- Module 3

Introduction

- In case of beams supporting uniformly distributed load, the maximum bending moment increases with the square of the span and hence they become uneconomical for long span structures. In such situations arches could be advantageously employed, as they would develop horizontal reactions, which

  in turn reduce the design bending moment.

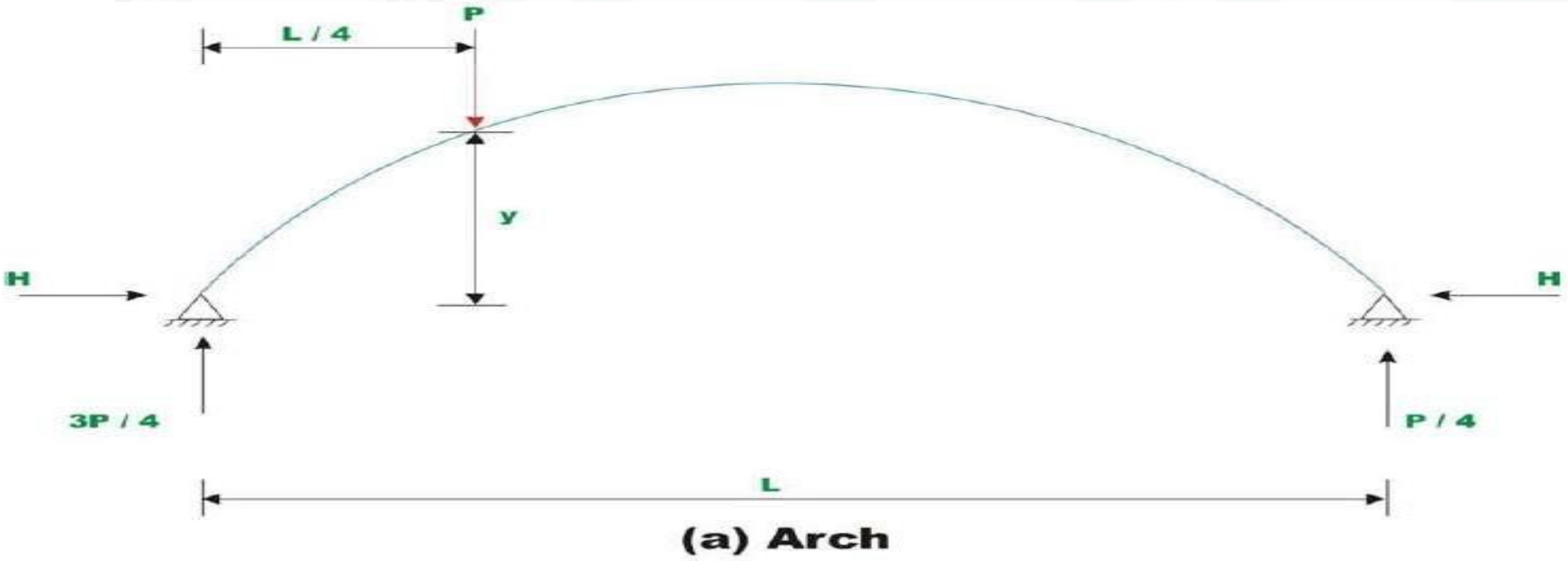


Figure 1: Beam and Arch comparison

- For example, in the case of a simply supported beam shown in Figure 1, the bending moment below the load is $3PL/16$.

- Now consider a two hinged symmetrical arch of the same span and subjected to similar loading as that of simply supported beam. The vertical reaction could be calculated by equations of statics. The horizontal reaction is determined by the method of least work. Now the bending moment below the load is $(3PL/16 - Hy)$.

- It is clear that the bending moment below the load is reduced in the case of an arch as compared to a simply supported beam.

- It is observed that, the cable takes the shape of the loading and this shape is termed as funicular shape.

- If an arch will constructed in an inverted funicular shape then it would be subjected to only compression for those loadings for which its shape is inverted funicular.

