
MODULE 5

Power System Stability

Course Objectives:

1. To discuss the dynamics of synchronous machine and derive the power angle equation for a synchronous machine
2. Discuss stability and types of stability for a power system and the equal area criterion for the evaluation of stability of a simple system.

5.1 INTRODUCTION

The stability of an interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. Conversely, instability means a condition denoting loss of synchronism or falling out of step. Stability considerations have been recognized as an essential part of power system planning for a long time. With interconnected systems continually growing in size and extending over vast geographical regions, it is becoming increasingly more difficult to maintain synchronism between various parts of a power system

The dynamics of a power system are characterised by its basic features given below:

1. Synchronous tie exhibits the typical behaviour that as power transfer is gradually increased a maximum limit is reached beyond which the system cannot stay in synchronism, i.e., it falls out of step.
2. The system is basically a spring-inertia oscillatory system with inertia on the mechanical side and spring action provided by the synchronous tie wherein power transfer is proportional to $\sin \delta$ or δ (for small δ ; δ being the relative internal angle of machines).
3. Because of power transfer being proportional to $\sin \delta$, the equation determining system dynamics is nonlinear for disturbances causing large variations in angle δ . Stability phenomenon peculiar to non-linear systems as distinguished from linear systems is therefore exhibited by power systems (stable up to a certain magnitude of disturbance and unstable for larger disturbances). Accordingly power system stability problems are classified into three basic types—steady state, dynamic and transient.

The study of steady state stability is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually. Dynamic instability is more probable than steady state instability. Small disturbances are continually occurring in a power system (variations in loadings, changes in turbine speeds, etc.) which are small enough not to cause the system to lose synchronism but do excite the system into the state of natural oscillations. The system is said to be dynamically stable if the oscillations do not acquire more than certain amplitude and die out quickly (i.e., the system is well-damped). In a dynamically unstable system, the oscillation amplitude is large and these persist for a long time (i.e., the system is underdamped). This kind of instability behaviour constitutes a serious threat to system security and creates very difficult operating conditions. Dynamic stability can be significantly improved through the use of power system stabilizers. Dynamic system study has to be carried out for 5–10 s and sometimes up to 30 s. Computer simulation is the only effective means of studying dynamic stability problems. The same simulation programmes are, of course, applicable to transient stability studies.

Following a sudden disturbance on a power system rotor speeds, rotor angular differences and power transfer undergo fast changes whose magnitudes are dependent upon the severity of disturbance. For a large disturbance, changes in angular differences may be so large as to cause the machines to fall out of step. This type of instability is known as transient instability and is a fast phenomenon usually occurring within 1 s for a generator close to the cause of disturbance. There is a large range of disturbances which may occur on a power system, but a fault on a heavily loaded line which requires opening the line to clear the fault is usually of greatest concern. The tripping of a loaded generator or the abrupt dropping of a large load may also cause instability.

The effect of short circuits (faults), the most severe type of disturbance to which a power system is subjected, must be determined in nearly all stability studies. During a fault, electrical power from nearby generators is reduced drastically, while power from remote generators is scarcely affected. In some cases, the system may be stable even with a sustained fault, whereas other systems will be stable only if the fault is cleared with sufficient rapidity. Whether the system is stable on occurrence of a fault depends not only on the system itself, but also on the type of fault, location of fault, rapidity of clearing and method of clearing, i.e., whether cleared by the sequential opening of two or more breakers or by simultaneous opening and whether or not the faulted line is reclosed. The transient stability limit is almost always lower than the steady state limit, but unlike the latter, it may exhibit different values depending on the nature, location and magnitude of disturbance. Modern power systems have many interconnected generating stations, each with several generators and many loads. The machines located at any one point in a system normally act in unison. It is, therefore, common practice in stability studies to consider all the machines at one point as one large machine. Also machines which are not separated by lines of high reactance are lumped together and considered as one equivalent machine.

