Air Standard ~~Gas Power~~ Cycles

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Gas power cycles are thermodynamics cycles, in which the working substances remains a gas throughout the cycle of operation.

In the thermodynamic analysis of power cycles, the chief interest lies in estimating the energy conversion efficiency (thermal efficiency) and how the major parameters (pressure, volume and temperature variations etc ...) of the cycles affect the performance of the heat engine. Such analysis are made based on the following assumptions:

(i) Air is used as the working substance, it behaves as perfect gas i.e it obeys the gas laws and has constant specific heats (C_p and C_v).

(ii) The engine operates in a closed cycle. The cylinder is filled with constant amount of air and the same air is used repeatedly.

(iii) No chemical reaction takes place in the engine cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with the cylinder head at appropriate time during the process.

(iv) Compression and expansion processes are adiabatic (insulated) and internally reversible (no mechanical or friction loss).

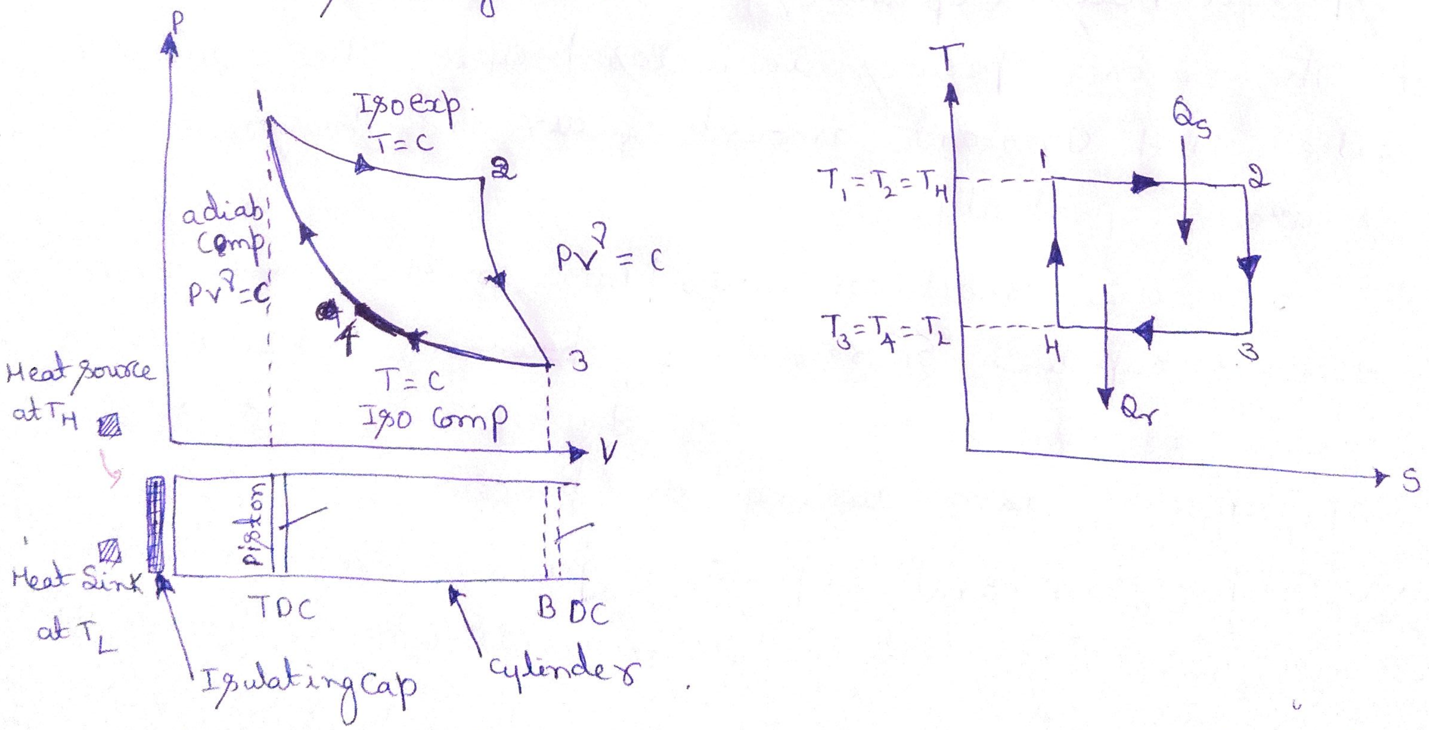
The efficiency calculated under the above discussed ideal conditions is known as ideal efficiency or air standard efficiency. However, under actual conditions variations occurs because air and fuel mixture is used as the working substance, intake and exhaust condition vary and also actual combustion process is different.

Also losses due to mechanical and friction effects need to be taken into account. Thus the actual efficiency of the cycle is always less than the air standard efficiency. and this is measured by a term known as relative efficiency.

$$\eta_{rel} = \frac{\text{actual thermal efficiency}}{\text{air standard efficiency}}$$

Carnot Cycle :

Carnot cycle consists of four process as shown in p-v and t-s diagram.



Let the cylinder contain 'm' kg of air at its initial condition represented by point 1 on p-v and t-s diagrams. The various processes involved in Carnot cycle are discussed below:

Process 1-2 Isothermal expansion :

The air in the cylinder is heated by bringing the hot body in contact with the cylinder head. The heat supplied by the hot body at constant temperature T_1 is fully absorbed by the air in the cylinder and this heat is utilized for doing external work (piston movement).

