FLUID MECHANICS

Module - 1 Unit - 1

FLUID AND THEIR PROPERTIES.

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CONCEPT OF FLUID

Any substance which exhibits the properties of flow is called fluid.
Liquids and gases together are called fluids

LIQUIDS :-

→ Intermolecular force is comparitively less and hence they exhibit definite volume and assume the shape of the container.

→ Liquids offer little resistance against tensile force and maximum resistance against compressive force.

→ When liquids are subjected to shear force, they undergo continuous (or) prolonged angular deformation (or) shear strain. This property of liquid is called flow of fluid.

GAS :-

→ Intermolecular force is very small and therefore the molecules are free to move in any direction. Hence, gases will occupy (or) assume the shape and volume of the container.

→ Gases offer little resistance against compressive force and therefore they are called 'compressible fluids'.

→ When subjected to shear force, gases undergo continuous (or) prolonged angular deformation (or) shear strain. This property of gas is called flow of gas.

SYSTEM OF UNITS

→ The international system of units is abbreviated as SI units.

→ Length, Mass, Time are the three fundamental quantities from which others can be derived. The units of these fundamental quantities are known as primary units, and others are called derived units.

→ The units of fundamental quantities are meter (m), kilogram (kg) and seconds (s) respectively.

SI PREFIXES

10^{-12}	pico (p)	10^{12}	tera (T)
10^{-9}	nano (n)	10^{9}	giga (G)
10^{-6}	micro (μ)	10^{6}	mega (M)
10^{-3}	milli (m)	10^{3}	kilo (k)
10^{-2}	centi (c)	10^{2}	hecto (h)

Imp PROPERTIES OF FLUID

1. DENSITY (OR) MASS DENSITY

→ Density or mass density of a fluid is defined as the ratio of the mass of fluid to its volume. Thus, mass per unit volume of a fluid is called density.

→ The unit of mass density in SI unit is 'kg/m³'. It is denoted by the symbol 'ρ' (rho).

→ Mass density of a fluid decreases with increasing temperature and it increases with increasing pressure.

Mathematically,

$$\text{Mass density, } \rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

The value of density of water is 1000 kg/m³.

2. SPECIFIC WEIGHT (OR) WEIGHT DENSITY

→ Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus, weight per unit volume of a fluid is called weight density.

→ It is denoted by the symbol γ and SI unit of weight density is 'N/m³'.

Mathematicaly,

$$\text{Specific weight, } \gamma = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$= \rho \times g$ $\left[\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right]$

Specific weight of water is,

$$\gamma_{WATER} = \rho_w \times g = 1000 \times 9.81 = 9810\ N/m^3$$

→ Since specific weight depends on acceleration due to gravity and mass density, it also depends on temperature and pressure. Hence weight density of a fluid decreases with increasing temperature and increases with increasing pressure.

3. SPECIFIC VOLUME

→ Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass (or) volume per unit mass of a fluid.

→ Specific volume is the reciprocal of mass density. It is expressed as m^3/kg

Mathematically,

$$\text{Specific volume, } V = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\rho}$$

→ For liquids, all the above properties vary only slightly with the variation in temperature and pressure.

For gases, all the above properties vary greatly with the variation in temperature and pressure.

4. SPECIFIC GRAVITY (OR) RELATIVE DENSITY

→ Specific gravity is defined as the ratio of the weight density of a fluid to the weight density of a standard fluid.

→ Specific gravity is the dimensionless quantity and it is denoted by 'S'

→ For liquids, the standard fluid is taken water and for gases, the standard fluid is taken as air

Mathematically,

$$S_{liquid} = \frac{\text{density of liquid}}{\text{density of water}} \text{ (or) } \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

$$S_{liq} = \frac{\rho_L}{\rho_w} \text{ (OR) } \frac{\gamma_L}{\gamma_w} \qquad \therefore \gamma_L = S_L \times \gamma_w \text{ (OR)}$$

$$\rho_L = S_L \times \rho_w$$

$$S_{(gas)} = \frac{\text{density (weight density) of gas}}{\text{density weight density of air}} = \frac{\rho_g}{\rho_w} \text{ (OR) } \frac{\gamma_g}{\gamma_w}$$

$$\therefore \rho_g = S_g \times \rho_w \quad \text{(OR)} \quad \gamma_g = S_g \times \gamma_w$$

PROBLEMS

1) Calculate the specific weight, specific volume, specific mass of a liquid having a volume of $4 m^3$ and weighing 29.43 KN.

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Soln:- Given,

Volume $= 4 m^3$

Weight $= 29.43 \times 10^3 N$

$\gamma = ?$, $\forall = ?$, $\rho = ?$, $S = ?$

(i) Specific weight

$$\gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{29.43 \times 10^3}{4}$$

$$\boxed{\gamma = 7357.5 \ N/m^3}$$

(ii) Specific mass

we have, $\gamma = \rho \times g$

$$\therefore \rho = \frac{\gamma}{g} = \frac{7357.5}{9.81}$$

$$\boxed{\rho = 750 \ Kg/m^3}$$

(iii) Specific volume

$$\forall = \frac{1}{\rho} = \frac{1}{750}$$

$$\boxed{\forall = 0.00133 \ m^3/Kg}$$

(iv) Specific gravity

$$S_L = \frac{\rho_L}{\rho_w} = \frac{750}{1000}$$

$$\boxed{S = 0.75}$$

2) Calculate specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

Soln:- Given,

Volume = 1 lt = $\frac{1}{1000}$ m^3

Weight = 7N

(i) Specific weight,

$$\gamma = \frac{\text{weight}}{\text{Volume}} = \frac{7}{1/1000}$$

$$\boxed{\gamma = 7000 N/m^3}$$

(ii) Density

$$\gamma = \rho \times g \Rightarrow \rho = \frac{\gamma}{g} = \frac{7000}{9.81}$$

$$\boxed{\rho = 713.56 \ Kg/m^3}$$

(iii) Specific gravity

$$S = \frac{\rho_L}{\rho_w} = \frac{713.56}{1000}$$

$$\boxed{S = 0.714}$$

3) Calculate the density, specific weight and weight of one litre of petrol of specific gravity 0.7

Soln:- Given,

volume of petrol = 1 lt = 1/1000 m^3

Specific gravity, S_p = 0.7

ρ_p = ?, γ_p = ?, weight = ?

(i) Density

$$S_p = \frac{\rho_p}{\rho_w} \Rightarrow \rho_p = S_p \times \rho_w = 0.7 \times 1000$$

$$\boxed{\rho_p = 700 \ Kg/m^3}$$

(ii) Specific weight

$$\gamma_p = \rho_p \times g = 700 \times 9.81$$

$$\boxed{\gamma_p = 6867 N/m^3}$$

(iii) Weight

$$\gamma_p = \frac{\text{Weight}}{\text{Volume}} \Rightarrow \text{Weight} = \gamma_p \times \text{Volume}$$

$$= 6867 \times 1/1000$$

$$\boxed{\text{Weight} = 6.867\,N}$$

4) $10\,m^3$ of mercury weighs 136×10^4 N. Calculate its specific weight, specific volume, specific gravity and specific mass.

Soln: Given,

Volume $= 10\,m^3$

Weight $= 136 \times 10^4\,N$

$\gamma_m = ?$, $\rho_m = ?$, $V_m = ?$, $S_m = ?$

(i) Specific weight

$$\gamma_m = \frac{\text{weight}}{\text{volume}} = \frac{136 \times 10^4}{10}$$

$$\boxed{\gamma_m = 136000\,N/m^3}$$

(ii) Specific mass

$$\gamma_m = \rho_m g \Rightarrow \rho_m = \frac{\gamma_m}{g} = \frac{136000}{9.81}$$

$$\boxed{\rho_m = 13863.4\,Kg/m^3}$$

(iii) Specific volume

$$V_m = \frac{1}{\rho_m} = \frac{1}{13863.4}$$

$$\boxed{V_m = 7.213 \times 10^5\,m^3/Kg}$$

(iv) Specific gravity

$$S_m = \frac{\rho_m}{\rho_w} = \frac{13863.4}{1000}$$

$$\boxed{S_m = 13.86}$$

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5) Calculate the specific weight, density, specific volume and specific gravity of two litres of a liquid which weighs 15N. (June/July 2018)

Soln: Given,

Volume = $2/1000 \ m^3$

Weight = 15N

$\gamma = ?$, $\forall = ?$, $\rho = ?$, $S = ?$

(i) Specific weight

$$\gamma = \frac{\text{weight}}{\text{Volume}} = \frac{15}{2/1000}$$

$$\boxed{\gamma = 7500 \ N/m^3}$$

(ii) Specific mass

$$\gamma = \rho g \Rightarrow \rho = \frac{\gamma}{g} = \frac{7500}{9.81}$$

$$\boxed{\rho = 764.53 \ Kg/m^3}$$

(iii) Specific volume

$$\forall = \frac{1}{\rho} = \frac{1}{764.53}$$

$$\boxed{\forall = 0.00131 \ m^3/Kg}$$

(iv) Specific gravity

$$S = \frac{\rho}{\rho_w} = \frac{764.53}{1000}$$

$$\boxed{S = 0.764}$$

6.) One litre of crude oil weighs 9.6N. Calculate its specific weight, density and specific gravity. (July/Aug. 2021)

Soln: Given,

Volume = 1 lt = $1/1000 \ m^3$

Weight = 9.6N

$\gamma = ?$, $\rho = ?$, $S = ?$

(i) Specific weight

$$\gamma = \frac{\text{weight}}{\text{Volume}} = \frac{9.6}{1/1000}$$

$$\boxed{\gamma = 9600 \ N/m^3}$$

(ii) Density

$\gamma = \rho \times g \Rightarrow \rho = \frac{\gamma}{g} = \frac{9600}{9.81}$

$\boxed{\rho = 978.59 \ Kg/m^3}$

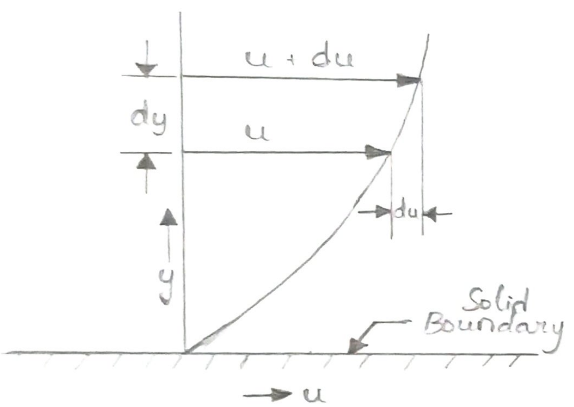
(iii) Specific gravity

$S = \frac{\rho}{\rho_w} = \frac{978.59}{1000}$

$\boxed{S = 0.979}$

VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.



Velocity Variation

→ When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say 'u' and 'u+du' as shown in figure.

→ The viscosity together with relative velocity causes a shear stress acting between the fluid layers.

→ The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to 'y'. It is denoted by 'τ' (tau)

Mathematically, $\tau \propto \frac{du}{dy}$ (OR) $\tau = \mu \frac{du}{dy}$ —(1)

where μ - constant of proportionality and know as the co-efficient of dynamic viscosity (or) viscosity.

$\frac{du}{dy}$ - rate of shear strain (OR) rate of shear deformation (OR) velocity gradient.

∴ From equation ①,

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

Thus, viscosity is also defined as the shear stress required to produce unit rate of shear strain.

UNITS OF VISCOSITY

The units of viscosity is obtained by putting the dimensions of the quantities in the equation,

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{\text{Shear stress}}{\left(\frac{\text{Change of velocity}}{\text{Change of distance}}\right)}$$

$$= \frac{\text{Force/Area}}{\left(\frac{\text{length}}{\text{time}} \times \frac{1}{\text{length}}\right)} = \frac{\text{Force}/(\text{length})^2}{1/\text{time}}$$

$$= \frac{\text{Force} \times \text{time}}{(\text{length})^2}$$

∴ → SI unit of viscosity is '$N\text{-}s/m^2$'

→ Unit of viscosity in MKS is '$Kgf \cdot sec/m^2$'

→ Unit of viscosity in CGS is 'Poise'

$$\text{Poise} = \frac{\text{dyne-sec}}{cm^2}, \quad 1\,\text{poise} = \frac{1}{10}\,N \cdot s/m^2$$

The viscosity of water at 20°C is 1/100 poise (OR) 1 centipoise

KINEMATIC VISCOSITY

It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the symbol 'ν' (nu).

Mathematically, $\nu = \frac{\text{viscosity}}{\text{density}} = \frac{\mu}{\rho}$

→ In MKS and SI, unit of kinematic viscosity is 'm^2/s'.

→ In CGS units it is 'cm^2/s' also known as stoke

$$1\ \text{stoke} = 10^{-4}\ m^2/s$$

$$\text{Centistoke} = 1/100\ \text{Stoke}.$$

NEWTON'S LAW OF VISCOSITY

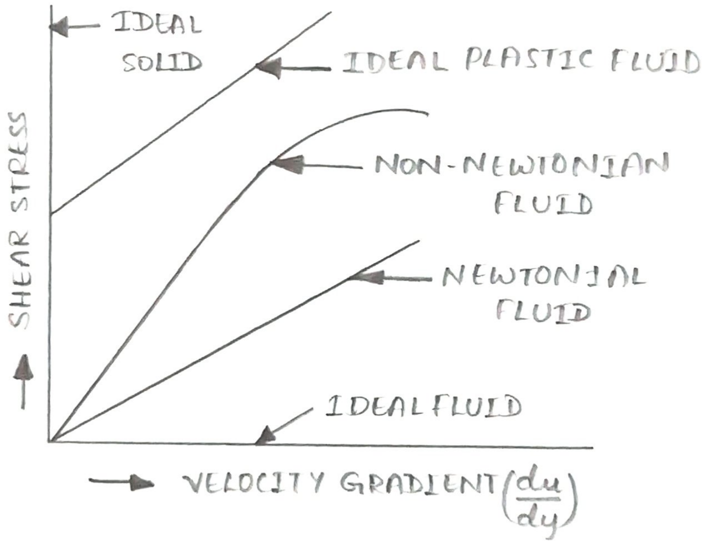
It states that "the shear stress 'τ' on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity".

Mathematically,
$$\tau = \mu \frac{du}{dy}$$

TYPES OF FLUIDS

The fluids may be classified into five types :

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non-Newtonian fluid
5. Ideal plastic fluid



1. IDEAL FLUID :

A fluid, which is incompoussible and is having no viscosity, is known as ideal fluid.

Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. REAL FLUID :

A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. NEWTONIAN FLUID :

A real fluid, in which the shear stress is directly proportional to the rate of shear strain is known as a Newtonian fluid. Ex : Kerosene, water, air.

4. NON-NEWTONIAN FLUID:

A real fluid, in which the shear stress is not proportional to the rate of shear strain, known as a non-newtonian fluid. Ex: Blood, mud flows, solutions etc

5. IDEAL PLASTIC FLUID:

A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain, is known as ideal plastic fluid.

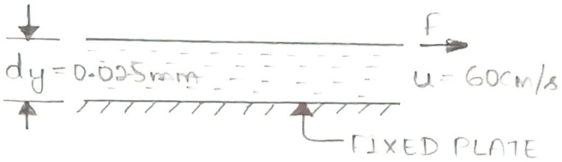
PROBLEMS

1) A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.

Soln: Given,

$dy = 0.025\,mm$

$= 0.025 \times 10^{-3}\,m$



$u = 60\,cm/s = 0.6\,m/s$

$F = 2\,N/m^2 = \tau$

we have, $\tau = \mu \frac{du}{dy}$ (OR) $2 = \mu \frac{0.6}{0.025 \times 10^{-3}}$

$\mu = \frac{2 \times 0.025 \times 10^{-3}}{0.6} = 8.333 \times 10^{-5}\,Ns/m^2$

$= 8.333 \times 10^{-5} \times 10$

$\therefore$ $\boxed{\mu = 8.333 \times 10^{-4}\ \text{poise}}$

2) A flat plate of area $1.5 \times 10^6\,mm^2$ is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.

Soln:- Given,

$A = 1.5 \times 10^6\,mm^2 = 1.5 \times 10^6 \times 10^{-6} = 1.5\,m^2$

$du = 0.4\,m/s$

$dy = 0.15\,mm = 0.15 \times 10^{-3}\,m$

$\mu = 1\,poise = 1/10\,Ns/m^2$

Force (F) = ? , Power (P) = ?

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we have,

$$\tau = \mu \frac{du}{dy} \quad \text{(OR)} \quad \frac{Force}{Area} = \mu \frac{du}{dy}$$

$$\frac{Force}{1.5} = \frac{1}{10}\left(\frac{0.4}{0.15\times10^{-3}}\right) \quad \text{(OR)} \quad Force = \frac{1}{10}\left(\frac{0.4}{0.15\times10^{-3}}\right)\times 1.5$$

$$\boxed{Force = 400N}$$

Power,

$$P = F\times du = 400\times 0.4$$

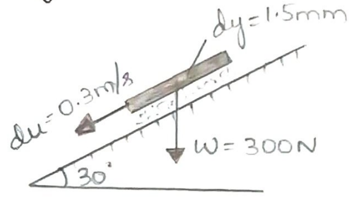
$$\boxed{P = 160 N.m/s} \quad \text{(OR)} \quad \boxed{P = 160W}$$

3) Calculate the dynamic viscosity of an oil which is used for lubricating between the square plates of sides 0.8m and an inclined plane with an angle of inclination of 30°. The weight of the square plate is 300N and it slides down with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5mm.