Thus a multimachine system can often be reduced to an equivalent few machine system. If synchronism is lost, the machines of each group stay together although they go out of step with other groups. Qualitative behaviour of machines in an actual system is usually that of a two machine system. Because of its simplicity, the two machine system is extremely useful in describing the general concepts of power system stability and the influence of various factors on stability. It will be seen in this chapter that a two machine system can be regarded as a single machine system connected to infinite system. Stability study of a multimachine system must necessarily be carried out on a digital computer.

5.2 DYNAMICS OF A SYNCHRONOUS MACHINE

The kinetic energy of the rotor at synchronous machine is

$$KE = \frac{1}{2} J \omega_{sm}^2 \times 10^{-6} \text{ MJ}$$

where J = rotor moment of inertia in kg-m²
 ω_{sm} = synchronous speed in rad (mech)/s

But $\omega_s = \left(\frac{P}{2}\right) \omega_{sm}$ = rotor speed in rad (elect)/s

where P = number of machine poles

$$\therefore KE = \frac{1}{2} \left(J \left(\frac{2}{P} \right)^2 \omega_s \times 10^{-6} \right) \omega_s$$

$$= \frac{1}{2} M \omega_s$$

where $M = J \left(\frac{2}{P} \right)^2 \omega_s \times 10^{-6}$

= moment of inertia in MJ-s/elect rad

We shall define the inertia constant H such that

$$GH = KE = \frac{1}{2} M \omega_s \text{ MJ}$$

where G = machine rating (base) in MVA (3-phase)

H = *inertia constant* in MJ/MVA or MW-s/MVA

$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{ MJ-s/elect rad}$$

$$= \frac{GH}{180f} \text{ MJ-s/elect degree}$$

M is also called the inertia constant.

Taking G as base, the inertia constant in pu is

$$M(\text{pu}) = \frac{H}{\pi f} \text{ s}^2/\text{elect rad}$$

$$= \frac{H}{180f} \text{ s}^2/\text{elect degree}$$

The inertia constant H has a characteristic value or a range of values for each class of machines. Table 7.1 lists some typical inertia constants.

Table 5.1 Typical inertia constants of synchronous machines*

<i>Type of Machine</i>		<i>Inertia Constant H</i>
		<i>Stored Energy in MW Sec per MVA**</i>
Turbine Generator		
Condensing	1,800 rpm	9-6
	3,000 rpm	7-4
Non-Condensing	3,000 rpm	4-3
Water wheel Generator		
Slow-speed (< 200 rpm)		2-3
High-speed (> 200 rpm)		2-4
Synchronous Condenser***		
Large		1.25
Small		1.00
Synchronous Motor with load varying from		
1.0 to 5.0 and higher for heavy flywheels		2.00

It is observed from Table 5.1 that the value of H is considerably higher for steam turbo-generator than for water wheel generator. Thirty to sixty per cent of the total inertia of a steam turbo-generator unit is that of the prime mover, whereas only 4 to 15% of the inertia of a hydroelectric generating unit is that of the waterwheel, including water.

5.3 Power Angle Equation Salient and Non – Salient pole Synchronous Machines

5.3.1 The Swing Equation

Figure 7.1 shows the torque, speed and flow of mechanical and electrical powers in a synchronous machine. It is assumed that the windage, friction and iron-loss torque is negligible. The differential equation governing the rotor dynamics can then be written as

$$J \frac{d^2\theta_m}{dt^2} = T_m - T_e \text{ Nm}$$

where θ_m = angle in rad (mech)

T_m = turbine torque in Nm; it acquires a negative value for a motoring machine

T_e = electromagnetic torque developed in Nm; it acquires negative value for a motoring machine