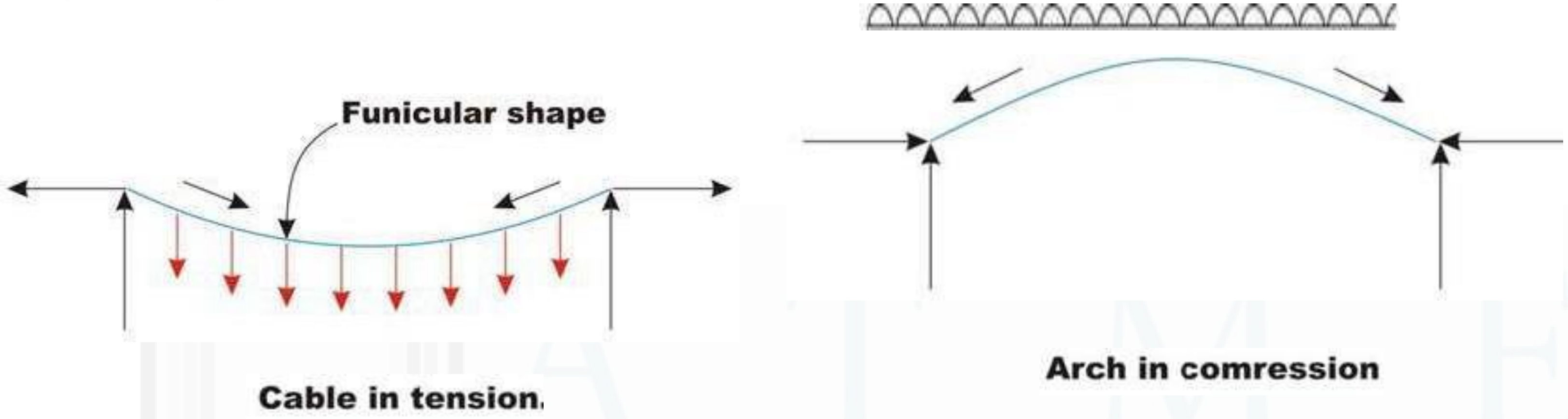


Figure 2: Cable and Arch structure

- Since in practice, the actual shape of the arch differs from the inverted funicular shape or the loading differs from the one for which the arch is an inverted funicular.
- Arches are also subjected to bending moment in addition to compression.
- As arches are subjected to compression, it must be designed to resist buckling.