Soln:- Given,

Side = 0.8m

∴ Area, $A = (0.8)^2 = 0.64 m^2$

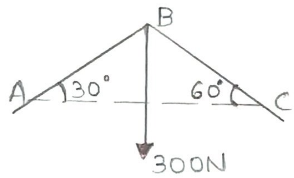


Angle of inclination, $\theta = 30°$

Weight = 300N

du = 0.3 m/s

$dy = 1.5mm = 1.5\times10^{-3} m$



From ΔABC

$F = W\cos 60° = 300 \cos 60°$

$$\boxed{F = 150N}$$

we have,

$$\tau = \mu \frac{du}{dy} \quad \text{(OR)}$$

$$\frac{Force}{Area} = \mu \frac{du}{dy} \quad \text{(OR)} \quad \frac{150}{0.64} = \mu\left(\frac{0.3}{1.5\times10^{-3}}\right)$$

$$\mu = \frac{150\times1.5\times10^{-3}}{0.64\times0.3} = 1.172 Ns/m^2 = 1.172\times10$$

$$\therefore \boxed{\mu = 11.72 \text{ Poise}}$$

4) The space between two square parallel plates is filled with a fluid each side of the plate is 60cm the thickness of the oil film is 12.5mm. The upper plate which moves with 2.5m/s requires a force of 98.1N to maintain the speed. Determine

(i) the dynamic viscosity of the oil in poise,

(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

(Dec.2019/Jan.2020)

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Soln:- Given,

side = 60cm = 0.6m

$\therefore$ Area $= (0.6)^2 = 0.36\,m^2$

$dy = 12.5\,mm = 12.5\times10^{-3}\,m$

$du = 2.5\,m/s$

Force = 98.1N

$S_o = 0.95$

$\mu = ?$ and $\nu = ?$

we have,
$$\tau = \mu\frac{du}{dy} \quad (OR) \quad \frac{Force}{Area} = \mu\frac{du}{dy}$$

$$\frac{98.1}{0.36} = \mu\frac{2.5}{12.5\times10^{-3}} \quad (OR) \quad \mu = \frac{98.1\times12.5\times10^{-3}}{2.5\times0.36}$$

$$\mu = 1.363\,Ns/m^2 = 1.363\times10$$

$$\therefore \boxed{\mu = 13.63\ Poise}$$

Kinematic viscosity
$$\nu = \frac{\mu}{\rho}$$

But,
$$S_o = \frac{\rho_o}{\rho_w} \quad (OR) \quad \rho_o = S_o\times\rho_w = 0.95\times1000$$

$$\rho_o = 950\,kg/m^3$$

$$\therefore \nu = \frac{1.363}{950} = 1.435\times10^{-3}\,m^2/s = \frac{1.435\times10^{-3}}{10^{-4}}$$

$$\therefore \boxed{\nu = 14.35\ stokes}$$

5) Find the kinematic viscosity of an oil having density of 981 kg/m³. The shear stress at a point on the oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second.

Soln: Given,

$\rho = 981 \, kg/m^3$

$\tau = 0.2452 \, N/m^2$

$\frac{du}{dy} = 0.2 \, s^{-1}$

We have, $\tau = \mu \frac{du}{dy}$ (OR) $0.2452 = \mu(0.2)$

$\mu = \frac{0.2452}{0.2} = 1.226 \, N\text{-}s/m^2$

∴ Kinematic viscosity,

$\nu = \frac{\mu}{\rho} = \frac{1.226}{981} = 1.25 \times 10^{-3} \, m^2/s$

$= 1.25 \times 10^{-3} \times 10^{-4}$

∴ $\boxed{\nu = 12.5 \text{ stokes}}$

6) Determine the viscosity of the liquid having kinematic viscosity of 6 stokes and specific gravity 1.9.

Soln: Given,

$\nu = 6 \text{ stokes} = 6 \times 10^{-4} \, m^2/s$

$S = 1.9$

$\mu = ?$

WKT, $\nu = \frac{\mu}{\rho}$ (OR) $\mu = \nu \times \rho$

But $S = \frac{\rho}{\rho_w}$ (OR) $\rho = S \times \rho_w$

$\rho = 1.9 \times 1000$

$= 1900 \, kg/m^3$

$\mu = 6 \times 10^{-4} \times 1900$

∴ $\mu = 1.14 \, N\text{-}s/m^2$

7) The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90mm. The thickness of the oil film is 1.5mm.

Soln:- Given,

$\mu = 6 \text{ poise} = 6/10 \ N\text{-}s/m^2$

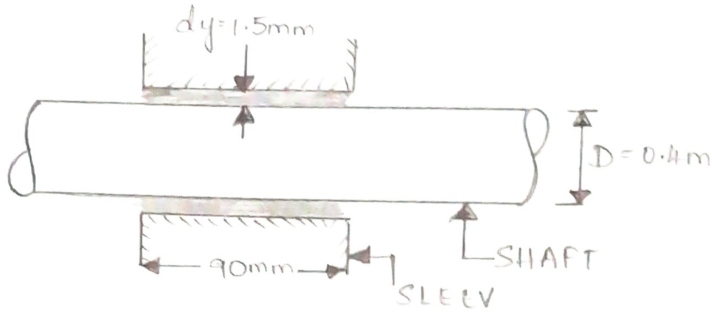
$D = 0.4m$

$N = 190 \ r.p.m$

$L = 90mm = 90 \times 10^{-3} m$

$dy = 1.5mm = 1.5 \times 10^{-3} m$

Power lost = ?



We have, Power loss, $P = \frac{2\pi NT}{60}$

But, $\tau = \mu \frac{du}{dy}$ (OR) $\frac{Force}{Area} = \mu \frac{du}{dy}$

→ A = Area of contact $= \pi DL = \pi \times 0.4 \times 90 \times 10^{-3}$

$\therefore A = 0.113 m^2$

→ $du = \frac{\pi DN}{60} = \frac{\pi \times 0.4 \times 190}{60}$

$\therefore du = 3.979 m/s$

$\therefore$ Force $= \mu \frac{du}{dy} \times Area = 0.6 \times \frac{3.979}{1.5 \times 10^{-3}} \times 0.113$

Force = 179.85 N

Also, Torque, T = Force × ⊥r distance

$= 179.85 \times 0.2$

$T = 35.97 Nm$

$\therefore$ Power lost, $P = \frac{2 \times \pi \times 190 \times 35.97}{60}$

$\therefore$ **P = 715.69 W**

8) A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10cm. Both cylinders are 25cm high. The space between the cylinders is fitted with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 r.p.m., determine the viscosity of the fluid.

Soln: Given

Inner dia. of cylinder, $D_i = 15cm = 0.15\,m$

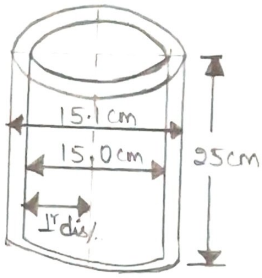
Outer dia. of cylinder, $D_o = 15.1cm = 0.151m$

length of cylinder, $L = 25\,cm = 0.25m$

Torque, $T = 12\ N\text{-}m$

Speed, $N = 100\ rpm$

Viscosity, $\mu = ?$



We have, $\tau = \mu \dfrac{du}{dy}$ (OR) $\dfrac{Force}{Area} = \mu \dfrac{du}{dy}$

But,

→ A = Area of contact $= \pi \times D_i \times L = \pi \times 0.15 \times 0.25$

$A = 0.118\,m^2$

→ $T = Force \times r$ distance (OR) $12 = Force \times \left(\dfrac{0.15}{2}\right)$

$Force = 160N$

→ $u = du = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.15 \times 100}{60}$

$du = 0.7854\ m/s$

→ $dy = \dfrac{D_o - D_i}{2} = \dfrac{0.151 - 0.15}{2} = 0.0005m$

$\therefore \dfrac{Force}{Area} = \mu\left(\dfrac{du}{dy}\right)$ (OR) $\mu = \dfrac{Force}{Area}\left(\dfrac{dy}{du}\right)$

$\mu = \dfrac{160}{0.118}\left(\dfrac{0.0005}{0.7854}\right) = 0.8632\ N\text{-}s/m^2$

$= 0.8632 \times 10$

$\therefore$ $\boxed{\mu = 8.632\ Poise}$

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9.) Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square meter between the two large plane surfaces at a speed of 0.6 m/s, if :
(i) the thin plate is in the middle of the two plane surfaces,
(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces ? Take the dynamic viscosity of glycerine as 8.10×10^{-1} Ns/m²

Soln :- given,

$H = 2.4\,cm = 0.024\,m$

$dy_1 = dy_2$

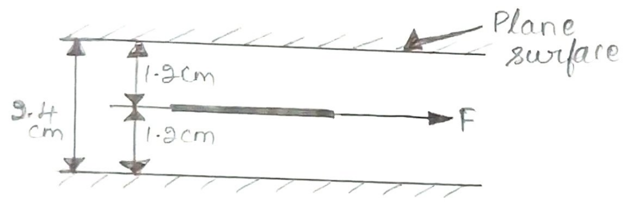
$\mu_1 = \mu_2 = 8.10 \times 10^{-1}\ Ns/m^2$

$du_1 = du_2 = 0.6\,m/s$

$A = 0.5\,m^2$

$F = ?$

Case i)



$dy_1 = 1.2\,cm$
$dy_2 = 1.2\,cm$

Let F_1 be the force required on the upper side of the plate and F_2 be the force required to drag the plate on the lower side.

Therefore, the total force required to drag the plate,

$$F = F_1 + F_2$$

Then,

$$\tau_1 = \mu\left(\frac{du}{dy}\right) \quad (OR) \quad \frac{F_1}{A} = \mu_1 \frac{du}{dy}$$

Here, $F_1 = F_2$

$$\therefore \quad F_1 = F_2 = \mu\left(\frac{du}{dy}\right)A = 8.10 \times 10^{-1}\left(\frac{0.6}{1.2 \times 10^{-2}}\right)(0.5)$$

$$F_1 = F_2 = 20.25\,N.$$

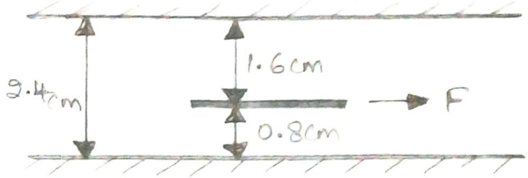
$$\therefore \quad F = F_1 + F_2 = 20.25 + 20.25$$

$$\therefore \quad \boxed{F = 40.50\,N}$$

case ii)

$$\tau_1 = \mu_1\left(\frac{du_1}{dy_1}\right) \quad \text{(OR)}$$

$$\frac{F_1}{A} = \mu_1\left(\frac{du_1}{dy_1}\right)$$



$dy_1 = 1.6\text{ cm}$
$dy_2 = 0.8\text{cm}$

$$F_1 = 8.10\times10^{-1}\left(\frac{0.6}{1.6\times10^{-2}}\right)\times 0.5$$

$$F_1 = 15.19\text{ N}$$

$|||^{ly}$ $$F_2 = \mu_2\left(\frac{du_2}{dy_2}\right)A = 8.10\times10^{-1}\left(\frac{0.6}{0.8\times10^{-2}}\right)0.5$$

$$F_2 = 30.38\text{N}$$

$$\therefore F = F_1 + F_2 = 15.19 + 30.38$$

$$\therefore \boxed{F = 45.57\text{N}}$$

10) A metal ~~pipe~~ plate 1.25m by 6mm weighing 90N is placed midway in 24mm gap between the two parallel surfaces the gap is filled with an oil of Specific gravity 0.85 and dynamic viscosity 3N.s/m². Determine the force required to lift the plate at a constant velocity of 0.15m/s.

Solⁿ:- Let F be the force required to lift the plate upwards. Let F_1 and F_2 be the forces required to lift the either side of the plate.
Let F_3 be the difference of self-weight and upward thrust.

Hence, $F = F_1 + F_2 + F_3$

$$F = F_1 + F_2 + (W - \text{upward thrust})$$

Given,

$$A = (1.25)^2 = 1.5625\text{m}^2$$

$$\text{Thickness} = 6\times10^{-3}\text{m}$$

$$\therefore \text{Volume} = \text{Area}\times\text{thickness} = 1.5625\times6\times10^{-3}$$

$$\text{Volume} = 9.375\times10^{-3}\text{m}^3$$

$S_o = 0.85$

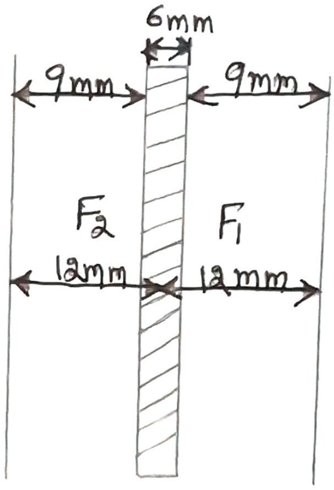
$S_o = \frac{\gamma_o}{\gamma_w}$ (OR) $\gamma_o = S_o \times \gamma_w$

$\gamma_o = 0.85 \times 9810$

$\gamma_o = 8338.5\ N/m^3$

$\mu = 3\ Ns/m^2$, $du = 0.15\ m/s$

From the figure,



$F_1 = F_2$

$\frac{F_1}{A} = \mu \frac{du}{dy}$

$F_1 = 3\left(\frac{0.15}{9\times10^{-3}}\right)(1.5625)$

$F_1 = F_2 = 78.125\ N$

$\left(\text{upward thrust}\right) = \gamma_o \times \text{Volume}$

Now,

$F_3 = W - \text{upward thrust}$

$= 90 - (8338.5 \times 9.375 \times 10^{-3})$

$F_3 = 11.827\ N$

$\therefore\ F = 78.125 + 78.125 + 11.827$

$\boxed{F = 168.077\ N}$

11.) A cube of 0.3 m sides and weight 30N slides down an inclined plane sloped at 30° to the horizontal. The plane is covered by an oil of viscosity 2.3×10^{-3} poise as with 0.03mm thickness. Determine the velocity with which the cube slides down.

(Dec. 2016/Jan. 2017)

Sol^n:- Given,

Side = 0.3m

$\therefore\ A = (0.3)^2 = 0.09\ m^2$

Weight = 30N

$\mu = 2.3\times10^{-3}$ Poise

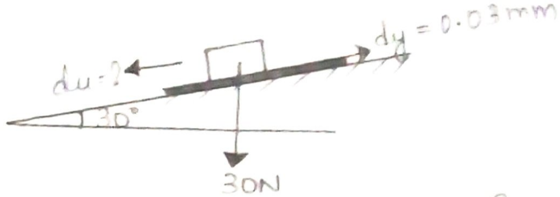
$= 2.3\times10^{-4}\ N\text{-}s/m^2$

$dy = 0.03\times10^{-3}\ m$, $du = ?$

$\theta = 30°$

we have,

$$\tau = \mu \frac{du}{dy} \quad \text{(OR)}$$

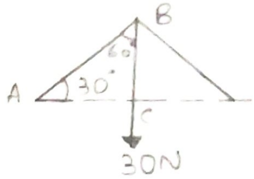


$$\frac{\text{Force}}{\text{Area}} = \mu \frac{du}{dy}$$

From ΔABC

$F = 30\cos 60°$

$\underline{F = 15N}$



$$du = \frac{\text{Force}}{\text{Area}} \times \frac{1}{\mu} \times dy$$

$$= \frac{15}{0.09} \times \frac{1}{2.3\times10^{-4}} \times 0.03\times10^{-3}$$

$$\therefore \boxed{du = 21.739\ m/s}$$

12.) A cylindrical shaft of 90mm diameter rotates about a vertical axis inside a fixed cylindrical tube of length 50cm and 95mm internal diameter. If the space between the tube and the shaft is filled by a lubricant of dynamic viscosity 8.0 Poise. Determine the power required to overcome viscous resistance, when the shaft is rotated at a speed of 240 rpm.

(June/July 2017)

Soln :- Given,

Inner dia., $D_i = 90mm = 90\times10^{-3}m$

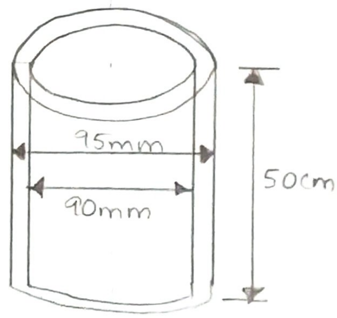
Outer dia., $D_o = 95mm = 95\times10^{-3}m$

length, $L = 50cm = 0.5m$

Speed, $N = 240\ rpm$

Viscosity, $\mu = 8\ \text{Poise} = 0.8\ Ns/m^2$

Power required, $P = ?$



WKT, $P = F.du$

But, $\frac{F}{A} = \mu \frac{du}{dy}$

→ A = Area of contact $= \pi D_i L = \pi \times 90\times10^{-3}\times0.5$

$A = 0.1414\ m^2$

→ $du = \frac{\pi D_i N}{60} = \frac{\pi\times90\times10^{-3}\times240}{60}$

$du = 1.131\ m/s$

$\rightarrow dy = \dfrac{0.095 - 0.09}{2} = 0.0025\ m$

$\therefore F = \mu\left(\dfrac{du}{dy}\right)A = 0.8\left(\dfrac{1.131}{0.0025}\right)0.1414$

$F = 51.175\ N$

Now, $P = F.du = 51.175 \times 1.131$

$\therefore$ Power required, $\boxed{P = 57.871\ W}$

13) State Newton's law of viscosity. The velocity distribution over a plate is given by $V = \frac{y}{3} - y^2$, in which 'v' is the velocity in m/sec, at a distance 'y' m above the plate. Find the shear stress at $y = 0$ and $y = 0.1m$, $\mu = 0.835\ N\text{-}s/m^2$. (June/July 2019)

Solⁿ:

Given,

$V = \frac{y}{3} - y^2 \quad \therefore \dfrac{dv}{dy} = \dfrac{1}{3} - 2y$

$\left(\dfrac{dv}{dy}\right)_{at\ y=0} = \dfrac{1}{3} - 2(0) = \dfrac{1}{3} = 0.333$

Also, $\left(\dfrac{dv}{dy}\right)_{y=0.1} = \dfrac{1}{3} - 2(0.1) = 0.133$

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Now, shear stress $\tau = \mu \dfrac{du}{dy}$

(i) Shear stress at $y = 0$

$\tau_0 = \mu\left(\dfrac{dv}{dy}\right)_{y=0} = 0.835 \times 0.333$

$\therefore \boxed{\tau_0 = 0.278\ N/m^2}$

(ii) Shear stress at $y = 0.1$

$\tau_{0.1} = \mu\left(\dfrac{dv}{dy}\right)_{y=0.1} = 0.835 \times 0.133$

$\therefore \boxed{\tau_{0.1} = 0.111\ N/m^2}$

COHESION

Cohesion is the attraction between the molecules of same fluid. The force of attraction is called cohesion.