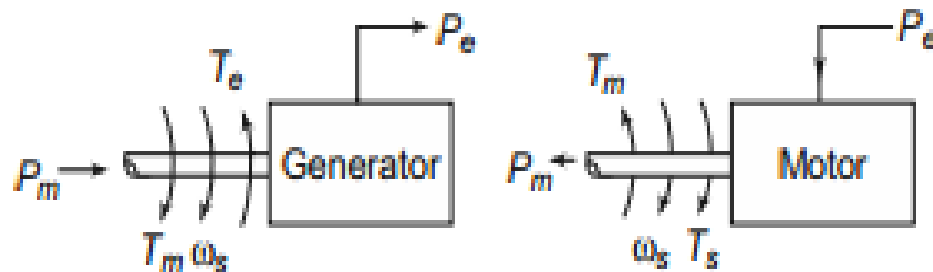


Fig 7.1: Flow of mechanical and electrical powers in a synchronous machine

$$J\omega_{sm} \frac{d^2\theta_m}{dt^2} \times 10^{-6} = P_m - P_e \text{ MW}$$

where P_m = mechanical power input in MW

P_e = electrical power output in MW; stator copper loss is assumed negligible.

$$\left(J\left(\frac{2}{p}\right)^2 \omega_s \times 10^{-6}\right) \frac{d^2\theta_e}{dt^2} = P_m - P_e \text{ MW}$$

where θ_e = angle in rad (elect)

$$\text{or} \quad M \frac{d^2\theta_e}{dt^2} = P_m - P_e$$

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let $\delta = \theta_e - \omega_s t$, rotor angular displacement from synchronously rotating reference frame (called torque angle/power angle)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2}$$

Hence Eq. (12.5) can be written in terms of δ as

$$M \frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW}$$

With M as defined in Eq. (12.1), we can write

$$\frac{GH}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW}$$

Dividing throughout by G , the MVA rating of the machine,

$$M(\text{pu}) \frac{d^2\delta}{dt^2} = P_m - P_e;$$

in pu of machine rating as base

where

$$M(\text{pu}) = \frac{H}{\pi f}$$

$$\text{or} \quad \frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu}$$

The above equation is called the swing equation and it describes the rotor dynamics for a synchronous machine (generating/motoring). It is a second-order differential equation where the damping term (proportional to $d\delta/dt$) is absent because of the assumption of a lossless machine and the fact that the torque of damper winding has been ignored. This assumption leads to pessimistic results in transient stability analysis damping helps to stabilize the system. Damping must of course be considered in a dynamic stability study. Since the electrical power P_e depends upon the sine of angle δ the swing equation is a non-linear second-order differential equation.

5.3.2 Multi machine System:

In a multi machine system a common system base must be chosen.

G_{mach} = machine rating (base)

G_{system} = system base

Equation (12.11) can then be written as

$$\frac{G_{\text{mach}}}{G_{\text{system}}} \left(\frac{H_{\text{mach}}}{f} \frac{d^2 \delta}{dt^2} \right) = (P_m - P_e) \frac{G_{\text{mach}}}{G_{\text{system}}}$$

or
$$\frac{H_{\text{system}}}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e \text{ pu in system base}$$

where
$$H_{\text{system}} = H_{\text{mach}} \left(\frac{G_{\text{mach}}}{G_{\text{system}}} \right)$$

= machine inertia constant in system base

Machines Swinging Coherently

Consider the swing equations of two machines on a common system base.

$$\frac{H_1}{\pi f} \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1} \text{ pu}$$

$$\frac{H_2}{\pi f} \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2} \text{ pu}$$

Since the machine rotors swing together (coherently or in unison)

$$\delta_1 = \delta_2 = \delta$$

Adding Eqs (12.14) and (12.15)

$$\frac{H_{eq}}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

where

$$P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

$$H_{eq} = H_1 + H_2$$

The two machines swinging coherently are thus reduced to a single machine

The equivalent inertia can be written as

$$H_{eq} = H_1 \text{ mach } G_1 \text{ mach } / G_{\text{system}} + H_2 \text{ mach } G_2 \text{ mach } / G_{\text{system}}$$

5.3.4 PROBLEMS:

1. A 50 Hz, four pole turbo-generator rated 100 MVA, 11 kV has an inertia constant of 8.0 MJ/MVA. (a) Find the stored energy in the rotor at synchronous speed. (b) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW, find rotor acceleration, neglecting mechanical and electrical losses. (c) If the acceleration calculated in part (b) is maintained for 10 cycles, find the change in torque angle and rotor speed in revolutions per minute at the end of this period.

