Course: Transmission and Distribution– BEE402

Module-2:Introduction to Line Parameters

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Introduction to line parameters

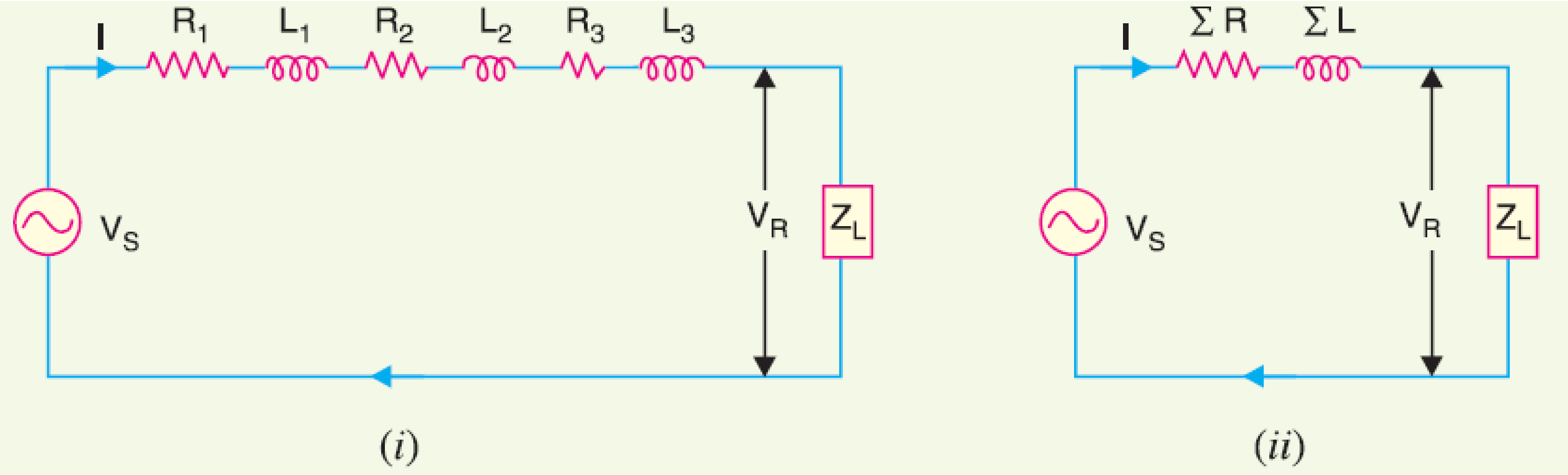
- We know that transmission of electric power is done by 3-phase, 3-wire overhead lines.
- An a.c. transmission line has resistance, inductance and capacitance uniformly distributed along its length. These are known as *constants or parameters of the line.*
- The performance of a transmission line depends to a considerable extent upon these constants.
- For instance, these constants determine whether the efficiency and voltage regulation of the line will be good or poor.

Constants of a Transmission Line

A transmission line has resistance, inductance and capacitance uniformly distributed along the whole length of the line.

(i) Resistance:

- It is the opposition of line conductors to current flow. The resistance is distributed uniformly along the whole length of the line as shown in Fig.(i).
- However, the performance of a transmission line can be analysed conveniently if distributed resistance is considered as lumped as shown in Fig.(ii).



(ii) Inductance :

- When an alternating current flows through a conductor, a changing flux is set up which links the conductor.
- Due to these flux linkages, the conductor possesses inductance.

Mathematically, inductance is defined as the flux linkages per ampere i.e.,

$$\text{Inductance,} \quad L = \frac{\psi}{I} \text{ henry}$$

where ψ = flux linkages in weber-turns

I = current in amperes

- The inductance is also uniformly distributed along the length of the line as shown in Fig.(i).
- Again for the convenience of analysis, it can be taken to be lumped as shown in Fig.(ii).

** The two parallel conductors of a transmission line form a rectangular loop of one turn. The changing flux in the line links the loop and hence the line has inductance.*

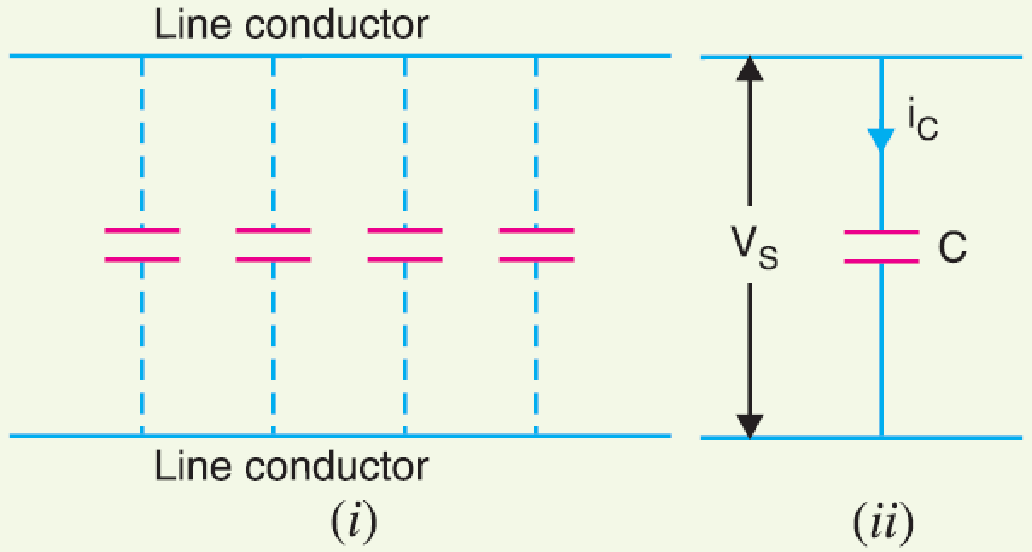
(iii) Capacitance :

- We know that any two conductors separated by an insulating material constitute a capacitor.
- As any two conductors of an overhead transmission line are separated by air which acts as an insulation, therefore, capacitance exists between any two overhead line conductors.
- The capacitance between the conductors is the charge per unit potential difference i.e.,

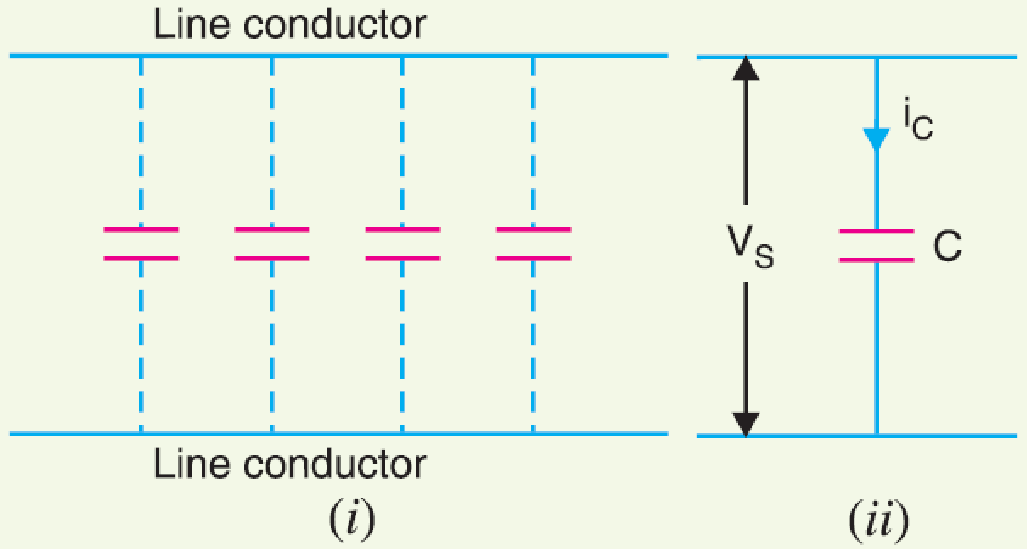
$$\text{Capacitance,} \quad C = \frac{q}{V} \text{ farad}$$

where q = charge on the line in coulomb

v = p.d. between the conductors in volts



- The capacitance is uniformly distributed along the whole length of the line and may be regarded as a uniform series of capacitors connected between the conductors as shown in Fig. (i).
- When an alternating voltage is impressed on a transmission line, the charge on the conductors at any point increases and decreases with the increase and decrease of the instantaneous value of the voltage between conductors at that point.
- The result is that a current (known as charging current) flows between the conductors as shown in Fig. (ii).



