ATME College of Engineering

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DEPARTMENT OF CIVIL ENGINEERING

(ACADEMIC YEAR 2024 -25)

SUBJECT NAME: ANALYSIS OF STRUCTURES

SUB CODE: BCV401

SEMESTER: IV

INSTITUTE

Vision of the Institute

Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

Mission of the Institute

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as tourch bearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence

DEPARTMENT

Vision of the Department

To develop globally competent Civil Engineers who excel in academics, research and are ethically responsible for the development of the society.

Mission of the Department

- To provide quality education through faculty and state of art infrastructure
- To identify the current problems in society pertaining to Civil Engineering disciplines and to address them effectively and efficiently
- To inculcate the habit of research and entrepreneurship in our graduates to address current infrastructure needs of society

Program outcomes (POs)

Engineering Graduates will be able to:

PO1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change

Program Specific Outcomes (PSOs)

PSO1: Provide necessary solutions to build infrastructure for all situations through Competitive plans, maps and designs with the aid of a thorough Engineering Survey and Quantity Estimation.

PSO2: Assess the impact of anthropogenic activities leading to environmental imbalance on land, in water & in air and provide necessary viable solutions revamping water resources and transportation for a sustainable development

Program Educational Objectives (PEOs)

PEO 1- Engaged in professional practices, such as construction, environmental, geotechnical, structural, transportation, water resource engineering by using technical, communication and management skills.

PEO 2- Engaged in higher studies and research activities in various civil engineering fields and life time commitment to learn ever changing technologies to satisfy increasing demand of sustainable infrastructural facilities.

PEO 3- Serve in a leadership position in any professional or community organization or local or state engineering board

PEO 4- Registered as professional engineer or developed a strong ability leading to professional licensure being an entrepreneur.

Module – 1
Introduction
Deflection of Beams

Structure

1.1 Introduction:

When a structure is subjected to the action of applied loads each member undergoes deformation due to which the axis of structure is deflected from its original position. The deflections also occur due to temperature variations and lack-of-fit of members. The deflections of structures are important for ensuring that the designed structure is not excessively flexible. The large deformations in the structures can cause damage or cracking of non-structural elements. The deflection in beams is dependent on the acting bending moments and its flexural stiffness. The computation of deflections in structures is also required for solving the statically indeterminate structures.

1.2 Objectives:

1. Computation of deflection using moment area method**.**

1.3 Moment area Method:

The moment-area method is one of the most effective methods for obtaining the bending displacement in beams and frames. In this method, the area of the bending moment diagrams is utilized for computing the slope and or deflections at particular points along the axis of the beam or frame.

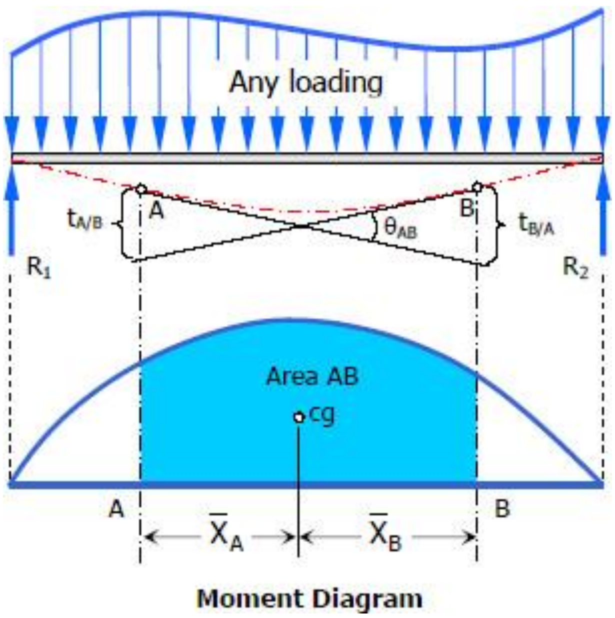


Figure-1: Moment Diagram

Theorems of Area-Moment Method

Theorem I

The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1/EI multiplied by the area of the moment diagram between these two points.

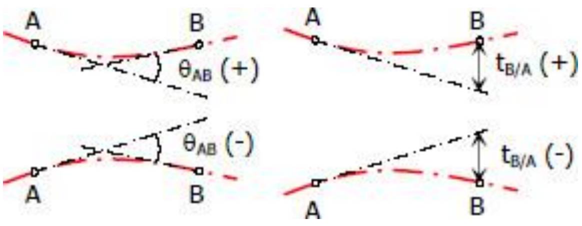
$$\theta_{AB} = \frac{1}{EI}(Area_{AB})$$

Theorem II

The deviation of any point B relative to the tangent drawn to the elastic curve at any other point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

$$t_{B/A} = \frac{1}{EI}(Area_{AB}) \cdot \bar{X}_B \qquad t_{A/B} = \frac{1}{EI}(Area_{AB}) \cdot \bar{X}_A$$

Rules of Sign



1. The deviation at any point is positive if the point lies above the tangent, negative if the point is below the tangent.

2. Measured from left tangent, if θ is counterclockwise, the change of slope is positive, negative if θ is clockwise.

1.4 Conjugate Beam Method:

The conjugate beam method is an extremely versatile method for computation of deflections in beams. The relationships between the loading, shear, and bending moments are given by

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} = -w(x)$$

where$M$ is the bending moment; V is the shear; and w (x) is the intensity of distributed laod.

Similarly, we have the following

$$\frac{d^2v}{dx^2} = \frac{d\theta}{dx} = \frac{M}{EI}$$

A comparison of two set of equations indicates that if M / EI is the loading on an imaginary beam, the resulting shear and moment in the beam are the slope and displacement of the real beam, respectively. The imaginary beam is called as the " **conjugate beam** " and has the same length as the original beam.

There are two major steps in the conjugate beam method. The first step is to set up an additional beam, called "**conjugate beam,**" and the second step is to determine the " **shearing forces** " and " **bendingmoments** " in the conjugate beam.

1.5 Recommended Questions:

1.) Draw the ILD for shear force and BM for a section at 5m from left end of a simply supported beam 20 mlong. Hence calculate the max SF and BM at the section due to an UDL of length of 8m and intensity of 10kN/m.

2.) A simply supported beam of span 20m and subjected to a moving load of 50kN from left and right end. Find the maximum BM at 7m from left end . Also find absolute Maximum BM . Use ILD.

3.4.1 Properties of Conjugate Beam

1. The length of a conjugate beam is always equal to the length of the actual beam.
2. The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
3. A simple support for the real beam remains simple support for the conjugate beam.
4. A fixed end for the real beam becomes free end for the conjugate beam.
5. The point of zero shear for the conjugate beam corresponds to a point of zero slope for the real beam.
6. The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.

Module-2

Energy Principles and Energy Theorems

Structure

2.1 Introduction

2.2 Objectives

2.3 Betti' law

2.4 Strain energy

2.5 Strain energy due to bending

2.**6 Strain energy due to torsion**

2.7 Strain energy due to transverse shear

2.8 Strain energy under axial load:

2.9 Principle of Virtual Work:

2.10 Principle of virtual displacements

2.11 Problems

2.12 Course outcomes

2.13 Recommended question

2.14 Further study

4.1 Introduction:

Deflection by Strain Energy Method

The concepts of strain, strain-displacement relationships are very useful in computing energy-related quantities such as work and strain energy. These can then be used in the computation of deflections. In the special case, when the structure is linear elastic and the deformations are caused by external forces only, (the complementary energy U^* is equal to the strain energy U) the displacement of structure in the direction of force P_j is expressed by

$$\Delta_j = \frac{\partial U}{\partial P_j}$$

2.2 Objectives:

1. Deflection by strain energy method

2. Evaluation of strain energy in member under different loading

2.3 Theorems:

2.3.1Betti's Law

The virtual work δU_{AB} done by a system of forces $\Sigma \mathbf{P}_B$ that undergo a displacement caused by a system of forces $\Sigma \mathbf{P}_A$ is equal to the virtual work δU_{BA} caused by the forces $\Sigma \mathbf{P}_A$ when the structure deforms due to the system of forces $\Sigma \mathbf{P}_B$, that is, $\delta U_{AB} = \delta U_{BA}$.

2.3.2 Maxwell's Theorem

The displacement of a point B on a structure due to a unit load acting at point A is equal to the displacement of point A when the unit load is acting at point B, that is, $f_{BA} = f_{AB}$.

$$f_{BA} = \int \frac{m_B m_A}{EI} dx$$

$$f_{AB} = \int \frac{m_A m_B}{EI} dx$$

The rotation of a point B on a structure due to a unit couple moment acting at point A is equal to the rotation of point A when the unit couple moment is acting at point B, that is, $\alpha_{BA} = \alpha_{AB}$.

These are useful for 2nd-degree-indeterminate and higher structures.

2.3.3Castigliano's first theorem – for forces in an elastic structure

Castigliano's method for calculating forces is an application of his first theorem, which states:

If the strain energy of an elastic structure can be expressed as a function of generalized displacement q_i; then the partial derivative of the strain energy with respect to generalized displacement gives the generalized force Q_i.

In equation form,

$$Q_i = \frac{\partial \mathbf{U}}{\partial q_i}$$

where U is the strain energy.

If the force-displacement curve is nonlinear then the complementary strain energy needs to be used instead of strain energy.

2.3.4 Castigliano's second theorem – for displacements in a linearly elastic structure.

Castigliano's method for calculating displacements is an application of his second theorem, which states:

If the strain energy of a linearly elastic structure can be expressed as a function of generalized force Qi; then the partial derivative of the strain energy with respect to generalized force gives the generalised displacement qi in the direction of Qi.

As above this can also be expressed as:

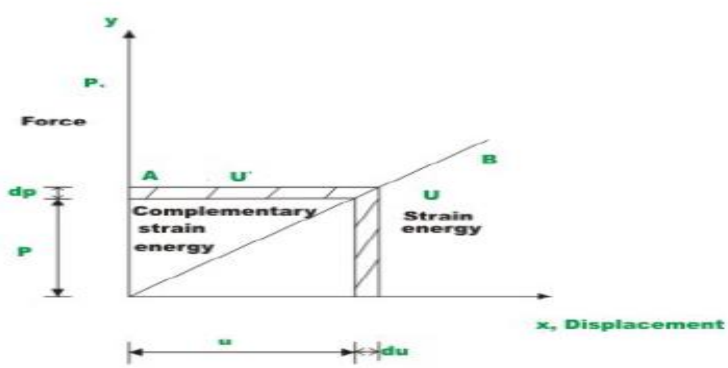
$$q_i = \frac{\partial \mathbf{U}}{\partial Q_i}.$$

2.4 Strain Energy:

Consider an elastic spring as shown in the Fig. When the spring is slowly pulled, it deflects by a small amount u_1. When the load is removed from the spring, it goes back to the original position. When the spring is pulled by a force, it does some work and this can be calculated once the load-displacement relationship is known. It may be noted that, the spring is a mathematical idealization of the rod being pulled by a force P axially. It is assumed here that the force is applied gradually so that it slowly increases from zero to a maximum value P. Such a load is called static loading, as there are no inertial effects due to motion. Let the load-displacement relationship be as shown in Fig. Now, work done by the external force may be calculated as,

$$W_{ext} = \frac{1}{2} P_1 u_1 = \frac{1}{2} (force \times displacement)$$

Fig. 1.0 Linear Spring



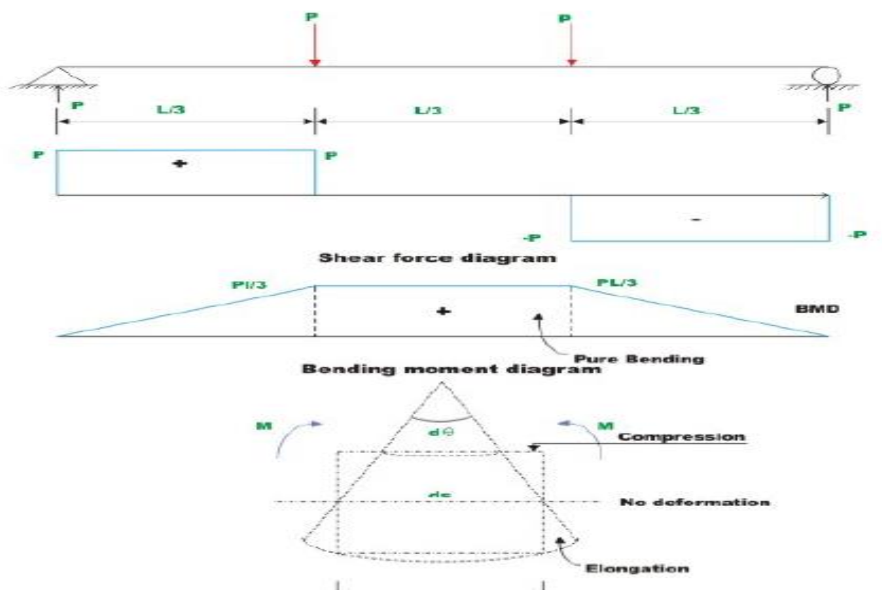
Force-displacement relation

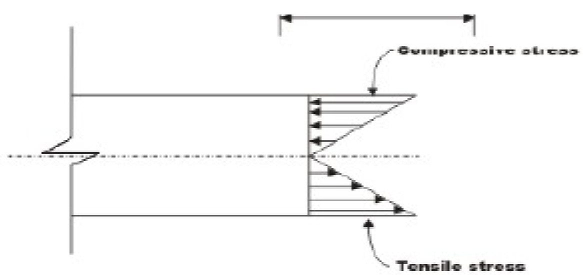
The area enclosed by force-displacement curve gives the total work done by the externally applied load. Here it is assumed that the energy is conserved i.e. the work done by gradually applied loads is equal to energy stored in the structure. This internal energy is known as strain energy. Now strain energy stored in a spring is

$$U = \frac{1}{2} P_1 u_1$$

2.5 Strain energy due to bending

Consider a prismatic beam subjected to loads as shown in the Fig. The loads are assumed to act on the beam in a plane containing the axis of symmetry of the cross section and the beam axis. It is assumed that the transverse cross sections (such as AB and CD), which are perpendicular to centroidal axis, remain plane and perpendicular to the centroidal axis of beam