(a) Stored energy = $GH = 100 \times 8 = 800 \text{ MJ}$

(b) $P_a = 80 - 50 = 30 \text{ MW} = M \frac{d^2\delta}{dt^2}$

$$M = \frac{GH}{180f} = \frac{800}{180 \times 50} = \frac{4}{45} \text{ MJ-s/elect deg}$$

$$\therefore \frac{4}{45} \frac{d^2\delta}{dt^2} = 30$$

or

$$\alpha = \frac{d^2\delta}{dt^2} = 337.5 \text{ elect deg/s}^2$$

(c) 10 cycles = 0.2 s

$$\text{Change in } \delta = \frac{1}{2}(337.5) \times (0.2)^2 = 6.75 \text{ elect degrees}$$

$$= 60 \times \frac{337.5}{2 \times 360^\circ} = 28.125 \text{ rpm/s}$$

\therefore Rotor speed at the end of 10 cycles

$$= \frac{120 \times 50}{4} + 28.125 \times 0.2$$

$$= 1505.625 \text{ rpm}$$

5.3.4 POWER ANGLE EQUATION:

In solving the swing equation, certain simplifying assumptions are usually made. These are:

1. Mechanical power input to the machine (P_m) remains constant during the period of electromechanical transient of interest. In other words, it means that the effect of the turbine governing loop is ignored being much slower than the speed of the transient. This assumption leads to pessimistic result-governing loop helps to stabilize the system.
2. Rotor speed changes are insignificant these have already been ignored in formulating the swing equation.

3. Effect of voltage regulating loop during the transient is ignored, as a consequence the generated machine emf remains constant. This assumption also leads to pessimistic results—voltage regulator helps to stabilize the system.

Before the swing equation can be solved, it is necessary to determine the dependence of the electrical power output (P_e) upon the rotor angle.

5.3.4.1 SIMPLIFIED MACHINE MODEL

For a nonsalient pole machine, the per phase induced emf-terminal voltage equation under steady conditions is

$$E = V + jX_d I_d + jX_q I_q; \quad X_d > X_q$$

Where $I = I_d + I_q$ and usual symbols are used

Under transient condition

$$X_d \rightarrow X'_d < X_d$$

$X'_q = X_q$ since the main field is on the d-axis

$X'_d < X_q$; but the difference is less

$$\begin{aligned} E' &= V + jX'_d I_d + jX_q I_q \\ &= V + jX_q (I - I_d) + jX'_d I_d \\ &= (V + jX_q I) + j(X'_d - X_q) I_d \end{aligned}$$

Since under transient condition, $X'_d < X_d$ but X_q remains almost unaffected, it is fairly valid to assume that, $X'_d = X_q$