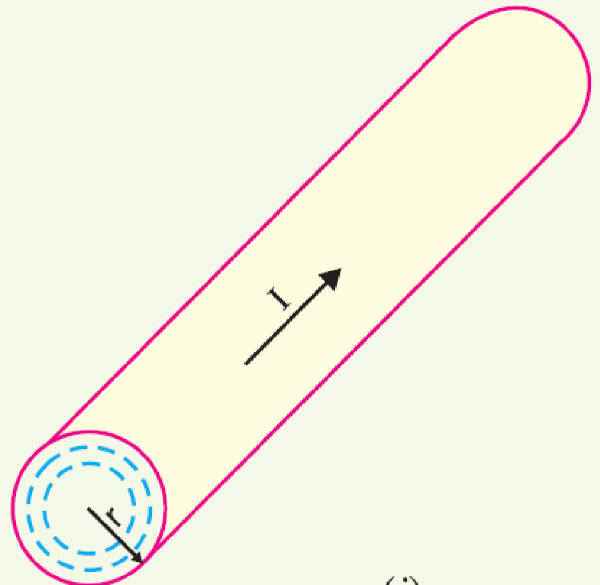
Flux Linkages

As stated earlier, the inductance of a circuit is defined as the flux linkages per unit current. Therefore, in order to find the inductance of a circuit, the determination of flux linkages is of primary importance.

We shall discuss two important cases of flux linkages.

1. Flux linkages due to a single current carrying conductor.

- Consider a long straight cylindrical conductor of radius *r* metres and carrying a current *I* amperes (r.m.s.) as shown in Fig. (*i*).
- This current will set up magnetic field. The magnetic lines of force will exist inside the conductor as well as outside the conductor.
- Both these fluxes will contribute to the inductance of the conductor.



(*i*)

(i) Flux linkages due to internal flux.

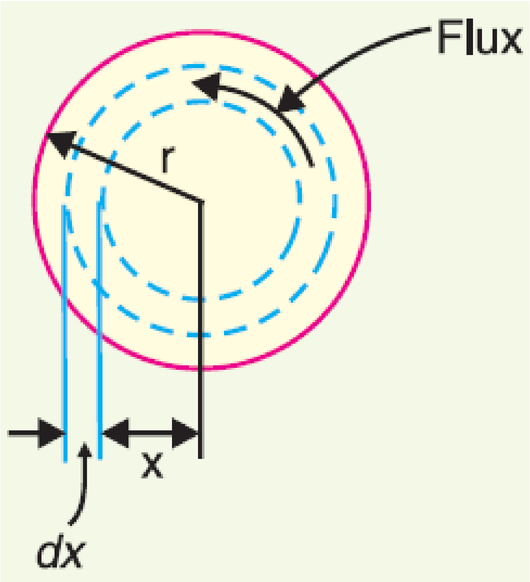
- Refer to Fig. (ii) where the X-section of the conductor is shown magnified for clarity.
- The magnetic field intensity at a point x metres from the centre is given by;

$$^{*}H_x = \frac{I_x}{2\pi x}$$

Assuming a uniform current density,

$$I_x = \frac{\pi x^2}{\pi r^2} I = \frac{x^2}{r^2} I$$

$$H_x = \frac{x^2}{r^2} \times I \times \frac{1}{2\pi x} = \frac{x}{2\pi r^2} I \text{ AT/m}$$



*According to Ampere's law, m.m.f. (ampere-turns) around any closed path equals the current enclosed by the path. The current enclosed by the path is I_x and m.m.f. = $H_x \times 2\pi x$. $\therefore$ $H_x \times 2\pi x = I_x$.

If μ $(= \mu_0\mu_r)$ is the permeability of the conductor, then flux density at the considered point is given by;

$$B_x = \mu_0 \mu_r H_x \text{wb/m}^2$$

$$= \frac{\mu_0 \mu_r x}{2 \pi r^2} I = \frac{\mu_0 xI}{2 \pi r^2} \text{ wb/m}^2 \qquad [\because \mu_r = 1 \text{ for non-magnetic material}]$$

Now, flux $d\phi$ through a cylindrical shell of radial thickness dx and axial length 1 m is given by;

$$d\phi = B_x \times 1 \times dx = \frac{\mu_0 \, x I}{2 \pi r^2} dx \text{ weber}$$

This flux links with current $I_x \left(= \frac{I \pi x^2}{\pi r^2}\right)$ only. Therefore, flux linkages per metre length of the conductor is

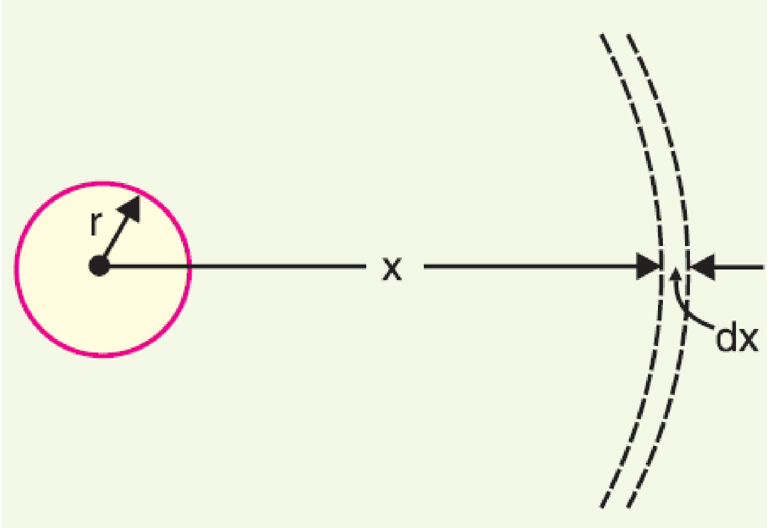
$$d\psi = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu_0 I x^3}{2\pi r^4} dx \text{ weber-turns}$$

Total flux linkages from centre upto the conductor surface is

$$\Psi_{int} = \int_0^r \frac{\mu_0 \, I x^3}{2\pi \, r^4} dx = \frac{\mu_0 \, I}{8\,\pi} \quad \text{weber-turns per metre length}$$

(ii) Flux linkages due to external flux

- Now let us calculate the flux linkages of the conductor due to external flux.
- The external flux extends from the surface of the conductor to infinity.
- Referring to Fig, the field intensity at a distance x metres (from centre) outside the conductor is given by ;



$$H_x = \frac{I}{2\pi x} \text{ AT / m}$$

Flux density, $$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x} \text{ wb/m}^2$$

Now, flux $d\phi$ through a cylindrical shell of thickness dx and axial length 1 metre is

$$d\phi = B_x \, dx = \frac{\mu_0 I}{2\pi x} dx \text{ webers}$$

The flux $d\phi$ links all the current in the conductor once and only once.

$$\therefore \quad \text{Flux linkages,} \quad d\psi = d\phi = \frac{\mu_0 I}{2\pi x} dx \text{ weber-turns}$$