∴ Heat absorbed (or) heat added (or) heat supplied = Q_s

$$Q_s = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

under ideal conditions, we have $PV = mRT$ i.e $P_1 V_1 = mRT_1$

$$\therefore Q_s = mRT_1 \ln\left(\frac{V_2}{V_1}\right) \longrightarrow (1)$$

Process 2-3 Adiabatic Expansion :

∴ The heat source (hot body) is removed and an insulating cap is brought in contact with the cylinder head. Air expands adiabatically and the temperature falls from T_2 to T_3.

W.K.T from adiabatic process,

heat transfer $Q = 0$ and $\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \longrightarrow (2)$

Process 3-4 Isothermal Compression :

The insulating cap is removed and a cold body is brought in contact with the cylinder head. The piston moves upwards compressing the air at constant temperature T_3. The heat rejected at constant temperature T_3 is given by :

$$\text{Heat rejected} = Q_r = P_3 V_3 \ln\left(\frac{V_3}{V_4}\right)$$

$$= mRT_3 \ln\left(\frac{V_3}{V_4}\right) \longrightarrow (3)$$

Process 4-1 Adiabatic Compression :

The cold body is removed and an insulating cap is brought in contact with the cylinder head. The piston moves to the TDC compressing the air adiabatically from temperature T_4 to T_1.

W.K.T for adiabatic process, heat transfer Q = 0 and

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \longrightarrow (4)$$

To find air standard efficiency (η_{air}):

W.K.T efficiency $\eta_{air} = \dfrac{\text{Work done (WD)}}{\text{Heat supplied } (Q_H)}$

$$= \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{Q_S - Q_r}{Q_S}$$

(or) $$\eta_{air} = 1 - \frac{Q_r}{Q_S} = 1 - \frac{m.RT_3 \ln\left(\frac{V_3}{V_4}\right)}{m.RT_1 \ln\left(\frac{V_2}{V_1}\right)} \longrightarrow (5)$$

Consider Eqn (2) $\dfrac{T_2}{T_3} = \left(\dfrac{V_3}{V_2}\right)^{\gamma-1}$

But, from t-s diagram, $T_2 = T_1$ and $T_3 = T_4$

$$\therefore \frac{T_1}{T_4} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \longrightarrow (6)$$

Comparing Eqn (4) and (6), we have $\left(\dfrac{V_1}{V_4}\right)^{\gamma-1} = \left(\dfrac{V_2}{V_3}\right)^{\gamma-1}$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3} \text{ (or) } \frac{V_3}{V_4} = \frac{V_2}{V_1} \longrightarrow (7)$$

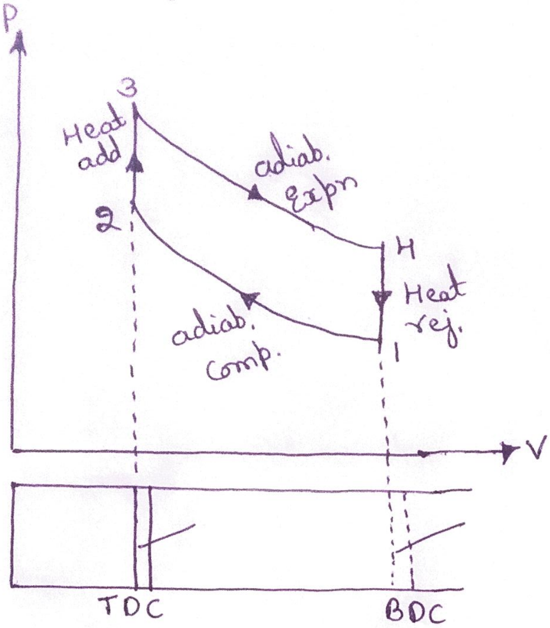
substituting Eqn (7) in (5), we get,

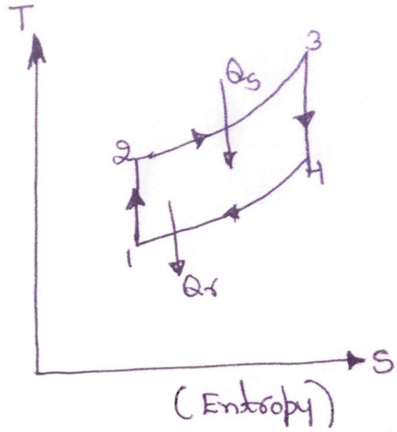
$$\eta_{air} = 1 - \frac{mRT_3 \ln\left(\frac{V_2}{V_1}\right)}{mRT_1 \ln\left(\frac{V_2}{V_1}\right)} = \boxed{1 - \frac{T_3}{T_1} = 1 - \frac{T_L}{T_H}}$$

$T_L = T_3 = T_4$ = Lowest temp. & $T_H = T_1 = T_2$ = Highest temp.

Otto Cycle:- Expression for Thermal Efficiency :-

Otto cycle consists of four processes as shown on p-v and t-s diagrams.





Let the cylinder ~~consist~~ contain 'm' kg of air at its initial condition represented by point 1 on p-v and t-s diagrams.

Process 1-2 Adiabatic Compression :

During this process, the insulating cap is brought in contact with the cylinder head. The piston moves from BDC to TDC compressing the air adiabatically in the cylinder. The temperature of air rises from T_1 to T_2.

