}{2}$

$$\bar{I}_{C_1} = j \frac{Y_C}{2} \bar{V}_R = j\,(2 \times 10^{-4})(76.21 \times 10^3 \angle 0°) = j\, 15.242 \text{ A}$$

$$\bar{I}_L = \bar{I}_{C_1} + \bar{I}_R = [j\, 15.242] + [328 \angle -36.86°] = 319.08 \angle -34.66° \text{ A}$$

$$\bar{V}_S = \bar{V}_R + \bar{I}_L \bar{Z} = \bar{V}_R + \bar{I}_L (R + j X_L) = (76210 + j\,0) + (319.08 \angle -34.66°)(28 + j63)$$

$$= 95.679 \angle 6.87 \text{ kV} \quad \therefore \quad V_S = \mathbf{95.67\ kV}$$

Line value of sending end voltage $= \sqrt{3} \times V_S \quad = \sqrt{3} \times 95.67 \quad = 165.70$ kV

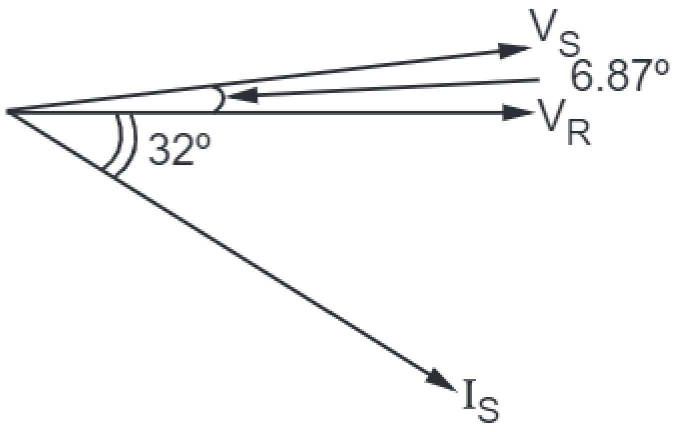
$$\bar{I}_{C_2} = j \frac{Y_C}{2} \bar{V}_S = j\,(2\times10^{-4})(94991.87 + j11451) = 19.12 \angle 96.87°A$$

$$\bar{I}_S = \bar{I}_L + \bar{I}_{C_2} = [262.43 - j\,181.51] + [-2.29 + j\,18.99] = 306.73 \angle -32° \text{ A}$$

$$\therefore \quad I_S = \mathbf{306.73\ A}$$

Angle between V_S and I_S $= 6.87° + 32°$

$$\therefore \quad \phi_s = 38.87°$$



$\therefore$ p.f. at sending end $= \cos \phi_s = \cos 38.87$

$$= \mathbf{0.7785\ lag}$$

Long Transmission Lines

- It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line.
- However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped.
- If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations.
- Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line.