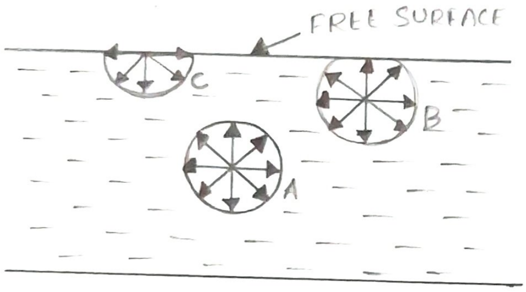
ADHESION

Adhesion is the attraction between the molecules of a fluid and the molecules of other substance.

SURFACE TENSION

→ Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas (or) on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

→ Surface tension is denoted by 'σ' (Sigma) and SI unit is 'N/m' while in MKS units, it is 'kgf/m'.



Consider three molecules A, B, C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force on A is zero.

→ But molecule B, which is situated near the free surface is acted upon by upward and downward forces which are unbalanced. Thus the net resultant force on molecule B is acting in downward direction.

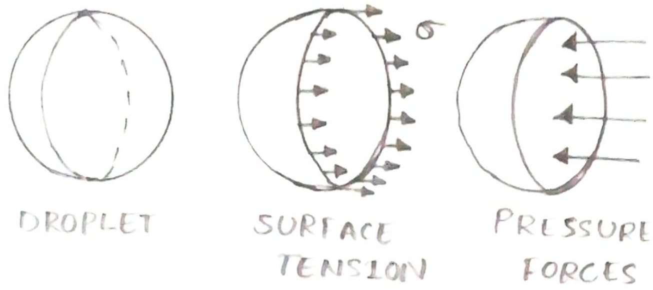
→ The molecule C, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force.

→ The property of liquid surface to offer resistance against tension is called surface tension.

CONSEQUENCES OF SURFACE TENSION

i) Liquid surface supports small loads.
ii) Formation of spherical droplets (or) bubbles of liquids.
iii) Rain drop falling over a leaf.
iv) Mercury spilling over the floor.

SURFACE TENSION ON LIQUID DROPLET



Consider a small spherical droplet of a liquid of radius 'r', On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension

P = pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = diameter of droplet.

Let the droplet is cut into two halves. The forces acting on one half will be

(i) tensile force due to surface tension acting around the circumference of the cut portion as shown in fig. (B) and this is equal to

$$= \sigma \times \text{circumference}$$
$$= \sigma \times \pi d$$

(ii) pressure force on the area $\frac{\pi}{4} d^2 = p \times \frac{\pi}{4} d^2$

These two forces will be equal and opposite under equilibrium conditions

i.e., $$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$p = \frac{4\sigma}{d}$$

SURFACE TENSION ON A HOLLOW BUBBLE

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

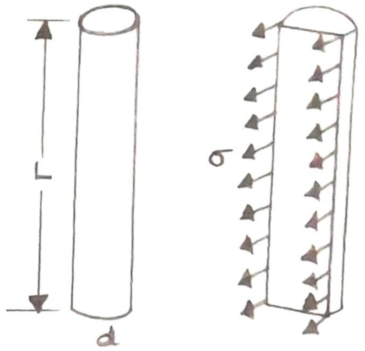
$$p = \frac{2\sigma \pi d}{\left(\frac{\pi}{4} \times d^2\right)} = \frac{8\sigma}{d}$$

SURFACE TENSION ON A LIQUID JET

Consider a liquid jet of diameter 'd' and length 'L' as shown in figure.

Let p = pressure intensity inside the liquid jet above the outside pressure.

σ = Surface tension of the liquid.



FORCES ON LIQUID JET

Consider the equilibrium of the semi jet, we have

Force due to pressure = p × Area of semi jet

$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L$

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$p = \frac{\sigma \times 2L}{L \times d} = \frac{2\sigma}{d}$$

PROBLEMS

1) The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure. Calculate the diameter of the droplet of water.

Solⁿ:- Given,

$\sigma = 0.0725 \, N/m$

$p = 0.02 \, N/cm^2 = 0.02 \times 10^4 \, N/m^2$

$d = ?$

We have, $p = \frac{4\sigma}{d}$ (OR) $0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$

$$d = \frac{4 \times 0.0725}{0.02 \times 10^4} = 0.00145 \, m$$

$\therefore$ $d = 1.45 \, mm$

2) Find the surface tension in a soap bubble of 40mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.

Soln :- Given,

$$d = 40mm = 40\times10^{-3} m$$

$$P = 2.5 N/m^2, \ \sigma = ?$$

we have, $p = \frac{8\sigma}{d}$ (OR) $2.5 = \frac{8\times\sigma}{40\times10^{-3}}$

$$\sigma = \frac{2.5\times40\times10^{-3}}{8}$$

$$\therefore \boxed{\sigma = 0.0125 N/m}$$

3) The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Soln: Given,

$$d = 0.04 mm = 0.04\times10^{-3} m$$

$$\sigma = 0.0725 N/m$$

$$p = ?$$

we have, $p = \frac{4\sigma}{d} = \frac{4\times0.0725}{0.04\times10^{-3}} = 7250 N/m^2$

$$p = 0.7250 N/cm^2$$

pressure outside the droplet $= 10.32 N/cm^2$ (given)

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$$= 0.7250 + 10.32$$

$$= 11.045 N/cm^2$$

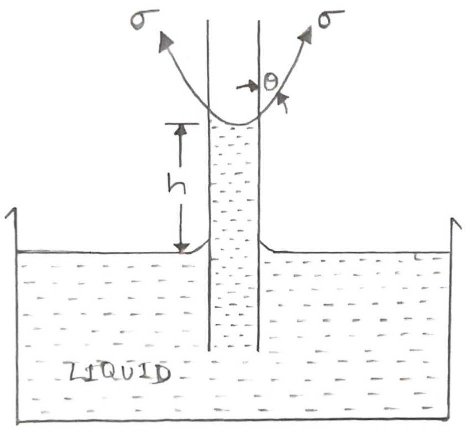
CAPILLARITY

→ Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

→ The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary fall (or) capillary depression.

→ It is expressed in terms of cm (or) mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

EXPRESSION FOR CAPILLARY RISE



Consider a glass tube of small diameter 'd' open at both ends and inserted in a container of liquid, (Say water).

The liquid will rise in the tube above the level of liquid.

Let h = height of liquid in the tube.

θ = Angle of contact between liquid and glass tube.

σ = Surface tension of liquid.

The weight of liquid of height 'h' in the tube

$$= (\text{Area of tube} \times h) \times \rho \times g$$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

Verticle component of the surface tensile force

$$= (\sigma \times \text{circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium, $\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi \times d \times \cos\theta$

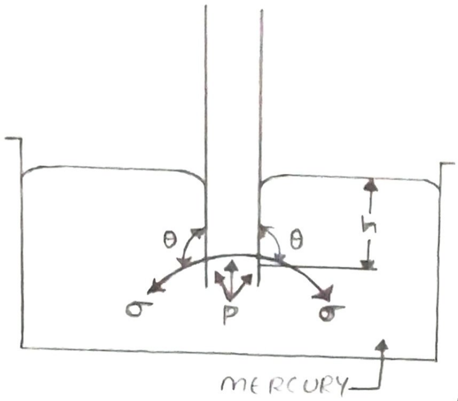
$$\therefore \left\langle h = \frac{4\sigma\cos\theta}{\rho g d} \right\rangle$$

The value of 'θ' between water and clean glass tube is approximately equal to zero. i.e., $\cos\theta = \cos 0 = 1$

Then rise of water is given by

$$h = \frac{4\sigma}{\rho g d}$$

EXPRESSION FOR CAPILLARY FALL



If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid.

Let h = Height of depression in the tube.

Then in equilibrium, two forces are acting on the mercury inside the tube.

(i) Due to surface tension acting in the downward direction & equal to, $\sigma \times \pi d \times \cos\theta$

(ii) Due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \qquad (\because p = \rho g h)$$

Equating (i) & (ii), we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \quad h = \frac{4\sigma\cos\theta}{\rho g d}$$

Value of θ for mercury and glass tube is 128°.

PROBLEMS

1.) Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in i) water and ii) mercury. Take surface tension $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.

Sol^n:- Given,

$d = 2.5\,mm = 2.5\times10^{-3}\,m$

$\sigma_w = 0.0725\,N/m$

$\sigma_m = 0.52\,N/m$

$S_m = 13.6$

$\therefore \rho_m = 13.6\times1000 = 13600\,Kg/m^3$

(i) capillary rise for water $(\theta = 0^\circ)$

$$h = \frac{4\sigma}{\rho g d} = \frac{4\times0.0725}{1000\times9.81\times2.5\times10^{-3}} = 0.0118\,m$$

$\therefore$ **h = 1.18 cm**

(ii) for mercury

$\theta = 130^\circ$

$$h = \frac{4\sigma\cos\theta}{\rho g d} = \frac{4\times0.52\times\cos130^\circ}{13600\times9.81\times2.5\times10^{-3}} = -0.004\,m$$

h = -0.4 cm

-ve sign indicates the capillary fall.

2) Calculate the capillary effect in mm in a glass tube of 4mm diameter, when immersed in i) water and ii) mercury. The temperature of the liquid is 20°C and the value of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130°. Take density of water at 20°C is equal to 998 Kg/m³.

Sol^n:- Given,

$d = 4\,mm = 4\times10^{-3}\,m$

$\sigma_w = 0.073575\,N/m$, $\sigma_m = 0.51\,N/m$

$\rho_w = 998\,Kg/m^3$, $\rho_m = 13.6\times1000 = 13600\,Kg/m^3$

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

i) Cappillary effect for water

$\theta = 0^\circ$

$$h = \frac{4\times0.073575\times1}{998\times9.81\times4\times10^{-3}} = 0.00751\,m$$

$\therefore$ **h = 7.51 mm**

ii) Capillary effect for mercury.

$$\theta = 130^\circ$$

$$h = \frac{4 \times 0.51 \times \cos 130^\circ}{13600 \times 9.81 \times 4 \times 10^{-3}} = -0.00246\,m$$

$$\therefore \boxed{h = -2.46\,mm}$$

-ve sign indicates the capillary depression.

3.) The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.

Soln: Given,

$$h = 0.2\,mm = 0.2 \times 10^{-3}\,m.$$

$$\sigma = 0.0725\,N/m$$

$$\rho_w = 1000\,kg/m^3$$

$$\theta = 0^\circ$$

$$d = ?$$

$$h = \frac{4\sigma \cos\theta}{\rho g d} \quad \text{(OR)} \quad d = \frac{4\sigma\cos\theta}{h\rho g}$$

$$d = \frac{4 \times 0.0725 \times 1}{0.2 \times 10^{-3} \times 1000 \times 9.81} = 0.148\,m$$

$$\therefore \boxed{d = 14.8\,cm}$$

4) If two tubes are dipped in water, one of 20mm diameter and other of 2mm diameter, what is the difference of level of liquids in two tubes. Take the surface tension of water as 0.073 N/m.

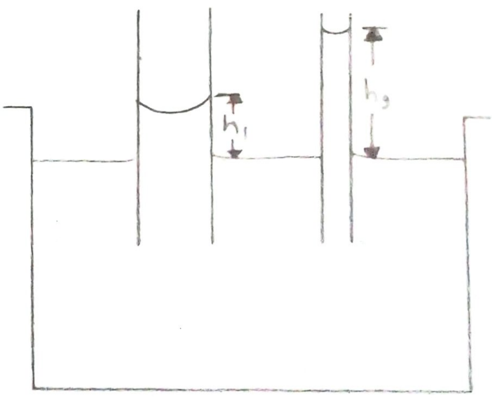
Soln:- Given,

$$\sigma = 0.073\,N/m$$

$$d_1 = 20\,mm = 20 \times 10^{-3}\,m$$

$$d_2 = 2\,mm = 2 \times 10^{-3}\,m$$

$$(h_2 - h_1) = ?$$



we have, $h = \frac{4\sigma \cos\theta}{\rho g d}$

Assume that water is pure and glass is clean i.e., $\theta = 0^\circ \Rightarrow \cos 0^\circ = 1$

$$h_1 = \frac{4\sigma \cos\theta}{\rho g d_1} = \frac{4 \times 0.073 \times 1}{1000 \times 9.81 \times 20 \times 10^{-3}} = 1.49\,mm$$

$$h_2 = \frac{4\sigma \cos\theta}{\rho g d_2} = \frac{4 \times 0.073 \times 1}{1000 \times 9.81 \times 2 \times 10^{-3}} = 14.9\,mm$$

$\therefore\ h_2 - h_1 = 14.9 - 1.49$

$$\boxed{h_2 - h_1 = 13.41\,mm}$$

5) A capillary tube having an inside diameter of 4mm is dipped in water. Determine the height of water which will raise in the tube. Take surface tension of water as 0.075 N/m and $\theta = 60^\circ$. What will be the percentage change when the tubed diameter is reduced to half.

Soln: Given,

$\sigma = 0.075\,N/m$

$\theta = 60^\circ$

$d_1 = 4\,mm = 4 \times 10^{-3}\,m \rightarrow$ Case-①.

$d_2 = \frac{d_1}{2} = 2 \times 10^{-3}\,m \rightarrow$ Case-②

$$h_1 = \frac{4\sigma \cos\theta}{\rho g d_1} = \frac{4 \times 0.075 \times \cos 60^\circ}{1000 \times 9.81 \times 4 \times 10^{-3}} = 3.823\,mm \rightarrow \text{Case-①}$$

$$h_2 = \frac{4\sigma \cos\theta}{\rho g d_2} = \frac{4 \times 0.075 \times \cos 60^\circ}{1000 \times 9.81 \times 2 \times 10^{-3}} = 7.645\,mm \rightarrow \text{Case-②}$$

$\therefore\ h_2 - h_1 = 7.645 - 3.823$

$\boxed{h_2 - h_1 = 3.822\,mm} \Rightarrow$ Change of height.

$\Rightarrow$ % change $= \frac{3.822\,mm \text{ (Change of height)}}{3.823 \text{ (original height)}} = 0.99973 \simeq 99.97 \simeq 100\,\%$

Now, $h \propto \frac{1}{d}$, here diameter reduced to half and hence new height becomes twice the previous one.

VAPOUR PRESSURE OF LIQUID AND CAVITATION

A change from the liquid state to the gaseous state is known as vapourization. The vapourization occurs because of continuous escaping of the molecules through the free liquid surface.

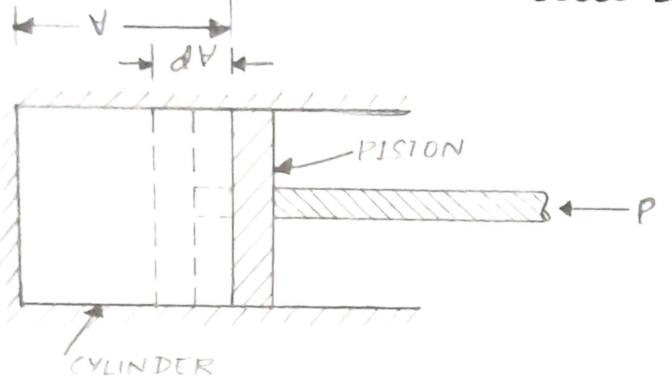
Consider a liquid which is confined in a closed vessel. Let the temperature of liquid is 20°C and pressure is atmospheric. This liquid will vaporise at 100°C. When vaporization takes place, the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as vapour pressure of the liquid (or) this is the pressure at which the liquid is converted into vapours.

Again consider the same liquid at 20°C at atmospheric pressure in the closed vessel. If the pressure above the liquid surface is reduced by some means, the boiling temperature will also reduce. If the pressure is reduced to such an extent that it becomes equal to (or) less than the vapour pressure, the boiling of the liquid will start, though the temperature of the liquid is 20°C. Thus a liquid may boil even at ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature.

Now consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to (or) less than the vapour pressure, the vapourization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known cavitation.

Hence the cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure.

COMPRESSIBILITY AND BULK MODULUS



Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in figure.

Let ∀ = volume of a gas enclosed in the cylinder

p = pressure of gas when volume is ∀

Let the pressure is increased to p+dp, the volume of gas decreases from '∀' to '∀−d∀'.

Then increase in pressure = dp kgf/m²

Decrease in volume = d∀

∴ Volumetric strain $= -\frac{d\forall}{\forall}$

−ve sign indicates the volume decreases with increase of pressure.

$$\therefore \text{Bulk modulus, } K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{dp}{\left(\frac{-d\forall}{\forall}\right)} = -\frac{dp \cdot \forall}{d\forall}$$

$$\text{Compressibility} = \frac{1}{K}$$

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RELATIONSHIP BETWEEN BULK MODULUS (K) AND PRESSURE (p) FOR A GAS.

The relationship between bulk modulus of elasticity (K) and pressure for a gas for two different processes of compression are as:

(i) FOR ISOTHERMAL PROCESS

If the change in density occurs at a constant temperature, then the process is called isothermal process.

And the relationship between pressure and density is given by, $\frac{p}{\rho}$ = constant

$$pV = \text{constant} \qquad \because V = 1/\rho$$

Differentiating this equation, (p and V are variables)

$$p\,dV + V\,dp = 0 \quad \text{(OR)} \quad p\,dV = -V\,dp \quad \text{(OR)} \quad p = \frac{-V\,dp}{dV}$$

But, $\frac{-V\,dp}{dV} = K$

$$\therefore \boxed{K = p}$$

(ii) FOR ADIABATIC PROCESS

If the change in density occurs with no heat exchange to and from the gas, the process is called as adiabatic process.