Total flux linkages of the conductor from surface to infinity,

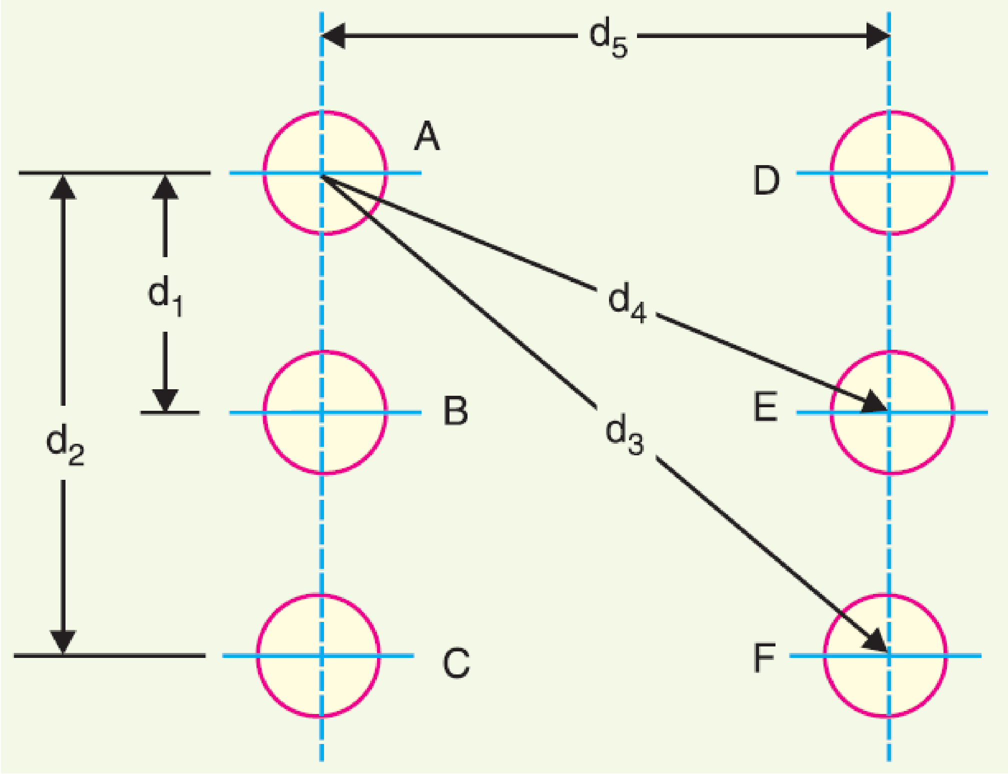
$$\psi_{ext} = \int_r^\infty \frac{\mu_0 I}{2\pi x} dx \text{ weber-turns}$$

$$\therefore \quad \text{Overall flux linkages,} \quad \psi = \psi_{int} + \psi_{ext} = \frac{\mu_0 I}{8\pi} + \int_r^\infty \frac{\mu_0 I}{2\pi x} dx$$

$$\therefore \quad \psi = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right] \text{ wb-turns/m length}$$

2. Flux linkages in parallel current carrying conductors

- We shall now determine the flux linkages in a group of parallel current carrying conductors.
- Fig. shows the conductors *A,B,C* etc. carrying currents I_A, I_B, I_C etc.
- Let us consider the flux linkages with one conductor, say conductor *A*.
- There will be flux linkages with conductor *A* due to its own current as discussed previously.
- Also there will be flux linkages with this conductor due to the mutual inductance effects of I_B, I_C, I_D etc.
- We shall now determine the total flux linkages with conductor *A*.



Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right] \qquad \ldots(i)$$

Flux linkages with conductor A due to current I_B

$$= * \frac{\mu_0 I_B}{2\pi} \int_{d_1}^{\infty} \frac{dx}{x} \qquad \ldots(ii)$$

Flux linkages with conductor A due to current I_C

$$= \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x} \qquad \ldots(iii)$$

$\therefore$ Total flux linkages with conductor A

$$= (i) + (ii) + (iii) + \ldots\ldots$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_{d_1}^{\infty} \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty} \frac{dx}{x} + \ldots$$

Similarly, flux linkages with other conductors can be determined. These relations provides the basis for evaluating inductance of any circuit.

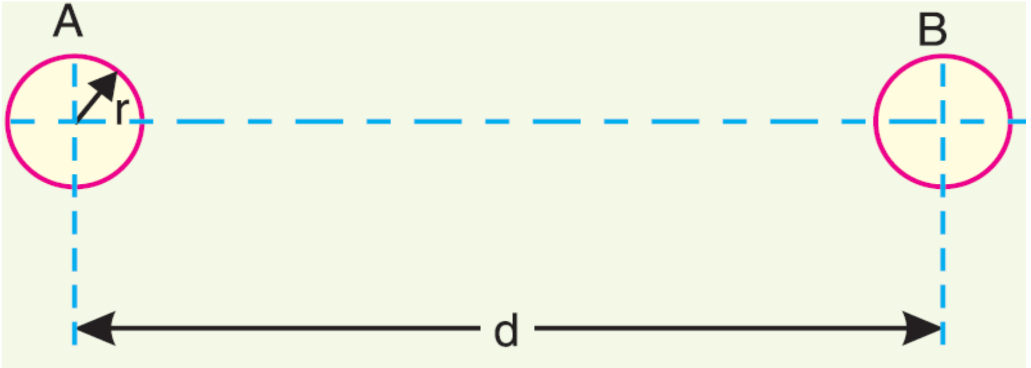
Inductance of a Single Phase Two-wire Line

- A single phase line consists of two parallel conductors which form a rectangular loop of one turn.
- When an alternating current flows through such a loop, a changing magnetic flux is set up.
- The changing flux links the loop and hence the loop (or single phase line) possesses inductance.
- It may appear that inductance of a single phase line is negligible because it consists of a loop of one turn and the flux path is through air of high reluctance.
- But as the X -sectional area of the loop is very **large, even for a small flux density, the total flux linking the loop is quite large and hence the line has appreciable inductance.

*** The conductors are spaced several metres and the length of the line is several kilometers. Therefore, the loop has a large X-sectional area.*

- Consider a single phase overhead line consisting of two parallel conductors *A* and *B* spaced *d* metres apart as shown in Fig.
- Conductors *A* and *B* carry the same amount of current (*i.e.* $I_A = I_B$), but in the opposite direction because one forms the return circuit of the other.

$$\therefore \qquad \mathbf{I_A + I_B = 0}$$



- In order to find the inductance of conductor A (or conductor B), we shall have to consider the flux linkages with it.
- There will be flux linkages with conductor A due to its own current I_A and also due to the mutual inductance effect of current I_B in the conductor B.

Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) \qquad \ldots(i)$$

Flux linkages with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi}\int_d^\infty \frac{dx}{x} \qquad \ldots(ii)$$

Total flux linkages with conductor A is

$$\psi_A = \text{exp. } (i) + \text{exp } (ii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_d^\infty \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right) I_A + I_B \int_d^{\infty} \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \log_e \infty - \log_e r\right) I_A + (\log_e \infty - \log_e d) I_B\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{I_A}{4} + \log_e \infty (I_A + I_B) - I_A \log_e r - I_B \log_e d\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \log_e r - I_B \log_e d\right] \qquad (\because I_A + I_B = 0)$$

Now, $I_A + I_B = 0$ or $-I_B = I_A$

$\therefore \quad -I_B \log_e d = I_A \log_e d$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e d - I_A \log_e r\right] \text{ wb-turns/m}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e \frac{d}{r}\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ wb-turns/m}$$

Inductance of conductor $A, L_A = \dfrac{\psi_A}{I_A}$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H / m}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H / m}$$

$$\therefore \quad L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{H/m} \qquad \ldots(i)$$

$$\text{Loop inductance} = 2\,L_A \text{H/m} = 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{H/m}$$

$$\therefore \quad \text{Loop inductance} = 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{H/m} \qquad \ldots(ii)$$

Note that eq. (*ii*) is the inductance of the two-wire line and is sometimes called loop inductance. However, inductance given by eq. (*i*) is the inductance per conductor and is equal to half the loop inductance.

Expression in alternate form.

The expression for the inductance of a conductor can be put in a concise form.