Consider a small segment of beam of length ds subjected to bending moment as shown in the Fig. Now one cross section rotates about another cross section by a small amount dθ. From the figure

$$d\theta = \frac{1}{R}ds = \frac{M}{EI}ds$$

Where R is the radius of curvature of the bent beam and EI is the flexural rigidity of the beam. Now the work done by the moment M while rotating through angle dθ will be stored in the segment of beam as strain energy dU. Hence,

$$dU = \frac{1}{2} M \, d\theta$$

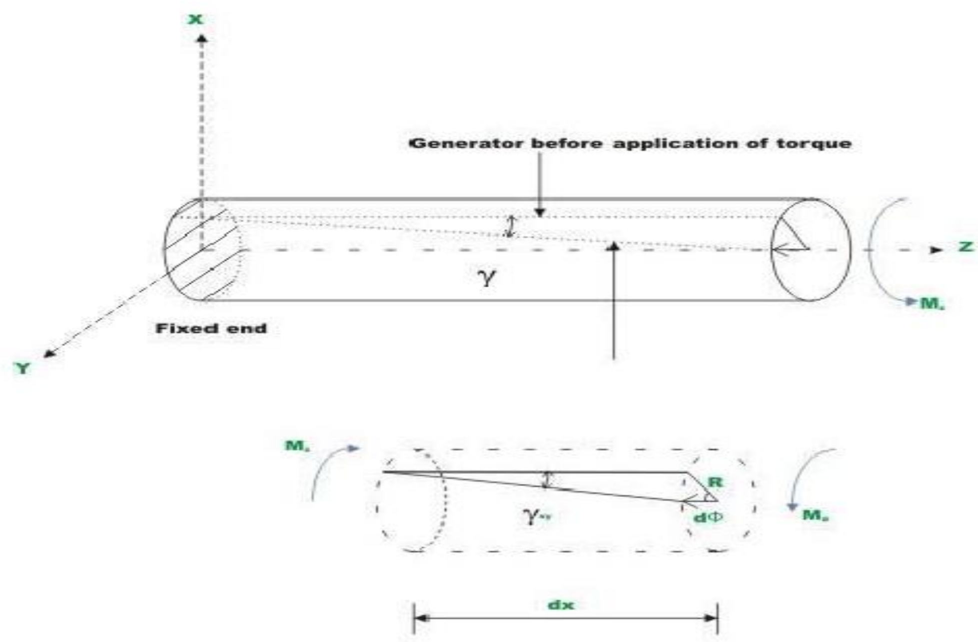
Substituting for dθ in equation

$$dU = \frac{1}{2} \frac{M^2}{EI} ds$$

Now, the energy stored in the complete beam of span L may be obtained by integrating equation Thus,

$$U = \int_0^L \frac{M^2}{2EI} ds$$

4.6 Strain energy due to torsion**:**



Consider a circular shaft of length L radius R, subjected to a torque T at one end (see Fig). Under the action of torque one end of the shaft rotates with respect to the fixed end by an angle dφ. Hence the strain energy stored in the shaft is,

$$U = \frac{1}{2} T\phi$$

Consider elemental length ds of the shaft. Let the one end rotates by a small amount dφ with respect to another end. Now the strain energy stored in the elemental length is

$$dU = \frac{1}{2} T d\phi$$

We know that,

$$d\phi = \frac{Tds}{GJ}$$

where, G is the shear modulus of the shaft material and is J the polar moment of area. Substituting for dφ from (3) in equation (2), we obtain

$$dU = \frac{T^2}{2GJ} ds$$

Now, the total strain energy stored in the beam may be obtained by integrating the above equation

$$U = \int_0^L \frac{T^2}{2GJ} ds$$

2.7 Strain energy due to transverse shear:

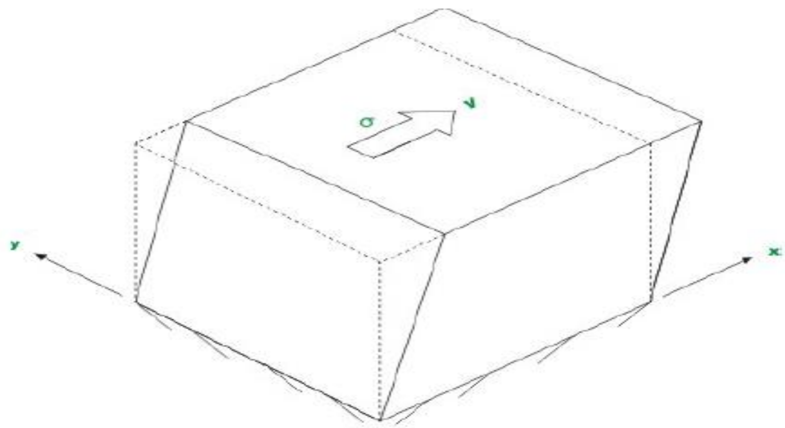
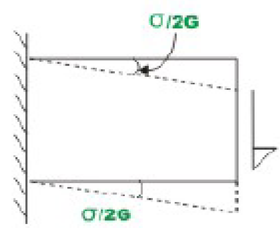


Fig. 1.0 (a) Shear Deformation



The shearing stress on a cross section of beam of rectangular cross section may be found out by the relation

$$\tau = \frac{VQ}{bI_{zz}}$$

where is the first moment of the portion of the cross-sectional area above the point where shear stress is required about neutral axis, Vis the transverse shear force b is the width of the rectangular cross-section and I_{zz} is the moment of inertia of the cross-sectional area about the neutral axis. Due to shear stress, the angle between the lines which are originally at right angle will change. The shear stress varies across the height in a parabolic manner in the case of a rectangular cross-section. Also, the shear stress distribution is different for different shape of the cross section. However, to simplify the computation shear stress is assumed to be uniform (which is strictly not correct) across the cross section. Consider a segment of length ds subjected to shear stress τ. The shear stress across the cross section may be taken as

$$\tau = k\frac{V}{A}$$

in which A is area of the cross-section and *k* is the form factor which is dependent on the shape of the cross section. One could write, the deformation du as

$$du = \Delta\gamma\, ds$$

where Δy is the shear strain and is given by

$$\Delta\gamma = \frac{\tau}{G} = k\frac{V}{AG}$$

Hence, the total deformation of the beam due to the action of shear force is

$$u = \int_0^L k\frac{V}{AG}ds$$

Now the strain energy stored in the beam due to the action of transverse shear force is given by,

$$U = \frac{1}{2}Vu = \int_0^L \frac{kV^2}{2AG}ds$$

The strain energy due to transverse shear stress is very low compared to strain energy due to bending and hence is usually neglected. Thus the error induced in assuming a uniform shear stress across the cross section is very small.

2.8 Strain energy under axial load:

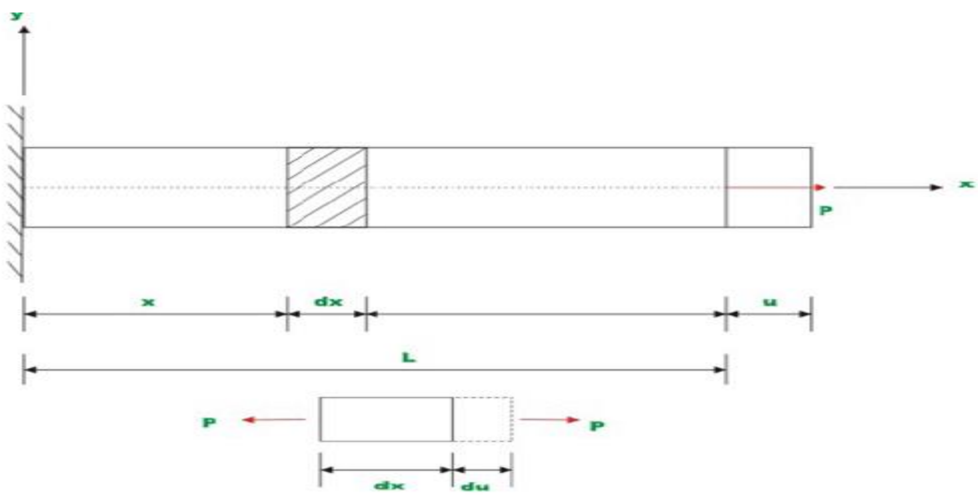
Consider a member of constant cross sectional area A, subjected to axial force P as shown in Fig Let E be the Young's modulus of the material. Let the member be under equilibrium under the action of this force, which is applied through the centroid of the cross section.

Now, the applied force P is resisted by uniformly distributed internal stresses given by average stress σ =P/A as shown by the free body diagram under the action of axial load P applied at one end gradually, the beam gets elongated

This may be calculated as follows. The incremental elongation of du small element of length dx of beam is given by

$$du = \varepsilon\, dx = \frac{\sigma}{E} dx = \frac{P}{AE} dx$$

$$u = \int_0^L \frac{P}{AE} dx$$



Now the work done by external loads W= 1/2XPu

In a conservative system, the external work is stored as the internal strain energy. Hence, the strain energy stored in the bar in axial deformation is

$$U = \frac{1}{2} Pu$$

$$U = \int_0^L \frac{P^2}{2AE} dx$$

2.9 Principle of Virtual Work:

The principle of virtual work states that in equilibrium the virtual work of the forces applied to a system is zero. Newton's laws state that at equilibrium the applied forces are equal and opposite to the reaction, or constraint forces. This means the virtual work of the constraint forces must be zero as well.