And if no heat is generated within the gas due to friction, the relationship between pressure and density is given by,

$$\frac{p}{\rho^k} = \text{constant} \qquad k = \text{ratio of specific heats}$$

$$p \cdot V^k = \text{constant}$$

On differentiation, we get

$$p \cdot d(V^k) + V^k \cdot dp = 0$$

$$p \cdot k \cdot V^{k-1}(dV) = -V^k \cdot dp$$

$$p \cdot k = \frac{-V^k\,dp}{V^{k-1}\,dV} = \frac{-V^{k-(k-1)}\,dp}{dV} = \frac{-V\,dp}{dV} = K$$

$$\therefore \boxed{K = p \cdot k}$$

FLUID AS A CONTINUUM

NOTE: Continuum means that the distance between fluid particles (or) molecules is small, i.e., small compared to any physical dimensions of the problem.

→ Fluid flows may be modelled either on a macroscopic level (or) on a microscopic level. The

→ The macroscopic model regards the fluid as a continuum and descriptions are in terms of macroscopic velocity, density, pressure and temperature with distance and time.

→ On the other hand, the microscopic (or) molecular model recognises the particulate structure of a fluid as a myriad of discrete molecules and provides information on the position and velocity of every molecule at all times.

→ All fluids are composed of molecules in constant motion. However, in most of the engineering applications, we are interested in the average (or) mean (or) the macroscopic effects of many molecules.

→ We thus treat a fluid as an infinitely divisible substance, a continuum, and do not concern with the behaviour of any individual molecules. The concept of a continuum forms the basis of classical fluid mechanics.

→ As a consequence of the 'continuum assumption' each fluid property is assumed to have a definite value at each point in space. Thus, fluid properties such as density, temperature, velocity etc. are considered to be the continuous functions of position and time.

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Module-1; Unit - 2

FLUID PRESSURE AND ITS MEASUREMENTS

FLUID PRESSURE

Fluid is a state of matter which exhibits the property of flow. When a certain mass of fluid is held in static equilibrium confirming it within a solid boundary, it excerts force along the direction perpendicular to the boundary in contact. This force is called Fluid Pressure.

FLUID PRESSURE AT A POINT

Intensity of pressure at a point is defined as the force excerted over the unit area considered around that point. In a small area 'dA' of large mass of stationary fluid, the force 'dF' excerted by the surrounding fluid on the area 'dA' will always be perpendicular to the surface area 'dA'. Then the ratio 'dF/dA' is known as the intensity of pressure (or) po and the ratio is represented by 'P'.

Therefore, The pressure at a point in a fluid at rest,

$$P = dF/dA$$

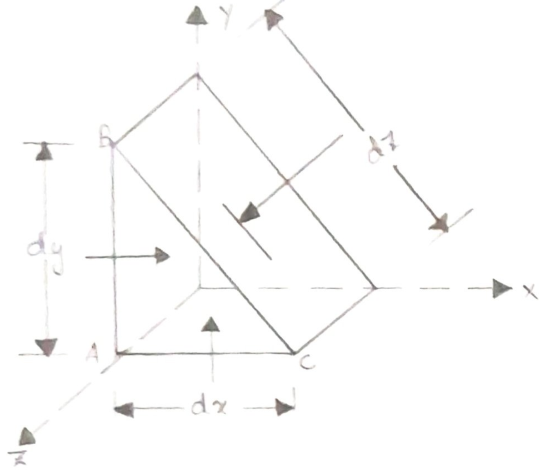
If the force 'F' is uniformly acting over an area 'A', then the pressure at any point is given by '$P = F/A$' and pressure force, $F = P.A$

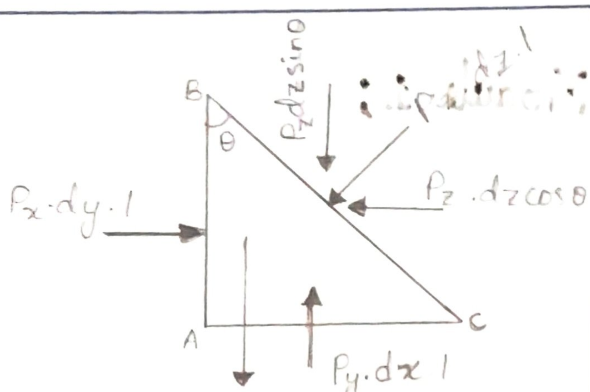
The unit of pressure is N/m^2, known as 'Pascal'.

PASCAL'S LAW

It states that "the pressure (or) intensity of pressure at a point in a static fluid is equal in all directions". This is proved as:

The fluid element is of very small dimensions i.e, dx, dy and ds





Self Weight $= \left(\frac{1}{2} \times dx\, dy\right) \times \rho g$

From △ABC,

$\sin\theta = \frac{dx}{dz} \Rightarrow dx = dz \sin\theta$

$\cos\theta = \frac{dy}{dz} \Rightarrow dy = dz \cos\theta$

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in figure.
Let the width of the element perpendicular to the plane of paper is unity. and,
P_x, P_y and P_z are the pressures (or) intensity of pressure acting on the face AB, AC and BC respectively.
Let ∠ABC = θ. Then the forces acting on the element are:

① Pressure forces normal to the surfaces
② Weight of element in the vertical direction

(i) The forces on the faces are:

Force on the face AB $= P_x \times$ Area of face AB

$= P_x \times dy \times 1$

Similarly,

Force on the face AC $= P_y \times dx \times 1$

Force on the face BC $= P_z \times ds \times 1$

(ii) Self weight of the element = Specific weight × Volume

$= \left[\rho g \left(\frac{1}{2} \times dx \times dy\right) \times 1\right]$

Now, resolving all the forces in x-directions, we have

$(P_x \times dy \times 1) - (P_z \times dz \cos\theta) = 0$

$\because \cos\theta = dy/dz$

$P_x\, dy - P_z\, dz\left(\frac{dy}{dz}\right) = 0$

$P_x\, dy - P_z\, dy = 0$ (OR) $dy(P_x - P_z) = 0$ (OR) $P_x - P_z = 0$

$\boxed{P_x = P_z}$ —— ①

Similarly, resolving all the forces in y-direction

$$(P_y \times dx \times 1) - (P_z \times dz \sin\theta) - (\rho g\, dx\, dy \cdot {}^{1}/_{2}) = 0$$

Since the element considered is small, self weight can be neglected.

$$P_y\, dx - P_z\, dz \sin\theta = 0 \qquad \because \sin\theta = dx/dz$$

$$P_y\, dx - P_z\, dz\left(\frac{dx}{dz}\right) = 0 \quad \text{(OR)}$$

$$P_y\, dx - P_z\, dx = 0 \quad \text{(OR)} \quad dx(P_y - P_z) = 0$$

$$P_y - P_z = 0$$

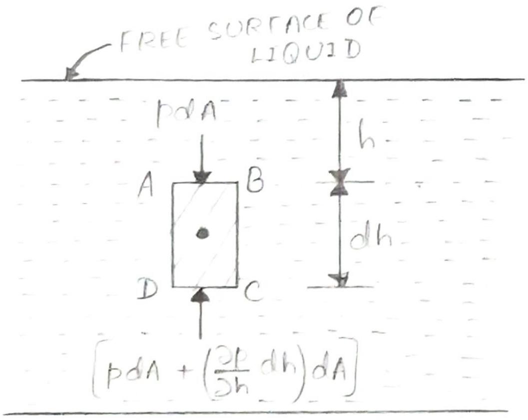
$$P_y = P_z \quad \text{——— (2)}$$

From equations (1) and (2), we get

$$\boxed{P_x = P_y = P_z}$$

This proves that the intensity of pressure at a point in a static fluid at all the directions is same.

VARIATION OF PRESSURE WITH DEPTH - HYDROSTATIC LAW



The pressure at any point in a fluid at rest is obtained by the Hydrostatic law which states that "the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point".

Consider a small fluid element ABCD of cross-sectional area 'dA' as shown in the figure.

Let 'dh = height of the fluid element

p = pressure on the face AB

h = distance of the fluid element from free surface.

The forces acting on the fluid element are:

① Pressure force on AB = $p \times dA$ acting perpendicular to the face AB in the downward direction.

② Pressure force on CD = $pdA + \frac{\partial p}{\partial h} dh\, dA$ acting perpendicular to the face CD in vertically upward direction.

③ Weight of fluid element = Density × g × volume

$= \rho \times g (dA \times dh)$

④ Pressure forces on surfaces BC and AD are equal and opposite.

For equilibrium of fluid element, we have

$$p\,dA - \left(p\,dA + \frac{\partial p}{\partial h} dh\, dA\right) + \rho g\, dA\, dh = 0$$

$$p\,dA - p\,dA - \frac{\partial p}{\partial h} dh\, dA + \rho g\, dA\, dh = 0$$

$$\frac{\partial p}{\partial h} dh\, dA = \rho g\, dA\, dh$$

$$\frac{\partial p}{\partial h} = \rho g$$

$\therefore\ \partial p = \rho g\, \partial h$ ← on integration we have

$$\int dp = \int \rho g\, dh$$

$$\therefore\ \boxed{p = \rho g h}$$

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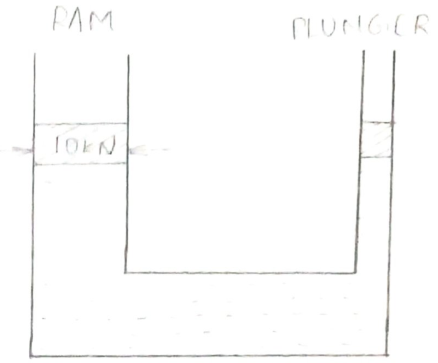
Also, pressure head, $\boxed{h = \frac{p}{\rho g}}$

PROBLEMS

1) A hydraulic press has a ram diameter of 12.5 cm and a plunger of diameter 1.25 cm. What force is required on the plunger to raise a weight 10 KN on the ram.

<u>Sol^n</u>: Given,



Dia. of plunger,

$D_P = 1.25cm = 0.0125m$

∴ Area, $A_P = \frac{\pi}{4}(D_P)^2 = 1.227\times10^{-4} m^2$

Dia. of Ram,

$D_R = 12.5cm = 0.125m$

∴ Area, $A_R = \frac{\pi}{4}(D_R)^2 = 0.0123 m^2$

$F_R = 10\times10^3 N$, $F_P = ?$

Pressure of plunger = Pressure of ram

i.e., $P_P = P_R$ (OR) ∵ $P = F/A$

$\frac{F_P}{A_P} = \frac{F_R}{A_R}$ (OR) $F_P = \frac{F_R \times A_P}{A_R} = \frac{10\times10^3\times1.227\times10^{-4}}{0.0123}$

∴ $\boxed{F_P = 99.76N}$ → Force on plunger.

2) A hydraulic press has a ram 20cm diameter and plunger of 3cm diameter it is used for lifting a weight of 30KN. Find the force required on the plunger.

<u>Sol^n</u>: Given,

Force of Ram, $F_R = 30\times10^3 N$

$D_R = 20\times10^{-2} m$

$D_P = 3\times10^{-2} m$

$F_P = ?$

From Pascal's law,

Pressure on plunger = Pressure on Ram

i.e, $P_P = P_R$ (OR) $\frac{F_P}{A_P} = \frac{F_R}{A_R}$

$$\frac{F_P}{\frac{\pi}{4}(3\times10^{-2})^2} = \frac{30\times10^3}{\frac{\pi}{4}(20\times10^{-2})^2} = 675N$$

∴ Force on plunger, $\boxed{F_P = 675N}$

3) A hydraulic press has a ram of 30cm diameter and a plunger of 4.5cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500N.

Soln :- Given,

$D_R = 30cm = 0.3m$

$\therefore A_R = \frac{\pi}{4}(D_R)^2 = 0.0707 m^2$

$D_P = 4.5cm = 0.045m$

$\therefore A_P = \frac{\pi}{4}(D_P)^2 = 0.00159 m^2$

$F_P = 500N$

Pressure intensity due to plunger $= \frac{F_P}{A_P} = \frac{500}{0.00159} = 314465.4 N/m^2$

From Pascal's law,

Pressure intensity at the plunger = Pressure intensity at the ram

$\therefore$ Pressure intensity at ram $= \frac{\text{weight}}{\text{Area of ram}} = \frac{W}{A_R}$

$$314465.4 = \frac{W}{0.0707}$$

$\therefore$ $W = 22232N$ (OR) $W = 22.232 kN$

4) Calculate the pressure due to a column of 0.3m of (i) water (ii) an oil of specific gravity 0.8, and (iii) mercury of specific gravity 13.6. Take density of water $\rho = 1000 kg/m^3$.

Soln: Given,

$h = 0.3m$

The pressure at any point in a liquid,

$$p = \rho g h$$

(i) water

$$p = 1000 \times 9.81 \times 0.3$$

$\therefore$ $p = 2943 N/m^2$

(ii) For oil of sp.gr. 0.8

$S_o = 0.8$

$\therefore \rho_o = S_o \times \rho_w = 0.8 \times 1000 = 800\ kg/m^3$

$\therefore p = \rho_o \times g \times h = 800 \times 9.81 \times 0.3$

$\therefore \boxed{p = 2354.4\ N/m^2}$

(iii) For mercury of sp. gr. 13.6

$S_m = 13.6$

$\therefore \rho_m = S_m \times \rho_w = 13.6 \times 1000 = 13600\ Kg/m^3$

$\therefore p = \rho_m \times g \times h = 13600 \times 9.81 \times 0.3$

$\boxed{p = 40024.8\ N/m^2}$

5) The pressure intensity at a point in a fluid is given by 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (i) water (ii) oil of sp.gr. 0.9

Soln: Given, $p = 3.924\ N/cm^2 = 3.924 \times 10^4\ /m^2$

(i) water

$p = \rho g h$

$\therefore h = \frac{p}{\rho g} = \frac{3.924 \times 10^4}{1000 \times 9.81}$

$\therefore$ h = 4 m of water.

(ii) For oil of sp.gr. 0.9

$S_o = 0.9 \Rightarrow \rho_o = S_o \times \rho_w = 0.9 \times 1000 = 900\ Kg/m^3$

$h = \frac{p}{\rho g} = \frac{3.924 \times 10^4}{900 \times 9.81}$

h = 4.44 m of oil

6) An oil of specific gravity 0.9 is contained in a vessel. At a point the height of oil is 40m. Find the corresponding height of water at the point.

Sol^n:- Given

$S_o = 0.9$

$h = 40m$

$\rho_o = S_o \times \rho_w = 0.9 \times 1000 = 900\ Kg/m^3$

$p = \rho_o \times g \times h = 900 \times 9.81 \times 40 = 353160\ N/m^3$

corresponding height of water $= \dfrac{P}{\text{density of water} \times g}$

$= \dfrac{353160}{1000 \times 9.81}$

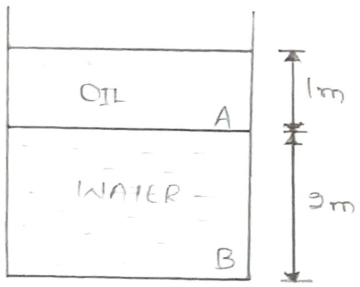
$= \underline{36m \text{ of water}}$

7) An open tank contains water upto a depth of 2m and above it an oil of specific gravity 0.9 for a depth of 1m. Find the pressure intensity

(i) at the interface of the two liquids, and

(ii) at the bottom of the tank.

Sol^n:- Given,



ht/ of water, $h_w = 2m$

ht/ of oil, $h_o = 1m$

Sp.gr. of oil, $S_o = 0.9$

$\therefore \rho_o = S_o \times \rho_w = 0.9 \times 1000$

$= 900\ kg/m^3$

(i) At interface, i.e., at A

$p = \rho_o \times g \times h_o = 900 \times 9.81 \times 1$

$\boxed{p = 8829\ N/m^2}$

(ii) At the bottom, i.e., at B

$p = (\rho_w \times g \times h_w) + (\rho_o \times g \times h_o)$

$= (1000 \times 9.81 \times 2) + 8829$

$\boxed{p = 28449\ N/m^2}$

8) The diameters of a small piston and a large piston of a hydraulic jack are 3cm and 10cm respectively. A force of 80N is applied on the small piston. Find the load lifted by the large piston when:

(a) the pistons are at the same level

(b) small piston is 40cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m³.

Soln:- Given,

Dia. of small piston, $d = 3cm = 0.03m$

$\therefore$ Area, $a = 7.069 \times 10^{-4} m^2$

Dia. of large piston, $D = 10cm = 0.1m$

$\therefore$ Area, $A = 7.854 \times 10^{-3} m^2$

Force on small piston, $F = 80N$

(a) When the pistons are at the same level

$$\left.\begin{array}{l}\text{Pressure intensity on}\\ \text{small piston}\end{array}\right\} = \frac{F}{a} = \frac{80}{7.069 \times 10^{-4}}$$

$$= 113170.2 \text{ N/m}^2$$

This is transmitted equally on the large piston.

$$\therefore \left.\begin{array}{l}\text{Pressure intensity on}\\ \text{large piston}\end{array}\right\} = 113170.2 \text{ N/m}^2$$

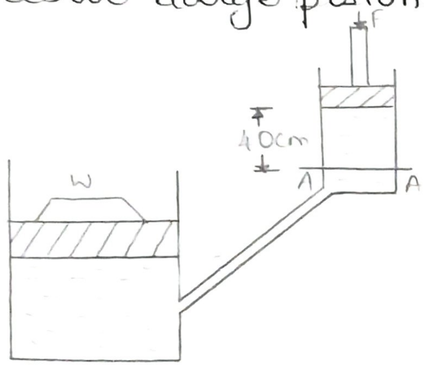
Now, Force on large piston = Pressure × Area

$$= 113170.2 \times 7.854 \times 10^{-3}$$

$$\therefore \boxed{F = 888.84 N}$$

(b) When the small piston is 40cm above large piston.

$$\left.\begin{array}{l}\text{Pressure intensity}\\ \text{on small piston}\end{array}\right\} = 113170.2 \text{ N/m}^2$$



∴ Pressure intensity at section A - A

= (Pressure intensity on the small piston) + (Pressure intensity due to height of 0.4m of liquid)

But, Pressure intensity due to 0.4 m of liquid } = $\rho \times g \times h$

$= 1000 \times 9.81 \times 0.4$

$= 3924 \text{ N/m}^2$

∴ Pressure intensity at section A - A

$= 113170.2 + 3924$

$= 117094.2 \text{ N/m}^2$

∴ Pressure intensity transmitted to the large piston } $= 117094.2 \text{ N/m}^2$

Now, Force on the large piston } = Pressure × Area of large piston

$= 117094.2 \times 7.854 \times 10^{-3}$

$= 919.66 \text{ N}$

TYPES OF PRESSURE - ABSOLUTE, GAUGE, ATMOSPHERIC AND VACCUM PRESSURES

1. ABSOLUTE PRESSURE

It is defined as the pressure which is measured with reference to absolute vaccum pressure.