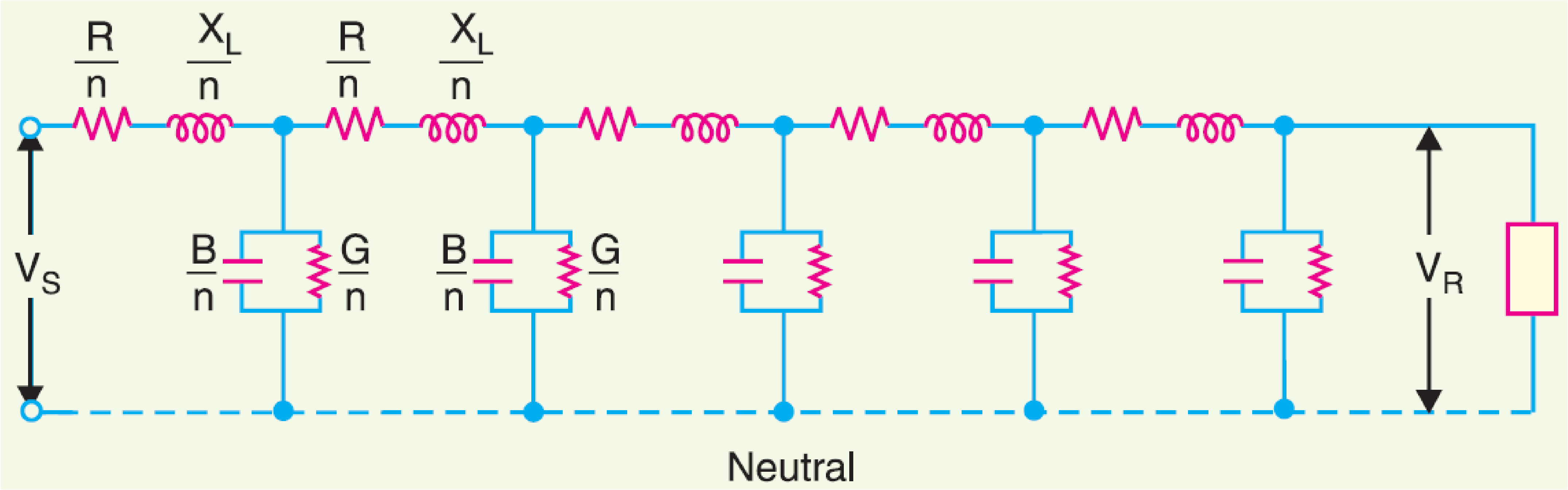


Fig. shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into *n* sections, each section having line constants $\frac{1}{n}th$ of those for the whole line.

The following points may by noted :

(*i*) The line constants are uniformly distributed over the entire length of line as is actually the case.

(*ii*) The resistance and inductive reactance are the series elements.

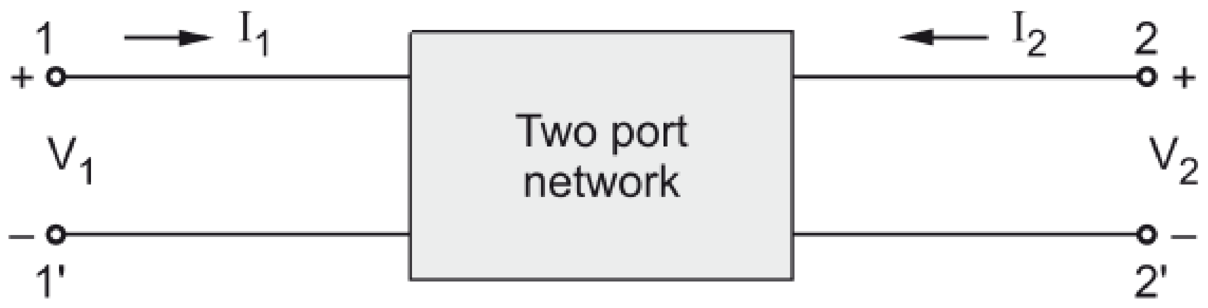
(*iii*) The leakage susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors.

$$\text{Admittance} = \sqrt{G^2 + B^2}$$

(*iv*) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

Generalised Circuit Constants of a Transmission Line

Consider a two port network shown in the Fig.



The network is a four terminal network with 4 variables; two on input side and two on output side. V_1 and I_1 are the voltage and current on input side whereas V_2 and I_2 are the voltage and current on output side.

The transmission system can also be assumed to be a four terminal network with two input terminals where power enters the network and two output terminals where power leaves the network.

The sending end voltage and current can be expressed in terms of receiving end voltage and current through the set of parameters known as transmission line parameters or ABCD parameters. Thus we have,

$$\bar{V}_S = \bar{A}\,\bar{V}_R + \bar{B}\,\bar{I}_R \quad \text{and} \quad \bar{I}_S = \bar{C}\,\bar{V}_R + \bar{D}\,\bar{I}_R$$

The parameters $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$ which are generally complex numbers are the constants also known as generalised circuit constants. The method which is used for analysis of transmission line has influence on these constants. With the knowledge of these constants, performance calculations of the line can be easily obtained.