2.10 Principle of virtual displacements:

There are many forces in a mechanical system that does no work during a virtual displacement, which means that they need not be considered in this analysis. The principle of virtual work states that in equilibrium the virtual work of the forces applied to a system is zero

Complimentary Energy: - The area enclosed by the inclined line and the vertical axis is called the complementary strain energy. For linearly elastic materials the complementary strain energy and elastic strain energy are the same

2.11 Problems:

1. Find the horizontal deflection at joint *C* of the pin-jointed frame as shown in Figure (a). *AE* is constant for all members

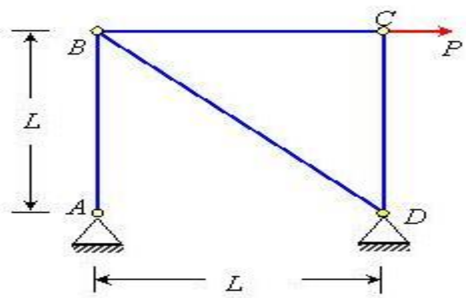


Figure (a)

Solution: The force in various members of the frame is shown in Figure 4.26(b). Calculation of strain energy of the frame is shown in Table

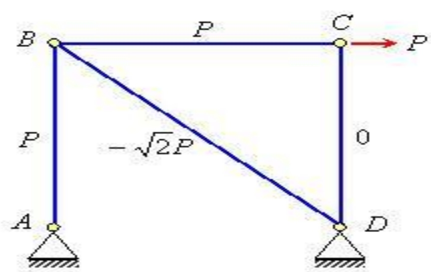


Figure (b). Forces due to applied load

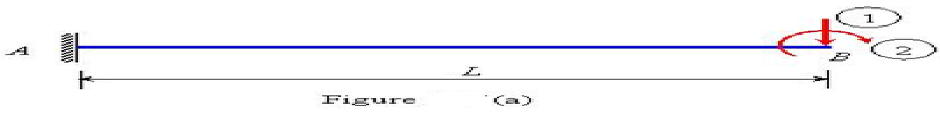
Member	Length (L)	Force (P)	$U = \dfrac{P^2 L}{2AE}$
AB	L	P	$P^2 L / 2AE$
BC	L	P	$P^2 L / 2AE$
BD	$\sqrt{2}L$	$-P\sqrt{2}$	$\sqrt{2}\ P^2 L / 2AE$
CD	L	0	0

$$\sum \quad (\sqrt{2}+1)P^2 L / AE$$

Horizontal displacement of joint *C*, $\Delta_{CH} = \dfrac{\partial U}{\partial P} = \dfrac{\partial}{\partial P}\left(\dfrac{(\sqrt{2}+1)P^2 L}{AE}\right)$

$$= \frac{2(\sqrt{2}+1)PL}{AE}(\rightarrow)$$

1. Verify Maxwell-Betti law of reciprocal displacement for the cantilever beam shown in Figure



Solution: Apply the forces and in the directions P 1 and P2, respectively. The total strain energy stored is calculated below

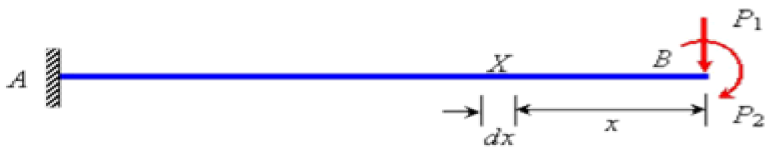
Consider any point *X* at a distance *x* from *B*

$$M_x = -(P_1 x + P_2)$$

$$U = \int_0^L \frac{M_x^2 dx}{2EI}$$

$$= \frac{1}{2EI} \int_0^L (P_1 x + P_2)^2 dx$$

$$= \frac{1}{2EI} \left(\frac{P_1^2 L^3}{3} + P_1 P_2 L^2 + P_2^2 L \right)$$



$$\Delta_{12} = \left.\frac{\partial U}{\partial P_1}\right|_{P_1=0, P_2=1}$$

$$= \frac{1}{2EI} \left(\frac{2P_1 L^3}{3} + P_2 L^2 + 0 \right)\Bigg|_{P_1=0, P_2=1}$$

$$= \frac{L^2}{2EI}$$

$$\Delta_{21} = \left.\frac{\partial U}{\partial P_2}\right|_{P_1=1, P_2=0}$$

$$= \frac{1}{2EI} (0 + P_1 L^2 + 2 P_2 L)\Big|_{P_1=1, P_2=0}$$

$$= \frac{L^2}{2EI}$$

$$\Delta_{12} = \Delta_{21}$$

2.12 Course outcomes:

1. Evaluation of strain energy in member under different loading.
2. Application of strain energy method for different types of structure.

2.13 **Recommended questions**:

1. Define Bettis law?

2. Define Maxwell's Theorem?

3. Calculate deflections of a statically determinate structure in any direction at a point where the load is not acting by fictious (imaginary) load method

2.14 **Further study:**

1. https://www.slideshare.net/scemd3/complimentary-energy-method-in-structural-analysis

2. https://www.studocu.com/en-nz/document/university-of-auckland/engineering-mechanics/lecture-notes/83-complementary-strain-energy/1243054/view

3. https://mae.ufl.edu/haftka/adv-elast/lectures/Sections%205.1-2.pdf

MODULE 3

Structure

3.1 Introduction:

An arch is a structure that spans a certain distance and supports the structure and weight above it. They are classified as three-hinged, two-hinged or hinge less. As shown in Figure 3.1, if the arch is three-hinged, two hinges are at the supports and the third hinge is anywhere within arch. This type of arch is commonly used in steel structures. Compared to fixed-end connections, three hinged arches are a better choice. This type of arch is also a statically derminate structure, therefore it can be easily solved by using the laws of equilibrium and statics**.**

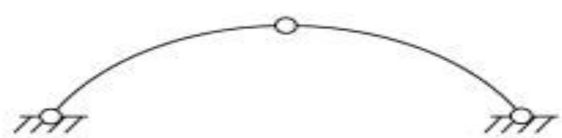


Figure 3.1: Three hinged arch

3.2 Objectives:

1. Familiarization with three-hinged arches (unsymmetrical and symmetrical)
2. Application of the method of sections and the conditions of equilibrium to calculate the support forces for
   point load, distributed load, moving load
3. Investigation of the influence of the load on the horizontal thrust in the supports

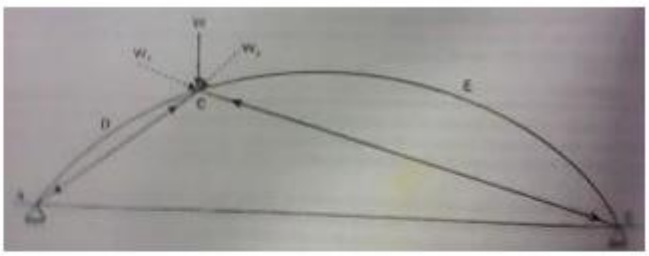
3.3 Arches and types:

Arches

An arch is defined as a curved girder, having convexity upwards and supported at its ends.
The supports must effectively arrest displacements in the vertical and horizontal directions. Only then there will be arch action.

Linear arch

If an arch is to take loads, say W_1, W_2, and W_3 and a vector diagram and funicular polygon are plotted as shown; the funicular polygon is known as the linear arch or theoretical arch.

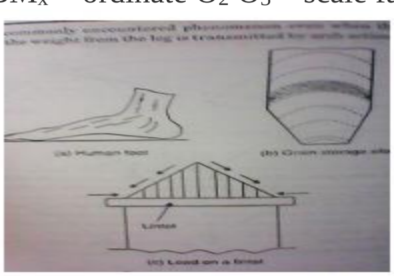


The polar distance at point represents the horizontal thrust.
The links AC, CD, DE and EB will br under compression and there will be no bending moment.
If an arch of this shape ACDEB is provided, there will be no bending moment.

Eddy theorem:

Eddy theorem states that The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the center line of the actual arch

BM_x = ordinate O_2 O_3 * scale factor

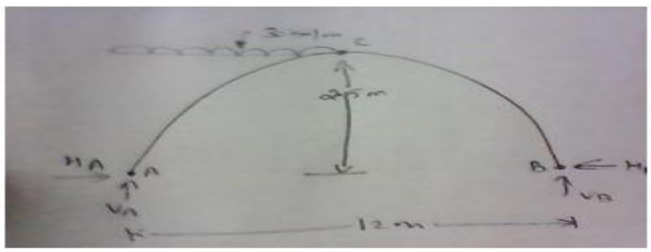


Degree of static indeterminacy of a three hinged parabolic arch

For a three-hinged parabolic arch, the degree of static indeterminacy is zero. It is statically determinate.

3.4 Problems:

1. A three hinged parabolic arch hinged at the crown and springing has a horizontal span of 12m and a central rise of 2.5m. it carries a udl of 30 kN/m run over the left hand half of the span. Calculate the resultant at the end hinges.



Let us take a section X of an arch. Let x be the inclination of the tangent at X. if H is the horizontal thrust and V the net vertical shear at X, from theb free body of the RHS of the arch, it is clear that V and H will have normal and radial components given by,

N = H cos θ+ V sin θ

R = V cos θ +H sin θ

The normal thrust and radial shear in an arch rib.

Parabolic arches are preferable to carry distributed loads. Because, both, the shape of the arch and the shape of the bending moment diagram are parabolic. Hence the intercept between the theoretical arch and actual arch is zero everywhere. Hence, the bending moment at every section of the arch will be zero. The arch will be under pure compression that will be economical.

Difference between the basic action of an arch and a suspension cable

An arch is essentially a compression member, which can also take bending moments and shears. Bending moment and shears will be absent if the arch is parabolic and the loading uniformly distributed.
A cable can take only tension. A suspension bridge will therefore have a cable and a stiffening girder. The girder will take the bending moment and shears in the bridge and the cable, only tension.

Because of the thrust in cables and arches, the bending moments are considerably reduced. If the load on the girder in uniform. The bridge will have only cable tension and no bending moment on the girder.

Under what conditions will the bending moment in an arch be zero throughout**?**

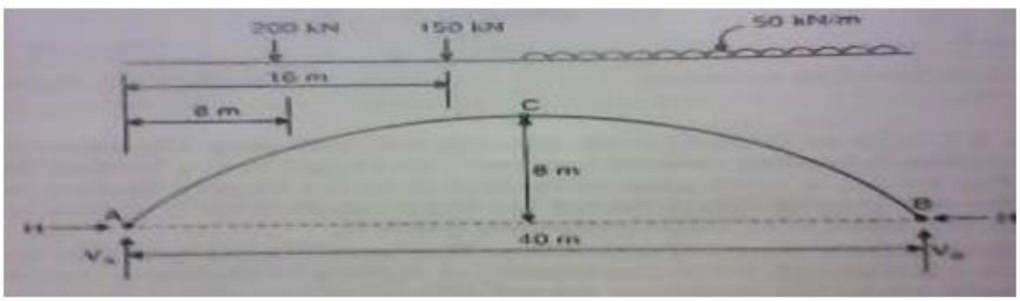
The bending moment in an arch throughout the span will be zero, if

(i) The arch is parabolic and

(ii) The arch carries udl throughout the span

3.4 Problems:

1. A 3 hinged arch of span 40m and rise 8m carries concentrated loads of 200 kN and 130 kN at a distance of 8m and 16m from the left end and an udl of 50 kN/m on the right half of the span. Find the horizontal thrust.



Solution:

(a) Vertical reactions V_A and V_B :

Taking moments about A,

$200(8) + 150(16) + 50 * 20 * (20 + 20/2) + V_B (40) = 0$

$1600 + 2400 + 30000 + 40 V_B = 0$

$V_B = = 850$ kN

V_A = Total load $V_B = 200 + 150 + 50 * 20 - 850 = 500$ kN

(b) Horizontal thrust (H)

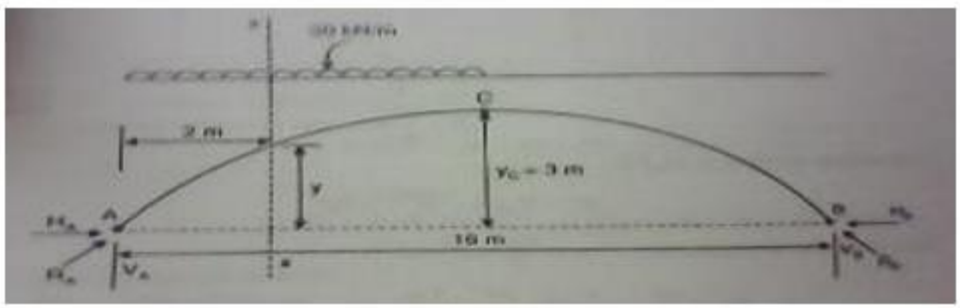
Taking moments about C,

$-H \times 8 + V_A (20) + 200 (20 * 8) + 150 (20 * 16) = 0$ $-8H + 500 * 20 * 200 (12) * 150 (4) = 0$

H = 875 kN

2. A parabolic 3-hinged arch carries a udl of 30kN/m on the left half of the span. It has a span of 16m and central rise of 3m. Determine the resultant reaction at supports. Find the bending moment, normal thrust and radial shear at xx, and 2m from left support

Solution:



(1) Reaction at A and B

Vertical components of reactions;

Taking moments about A,

$-V_B (16) + 30 \times 8^2 /2 = 0$

$- V_B (16) + 30 * 32 = 0$ $V_B = 60$ kN

V_A = Total load $V_B = 30 * 8 = 60$ kN $V_A = 180$ kN

(ii) Horizontal components of reactions at A and

Taking moments about the crown point C,

$V_A * 8 - 30 * 8 * 8/2 - H_A * y_c = 0$

$180 * 8 - 30 * 32 = H_A * 3$

$H_A = 160$ kN

$H_B = H_A =$ since $\sum H = 0$

$H_B = 160$ kN

(iii) Resultants reactions at A and B;

$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(180)^2 + (160)^2} = 240.83 \ kN$

$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(60)^2 + (160)^2} = 170.88 \ kN$

Bending moment at x = 2m from A:

Bending moment = V_A (2) +30 * 2 *1 +H_A(y) ---- (1)

Where, y = Rise of the arch at x = 2m from 'A':

For parabolic arches, $y = \frac{4r}{l^2} * x(l-x)$ at a distance of 'x' from thje support

Where, r = rise of the arch at Crown Point = 3m

$$y = \frac{4*3}{(16)^2} * 2(16-2)$$

Substitute in (1) y = 1.3125 m at x = 2m from A.