2. GAUGE PRESSURE

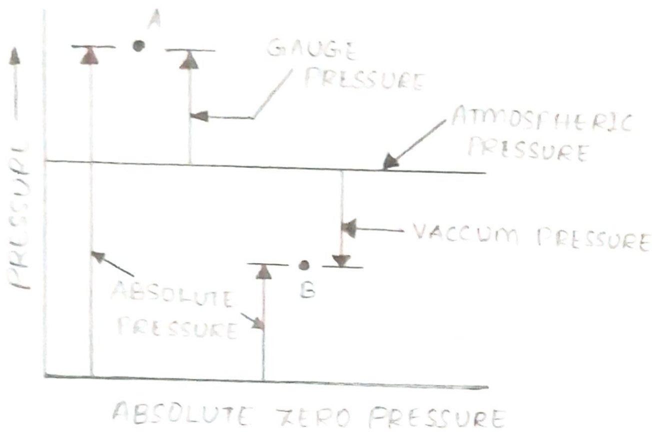
It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. ATMOSPHERIC PRESSURE

The force exerted per unit area against a surface by the weight of the air above the atmosphere is called atmospheric pressure.

4. VACUUM PRESSURE

It is defined as the pressure below the atmospheric pressure.



Relationship between pressures.

(i) Absolute pressure = (Atmospheric pressure) + (Gauge pressure)

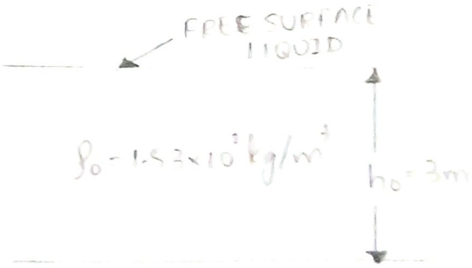
i.e., $P_{ab} = P_{atm} + P_{gauge}$

(ii) Vaccum pressure = (Atmospheric pressure) − (Absolute pressure)

PROBLEM

1) What are the gauge pressure and absolute pressure at a point 3m above the free surface of a liquid having density of 1.53×10^3 kg/m³ if the atmospheric pressure is equivalent to 750 mm of mercury? The specific gravity of mercury is 13.6 and density of water is 1000 kg/m³.

Soln :- Given,



$h_0 = 3$m

$\rho_0 = 1.53 \times 10^3$ kg/m³

$\rho_w = 1000$ kg/m³

we have, $p_o = \rho_o g h_o = 1.53\times10^3 \times 9.81 \times 3$

$P_o = 45027.9\ N/m^2 \Rightarrow$ Gauge pressure.

WKT, $P_{ab} = P_{atm} + P_{gauge}$

But, $P_{atm} = 750mm$ of Hg $= 0.75m$ of Hg

$P_m = \rho_m g h_m$

$= (13.6\times1000)\times9.81\times0.75$

$P_m = 100062\ N/m^2 = P_{atm}$

$\therefore P_{abs} = 100062 + 45027.9$

$$P_{abs} = 145089.9\ N/m^2$$

MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices:

(1) Manometers (2) Mechanical Gauges

MANOMETERS

Manometers are defined as the devices used for measuring the pressures at a point in a fluid by balancing the column of fluid by the same (or) another column of the fluid. They are classified as:

(a) Simple Manometers (b) Differential Manometers

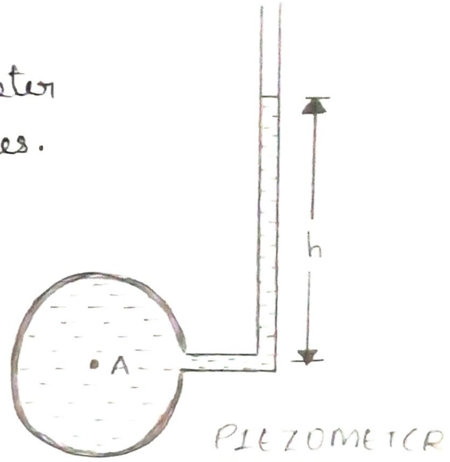
⊛ SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are:

1. Piezometer
2. U-tube manometer
3. Single Column Manometer

→ PIEZOMETER

- It is the simplest form of manometer used for measuring gauge pressures.
- One end of this manometer is connected to the point where pressure is to be measured and other end is open to atmosphere.
- The rise of liquid gives the pressure head at that point.

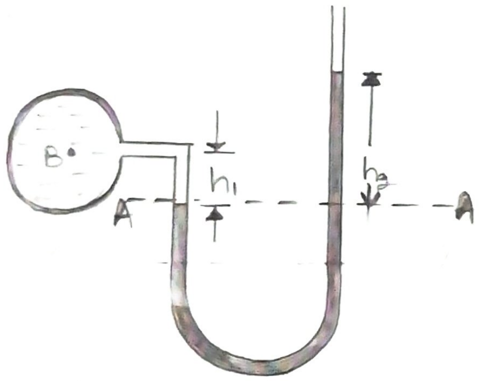


PIEZOMETER

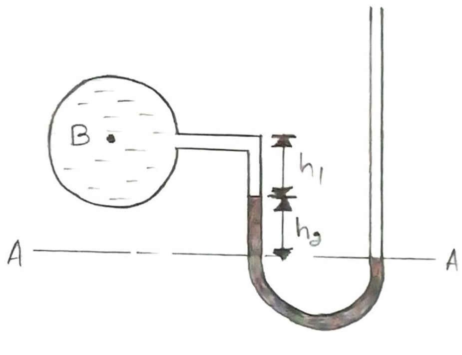
- If at a point A, the height of liquid say water is 'h' in piezometer tube, then pressure at A $= \rho g h \ N/m^2$

→ U-TUBE MANOMETER

- It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.
- The tube generally contains mercury (or) any other liquid whose specific gravity is greater than the specific gravity of liquid whose pressure is to be measured.



(a) FOR GAUGE PRESSURE



(b) FOR VACUUM PRESSURE

(a) For gauge pressure

Let B is the point at which pressure is to be measured, whose value is 'p'. The datum line is A-A

Let h_1 = height of liquid (light) above the datum line

h_2 = height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = density of light liquid $= 1000 \times S_1$

S_2 = Sp. gr. of heavy liquid

ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in left column = $p + \rho_1 g h_1$

Pressure above A-A in right column = $\rho_2 g h_2$

Equating the pressures,

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$\therefore \quad p = (\rho_2 g h_2) - (\rho_1 g h_1)$$

(b) For vacuum pressure

For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in fig (b).

Pressure above A-A in left column = $\rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in right column above A-A = 0

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$p = -(\rho_2 g h_2 + \rho_1 g h_1)$$

→ **SINGLE COLUMN MANOMETER**

Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to the one of the limbs (say left limb) of the manometer. There are two types of single column manometer :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

VERTICAL SINGLE COLUMN MANOMETER

Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at 'A', the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

P_A = pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

a = cross-sectional area of the right limb.

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir & right limb

ρ_1 = density of liquid in pipe

ρ_2 = density of liquid in reservoir.

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A} \quad \text{--- (1)}$$

Now consider the datum line Y-Y.

Then pressure in the right limb above Y-Y,

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb $= \rho_1 \times g \times (\Delta h + h_1) + P_A$

Equating the pressures,

MRN

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = \rho_2 g(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

But from equation ①

$$\Delta h = \frac{a h_2}{A}$$

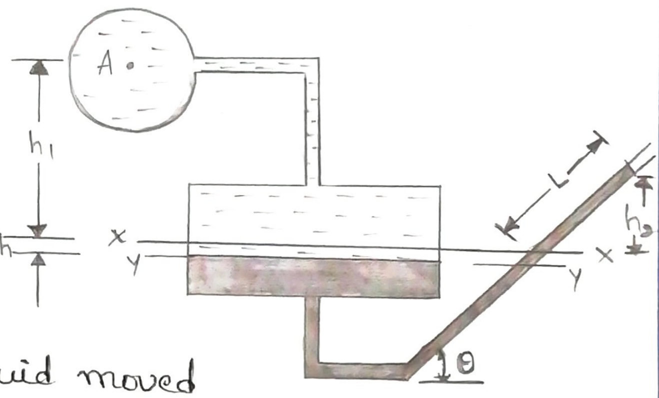
$$\therefore P_A = \frac{a \times h_2}{A}[\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

As the area 'A' is very large as compared to 'a', hence ratio a/A becomes very small and can be neglected.

Then $\langle P_A = h_2 \rho_2 g - h_1 \rho_1 g \rangle$

INCLINED SINGLE COLUMN MANOMETER

Inclined single column manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.



Let L = length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-X = $L \times \sin\theta$.

The pressure at A is

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

Substituting the value of h_2, we get

$$P_A = \sin\theta \times \rho_2 g - (h_1 \rho_1 g).$$

⊛ DIFFERENTIAL MANOMETERS

→ Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe (or) in two different pipe.

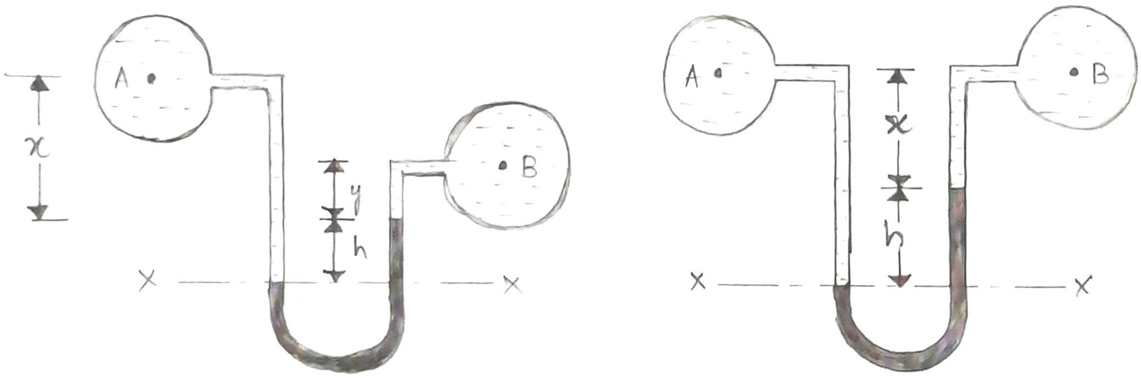
→ A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured.

Most commonly types of differential manometers are:

① U-tube differential manometer

② Inverted U-tube differential manometer.

→ U-TUBE DIFFERENTIAL MANOMETER



TWO PIPES AT DIFFERENT LEVELS. 'A' AND 'B' ARE AT THE SAME LEVEL

The two points 'A' and 'B' are at different level and also contains liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at 'A' and 'B' are P_A and P_B.

Let h = difference of mercury level in the U-tube

y = distance of the centre of B, from the mercury level in the right limb.

x = distance of the centre of A, from the mercury level in the right limb.

ρ_1 = density of liquid at A.

ρ_2 = density of liquid at B.

ρ_g = density of heavy liquid (or) mercury.

Taking datum line at X-X

Pressure above X-X in left limb $= \rho_1 g(h+x) + P_A$

Pressure above X-X in right limb $= (\rho_g \times g \times h) + (\rho_2 \times g \times y) + P_B$

Equating the two pressure,

$$\rho_1 g(h+x) + P_A = \rho_g g h + \rho_2 g y + P_B$$

$$(P_A - P_B) = \rho_g g h + \rho_2 g y - \rho_1 g(h+x)$$

$$\therefore (P_A - P_B) = hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

The two points A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= (\rho_g \times g \times h) + (\rho_1 \times g \times x) + P_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h+x) + P_A$

Equating the pressures,

$$(\rho_g \times g \times h) + (\rho_1 \times g \times x) + P_B = \rho_1 \times g \times (h+x) + P_A$$

$$\therefore P_A - P_B = (\rho_g \times g \times h) + \rho_1 g x - \rho_1 g(h+x)$$

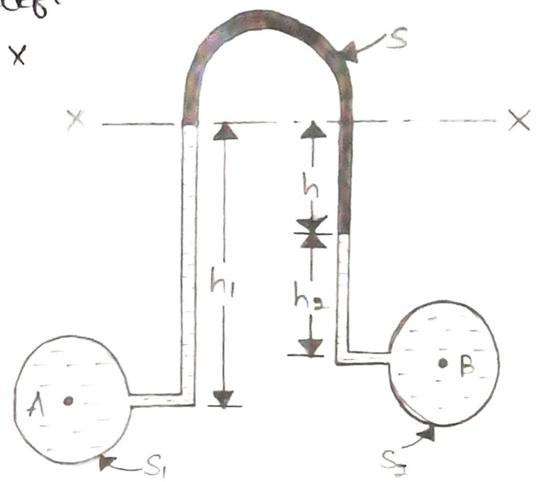
$$\therefore (P_A - P_B) = gh(\rho_g - \rho_1)$$

→ INVERTED U-TUBE DIFFERENTIAL MANOMETER

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures.

Let the pressure at A is more than the pressure at B.

Let h_1 = height of liquid in left limb below the datum X-X



h_2 = height of liquid in right limb

h = difference of light liquid

ρ_1 = density of liquid at A

ρ_2 = density of liquid at B

ρ_s = density of light liquid

P_A = pressure at A

P_A = Pressure at B

Taking X-X as datum line. Then pressure in the left limb below X-X $= P_A - \rho_1 g h_1$

Pressure in the right limb below X-X $\Big\} = P_B - \rho_2 g h_2 - \rho_s g h$

Equating the two pressure

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_s g h$$

$$(P_A - P_B) = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h$$

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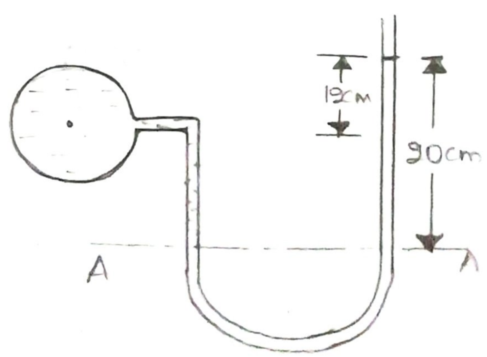
PROBLEMS

1) The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20cm.

Soln :- Given,

Sp gr of fluid, $S_1 = 0.9$

$\therefore \rho_1 = S_1 \times 1000 = 900\,kg/m^3$



Sp gr of mercury, $S_2 = 13.6$

$\therefore \rho_2 = S_2 \times 1000 = 13600\,kg/m^3$

$h_2 = 20cm = 0.2\,m$

Height of fluid from A-A } $h_1 = 20 - 12 = 8cm = 0.08m$

Equating the pressure above A-A, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

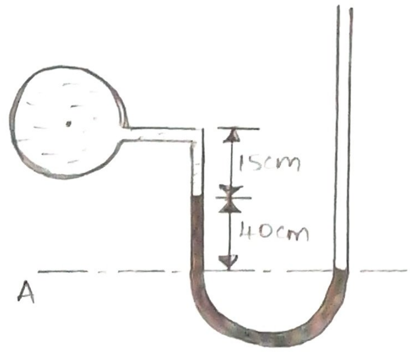
$$p + (900 \times 9.81 \times 0.08) = 13600 \times 9.81 \times 0.2$$

$$p = 26683.2 - 706.32$$

$$\therefore \boxed{p = 25976.88 N/m^2}$$

2) A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp.gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm and the height of fluid in the left from the centre of pipe is 15cm below.

Soln: Given,



Sp gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

$\rho_1 = 800 kg/m^3$

$\rho_2 = 13600 kg/m^3$

$h_2 = 40cm = 0.4m$

$h_1 = 15cm = 0.15m$

Equating the pressure above datum line, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$p = -[\rho_2 g h_2 + \rho_1 g h_1]$$

$$= -[(13600 \times 9.81 \times 0.4) + (800 \times 9.81 \times 0.15)]$$

$$= -[53366.4 + 1177.2] = -54543.6 N/m^2$$

$$\boxed{p = -54543.6 Pa}$$

-ve sign indicates that the pressure is vacuum.

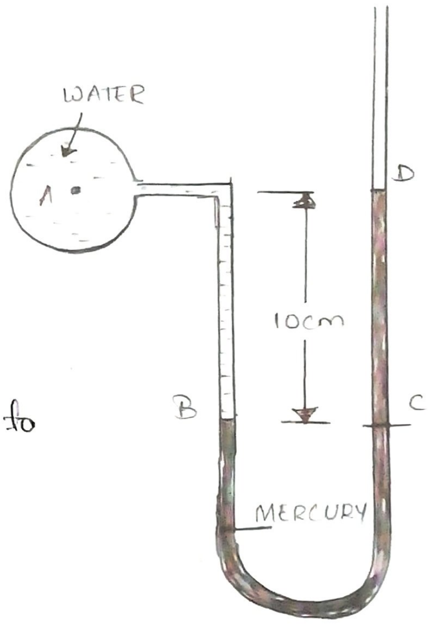
3.) A U-tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to $9810\ N/m^2$, calculate the new difference in the level of mercury. Sketch the arrangements in both cases.

Soln :- Given,

Difference of mercury $= 10cm$

$= 0.1m$

Case(1)



$\rho = 1000\ kg/m^3$

$h = 0.1m$

Let P_A = pressure of water in pipe line (i.e at point A)

pressure at B,

P_B = pressure at A + pressure due to 10cm

$= P_A + \rho g h$

$= P_A + 1000 \times 9.81 \times 0.1$

$P_B = P_A + 981$ —— ①

pressure at C,

P_C = pressure at D + pressure due to 10cm

$= 0 + \rho_0 \times g \times h_0 = 13600 \times 9.81 \times 0.1$ $(\because \rho_0 = 13.6 \times 1000)$

$P_B = 13341.6$ —— ②

But pressure at B is equal to pressure at C. Hence equating the equations ① & ② we get,

$P_A + 981 = 13341.6$

$\therefore$ $\boxed{P_A = 12360.6\ N/m^2}$

Case (2)

When the pressure is reduced to $9810\ N/m^2$ the level of mercury in left limb will rise.