W.K.T for adiabatic process,

heat transfer $Q = 0$, and $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $\longrightarrow (1)$

Process 2-3 Constant Volume heat addition :-

Heat is supplied at constant volume resulting in increase in pressure (P_3) and temperature (T_3).

Heat supplied $Q_s = mC_v(T_3 - T_2) \longrightarrow (2)$

Process 3-4 Adiabatic Expansion :-

Air expands adiabatically and its temperature falls from T_3 to T_4. The piston moves towards BDC.

W.K.t for adiabatic process,

heat transfer $Q = 0$, and $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \longrightarrow (3)$

Process 4-1 Constant Volume heat rejection:

Heat is rejected (transferred) to the cold body at constant volume.

Heat rejected $Q_r = mC_v(T_4 - T_1) \longrightarrow (4)$

To find air standard efficiency (η_{air}):

W.K.t efficiency $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}}$

$$\eta_{air} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s}$$

i.e $\eta_{air} = 1 - \frac{Q_r}{Q_s} = 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$

$$\eta_{air} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \longrightarrow (5)$$

$$= 1 - \frac{\left(\frac{T_4}{T_1} - 1\right)T_1}{\left(\frac{T_3}{T_2} - 1\right)T_2} \longrightarrow (6)$$

From Eqn (1), We have $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ But $V_2 = V_3$ and $V_1 = V_4$

$\therefore \frac{T_1}{T_2} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$ (or) $\frac{T_2}{T_1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \longrightarrow (7)$

Comparing Eqⁿ (7) and (3), we have $\frac{T_3}{T_4} = \frac{T_2}{T_1}$

or $\frac{T_4}{T_1} = \frac{T_3}{T_2} \longrightarrow (8)$

Substituting Eqⁿ (8) in (6), we have

$$\eta_{air} = 1 - \frac{\left(\frac{T_3}{T_2} - 1\right) T_1}{\left(\frac{T_3}{T_2} - 1\right) T_2} = 1 - \frac{T_1}{T_2} \longrightarrow (9)$$

Defining Compression ratio $R_c = \frac{V_1}{V_2}$, we have from Eqⁿ(1)

$$\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{1}{(R_c)^{\gamma-1}} \longrightarrow (10)$$

Substituting Eqⁿ (10) in (9), we get

$$\boxed{\eta_{air} = 1 - \frac{1}{(R_c)^{\gamma-1}}}$$ for otto cycle.

Mean Effective Pressure for otto Cycle :-

Mean Effective pressure (MEP) is defined as the mean or average pressure acting on the piston during the power stroke of the working cycle.

$$MEP = P_m = \frac{\text{Work done / cycle}}{\text{Swept Volume}}$$

$$= \frac{Q_S - Q_R}{V_1 - V_2} \quad \text{[Refer p-v diagram]}$$

$$= \frac{mC_v(T_3 - T_2) - mC_v(T_4 - T_1)}{V_1 - V_2}$$

$$MEP = \frac{mC_v[(T_3 - T_2) - (T_4 - T_1)]}{V_1 - V_2} \longrightarrow (1)$$

Express temperatures T_2, T_3 and T_4 in terms of T_1 :

For adiabatic process 1-2, we have

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \text{ (or) } \frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1} \quad \left[\because R_c = \frac{V_1}{V_2}\right]$$

$$\therefore T_2 = T_1 \cdot (R_c)^{\gamma-1} \longrightarrow (2)$$

For Constant volume process 2-3, we have $\frac{P}{T}$ = constant

i.e $\frac{P_2}{T_2} = \frac{P_3}{T_3}$ (or) $\frac{P_3}{P_2} = \frac{T_3}{T_2}$

$$\therefore T_3 = T_2 \cdot \left(\frac{P_3}{P_2}\right) = T_2 \cdot \alpha$$

Where α = Explosion ratio (or) pressure ratio = $\frac{P_3}{P_2}$

(or) $T_3 = T_1 (R_c)^{\gamma-1} \cdot \alpha$ from Eqn (2) $\longrightarrow (3)$

For adiabatic process 3-4, we have, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$

But $V_4 = V_1$ & $V_3 = V_2$

$$\therefore \frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \text{ (or) } \frac{T_3}{T_4} = (R_c)^{\gamma-1}$$

$$T_4 = \frac{T_3}{(R_c)^{\gamma-1}} = \frac{T_1 \cdot (R_c)^{\gamma-1} \cdot \alpha}{(R_c)^{\gamma-1}} \quad \text{from Eqn (3)}$$

$$\therefore T_4 = T_1 \cdot \alpha \longrightarrow (4)$$

Substituting Eqn (2) & (3) and (4) in (1) we have

$$MEP = \frac{m C_v \left\{ \left[T_1 (R_c)^{\gamma-1} \cdot \alpha - T_1 (R_c)^{\gamma-1}\right] - \left(T_1 \alpha - T_1\right) \right\}}{(V_1 - V_2)}$$

$$= \frac{m C_v \cdot T_1 \left[\left(\alpha \cdot R_c^{\gamma-1} - R_c^{\gamma-1}\right) - (\alpha - 1)\right]}{V_1 - V_2} \longrightarrow (5)$$

under ideal Conditions, at point (1), we have $P_1 V_1 = m R T_1$

(or) $V_1 = \frac{m R T_1}{P_1} \longrightarrow (6)$

W.K.T Compression ratio $R_c = \frac{V_1}{V_2}$

$$\therefore V_2 = \frac{V_1}{R_c} = \frac{mRT_1}{P_1 R_c} \text{ from (Eq}^n\text{ 6)} \longrightarrow (7)$$