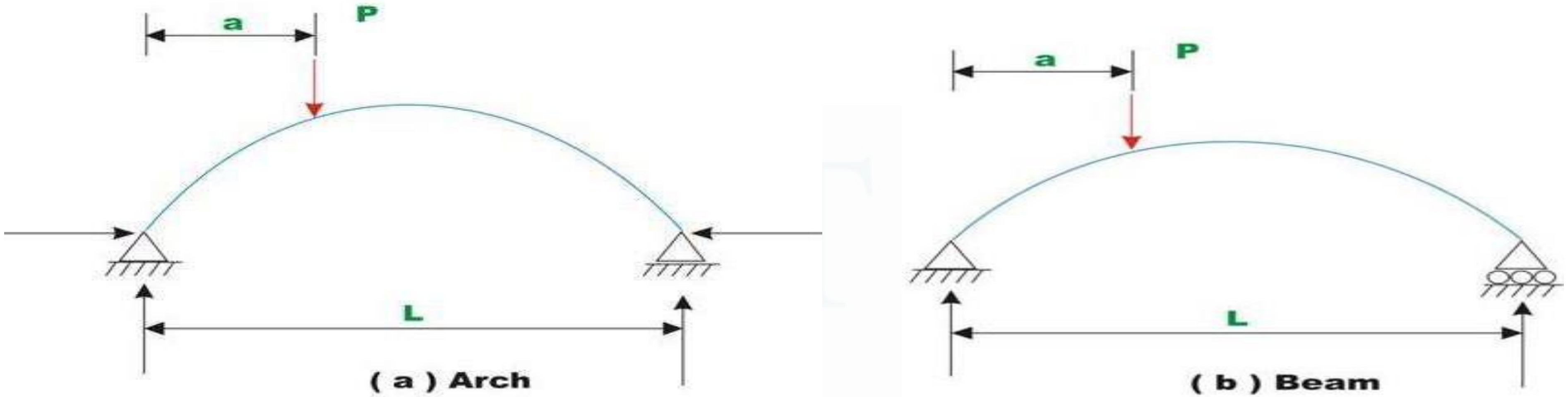


Figure 3: (a) Arch **(b)** Beam

- A structure is classified as an arch not based on its shape but the way it supports the lateral load. Arches support load primarily in compression.

- For example in Figure 3(b), no horizontal reaction is developed. Consequently bending moment is not reduced. It is important to appreciate the point that the definition of an arch is a structural one, not geometrical.

- For large spans like bridges, arches are provided in stead of beams. Arches are curved beams that transfer loads to their planes.
- Arches transfer loads to abutments at springing points. Hinges may be provided at springing points.
- The top most point is called crown.

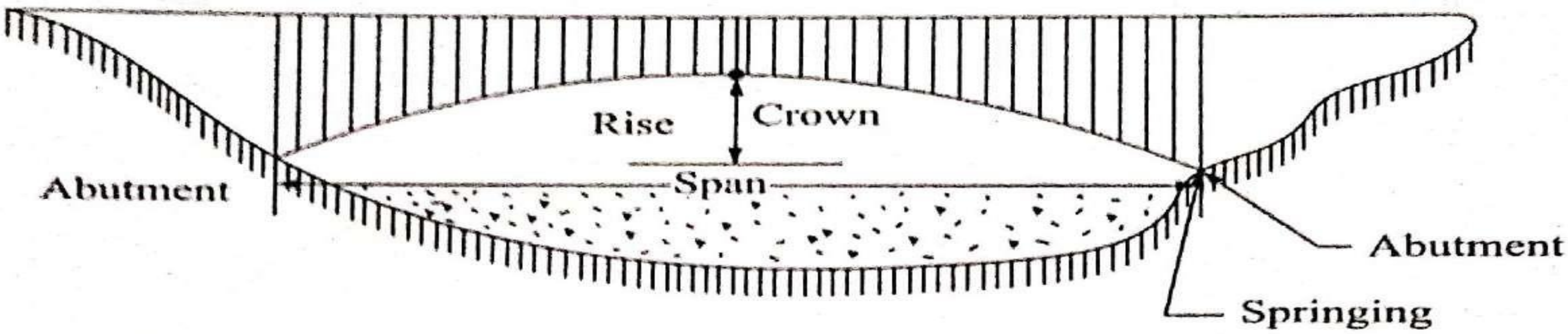


Figure 4: Typical arch bridges

- Due to curved nature of arches, they give rise to horizontal forces.
- Abutments are designed for horizontal forces also.
- Any section in the arch is subjected to normal thrust, radial shear and bending moment.
- Loads in the arches partly transferred by axial compression and partly by flexure.