Bending moment at x = 2m from A = 180 (2) +30 * 2 * 1 +160 * 1.3125

Bending moment at xx = 90 kNm

Radial shear force at x = 2m from A

H = Horizontal shear force = 160 kN

$$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l-2x)\right]$$

$$\theta = \tan^{-1}\left[\frac{4*3}{(16)^2}(16-2(2))\right]$$

θ = 29º21'

R = 120 cos 29º21' – 160 sin 29º21'

R = 26.15 kN

Normal thrust at x = 2m from A:

Normal thrust $P_N = V_x \sin\theta + H\cos\theta$ = 120 sin 29 .21+160cos 29θ21

P_N = 198.28 kN.

3.5 Course Outcome:

1. Define an arch.

2. Identify three-hinged, two-hinged and hinge less arches.

3. State advantages of arch construction.

4. Analyze three-hinged arch.

5. Evaluate horizontal thrust in three-hinged arch.

3.6 **Recommended questions:**

1. Differentiate between two hinged arch and three hinged arch?

3.7 Further **study**

nptel.ac.in/courses/105105109/pdf/m5l32.pdf

www.gecdelhi.ac.in/pdf_files/three.DOC

Module – 4
Introduction

Structure

4.0 Introduction

By forming **slope deflection equations** and applying joint and shear equilibrium conditions, the rotation angles (or the slope angles) are calculated. Substituting them back into the slope deflection equations, member end moments are readily determined. Deformation of member is due to the bending moment.

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4.1Objectives

To obtain slope and deflection of beam and frame structures using slope deflection method

Equilibrium conditions

The slope-deflection equations give us the moment at either end of each element within a structure as a function of end rotations, the chord rotation, and the fixed end moments caused by the external loads between the nodes

$$\boxed{M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi) + \mathrm{FEM}_{AB}}$$

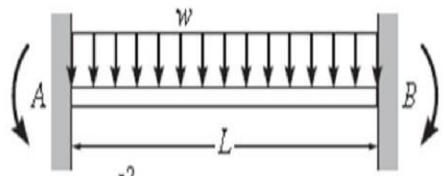
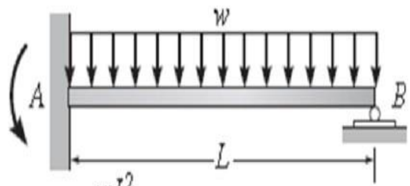
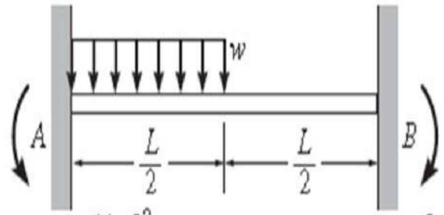
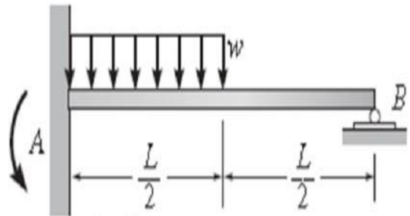
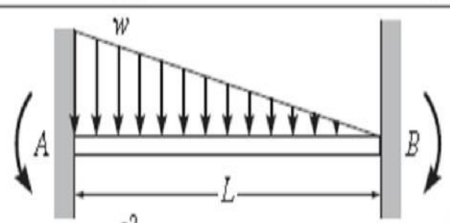
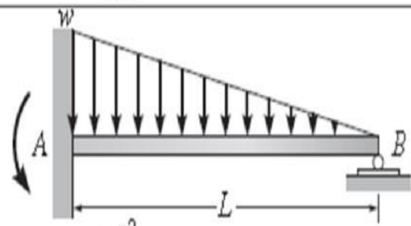
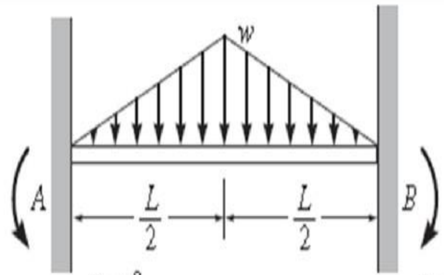
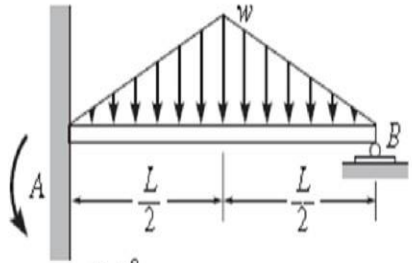
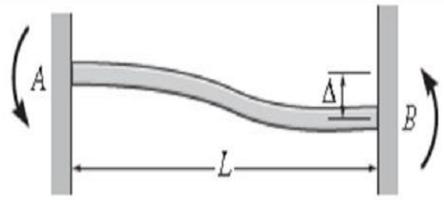
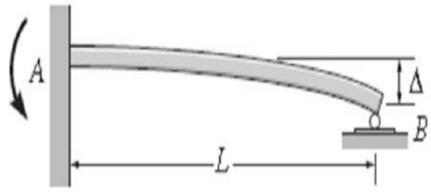
$$\boxed{M_{BA} = \frac{2EI}{L}(\theta_A + 2\theta_B - 3\psi) + \mathrm{FEM}_{BA}}$$

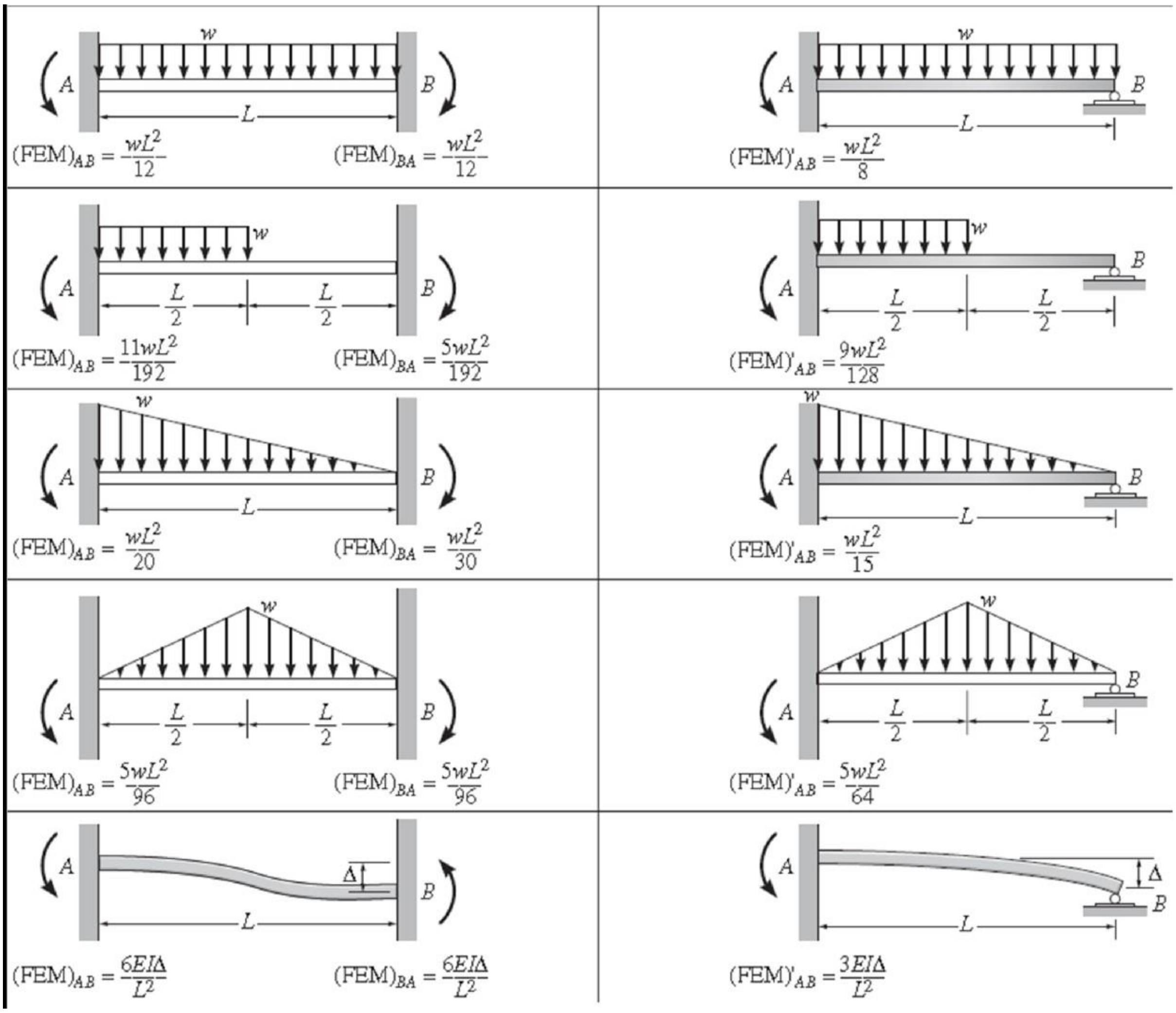
$$\boxed{M_{nf} = \frac{2EI}{L}(2\theta_n + \theta_f - 3\psi) + \mathrm{FEM}_{nf}}$$

$$\boxed{M_{rh} = \frac{3EI}{L}(\theta_r - \psi) + \left(\mathrm{FEM}_{rh} - \frac{\mathrm{FEM}_{hr}}{2}\right)}$$

$$\boxed{M_{hr} = 0}$$

Fixed end table

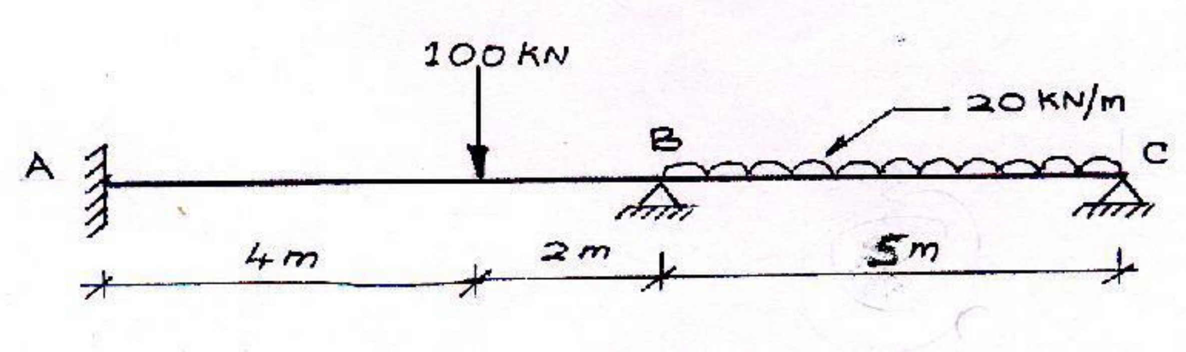
 $(FEM)_{AB} = \frac{wL^2}{12}$ $(FEM)_{BA} = \frac{wL^2}{12}$	 $(FEM)'_{AB} = \frac{wL^2}{8}$
 $(FEM)_{AB} = \frac{11wL^2}{192}$ $(FEM)_{BA} = \frac{5wL^2}{192}$	 $(FEM)'_{AB} = \frac{9wL^2}{128}$
 $(FEM)_{AB} = \frac{wL^2}{20}$ $(FEM)_{BA} = \frac{wL^2}{30}$	 $(FEM)'_{AB} = \frac{wL^2}{15}$
 $(FEM)_{AB} = \frac{5wL^2}{96}$ $(FEM)_{BA} = \frac{5wL^2}{96}$	 $(FEM)'_{AB} = \frac{5wL^2}{64}$
 $(FEM)_{AB} = \frac{6EI\Delta}{L^2}$ $(FEM)_{BA} = \frac{6EI\Delta}{L^2}$	 $(FEM)'_{AB} = \frac{3EI\Delta}{L^2}$



General Procedure OF Slope-Deflection Method

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

4.2 Analyze two span continuous beam ABC by slope deflection method. Then draw Bending moment & Shear force diagram. Take EI constant.