These parameters are given by

$$\bar{A} = \left.\frac{\bar{V}_S}{\bar{V}_R}\right|_{\bar{I}_R = 0} \quad ; \quad \bar{C} = \left.\frac{\bar{I}_S}{\bar{V}_R}\right|_{\bar{I}_R = 0}$$

$$\bar{B} = \left.\frac{\bar{V}_S}{\bar{I}_R}\right|_{\bar{V}_R = 0} \quad ; \quad \bar{D} = \left.\frac{\bar{I}_S}{\bar{I}_R}\right|_{\bar{V}_R = 0}$$

The constants $\bar{A}$ and $\bar{D}$ are dimensionless as they are simply ratios whereas constants $\bar{B}$ and $\bar{C}$ are having units of ohm and mho respectively.

If the network is **symmetrical** then we have $\bar{A} = \bar{D}$

If the network is reciprocal then we have $\bar{A}\,\bar{D} - \bar{B}\,\bar{C} = 1$

Determination of Generalised Constants for Transmission Lines

As stated previously, the sending end voltage (V_S) and sending end current (I_S) of a transmission line can be expressed as :

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R} \qquad \ldots(i)$$

$$\vec{I_S} = \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R} \qquad \ldots(ii)$$

We shall now determine the values of these constants for different types of transmission lines.

(i) Short lines.

- In short transmission lines, the effect of line capacitance is neglected.
- Therefore, the line is considered to have series impedance. Fig. shows the circuit of a 3-phase transmission line on a single phase basis.

Here, $\vec{I}_S = \vec{I}_R$ ...(*iii*)

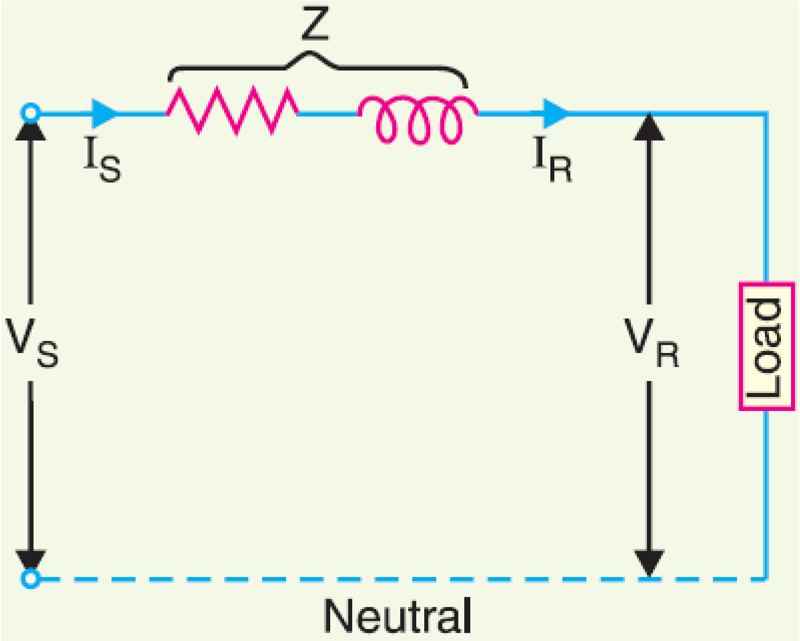
$\vec{V}_S = \vec{V}_R + \vec{I}_R \vec{Z}$ ...(*iv*)

Comparing these with eqs. (*i*) and (*ii*), we have,

$\vec{A} = 1$; $\vec{B} = \vec{Z}$, $\vec{C} = 0$ and $\vec{D} = 1$

Incidentally ; $\vec{A} = \vec{D}$

$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = 1 \times 1 - \vec{Z} \times 0 = 1$



(*ii*) Medium lines – Nominal T method.

- In this method, the whole line to neutral capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on either side as shown in Fig.