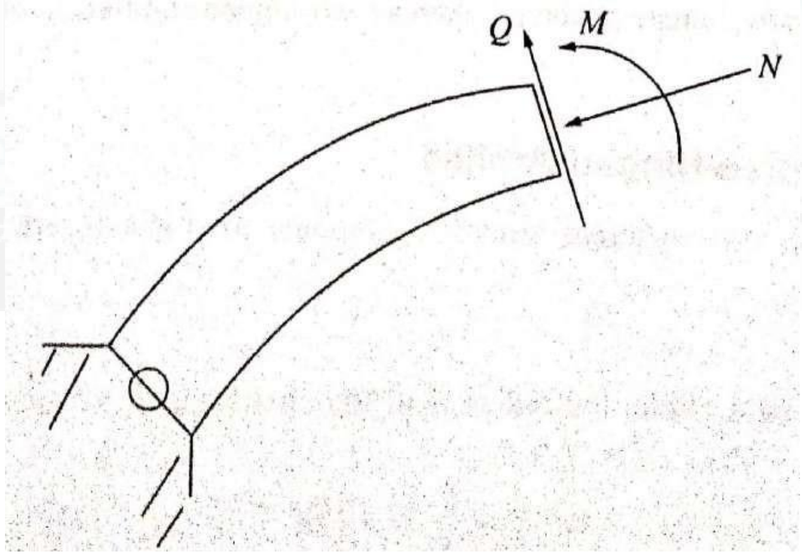


Figure 5: Arch with section showing forces

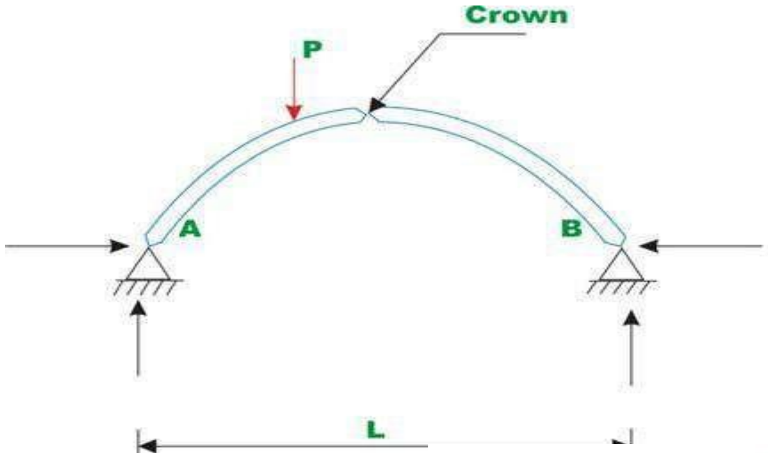
Types of Arches

There are mainly three types of arches that are commonly used in practice:

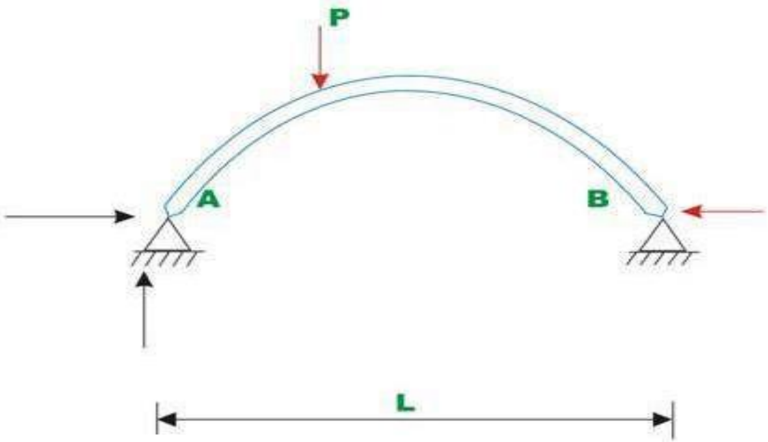
- Three hinged arch,
- Two-hinged arch
- Fixed-fixed arch or Hingeless arch

- Three-hinged arch is statically determinate structure and its reactions / internal forces are evaluated by static equations of equilibrium.

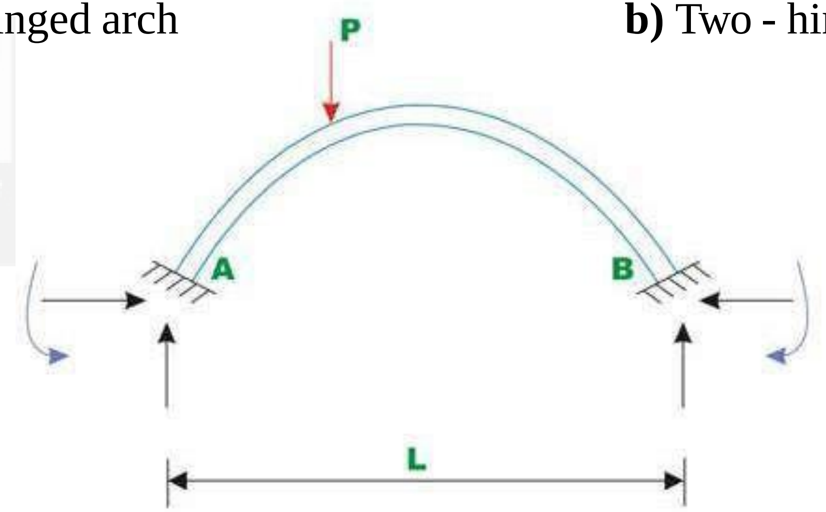
- Two-hinged arch and fixed-fixed arch are statically indeterminate structures. The indeterminate reactions are determined by the method of least work or by the flexibility matrix method.



