DESIGN OF MACHINE ELEMENTS
[AS PER CHOICE BASED CREDIT SYSTEM (CBCS) SCHEME]
SEMESTER – V

Subject Code			18ME52	
Teaching Hours / Week			IA Marks	40
Lecture	Tutorial	Practical	Exam Marks	60
03	02	00	Exam Hours	03
CREDITS – 04				

Course Objectives
1. Able to understand mechanical design procedure, materials, codes and use of standards
2. Able to design machine components for static, impact and fatigue strength.
3. Able to design fasteners, shafts, joints, couplings, keys, threaded fasteners riveted joints, welded joints and power screws.

Course outcomes
On completion of the course the student will be able to
1. Describe the design process, choose materials. Apply the codes and standards in design process.
2. Analyze the behaviour of machine components under static, impact, fatigue loading using failure theories.
3. Design shafts, joints, couplings.
4. Design of riveted and welded joints.
5. Design of threaded fasteners and power screws

Module-1

Fundamentals of Mechanical Engineering Design
Mechanical engineering design, Phases of design process, Design considerations, Engineering Materials and their Mechanical properties, Standards and Codes, Factor of safety, Material selection.

Static Stresses: Static loads .Normal, Bending, Shear and Combined stresses.Stress concentration and determination of stress concentration factor. **10 Hours**

Module -2
Design for Impact and Fatigue Loads
Impact stress due to Axial, Bending and Torsional loads.
Fatigue failure: Endurance limit, S-N Diagram, Low cycle fatigue, High cycle fatigue, modifying factors: size effect, surface effect. Stress concentration effects, Notch sensitivity, fluctuating stresses, Goodman and Soderberg relationship, stresses due to combined loading, cumulative fatigue damage.
1**0 Hours**

Module -3

Design of Shafts, Joints, Couplings and Keys
Torsion of shafts, design for strength and rigidity with steady loading, ASME codes for power

Design of Cotter and Knuckle joints, Rigid and flexible couplings, Flange coupling, Bush and Pin type coupling and Oldham's coupling. Design of keys-square, saddle, flat and feather.

10 Hours

Module - 4

Riveted Joints and Weld Joints

Rivet types, rivet materials, failures of riveted joints, Joint Efficiency, Boiler Joints, Lozanze Joints, Riveted Brackets, eccentrically loaded joints.

Types of welded joints, Strength of butt and fillet welds, welded brackets with transverse and parallel fillet welds, eccentrically loaded welded joints. **10 Hours**

Module - 5

Threaded Fasteners and Power Screws

Stresses in threaded fasteners, Effect of initial tension, Design of threaded fasteners under static loads, Design of eccentrically loaded bolted joints.

Types of power screws, efficiency and self-locking, Design of power screw, Design of screw jack: (Complete Design). **10 Hours**

Text Books:

1. Design of Machine Elements, V.B. Bhandari, Tata McGraw Hill Publishing Company Ltd., New Delhi, 2nd Edition 2007.
2. Mechanical Engineering Design, Joseph E Shigley and Charles R. Mischke. McGraw Hill International edition, 6th Edition, 2009.

Design Data Handbook:

1. Design Data Hand Book, K. Lingaiah, McGraw Hill, 2nd Ed.
2. Data Hand Book, K. Mahadevan and Balaveera Reddy, CBS Publication
3. Design Data Hand Book, S C Pilli and H. G. Patil, I. K. International Publisher, 2010.

Reference Books:

1. Machine Design, Robert L. Norton, Pearson Education Asia, 2001.
2. Engineering Design, George E. Dieter, Linda C Schmidt, McGraw Hill Education, Indian Edition, 2013.
3. Design of Machined Elements, S C Pilli and H. G. Patil, I. K. International Publisher, 2017.
4. Machine Design, Hall, Holowenko, Laughlin (Schaum's Outline series) adapted by S.K Somani, tata McGraw Hill Publishing company Ltd., New Delhi, Special Indian Edition, 2008

CONTENTS

Module 1

INTRODUCTION

Instructional Objectives

- Explain what is design?
- Describe the machine and its designer,
- Illustrate the procedure of design,
- Know materials used in mechanical design, and
- Understand the considerations for manufacturing.

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

If the end product of the engineering design can be termed as mechanical then this may be termed as Mechanical Engineering Design. Mechanical Engineering Design may be defined as: **―Mechanical Engineering Design is defined as iterative decision making process to describe a machine or mechanical system to perform specific function with maximum economy and efficiency by using scientific principles, technical information, and imagination of the designer.‖** A designer uses principles of basic engineering sciences, such as Physics, Mathematics, Statics, Dynamics, Thermal Sciences, Heat Transfer, Vibration etc.

Classifications of Machine Design:

The machine design may be classified as follows:

1. ***Adaptive design.*** In most cases, the designer‟s work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. *Development design.* This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. *New design.* This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified asfollows:

(a) *Rational design.* This type of design depends upon mathematical formulae of principle of mechanics.

(b) *Empirical design.* This type of design depends upon empirical formulae based on the practice and past experience.

(c) *Industrial design.* This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) *Optimum design.* It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) *System design.* It is the design of any complex mechanical system like a motor car.

(f) *Element design.* It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) *Computer aided design.* This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

1. *Type of load and stresses caused by the load.* The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

2. *Motion of the parts or kinematics of the machine.* The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be : (*a*) Rectilinear motion which includes unidirectional and reciprocating motions. (*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic. (*c*) Constant velocity. (*d*) Constant or variable acceleration.

3. *Selection of materials.* It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. *Form and size of the parts.* The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. *Frictional resistance and lubrication.* There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. *Convenient and economical features.* In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. *Use of standard parts.* The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. *Safety of operation.* Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. *Workshop facilities.* A design engineer should be familiar with the limitations of this employer‟s workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. *Number of machines to be manufactured.* The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Manufacturing considerations in Machine design

Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer.

The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary shaping processes.*** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes.*** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes.*** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. ***Joining processes.*** The processes used for joining machine components are known as joining processes. The common op e r a t i o ns u s ed for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties*.** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

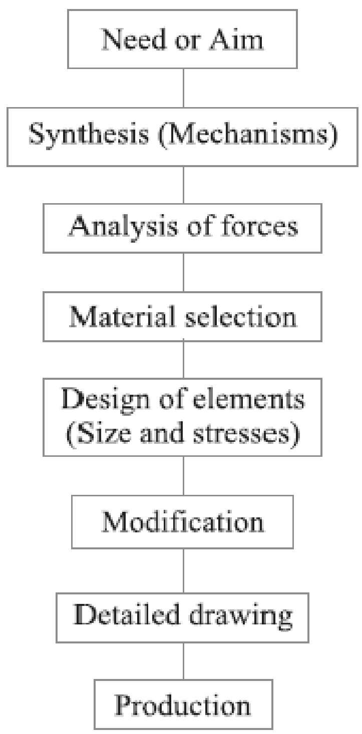


Fig.1. General Machine Design Procedure

1. *Recognition of need.* First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. *Synthesis* (*Mechanisms*). Select the possible mechanism or group of mechanisms which will give the desired motion.

3. *Analysis of forces.* Find the forces acting on each member of the machine and the energy transmitted by each member.

4. *Material selection.* Select the material best suited for each member of the machine.

5. ***Design of elements* (*Size and Stresses*).** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. ***Modification.*** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. ***Detailed drawing.*** Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. ***Production.*** The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design is shown in Fig.1.

Standards and Standardization

Standards in Design:

Standard is a set of specifications, defined by a certain body or an organization, to which various characteristics of a component, a system, or a product should conform. The characteristics may include: dimensions, shapes, tolerances, surface finish etc.

Types of Standards Used In Machine Design:

Based on the defining bodies or organization, the standards used in the machine design can be divided into following three categories:

(i) Company Standards: These standards are defined or set by a company or a group of companies for their use.

(ii) National Standards: These standards are defined or set by a national apex body and are normally followed throughout the country. Like BIS, AWS.

(iii) International Standards: These standards are defined or set by international apex body and are normally followed throughout the world. Like ISO, IBWM.

Advantages:

- Reducing duplication of effort or overlap and combining resources
- Bridging of technology gaps and transferring technology
- Reducing conflict in regulations
- Facilitating commerce
- Stabilizing existing markets and allowing development of new markets
- Protecting from litigation

B.I.S DESIGNATIONS OF THE PLAIN CARBON STEEL:

Plain carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.
2. Followed by letter "C"
3. Followed by digits indicates 10 times the average percentage content of Manganese "Mn".

B.I.S DESIGNATIONS OF ALLOY STEEL:

Alloy carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.

2. Followed by the chemical symbol of chief alloying element.

3. Followed by the rounded off the average percentage content of alloying element as per international standards.

4. Followed by the chemical symbol of alloying elements followed by their average percentage content rounded off as per international standards in the descending order.

5. If the average percentage content of any alloying element is less than 1%, it should be written with the digits up to two decimal places and underlined.

Engineering materials and their properties:

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(*a*) Ferrous metals and (*b*) Non-ferrous metals.

The ****ferrous metals*** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

Stress

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress*.** It is denoted by a Greek letter sigma (ζ). Mathematically,

$$\text{Stress}, \zeta = P/A$$

Where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress *i.e*. megapascal (MPa) and gigapascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

$$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (ε). Mathematically,

Strain, $\varepsilon = \delta l / l$ or $\delta l = \varepsilon . l$

Where δl = Change in length of the body,

l = Original length of the body.

Tensile Stress and Strain

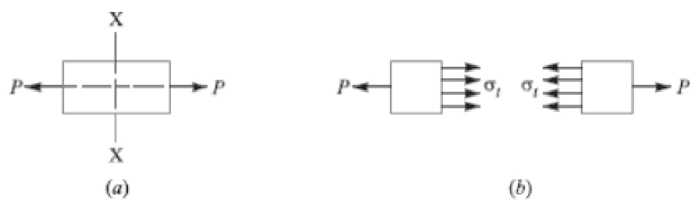


Fig. Tensile stress and strain

When a body is subjected to two equal and opposite axial pulls *P* (also called tensile load) as shown in Fig. (*a*), then the stress induced at any section of the body is known as ***tensile stress*** as shown in Fig. (*b*). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as ***tensile strain.***

Let P = Axial tensile force acting on the body,
A = Cross-sectional area of the body,
l = Original length, and δl= Increase in length.

Then, Tensile stress, $\zeta_t = P/A$ and tensile strain, $\varepsilon_t = \delta l / l$

Young's Modulus or Modulus of Elasticity

Hooke's law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain, *i.e* . $\sigma \propto \varepsilon$ or $\sigma = E.\varepsilon$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where *E* is a constant of proportionality known as ***Young's modulus*** or ***modulus of elasticity.*** In S.I. units, it is usually expressed in GPa *i.e*. GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (*E*) for the materials commonly used in engineering practice.

Values of „E" for the commonly used engineering materials.

Material	Modulus of elasticity (E) GPa
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Stress-Strain Curves

Properties are quantitative measure of materials behavior and mechanical properties pertain to material behaviors under load. The load itself can be **static** or **dynamic**. A gradually applied load is regarded as static. Load applied by a universal testing machine upon a specimen is closet example of gradually applied load and the results of tension test from such machines are the basis of defining mechanical properties. The dynamic load is not a gradually applied load – then how is it applied. Let us consider a load *P* acting at the center of a beam, which is simply supported at its ends. The reader will feel happy to find the stress (its maximum value) or deflection or both by using a formula from Strength of Materials. But remember that when the formula was derived certain assumptions were made. One of them was that the load *P* is gradually applied. Such load means that whole of *P* does not act on the beam at a time but applied in instalments. The instalment may be, say *P*/100 and thus after the 100th instalment is applied the load *P* will be said to be acting on the beam. If the whole of *P* is placed upon the beam, then it comes under the category of the dynamic load, often referred to as **Suddenly Applied Load**. If the load *P* falls from a height then it is a **shock load**. A fatigue load is one which changes with time. Static and dynamic loads can remain unchanged with time after first application or may alter with time (increase or reduce) in which case, they are fatigue load. A load which remains constantly applied over a long time is called creep load.

All Strength of Material formulae are derived for static loads. Fortunately the stress caused by a suddenly applied load or shock load can be correlated with the stress caused by gradually applied load. We will invoke such relationships as and when needed. Like stress formulae, the mechanical properties are also defined and determined under gradually applied loads because such determination is easy to control and hence economic. The properties so determined are influenced by sample geometry and size, shape and surface condition, testing machines and even operator. So the properties are likely to vary from one machine to another and from one laboratory to another. However, the static properties carry much less influence as compared to dynamic (particularly fatigue) properties. The designer must be fully aware of such influences because most machines are under dynamic loading and static loading may only be a dream.

It is imperative at this stage to distinguish between **elastic constants** and mechanical properties. The elastic constants are dependent upon type of material and not upon the sample. However, strain rate (or rate of loading) and temperature may affect elastic constants. The materials used in machines are basically **isotropic** (or so assumed) for which two independent elastic constants exist whereas three constants are often used in correlating stress and strains. The three constants are Modules of Elasticity (*E*), Modulus of Rigidity (*G*) and Poisson"s Ratio (*v*). Any one constant can be expressed in terms of other two.

An isotropic material will have same value of *E* and *G* in all direction but a natural material like wood may have different values of *E* and *G* along fibres and transverse to fibre. Wood is non-isotropic. Most commonly used materials like iron, steel, copper and its alloys, aluminum and its alloys are very closely isotropic while wood and plastic are non-isotropic. The strength of material formulae are derived for isotropic materials only.

The leading mechanical properties used in design are ultimate tensile strength, yield strength, percent elongation, hardness, impact strength and fatigue strength. Before we begin to define them, we will find that considering tension test is the most appropriate beginning.

Tension Test

The tension test is commonest of all tests. It is used to determine many mechanical properties. A cylindrical machined specimen is rigidly held in two jaws of universal testing machine. One jaw is part of a fixed cross-head, while other joins to the part of moving cross-head. The moving cross-heads moves slowly, applying a gradually applied load upon the specimen.