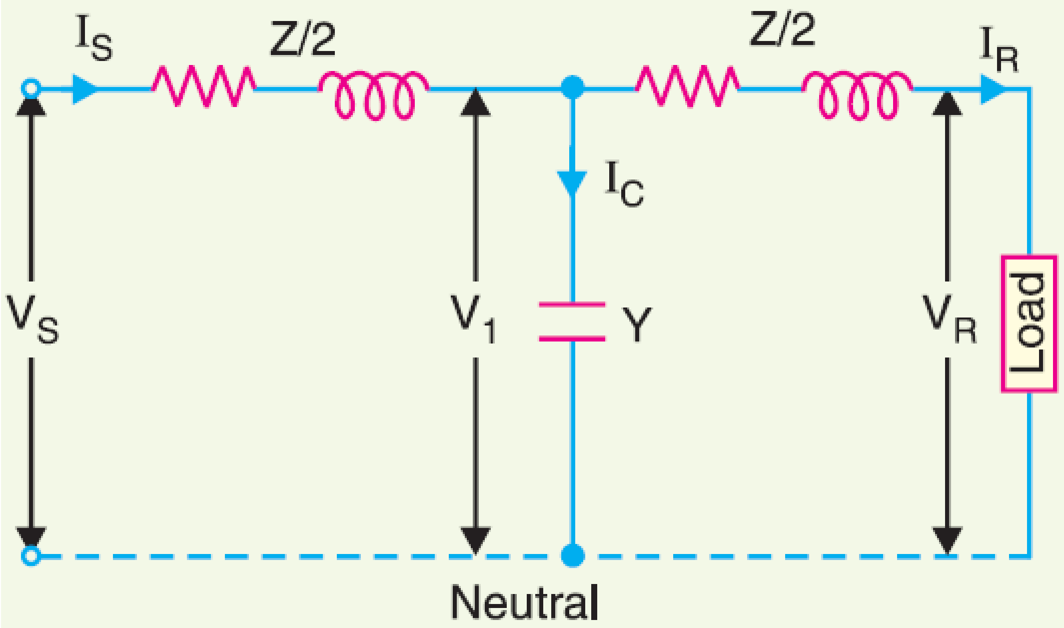
Here, $\vec{V}_S = \vec{V}_1 + \vec{I}_S \vec{Z}/2 \quad \ldots(v)$

and $\vec{V}_1 = \vec{V}_R + \vec{I}_R \vec{Z}/2$

Now, $\vec{I}_C = \vec{I}_S - \vec{I}_R$

$= \vec{V}_1 \vec{Y}$ where Y = shunt admittance

$$= \vec{Y}\left(\vec{V}_R + \frac{\vec{I}_R \vec{Z}}{2}\right)$$



$$\vec{I}_S = \vec{I}_R + \vec{Y}\,\vec{V}_R + \vec{Y}\,\frac{\vec{I}_R\,\vec{Z}}{2}$$

$$= \vec{Y}\,\vec{V}_R + \vec{I}_R\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) \qquad \ldots(vi)$$

Substituting the value of V_1 in eq. (v), we get,

$$\vec{V}_S = \vec{V}_R + \frac{\vec{I}_R\,\vec{Z}}{2} + \frac{\vec{I}_S\,\vec{Z}}{2}$$

Substituing the value of I_S, we get,

$$\vec{V}_S = \left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right)\vec{V}_R + \left(\vec{Z} + \frac{\vec{Y}\,\vec{Z}^2}{4}\right)\vec{I}_R \qquad \ldots(vii)$$

Comparing eqs. (vii) and (vi) with those of (i) and (ii), we have,

$$\vec{A} = \vec{D} = 1 + \frac{\vec{Y}\,\vec{Z}}{2} \;;\quad \vec{B} = \vec{Z}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right) \;;\; \vec{C} = \vec{Y}$$