a) Three – hinged arch



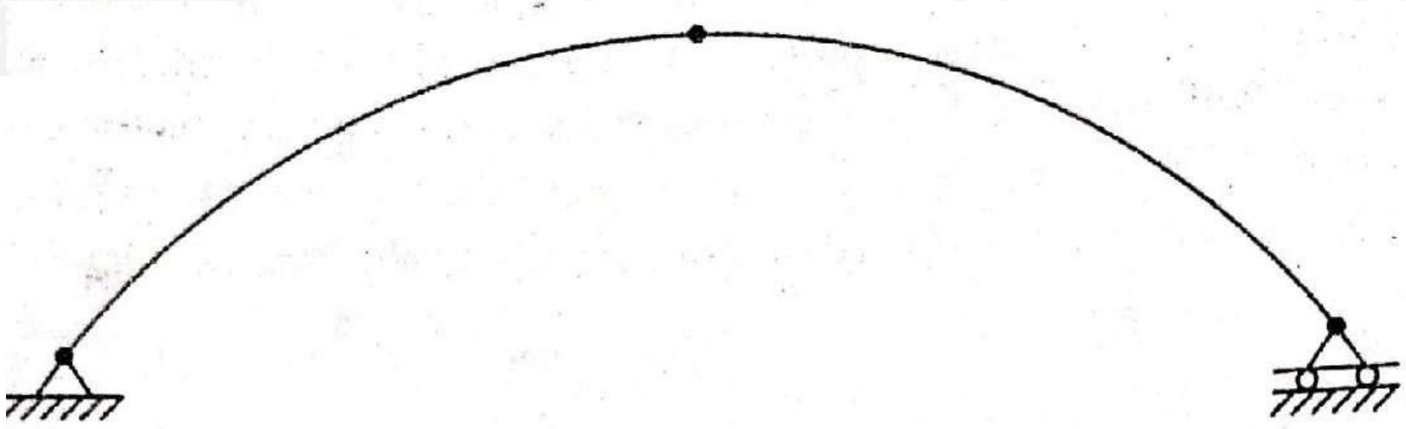
b) Two - hinged arch



c) Fixed – Fixed arch

Figure 6: Types of arches

- Three hinged arches is statically determinate structure and its reaction and internal forces are evaluated by static equations of equilibrium.
- Arches transfer loads to abutments at springing points.
- The horizontal distance between one support to another support is called span.
- The Top most point of the arch is called crown.
- The height of the crown above the support level is called rise



Types of three-hinged arches

- There are two types of three-hinged arches according to the shape of the structure.
- Circular three-hinged arch
- Parabolic three-hinged arch