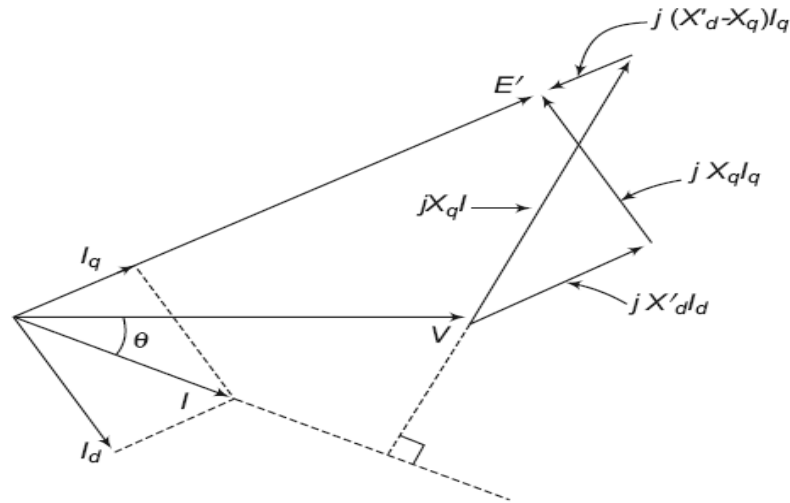


Fig 5.2: Phasor diagram – salient pole machine

$$E' = V + jX_q I$$

$$= V + jX'_d I$$

The machine model corresponding to above equation drawn in Fig. 7.3 which also applies to a cylindrical rotor machine where $X'_d = X'_q = X'_s$ (transient synchronous reactance)

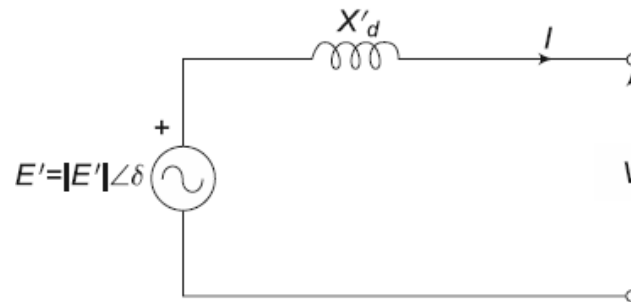


Fig. 5.3 Simplified machine model

The simplified machine of Fig.5.3 will be used in all stability studies.

5.3.5 POWER ANGLE CURVE

For the purposes of stability studies $|E'|$, transient emf of generator motor, remains constant or is the independent variable determined by the voltage regulating loop but V , the generator determined terminal voltage is a dependent variable. Therefore, the nodes (buses) of the stability study network pertain to the emf terminal in the machine model as shown in Fig. 7.4, while the machine reactance (X_d') is absorbed in the system network as different from a load flow study. Further, the loads (other than large synchronous motors) will be replaced by equivalent static admittances (connected in shunt between transmission network buses and the reference bus). This is so because load voltages vary during a stability study (in a load flow study, these remain constant within a narrow band).

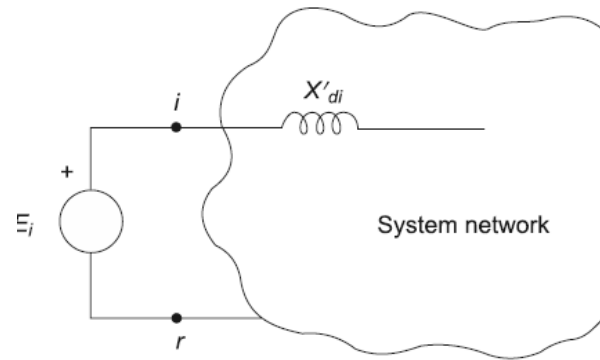


Fig 5.4: Nodes (buses) of the stability study network pertain to the emf terminal in the machine model

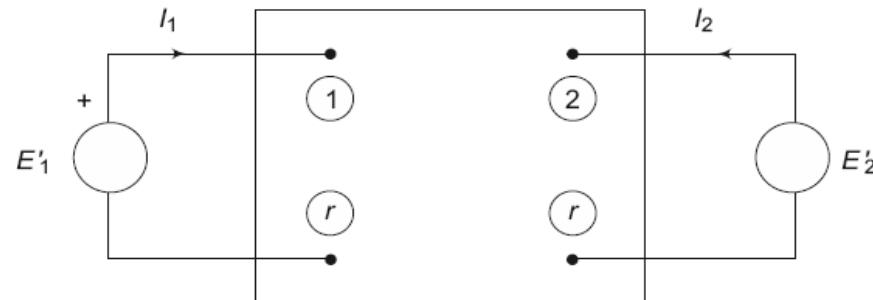


Fig. 5.5: Two-bus stability study network

For the 2-bus system

$$\mathbf{Y}_{\text{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}; Y_{12} = Y_{21}$$