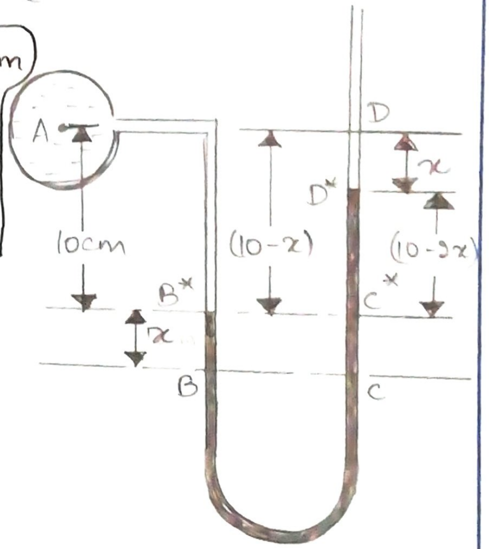
The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x = rise of Hg in left limb then,

fall of Hg in right limb = x cm.

→ The pressure at B^* = pressure at C^*

→ pressure at A + pressure due to $(10-x)$ cm of water = Pressure at D^* + (pressure due to $(10-2x)$ cm of Hg)



→ i.e., $P_A + \rho_1 g h_1 = P_D^* + \rho_2 g h_2$

→ $$9810 + 1000 \times 9.81 \times \left(\frac{10-x}{100}\right) = 0 + 13600 \times 9.81 \times \left(\frac{10-2x}{100}\right)$$

→ ÷ by 9.81 to simply.

$1000 + 10(10-x) = 136(10-2x)$

$1000 + 100 - 10x = 1360 - 272x$

$262(x) = 260$

∴ $x = 0.9923$ m (or) 99.23 cm

Wrong Data & Conversion Problem

Understanding the concept is more imp.

∴ New difference of Mercury $= 10 - 2x$

$= 10_{cm} - 2(99.23_{cm}) = 188.46$ cm (or) 1.8846 m.

4) A figure shows a conical vessel having it outlet at 'A' to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the water is completely filled with water.

Solⁿ: Given,

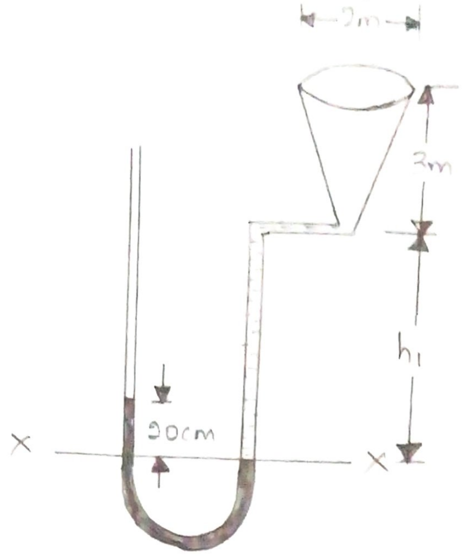
Sp gr of Hg = S_2 = 13.6

$\therefore \rho_2 = 13600\ kg/m^3$

Sp gr of water = S_1 = 1

$\therefore \rho_1 = 1000\ kg/m^3$

h_2 = 20cm = 0.2m



Equating the pressure above datum line X-X,

$\rho_2 g h_2 = \rho_1 g h_1$

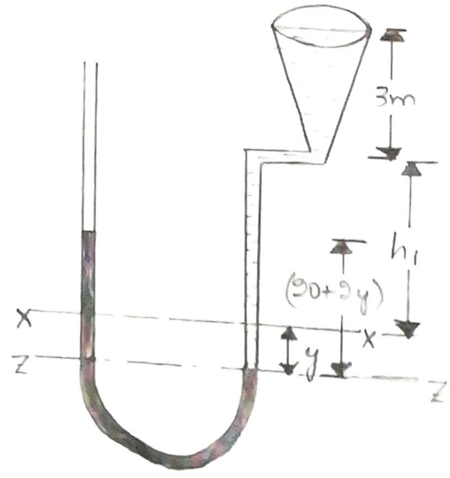
$(13600 \times 9.81 \times 0.2) = 1000 \times 9.81 \times h_1$

$\therefore h_1 = 2.72m$

Vessel is full of water

When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. The mercury will rise in the left limb by a distance y

Now, Equating the pressure above the datum Z-Z



Pressure in left limb = Pressure in right limb

$13600 \times 9.81 \times \left(0.2 + \frac{2y}{100}\right) = 1000 \times 9.81 \times (3 + h_1 + y/100)$

(h_1 ⇒ 2.72m)

$2720 + 272y = 1000\left(5.72 + \frac{y}{100}\right)$

$2720 + 272y = 5720 + 10y$

$262y = 3000$

$\therefore y = 11.45m$ ← {Practically impossible}

{Conversion Problem}

{Understanding the Concept is more imp.}

∴ The difference of Mercury level in two limbs

$= (20 + 2y)$ cm of mercury.

$= 20 + 2 \times 11.45 = 42.90$ m of Mercury.

∴ Reading of manometer = 42.90m of Mercury.

5) A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.

Soln: Given,

Sp. gr. of oil, $S_1 = 0.9$

$\therefore \rho_1 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_g = 13.6$

$\therefore \rho_g = 13.6 \times 1000 = 13600 \text{ kg/m}^3$

Difference in mercury level, $h = 15\text{cm} = 0.15\text{m}$

The difference of pressure is given by,

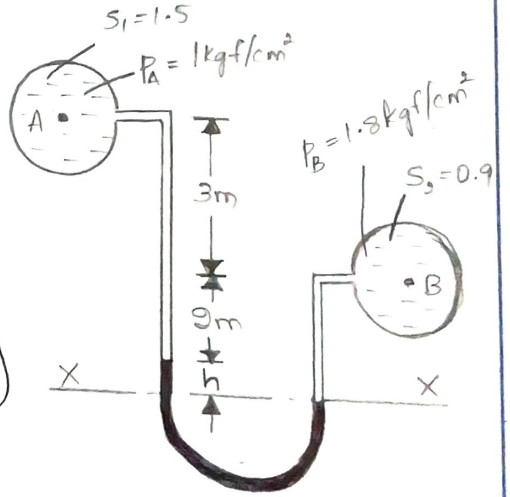
$$p_A - p_B = (\rho_g - \rho_1) g h$$

$$(p_A - p_B) = (13600 - 900)(9.81 \times 0.15)$$

$$\therefore \boxed{(p_A - p_B) = 18688.1 \text{ N/m}^2}$$

6) A differential manometer is connected at the two points 'A' and 'B' of two pipes as shown in figure. The pipe 'A' contains a liquid of sp. gr. = 1.5 while pipe 'B' contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.

Soln:- Given,



Sp. gr. of liquid A, $S_1 = 1.5$

$\therefore \rho_1 = 1500 \text{ kg/m}^3$

Sp. gr. of liquid B, $S_2 = 0.9$

$\therefore \rho_2 = 900 \text{ kg/m}^3$

$P_A = 1 \text{ kgf/cm}^2 = 1 \times 10^4 \text{ kgf/m}^2$

$= 10^4 \times 9.81 \text{ N/m}^2 \quad (\because 1\text{kgf} = 9.81\text{N})$

$P_B = 1.8 \text{ kgf/cm}^2 = 1.8 \times 10^4 \text{ kgf/m}^2$

$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

Taking x-x as datum line

Pressure above x-x in the left limb

$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + p_A$

$= 133416 h + 73575 + 9.81 \times 10^4$

$= 133416 \times h + 171675$

Pressure above x-x in the right limb

$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$

$= 8829 \times h + 17658 + 17.658 \times 10^4$

Equating the two pressures, we get

$133416 \times h + 171675 = 8829 \times h + 17658 + 17.658 \times 10^4$

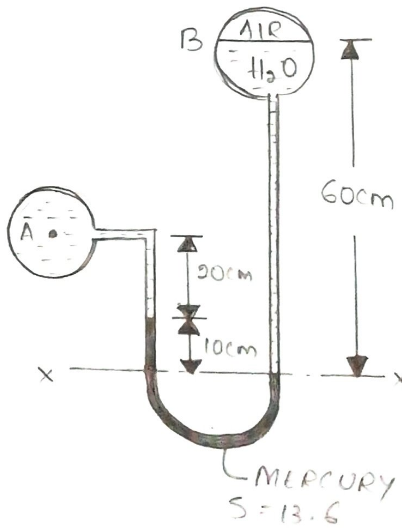
$124587 \times h = 22563$

$h = 0.181 m$

$\boxed{h = 18.1 cm}$

7) A differential manometer is connected at the two points A and B as shown in figure. At 'B' air pressure is 9.81 N/cm² (abs), find the absolute pressure at A.

Soln: Given,



$p_B = 9.81 \times 10^4 N/m^2$

Density of oil $= 0.9 \times 1000 = 900 kg/m^3$

Density of Hg $= 13.6 \times 1000 kg/m^3$

Pressure above x-x in right limb

$= 1000 \times 9.81 \times 0.6 + p_B$

$= 5886 + 9.81 \times 10^4$

$= 103986 N/m^2$

Pressure above x-x in left limb

$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$

$= 13341.6 + 1765.8 + p_A$

Equating two pressures

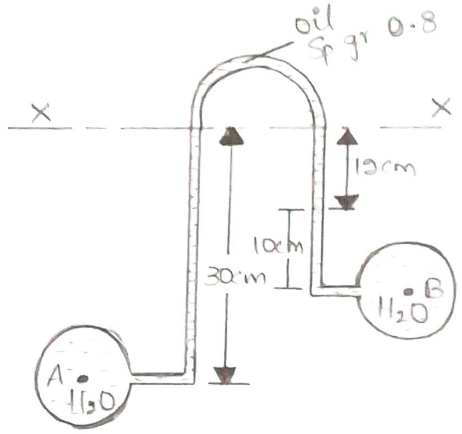
$$103986 = 13341.6 + 1765.8 + P_A$$

$$P_A = -15107.4 + 103986$$

$$\boxed{P_A = 88878.6 \ N/m^2}$$

8) Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2m of water, find the pressure in the pipe B for the manometer readings.

Soln: Given,



pressure head at A $= \frac{P_A}{\rho g} = 2m$

$\therefore \ P_A = 2 \times 9.81 \times 1000$

$= 19620 \ N/m^2$

Pressure below X-X in the left limb

$= P_A - \rho_1 g h_1$

$= 19620 - (1000 \times 9.81 \times 0.3)$

$= 16677 \ N/m^2$

Pressure below X-X in the right limb

$= P_B - 1000 \times 9.8 \times 0.1 - 800 \times 9.81 \times 0.12$

$= P_B - 981 - 941.76 = P_B - 1922.76$

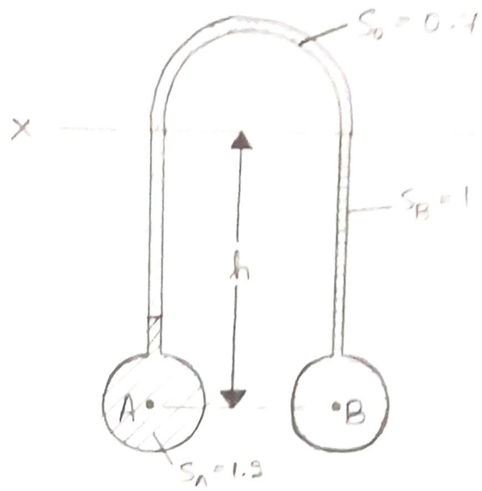
Equating the two pressures.

$$16677 = P_B - 1922.76$$

$$\boxed{P_B = 18599.76 \ N/m^2}$$

9) Find out the differential reading 'h' of an inverted U-tube manometer containing oil of sp. gr. 0.7 as the manometric fluid when connected across pipes A and B as shown in figure, conveying liquids of sp. gr. 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal.

Soln: Given,



$S_A = 1.2$

$\therefore \rho_A = 1.2 \times 1000 = 1200\ kg/m^3$

$S_B = 1$

$\therefore \rho_B = 1000\ kg/m^3$

Sp. gr. of oil, $S_o = 0.7$

$\therefore \rho_o = 0.7 \times 1000 = 700\ kg/m^3$

Pressure below x-x in left limb

$= P_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h$

Pressure below x-x in right limb

$= P_B - 1000 \times 9.81 \times (h + 0.3)$

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Equating the pressures,

$P_A - (1200 \times 9.81 \times 0.3) - (700 \times 9.81 \times h) = P_B - 1000 \times 9.81 \times (h + 0.3)$

But $P_A = P_B$

$\therefore -3531.6 - 6867 \times h = -9810 \times h - 2943$

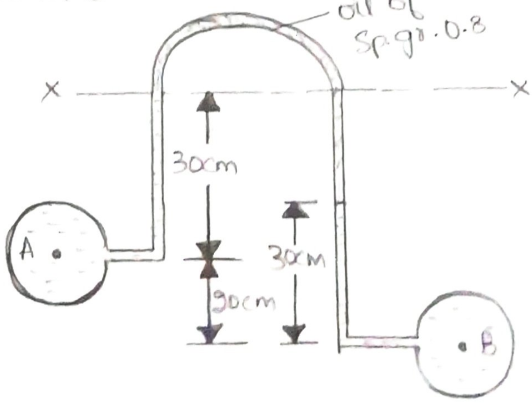
$-3531.6 - 6867 \times h = -9810 \times h - 2943$

$-588.6 = -2943 \times h$

$\therefore \boxed{h = 0.2\ m}$ (OR) $\boxed{h = 20\ cm}$

10) In the figure, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of Sp. gr. 0.8. For the manometer readings shown in figure, find the pressure difference between A and B.

Soln:- Given,



Sp. gr. of oil = 0.8

$\therefore \rho_s = 800\ kg/m^3$

Difference of oil in the two limbs $= (30 + 20) - 30$

$= 20\ cm$

$= 0.2\ m$

Pressure in the left limb below X-X

$= P_A - 1000 \times 9.81 \times 0.3$

$= P_A - 2943$

Pressure in the right limb below X-X

$= P_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= P_B - 2943 - 1569.6$

$= P_B - 4512.6$

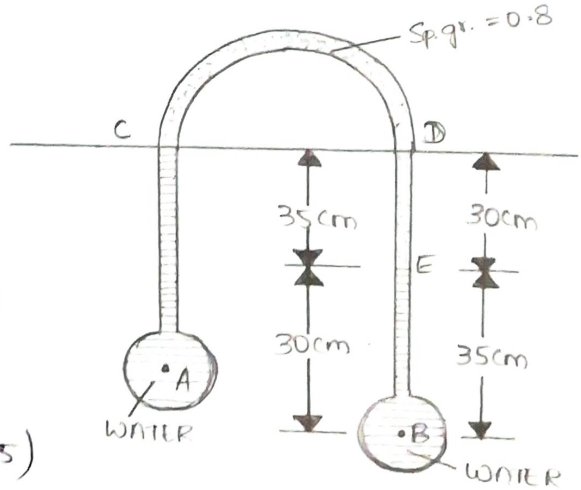
Equating the two pressures, we get

$P_A - 2943 = P_B - 4512.6$

$$P_A - P_B = 1596.6 \text{ N/m}^2$$

11.) An inverted U-tube manometer is connected to two horizontal pipes A and B through which water is flowing. The vertical distance between the axes of these pipes is 30cm. When an oil ~~oil~~ of sp.gr. 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from the respective centre lines of the pipes) are found to be same and equal to 35cm. Determine the difference of pressure between the pipes.

Soln :- Given,



Sp. gr. = 0.8 $\therefore \rho = 800$ kg/m³

The points C and D lie on the same horizontal line.