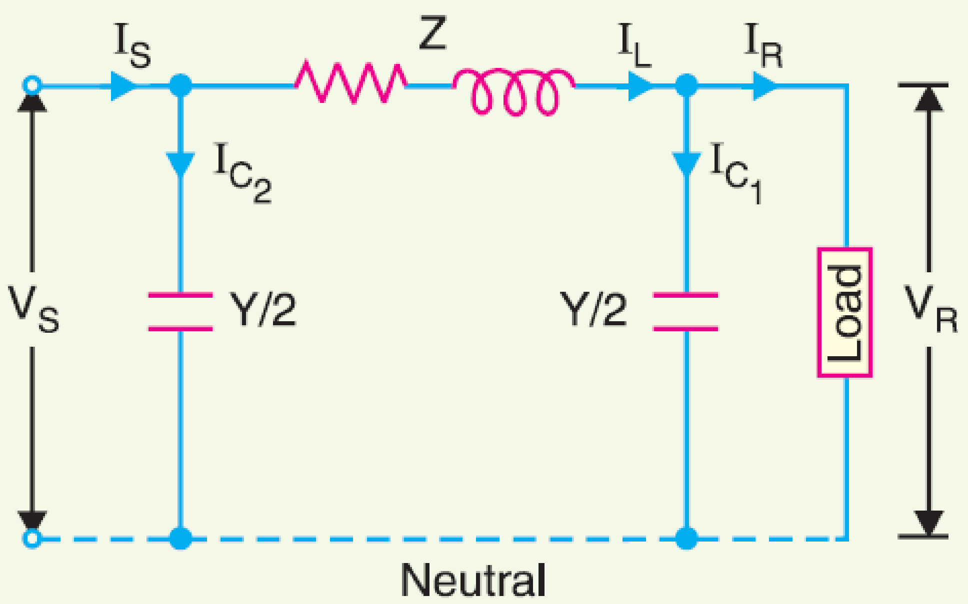
$$Incidentally:\; \vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1 + \frac{Y\,Z}{2}\right)^2 - Z\left(1 + \frac{Y\,Z}{4}\right)Y$$

$$= 1 + \frac{Y^2 Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2 Y^2}{4}$$

$$= 1$$

(*iii*) Medium lines—Nominal π method.

- In this method, line-to-neutral capacitance is divided into two halves ; one half being concentrated at the load end and the other half at the sending end as shown in Fig.



Here, $\vec{Z} = R + jX_L$ = series impedenace/phase

$\vec{Y} = j\,\omega\,C$ = shunt admittance

$$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$$

or $\vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2 \quad \ldots(viii)$

Also $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = \vec{I_R} + \vec{V_R}\,\vec{Y}/2 \quad \ldots(ix)$

$$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z}$$

$$= \vec{V_R} + \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right)\vec{Z} \quad \text{(Putting the value of } \vec{I_L}\text{)}$$

$$\vec{V}_S = \vec{V}_R\left(1+\frac{\vec{Y}\,\vec{Z}}{2}\right)+\vec{I}_R\,\vec{Z} \qquad \ldots(x)$$

$$\vec{I}_S = \vec{I}_L+\vec{V}_S\,\vec{Y}/2 = \left(\vec{I}_R+\vec{V}_R\,\vec{Y}/2\right)+\vec{V}_S\,\vec{Y}/2 \qquad \text{(Putting the value of } \vec{I}_L\text{)}$$

Putting the value of $\vec{V}_S$ from eq. (x), we get,

$$\vec{I}_S = \vec{I}_R+\vec{V}_R\frac{\vec{Y}}{2}+\frac{\vec{Y}}{2}\left\{\vec{V}_R\left(1+\frac{\vec{Y}\,\vec{Z}}{2}\right)+\vec{I}_R\,\vec{Z}\right\}$$

$$= \vec{I}_R+\vec{V}_R\frac{\vec{Y}}{2}+\frac{\vec{V}_R\,\vec{Y}}{2}+\frac{\vec{V}_R\,\vec{Y}^2\,\vec{Z}}{4}+\frac{\vec{Y}\,\vec{I}_R\,\vec{Z}}{2}$$

$$= \vec{I}_R\left(1+\frac{\vec{Y}\,\vec{Z}}{2}\right)+\vec{V}_R\,\vec{Y}\left(1+\frac{\vec{Y}\,\vec{Z}}{4}\right) \qquad \ldots(xi)$$

Comparing equations (*x*) and (*xi*) with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \left(1 + \frac{\vec{Y}\vec{Z}}{2}\right) ; \quad \vec{B} = \vec{Z}; \quad \vec{C} = \vec{Y}\left(1 + \frac{\vec{Y}\vec{Z}}{4}\right)$$

Also

$$\vec{A}\vec{D} - \vec{B}\vec{C} = \left(1 + \frac{Y Z}{2}\right)^2 - Z Y \left(1 + \frac{Y Z}{4}\right)$$

$$= 1 + \frac{Y^2 Z^2}{4} + Y Z - Z Y - \frac{Z^2 Y^2}{4} = 1$$

Example 5.9.1 *A 110 kV, 50 Hz, 3 phase transmission line delivers a load of 40 MW at 0.85 lagging p.f. at the receiving end. The generalised constants of the transmission line are A = D = 0.95∠ 1.4°, B = 96 ∠ 78° ohm, C = 0.0015 ∠ 90° mho*

Find the regulation of the line and charging current use nominal T method.