Fixed end moments are

$$M_{AB=}\ -\frac{Wab^2}{L^2} = -\frac{100\ \times\ 4\ \times 2^2}{6^2} = -44.44\ \text{KNm}$$
$$M_{BA=}\ \frac{Wa^2b}{L^2} = \frac{100\ \times\ 4^2 \times\ 2}{6^2} = 88.89\text{KNm}$$
$$M_{BC=}\ -\frac{wL^2}{12} = -\frac{20\ \times\ 5^2}{12} = -41.67\text{KNm}$$
$$M_{CB\ =}\ \frac{wL^2}{12} = \frac{20\ \times\ 5^2}{12} = 41.67\ \text{KNm}$$

Since A is fixed $\Theta_A = 0$ & Θ_B& $\Theta_c \neq 0$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$
$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \quad \cdots\cdots(3)$$
$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \quad \cdots\cdots(4)$$

In all the above 4 equations there are only 2 unknowns θ_B& θ_C and accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$

$M_{CB} = 0$ as end C is simply supported.

$$M_{BA} + M_{BC} = 88.89 + \tfrac{2EI}{3}\theta_B \quad - 41.67 + \tfrac{4EI}{5}\theta_B + \tfrac{2EI}{5}\theta_C = 47.22 + \tfrac{22}{15}EI\theta_B + \tfrac{2}{5}EI\theta_C = 0 \quad \cdots\cdots(5)$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B = 0 \quad \cdots\cdots(6)$$

Solving the equations (5) & (6), we get

$$\theta_B = -\frac{20.83}{EI}$$

$$\theta_C = -\frac{41.67}{EI}$$

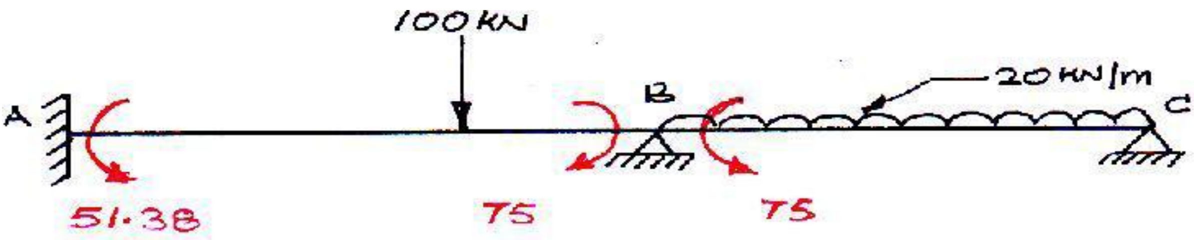
Substituting the values in the slope deflections we have,

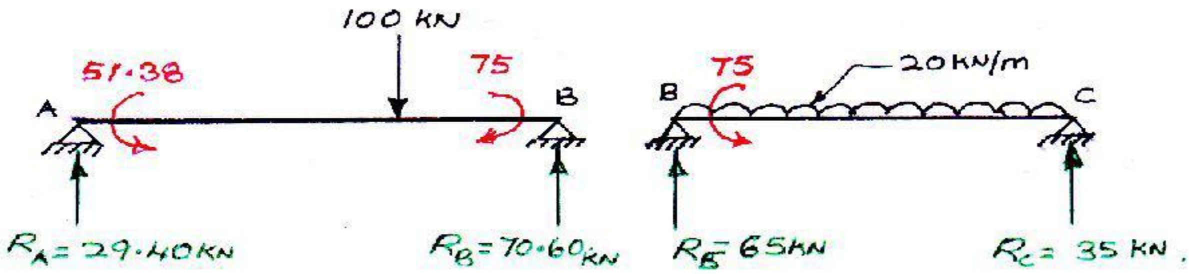
$$M_{AB} = -44.44 + \frac{EI}{3}\left(-\frac{20.83}{EI}\right) = -51.38 \text{ KNm}$$

$$M_{BA} = 88.89 + \frac{2EI}{3}\left(-\frac{20.83}{EI}\right) = 75 \text{ KNm}$$

$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{20.83}{EI}\right) + \frac{2EI}{5}\left(-\frac{41.67}{EI}\right) = -75 \text{ KNm}$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\left(-\frac{41.67}{EI}\right) + \frac{2EI}{5}\left(-\frac{20.83}{EI}\right) = 0$$





Find reactions using equations of equilibrium.

Span AB: MA = 0 , RB×6 = 100×4+75-51.38

RB = 70.60 KN

V = 0 ,

RA+RB = 100KN

RA = 100-70.60=29.40 KN

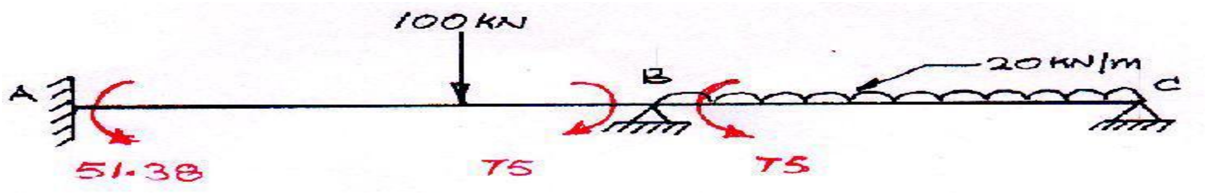
Span BC: MC = 0, RB×5 = 20×5× +75

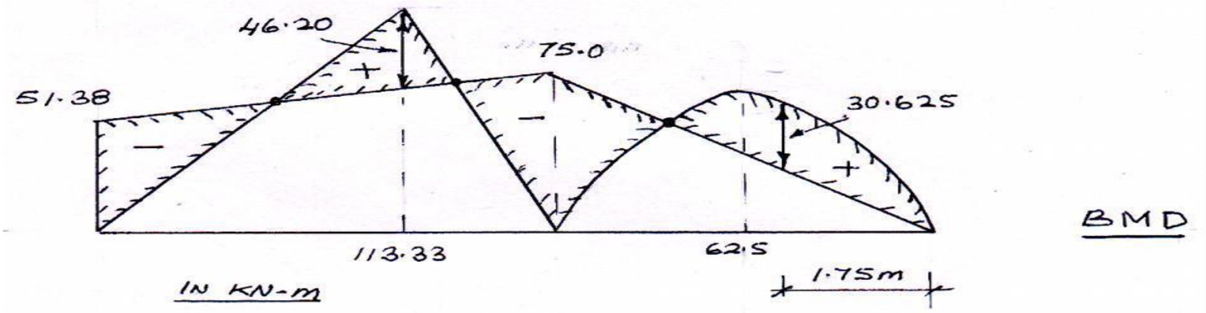
RB = 65 KN

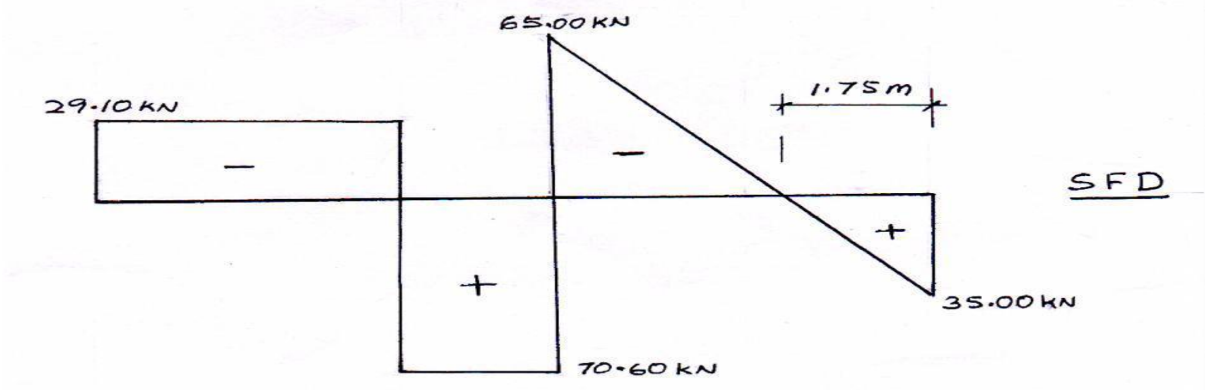
V=0 RB+RC = 20×5 = 100KN

RC = 100-65 = 35 KN

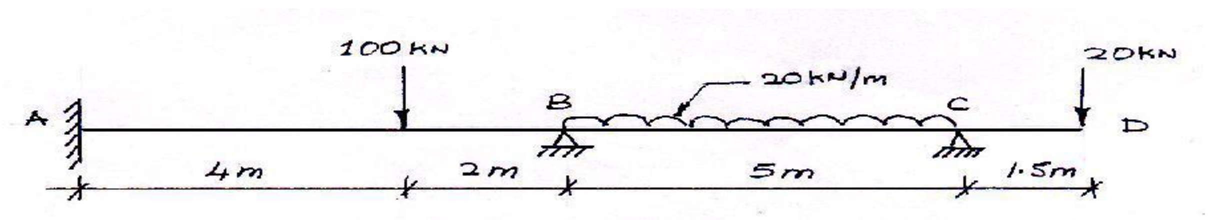
Using these data BM and SF diagram can be drawn







2. Analyze continuous beam ABCD by slope deflection method and then draw bending moment diagram. Take EI constant**.**



$$\Theta_A = 0 \ \& \ \Theta_B \& \ \Theta_c \neq 0$$

$$M_{AB=} \ -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \text{ KNm}$$

$$M_{BA=} \ \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89\text{KNm}$$

$$M_{BC=} \ -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67\text{KNm}$$

$$M_{CB=} \ \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67 \text{ KNm}$$

$$M_{CD} = -20 \times 1.5 = -30 \text{ KNm}$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \quad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \quad \cdots\cdots(4)$$

$$M_{CD} = -20 \times 1.5 = -30 \text{ KNm}$$

In all the above equations there are only 2 unknowns $\theta_B \& \theta_C$ and accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$M_{BA} + M_{BC} = 88.89 + \frac{2EI}{3}\theta_B - 41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C = 47.22 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \quad \cdots\cdots \quad (5)$$

$$M_{CB} + M_{CD} = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B - 30 = 11.67 + \frac{2EI}{5}\theta_B + \frac{4EI}{5}\theta_C = 0 \quad \cdots\cdots \quad (6)$$

Solving equations (5) & (6),

$$\theta_B = -\frac{32.67}{EI}$$

$$\theta_C = \frac{1.75}{EI}$$

Substituting the values in the slope deflections we have,

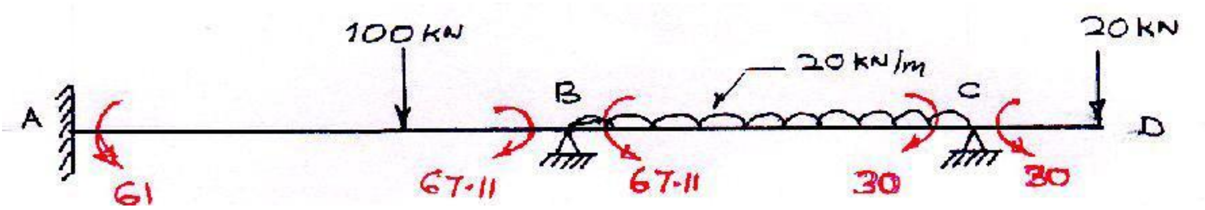
$$M_{AB} = -44.44 + \frac{EI}{3} \times \left(-\frac{32.67}{EI}\right) = -61 \text{ KNm}$$

$$M_{BA} = 88.89 + \frac{2EI}{3} \times \left(-\frac{32.67}{EI}\right) = 67.11\text{KNm}$$

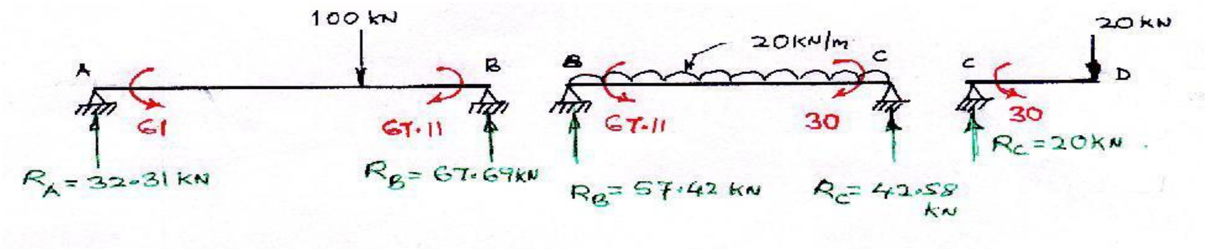
$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{32.67}{EI}\right) + \frac{2EI}{5}\left(\frac{1.75}{EI}\right) = -67.11\text{KNm}$$