Substituting Eq^n (6) & (7) in (5), We have

$$MEP = \frac{m.C_v.T_1\left[(\alpha R_c^{\gamma-1} - R_c^{\gamma-1}) - (\alpha-1)\right]}{\frac{mRT_1}{P_1} - \frac{mRT_1}{P_1 R_c}}$$

$$= \frac{m.C_v.T_1\left[R_c^{\gamma-1}(\alpha-1) - (\alpha-1)\right]}{\frac{mRT_1}{P_1}\left[1 - \frac{1}{R_c}\right]}$$

$$= \frac{P_1 C_v (\alpha-1)\left[R_c^{\gamma-1} - 1\right]}{R\left[\frac{R_c-1}{R_c}\right]} \longrightarrow (8)$$

W.K.t Gas Constant $R = C_p - C_v$ (or) $\frac{R}{C_v} = \frac{C_p}{C_v} - 1$

$$\frac{R}{C_v} = \gamma - 1 \qquad \therefore \frac{C_p}{C_v} = \gamma$$

$$\therefore \frac{C_v}{R} = \frac{1}{\gamma-1} \longrightarrow (9)$$

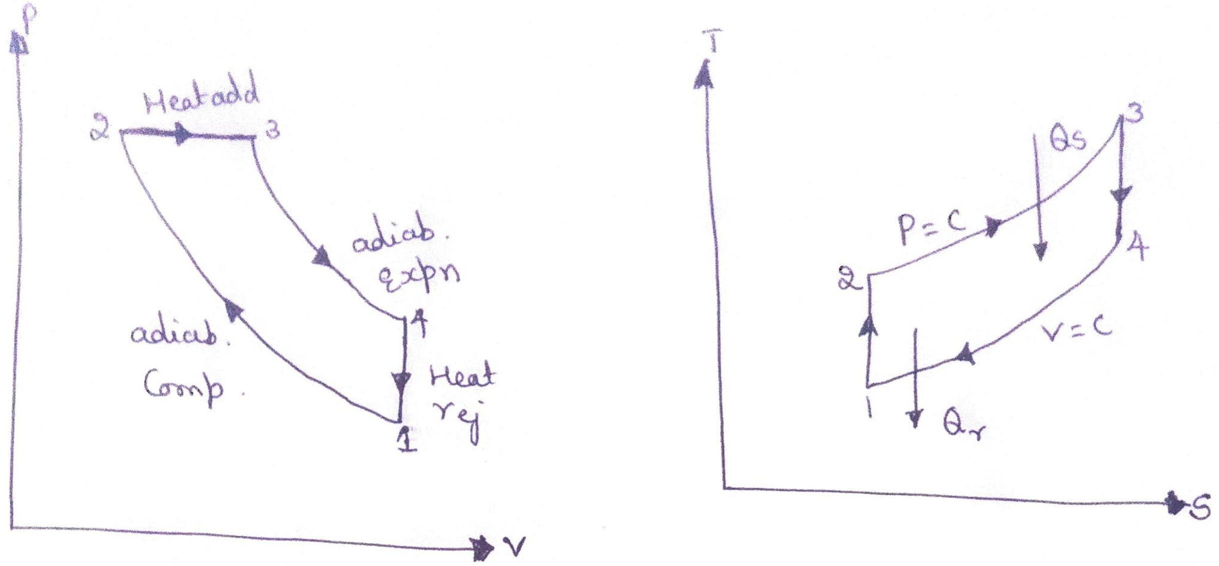
Substituting Eq^n (9) in (8), we have

$$\boxed{MEP = P_m = \frac{P_1 R_c (\alpha-1)(R_c^{\gamma-1}-1)}{(\gamma-1)(R_c-1)}}$$

Diesel Cycle [Constant Pressure Cycle]

Diesel cycle consists of four processes as shown on P-V and T-S diagrams.

Let the cylinder contain m kg of air at its initial condition represented by point 1 on p-v and t-s diagrams



Process 1-2 Adiabatic Compression :

The piston moves from BDC to TDC Compressing the air adiabatically in the cylinder. The temperature of air rises from T_1 to T_2.

W.K.T. for adiabatic process, heat transfer $Q=0$,

$$\text{and } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \longrightarrow (1)$$

Process 2-3 Constant pressure heat addition :

Heat is supplied at constant pressure resulting in an increase in the temperature from T_2 to T_3. At point 3, the supply of heat is stopped and this point is called as cut off.

∴ Heat supplied at constant pressure $= Q_S = mC_p\,(T_3 - T_2) \longrightarrow (2)$

Process 3-4 Adiabatic Expansion :

Air expands adiabatically and its temperature falls from T_3 to T_4. The piston moves towards BDC.

W.K.T for adiabatic process,

heat transfer $Q = 0$ and $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$ $\longrightarrow (3)$

Process 4-1 Constant Volume heat rejection :

Heat rejected to the cold body at Constant Volume.