VTU : July-14, Jan.-18, Marks 7

Solution : Receiving end voltage, $V_R = 110$ kV

$$\text{Receiving end phase voltage} = \frac{110\times10^3}{\sqrt{3}} = 63.50 \text{ kV}$$

$$\text{Receiving end current, } I_R = \frac{\text{Power at receiving end}}{\sqrt{3}\times V_R \times \cos\phi_R} = \frac{40\times10^6}{\sqrt{3}\times110\times10^3\times0.85} = 247 \text{ A}$$

$$\bar{I}_R = 247\angle -\cos^{-1} 0.85 = 247\angle -31.78^\circ \text{ A}$$

Now, $\bar{V}_S = \bar{A}\,\bar{V}_R + \bar{B}\,.\,\bar{I}_R$

$$= (0.95 \angle 1.4°)\ (63508.53 \angle 0°) + (96 \angle 78°)\ (247 \angle -31.78°)$$

$$= 78942.301 \angle 13.62°\ V$$

$$\therefore \quad V_S = 78.942\ kV$$

Sending end current,

$$\bar{I}_S = \bar{C}\,\bar{V}_R + \bar{D}\,\bar{I}_R$$

$$= (0.0015 \angle 90°)\ (63508.53 \angle 0°) + (0.95 \angle 1.4°)\ (247 \angle -31.78°)$$

$$= 203.77 \angle -6.5959°\ A$$

We have for nominal T method

$$\bar{I}_S = \bar{I}_C + \bar{I}_R$$

$$\therefore \text{ Charging current, } \bar{I}_C = \bar{I}_S - \bar{I}_R = [203.77 \angle -6.5959°] - [247 \angle -31.78°]$$

$$= 106.93 \angle 94.04° \text{ A}$$

$$I_C = \mathbf{106.93 \ A}$$

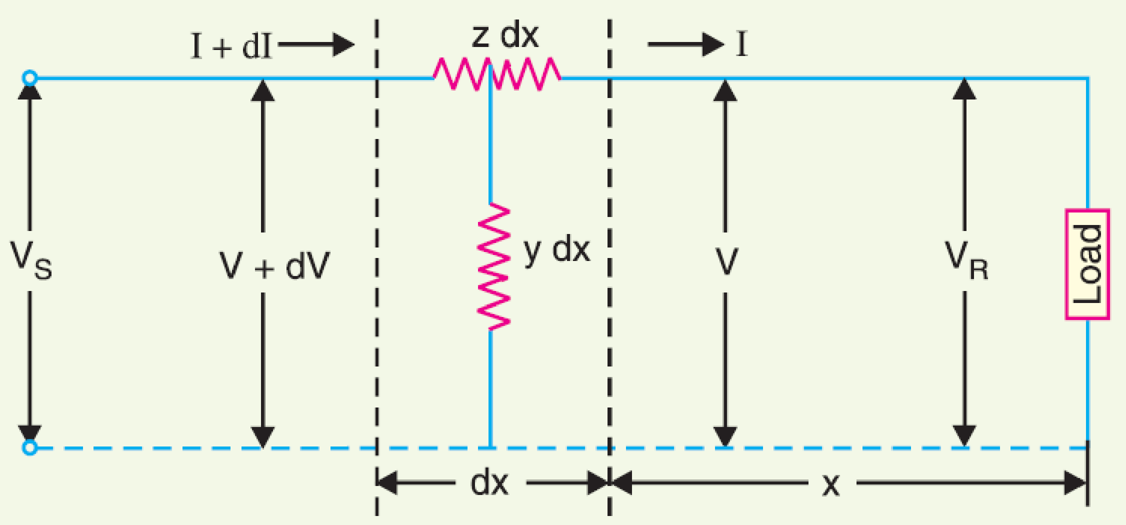
$$\% \text{ Voltage regulation} = \frac{V_{Ro} - V_R}{V_R} \times 100 = \frac{(V_S / A) - V_R}{V_R} \times 100$$

$$= \frac{(78.942 / 0.95) - 63.50}{63.50} \times 100 = \mathbf{30.86 \ \%}$$

Analysis of Long Transmission Line (Rigorous method)