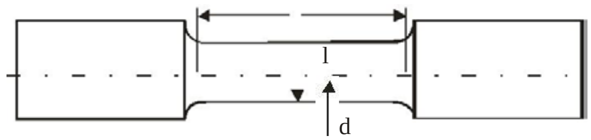


Figure 1.1 : Tension Test Specimen

The specimen is shown in Figure 1.1. The diameter of the specimen bears constant ratio with the gauge length which is shown is Figure 1.1 as distance between two gauge points marked at the ends of uniform diameter length. In a standard specimen □ $\lambda = 5$. The diameter, *d*, and gauge length, *l*, are measured before the specimen is placed in the machine. As the axial force increases upon the specimen, its length increases, almost imperceptibly in the beginning. But if loading continues the length begins to increase perceptibly and at certain point reduction in diameter becomes visible, followed by great reduction in diameter in the local region of the length. In this localized region the two parts of the specimen appear to be separating as the machine continues to operate but the load upon the specimen begins to reduce. Finally at some lesser load the specimen breaks, with a sound, into two pieces. However, the increase in length and reduction of load may not be seen in all the materials. Specimens of some materials show too much of extension and some show too little. The reader must be conversant with the elastic deformation, which is recoverable and plastic deformation, which is irrecoverable. Both type of deformations occur during the test. The appearance of visible decrease in the diameter in the short portion of length (called necking) occurs when the load on the specimen is highest. The machines of this type have arrangement (devices) for the measurement of axial force, *P*, and increase in length, δ. The values of force, *P* and extensions, δ can be plotted on a graph. Many machines have *x*-*y* recorder attached and direct output of graph is obtained. The stress is denoted by σ and calculated as P/A where, *A* is the original area of cross-section. Although the area of cross-section of specimen begins to change as the deformations goes plastic, this reduction is seen at and after the maximum load. The separation or fracture into two pieces can be seen to have occurred on smaller diameter. Yet, the stress all through the test, from beginning to end, is represented by $\sigma = P/A$. The strain is defined as the ratio of change in length at any load *P* and original length *l* and represented by ε, i.e. $\varepsilon = \delta/l$ at all loads. Since *A* and *l* are constants hence nature of graph between *P* and δ (load-extension) or between σ and ε (stress-strain) will be same. Figure 1.2 shows a stress-strain diagram, typically for a material, which has extended much before fracture occurred.

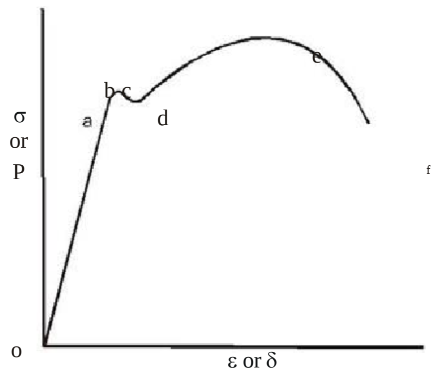


Figure 1.2: Typical σ – ε Diagram

At first we simply observe what this diagram shows. In this diagram *o* is the starting point and *oa* is straight line. Along line *oa*, stress (σ) is directly proportional to strain (ε). Point *b* indicates the elastic limit, which means that if specimen is unloaded from any point between *o* and *b* (both inclusive) the unloading curve will truly retrace the loading curve. Behaviour of specimen material from point *b* to *c* is not elastic. In many materials all three points of *a, b* and *c* may coincide. At *c* the specimen shows deformation without any increase in load (or stress). In some materials (notably mild or low carbon steel) the load (or stress) may reduce perceptibly at *c*, followed by considerable deformation at the reduced constant stress. This will be shown in following section. However, in most materials *cd* may be a small (or very small) region and then stress starts increasing as if the material has gained strength. Of course the curve is more inclined toward ε axis. This increase in stress from *d* to *e* is due to strain hardening. Also note again that *ob* is elastic deformation zone and beyond *b* the deformation is elastic and plastic – meaning that it is part recoverable and part irrecoverable. As the deformation increases plastic deformation increases while elastic deformation remains constant equal to that at *b*. If the specimen is unloaded from any point in the plastic deformation region the unloading curve will be parallel to elastic deformation curve as shown in Figure 1.3.

e

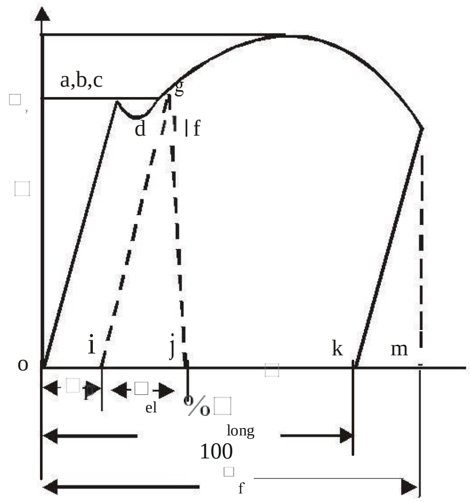


Figure 1.3 : σ – ε Diagram for a Ductile Material

Percent Elongation

From any point *g* the unloading will be along *gi* where *gi* is parallel to *oa*. *oi* is the strain which remains in the specimen or the specimen is permanently elongated by $l\ \varepsilon_p$. The total strain at *g* when the specimen is loaded is $oj = \varepsilon_p + \varepsilon_{el}$ where ε_{el} is recoverable part. At fracture, i.e. at point *f*, if one is able to control and unload the specimen just before fracture, the unloading will follow *f k*. The strain *ok* is an important property because deformation is defined as percent elongation. Hence, *ok* = % elongation/100. Percent elongation is important property and is often measured by placing two broken pieces together and measuring the distance between the gauge points. You can easily see that after the fracture has occurred, the specimen is no more under load, hence elastic deformation (which is equal to km) is completely recovered. However, in a so-called ductile material *km* << *om*. If the distance between gauge points measured on two broken halves placed together is l_f, then

$$\%\ \text{Elongation} = \frac{l_f - l}{l} \times 100$$

The gauge length has pronounced effect on % elongation. Since the major amount of deformation occurs locally, i.e. over very small length smaller gauge length will result in higher % elongation. After l/d > 5 the % elongation becomes independent of gauge length. % elongation is an indication of very important property of the material called **ductility**. The ductility is defined as the property by virtue of which a material can be drawn into wires which means length can be increased and diameter can be reduced without fracture. However, a ductile material deforms plastically before it fails. The property opposite to ductility is called **brittleness**. A brittle material does not show enough plastic deformation. Brittle materials are weak under tensile stress, though they are stronger than most ductile materials in compression

Ultimate Tensile Strength, Yield Strength and Proof Stress

The maximum stress reached in a tension test is defined as **ultimate tensile strength**. As shown in Figure 1.3 the highest stress is at point *e* and ultimate tensile stress (UTS) is represented by σ_u. Some authors represent it by S_u. The point *c* marks the beginning while *d* marks the end of yielding. *c* is called upper yield point while *d* is called the lower yield point. The stress corresponding to lower yield point is defined as the **yield strength**. For the purposes of machines, the part has practically failed if stress reaches yield strength, (σ_Y), for this marks the beginning of the plastic deformation. Plastic deformation in machine parts is not permissible. Hence one may be inclined to treat σ_Y as failure criterion. We will further discuss this later in the unit.

It is unfortunate to note that many practical materials show $\sigma - \varepsilon$ diagrams which do not have such well defined yielding as in Figures 1.2 and 1.3. Instead they show a continuous transition from elastic to plastic deformation. In such cases yield strength (σ_Y) becomes difficult to determine. For this reason an alternative, called **proof stress**, is defined which is a stress corresponding to certain predefined strain. The proof stress is denoted by σ_p. A $\sigma - \varepsilon$ diagram for a material, which shows no distinct yield is shown in Figure 1.5. The proof stress is determined corresponding to proof strain ε_p which is often called offset. By laying ε_p on strain axis to obtain a point *q* on ε axis and drawing a line parallel to elastic line to cut the $\sigma - \varepsilon$ curve at *p* the proof stress σ_p is defined. Then σ_p is measured on stress axis. The values of proof strain or offset have been standardized for different materials by American Society for Testing and Materials (*ASTM*). For example, offset for aluminum alloys is 0.2%, same is for steels while it is 0.05% for cast iron (CI) and 0.35% for brass and bronze

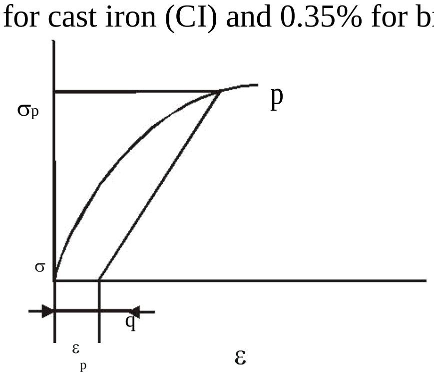


Figure 1.5: Proof Stress (σ_p) Corresponding to Offset ε_p

Toughness and Resilience

Since the force, which pulls the tension test specimen, causes movement of its point of application, the work is done. This work is stored in the specimen and can be measured as energy stored in the specimen. It can be measured as area under the curve between load (P) and elongation (Δl). In case of $\sigma - \varepsilon$ curve area under the curve represents energy per unit volume.

Toughness is regarded as ability of a material to absorb strain energy during elastic and plastic deformation. The resilience is same capacity within elastic range. The maximum toughness will apparently be at fracture, which is the area under entire $\sigma - \varepsilon$ diagram. This energy is called **modulus of toughness**. Likewise the maximum energy absorbed in the specimen within elastic limit is called **modulus of resilience**. This is the energy absorbed in the tension specimen when the deformation has reached point *a* in Figure 1.2. But since in most materials the proportional limit, elastic limit (points *a* and *b* in Figures 1.2 and 1.3) seem to coincide with yield stress as shows in Figure 1.3, the modules of resilience is the area of triangle as shown in Figure 1.6.

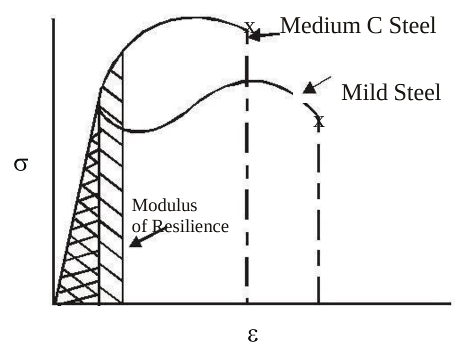


Figure 1.6 : Resilience and Toughness for Two Materials

It can be seen that modulus of resilience is greater for medium carbon steel than for mild steel, whereas modulus of toughness of two materials may be closely same. Medium carbon steel apparently has higher UTS and YS but smaller percent elongation with respect to mild steel. High modulus of resilience is preferred for such machine parts, which are required to store energy. Springs are good example. Hence, springs are made in high yield strength materials.

Stress Strain Diagram for Mild Steel

Mild steel as steel classification is no more a popular term. It was in earlier days that group of steel used for structural purposes was called mild steel. Its carbon content is low and a larger group of steel, named low carbon steel, is now used for the same purposes. We will read about steel classification later. Mild steel was perhaps developed first out of all steels and it was manufactured from Bessemer process by blowing out carbon from iron in a Bessemer converter. It was made from pig iron. The interesting point to note is that this steel was first studied through $\sigma - \varepsilon$ diagram and most properties were studied with respect to this material.

The term **yield strength** (YS) is frequently used whereas yield behavior is not detectable in most steel varieties used today. It is mild steel, which very clearly shows yield behavior and upper and lower, yield points. Figure 1.7 shows a typical $\sigma - \varepsilon$ diagram for mild steel.

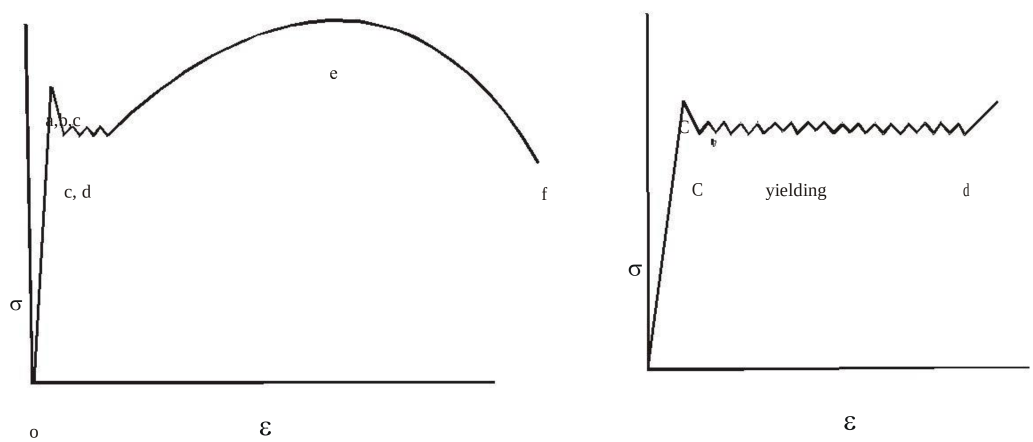


Figure 1.7 : $\sigma - \varepsilon$ Diagram for Mild Steel

The proportional limit, elastic limit and upper yield point almost coincide. *d* is lower yield point and deformation from *c′* to *d* is at almost constant stress level. There is perceptible drop in stress from *c* to *c′*. The deformation from *c′* to *d* is almost 10 times the deformation upto *c*. It can be seen effectively if strain is plotted on larger scale, as shown on right hand side in Figure 1.7, in which the ε scale has been doubled.

The mechanism of yielding is well understood and it is attributed to line defects, dislocations.

The UTS normally increases with increasing strain rate and decreases with increasing temperature. Similar trend is shown by yield strength, particularly in low carbon steel.