Complex power into bus is given by

$$P_i + jQ_i = E_i I_i^*$$

At bus 1

$$P_1 + jQ_1 = E_1' (Y_{11} E_1')^* + E_1 (Y_{12} E_2')^*$$

$$E_1' = |E_1'| \angle \delta_1; E_2' = |E_2'| \angle \delta_2$$

$$Y_{11} = G_{11} + jB_{11}; Y_{12} = |Y_{12}| \angle \theta_{12}$$

$$P_1 = |E_1'|^2 G_{11} + |E_1'| |E_2'| |Y_{12}| \cos (\delta_1 - \delta_2 - \theta_{12})$$

At bus 2

$$|E_1'|^2 G_{11} = P_c$$

$$|E_1'| |E_2'| |Y_{12}| = P_{\text{max}}$$

$$\delta_1 - \delta_2 = \delta$$

$$\phi_{12} = \pi/2 - \gamma$$

$$P_1 = P_c + P_{\text{max}} \sin (\delta - \gamma); \text{Power Angle Equation}$$

For a purely reactive network

$G_{11} = 0$ ($\because P_c = 0$); lossless network

$$\theta_{12} = \pi/2, \therefore \gamma = 0$$

$$P_e = P_{\max} \sin \delta$$

$$P_{\max} = \frac{|E_1'| |E_2'|}{X}$$

simplified power angle equation

where X = transfer reactance between nodes

The graphical plot of power angle equation is shown below

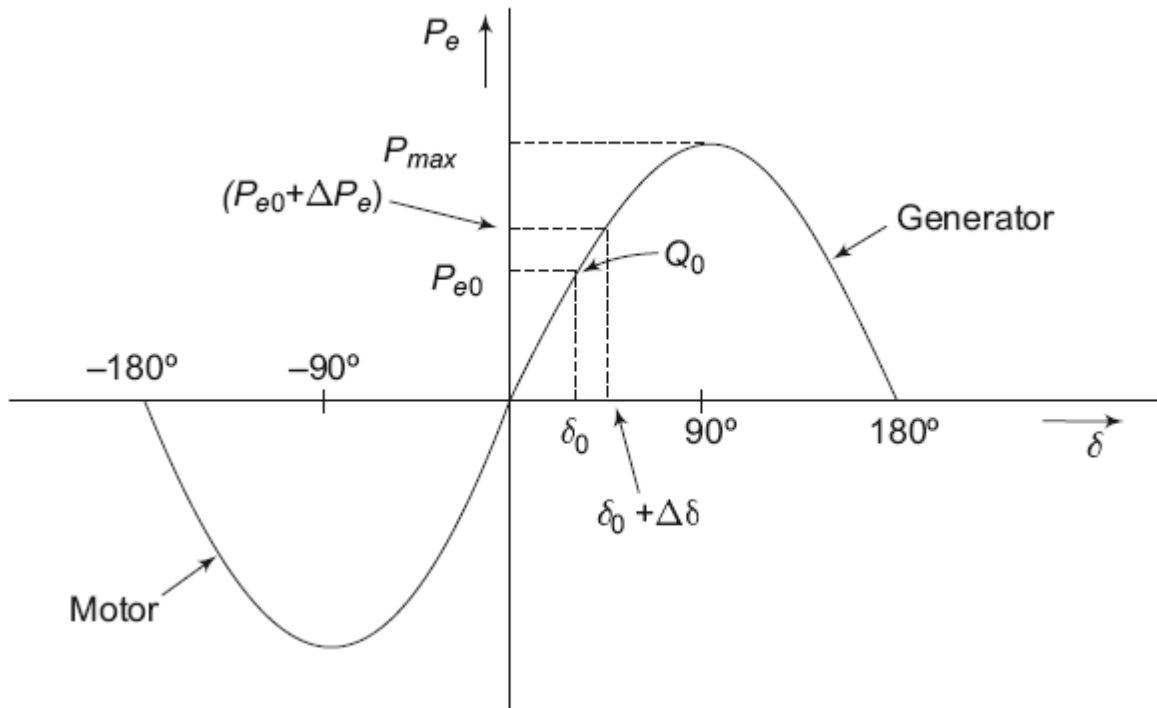


Fig. 5.6 Power angle curve

The swing equation can now be written as

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_{\max} \sin \delta \text{ pu}$$

which, as already stated, is a non-linear second-order differential equation with no damping.