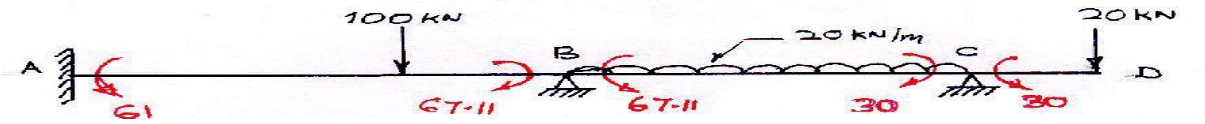
$$M_{CB} = 41.67 + \frac{4EI}{5}\left(\frac{1.75}{EI}\right) + \frac{2EI}{5}\left(-\frac{32.67}{EI}\right) = 30\text{KNm}$$

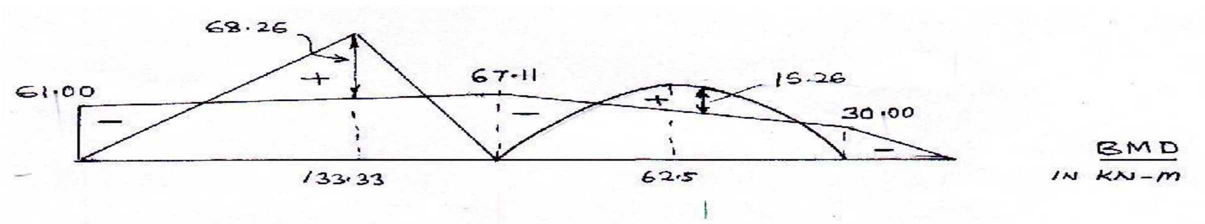
$$M_{CD} = -30\text{KNm}$$

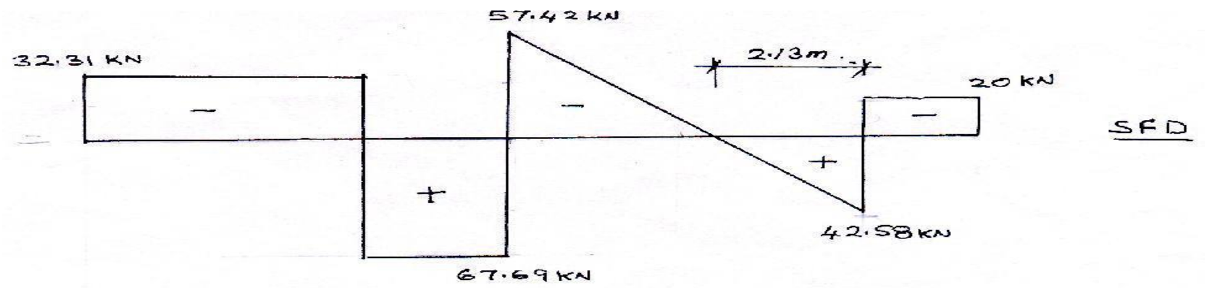


Reactions: Consider free body diagram of beam AB, BC and CD as shown









Span AB: RB 6 =100 4+ 67.11- 61

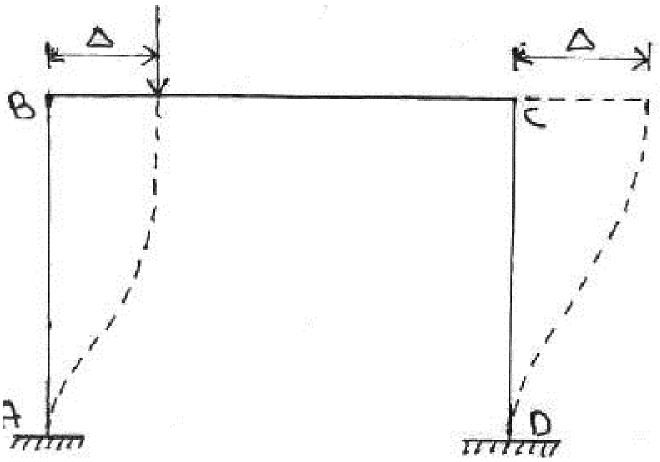
RB = 67.69KN

RA =100- RB =32.31 KN

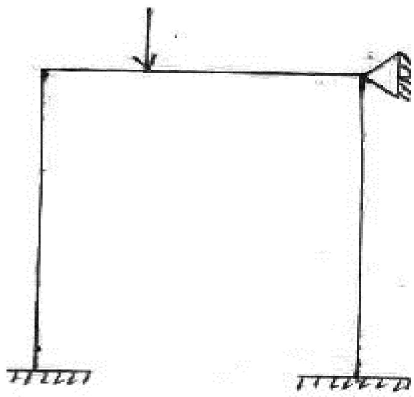
Span BC: RC 5 = 20

4.4 Analysis of frames (without & with sway)

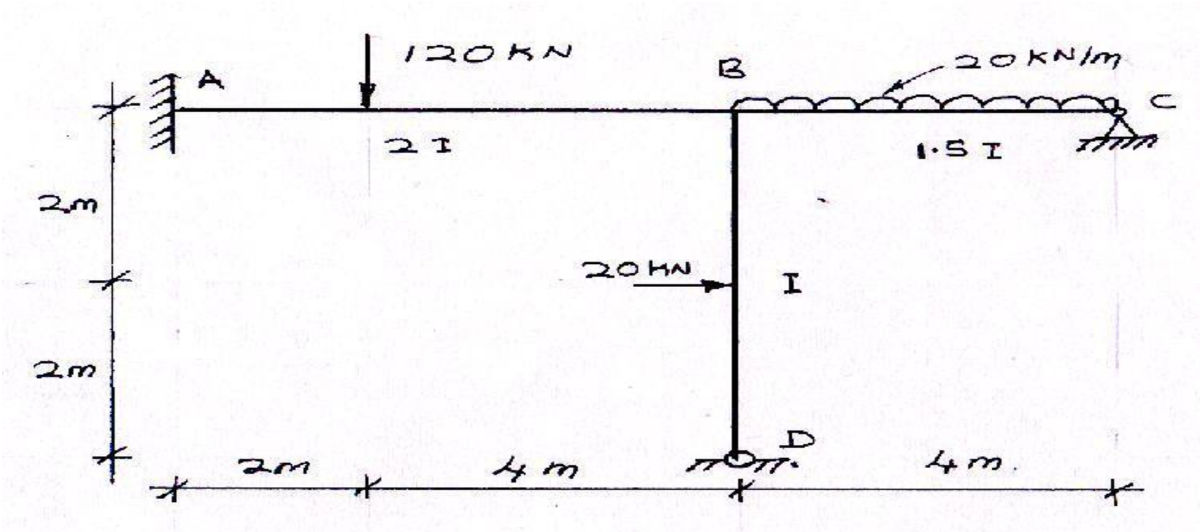
The side movement of the end of a column in a frame is called sway. Sway can be prevented by unyielding supports provided at the beam level as well as geometric or load symmetry about vertical axis.



Frame with sway



3. Analyse the simple frame shown in figure. End A is fixed and ends B & C are hinged. Draw the bending moment diagram**.**



$$\Theta_A = 0 \ \& \ \Theta_B, \Theta_c \ \& \ \Theta_D \neq 0$$

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{120 \times 2 \times 4^2}{6^2} = -106.67\ KNm$$

$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{120 \times 2^2 \times 4}{6^2} = 53.33KNm$$

$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 4^2}{12} = -26.67KNm$$

$$M_{CB} = \frac{wL^2}{12} = \frac{20 \times 4^2}{12} = 26.67\ KNm$$

$$M_{DB} = -\frac{wL}{8} = -\frac{20 \times 4}{8} = -10\ KNm$$

$$M_{BD} = \frac{wL}{8} = \frac{20 \times 4}{8} = 10\ KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -106.67 + \frac{2E(2I)}{6}\theta_B$$

$$= -106.67 + \frac{2EI}{3}\theta_B \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 53.33 + \frac{2E(2I)}{6}2\theta_B = 53.33 + \frac{4EI}{3}\theta_B \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B \qquad \cdots\cdots(4)$$

$$M_{BD} = M_{FBD} + \frac{2EI}{L}[2\theta_B + \theta_D] = 10 + \frac{2EI}{4}2\theta_B + \frac{2EI}{4}\theta_D$$

$$= 10 + EI\theta_B + \frac{EI}{2}\theta_D \qquad \cdots\cdots(5)$$

$$M_{DB} = M_{FDB} + \frac{2EI}{L}[2\theta_D + \theta_B] = -10 + \frac{2EI}{4}2\theta_D + \frac{2EI}{4}\theta_B$$

$$= -10 + EI\theta_D + \frac{EI}{2}\theta_B \qquad \cdots\cdots(6)$$

In all the above equations there are only 3 unknowns $\theta_B \& \theta_C \& \theta_D$ and accordingly the boundary conditions are

$$M_{BA} + M_{BC} + M_{BD} = 0$$
$$M_{CB} = 0$$
$$M_{DB} = 0$$

$M_{BA} + M_{BC} + M_{BD} = 53.33 + \frac{4EI}{3}\theta_B - 26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C + 10 + EI\theta_B + \frac{EI}{2}\theta_D =$
$36.66 + \frac{23}{6}EI\theta_B + \frac{3}{4}EI\theta_C + \frac{EI}{2}\theta_D = 0 \qquad \cdots\cdots \quad (7)$

$M_{CB} = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B = 0 \qquad \cdots\cdots \quad (8)$

$M_{DB} == -10 + EI\theta_D + \frac{EI}{2}\theta_B$ =0 $\qquad \cdots\cdots \quad (9)$

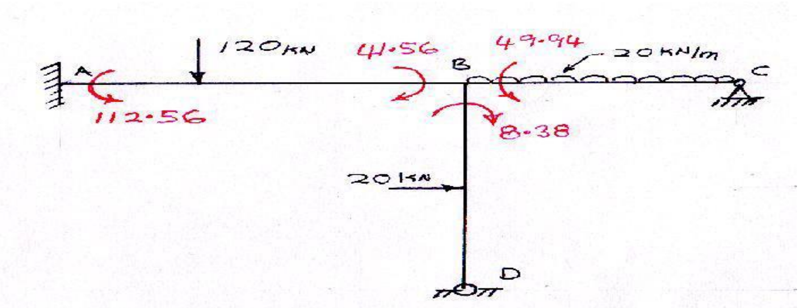
Solving equations (7) & (8) & (9),

$$\theta_B = -\frac{8.83}{EI}$$
$$\theta_C = -\frac{13.36}{EI}$$
$$\theta_D = \frac{14.414}{EI}$$

Substituting the values in the slope deflections we have,

$$M_{AB} = -106.67 + \frac{2}{3}(-8.83) = -112.56KNm$$
$$M_{BA} = 53.33 + \frac{4}{3}(-8.83) = 41.56KNm$$
$$M_{BC} = -26.67 + \frac{3}{2}(-8.83) + \frac{3}{4}(-13.36) = -49.94KNm$$
$$M_{CB} = 26.67 + \frac{3}{2}(-13.36) + \frac{3}{4}(-8.83) = 0$$

$$M_{BD} = 10 - 8.83 + \frac{1}{2}(14.414) = 8.38KNm$$
$$M_{DB} = -10 + \frac{1}{2}(-8.83) + (14.414) = 0$$



REACTIONS:
SPAN AB:

$$R_B = \frac{41.56 - 112.56 + 120 \times 2}{6} = 28.17KN$$
$$R_A = 120 - R_B = 91.83KN$$

SPAN BC:

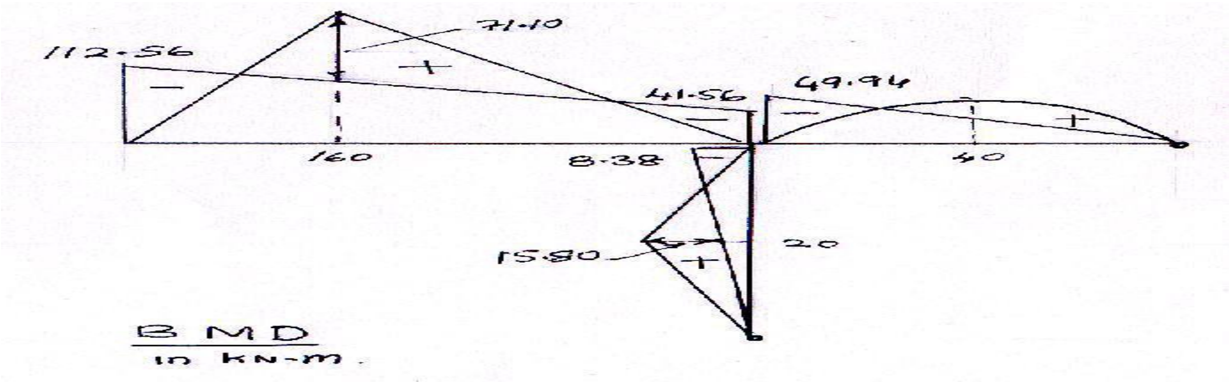
$$R_B = \frac{49.94 + 20 \times 4 \times 2}{4} = 52.485KN$$

$$R_C = 20 \times 4 - R_B = 27.515KN$$

Column BD:

$$H_D = \frac{20 \times 2 - 8.38}{4} = 7.92KN$$

$$H_B = 12.78KN$$



4.5 Assignment questions

1 What is slope deflection equation? Develop slope deflection equation?

2 Write the sign conventions considered while analysing the slope deflection Equation?

3 Analyse the continuous beams and frames by slope deflection method considering Different loadings and draw SFD and BMD?