$\therefore$ Heat rejected $= Q_r = mC_v\,(T_4 - T_1)$ $\longrightarrow (4)$

To find Air standard Efficiency (η_{air}) :

W.K.t $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$$\eta_{air} = 1 - \frac{m.C_v\,(T_4 - T_1)}{m.C_p\,(T_3 - T_2)}$$

But $\frac{C_p}{C_v} = \gamma$; $\therefore$ $\eta_{air} = 1 - \frac{1\,(T_4 - T_1)}{\gamma\,(T_3 - T_2)}$ $\longrightarrow (5)$

Express temperature T_2, T_3 and T_4 in terms of T_1

From Eqn (1), $\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1}$ $\because R_c = \frac{V_1}{V_2}$ = Compression Ratio.

$\therefore$ $T_2 = T_1\,(R_c)^{\gamma-1}$ $\longrightarrow (6)$

From Eqn (3), $\frac{T_3}{T_4} = \left(\frac{V_1}{V_3}\right)^{\gamma-1}$ $(\because V_4 = V_1)$

(or) $\frac{T_3}{T_4} = \left(\frac{V_1}{V_2} \cdot \frac{V_2}{V_3}\right)^{\gamma-1}$

Defining Cut-off ratio $\rho = \frac{V_3}{V_2}$, we have $\frac{T_3}{T_4} = R_c^{\gamma-1} \cdot \frac{1}{\rho^{\gamma-1}}$

(or) $T_4 = T_3 \cdot \frac{\rho^{\gamma-1}}{(R_c)^{\gamma-1}}$ $\longrightarrow (7)$

Note that T_4 has to be Expressed in terms of T_1.

For Constant pressure process 2-3, $\frac{V}{T}$ = Constant i.e $\frac{V_2}{T_2} = \frac{V_3}{T_3}$ or $\frac{T_3}{T_2} = \frac{V_3}{V_2}$

$$T_3 = T_2 \left(\frac{V_3}{V_2}\right) = T_2 \rho$$

$$T_3 = T_1 (R_c)^{\gamma-1} \cdot \rho \quad \text{from Eq}^n (6) \longrightarrow (8)$$

Substituting Eq^n (8) in (7), we have $T_4 = T_1 (R_c)^{\gamma-1} \cdot \rho \cdot \frac{\rho^{\gamma-1}}{(R_c)^{\gamma-1}}$

$$\therefore T_4 = T_1 \cdot \rho^\gamma \longrightarrow (9)$$

Substituting Eq^n (6), (7), (8) and (9) in (5), we have

$$\eta_{air} = 1 - \frac{1}{\gamma} \frac{(T_1 \rho^\gamma - T_1)}{(T_1 R_c^{\gamma-1}\rho - T_1 R_c^{\gamma-1})}$$

$$\eta_{air} = 1 - \frac{1}{\gamma} \frac{T_1 (\rho^\gamma - 1)}{T_1 R_c^{\gamma-1} (\rho - 1)}$$

$$\boxed{\eta_{air} = 1 - \frac{1}{\gamma \cdot (R_c)^{\gamma-1}} \cdot \frac{(\rho^\gamma - 1)}{(\rho - 1)}}$$

Mean Effective Pressure for Disel cycle :-

Mean Effective pressure, $P_m = \frac{\text{Work done / Cycle}}{\text{Swept Volume}}$

$$= \frac{Q_S - Q_r}{V_1 - V_2} \quad \text{(Refer P-V diagram)}$$

$$\text{MEP, } P_m = \frac{mC_p (T_3 - T_2) - mC_v (T_4 - T_1)}{V_1 - V_2} \longrightarrow (1)$$

Express all temperature in terms of T_1:

Reffering to Equations derived in section 2-6, we have heat transfer $Q = 0$ and $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$ $\longrightarrow (3)$

Process 4-1 Constant Volume heat rejection:

Heat rejected to the cold body at constant volume.

$\therefore$ Heat rejected $= Q_r = mC_v (T_4 - T_1)$ $\longrightarrow (4)$

To find Air standard Efficiency (η_{air}):

W.K.T $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_S - Q_r}{Q_S} = 1 - \frac{Q_r}{Q_S}$

$$\eta_{air} = 1 - \frac{m C_v (T_4 - T_1)}{m C_p (T_3 - T_2)}$$

But $\frac{C_p}{C_v} = \gamma$; $\therefore \eta_{air} = 1 - \frac{1 (T_4 - T_1)}{\gamma (T_3 - T_2)}$ $\longrightarrow (5)$

Express temperature T_2, T_3 and T_4 in terms of T_1

From Eqn (1), $\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1}$ $\therefore R_c = \frac{V_1}{V_2}$ = Compression Ratio.

$\therefore T_2 = T_1 (R_c)^{\gamma-1}$ $\longrightarrow (6)$

From Eqn (3), $\frac{T_3}{T_4} = \left(\frac{V_1}{V_3}\right)^{\gamma-1}$ $(\because V_4 = V_1)$

or $\frac{T_3}{T_4} = \left(\frac{V_1}{V_2} \cdot \frac{V_2}{V_3}\right)^{\gamma-1}$

Defining Cut-off ratio $\rho = \frac{V_3}{V_2}$, we have $\frac{T_3}{T_4} = R_c^{\gamma-1} \cdot \frac{1}{\rho^{\gamma-1}}$

or $T_4 = T_3 \cdot \frac{\rho^{\gamma-1}}{(R_c)^{\gamma-1}}$ $\longrightarrow (7)$

Note that T_4 has to be expressed in terms of T_1.