7.6 Machine Connected to Infinite Bus

Figure 7.7 is the circuit model of a single machine connected to infinite bus through a line of reactance X_e . In this simple case

$$X_{\text{transfer}} = X'_d + X_e$$

$$P_e = \frac{|E'| |V|}{X_{\text{transfer}}} \sin \delta = P_{\text{max}} \sin \delta$$

The dynamics of this system are described as

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e \text{ pu}$$

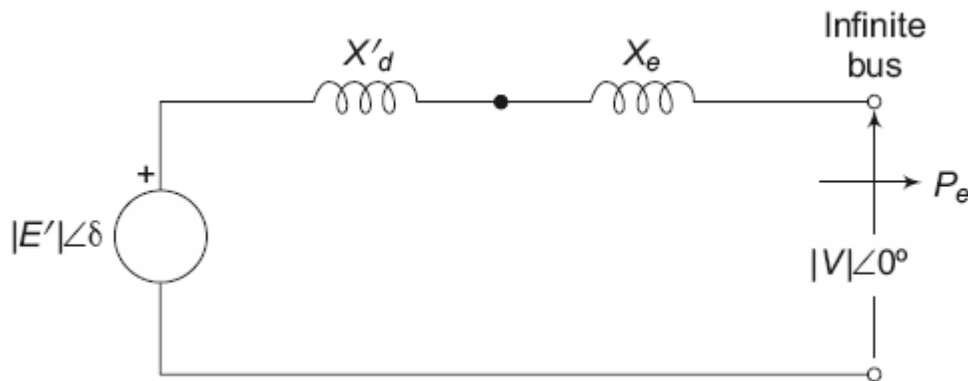


Fig. 7.7 Machine connected to infinite bus

5.4 TWO MACHINE SYSTEM

The case of two finite machines connected through a line (X_e) is illustrated in Fig. 7.8 where one of the machines must be generating and the other must be motoring. Under steady condition, before the system goes into dynamics and the mechanical

input/output of the two machines is assumed to remain constant at these values throughout the dynamics (governor action assumed slow). During steady state or in dynamic condition, the electrical power output of the generator must be absorbed by the motor (network being lossless).

$$P_{m1} = -P_{m2} = P_m$$

$$P_{e1} = -P_{e2} = P_e$$

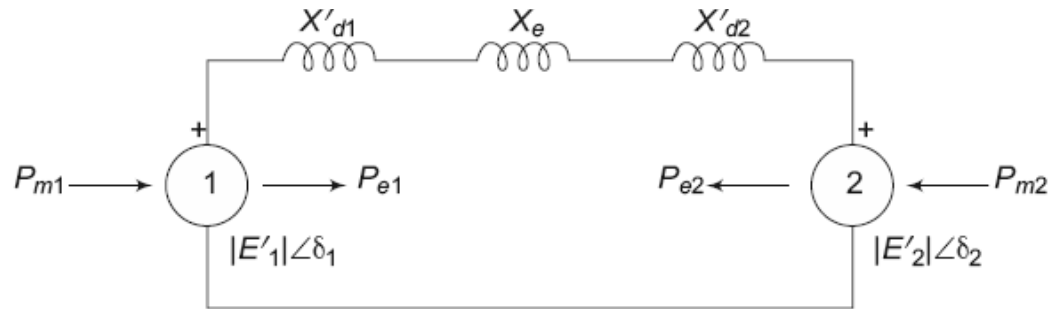


Fig 7.8: Two-machine system

The swing equations for the two machines can now be written as

$$\frac{d^2\delta_1}{dt^2} = \pi f \left(\frac{P_{m1} - P_{e1}}{H_1} \right) = \pi f \left(\frac{P_m - P_e}{H_1} \right)$$

$$\frac{d^2\delta_2}{dt^2} = \pi f \left(\frac{P_{m2} - P_{e2}}{H_2} \right) = \pi f \left(\frac{P_e - P_m}{H_2} \right)$$