Compression Strength

Compression test is often performed upon materials. The compression test on ductile material reveals little as no failure is obtained. Brittle material in compression shows specific fracture failure, failing along a plane making an angel greater than 45^o with horizontal plane on which compressive load is applied. The load at which fracture occurs divided by area of *X*-section is called compressive strength. For brittle material the stress-strain curves are similar in tension and compression and for such brittle materials as CI and concrete modulus of elasticity in compression are slightly higher than that in tension.

Torsional Shear Strength

Another important test performed on steel and CI is *torsion test*. In this test one end of specimen is rigidly held while twisting moment or torque is applied at the other end. The result of test is plotted as a curve between torque (*T*) and angle of twist or angular displacement θ. The test terminates at fracture. The $T - \theta$ curves of a ductile material is very much similar to load extension or $\sigma - \varepsilon$ curve of tensile test except that the torque does not reduce after attaining a maximum value but fracture occurs at maximum torque. It is because of the fact that there is no reduction in the sectional area of the specimen during the plastic deformation. The elastic limit in this case can be found as the point where straight line terminates and strain hardening begins, marked as point *b* in Figure 1.8. Mild steel will show a marked yielding while other ductile materials show a smooth transition from elastic to plastic deformation. The plastic deformation zone in torsion is much larger than in torsion because the plastic deformation beginning from outer surface and spreads inside while in tension the stress being uniform over the *X*-section the plastic deformation spreads over entire section at the same time.

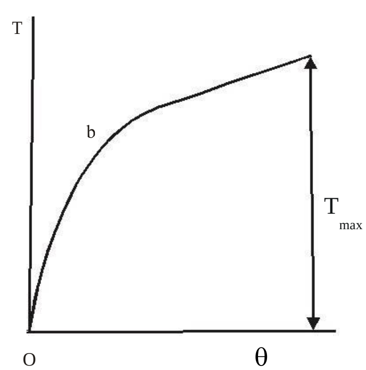


Figure 1.8 : Torque-twist Diagram in Torsion

The *modulus of rapture or ultimate torsional shear strength* is calculated form

$$\tau_u = \frac{3}{4}\frac{T_{max}}{J}\frac{d}{2}$$

where T_{max} is maximum torque, *j* is polar moment of inertia of the specimen section of diameter *d*. From the *T* diagram the slope of linear region can be found as proportional to modulus of rigidity, which is ratio of shearing stress to shearing strain.

Elastic Constants

Within elastic limit the stress is directly proportional to strain. This is the statement of Hooke‟s law and is true for direct (tensile or compressive) stress and strain as well as for shearing (including torsional shearing) stress and strain. The ratio of direct stress to direct strain is defined as *modulus of elasticity* (*E*) and the ratio of shearing stress and shearing strain is defined as *modulus of rigidity* (*G*). Both the modulus is called elastic constants. For isotropic material *E* and *G* are related with Poisson‟s ratio

$$G = \frac{E}{2\,(1 \;\; v)}$$

Poisson‟s ratio which is the ratio of transverse to longitudinal strains (only magnitude) in tensile test specimen is yet another elastic constant. If stress σ acts in three directions at a point it is called volumetric stress and produces volumetric strain. The ratio of volumetric stress to volumetric strain according to Hooke‟s law is a constant, called *bulk modulus* and denoted by *K*. It is important to remember that out of four elastic constants, for an isotropic material only two are independent and other two are dependent. Thus *K* can also be expressed as function of any two constants.

$$K = \frac{E}{3\,(1 - 2v)}$$

It may be understood that elastic constants *E* and *G* are not determined from tension or torsion test because the machines for these tests undergo adjustment of clearance and also some deformation, which is reflected in diagram ordinarily. The constants are determined from such devices, which show large deformation for comparatively smaller load. For example, *E* is determined by measuring deflection of a beam under a central load and *G* is determined by measuring deflection of a close-coiled helical spring an axial load. Poisson‟s ratio is normally not measured directly but is calculated from above equation.

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called ***shear stress.***

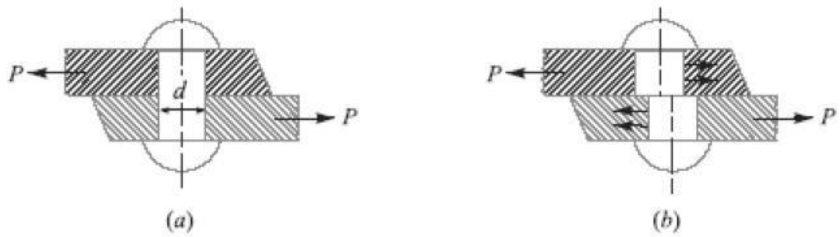


Fig. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (φ) respectively. Mathematically,

$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown in Fig. (*a*). In this case, the tangential force *P* tends to shear off the rivet at one cross-section as shown in Fig. (*b*). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in ***single shear.*** In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

And shear stress on the rivet cross-section

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi d^2}$$

Now let us consider two plates connected by the two cover plates as shown in Fig. *(a)*. In this case, the tangential force *P* tends to shear off the rivet at two cross-sections as shown in Fig.

rivet (or when the shearing takes place at Two cross-sections of the rivet), then the rivets are said to be in *double shear*. In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} \times d^2 \qquad \text{(For double shear)}$$

and shear stress on the rivet cross-section.

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

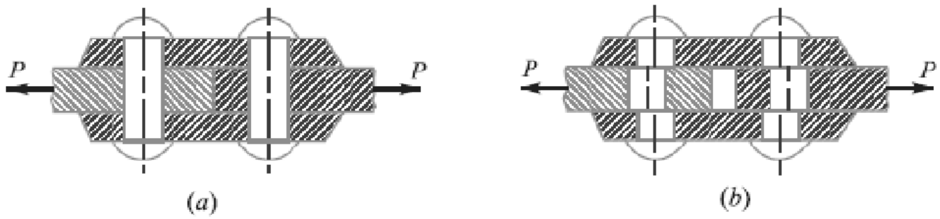


Fig. Double shearing of a riveted joint.

Notes:

1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter '*d*' is to be punched in a metal plate of thickness '*t*', then the area to be sheared,

$$A = \pi d \times t$$

And the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi\, d \times t \times \tau_u$$

Where σ_u= Ultimate shear strength of the material of the plate.

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \,.\, \phi \quad \text{or} \quad \tau / \phi = C$$

Where, η= Shear stress,

φ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by *N* or *G*.

The following table shows the values of modulus of rigidity (*C*) for the materials in everyday use:

Values of C for the commonly used materials

Material	*Modulus of rigidity (C) GPa*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Linear and Lateral Strain

Consider a circular bar of diameter *d* and length *l*, subjected to a tensile force *P* as shown in Fig. (*a*).

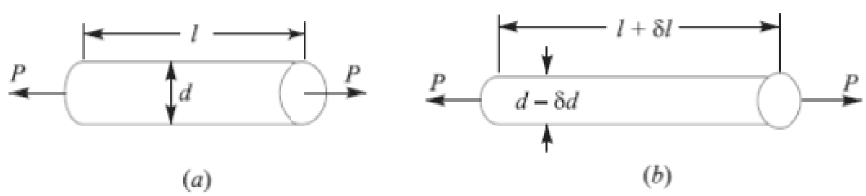


Fig. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δ*l* and the diameter decreases by an amount δ*d*, as shown in Fig. (*b*). similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{LateralStrain}}{\text{LinearStrain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Values of Poisson"s ratio for commonly used materials

S.No.	*Material*	*Poisson 's ratio (1/m or μ)*
1	**Steel** Cast	0.25 **to** 0.33
2	iron Copper	0.23 **to** 0.27
3	Brass	0.31 to 0.34
4	Aluminium	0.32 **to** 0.42
5	Concrete	0.32 to 0.36
6	Rubber	0.08 to 0.18

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

Where δV = Change in volume, and V = Original volume

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$. Where, ε_x, ε_y and ε_z are the strains in the directions *x*-axis, *y*-axis and *z*-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by *K*. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

Relation between Young's Modulus and Modulus of Rigidity

The Young's modulus (*E*) and modulus of rigidity (*G*) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as principal planes and the direct stresses along these planes are known as principal stresses. The planes on which the maximum shear known as planes of maximum shear.

Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (i.e. direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body ABCD of uniform cross-sectional area and unit thickness subjected to normal stresses ζ1 and ζ2 as shown in Fig. (a). In addition to these normal stresses, a shear stress η also acts. It has been shown in books on „Strength of Materials" that the normal stress across any oblique section such as EF inclined at an angle θ with the direction of ζ2, as shown in Fig. (a), is given by

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad ...(i)$$

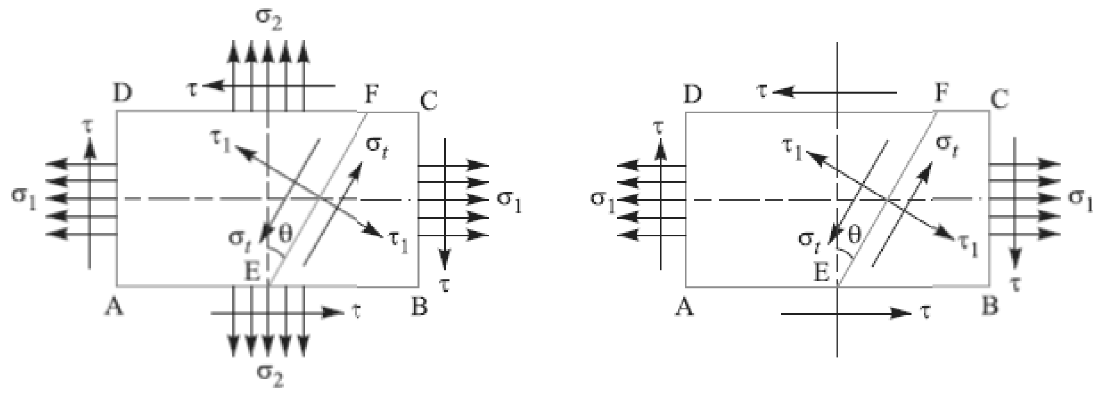
And tangential stress (i.e. shear stress) across the section EF,

Since the planes of maximum and minimum normal stress (i.e. principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating η1 = 0 in the above equation (ii), i.e.

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta \qquad ...(ii)$$

$$\frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} \qquad ...(iii)$$



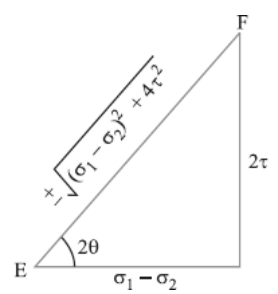
(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

Fig. Principal stresses for a member subjected to bi-axial stress

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section. From the following Fig., we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$



$$\therefore \qquad \sin 2\theta_1 = + \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{and} \qquad \sin 2\theta_2 = - \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{Also} \qquad \cos 2\theta = \pm \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\therefore \qquad \cos 2\theta_1 = + \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\text{and} \qquad \cos 2\theta_2 = - \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation (i).

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

So, Maximum principal (or normal) stress, and minimum principal (or noral) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by one-half the algebraic difference between the principal stresses, i.e.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

Notes: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress, then the principal stresses are obtained by substituting ζ2 = 0 in above equations.

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

2. In the above expression of ζt2, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$ is more than ζ1/2

Therefore the nature of ζt2 will be opposite to that of ζt1, i.e. if ζt1 is tensile then ζt2 will be compressive and vice-versa.

Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

Where, ζt = Tensile stress due to direct load and bending,

ζc = Compressive stress, and

η = Shear stress due to torsion.

Notes: 1. When η = 0 as in the case of thin cylindrical shell subjected in pressure, then ζtmax = ζt.

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (ζb). This will give the resultant tensile stress or compressive stress (ζt or ζc) depending upon the type of axial load (i.e. pull or push).

Problems:

1. A shaft, as shown in Fig., is subjected to a bending load of 3 kN, pure torque of 1000Nm and an axial pulling force of 15 kN stresses. Calculate the stresses at A and B.

Solution. Given : $W = 3$ kN $= 3000$ N ; $T = 1000$ N-m $= 1 \times 10^6$ N-mm ; $P = 15$ kN $= 15 \times 10^3$ N ; $d = 50$ mm; $x = 250$ mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2$$

∴ Tensile stress due to axial pulling at points A and B,

$$\sigma_o - \frac{P}{A} - \frac{15 \times 10^3}{1964} - 7.64 \text{ N/mm}^2 - 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

∴ Bending stress at points A and B,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point A and compressive at point B.

$\therefore$ Resultant tensile stress at point *A*,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$
$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point *B*,

$$\sigma_B - \sigma_b \quad \sigma_o - 61.1 \quad 7.64 - 53.46 \text{ MPa}$$

We know that the shear stress at points *A* and *B* due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 1 \times 10^6}{\pi\,(50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$
$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$
$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile)}$$ **Ans.**

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -18.93 \text{ MPa}$$
$$= 18.93 \text{ MPa (compressive)}$$ **Ans.**

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$
$$= 53.3 \text{ MPa}$$ **Ans.**

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$
$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$
$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive)}$$ **Ans.**

Minimum principal (or normal) stress at point *B*,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$
$$= 26.73 - 48.73 = -22 \text{ MPa}$$
$$= 22 \text{ MPa (tensile)}$$ **Ans.**

and maximum shear stress at point *B*,

$$\tau_{B(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$
$$= 48.73 \text{ MPa}$$ **Ans.**

Module 2

DESIGN FOR STATIC AND IMPACT STRENGTH

Instructional Objectives

- *Types of loading on machine elements and allowable stresses.*
- *Concept of yielding and fracture.*
- *Different theories of failure.*
- *Construction of yield surfaces for failure theories.*
- *Optimize a design comparing different failure theories*

2.1 Introduction

Machine parts fail when the stresses induced by external forces exceed their strength. The external loads cause internal stresses in the elements and the component size depends on the stresses developed. Stresses developed in a link subjected to uniaxial loading are shown in figure. Loading may be due to:

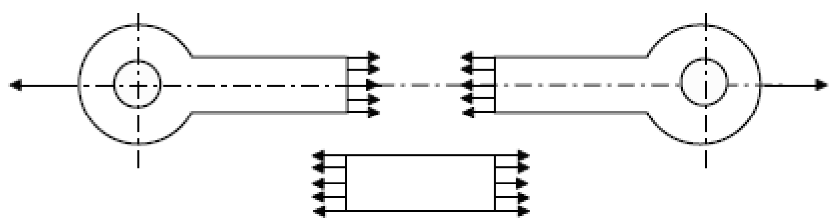
a) The energy transmitted by a machine element.

b) Dead weight.

c) Inertial forces.

d) Thermal loading.