For constant pressure process 2-3, $\frac{V}{T}$ = Constant

From Eq^n (6), $T_2 = T_1 (R_c)^{\gamma-1}$

From Eq^n (8), $T_3 = T_1\, R_c^{\gamma-1} \cdot \rho$

From Eq^n (9), $T_4 = T_1 \cdot \rho^{\gamma}$

Substituting T_2, T_3 and T_4 in Eq^n (1), We have

$$MEP = \frac{\left[m C_p \left(T_1 \cdot R_c^{\gamma-1} \cdot \rho - T_1 \cdot R_c^{\gamma-1}\right) - m C_v \left(T_1 \rho^{\gamma} - T_1\right)\right]}{V_1 - V_2}$$

$$MEP = \frac{\left[m C_p \left(T_1\, R_c^{\gamma-1} (\rho - 1)\right) - m C_v\, T_1 \left(\rho^{\gamma} - 1\right)\right]}{V_1 - V_2} \longrightarrow (2)$$

Under ideal Conditions, at point 1 on P-V diagram

We have $P_1 V_1 = m R T_1$

$$\therefore V_1 = \frac{m R T_1}{P_1} \longrightarrow (3)$$

W.K.T Compression ratio, $R_c = \frac{V_1}{V_2}$

$$\therefore V_2 = \frac{V_1}{R_c} = \frac{m R T_1}{P_1 R_c} \quad \text{from Eq}^n\ (3)$$

$$\therefore V_1 - V_2 = \frac{m R T_1}{P_1} - \frac{m R T_1}{P_1 R_c}$$

$$V_1 - V_2 = \frac{m R T_1}{P_1}\left(1 - \frac{1}{R_c}\right) \longrightarrow (4)$$

Substituting Eq^n (4) in (2) We have,

$$MEP = \frac{m \cdot C_p \cdot T_1\, R_c^{\gamma-1} (\rho - 1) - m C_v T_1 (\rho^{\gamma} - 1)}{\frac{m R T_1}{P_1}\left(1 - \frac{1}{R_c}\right)}$$

$$= \frac{P_1 R_c \left\{C_p\, R_c^{\gamma-1} (\rho - 1) - C_v (\rho^{\gamma} - 1)\right\}}{R\left[R_c - 1\right]}$$

Divide both Numerator and Denominator by C_v

$$MEP = \frac{\frac{P_1 R_c}{C_v}\left\{C_p \cdot R_c^{\gamma-1}(\rho-1) - C_v(\rho^{\gamma}-1)\right\}}{\frac{R}{C_v}[R_c - 1]}$$

$$MEP = \frac{P_1 R_c\left\{\gamma R_c^{\gamma-1}(\rho-1) - (\rho^{\gamma}-1)\right\}}{\frac{R}{C_v}(R_c-1)}$$

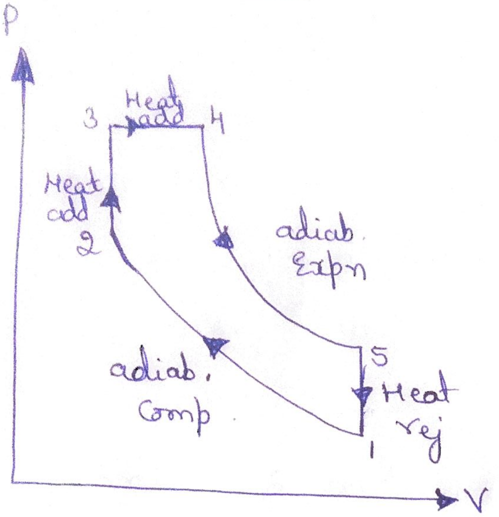
$$\because \frac{C_p}{C_v} = \gamma \longrightarrow (5)$$

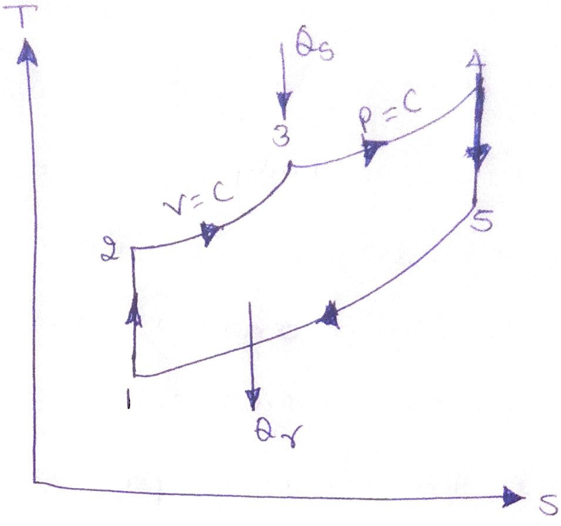
W.K.T $\quad R = C_p - C_v$

$$\frac{R}{C_v} = \frac{C_p}{C_v} - 1 \quad \text{or} \quad \frac{R}{C_v} = \gamma - 1$$

$\therefore$ Eqn (5) becomes, $\boxed{MEP = \frac{P_1 R_c\left\{\gamma R_c^{\gamma-1}(\rho-1) - (\rho^{\gamma}-1)\right\}}{(\gamma-1)(R_c-1)}}$

Dual Combustion Cycle (Semi-diesel cycle (or) Limited pressure cycle):





Let the cylinder contain m kg of air at its initial condition represented by point 1 on P-V and t-s diagrams. The dual combustion cycle consists of the following processes.