4.6 Further Readings

2. https://en.wikipedia.org/wiki/Slope_deflection_method
3. www.engineeringwiki.org/wiki/Slope-Deflection_Method_for_Continuous_Beams
4. nptel.ac.in/courses/105105109/15
5. https://www.youtube.com/watch?v=gQtII2rL8PQ

Module 5
MOMENT DISTRIBUTION METHOD

Structure

5.0 Introduction:

This method of analyzing beams and frames was developed by Hardy Cross in 1930. Moment distribution method is basically a displacement method of analysis. But this method side steps the calculation of the displacement and instead makes it possible to apply a series of converging corrections that allow direct calculation of the end moments.

This method of consists of solving slope deflection equations by successive approximation that may be carried out to any desired degree of accuracy. Essentially, the method begins by assuming each joint of a structure is fixed. Then by unlocking and locking each joint in succession, the internal moments at the joints are distributed and balanced until the joints have rotated totheir final or nearly final positions. This method of analysis is both repetitive and easy toapply. Before explaining the moment distribution method certain definitions and concepts must be understood.

Sign convention: In the moment distribution table clockwise moments will be treated +ve and anti-clockwise moments will be treated –ve. But for drawing BMD moments causing concavity upwards (sagging) will be treated +ve and moments causing convexity upwards (hogging) will be treated –ve.

Fixed end moments: The moments at the fixed joints of loaded member are called fixed end moment.

Member stiffness factor:

a) Consider a beam fixed at one end and hinged at other as shown in figure subjected to a clockwise couple M at end B. The deflected shape is shown by dotted line.

BM at any section xx at a distance x from 'B' is given by

$$EI\frac{d^2y}{dx^2} = R_B x - M$$

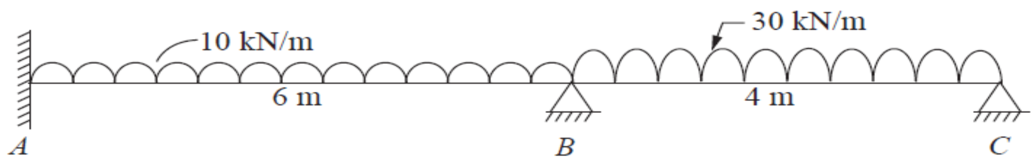
5.1 Objectives

In the moment distribution method, initially the structure is rigidly fixed at every joint or support. The fixed end moments are calculated for any loading under consideration. Subsequently, one joint at a time is then released. When the moment is released at the joint, the joint moment becomes unbalanced. The equilibrium at this joint is maintained by distributing the unbalanced moment. This joint is temporarily fixed again until all other joints have been released and restrained in the new position. This procedure of fixing the moment and releasing them is repeated several times until the desired accuracy is obtained. The experience of designers points that about fi ve cycles of moment distribution lead to satisfactory converging results.

5.3 Problems on Continuous beams

Numerical Problems:

1 Analyze the continuous beam shown in Fig.by the moment distribution method. Draw the bending moment diagram and shear force diagram.The beam is of uniform section.



Step 1
The distribution factors at joint *B* are evaluated as follows

Distribution factors

Joint	Members	Relative Stiffness (k)	Sum Σk	Distribution Factors (k/Σk)
	BA	$I/6 = 0.167I$		0.47
B			$0.3555I$	
	BC	$\frac{3}{4} \times \frac{I}{4} = 0.188I$		0.53

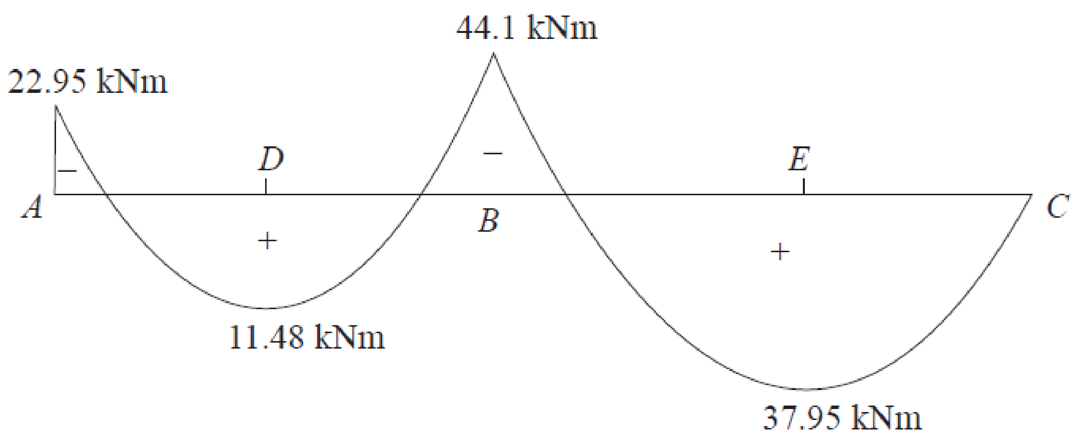
Fixed End Moments

$$M_{FAB} = \frac{-10 \times 6^2}{12} = -30 \text{ kNm}, \; M_{FBA} = +30 \text{ kNm}$$

$$M_{FBC} = \frac{-30 \times 4^2}{12} = -40 \text{ kNm}, \; M_{FCB} = +40 \text{ kNm}$$

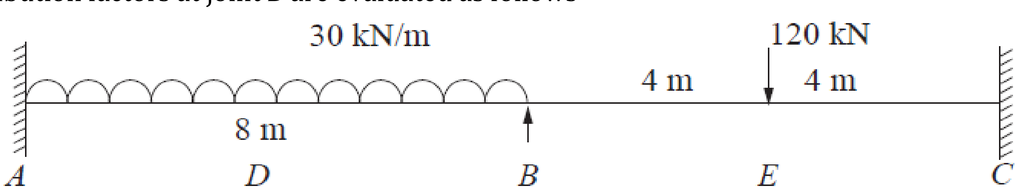
Moment Distribution Table
As the joint *C* is a hinged end, the moment is zero. Hence, it is balanced first. Then half of this moment is carried over. Then joint *B* is balanced. From the joint *B*, the moment is carried over to *A*.

Joint	A	B		C
Members	AB	BA	BC	CB
DF	0	0.47	0.53	1
FEMS	−30.00	+30.00	−40.00	+40.00
Bal	–	–	–	−40.00
Co	–	–	−20.00	
Total	−30.00	+30.00	−60.00	0.00
Bal	–	+14.10	+15.90	–
Co	+7.05	–	–	–
Final	−22.95	+44.10	−44.10	0.00



2. Analyze the continuous beam by the moment distribution method. Drawthe shear force diagram and bending moment diagram

The distribution factors at joint *B* are evaluated as follows



The distribution factors at joint *B* are evaluated as follows

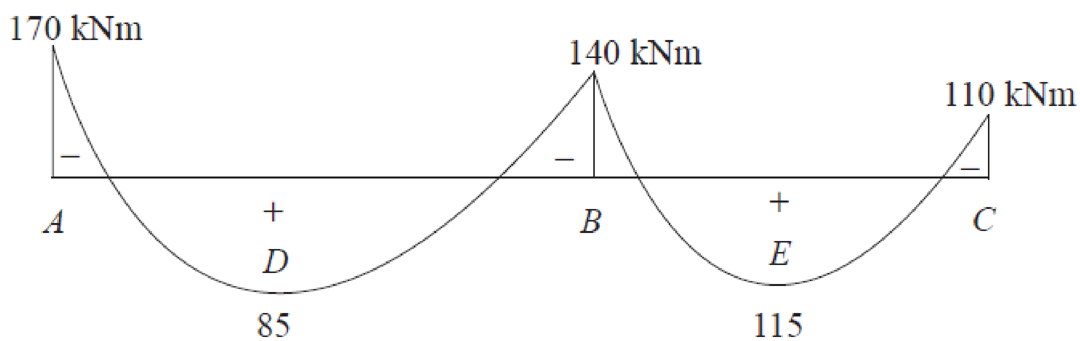
Joint	Members	Relative Stiffness (k)	Sum Σk	Distribution Factors $(k/\Sigma k)$
	BA	$I/8 = 0.125I$		0.5
B			$0.25I$	
	BC	$I/8 = 0.125I$		0.5

$M_{AB} = -160$kN-m
$M_{BA} = +160$ kNm;

Mbc=-120 kn-m
$M_{CB} = +120$ kNm

Moment distribution table

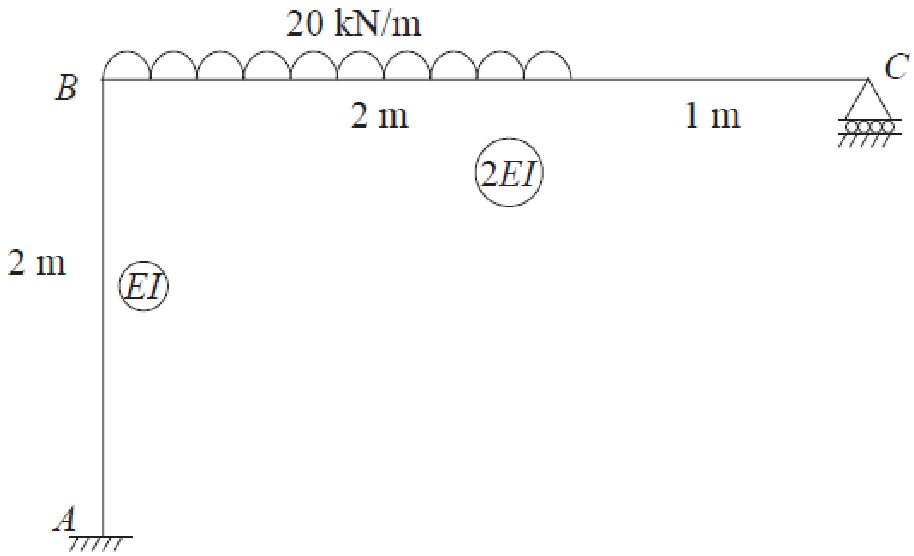
Joint	A	B		C
Members	AB	BA	BC	CB
DF	0	0.5	0.5	0
FEMS	−160	+160	−120	+120
Bal	–	−20	−20	–
Co	−10	–	–	−10
Final	−170	+140	−140	+110



5.4 Problem on Frames

ANALYSIS OF RECTILINEAR FRAMES

Analyze the frame shown in Fig. by moment distribution method. Draw the bending moment diagram



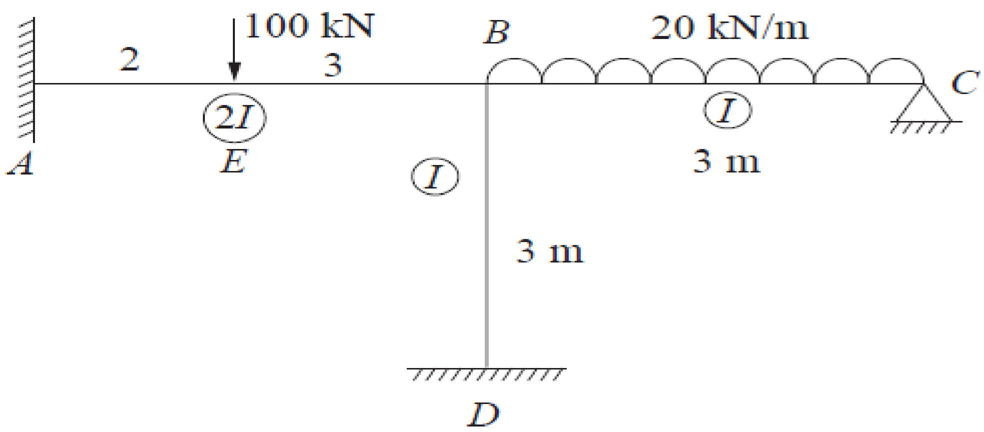
Distribution factors

Joint	*Members*	*Relative Stiffness Values (I/l)*	Σk	$k/\Sigma k$
	BA	$I/2$		0.5
B			I	
	BC	$\frac{3}{4}\left(\frac{2I}{3}\right)=I/2$		0.5