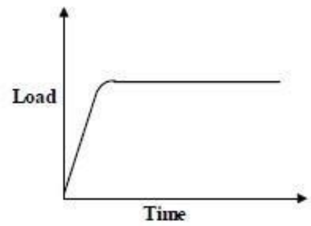
e) Frictional forces.



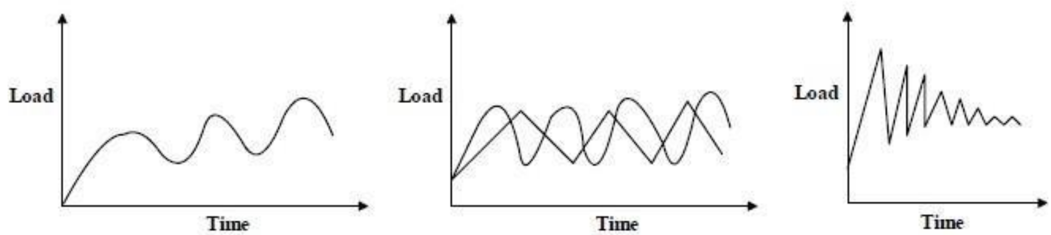
2.2 Load may be classified as:

a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.

b) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge. Load may change in direction, for example, load on piston rod of a double acting cylinder. Vibration and shock are types of dynamic loading. **Figure** shows load v/s time characteristics for both static and dynamic loading of machine elements.



Static Loading



Dynamic Loading

2.3 Factor of Safety

Determination of stresses in structural or machine components would be meaningless unless they are compared with the material strength. If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe and an indication about the size of the component is obtained. The strength of various materials for engineering applications is determined in the laboratory with standard specimens. For example, for tension and compression tests a round rod of specified dimension is used in a tensile test machine where load is applied until fracture occurs. This test is usually carried out in a **Universal testing machine**. The load at which the specimen finally ruptures is known as Ultimate load and the ratio of load to original cross-sectional area is the Ultimate tress.

Similar tests are carried out for bending, shear and torsion and the results for different materials are available in handbooks. For design purpose an allowable stress is used in place of the critical stress to take into account the uncertainties including the following:

1) Uncertainty in loading.

2) In-homogeneity of materials.

3) Various material behaviors. e.g. corrosion, plastic flow, creep.

4) Residual stresses due to different manufacturing process.

5) Fluctuating load (fatigue loading): Experimental results and plot- ultimate strength depends on number of cycles.

6) Safety and reliability.

For ductile materials, the yield strength and for brittle materials the ultimate strength are taken as the critical stress. An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e.

$$\textbf{FOS} = \frac{\textbf{Ultimate stress}}{\textbf{Allowable stress}}$$

The ratio must always be greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

Factor of safety = Maximum stress/ Working or design stress In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases, Factor of safety = Yield point stress/ Working or design stress In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress. Factor of safety = Ultimate stress/ Working or design stress.

2.4 Static Strength

Ideally, in designing any machine element, the engineer should have available the results of a great many strength tests of the particular material chosen. These tests should be made on specimens having the same heat treatment, surface finish, and size as the element the engineer proposes to design; and the tests should be made under exactly the same loading conditions as the part will experience in service. This means that if the part is to experience a bending load, it should be tested with a bending load. If it is to be subjected to combined bending and torsion, it should be tested under combined bending and torsion. If it is made of heat-treated

AISI 1040 steel drawn at 500◦C with a ground finish, the specimens tested should be of the same material prepared in the same manner. Such tests will provide very useful and precise information. Whenever such data are available for design purposes, the engineer can be assured of doing the best possible job of engineering.

The cost of gathering such extensive data prior to design is justified if failure of the part may endanger human life or if the part is manufactured in sufficiently large quantities. Refrigerators and other appliances, for example, have very good reliabilities because the parts are made in such large quantities that they can be thoroughly tested in advance of manufacture. The cost of making these tests is very low when it is divided by the total number of parts manufactured.

You can now appreciate the following four design categories:

1. Failure of the part would endanger human life, or the part is made in extremely large quantities; consequently, an elaborate testing program is justified during design.

2. The part is made in large enough quantities that a moderate series of tests is feasible.

3. The part is made in such small quantities that testing is not justified at all; or the design must be completed so rapidly that there is not enough time for testing.

4 The part has already been designed, manufactured, and tested and found to be unsatisfactory. Analysis is required to understand why the part is unsatisfactory and what to do to improve it.

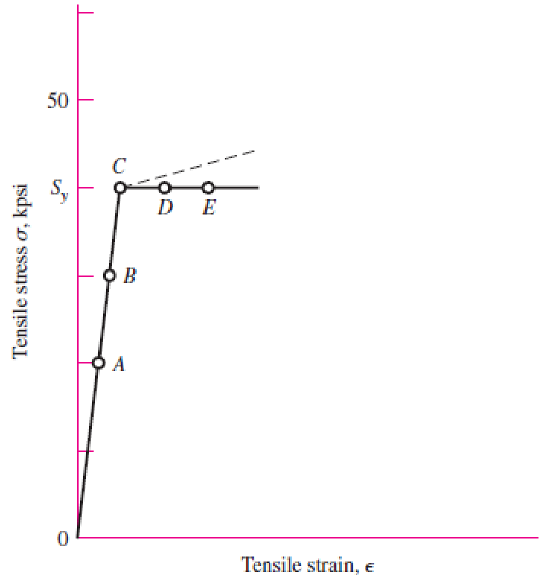
More often than not it is necessary to design using only published values of yield strength, ultimate strength, and percentage reduction in area, and percentage elongation, such as those listed in Appendix A. How can one use such meager data to design against both static and dynamic loads, two- and three-dimensional stress states, high and low temperatures, and very large and very small parts? These and similar questions will be addressed in this chapter and those to follow, but think how much better it would be tohave data available that duplicate the actual design situation.

2.5 Stress Concentration

Stress concentration is a highly localized effect. In some instances it may be due to a surface scratch. If the material is ductile and the load static, the design load may cause yielding in the critical location in the notch. This yielding can involve strain strengthening of the material and an increase in yield strength at the small critical notch location. Since the loads are static and the material is ductile, that part can carry the loads satisfactorily with no general yielding. In these cases the designer sets the geometric (theoretical) stress concentration factor *Kt* to unity.

The rationale can be expressed as follows. The worst-case scenario is that of an idealized non–strain-strengthening material shown in Fig. 5–6. The stress-strain curve rises linearly to the yield strength *Sy* , then proceeds at constant stress, which is equal to *Sy* . Consider a filleted rectangular bar as depicted in Fig. A–15–5, where the crosssection area of the small shank is 1 in2. If the material is ductile, with a yield point of 40 kpsi, and the theoretical stress-concentration factor (SCF) *Kt* is 2,

- A load of 20 kip induces a tensile stress of 20 kpsi in the shank as depicted at point *A* in Fig. 5–6. At the critical location in the fillet the stress is 40 kpsi, and the SCF is *K* = ζmax/ζnom = 40/20 = 2.
- A load of 30 kip induces a tensile stress of 30 kpsi in the shank at point *B*. The fillet stress is still 40 kpsi (point*D*), and the SCF *K* = ζmax/ζnom = *Sy*/ζ = 40/30 = 1.33.
- At a load of 40 kip the induced tensile stress (point *C*) is 40 kpsi in the shank. At the critical location in the fillet, the stress (at point *E*) is 40 kpsi. The SCF *K* = ζmax/ζnom = *Sy*/ζ = 40/40 = 1.



For materials that strain-strengthen, the critical location in the notch has a higher *Sy* . The shank area is at a stress level a little below 40 kpsi, is carrying load, and is very near its failure-by-general-yielding condition. This is the reason designers do not apply *Kt* in *static loading* of a *ductile material* loaded elastically, instead setting *Kt* = 1. When using this rule for ductile materials with static loads, be careful to assure yourself that the material is not susceptible to brittle fracture (see Sec. 5–12) in the environment of use. The usual definition of geometric (theoretical) stress-concentration factor for normal stress *Kt* and shear stress *Kts* is,

$$\zeta_{max} = K_t \zeta_{nom} \qquad (a)$$
$$\eta_{max} = K_{ts} \eta_{nom} \qquad (b)$$

Since your attention is on the stress-concentration factor, and the definition of ζnom or ηnom is given in the graph caption or from a computer program, be sure the value of nominal stress is appropriate for the section carrying the load. Brittle materials do not exhibit a plastic range. A brittle material "feels" the stress concentration factor *Kt* or *Kts* , which is applied by using Eq. (*a*) or (*b*). An exception to this rule is a brittle material that inherently contains micro-discontinuity stress concentration, worse than the macro-discontinuity that the designer has in mind. Sand molding introduces sand particles, air, and water vapor bubbles. The grain structure of cast iron contains graphite flakes (with little strength), which are literally cracks introduced during the solidification process. When a tensile test on a cast iron is performed, the strength reported in the literature *includes* this stress concentration. In such cases *Kt* or *Kts* need not be applied.

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as *impact stress.* Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig.

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

And potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W\,(h + \delta l)$$

$$\frac{1}{2}\,\sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right) \qquad \ldots\left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \qquad \frac{Al}{2E}(\sigma_i)^2 - \frac{Wl}{E}(\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hAE}{Wl}}\right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Problem:

An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm^2.

Solution. Given : h = 10 mm ; l = 3 m = 3000 mm ; A = 600 mm^2 ; δl = 2 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\therefore \quad \sigma = \frac{E . \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3 \text{ N/mm}^2 \quad \text{Ans.}$$

Value of the unknown weight

Let W = Value of the unknown weight.

We know that

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2h\,A\,E}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3\,W} = 1 + \sqrt{1 + \frac{800\,000}{W}}$$

$$\frac{80\,000}{W} - 1 = \sqrt{1 + \frac{800\,000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

$$\therefore \quad W = 6400 \times 10^2 / 96 = 6666.7 \text{ N} \quad \text{Ans.}$$

Problem:

A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m^2.

Solution. Given : $d = 50$ mm ; $l = 2.5$ m = 2500 mm ; $U = 100$ N-m = 100×10^3 N-mm ; $E = 200$ GN/m^2 = 200×10^3 N/mm^2

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

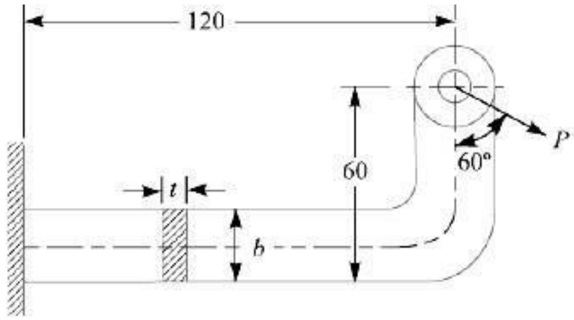
$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} (50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\,\sigma^2$$

$$\therefore \quad \sigma^2 = 100 \times 10^3 / 12.25 = 8163 \quad \text{or} \quad \sigma = 90.3 \text{ N/mm}^2 \text{ Ans.}$$

Q. A wall bracket, as shown in following figure, is subjected to a pull of $P = 5$ kN, at 60° to the vertical. The cross-section of bracket is rectangular having $b = 3t$. Determine the dimensions b and t if the stress in the material of the bracket is limited to 28 MPa.



All dimensions in mm.

Solution: Given : $P = 6000$ N ; $\theta = 45°$; $\sigma = 60$ MPa = 60 N/mm2

Let t = Thickness of the section in mm, and

b = Depth or width of the section = 3 t

Aarea of cross-section,

$A = b \times t = 3\,t \times t = 3\,t^2\ \text{mm}^2$

and section modulus,

$$Z = \frac{tb^2}{6} = \frac{3t^3}{2}$$

Horizontal component of the load,

$PH = 5000 \sin 60°$

$= 5000 \times 0.866 = 4330.13$ n

Bending moment due to horizontal component of the load,

$MH = PH \times 60 = 4330.13 \times 60 = 259807.62$ N-mm

Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bh} = \frac{MH}{z} = \frac{259807.62 \times 2}{3t^2} = \frac{173205.81}{t^2} N/mm^2$$

Vertical component of the load,

$PV = 5000 \cos 60° = 6000 \times 0.5 = 2500$ N

Direct Shear;

$$\tau = \frac{PV}{A} = \frac{2500}{3t^2} = \frac{833.33}{t^2}\ N/mm^2$$

Bending moment due to vertical component of the load,

$MV = PV \times 60 = 2500 \times 120 = 300000$ N-mm

Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bV} = \frac{MV}{z} = \frac{300000 \times 2}{3t^2} = \frac{200000}{t^2} N/mm^2$$

Direct tensile stress due to horizontal component

$$\sigma_d = \frac{PH}{A} = \frac{4330.13}{3t^2} = \frac{1443.38}{t^2} \; N/mm^2$$

Net normal stress

$$\sigma = \frac{173205.81}{t^2} + \frac{200000}{t^2} + \frac{1443.38}{t^2} = \frac{374649.1867}{t^2} N/mm^2$$

Now applying the maximum shear stress theory

$$\frac{1}{2}\sqrt{\left(\sigma^2 + 4\tau^2\right)} \le 28$$

putting the values and solving the above equation for "t"

$t = 25mm$ *and* $b = 3t = 75\ mm$

Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion***. The stress set up by torsion is known as ***torsional shear stress***. It is zero at the centroidal axis and maximum at the outer surface. Consider a shaft fixed at one end and subjected to a torque (*T*) at the other end as shown in Fig. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{-------- (i)}$$

Where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

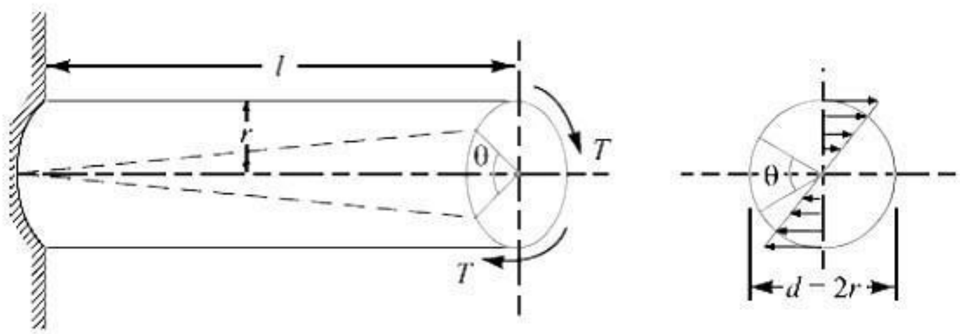
T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.