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{H/m}$$

$$= 2\times 10^{-7}\left[\frac{1}{4} + \log_e \frac{d}{r}\right]$$

$$= 2\times 10^{-7}\left[\log_e e^{1/4} + \log_e \frac{d}{r}\right]$$

$$\therefore \quad L_A = 2\times 10^{-7}\log_e \frac{d}{r\,e^{-1/4}}$$

If we put $r\,e^{-1/4} = r'$, then,

$$L_A = 2\times 10^{-7}\log_e \frac{d}{r'} \text{ H/m} \qquad \ldots(iii)$$

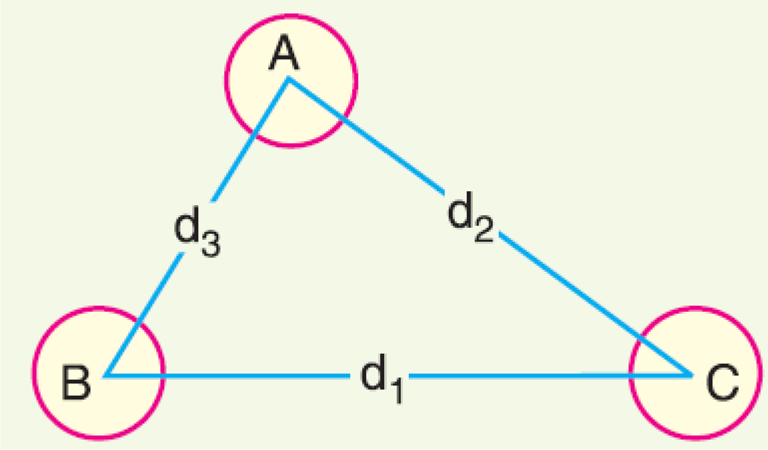
The radius r' is that of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor of radius r. The quantity $e^{-1/4} = 0{\cdot}7788$ so that

$$r' = r\,e^{-1/4} = 0{\cdot}7788\ r$$

The term r' $(= r\,e^{-1/4})$ is called *geometric mean radius* (GMR) of the wire.

Inductance of a 3-Phase Overhead Line

- Figure shows the three conductors *A*, *B* and *C* of a 3-phase line carrying currents I_A, I_B and I_C respectively.
- Let d_1, d_2 and d_3 be the spacings between the conductors as shown.
- Let us further assume that the loads are balanced *i.e.* $I_A + I_B + I_C = 0$. Consider the flux linkages with conductor *A*. There will be flux linkages with conductor *A* due to its own current and also due to the mutual inductance effects of I_B and I_C.



Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) \qquad \ldots(i)$$

Flux linkages with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi}\int_{d_3}^\infty \frac{dx}{x} \qquad \ldots(ii)$$

Flux linkages with conductor A due to current I_C

$$= \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x} \qquad \ldots(iii)$$

Total flux linkages with conductor A is

$$\psi_A = (i) + (ii) + (iii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_{d_3}^\infty \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) I_A + I_B \int_{d_3}^\infty \frac{dx}{x} + I_C \int_{d_2}^\infty \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2 + \log_e \infty \left(I_A + I_B + I_C\right)\right]$$

As $\quad I_A + I_B + I_C = 0,$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

(i) Symmetrical spacing

If the three conductors A, B and C are placed symmetrically at the corners of an equilateral triangle of side d, then, d1 = d2 = d3 = d. Under such conditions, the flux linkages with conductor A become :

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d - I_C \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - \left(I_B + I_C\right)\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A + I_A \log_e d\right] \qquad (\because I_B + I_C = -I_A)$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ werber-turns/m}$$

Inductance of conductor A, $L_A = \dfrac{\Psi_A}{I_A}$ H / m $= \dfrac{\mu_0}{2\pi}\left[\dfrac{1}{4} + \log_e \dfrac{d}{r}\right]$ H/m

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m}$$

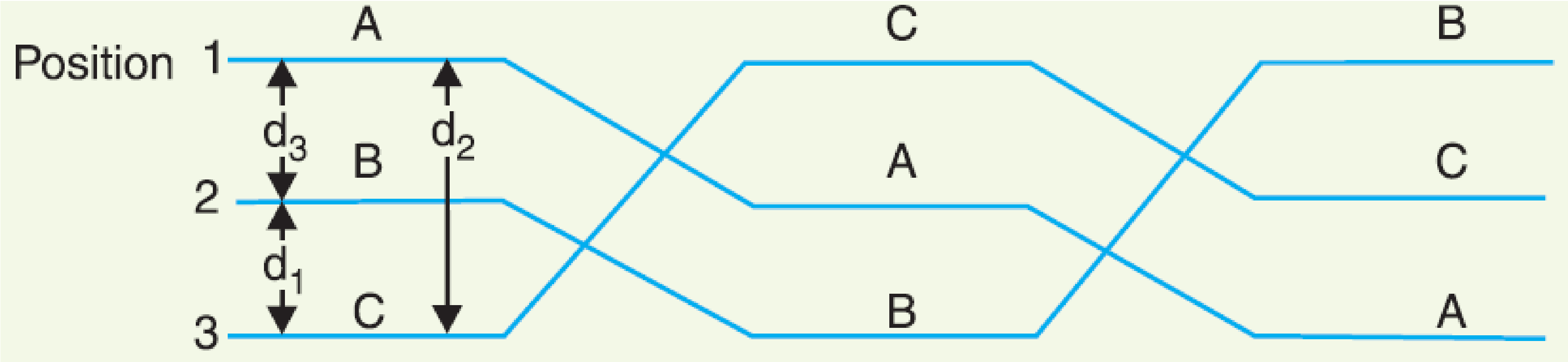
$$\therefore \quad L_A = 10^{-7}\left[0\cdot 5 + 2\log_e \frac{d}{r}\right] \text{H/m}$$

Derived in a similar way, the expressions for inductance are the same for conductors B and C.

(*ii*) Unsymmetrical spacing

- When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical.
- Under such conditions, the flux linkages and inductance of each phase are not the same.
- A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced.
- Therefore, the voltage at the receiving end will not be the same for all phases.
- In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as *transposition.*

- Figure shows the transposed line. The phase conductors are designated as A, B and C and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.
- Figure shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions i.e., $I_A + I_B + I_C = 0$. Let the line currents be :



$$I_A = I\,(1 + j\,0)$$
$$I_B = I\,(-0{\cdot}5 - j\,0{\cdot}866)$$
$$I_C = I\,(-0{\cdot}5 + j\,0{\cdot}866)$$

As proved above, the total flux linkages per metre length of conductor A is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

Putting the values of I_A, I_B and I_C, we get,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I - I(-0\cdot 5 - j\,0\cdot 866)\log_e d_3 - I(-0\cdot 5 + j\,0\cdot 866)\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I - I \log_e r + 0\cdot 5\, I \log_e d_3 + j\,0\cdot 866 \log_e d_3 + 0\cdot 5\, I \log_e d_2 - j\,0\cdot 866\, I \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I - I \log_e r + 0\cdot 5\, I\left(\log_e d_3 + \log_e d_2\right) + j\,0\cdot 866\, I\left(\log_e d_3 - \log_e d_2\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I - I \log_e r + I^{*} \log_e \sqrt{d_2 d_3} + j\,0\cdot 866\; I \log_e \frac{d_3}{d_2}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I + I \log_e \frac{\sqrt{d_2\, d_3}}{r} + j\,0\cdot 866\; I \log_e \frac{d_3}{d_2}\right]$$

$$= \frac{\mu_0\, I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j\,0\cdot 866 \log_e \frac{d_3}{d_2}\right]$$

$\therefore$ Inductance of conductor A is

$$L_A = \frac{\Psi_A}{I_A} = \frac{\Psi_A}{I}$$

$*$ $0\cdot 5\, I\,(\log_e d_3 + \log_e d_2) = 0\cdot 5\, I \log_e d_2 d_3 = I \log_e (d_2 d_3)^{0\cdot 5} = I \log_e \sqrt{d_2 d_3}$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\ d_3}}{r} + j\,0\cdot 866 \log_e \frac{d_3}{d_2}\right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\ d_3}}{r} + j\,0\cdot 866 \log_e \frac{d_3}{d_2}\right] \text{H/m}$$

$$= 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_2\ d_3}}{r} + j\,1\cdot 732 \log_e \frac{d_3}{d_2}\right] \text{H/m}$$

Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3\ d_1}}{r} + j\,1\cdot 732 \log_e \frac{d_1}{d_3}\right] \text{H/m}$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_1\ d_2}}{r} + j\,1\cdot 732 \log_e \frac{d_2}{d_1}\right] \text{H/m}$$

Inducance of each line conductor

$$= \frac{1}{3}(L_A + L_B + L_C)$$

$$= {}^{*}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

$$= \left[0 \cdot 5 + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

If we compare the formula of inductance of an unsymmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$.