Process 1-2 Adiabatic Compression:

The insulating cap is brought in contact with the cylinder head. The piston moves from BDC to TDC compressing the air adiabatically in the cylinder.

W.K.T for adiabatic process, heat transfer $Q=0$ and $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $\longrightarrow (1)$

Process 2-3 Constant volume heat addition:

The insulating cap is removed and the hot body is brought in contact with the cylinder head. Heat is supplied at constant volume up to point 3.

$\therefore$ Heat supplied $(Q_S)_{2-3} = mC_V(T_3 - T_2)$ $\longrightarrow (2)$

Process 3-4 Constant pressure heat addition:

At point 3, heat is supplied at constant pressure up to condition 4 is reached.

$\therefore$ Heat supplied $(Q_s)_{3-4} = mC_p(T_4 - T_3) \longrightarrow (3)$

Process 4-5 Adiabatic Expansion:

The hot body is removed and the insulating cap is brought in contact with the cylinder head. Air expands adiabatically and its temperature falls from T_4 and T_5. The piston moves towards BDC.

W.K.t for adiabatic process, heat transfer, $Q = 0$ and $\left(\frac{T_4}{T_5}\right) = \left(\frac{V_5}{V_4}\right)^{\gamma-1}$

$\longrightarrow (4)$

Process 5-1 Constant volume heat rejection:

The insulating cap is removed and the cold body is brought in contact with the cylinder head. Heat is rejected to the cold body at constant volume.

$\therefore$ Heat rejected $Q_r = mC_v(T_5 - T_1) \longrightarrow (5)$

To find air standard efficiency (η_{air})

W.K.t $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$= 1 - \frac{Q_r}{(Q_s)_{2-3} + (Q_s)_{3-4}}$

i.e $\eta_{air} = 1 - \frac{mC_v(T_5 - T_1)}{mC_v(T_3 - T_2) + mC_p(T_4 - T_3)}$

$= 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)} \longrightarrow (6)$

Express all temperatures in terms of T_1:

From eqⁿ (1), we have $\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1}$ where $R_c = \frac{V_1}{V_2}$ = Compression ratio

$$T_2 = T_1 (R_c)^{\gamma-1} \longrightarrow (7)$$

From eqⁿ (4), we have $\frac{T_4}{T_5} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$ $\because V_5 = V_1$

$$\frac{T_4}{T_5} = \left(\frac{V_1}{V_2} \times \frac{V_2}{V_4}\right)^{\gamma-1}$$

(or) $$\frac{T_4}{T_5} = \left(\frac{V_1}{V_2} \times \frac{V_3}{V_4}\right)^{\gamma-1} \quad \because V_2 = V_3$$

$$\frac{T_4}{T_5} = \left(R_c \times \frac{1}{\rho}\right)^{\gamma-1} \quad \because \frac{V_4}{V_3} = \rho \text{ cut-off ratio}$$

$$\therefore T_4 = T_5 \left(R_c^{\gamma-1} \cdot \rho^{\gamma-1}\right) \longrightarrow (8)$$

For constant volume process 2-3, we have $\frac{P}{T}$ = Constant

i.e $\frac{P_2}{T_2} = \frac{P_3}{T_3}$ (or) $T_3 = T_2 \left(\frac{P_3}{P_2}\right)$

$= T_2 (\alpha)$ where α = Explosion ratio $= \frac{P_3}{P_2}$

using eqⁿ (7), we have, $T_3 = T_1 \cdot (R_c)^{\gamma-1} \cdot \alpha \longrightarrow (9)$

For constant pressure process 3-4, we have $\frac{V}{T}$ = Constant

i.e $\frac{V_4}{T_4} = \frac{V_3}{T_3}$ (or) $T_4 = T_3 \left(\frac{V_4}{V_3}\right) = T_3 (\rho)$

using eqⁿ (9), we have $T_4 = T_1 \cdot (R_c)^{\gamma-1} \cdot \alpha \cdot \rho \longrightarrow (10)$

(8)

Substituting Eqn (10) in (8) we have $T_1 \cdot (R_c)^{\gamma-1} \times \alpha \times \rho = T_5 \times (R_c)^{\gamma-1} \times \rho^{1-\gamma}$

$$T_5 = \frac{T_1 \alpha \rho}{\rho^{1-\gamma}}$$

$$T_5 = T_1 \alpha \rho^{\gamma} \longrightarrow (11)$$

Substituting $T_2 \cdot T_3 \cdot T_4$ and T_5 in Eqn (6), we have

$$\eta_{air} = 1 - \frac{C_v(T_1 \alpha \rho^{\gamma} - T_1)}{C_v(T_1 R_c^{\gamma-1} \alpha - T_1 R_c^{\gamma-1}) + C_p(T_1 R_c^{\gamma-1} \alpha \rho - T_1 R_c^{\gamma-1} \alpha)}$$