The above equation is known as ***torsion equation***. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft is uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Problem

A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\pi N \cdot T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm } \textbf{Ans.}$$

Bending Stress

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment M as shown in Fig.

The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).

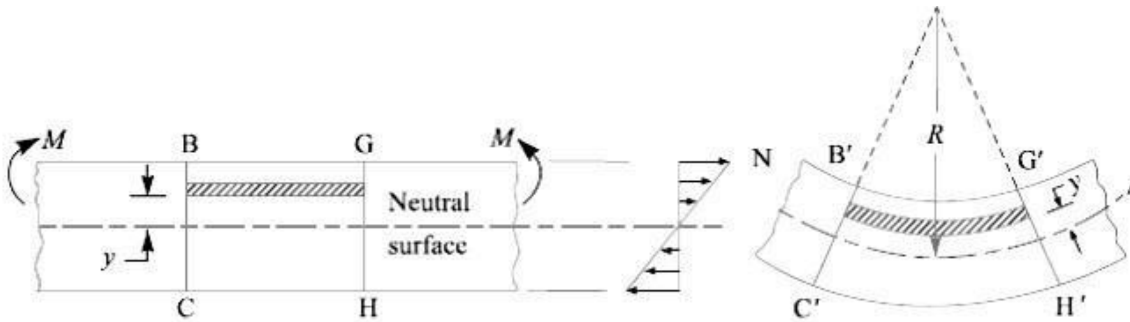
2. The material of the beam obeys Hooke's law.

3. The transverse sections (*i.e.* *BC* or *GH*) which were plane before bending remain plane after bending also.

4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

5. The Young's modulus (E) is the same in tension and compression.

6. The loads are applied in the plane of bending.



A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface***. The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis***. The stress distribution of a beam is shown in Fig. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I / y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

∴ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

∴ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Problem:

A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

Solution. Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m $= 1200$ mm or $R = 600$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\,\pi\, N\,.\,T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

$$\therefore \quad T = 10 \times 10^3/42 = 238 \text{ N-m} = 238 \times 10^3 \text{ N-mm}$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm $= 2 \times 2b = 4b$ ...(Given)

$\therefore$ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}\,(2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\, b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

$\therefore$ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

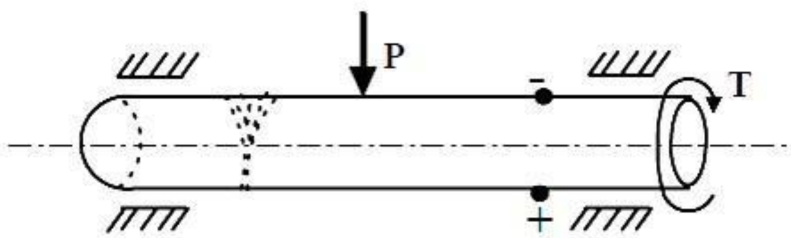
DESIGN FOR FATIGUE STRENGTH

Instructional Objectives

- *Mean and variable stresses and endurance limit.*
- *S-N plots for metals and non-metals and relation between endurance limit and ultimate tensile strength.*
- *Low cycle and high cycle fatigue with finite and infinite lives.*
- *Endurance limit modifying factors and methods of finding these factors*
- *Design of components subjected to low cycle fatigue; concept and necessary formulations.*
- *Design of components subjected to high cycle fatigue loading with finite life; concept and necessary formulations.*
- *Fatigue strength formulations; Gerber, Goodman and Soderberg equations.*

Introduction

Conditions often arise in machines and mechanisms when stresses fluctuate between a upper and a lower limit. For example in **figure-3.3.1.1**, the fiber on the surface of a rotating shaft subjected to a bending load, undergoes both tension and compression for each revolution of the shaft.



3.3.1.1F- *Stresses developed in a rotating shaft subjected to a bending load.*

Any fiber on the shaft is therefore subjected to fluctuating stresses. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too. These failures occur due to very large number of stress cycle and are known as fatigue failure. These failures usually begin with a small crack which may develop at the points of discontinuity, an existing subsurface crack or surface faults. Once a crack is developed it propagates with the increase in stress cycle finally leading to failure of the component by fracture. There are mainly two characteristics of this kind of failures:

(a) Progressive development of crack.

(b) Sudden fracture without any warning since yielding is practically absent.

Fatigue failures are influenced by

(i) Nature and magnitude of the stress cycle.

(ii) Endurance limit.

(iii) Stress concentration.

(iv) Surface characteristics.

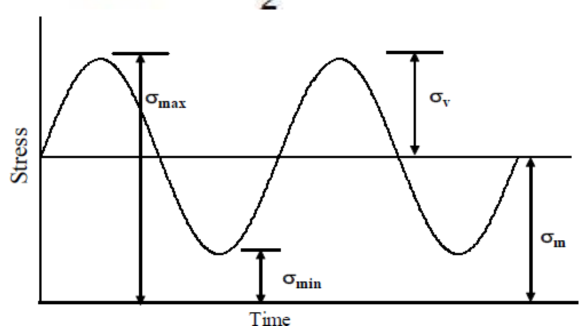
These factors are therefore interdependent. For example, by grinding and polishing, case hardening or coating a surface, the endurance limit may be improved. For machined steel endurance limit is approximately half the ultimate tensile stress. The influence of such parameters on fatigue failures will now be discussed in sequence.

Stress Cycle

A typical stress cycle is shown in **figure- 3.3.2.1** where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

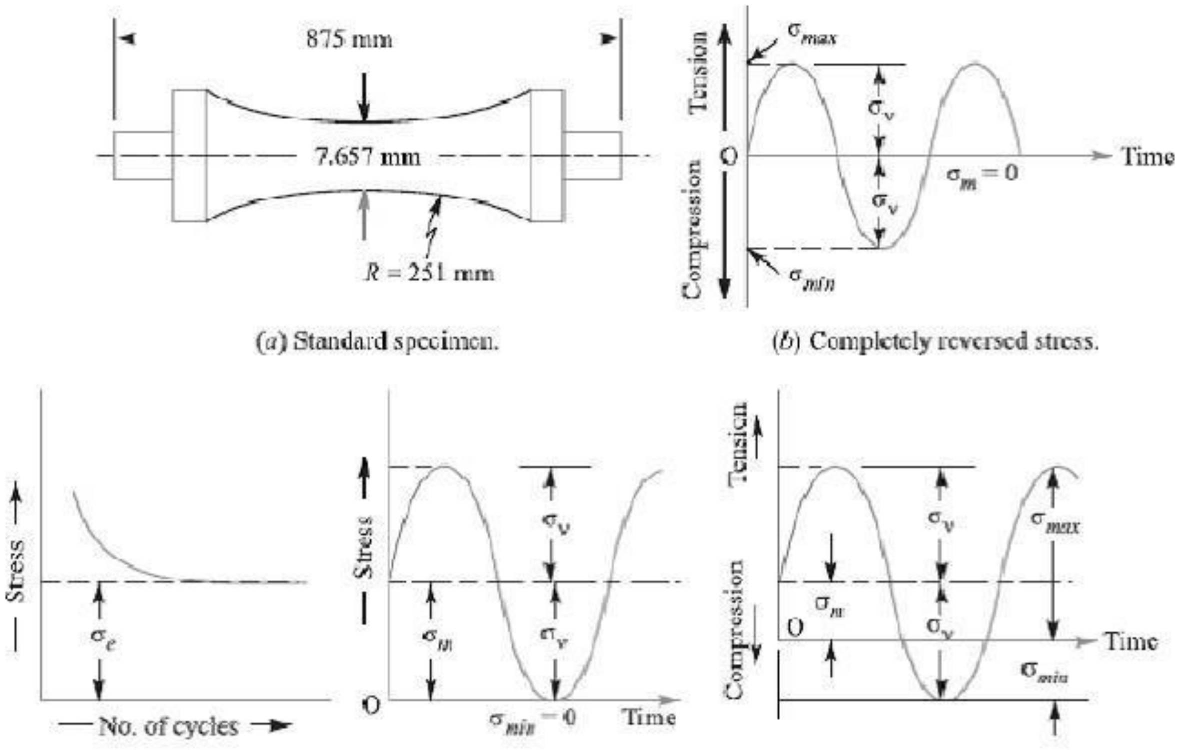
$$\sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$



3.3.2.1F- A typical stress cycle showing maximum, mean and variable stresses.

Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(a) Standard specimen. (b) Completely reversed stress. (c) Endurance or fatigue limit. (d) Repeated stress. (e) Fluctuating stress.

Fig.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig.2 (*a*), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as endurance or ***fatigue limit*** (ζe). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress verses ***time*** diagram for fluctuating stress having values ζ*min* and ζmax is shown in Fig.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component ζ*v*. The following relations are derived from Fig. 2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading, the stress varies from maximum to zero (*i.e*. ζ*min* = 0) in each cycle as shown in Fig.2 (*d*).

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, *R* = ζmax/ζmin. For completely reversed stresses, *R* = – 1 and for repeated stresses, *R* = 0. It may be noted that *R* cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (ζe) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let, Kb = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

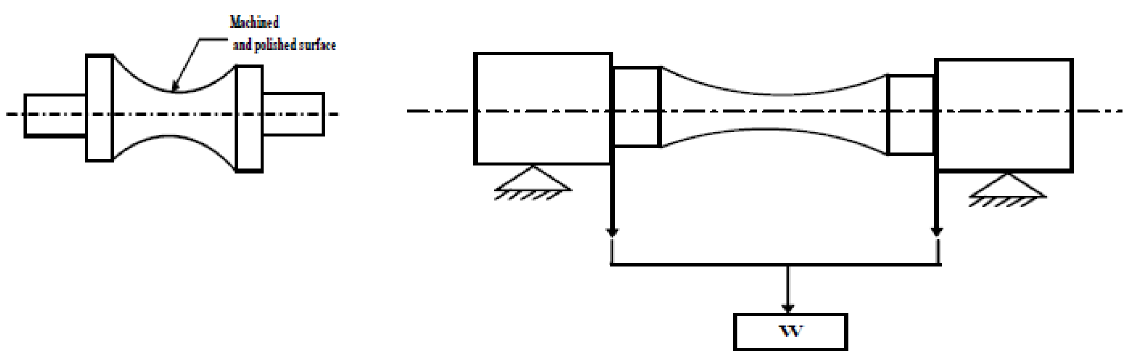
Ks = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e K_b = \sigma_e$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e K_s$

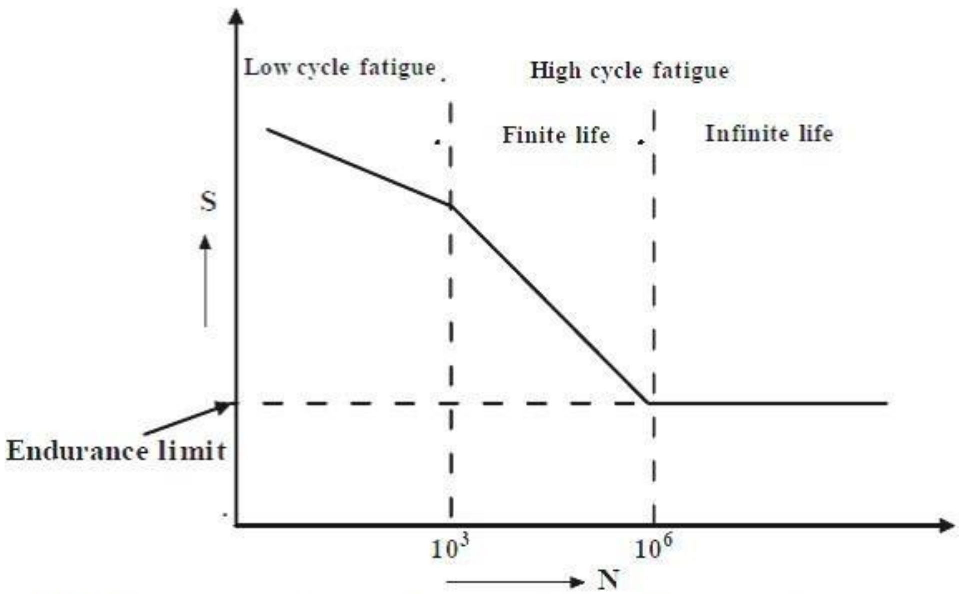
Figure- 3.3.3.1 shows the rotating beam arrangement along with the specimen.