Circular three-hinged arch

- From the property of a circle, the radius '*R*' of the circular arch of span '*L*' and rise '*h*'

$$\frac{L}{2} \times \frac{L}{2} = h(2R - h)$$

$$R = \frac{L^2}{8h} + \frac{h}{2}$$

Circular three-hinged arch

- Taking '*A*' as origin at support, the coordinates of any point '*D*' on the arch is:

$$x = \left[\frac{L}{2} - R\sin\theta\right]$$

$$y = R\cos\theta - (R - h)$$
$$= h - R(1 - \cos\theta)$$

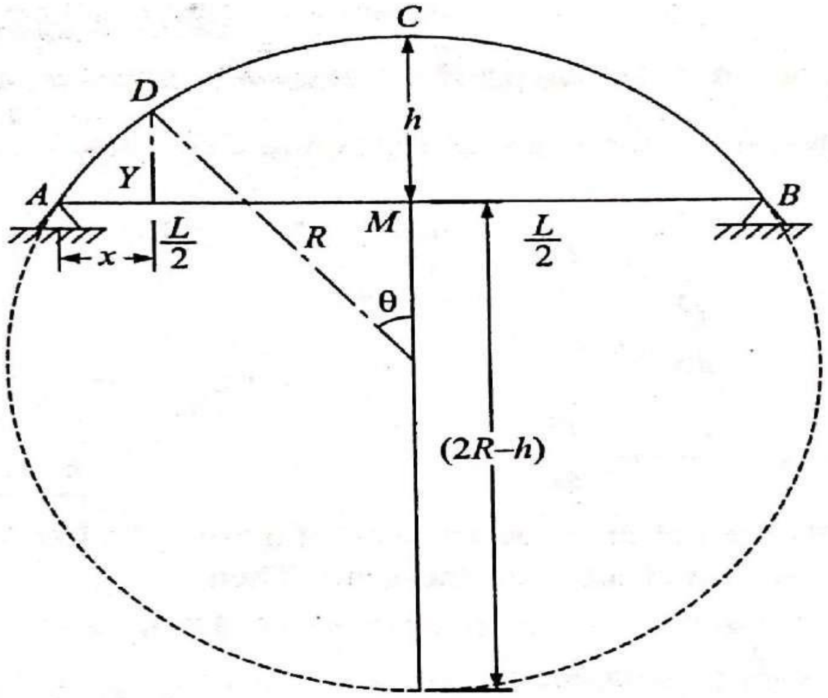


Figure 2: Coordinates of '*D*'

Parabolic three-hinged arch

- In the case of a parabolic arch, taking the springing point as the origin its equation = $y = \frac{4hx}{L^2}(L-x)$