The distance d is known as equivalent equilateral spacing for unsymmetrically transposed line.

Concept of Self-GMD and Mutual-GMD

The use of Self Geometrical Mean Distance (abbreviated as **Self-GMD**) and Mutual Geometrical Mean Distance (**Mutual-GMD**) simplifies the inductance calculations, particularly relating to multi conductor arrangements. The symbols used for these are respectively Ds and Dm.

(i) Self-GMD (Ds)

In order to have concept of self-GMD (also sometimes called Geometrical Mean Radius ; **GMR**), consider the expression for inductance per conductor per metre,

$$\text{Inductance/conductor/m} = 2 \times 10^{-7}\left(\frac{1}{4} + \log_e \frac{d}{r}\right)$$

$$= 2 \times 10^{-7} \times \frac{1}{4} + 2 \times 10^{-7} \log_e \frac{d}{r} \quad \ldots(i)$$

- In this expression, the term $2 \times 10^{-7} \times (1/4)$ is the inductance due to flux within the solid conductor.
- For many purposes, it is desirable to eliminate this term by the introduction of a concept called **Self-GMD or GMR**.
- If we replace the original solid conductor by an equivalent hollow cylinder with extremely thin walls, the current is confined to the conductor surface and internal conductor flux linkage would be almost zero.
- Consequently, inductance due to internal flux would be zero and the term $2{\times}10^{-7}{\times}\ (1/4)$ shall be eliminated.
- The radius of this equivalent hollow cylinder must be sufficiently smaller than the physical radius of the conductor to allow room for enough additional flux to compensate for the absence of internal flux linkage.

It can be proved mathematically that for a solid round conductor of radius *r*, the self-GMD or GMR = 0·7788 *r*. Using self-GMD, the equ. (*i*) becomes :

$$\text{Inductance/conductor/m} = 2 \times 10^{-7} \log_e d/Ds$$

where Ds = GMR or self-GMD = 0·7788 r

It may be noted that self-GMD of a conductor depends upon the size and shape of the conductor and is independent of the spacing between the conductors.

(ii) Mutual-GMD

The mutual-GMD is the geometrical mean of the distances form one conductor to the other and, therefore, must be between the largest and smallest such distance. In fact, mutual-GMD simply represents the equivalent geometrical spacing.

(*a*) The mutual-GMD between two conductors (assuming that spacing between conductors is large compared to the diameter of each conductor) is equal to the distance between their centres *i.e.*

$$Dm = \text{spacing between conductors} = d$$

(*b*) For a single circuit 3-φ line, the mutual-GMD is equal to the equivalent equilateral spacing *i.e.*, $(d_1\ d_2\ d_3)^{1/3}$.

$$\therefore\quad D_m = (d_1\ d_2\ d_3)^{1/3}$$

(c) The principle of geometrical mean distances can be most profitably employed to 3-φ double circuit lines. Consider the conductor arrangement of the double circuit shown in Fig. Suppose the radius of each conductor is *r*.

Self-GMD of conductor = 0·7788 *r*

Self-GMD of combination *aa′* is

$$D_{s1} = (**D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$$

Self-GMD of combination *bb′* is

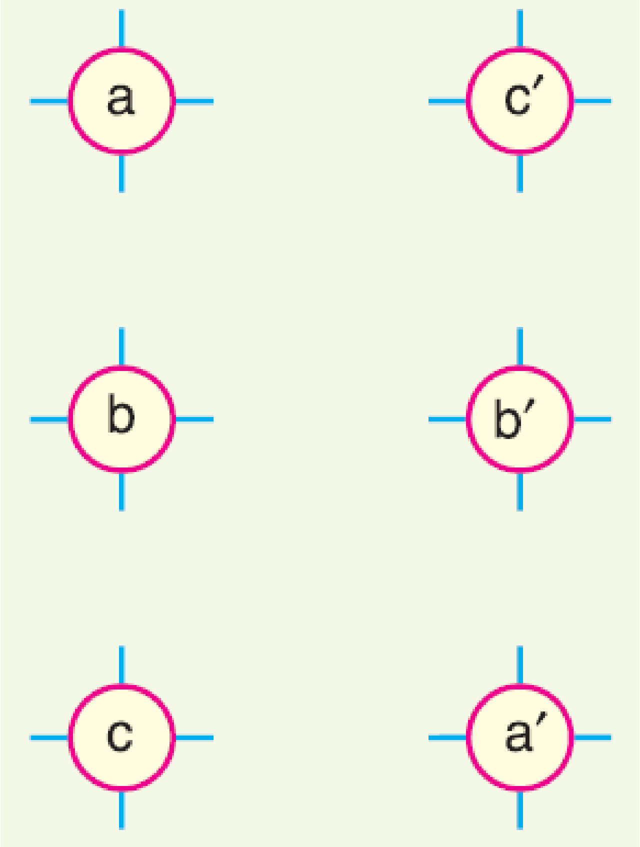
$$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4}$$

Self-GMD of combination *cc′* is

$$D_{s3} = (D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c})^{1/4}$$

Equivalent self-GMD of one phase

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3}$$



The value of *Ds* is the same for all the phases as each conductor has the same radius.

Mutual-GMD between phases A and B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$$

Mutual-GMD between phases B and C is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4}$$

Mutual-GMD between phases C and A is

$$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4}$$

It is worthwhile to note that mutual GMD depends only upon the spacing and is substantially independent of the exact size, shape and orientation of the conductor.

Electric Potential

- The electric potential at a point due to a charge is the work done in bringing a unit positive charge from infinity to that point.
- The concept of electric potential is extremely important for the determination of capacitance.

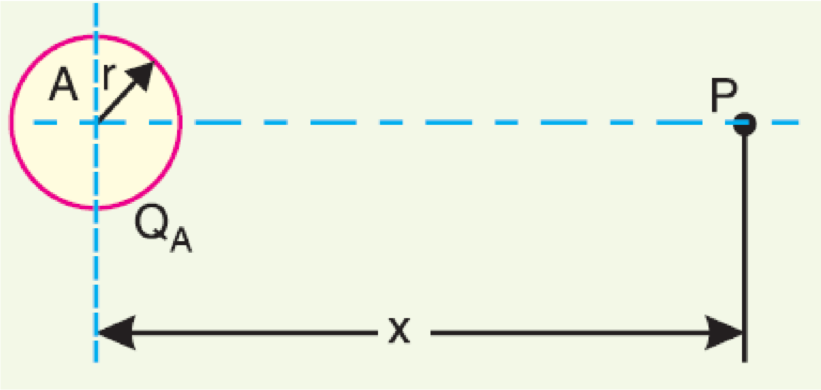
(i) Potential at a charged single conductor:

Consider a long straight cylindrical conductor A of radius r metres. Let the conductor operate at such a potential (V_A) that charge Q_A coulombs per metre exists on the conductor. It is desired to find the expression for V_A. The electric intensity E at a distance x from the centre of the conductor in air is given by:

$$E = \frac{Q_A}{2\pi x \varepsilon_0} \text{ volts/m}$$

where Q_A = charge per metre length

ε_0 = permittivity of free space



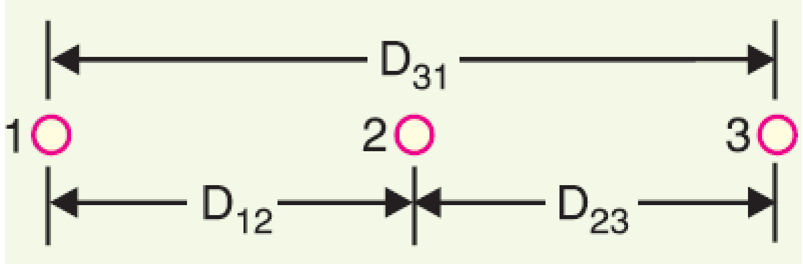
As x approaches infinity, the value of E approaches zero. Therefore, the potential difference between conductor A and infinity distant neutral plane is given by :

$$\dagger V_A = \int_r^\infty \frac{Q_A}{2\pi\, x\, \varepsilon_0}\, dx = \frac{Q_A}{2\pi\varepsilon_0}\int_r^\infty \frac{dx}{x}$$

Example 9.5. *Calculate the inductance of each conductor in a 3-phase, 3-wire system when the conductors are arranged in a horizontal plane with spacing such that $D_{31} = 4$ m ; $D_{12} = D_{23} = 2$m. The conductors are transposed and have a diameter of 2·5 cm.*

Solution. Fig. shows the arrangement of the conductors of the 3phase line. The conductor radius $r = 2{\cdot}5/2 = 1{\cdot}25$ cm.