(a) Beam specimen (b) Loading arrangement

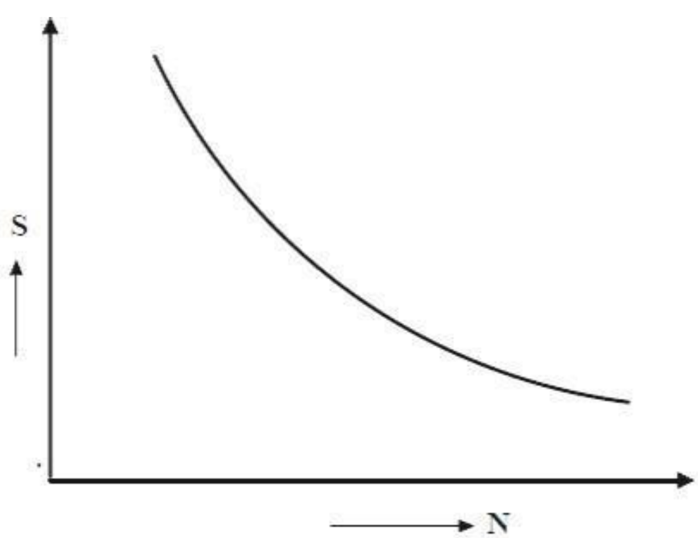
3.3.3.1F- A typical rotating beam arrangement.

The loading is such that there is a constant bending moment over the specimen length and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear since bending moment is constant over the length. Large number of tests with varying bending loads are carried out to find the number of cycles to fail. A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in **figure-3.3.3.2**. The zone below 10^3 cycles is considered as low cycle fatigue, zone between 10^3 and 10^6 cycles is high cycle fatigue with finite life and beyond 10^6 cycles, the zone is considered to be high cycle fatigue with infinite life.



3.3.3.2F- A schematic plot of reversed stress (S) against number of cycles to fail (N) for steel.

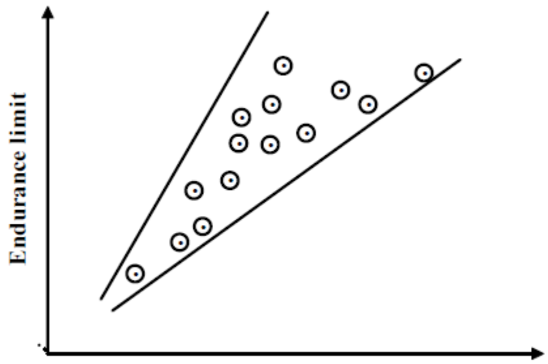
The above test is for reversed bending. Tests for reversed axial, torsional or combined stresses are also carried out. For aerospace applications and non-metals axial fatigue testing is preferred. For non-ferrous metals there is no knee in the curve as shown in **figure- 3.3.3.3** indicating that there is no specified transition from finite to infinite life.



3.3.3.3F- A schematic plot of reversed stress (S) against number of cycles to fail (N) for non-metals, showing the absence of a knee in the plot.

A schematic plot of endurance limit for different materials against the ultimate tensile strengths (UTS) is shown in **figure- 3.3.3.4**. The points lie within a narrow band and the following data is useful:

Steel	Endurance limit	~	35-60 % UTS
Cast Iron	Endurance limit	~	23-63 % UTS



3.3.3.4 F- A schematic representation of the limits of variation of endurance limit with ultimate tensile strength.

The endurance limits are obtained from standard rotating beam experiments carried out under certain specific conditions. They need be corrected using a number of factors. In general the modified endurance limit σ_e' is given by

$$\sigma_e' = \sigma_e C_1 C_2 C_3 C_4 C_5 / K_f$$

C_1 is the size factor and the values may roughly be taken as

$$C_1 = 1, \quad d \le 7.6 \text{ mm}$$
$$= 0.85, \quad 7.6 \le d \le 50 \text{ mm}$$
$$= 0.75, \quad d \ge 50 \text{ mm}$$

For large size C_1= 0.6. Then data applies mainly to cylindrical steel parts. Some authors consider 'd' to represent the section depths for non-circular parts in bending.

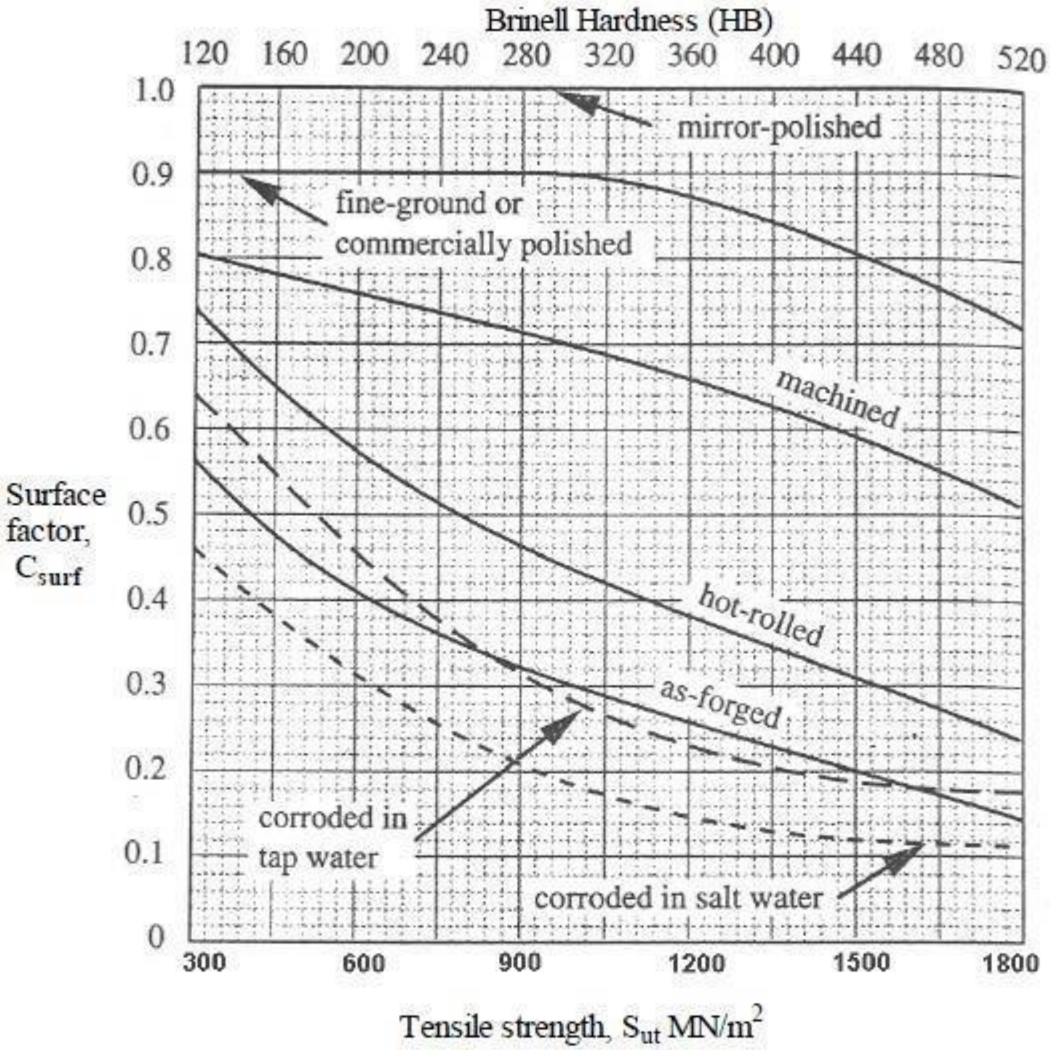
C_2 is the loading factor and the values are given as

C_2 = 1, for reversed bending load.

= 0.85, for reversed axial loading for steel parts

= 0.78, for reversed torsional loading for steel parts.

C_3 is the surface factor and since the rotating beam specimen is given a mirror polish the factor is used to suit the condition of a machine part. Since machining process rolling and forging contribute to the surface quality the plots of C_3 versus tensile strength or Brinnel hardness number for different production process, in **figure- 3.3.3.5**, is useful in selecting the value of C_3.



3.3.3.5 F- Variation of surface factor with tensile strength and Brinnel hardness for steels with different surface conditions

C_4 is the temperature factor and the values may be taken as follows:

$$C_4 = 1, \qquad \text{for } T \leq 450^\circ C.$$

$$= 1\text{-}0.0058(T\text{-}450) \qquad \text{for } 450^\circ C < T \leq 550^\circ C.$$

C_5 is the reliability factor

K_f is the fatigue stress concentration factor

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

Then, Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$
$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$
$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$
$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions:

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u).

For steel, $\sigma_e = 0.5\ \sigma_u$;
For cast steel, $\sigma_e = 0.4\ \sigma_u$;
For cast iron, $\sigma_e = 0.35\ \sigma_u$;
For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$

σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

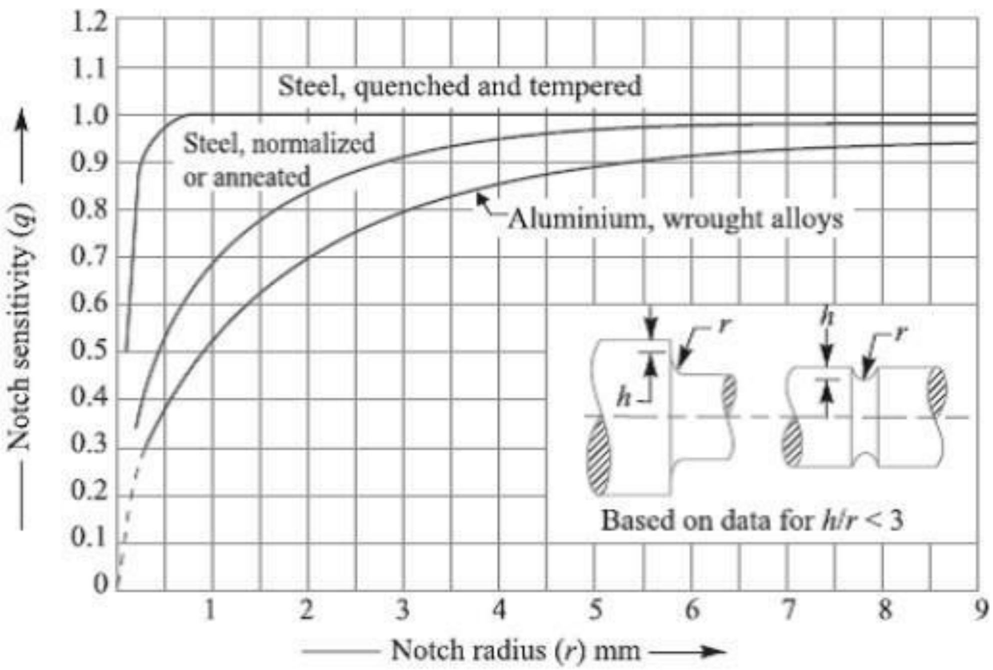
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig., may be used for determining the values of q for two steals. When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

Or



$K_f = 1 + q\,(K_t - 1)$ ...[For tensile or bending stress]

And

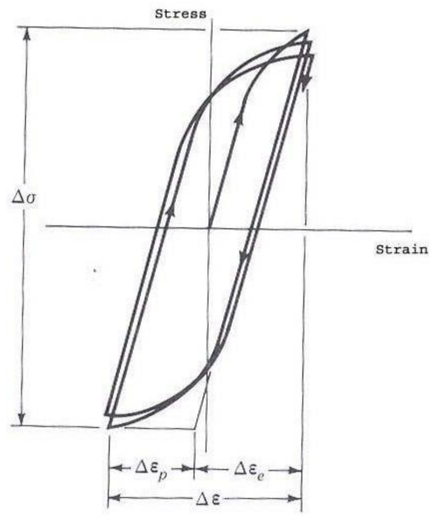
$K_{fs} = 1 + q\,(K_{ts} - 1)$ ...[For shear stress]

Where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Low Cycle Fatigue

This is mainly applicable for short-lived devices where very large overloads may occur at low cycles. Typical examples include the elements of control systems in mechanical devices. A fatigue failure mostly begins at a local discontinuity and when the stress at the discontinuity exceeds elastic limit there is plastic strain. The cyclic plastic strain is responsible for crack propagation and fracture. Experiments have been carried out with reversed loading and the true stress strain hysteresis loops are shown in **figure-3.4.1.1**. Due to cyclic strain the elastic limit increases for annealed steel and decreases for cold drawn steel. Low cycle fatigue is investigated in terms of cyclic strain. For this purpose we consider a typical plot of strain amplitude versus number of stress reversals to fail for steel as shown in **figure-3.4.1.2**.



3.4.1.1F- A typical stress-strain plot with a number of stress reversals (Ref.[4]).

Here the stress range is $\Delta\sigma$. $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$ are the plastic and elastic strain ranges, the total strain range being $\Delta\varepsilon$. Considering that the total strain amplitude can be given as

$$\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$$

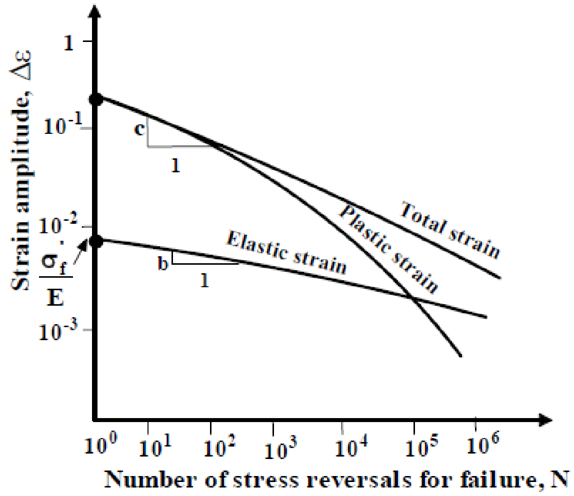
A relationship between strain and a number of stress reversals can be given as

$$\Delta\varepsilon = \frac{\sigma_f'}{E}(N)^a + \varepsilon_f'(N)^b$$

where σ_f and ε_f are the true stress and strain corresponding to fracture in one cycle and a, b are systems constants. The equations have been simplified as follows:

$$\Delta\varepsilon = \frac{3.5\sigma_u}{EN^{0.12}} + \left(\frac{\varepsilon_p}{N}\right)^{0.6}$$

In this form the equation can be readily used since σ_u, ε_p and E can be measured in a typical tensile test. However, in the presence of notches and cracks determination of total strain is difficult.