Subtracting the above equations

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \pi f \left(\frac{H_1 + H_2}{H_1 H_2} \right) (P_m - P_e)$$

$$\frac{H_{eq}}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

Where

$$\delta = \delta_1 - \delta_2$$

$$H_{eq} = \frac{H_1 H_2}{H_1 + H_2}$$

The electrical power interchange is given by expression

$$P_e = \frac{|E'_1| |E'_2|}{X'_{d1} + X_e + X'_{d2}} \sin \delta$$

The swing equation and the power angle equation have the same form as for a single machine connected to infinite bus. Thus a two-machine system is equivalent to a single machine connected to infinite bus. Because of this, the single-machine (connected to infinite bus) system would be studied extensively in this chapter.

5.6 STEADY STATE STABILITY

The steady state stability limit of a particular circuit of a power system is defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Consider the simple system of Fig. 7.7 whose dynamics is described by equations

$$M \frac{d^2 \delta}{dt^2} = P_m - P_e \text{ MW}$$

$$M = \frac{H}{\pi f} \text{ in pu system}$$

$$P_e = \frac{|E||V|}{X_d} \sin \delta = P_{\max} \sin \delta$$

For determination of steady state stability, the direct axis reactance (X_d) and voltage behind X_d are used in the above equations. Let the system be operating with steady power transfer of $P_{e0} = P_m$ with torque angle δ_0 . Assume a small increment ΔP in the electric power with the input from the prime mover remaining fixed at P_m (governor response is slow compared to the speed of energy dynamics), causing the torque angle to change to $(\delta_0 + \Delta\delta)$. Linearizing about the operating point $Q_0 (P_{e0}, \delta_0)$ we can write

$$\Delta P_e = \left(\frac{\partial \mathbf{P}_e}{\partial \delta} \right)_0 \Delta \delta$$

The excursions of $\Delta\delta$ are then described by

$$M \frac{d^2 \Delta \delta}{dt^2} = P_m - (P_{e0} + \Delta P_e) = - \Delta P_e$$

$$M \frac{d^2 \Delta \delta}{dt^2} + \left[\frac{\partial \mathbf{P}_e}{\partial \delta} \right]_0 \Delta \delta = 0$$

$$\left[Mp^2 + \left(\frac{\partial P_e}{\partial \delta} \right)_0 \right] \Delta \delta = 0$$

Where

$$P = \frac{d}{dt}$$

The system stability to small changes is determined from the characteristic equation

$$Mp^2 + \left[\frac{\partial P_e}{\partial \delta} \right]_0 = 0$$

Whose two roots are

$$P = \pm \left[\frac{-(\partial P_e / \partial \delta)_0}{M} \right]^{\frac{1}{2}}$$

As long as $(\partial P_e / \partial \delta)_0$ is positive, the roots are purely imaginary and conjugate and the system behaviour is oscillatory about δ_0 . Line resistance and damper windings of machine, which have been ignored in the above modelling, cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as

$$(\partial P_e / \partial \delta)_{0>0}$$

When $(\partial P_e / \partial \delta)_0$ is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment (disturbance) and the synchronism is soon lost. The system is therefore unstable for

$$(\partial P_e / \partial \delta)_{0<0}$$

$(\partial P_e / \partial \delta)_{0<0}$ is known as synchronizing coefficient. This is also called stiffness (electrical) of synchronous machine.

Course Outcome

At the end of the module, students will be able to:

Analyze the dynamics of synchronous machine and transient stability [L4]