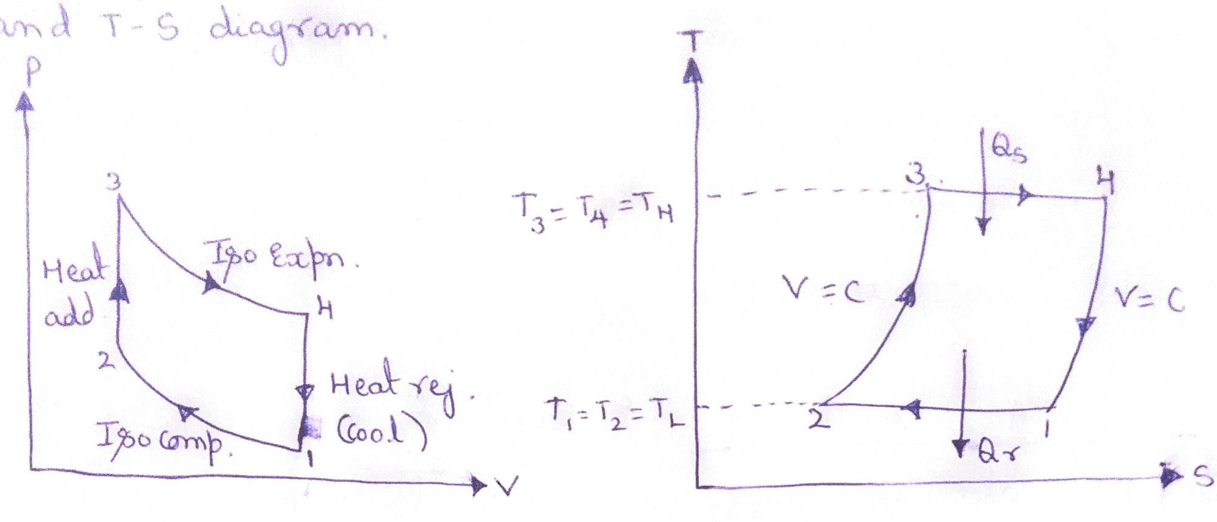
$$= 1 - \frac{T_1[\alpha \rho^{\gamma} - 1]}{T_1(R_c^{\gamma-1} \alpha - R_c^{\gamma-1}) + \frac{C_p}{C_v} \cdot T_1 (R_c^{\gamma-1} \cdot \alpha \cdot \rho - R_c^{\gamma-1} \cdot \alpha)}$$

$$\eta_{air} = 1 - \frac{\alpha \rho^{\gamma} - 1}{R_c^{\gamma-1}(\alpha - 1) + \gamma \cdot R_c^{\gamma-1} \cdot \alpha(\rho - 1)}$$

(or) $$\eta_{air} = 1 - \frac{1}{R_c^{\gamma-1}} \frac{\alpha \cdot \rho^{\gamma} - 1}{[(\alpha - 1) + \alpha \cdot \gamma(\rho - 1)]}$$

Stirling Cycle :-

Stirling cycle consists of four process as shown in P-V and T-S diagram.



Let us consider m kg of air at its initial conditions represented by point 1 on P-V and t-s diagrams. The various processes involved in stirling cycle are discussed below.

Process 1-2 Isothermal Compression :

During this process the cylinder head is kept in contact with the heat reservor at temperature T_1. The piston moves towards the TDC ; the air in the cylinder is compressed isothermally and heat is rejected from the working medium to the external sink.

Heat rejected $= Q_r$ Work done $= P_1 V_1 \ln\left(\frac{V_1}{V_2}\right)$

$= mRT_1 \ln(R_c)$ $\quad \|$ $\because P_1V_1 = mRT_1$ & $\frac{V_1}{V_2} = R_c$

Since $T_1 = T_L$ we can write, $Q_r = mRT_L \ln(R_c) \longrightarrow (1)$

Process 2-3 Constant volume heat addition :-

At condition 2, the air is made to enter the regen -erator from its top. The air while passing through the regenerator matrix gets heated from T_2 to T_3 (T_L to T_H). During constant volume process, work done = 0.

But heat transfer $Q_{2-3} = mC_v(T_3 - T_2)$

Process 3-4 Isothermal Expansion :-

The hot air is now admitted to the engine cylinder from the bottom portion of the regenerator. Expansion of air takes place isothermally and work is produced. The piston moves towards BDC. During the process, the cylinder head is kept in contact with the heat source at T_H.

$$\text{Heat supplied } Q_s = \text{Workdone} = P_3V_3 \ln\left(\frac{V_4}{V_3}\right)$$

$$= mRT_3 \ln\left(\frac{V_1}{V_2}\right) \quad \| \because V_4 = V_1 \ \& \ V_3 = V_2$$

Since $T_3 = T_H$ we can write, $Q_s = mRT_H \ln(R_c)$.

Process 4-1 Constant volume heat rejection :-

At condition 4, the air is made to enter the regenerator from the bottom and gets cooled while passing through the regenerator matrix at constant volume.

The process is so controlled that ultimately the air comes to its initial condition 1 and the cycle is completed.

During constant volume process, work done = 0

$\therefore$ But, Heat transfer $Q_{4-1} = mC_v(T_4 - T_1)$

To find air standard efficiency (η_{air}):-

W.K.T $\eta_{air} = \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{Q_s - Q_r}{Q_s} = 1 - \dfrac{Q_r}{Q_s}$

$$\eta_{air} = 1 - \frac{mRT_L \ln(R_c)}{mRT_H \ln(R_c)}$$

Thus for stirling cycle, $\eta_{air} = 1 - \dfrac{T_L}{T_H}$

It is clear that the efficiency of stirling cycle is equal to that of Carnot cycle operating between the same temperature limits.