Equivalent equilateral spacing, $D_{eq} = \sqrt[3]{D_{12} \times D_{23} \times D_{31}} = \sqrt[3]{2 \times 2 \times 4} = 252$ cm

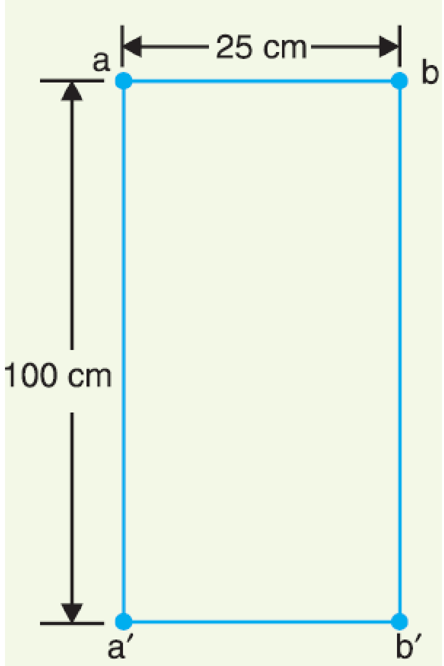


$$\text{Inductance/phase/m} = 10^{-7}\,(0{\cdot}5 + 2\log_e D_{eq}/r)\text{ H}$$
$$= 10^{-7}\,(0{\cdot}5 + 2\log_e 252/1{\cdot}25)\text{ H}$$
$$= 11{\cdot}1 \times 10^{-7}\text{H}$$
$$\text{Inductance/phase/km} = 11{\cdot}1 \times 10^{-7} \times 1000$$
$$= 1{\cdot}11 \times 10^{-3}\text{ H}$$
$$= \mathbf{1{\cdot}11\ mH}$$

Example 9.6. *Two conductors of a single phase line, each of 1 cm diameter, are arranged in a vertical plane with one conductor mounted 1 m above the other. A second identical line is mounted at the same height as the first and spaced horizontally 0·25 m apart from it. The two upper and the two lower conductors are connected in parallel. Determine the inductance per km of the resulting double circuit line.*

Solution. Fig. shows the arrangement of double circuit single phase line. Conductors *a*, *a′* form one connection and conductors *b*, *b′* form the return connection.

The conductor radius, $r = 1/2 = 0{\cdot}5$ cm.



$$\text{G.M.R. of conductor} = 0{\cdot}7788\,r = 0{\cdot}7788 \times 0{\cdot}5 = 0{\cdot}389 \text{ cm}$$

$$\text{Self G.M.D. of } aa' \text{ combination is } D_s = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$$

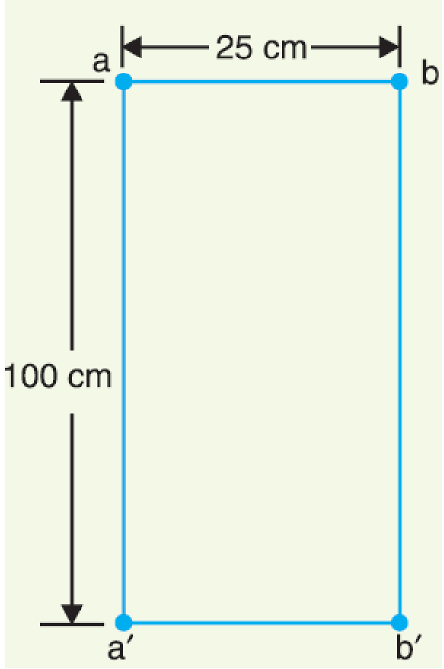
$$= \sqrt[4]{(0 \cdot 389 \times 100)^2} = 6{\cdot}23 \text{ cm}$$

Mutual G.M.D. between a and b is $D_m = \sqrt[4]{D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'}}$

$$[\because D_{ab'} = D_{a'b} = \sqrt{25^2 + 100^2} = 103 \text{ cm}]$$

$$= \sqrt[4]{(25 \times 103 \times 103 \times 25)}$$

$$= 50{\cdot}74 \text{ cm}$$



Inductance per conductor per metre

$$= 2 \times 10^{-7} \log_e D_m/D_s$$

$$= 2 \times 10^{-7} \log_e 50{\cdot}74/6{\cdot}23 \text{ H}$$

$$= 0{\cdot}42 \times 10^{-6} \text{ H}$$

$\therefore$ Loop inductance per km of the line

$$= 2 \times 0{\cdot}42 \times 10^{-6} \times 1000 \text{ H}$$

$$= \mathbf{0{\cdot}84 \text{ mH}}$$

Example 9.8. *Find the inductance per phase per km of double circuit 3-phase line shown in Fig. 9·16. The conductors are transposed and are of radius 0·75 cm each. The phase sequence is ABC.*

Solution. G.M.R. of conductor $= 0{\cdot}75 \times 0{\cdot}7788 = 0{\cdot}584$ cm

Distance a to b $= \sqrt{3^2 + (0\cdot75)^2} = 3{\cdot}1$ m

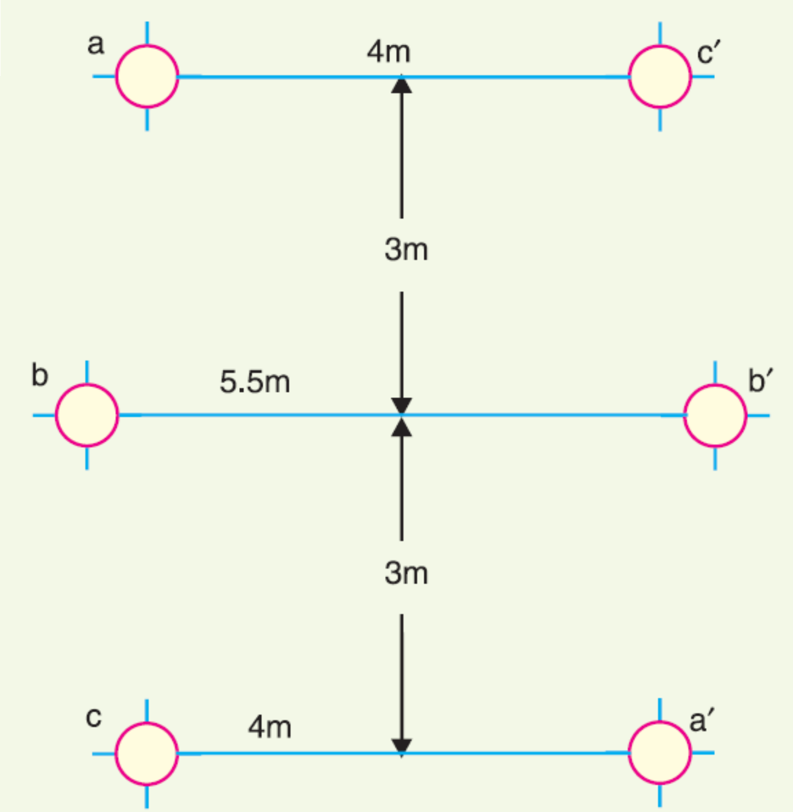
Distance a to b' $= \sqrt{3^2 + (4\cdot75)^2} = 5{\cdot}62$ m

Distance a to a' $= \sqrt{6^2 + 4^2} = 7{\cdot}21$ m

Equivalent self G.M.D. of one phase is

$$D_s = \sqrt[3]{D_{s1} \times D_{s2} \times D_{s3}}$$

where $$D_{s1} = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$$



$$= \sqrt[4]{\left(0\cdot 584\times 10^{-2}\right)\times(7\cdot 21)\times\left(0\cdot 584\times 10^{-2}\right)\times(7\cdot 21)}$$