Hence,

pressure at C = pressure at D

Pressure at C $= P_A - \rho g h$

$= P_A - (1000 \times 9.81 \times 0.35)$

$= P_A - 3433.5$

Pressure at D $= P_B - \rho_1 g h_1 - \rho_2 g h_2$

$= P_B - (1000 \times 9.81 \times 0.35) - (800 \times 9.81 \times 0.3)$

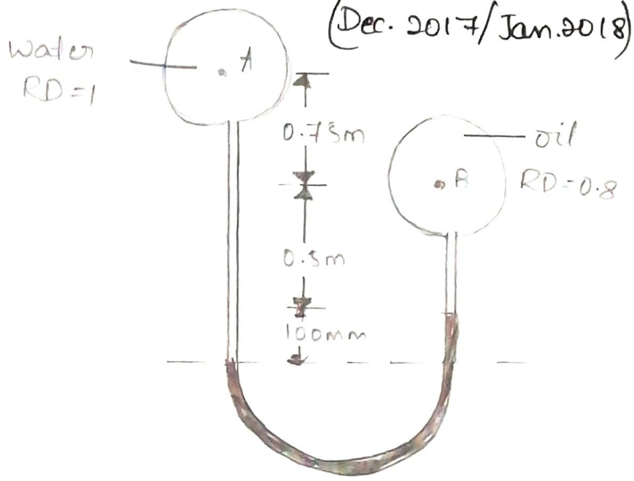
$= P_B - 3433.5 - 2354.4 = P_B - 1079.1$

pressure at C = pressure at D

$P_A - 3433.5 = P_B - 1079.1$

$\therefore \boxed{(P_A - P_B) = 2354.4 \ N/m^2}$

12) Determine the pressure difference $(P_A - P_B)$ in the below given figure. (Dec. 2017/Jan. 2018)



Sol^n: Given,

$S_A = 1$

$\therefore \rho_A = 1000 \ kg/m^3$

$S_B = 0.8$

$\therefore \rho_B = 800 \ kg/m^3$

$S_m = 13.6$

$\therefore \rho_m = 13600 \ kg/m^3$

From Pascal's law,

$P_A + \rho_A \times g \times 1.35 - \rho_m \times g \times 0.1 - \rho_B \times g \times 0.5 - P_B = 0$

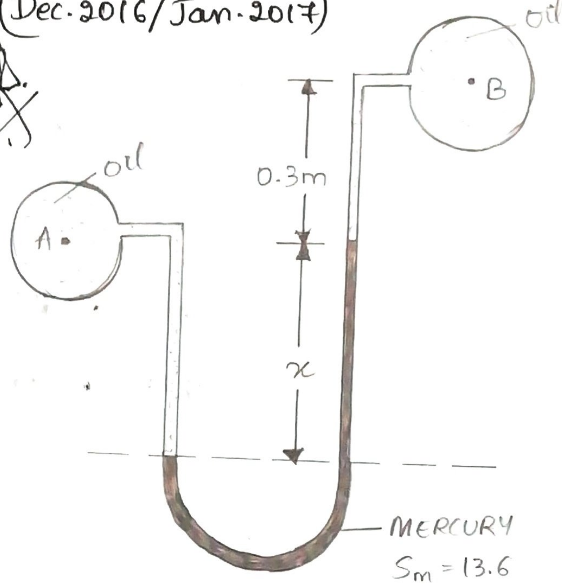
$P_A - P_B + 1000 \times 9.81 \times 1.35 - 13600 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.5 = 0$

$\therefore \boxed{P_A - P_B = 4022.1 \ N/m^2}$

13) Petrol of specific gravity 0.8 flows upwards through a vertical pipe. 'A' and 'B' are two points in the pipe, 'B' being 0.3 m higher than 'A', connections are led from 'A' and 'B' to a U-tube containing mercury. If the difference of pressure between 'A' and 'B' is 0.18 kgf/cm². Find the difference in the mercury level in the differential manometer. (Dec. 2019/Jan. 2020)

(Dec. 2016/Jan. 2017)

Solⁿ: Given,



Sp gr. of petrol, $S_A = 0.8$

$\therefore \rho_A = 800 \ kg/m^3$

$h = 0.3 m$

Sp. gr. of momometric liquid, $S_m = 13.6$

$\therefore \rho_m = 13600 \ kg/m^3$

$(P_A - P_B) = 0.18 \ kgf/cm^2$

$= 0.18 \times 10^4 \ kgf/m^2$

$= 0.18 \times 9.81 \times 10^4 \ N/m^2$

$= 1.7658 \times 10^4 \ N/m^2$

$x = ?$

$$P_A + (\rho_A \times g \times x) - (\rho_m \times g \times x) - (\rho_B \times g \times 0.3) - P_B = 0$$

$$(P_A - P_B) + (\rho_A g x) - (\rho_m g x) - (\rho_B g \times 0.3) = 0$$

$$(1.7658 \times 10^4) + (800 \times 9.81 \times x) - (13600 \times 9.81 \times x) - (800 \times 9.81 \times 0.3) = 0$$

$$1.7658 \times 10^4 + 7848 \times x - 133416 \times x - 2354.4 = 0$$

$$15303.6 = 125568 \times x$$

$$\therefore \boxed{x = 0.1219 m}$$

14) A U-tube differential manometer connects two pipes 'A' and 'B'. Pipe 'A' contains carbontetrachloride (CCl_4) having Sp. gr. 1.594 under a pressure of 117.72 kN/m² and pipe 'B' contains oil of sp. gr. 0.8 under a pressure of 117.72 kN/m². The pipe 'A' lies 2.5 m above pipe 'B'. Find the difference in pressure "measured by mercury" as fluid filling U-tube. Assume mercury in the right limb is 50 cm below centre of pipe 'B'. (Aug./Sept. 2020) (June/July 2018)

Sol^n: Given,

$S_A = 1.594$,

$\therefore \rho_A = 1549 \text{ kg/m}^3$

$S_m = 13.6$

$\therefore \rho_m = 13600 \text{ kg/m}^3$

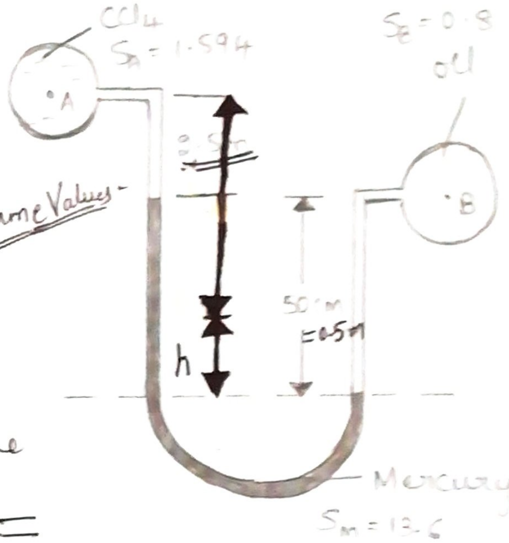
$S_B = 0.8$

$\therefore \rho_B = 800 \text{ kg/m}^3$

~~$h_A = 2.5$ m~~

~~$h_B = 0.5$ m~~

$P_A = 117.72 \text{ kN/m}^2$
$P_B = 117.72 \text{ kN/m}^2$ } Same Values



Equate the pressures in both the limbs –

$$\cancel{P_A} + (1549 \times 9.81 \times 2.5) + (13600 \times 9.81 \times h) = (800 \times 9.81 \times 0.5) + \cancel{P_B}$$

$$37989.225 + 133416(h) = 3924$$

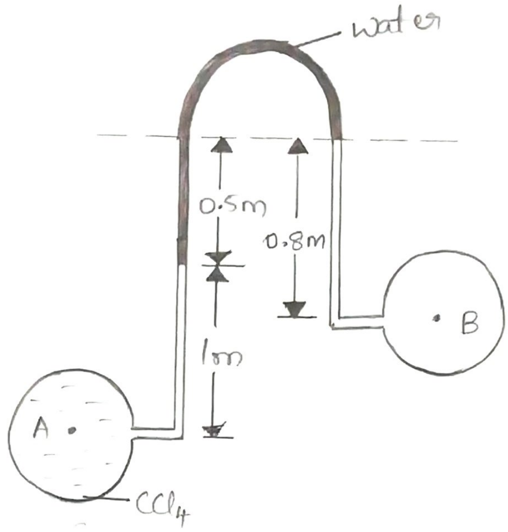
$$34065.225 = 133416(h)$$

$$\therefore h = 0.2553 \text{ m} \text{ or } 25.53 \text{ cm}.$$

15) Using an inverted U-tube manometer, find the intensity of pressure at 'B' for the given condition shown in below figure. Carbon tetrachloride of relative density 1.6 is flowing through the pipe 'A' and 'B'. Water is used as manometer fluid. The pressure at 'A' is 294.33 kN/m².

(June/July 2019)

Soln :- Given,



$S_A = 1.6 = S_B$

$\therefore \rho_A = 1600\ kg/m^3$

$S_m = 1$

$\therefore \rho_m = 1000\ kg/m^3$

$P_A = 294.33\ kN/m^2$

$= 294.33 \times 10^3\ N/m^2$

$P_B = ?$

From Pascals law,

$$-P_A - (\rho_A \times g \times 1) - (\rho_m \times g \times 0.5) + (\rho_B \times g \times 0.8) + P_B = 0$$

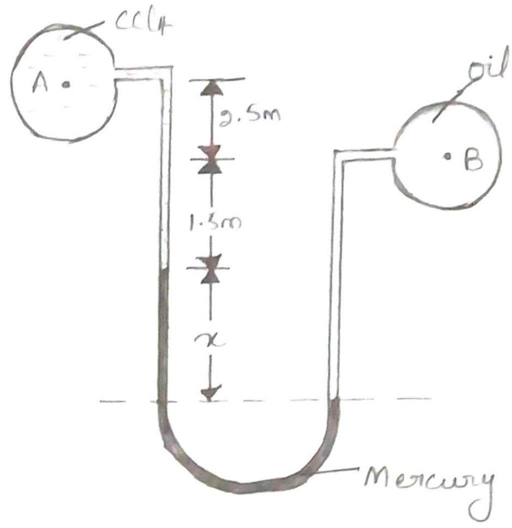
$$-(294.33 \times 10^3) - (1600 \times 9.81 \times 1) - (1000 \times 9.81 \times 0.5) + (1600 \times 9.81 \times 0.8) + P_B = 0$$

$\therefore P_B = 302374.2\ N/m^2$ (OR)

$$\boxed{P_B = 302.37\ kN/m^2}$$

16) The figure shows a differential manometer connecting two points 'A' and 'B'. Pipe 'A' contains carbon tetra-chloride of sp. gr. 1.594 under a pressure of 1.05 kgf/cm² and pipe 'B' contains oil of sp. gr. 0.8 under a pressure of 1.75 kgf/cm². If the manometer liquid is mercury, find the difference 'x' between the mercury levels.

(June/July 2017)



Soln: Given,

$S_A = 1.594$

$\therefore \rho_A = 1594 \, kg/m^3$

$S_m = 13.6$

$\therefore \rho_m = 13600 \, kg/m^3$

$S_B = 0.8$

$\therefore \rho_B = 800 \, kg/m^3$

$P_A = 1.05 \, kgf/cm^2 = 10.3 \times 10^4 \, N/m^2$

$P_B = 1.75 \, kgf/cm^2 = 17.17 \times 10^4 \, N/m^2$

$x = ?$

from Pascal's Law,

WKT,

$$P_A + (\rho_A \cdot g \cdot 4) + (\rho_m g \cdot x) - [\rho_r \, g (x + 1.5)] - P_B = 0$$

$$(10.3 \times 10^4) + (1594 \times 9.81 \times 4) + (13600 \times 9.81 \times x) - (800 \times 9.81 \times x) - (800 \times 9.81 \times 1.5) - (17.17 \times 10^4) = 0$$

$$17993.44 = 125568 \times x$$

$$\therefore \boxed{x = 0.143 \, m}$$

INTRODUCTION TO MECHANICAL GAUGES

→ Mechanical gauges are the pressure measuring devices which has an elastic element that deflects under the action of applied pressure. This deflection (or) movement when magnified mechanically will operate a pointer moving against a graduated circumferential scale.

→ Generally these gauges are used for measuring high pressure and where high ~~pere~~ precision is not required.

→ The commonly used mechanical pressure gauges are:

1. Diaphragm pressure gauge,
2. Bourdon tube pressure gauge,
3. Dead-weight pressure gauge,
4. Bellows pressure gauge.

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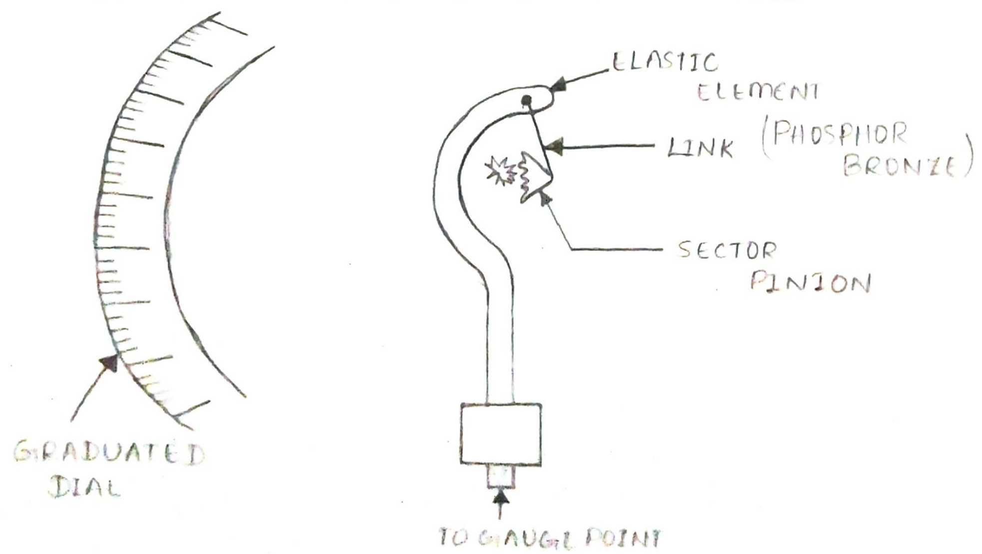
1. DIAPHRAGM PRESSURE GAUGE

The pressure responsive element in this gauge is a elastic steel corrugated diaphragm. The elastic deformation of the diaphragm under pressure is transmitted to a pointer by a similar arrangement as in the case of Bourdon tube pressure gauge. This gauge is used to measure relatively low pressure intensities.

2. BOURDON TUBE PRESSURE GAUGE

→ This ~~is~~ the most common type of pressure gauge which was invented by E-Bourdon.

→ The pressure responsive element in this gauge is a tube of steel (or) bronze which is of elliptic cross-section and is curved into a circular arc.



→ The tube is closed at its outer end and this end of the tube is free to move.
→ The other end of the tube, through which the fluid enters, is rigidly fixed to the frame as shown in fig/.
→ When the gauge is connected to the gauge point, fluid under pressure enters the tube Due to increase in pressure, the elliptical cross-section of the tube tends to become circular, thus causing be to straighten out slightly.
→ The small outward movement of the free end of the tube is transmitted to a pointer through the link and sector pinion.
→ The pointer moves clockwise on the graduated circular dial which indicates the pressure intensity of the fluid.
→ The dial of the gauge is so calibrated that it reads zero when the pressure inside the tube is equal to local atmospheric pressure and electric deformation of the tube causes the pointer to be displaced on the dial in proportion to the pressure intensity of the fluid.

3. DEAD-WEIGHT PRESSURE GAUGE

→ A simple form of a dead-weight pressure gauge consists of a plunger of diameter 'd', which can slide within a vertical cylinder.
→ The fluid under pressure entering the cylinder, exerts a force on the plunger which is balanced by the weights loaded on the top of the plunger.
→ If the weight required to balance the fluid under pressure is 'W', then the pressure intensity 'p' of the fluid may be determined as,

$$p = \frac{W}{(\frac{\pi}{4})d^2}$$

4. BELLOWS PRESSURE GAUGE

In this gauge, the pressure responsive element is made up of a thin metallic tube having deep circumferential corrugations. In response to the pressure changes the elastic element expands (or) contracts, thereby moving the pointer on the graduated circular dial.

INTRODUCTION TO ELECTRONIC PRESSURE GAUGE

Electronic pressure transducers convert pressure into an electrical output. These devices consists of a sensing element, transduction element and signal conditioning device to convert pressure reading to digital values on the display panal.

The main types of sensing elements are:

* Bourdan tubes
* Capsules
* Diaphragm
* Bellows

A pressure transducer is a device that turns a mechanical signal into an electrical signal (or) an electrical signal into a mechanical signal response.

For example, the Bourdan gauge transfer pressure to displacement.

There are number of ways to accomplish this kind of conversion.

→ Strain gauge
→ Variable reluctance
→ Capacitance
→ Optical method.

Normally, electronic pressure transducers are costly compared to conventional mechanical gauges and need to be calibrated at National laboratories before put into use.

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FLUID MECHANICS

Some specific fluid properties

1. Density = $\frac{\text{mass}}{\text{volume}}$ (measured in kg/m³).
2. Density of liquid & gas is directly proportional to pressure and inversely to temperature
3. Specific gravity/relative density

$$= \frac{\text{Density of liquid}}{\text{Density of water at } 4^\circ C}$$

4. If R.D < 1, then fluid is lighter than water.
5. Specific weight = $\frac{\text{Weight of substance}}{\text{Volume of substance}}$, ($g = \rho g$ in N/m³)
6. Some Important Relation

   1 milibar = 10^{-3} bar = 100 N/m²

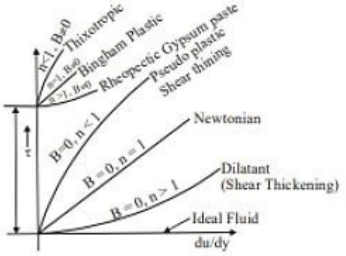
   1 mm of Hg = 10^{-3} m of Hg = $10^{-3} \times 13.6$ m of water = $10^{-3} \times 13.6 \times 9810$ N/m² = 133.42 N/m²

   1 N/mm² = 10^6 N/m²

   $$1\ \text{Kgf/cm}^2 = \frac{9.81\ \text{N}}{10^{-4}\text{m}^2} = 98.1 \times 10^3\ \text{N/m}^2$$

7. $g_{water} = 9810\frac{N}{m^3} = 9.81\frac{KN}{m^3}$
8. $g_{mercury} = 13.6\ g_w$
9. Specific volume = $\frac{1}{\text{Density}}$

- **Viscosity:** $\boxed{\mu = \frac{\tau}{\frac{d\theta}{dt}}}$ $\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$
- **Kinematic Visocity:** $\boxed{V = \frac{\mu}{\rho}}$ m²/sec.
- **Newton's Law Of Viscosity:** $\boxed{\tau = \mu\frac{du}{dy}}$
- **Non- Newtonian Fluid:** $\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$



- Ex.

(a) Thixotropic — Ink, Ketchup, Enamels etc.

(b) Bingham plastic — Sewage, Sludge, Drilling mud, Gel, Toothpaste, Cream

(c) Rheopectic — Gypsum in water & Bentonite slurry.

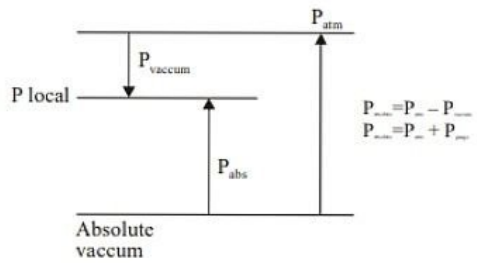
(d) Pseudo Plastic — Paint, Paper, Pulp, Blood, Syrup, Polymer, Lipstick, Nailpaint

(e) Dilatant — Quick sand, Sugar in water, Butter

Special Points:

1. Wetting property is due to surface tension.
2. Ideal fluids ® No-viscosity ® no "No slip" condition
3. No slip condition is due to fluid viscosity.