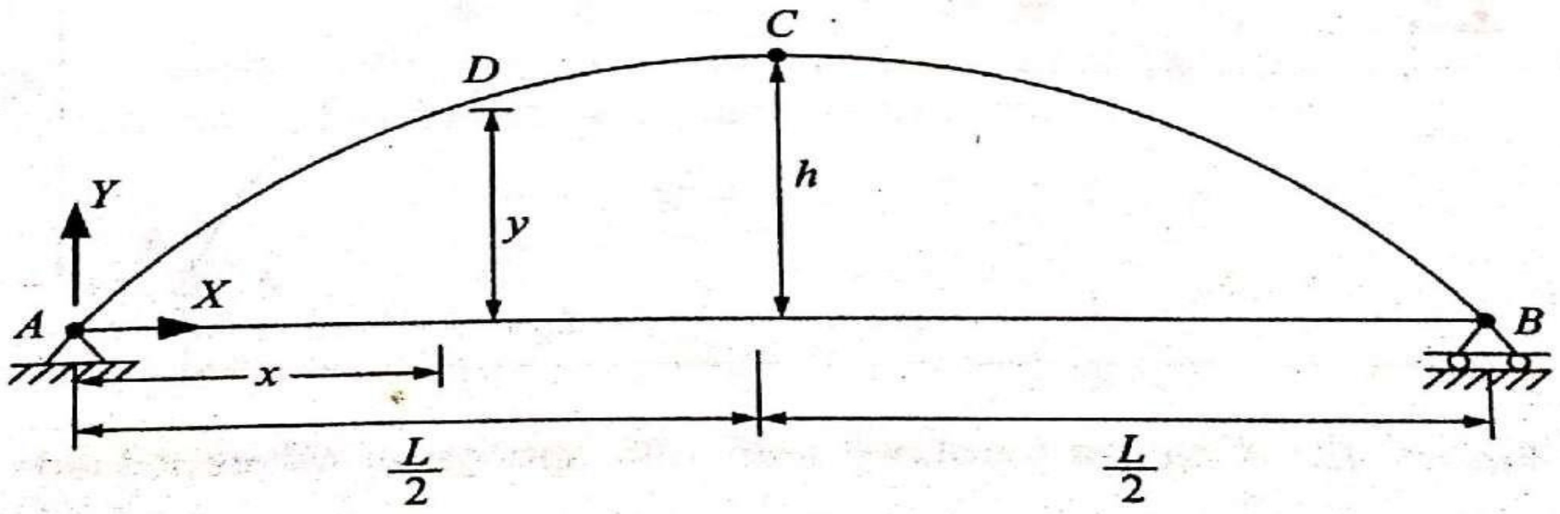


Figure 3: Parabolic arch with origin at '*A*'

- If the crown is taken as the origin, the equation of the parabolic curve $= \frac{x^2}{y} = a$, where a = Constant

If the springing point are at the same level:

$$x = \frac{L}{2},\ y = h,$$

$$\frac{L^2}{4h} = a$$

Hence, the equation is $\dfrac{x^2}{y} = \dfrac{L^2}{4h}$

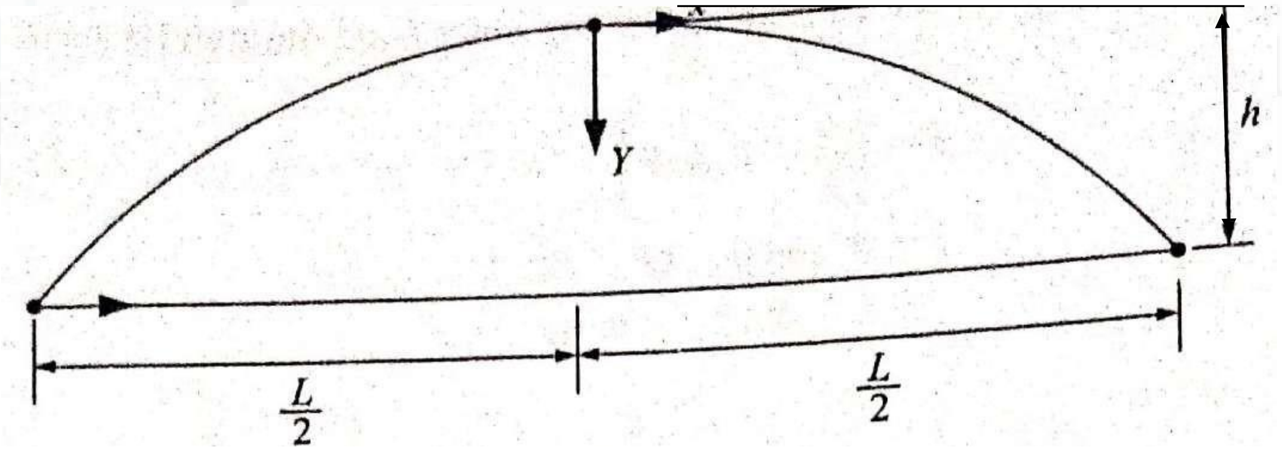


Figure 4: Parabolic arch with origin at crown

If the springing points are not at the same level:

- Let h_1 and h_2 is the depth of the abutments from the crown and let 'L' is the span.