***3.4.1.2F-** Plots of strain amplitude vs number of stress reversals for failure.*

High Cycle Fatigue

This applies to most commonly used machine parts and this can be analyzed by idealizing the S-N curve for, say, steel, as shown in **figure- 3.4.2.1** .

The line between 10^3 and 10^6 cycles is taken to represent high cycle fatigue with finite life and this can be given by

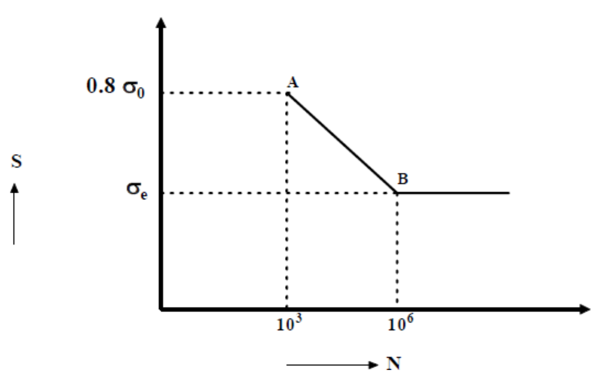
$$\log S = b \log N + c$$

where S is the reversed stress and b and c are constants.

At point A $\log(0.8\sigma_u) = b \log 10^3 + c$ where σ_u is the ultimate tensile stress

and at point B $\log \sigma_e = b \log 10^6 + c$ where σ_e is the endurance limit.

This gives $$b = -\frac{1}{3}\log\frac{0.8\sigma_u}{\sigma_e} \quad \text{and} \quad c = \log\frac{(0.8\sigma_u)^2}{\sigma_e}$$



***3.4.2.1F-** A schematic plot of reversed stress against number of cycles to fail.*

Goodman Method for Combination of Stresses:

A straight line connecting the endurance limit (σe) and the ultimate strength (σ*u*), as shown by line *AB* in figure given below follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

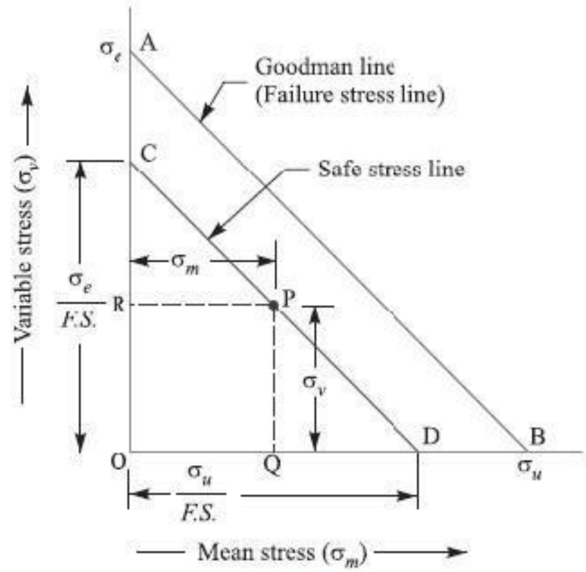


Figure 2.8

Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad \ldots(\because \; QD = OD - OQ)$$

$$\therefore \qquad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e \left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad \ldots(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads. Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (*i*) must be altered to include this effect. In such cases, the fatigue stress concentration factor (*Kf*) is used to multiply the variable stress (σv). The equation **(*i*)** may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

where $F.S.$ = Factor of safety,
σ_m = Mean stress,
σ_u = Ultimate stress,
σ_v = Variable stress,
σ_e = Endurance limit for reversed loading, and
K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation (*ii*) may be written as

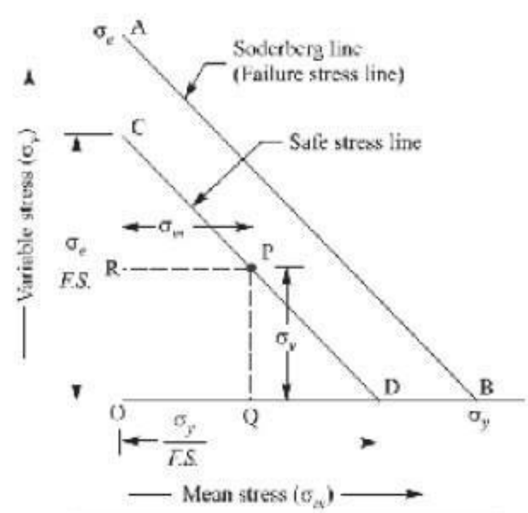
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad ...(\because \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where K_b = Load factor for reversed bending load,
K_{sur} = Surface finish factor, and
K_{sz} = Size factor.

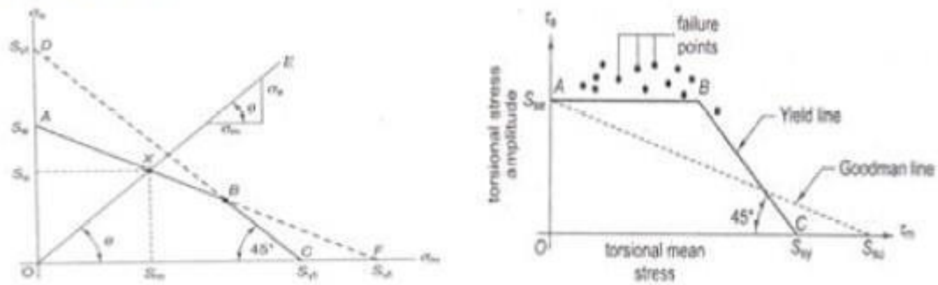
Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ*e*) and the yield strength (σ*y*), as shown by the line *AB* in following figure, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength. the line *AB* connecting σ*e* and σ*y*, as shown in following figure, is called ***Soderberg's failure stress line.*** If a suitable factor of safety (*F.S.*) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*. Now from similar triangles *COD* and *PQD*,



Modified Goodman Diagram:

In the design of components subjected to fluctuating stresses, the Goodman diagram is slightly modified to account for the yielding failure of the components, especially, at higher values of the mean stresses. The diagram known as modified Goodman diagram and is most widely used in the design of the components subjected to fluctuating stresses. There are two modified Goodman diagrams for the axial, normal or bending stresses and shear or torsion shear stresses separately as shown below. In the following diagrams the safe zones are ABCOA.



Modified Goodman Diagram for the Axial and Bending Stresses

Modified Goodman Diagram for the Torsional Shear Stresses

Figure 2.10

DESIGN APPROACH FOR FATIGUE LOADINGS

Design for Infinite Life

It has been noted that if a plot is made of the applied stress amplitude verses the number of reversals to failure to (S-N curve) the following behaviour is typically observed.

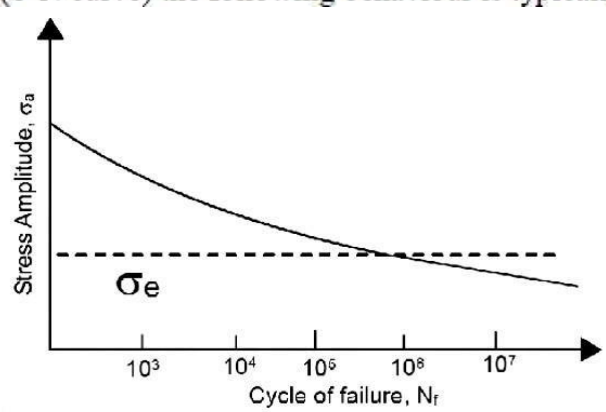


Figure 2.11

Completely Reversible Loading

If the stress is below the (the endurance limit or fatigue limit), the component has effectively infinite life. for the most steel and copper alloys. If the material does not have a well defined σ_e. Then, endurance limit is arbitrarily defined as Stress(0.35- 0.50) that gives For a known load (Moment) the section area/(modulus) will be designed such that the resulting amplitude stress will be well below the endurance limit.

Design approach can be better learnt by solving a problem.

Stress Concentration Factor

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. His irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

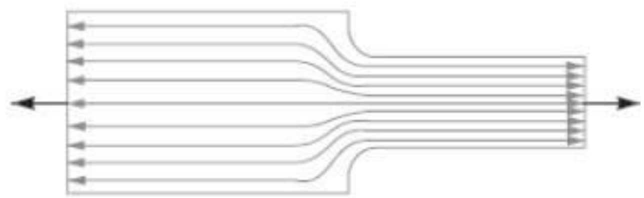


Fig. Stress concentration

<u>Theoretical or Form Stress Concentration Factor</u>

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \text{Maximum stress/ Nominal stress}$$

The value of K_t depends upon the material and geometry of the part. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig.1(a). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

And the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{r}\right)$$

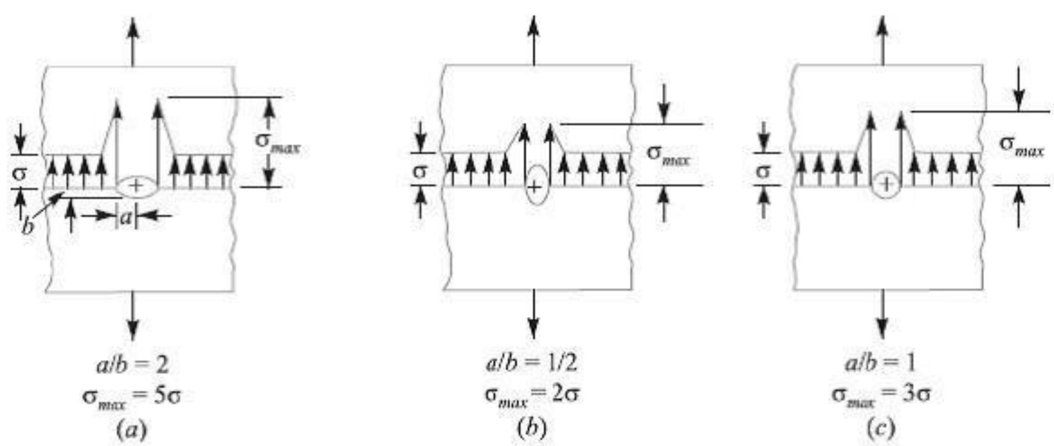


Fig.1. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 2, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

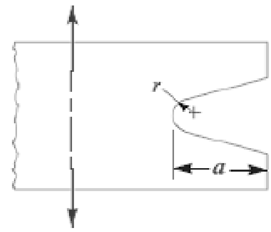


Fig.2. Stress concentration due to notches.

Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig.3. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

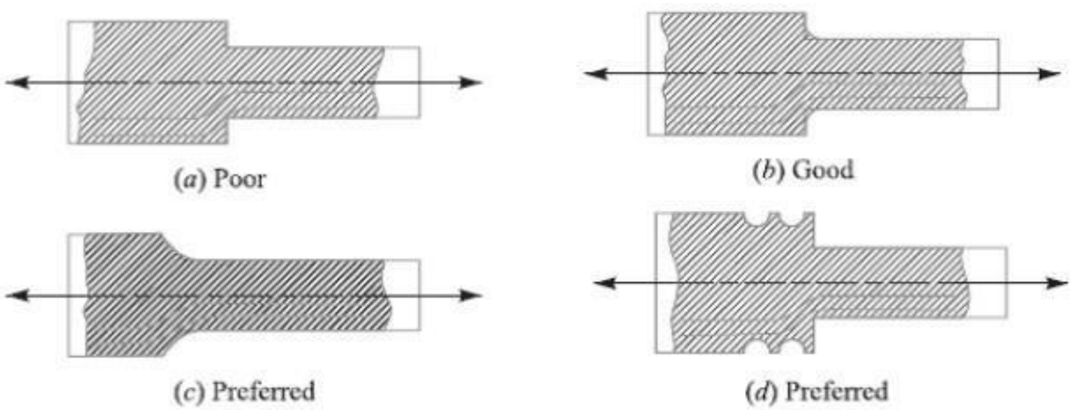


Fig.3

In Fig. 3 (a) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 3 (b) and (c) to give more equally spaced flow lines.

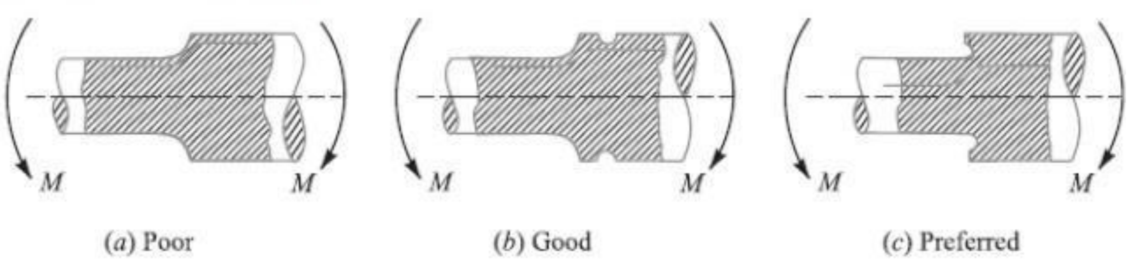
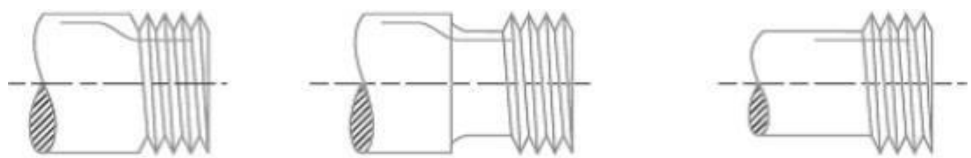


Fig. reducing stress concentration in cylindrical members with shoulders

(a) Poor (b) Preferred

Fig. Reducing stress concentration in cylindrical members with holes.