Moment distribution table

Joint	*A*	*B*		*C*
Members	*AB*	*BA*	*BC*	*CB*
DF	0	0.5	0.5	1.0
FEMS			−13.33	+8.89
Bal		+6.66	+6.67	−8.89
Co	+3.33		−4.44	
Bal		+2.22	+2.22	
Co	+1.11			
	+4.44	+8.88	−8.88	0.00

Analyze the frame by the moment distribution method. Draw the bending moment diagram

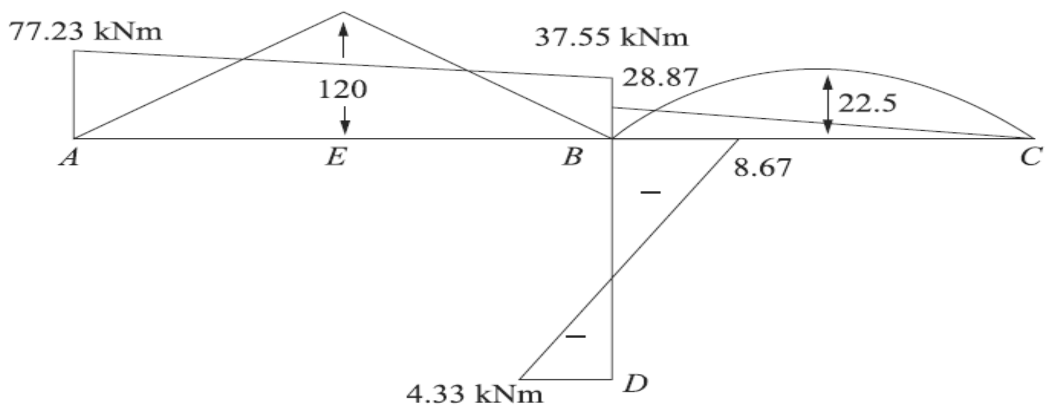


Distribution factors

Joint	Members	Relative Stiffness (k)	Σk	DF = k/Σk
	BA	2I/5 = 0.4I		0.41
B	BD	I/3 = 0.33I	0.98I	0.34
	BC	$\frac{3}{4}(I/3) = 0.25I$		0.25

Moment distribution table

Joint	A		B		C	D
Members	AB	BA	BD	BC	CB	DB
DF	0	0.41	0.34	0.25	1	0
FEMS	−72.00	+48.00	–	−15.00	+15.00	
Bal	–	−13.53	−11.22	−8.25	−15.00	
Co	−6.77			−7.50		−5.61
Bal		+3.08	+2.55	+1.88		
Co	+1.54					+1.28
Total	−77.23	+37.55	−8.67	−28.87	0.00	−4.33



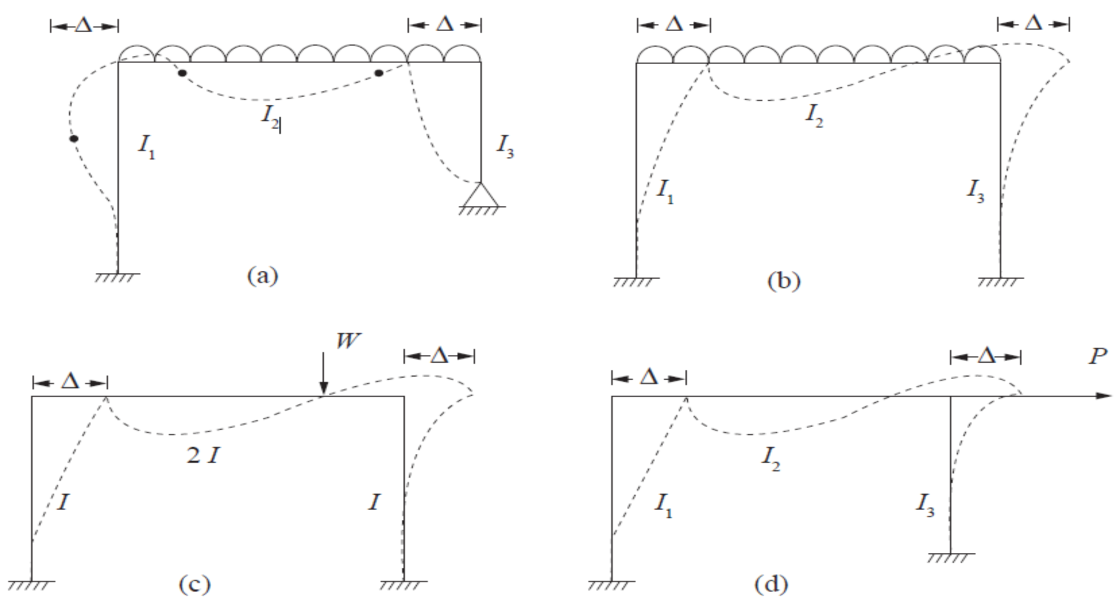
ANALYSIS OF UNSYMMETRICAL FRAMES

1 Portal frames with unequal columns

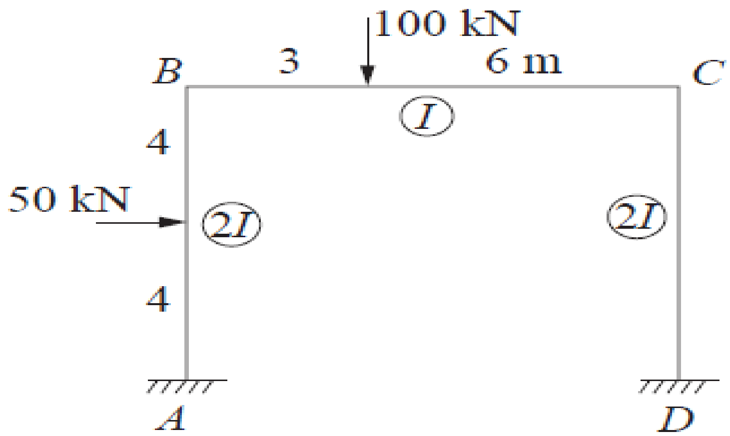
2 Portal frames whose columns are of different moment of inertia

3 Unsymmetrical loading on the beams

4 Lateral loading



3 Analyze the given frame and draw the bending moment diagram



Distribution factors

Joint	*Members*	*Relative Stiff Values (I/l)*	*Σk*	*k/Σk*
	BA	$2I/8 = 0.25I$		0.7
B			$0.36I$	
	BC	$I/9 = 0.11I$		0.3
	CB	$I/9 = 0.11I$		0.3
C			$0.36I$	
	CD	$2I/8 = 0.25I$		0.7

Fixed end moments

$$M_{FAB} = -\frac{Wl}{8} = -50 \times \frac{8}{8} = -50\ \text{kNm}$$

$$M_{FBA} = +\frac{Wl}{8} = +50 \times \frac{8}{8} = +50\ \text{kNm}$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -100 \times 3 \times \frac{6^2}{9^2} = -133.33\ \text{kNm}$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +100 \times 3^2 \times \frac{6}{9^2} = +66.67\ \text{kNm}$$

Non sway Analysis

Joint	A	B		C		D
Members	AB ←	BA	BC ↔	CB	CD →	DC
DF	0	0.70	0.30	0.30	0.70	0
FEMS	−50.00	+50.00	−133.33	+66.67		
Bal		+58.33	+25.00	−20.00	−46.67	
Co	+29.20		−10.00	+12.50		−23.34
Bal		+7.00	+3.00	−3.75	−8.75	
Co	+3.50		−1.88	+1.50		−4.38
Bal		+1.32	+0.56	−0.45	−1.05	
Co	+0.66		−0.23	+0.28		−0.53
Bal		+0.16	+0.07	−0.08	−0.20	
Co	+0.08		−0.04	+0.04		−0.10
Bal		+0.03	+0.01	−0.01	−0.03	
	−16.56	+116.84	−116.84	56.70	−56.70	−28.35
	M'_{AB}	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}	M'_{DC}

Sway Analysis

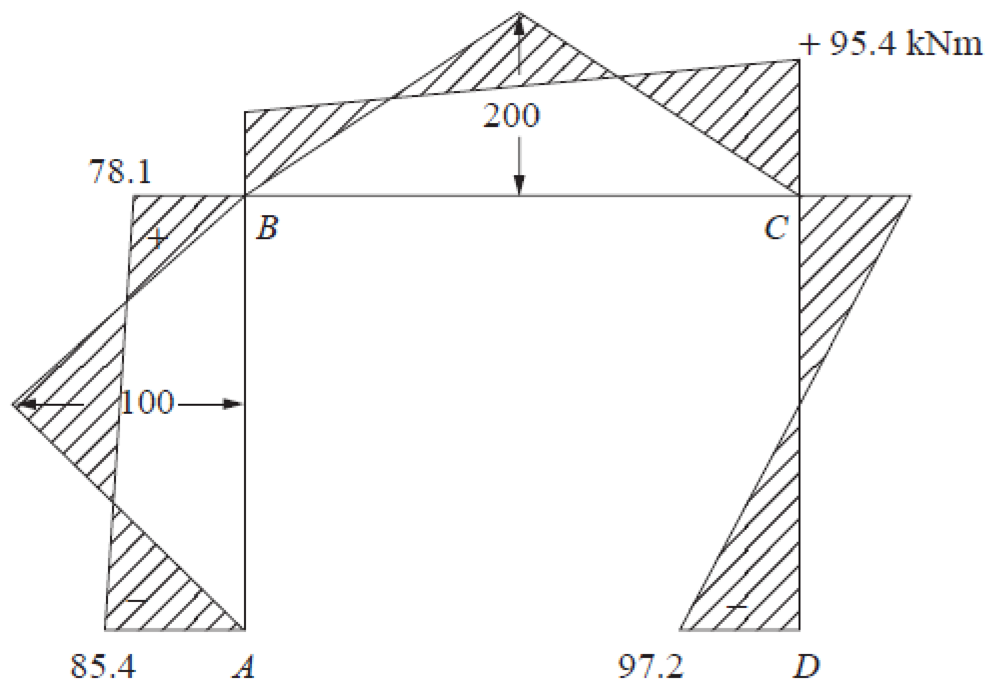
Joint	A	B		C		D
Members	AB ←	BA	BC ↔	CB	CD →	DC
DF	0	0.70	0.30	0.30	0.70	0
FEMS	+100.00	+100.00			+100.00	+100.00
Bal		−70.00	−30.00	−30.00	−70.00	
CO	−35.00		−15.00	−15.00		−35.00
Bal		+10.50	+4.50	+4.50	+10.50	
Co	+5.25		+2.25	+2.25		+5.25
Bal		−1.57	−0.68	−0.68	−1.57	
CO	−0.79		−0.34	−0.34		−0.79
Bal		+0.24	+0.10	+0.10	+0.24	
CO	+0.12		+0.05	+0.05		+0.12
Bal		−0.04	−0.01	−0.02	−0.03	
Final	69.58	39.13	−39.13	−39.13	+39.13	+69.58
	M''_{AB}	M''_{BA}	M''_{BC}	M''_{CB}	M''_{CD}	M''_{DC}

$, M_{AB} = M'_{AB} + k\, M''_{AB}.$

$MAB = -16.56 + 69.58\,k$
$MBA = +116.84 + 39.13\,k$
$MBC = -116.84 - 39.13\,k$
$MCB = +56.70 - 39.13\,k$
$MCD = -56.70 + 39.13\,k$
$MDC = -28.35 + 69.58\,k$

$23.096 - 27.18\,k = 50$
$k = -0.99$

$MAB = -16.56 + 69.58(-0.99) = -85.4$ kNm
$MBA = +116.84 + 39.13(-0.99) = +78.1$ kNm
$MBC = -116.84 - 39.13(-0.99) = -78.1$ kNm
$MCB = +56.70 - 39.13(-0.99) = +95.4$ kNm
$MCD = -56.70 + 39.13(-0.99) = -95.4$ kNm
$MDC = -28.35 + 69.58(-0.99) = -97.2$ kNm



5.5 Further Reading

1. www.civil.iitb.ac.in › ~minamdar › Lectures › Moment-Distribution

2 .https://www.mathalino.com › reviewer › moment-distribution-method

3.www.engr.mun.ca › ~swamidas › ENGI6705-ClassNotesHandout7