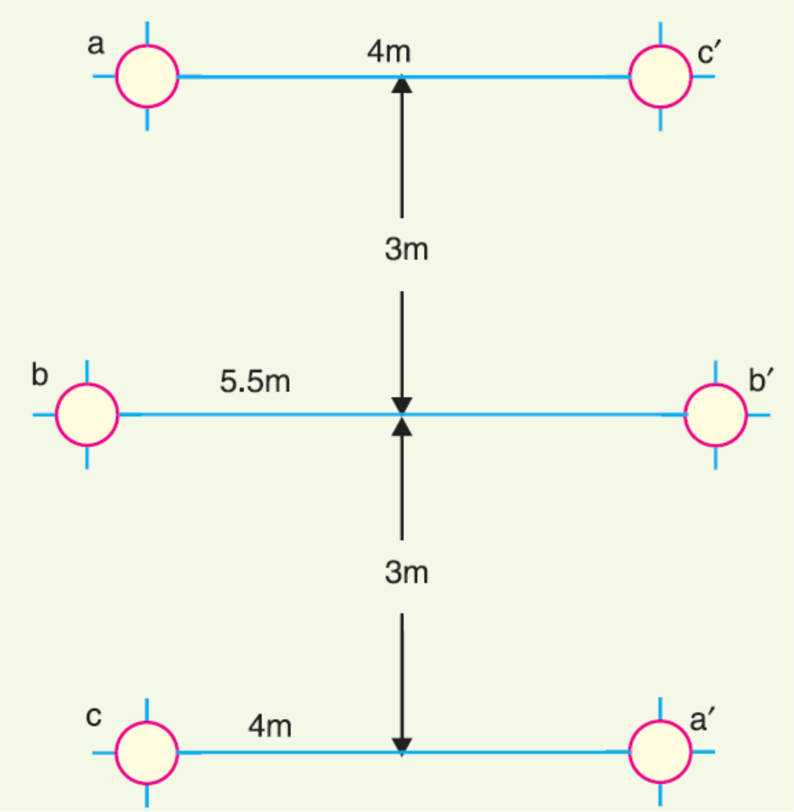
$$= 0{\cdot}205 \text{ m} = D_{s3}$$

$$D_{s2} = \sqrt[4]{\left(D_{bb}\times D_{bb'}\times D_{b'b'}\times D_{b'b}\right)}$$

$$= \sqrt[4]{\left(0\cdot 584\times 10^{-2}\right)\times(5\cdot 5)\times\left(0\cdot 584\times 10^{-2}\right)\times 5\cdot 5}$$

$$= 0\cdot 18 \text{ m}$$

$$\therefore \quad D_s = \sqrt[3]{0\cdot 205\times 0\cdot 18\times 0\cdot 205} = 0{\cdot}195 \text{ m}$$



Equivalent mutual G.M.D. is

$$D_m = \sqrt[3]{D_{AB}\times D_{BC}\times D_{CA}}$$

where $D_{AB} = \sqrt[4]{D_{ab}\times D_{ab'}\times D_{a'b}\times D_{a'b'}} = \sqrt[4]{3\cdot 1\times 5\cdot 62\times 5\cdot 62\times 3\cdot 1}$

$$= 4{\cdot}17 \text{ m} = D_{BC}$$

$$D_{CA} = \sqrt[4]{D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'}}$$

$$= \sqrt[4]{6 \times 4 \times 4 \times 6} \quad = 4{\cdot}9 \text{ m}$$

$$\therefore \quad D_m = \sqrt[3]{4.17 \times 4.17 \times 4.9} \quad = 4{\cdot}4 \text{ m}$$

$$\text{Inductance/phase/m} = 10^{-7} \times 2 \log_e D_m/D_s \quad = 10^{-7} \times 2 \log_e 4{\cdot}4/0{\cdot}195 \text{ H}$$

$$= 6{\cdot}23 \times 10^{-7} \text{ H} = 0{\cdot}623 \times 10^{-3} \text{ mH}$$

$$\text{Inductance/phase/km} = 0{\cdot}623 \times 10^{-3} \times 1000 \quad = \mathbf{0{\cdot}623\ mH}$$

Capacitance of a Single Phase Two-wire Line

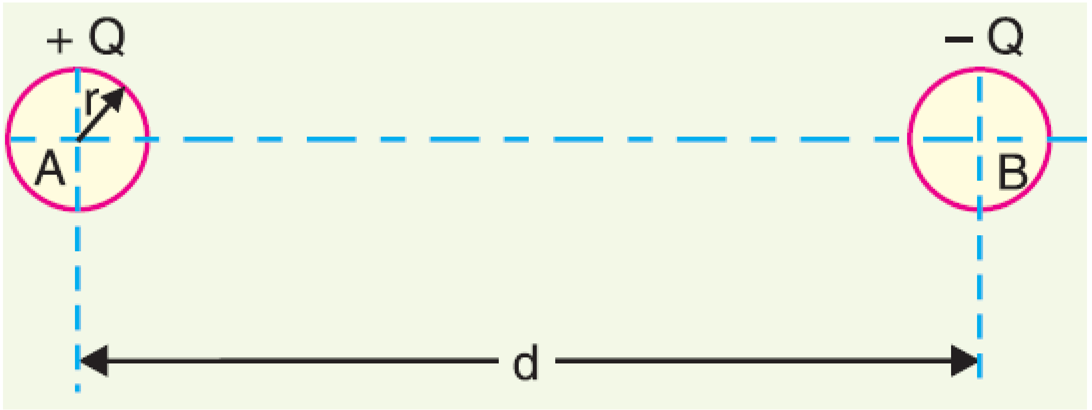
- Consider a single phase overhead transmission line consisting of two parallel conductors A and B spaced d metres apart in air.
- Suppose that radius of each conductor is r metres. Let their respective charge be + Q and − Q coulombs per metre length.

The total p.d. between conductor A and neutral "infinite" plane is

$$V_A = \int_r^{\infty} \frac{Q}{2\pi x \varepsilon_0} dx + \int_d^{\infty} \frac{-Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] \text{volts}$$

$$= \frac{Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$



Similarly, p.d. between conductor B and neutral "infinite" plane is

$$V_B = \int_r^{\infty} \frac{-Q}{2\pi x\, \varepsilon_0}\, dx + \int_d^{\infty} \frac{Q}{2\pi x\, \varepsilon_0}\, dx$$

$$= \frac{-Q}{2\pi\, \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] \qquad = \frac{-Q}{2\pi\, \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Both these potentials are *w.r.t.* the same neutral plane. Since the unlike charges attract each other, the potential difference between the conductors is

$$V_{AB} = 2\, V_A = \frac{2Q}{2\pi\, \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$$\therefore \quad \text{Capacitance, } C_{AB} = Q/V_{AB} = \frac{Q}{\dfrac{2Q}{2\pi\, \varepsilon_0} \log_e \dfrac{d}{r}} \text{ F/m} \quad \therefore \quad C_{AB} = \frac{\pi\, \varepsilon_0}{\log_e \dfrac{d}{r}} \text{ F / m} \qquad \ldots(i)$$

Capacitance to Neutral

- Equation (*i*) gives the capacitance between the conductors of a two wire line [See Fig.A].
- Often it is desired to know the capacitance between one of the conductors and a neutral point between them. Since potential of the mid-point between the conductors is zero, the potential difference between each conductor and the ground or neutral is half the potential difference between the conductors.
- Thus the *capacitance to ground* or capacitance to neutral for the two wire line is *twice* the line-to-line capacitance (capacitance between conductors as shown in Fig.B).