VTU : Feb.-05, July-11,12,17,19, Dec.-11, Jan.-05,12

- Fig. shows one phase and neutral connection of a 3-phase line with impedance and shunt admittance of the line uniformly distributed.
- Consider a small element in the line of length *dx* situated at a distance *x* from the receiving end.



Let

z = series impedance of the line per unit length

y = shunt admittance of the line per unit length

V = voltage at the end of element towards receiving end

$V + dV$ = voltage at the end of element towards sending end

$I + dI$ = current entering the element dx

I = current leaving the element dx

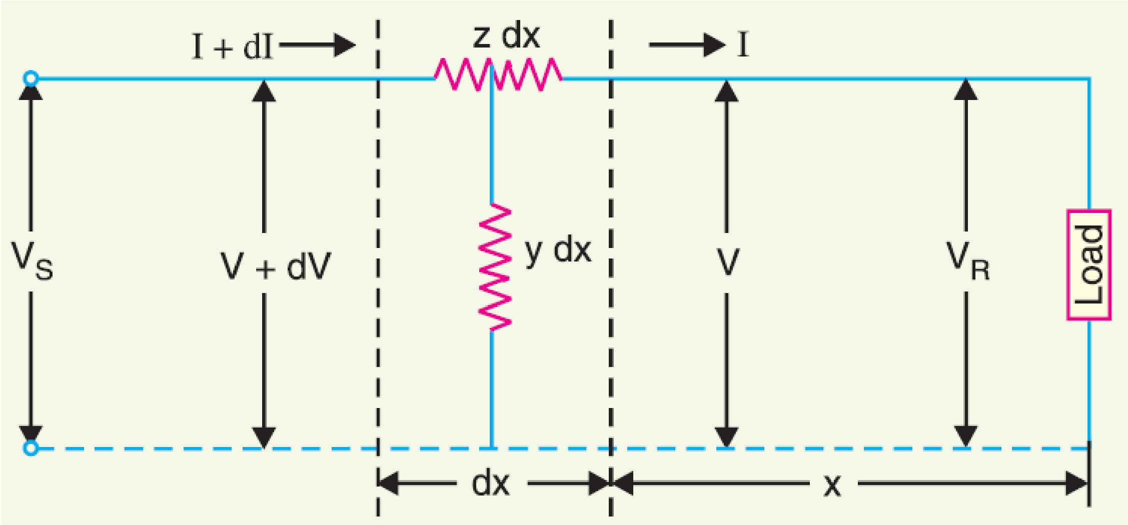
Then for the small element *dx*,

z dx = series impedance

y dx = shunt admittance

Obviously, $dV = I\, z\, dx$

$$\frac{dV}{dx} = I\,z \qquad \ldots(i)$$



Now, the current entering the element is I + dI whereas the current leaving the element is I. The difference in the currents flows through shunt admittance of the element i.e.,

dI = Current through shunt admittance of element = V y dx

$$\frac{dI}{dx} = V\,y \qquad \ldots(ii)$$

Differentiating eq. (*i*) w.r.t. *x*, we get,

$$\frac{d^2V}{dx^2} = z\,\frac{dI}{dx} = z\,(V\,y) \qquad \left[\because \frac{dI}{dx} = V\ y\ from\ \exp.\ (ii)\right]$$

$$\frac{d^2V}{dx^2} = y\,z\,V \qquad \ldots (iii)$$

The solution of this differential equation is

$$V = k_1 \cosh\left(x\sqrt{y\,z}\right) + k_2 \sinh\left(x\sqrt{y\,z}\right) \qquad \ldots(iv)$$