$$\frac{x^2}{y} = \text{constant}$$

$$\frac{x}{\sqrt{y}} = \text{constant}$$

Applying this equation to points A and B, we get

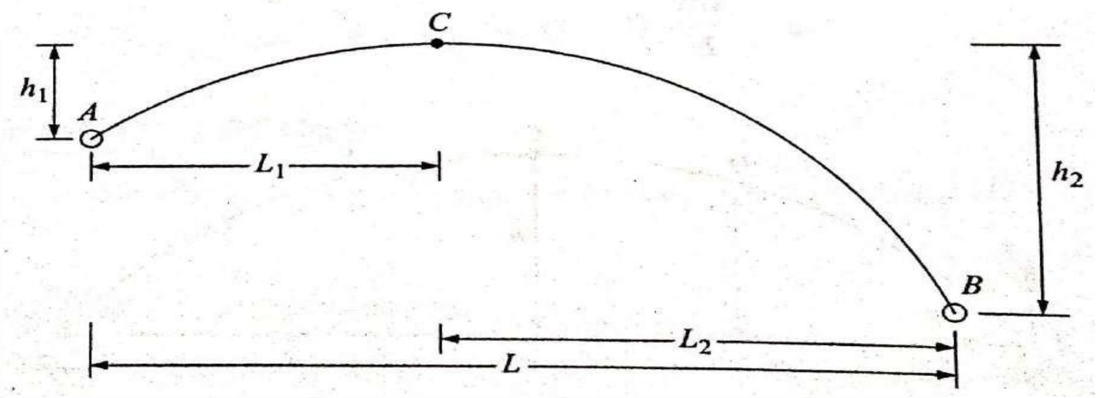


Figure 5: Parabolic arch with springing at different level

$$\text{Constant} = \frac{L_1}{\sqrt{h_1}} = \frac{L_2}{\sqrt{h_2}} = \frac{L_1+L_2}{\sqrt{h_1}+\sqrt{h_2}} = \frac{L}{\sqrt{h_1}+\sqrt{h_2}}$$

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$$

$$L_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

Analysis for Static Loads

- Consider a three-hinged arches as shown in Figure 6.
- The ends are hinged there is two reaction components at each end namely vertical and horizontal.
- There are four reaction components i.e. V_A, H_A, V_B and H_B.

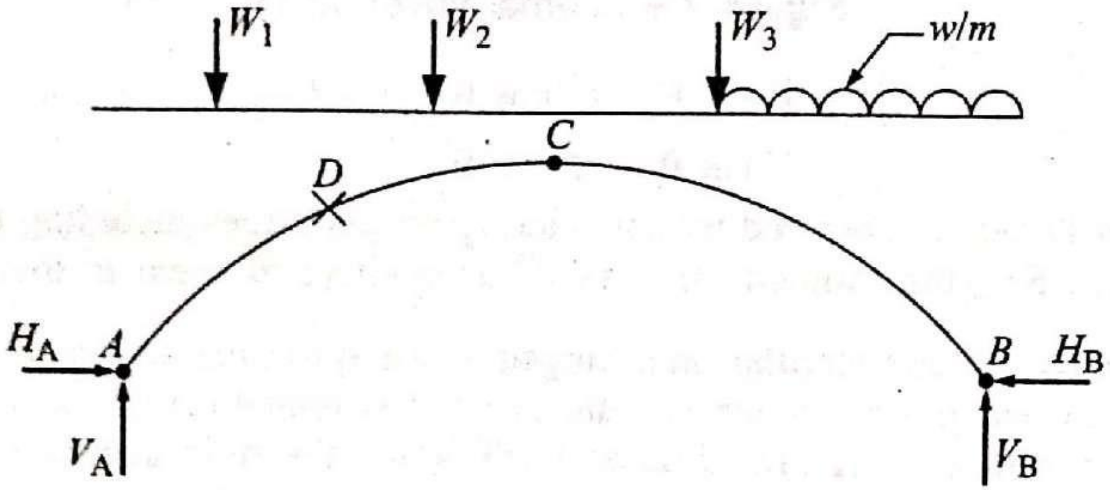


Figure 6: Three-hinged arch with load

- For any plane structure, there are three independent equilibrium equations, i.e.

$$\sum F_H = 0$$

$$\sum F_V = 0$$

$$M_A \text{ or } M_B = 0$$

In this case, the fourth equation is $M_c = 0$, since C is a hinge

- If horizontal load is not acting, $H_A = H_B = H$
- In this case the following equations are used

$$\sum F_V = 0$$

$$M_A \text{ or } M_B = 0$$

$$M_C = 0$$