(a) Poor (b) Good (c) Preferred

Fig. Reducing stress concentration in cylindrical members with holes

Problems:

Q. A machine component is subjected to bending stress which fluctuates between 300 N/mm² tensile and 150 N/mm² compressive in cyclic manner. Using the Goodman and Soderberg criterion, calculate the minimum required ultimate tensile strength of the material. Take the factor of safety 1.5 and the endurance limit in reversed bending as 50% of ultimate tensile strength.

Solution:

Assuming the yield strength $S_{yt} = 0.55 \times$ ultimate strength S_{ut}

Mean stress " $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{300 + (-150)}{2} = 75\text{MPa};$

Amplitude stress " $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{300 - (-150)}{2} = 225\text{MPa};$

As per Goodman Relation :

$$\frac{1}{f.o.s} = \frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e}$$

As given $S_e = 0.5S_{ut}$

$$\frac{1}{1.5} = \frac{75}{S_{ut}} + \frac{225}{0.5S_{ut}} \Rightarrow \frac{1}{1.5} = \frac{525}{S_{ut}} \Rightarrow S_{ut} = 787.5\text{MPa};$$

As per Soderberg Relation :

$$\frac{1}{f.o.s} = \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e}$$

As given $S_{yt} = 0.55S_{ut}$

$$\frac{1}{1.5} = \frac{75}{0.55S_{ut}} + \frac{225}{0.5S_{ut}} \Rightarrow \frac{1}{1.5} = \frac{586.36}{S_{ut}} \Rightarrow S_{ut} = 879.545\text{MPa};$$

Q. A circular bar is subjected to a completely reversed axial load of 150 kN. Determine the size of the bar for infinite life, if it is made of plain carbon steel having ultimate tensile strength of 800 N/mm^2 and yield point in tension of 600 N/mm^2. Assuming the surface finish factor as 0.80, size factor 0.85, reliability as 90%, and modifying factor for the stress concentration as 0.9.

Solution:

GIVEN:

Maximum Axial Load " P_{max} *" = +150kN; Minimum Axial Load "* P_{min} *" = −150kN;*

Ultimate tensile strength of the material of the bar " S_{ut} *" = 800 N/mm*^{2}*;*

Yield point in tension of the material of the bar " S_{yt} *" = 600 N/mm*^{2}*;*

Surface finish factor " k_a *" = 0.80; Size factor "* k_b *" = 0.85; Reliability factor "* k_c *" = 0.90; Modifying the stress concentration*

factor " k_e *" = 0.90.*

ASSUMING:

The temperature factor $k_d = 1.0$; & miscelleneous factor $k_g = 1.0$;

Factor of safety = 1.0;

Endurance Limit of the material:

$$S_e' = 0.5 \times S_{ut} = 0.5 \times 800 = 400\text{MPa}$$

Modified Endurance Limit of the Material of the Bar:

$$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times k_g \times S_e' = 0.80 \times 0.85 \times 0.90 \times 1.0 \times 0.9 \times 1.0 \times 400 = 220.32\text{ MPa};$$

Amplitude and Mean Normal Stresses:

$$\text{Amplitude Load "}P_a\text{"} = \frac{P_{max} - P_{min}}{2} = \frac{150-(-150)}{2} = 150\text{ kN};$$

$$\text{Mean Load "}P_m\text{"} = \frac{P_{max} + P_{min}}{2} = \frac{150+(-150)}{2} = 0\text{ kN};$$

$$\text{Amplitude Stress "}\sigma_a\text{"} = \frac{4 \times P_a}{\pi \times d^2} = \frac{4 \times 150}{\pi \times d^2} = \frac{190.9859}{d^2}\text{kN/mm}^2 = \frac{190985.9}{d^2}\text{N/mm}^2;$$

$$\text{Mean Stress "}\sigma_m\text{"} = 0\ \text{N/mm}^2;$$

Using Modified Goodman Diagram:

The Load Line becomes the amplitude axis.

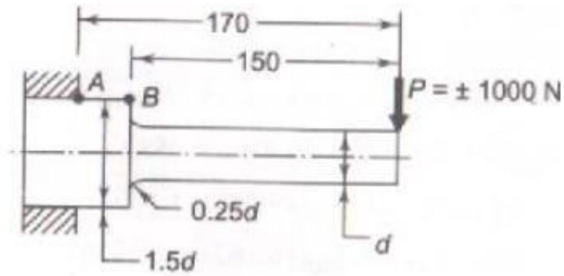
Hence the design equation may be written for infinite life as:

$$S_e \geq \sigma_a \Rightarrow 220.32 = \frac{190985.9}{d^2};$$

$$d \geq \sqrt{\frac{190985.9}{220.32}} \Rightarrow d \geq 29.44$$

$$d = 30\text{mm}.$$

(b) **A cantilever beam made of cold drawn steel 20C8 (S_{ut} =540 N/mm²) is subjected to a completely reversed load of 1000 N as shown in below figure. The corrected endurance limit for the material of the beam may be taken as 123.8 N/mm². Determine the diameter "d" of the beam for a life of 10000 cycles.**



Solution:

GIVEN:

Maximum Axial Load "P_{max}" = +1000 N;

Minimum Axial Load "P_{min}" = −1000 N;

Ultimate tensile strength of the material of the bar "S_{ut}" = 540 N/mm²;

Corrected Endurance Limit "S_e" = 123.8 N/mm²

USING THE S-N DIAGRAM:

The values of various points

$0.9S_{ut} = 0.9 \times 540 = 486\ N/mm^2;$

$\log_{10}(0.9S_{ut}) = \log_{10}(486) = 2.6866;$

$\log_{10}(S_e) = \log_{10}(123.8) = 2.0927;$

$\log_{10}(N) = \log_{10}(10000) = 4.0$

From the S-N Diagram:

$$\overline{AE} = \frac{\overline{AD} \times \overline{EF}}{\overline{DB}} = \frac{(2.6866 - 2.0927) \times (4-3)}{(6-3)} = 0.198$$

Therefor

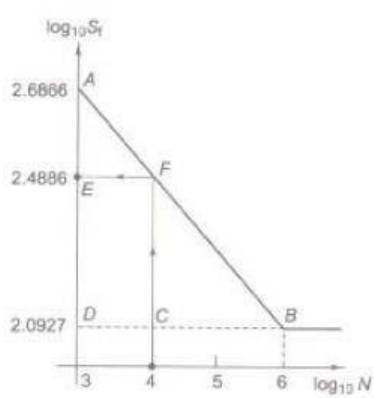
$\log_{10} S_f = 2.6866 - \overline{AE} = 2.6866 - 0.198 = 2.4886;$

$S_f = 308.03\ N/mm^2;$

And

$$S_f = \sigma_b = \frac{32M_b}{\pi d^3} \Rightarrow d^3 = \frac{32 \times M_b}{\pi \times S_f} = \frac{32 \times (1000 \times 150)}{\pi \times 308.3}$$

d = 17.05 mm.



Q. **A flat bar as shown in the figure 1 is subjected to an axial load F equal to 500 N. Assuming that the stress in the bar is limited to 200 MPa, determine the thickness of the bar. All dimensions are in mm.**

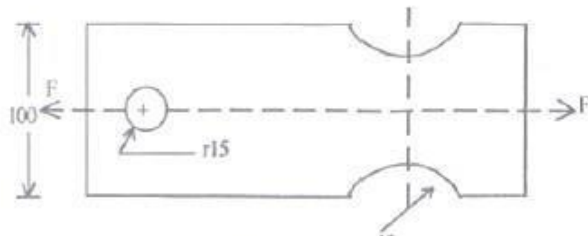


Figure 1

Solution: The stress concentration factor at circular hole k_t =2.35

The stress concentration factor at circular hole k_t =1.78

Hence the critical section is at the section of circular hole. The stress magnitude induced at this location

$$\sigma = \frac{F}{t \times (100-30)} = \frac{500}{t \times 70} = \frac{7.143}{\text{t}} N/mm^2;$$

For successful design

$$\frac{7.143}{\text{t}} \leq 200;$$
$$t \geq 0.036mm;$$

However the minimum cross section area "A" =62t mm^2;

$$\sigma = \frac{F}{62t} = \frac{500}{t \times 62} = \frac{8.06452}{\text{t}} N/mm^2;$$

For successful design

$$\frac{8.06452}{\text{t}} \leq 200;$$
$$t \geq 0.0403mm;$$

Hence the thickness of the plate = 0.0403 mm.

Q. A forged steel bar 50 mm in diameter is subjected to a reversed bending stress of 300 MPa. The bar is made of 40C8. Calculate the life of the bar for a reliability of 90%

Given: ***The material 40C8,***

The diameter of the shaft = d=50mm;

Reversed bending stress = 300MPa.

Reliability = 90%

Assuming: *σ_{ut}= 600MPa ; σ_{yt}=380 MPa Fatigue stress concentration factor=1.612*

Assuming the size factor $Ksz = 0.85$;

Surface finish factor $Ksur = 0.89$

Reliability factor K_{re}=0.892

Endurance Limit:

$$S'_e = 0.5\sigma_{ut} \qquad \because \sigma_{ut} < 1400MPa;$$
$$S'_e = 0.5 \times 600 = 300MPa;$$

Modified Endurance Limit:

$$S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S'_e$$
$$= 0.85 \times 0.89 \times 0.892 \times \frac{1}{1.612} \times 300 = 125.583\,\text{MPa}$$

Using the S-N diagram

$$\log_{10}(S_{ut}) = \log_{10}(600) = 2.778;$$
$$0.9 \times \log_{10}(S_{ut}) = 2.500$$
$$\log_{10}(S_e) = \log_{10}(125.583) = 2.099;$$
$$\log_{10}(\sigma_a) = \log_{10}(300) = 2.477;$$
$$\frac{2.50 - 2.099}{6-3} = \frac{2.477 - 2.099}{6 - \log_{10} N}$$
$$6 - \log_{10} N = 2.82793$$
$$\log_{10} N = 6 - 2.82793 = 3.17207$$
$$N = 1486.1752\,\text{reversals}$$

Q. A shaft subjected to bending moment varying from -200 N m to +500 N m and a varying torque from 50 N m to 175 N m. If material of the shaft is 30C8, stress concentration factor is 1.85, notch sensitivity is 0.95 reliability 99.9% and factor of safety is 1.5, find the diameter of the shaft.

Solution: Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500 + (-200)}{2} = 150 N - m\,;$$

Amplitude or variable bending moment

$$M_a = \frac{M_{max} - M_{min}}{2} = \frac{500 - (-200)}{2} == 350 N - m$$

Mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{175 + (50)}{2} = 112.5 N - m\,;$$

Amplitude or variable torque

$$T_a = \frac{T_{max} - T_{min}}{2} = \frac{175 - (50)}{2} == 62.5 N - m$$

Equivalent mean and amplitude bending moments

$$M_{em} = \frac{1}{2}\left[M_m + \sqrt{M_m^2 + T_m^2}\right] = \frac{1}{2}\left[150 + \sqrt{150^2 + 112.5^2}\right] = 168.75\text{N-m} = 168750\text{ N-mm};$$

$$M_{am} = \frac{1}{2}\left[M_a + \sqrt{M_a^2 + T_a^2}\right] = \frac{1}{2}\left[350 + \sqrt{350^2 + 62.5^2}\right] = 352.77\text{N-m} = 352770\text{ N-mm};$$

Mean or average bending stress,

$$\sigma_m = \frac{32M_{em}}{\pi \times d^3} = \frac{32 \times 168750}{\pi \times d^3} = \frac{1718874}{d^3};\,;$$

Amplitude or variable bending moment

$$\sigma_a = \frac{32M_a}{\pi \times d^3} = \frac{32 \times 352.77}{\pi \times d^3} = \frac{3593286}{d^3};$$

Material properties: σ_{ut}= 490MPa ; σ_{yt}=270 MPa (assumed)

Given Notch sensitivity q = 0.95 ;

Assuming the size factor *Ksz* = 0.85 ;

Surface finish factor *Ksur* = 0.89

Reliability factor K_{re}=0.75 corresponding to 99.9% reliability (Assumed)

The fatigue stress concentration factor

$$K_f = 1 + q(K_t - 1) = 1 + 0.95(1.85 - 1) = 1.8075$$

Endurance Limit:

$$S'_e = 0.5\sigma_{ut} \qquad \because \sigma_{ut} < 1400MPa;$$
$$S'_e = 0.5 \times 490 = 245MPa;$$

Modified Endurance Limit:

$$S_e = K_{sz} \times K_{sur} \times K_{re} \times \frac{1}{K_f} \times S'_e = 0.85 \times 0.89 \times 0.75 \times \frac{1}{1.8075} \times 245 = 76.91\text{MPa}$$

We know that according to Soderberg formula;

$$\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_a}{S_e};$$
$$\frac{1}{1.5} = \frac{\frac{1718874}{d^3}}{270} + \frac{\frac{3593286}{d^3}}{76.91};$$
$$\frac{d^3}{1.5} = \frac{1718874}{270} + \frac{3593286}{76.91};$$
$$d^3 = 1.5 \times [100519 + 6083178] = 79630.291$$
$$d = 43.022 \text{ mm} \approx 45 \text{ mm};$$

We know that according to Goodman's formula

$$\frac{1}{f.o.s.} = \frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_a}{S_e};$$
$$\frac{1}{1.5} = \frac{\frac{1718874}{d^3}}{490} + \frac{\frac{3593286}{d^3}}{76.91};$$
$$\frac{d^3}{1.5} = \frac{1718874}{490} + \frac{3593286}{76.91};$$
$$d^3 = 75342.85$$
$$d = 42.235\text{mm} \approx 45 \text{ mm};$$

Hence the shaft diameter is 45 mm. Ans.