Differentiating exp. (iv) *w.r.t.* x, we have,

$$\frac{dV}{dx} = k_1\sqrt{y\,z}\,\sinh(x\sqrt{y\,z}) + k_2\sqrt{y\,z}\,\cosh(x\sqrt{y\,z})$$

But $\quad \dfrac{dV}{dx} = I\,z \qquad$ [from exp. (i)]

$$\therefore \quad I\,z = k_1\sqrt{y\,z}\,\sinh(x\sqrt{y\,z}) + k_2\sqrt{z\,y}\,\cosh(x\sqrt{y\,z})$$

$$I = \sqrt{\frac{y}{z}}\left[k_1 \sinh\;(x\sqrt{y\,z}) + k_2 \cosh\left(x\sqrt{y\,z}\right)\right] \qquad \ldots(v)$$

Equations (*iv*) and (*v*) give the expressions for V and I in the form of unknown constants k_1 and k_2. The values of k_1 and k_2 can be found by applying end conditions as under :

$$\text{At} \quad x = 0, \qquad V = V_R \text{ and } I = I_R$$

Putting these values in eq. (*iv*), we have,

$$V_R = k_1 \cosh 0 + k_2 \sinh 0 = k_1 + 0$$

$$\therefore \quad V_R = k_1$$

Similarly, putting $x = 0$, $V = V_R$ and $I = I_R$ in eq. (*v*), we have,

$$I_R = \sqrt{\frac{y}{z}}\left[k_1 \sinh 0 + k_2 \cosh 0\right] = \sqrt{\frac{y}{z}}\left[0 + k_2\right]$$

$$\therefore \quad k_2 = \sqrt{\frac{z}{y}}\, I_R$$

Substituting the values of k_1 and k_2 in eqs. (iv) and (v), we get,

$$V = V_R \cosh(x\sqrt{y\,z}) + \sqrt{\frac{z}{y}}\, I_R \sinh(x\sqrt{y\,z})$$

$$I = \sqrt{\frac{y}{z}}V_R \sinh(x\sqrt{y\,z}) + I_R \cosh(x\sqrt{y\,z})$$

The sending end voltage (V_S) and sending end current (I_S) are obtained by putting $x = l$ in the above equations $i.e.$,

$$V_S = V_R \cosh(l\,\sqrt{y\,z}) + \sqrt{\frac{z}{y}} I_R \sinh(l\sqrt{y\,z})$$

$$I_S = \sqrt{\frac{y}{z}}\, V_R \sinh(l\sqrt{y\,z}) + I_R \cosh(l\sqrt{y\,z})$$

Now, $\quad l\sqrt{y\,z} = \sqrt{l\,y \cdot l\,z} = \sqrt{Y\,Z}$

$$\sqrt{\frac{y}{z}} = \sqrt{\frac{y\,l}{z\,l}} = \sqrt{\frac{Y}{Z}}$$

where Y = total shunt admittance of the line

Z = total series impedance of the line

Therefore, expressions for V_S and I_S become :

$$\mathbf{V}_S = \mathbf{V}_R \cosh \sqrt{\mathbf{YZ}} + \mathbf{I}_R \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} \sinh \sqrt{\mathbf{YZ}}$$

$$\mathbf{I}_S = \mathbf{V}_R \sqrt{\frac{\mathbf{Y}}{\mathbf{Z}}} \sinh \sqrt{\mathbf{YZ}} + \mathbf{I}_R \cosh \sqrt{\mathbf{YZ}}$$

(*iv*) Long lines—Rigorous method.

- By rigorous method, the sending end voltage and current of a long transmission line are given by :

$$V_S = V_R \cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}$$

$$I_S = V_R\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} + I_R\cosh\sqrt{YZ}$$

Comparing these equations with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \cosh\sqrt{YZ} \quad ; \quad \vec{B} = \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ} \; ; \quad \vec{C} = \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$$

Incidentally

$$\vec{A}\vec{D} - \vec{B}\vec{C} = \cosh\sqrt{YZ} \times \cosh\sqrt{YZ} - \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ} \times \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$$

$$= \cosh^2\sqrt{YZ} - \sinh^2\sqrt{YZ} = 1$$

THANK YOU