- **Pressure Inside The Liquid Drop:** $\boxed{P_{ld} = \frac{4\sigma}{d}}$
- **Pressure Inside The Liquid Jet:** $\boxed{P_{lj} = \frac{2\sigma}{d}}$
- **Pressure Inside the Soap Bubble:** $\boxed{P_{sb} = \frac{8\sigma}{d}}$
- **Expression For Capillary Rise:** $\boxed{h = \frac{4\sigma\cos\theta}{wd}}$



Special Points:

- Buoyant force is independent of distance of body from free surface of liquid and also the density of solid body.
- Mechanical gauges are used for measuring high pressure values which does not requires high precision.
- Air cavitation is less damaging than vapour cavitation.

Measurement of fluid pressure

- **Manometer** — Based on principle of balancing a column of fluid by the same or other column
  - **Simple manometer** — To measure pressure at a point
    - U-Tube manometer
    - Single column manometer
    - Piezometer
  - **Differential manometer** — To measure the pressure difference
    - Inverted differential manometer
    - Micro manometer
- **Mechanical gauges** — Mechanical pressure measuring instruments with a deflecting needle (used in filling air in tyres)

No.	Type of Manometer	Fluid Types	Pressure measurement
1.	Piezometer	Liquid	Positive (Gauge pressure)
2.	U-tube Manometer	Both liquid & gases	Both positive & Negative Pressure
3.	Inclined Tube Manometer	Gases (for very low pressure)	Both (+ve & -ve) (mostly +ve)
4.	Differential & Inverted Differential	Both liquid & gases	Pressure difference Between 2 points
5.	Bourdon Pressure gauge	Both liquid & gases	It measures pressure at a point

Facts about pressure

1.Longer runway's needed at higher altitude due to reduced drag and lift.

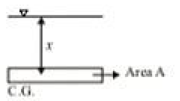
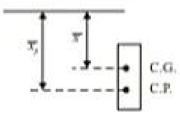
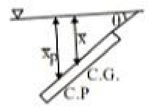
2.Nose bleeding starts at higher altitude because of difference in body's blood pressure and atmosphere pressure.

3.Motor capacity reduces at higher altitude.

4.Cooking takes longer time at higher altitudes.

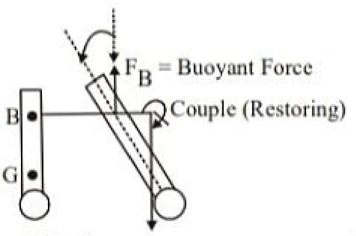
- **Buoyancy And Floatation**

$$\boxed{\text{Weight of body} = \text{weight of fluid displaced}}$$

Horizontal Plane Surface	Vertical Plane Surface	Inclined Plane Surface
		
$F = \gamma A\bar{x}$	$F = \gamma A\bar{x}$	$F = \gamma A\bar{x}$
	$x_p = \bar{x} + \frac{I_g}{A\bar{x}}$	$x_p = \bar{x} + \frac{I_g \sin^2\theta}{A\bar{x}}$

$\bar{x}$ & $\bar{x}_p$ for same horizontal plane surface from liquid surface

Rotational Stability: When a small angular displacement sets up a restoring couple, then stability is known as rotational stability.



Submerged body	Floating body
Stable equilibrium G below B	M above G BM > BG GM = MB – BG = +Ve
Unstable equilibrium	G above B M below G BM < MG GM = MB – BG = –Ve
Neutral equilibrium	G and B coincide M and G GM = 0

Metacentre (M) is the point of intersection of lines of action of buoyant force before and after rotation.

- **Continuity Equation:** $\boxed{A_1V_1 = A_2V_2}$
- **Hydrostatic Force**

Horizontal $\boxed{F = WA\bar{x}}$ $\boxed{h = \bar{x}}$

Vertical $\boxed{F = WA\bar{x}}$ $\boxed{h = \bar{x} + \frac{I_g}{A\bar{x}}}$

Inclined $\boxed{F = WA\bar{x}}$ $\boxed{h = \bar{x} + \frac{Ig}{Ax}\sin^2\theta}$

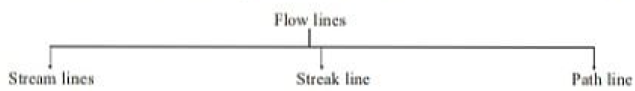
Note: We generally follow Eulerian concept, as its difficult to keep the track of a single fluid particle.

Types of fluid show:

1. **Steady and Unsteady Flow:** At any given location, the flow and fluid properties do not change with time, then its steady flow otherwise unsteady.

$$\frac{\partial v}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial f}{\partial t} = 0 \Rightarrow \text{Steady flow}$$

2. **Uniform and Non-Uniform Flow:** A flow is said to be uniform flow in which velocity & flow both in magnitude and direction do not change along the direction of flow for given instant of time.
3. **One, two or three Dimensional Flow:** If flow parameters varies in one dimension wrt space only then its one dimensional otherwise its 2 or 3 dimension respectively.
   $V = V(x, t)$ ® one dimensional
   $V = V(x, y, t)$ ® two dimensional
   $V = V(x, y, z, t)$ ® three dimensional
4. **Laminar and Turbulent Flow:** In Laminar flow, the particles moves in layers sliding smoothly over the adjacent layers while in turbulent flow particles have the random and erratic movement, intermixing in the adjacent layers. Which causes continuous momentum transfer.
   Flow of blood in veins and arteries occurs as a viscous flow. Hence, Laminar flow.
   A water supply pipe carries water at high speed leading to rapid mixing which causes highly turbulent conditions.
5. **Rotational and Irrotational Flow:** When fluid particles rotate about their mass centre during movement. Flow is said to be rotational otherwise irrotational.
   Rotational Flow ® Forced Vortex, Flow inside boundary layer.
   Irrotational Flow ® Free Vortex, Flow outside boundary layer.
   In a straight tube of uniform diameter and uniform roughness, the flow properties does not vary across the length of the pipe. Hence, Uniform flow.
   Flow above the drain having a wash basin is a free vortex motion (Irrotational flow).
6. **Compressible and Incompressible Flow:** In compressible flow density of fluid changes from time to time while in Incompressible flow it remains constant.



- **Stream Line:** There are a set of concentric circle with origin at centre.
- Stream lines neither touch nor cross each other.
- Tracing of motion of different fluid particle.

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \text{ Equation of stream line}$$

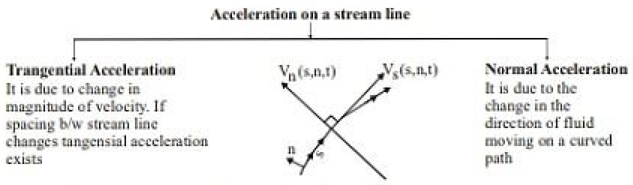
- **Streak Line:** It is line traced by series of fluid particles passing through a fixed point. It is formed by continous introduction of dye or smoke from a point in the flow.
- **Path Lines:** It is actual path traced by a fluid particle over a

period of time. It is based on lagrangian concept

- Two path lines can intersect each other.

Total Acceleration = Convective acceleration with respect to space + local acceleration with respect to time.

Type of flow	Convective Acceleration	Temporal Acceleration
Steady & uniform	0	0
Steady & non-uniform	Exists	0
Unsteady & uniform	0	Exists
Unsteady & non-uniform	Exists	Exists

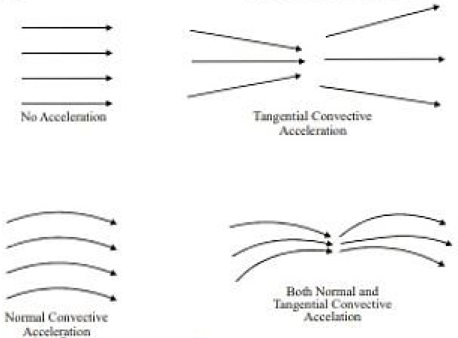


- **Acceleration Of A Fluid Particle**

$$a_x = \underbrace{\frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y} + \frac{w\partial u}{\partial z}}_{\text{Convective acceleration}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{Temporal acceleration}}$$

$$a_s = V_s\frac{\partial V_s}{\partial s} + \frac{\partial V_s}{\partial t} \qquad a_n = V_s\frac{\partial V_n}{\partial s} + \frac{\partial V_s}{\partial t}$$

convective tangential acceleration, local tangential acceleration; convective normal acceleration, local normal acceleration



- **Rotational Component**

$$w_x = \frac{1}{2}\left(\frac{\partial w}{dy} - \frac{\partial v}{dz}\right), \quad w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right), \quad w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

Special points:

1.Velocity potential exists only for ideal and irrotational flow.

2.Velocity of flow is in direction of decreasing potential function.

3.Equipotential line is the line joining points having same potential function.

- **Velocity Potential Function (f):** $-\frac{\partial \phi}{\partial x} = u$ $\quad -\frac{\partial \phi}{\partial y} = v$

- **Stream Function (v):** $u = -\frac{\partial \psi}{\partial y}$ $\quad v = \frac{\partial \psi}{\partial x}$

It is the study of motion of fluid along with the forces causing the motion.

(*i*) Newton's equation of motion

$$\vec{F_g} + \vec{F_P} + \vec{F_V} + \vec{F_t} + \vec{F_c} + \vec{F_\sigma} = m\vec{a}$$

(*ii*) Reynold's equation of motion

$$\vec{F_g} + \vec{F_P} + \vec{F_V} + \vec{F_t} = m\vec{a}$$

(*iii*) Navier-stock equation of motion

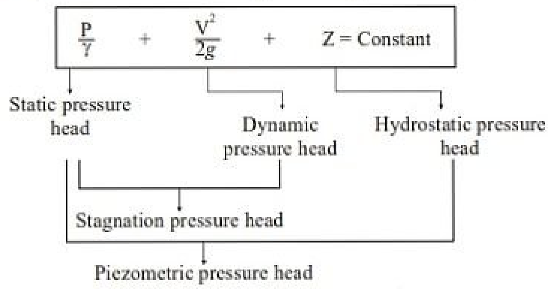
$$\vec{F_g} + \vec{F_P} + \vec{F_V} = m\vec{a}$$

(*iv*) Euler's equation of motion

$$\vec{F_g} + \vec{F_P} = m\vec{a}$$

Special points:

- Energy equation can be used to find the pressure at a point in a pipeline using Bernoulli's eq.
- Continuity eq. is used to find out the flow at two sections of tapering pipes.
- Euler equation based on momentum conservation while Bernoulli is based on energy conservation.
- Impulse momentum principle is used to find out the force on a moving vane.
- Concept of moment of momentum (Angular momentum principle is used in lawn sprinkler problems)



- **Euler's Equation:** $\frac{dp}{p} + gd_z + vdv = 0$

- **Bernoullies Equation:** $\frac{P}{w} + z + \frac{v^2}{2g} =$ constant.

- **Rotameter** is used to measure discharge while current meter is used to measure velocity in open channel.

- **Hot Wire Anemometer:** Used for measurement of Instantaneous velocity and temperature at a point in flow.

- **Theoretical Discharge:**

$$Q_{th} = \frac{A_1 A_2\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \qquad C_d = \frac{qA_{ct}}{q_{th}} = \sqrt{\frac{h - h_L}{h}}$$

- **Percentage Error In Discharge:**

$$\%\ \text{error} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 \qquad \%\ \text{error} = (1 - C_d) \times 100$$

NOTE.

When Pressure Difference Measured by Manometer

When heavier fluid in manometer & lighter fluid in pipe.

$$h = x\left(\frac{g_h}{g_l} - 1\right)$$

g_h ® Specific gravity of heavier liquid
g_l ® Specific gravity of lighter liquid
x ® Reading Manometer
h ® Reading Piezometer.

- **Orificemeter:**

$$\theta = \frac{Cd_0\, A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \quad C_C = \frac{A_C}{A_0} \quad C_d = C_c \times C_v$$

Where
C_c ® Coefficient of Contraction.
C_d ® Coefficient of Discharge
C_V ® Coefficient of Velocity.

- **Pitot Tube –Velocity Of Flow:** $\frac{P_1}{w} + \frac{V_1^2}{2g} = \text{Constant}$
- **Reynold's Number:** $R_e = \frac{\rho v d}{\mu}$

Nature of flow according to Reynold's number (R_e)

	Laminar	Transition	Turbulent
Flow in pipe	$R_e < 2000$	$2000 < R_e < 4000$	$R_e > 4000$
Flow between parallel plate	$R_e < 1000$	$1000 < R_e < 2000$	$R_e > 2000$
Flow in open channel	$R_e < 500$	$500 < R_e < 2000$	$R_e > 2000$
Flow through soil	$R_e < 1$	$1 < R_e < 2$	$R_e > 2$

- **Laminar Flow Through Circular Pipe:** $\tau = -\frac{r}{2}\left(\frac{dp}{dx}\right)$
- **Velocity Distribution:**

$$U_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2 \quad U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

- **Discharge:** $Q = \frac{\pi U_{max} R^2}{2}$ $\quad Q = \frac{\pi}{128\mu}\left(-\frac{\partial P}{\partial x}\right)D^4$
- **Friction Factor:** $F = 4f$ $\quad f = \frac{16}{R_e}$ $\quad f = \frac{8\tau_0}{\rho u^2}$
- **Trapezoidal Notch:**

$$Q = \frac{2}{3}C_{d_1}\sqrt{2g}LH^{3/2} + \frac{8}{15}C_{d_2}\sqrt{2g}\tan\frac{\theta}{2}H^{5/2}$$

- **Entrance length** in a pipe is the length where boundary layer increases and flow is fully developed.
  For Laminar Flow $L = 0.07\, R_e D$
  For Turbulent Flow $L_e = 50\, D$

Note:

- **Hele Show flow:** Laminar flow between parallel plates
- **Stoke's Law:** Settling of fine particles.
- **Hagen Poiseuille flow:** Laminar flow in Tubes/pipes.
- **Major Losses   Head/Loses**

$$h_L = \frac{fLQ^2}{12D^5} \quad h_f = \frac{fLv^2}{2gD}$$

Number	Equation	Uses
Reynolds No.	$\frac{F_i}{F_v} = \frac{\rho VL}{\mu}$	Aeroplanes, submarines, pipe flow
Eulers No.	$\sqrt{\frac{F_i}{F_p}} = \frac{V}{\sqrt{\frac{p}{\rho}}}$	Cavitation problem
Mach No.	$\sqrt{\frac{F_i}{F_e}} = \frac{V}{C}$	Aerodynamic testing, rocket, missile
Froude No.	$\sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{gL}}$	OCF, spillway, weir
Weber No.	$\sqrt{\frac{F_i}{F_s}} = \frac{v}{\sqrt{\sigma/\rho L}}$	Veins, arteries, rising bubble

Water hammer Pressure: Rapid/Sudden closure of valve in a pipe carying flowing liquid destroys the momentum of flowing liquid and sets up a high pressure wave. This pressure wave travels with the speed of sound and causes hammering action in pipe called Knocking water hammer.

- Surge tanks are used to absorb the Increase in the pressure due to water hammer phenomenon.

Chezy's Formula: $V = C\sqrt{RS}$, Manning equation $V = \frac{1}{n}R^{2/3}S_0^{1/2}$

Dimension of $C = L^{1/2}\,T^{-1}$, $n = L^{-1/3}\,T^{1}$, f = Dimensionless

Open-channel Flow						
Steady				unsteady		
Uniform Canal Flow	Gradually Varied (GVF)	Rapidly Varied (RVF)	Spatially Varied (SVF)	Gradually Varied (GVUF)	Rapidly Varied (RVUF)	Spatially Varied (SVUF)
Flow in river U/S of a weir during winter		Flow D/S of an overflow spillway.	Flow over side weir	River Flow in alluvial reach during rising flood	A surge moving upstream	Surface runoff due to rainfall

Type of flow	Depth of flow	Velocity of flow	Froude No	Comments
Subcritical	$y > y_c$	$v < v_c$	$F_r < 1$	Also called as streaming or transquil flow
Critical	$y = y_c$	$v = v_c$	$F_r = 1$	
Super Critical	$y < y_c$	$v > v_c$	$F_r > 1$	Shooting flow, rapid flow, torrential flow

Dynamic eq. for G.V.F.: $\frac{dy}{dx} = \left(\frac{S_o - S_f}{1 - \frac{q^2}{gy^3}}\right)$

Hydraulic Jump Eq.

1. $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$
2. Energy Loss $E_L = \frac{(y_2 - y_1)^3}{4y_1 y_2}$

3. $\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right)$ 4. $y_c^3 = \frac{y_1 y_2 (y_1 + y_2)}{2}$

Types of Jump	Fr	E_L/E_1	Water surface
Undular	1-1.7	» 0	Undulating
Weak	1.7-2.5	5–18%	Small rollers form
Oscillating	2.5-4.5	18–45%	Water oscillates in random manner
Steady	4.5-9	45–70%	Roller and jump action
strong	^{3} 9	^{3} 70%	Very rough and choppy

$$N_S = \frac{N\sqrt{P}}{(H)^{5/4}} \text{ (for Turbine)}, \quad N_S = \frac{N\sqrt{Q}}{(H_m)^{3/4}} \text{ (for Pump)}$$

Classificationaccording to energy available at input

	Impulse turbine	Reaction Turbine
1.	Input energy is only kinetic energy	1. Input energy is kinetic energy + pressure energy
2.	Pressure remains constant throughout the working & which is equal to atmospheric.	2. Pressure drop takes place.
3.	Useful for high head & low discharge	3. Useful for low head & high discharge
4.	Degree of reaction is zero.	4. Degree of reaction not zero.
5.	No draft tabe	5. Draft tube is present.
6.	Example Pelton wheel	6. Example (i) Francis Turbine (ii) kaplan & propeller Turbine.

Turbine						
Name	Type	Type of Energy	N_s (MKS)	Head	Discharge	Direction of flow
Pelton wheel turbine (Single jet)	Impulse	Kinetic	10-35	High (250 to 1000m)	Low(Q<1000 LPM)	Tangential
Peloton wheel turbine (multiple jet)	Impulse	Kinetic	35-60	High (250 to 1000 m)	Low	Tangential flow runner
Francis Turbine	Reaction	Kinetic + Pressure	60-300	Medium (60 to 150 m)	Medium (1000-10000) LPM	Inward Radial Mixed flow (Modern Francis)
Kaplan & Propeller turbine	Reaction	Kinetic + Pressure	300-1000	Low (< 30 m)	High(Q>10000LPM)	Axial flow