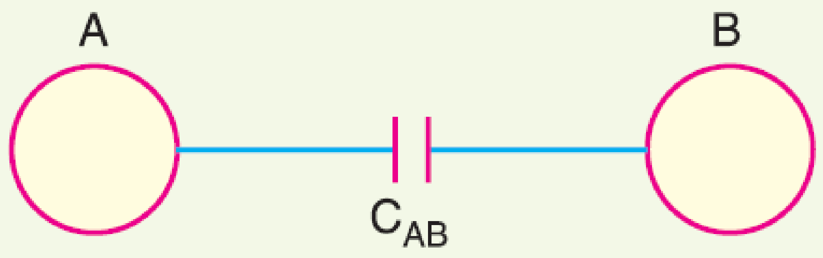


Fig.A

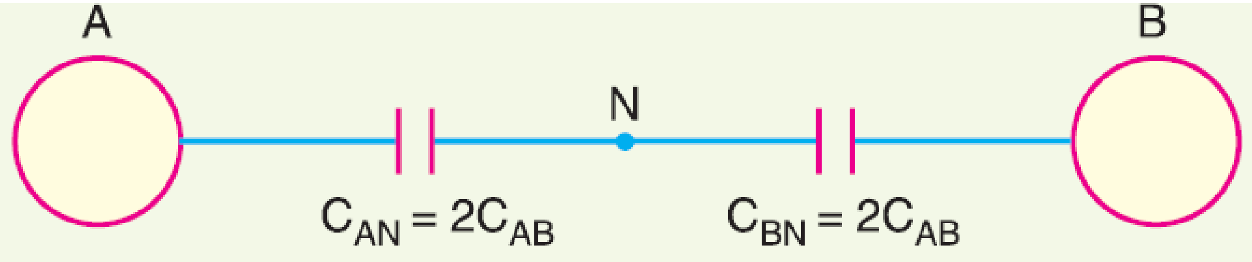


Fig.B

Capacitance of a 3-Phase Overhead Line

In a 3-phase transmission line, the capacitance of each conductor is considered instead of capacitance from conductor to conductor.

Here, again two cases arise viz., symmetrical spacing and unsymmetrical spacing.

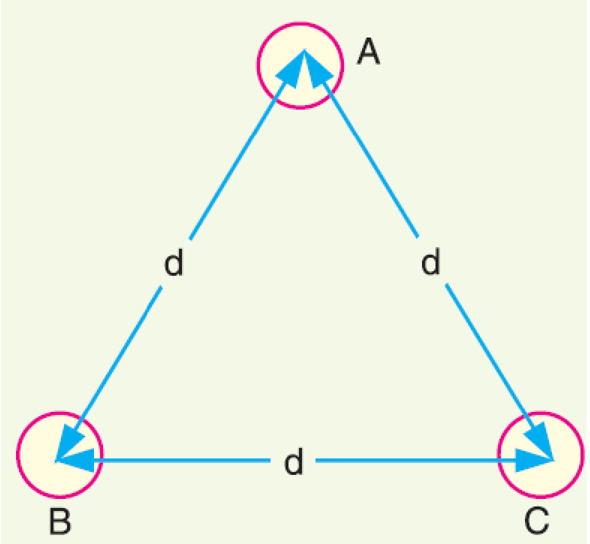
(i) Symmetrical Spacing

Fig. shows the three conductors *A*, *B* and *C* of the 3-phase overhead transmission line having charges Q_A, Q_B and Q_C per metre length respectively. Let the conductors be equidistant (*d* metres) from each other. We shall find the capacitance from line conductor to neutral in this symmetrically spaced line. Referring to Fig., overall potential difference between conductor *A* and infinite neutral plane is given by ;

$$V_A = \int_r^{\infty} \frac{Q_A}{2\,\pi x\varepsilon_0}\,dx + \int_d^{\infty} \frac{Q_B}{2\,\pi x\varepsilon_0}\,dx + \int_d^{\infty} \frac{Q_C}{2\,\pi x\varepsilon_0}\,dx$$

$$= \frac{1}{2\pi\,\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d}\right]$$

$$= \frac{1}{2\pi\,\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d}\right]$$



Assuming balanced supply, we have, $Q_A + Q_B + Q_C = 0$

$$\therefore \quad Q_B + Q_C = -Q_A$$

$$\therefore \quad V_A = \frac{1}{2\pi\,\varepsilon_0}\left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d}\right] = \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$\therefore$ Capacitance of conductor A w.r.t neutral,

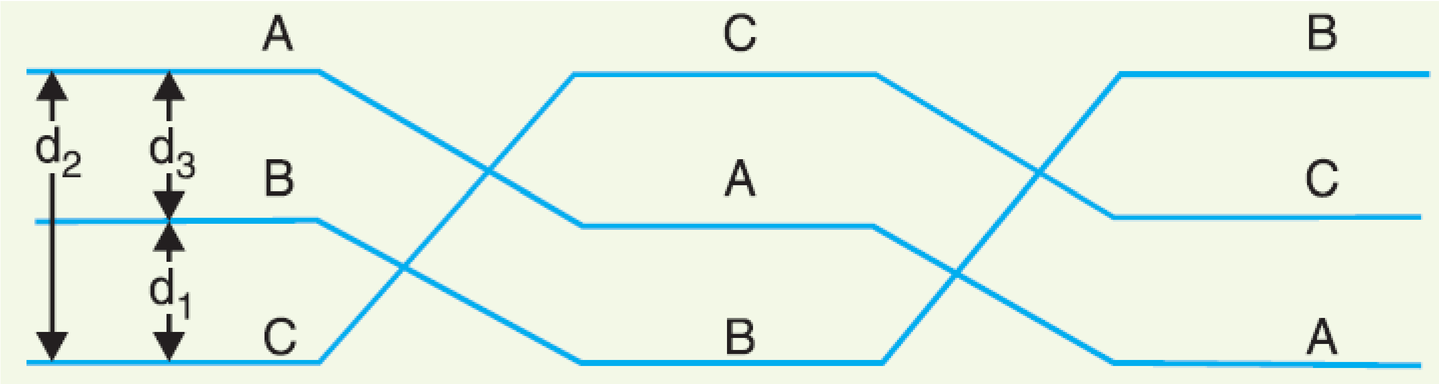
$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\dfrac{Q_A}{2\pi\,\varepsilon_0}\log_e \dfrac{d}{r}} \text{ F / m} = \frac{2\pi\,\varepsilon_0}{\log_e \dfrac{d}{r}} \text{ F / m}$$

$$\therefore \quad C_A = \frac{2\pi\,\varepsilon_0}{\log_e \dfrac{d}{r}} \text{ F / m}$$

Note that this equation is identical to capacitance to neutral for two-wire line. Derived in a similar manner, the expressions for capacitance are the same for conductors B and C.

(ii) Unsymmetrical spacing

Fig. shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions i.e. $Q_A + Q_B + Q_C = 0$.



Considering all the three sections of the transposed line for phase *A*,

$$\text{Potential of 1st position,} \quad V_1 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_3} + Q_C \log_e \frac{1}{d_2}\right)$$

Potential of 2nd position, $V_2 = \dfrac{1}{2\pi\varepsilon_0}\left(Q_A \log_e \dfrac{1}{r} + Q_B \log_e \dfrac{1}{d_1} + Q_C \log_e \dfrac{1}{d_3}\right)$

Potential of 3rd position, $V_3 = \dfrac{1}{2\pi\varepsilon_0}\left(Q_A \log_e \dfrac{1}{r} + Q_B \log_e \dfrac{1}{d_2} + Q_C \log_e \dfrac{1}{d_1}\right)$

Average voltage on condutor A is

$$V_A = \frac{1}{3}(V_1 + V_2 + V_3)$$

$$= \frac{1}{3 \times 2\pi\varepsilon_0} * \left[Q_A \log_e \frac{1}{r^3} + (Q_B + Q_C) \log_e \frac{1}{d_1 d_2 d_3}\right]$$

As $Q_A + Q_B + Q_C = 0$, therefore, $Q_B + Q_C = -Q_A$

$$\therefore \quad V_A = \frac{1}{6\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r^3} - Q_A \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$= \frac{Q_A}{6\pi\varepsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{1}{3} \times \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{Q_A}{2\pi\varepsilon_0} \log_e \left(\frac{d_1 d_2 d_3}{r^3} \right)^{1/3}$$

$$= \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{r}$$

$\therefore$ Capacitance from conductor to neutral is

$$C_A = \frac{Q_A}{V_A} = \frac{2\,\pi\,\varepsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} \; F/m$$

THANK YOU