ATME College of Engineering

BEE306B:Electrical Measurements and Instrumentation

ATME College of Engineering

MODULE 2 : MEASUREMENT OF RESISTANCE AND MEASUREMENT OF INDUCTANCE AND CAPACITANCE

MODULE 2 :MEASUREMENT OF RESISTANCE

MEASUREMENT OF RESISTANCE

LOW, MEDIUM AND HIGH

Classification of Resistances

Low Resistances	upto and including **one ohm**
Medium Resistances	**one ohm upwards** to about **0.1 mega ohm**
High Resistances	above **0.1 Mega-ohm**

Measurement of Low Resistance and Medium Resistance

The methods used for measurement

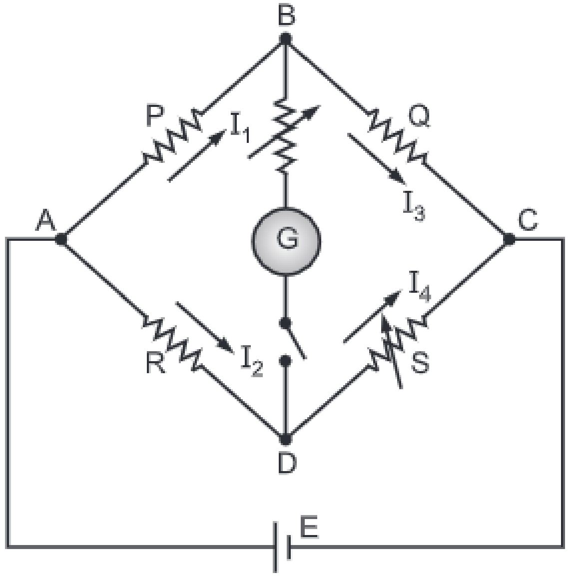
- Ammeter voltmeter method
- Kelvin's double bridge method
- Potentiometer method
- Wheatstone's bridge

Wheatstone's Bridge Method

measuring an unknown resistance

medium resistance in industry

very accurate and reliable method of measuring



Wheatstone's bridge

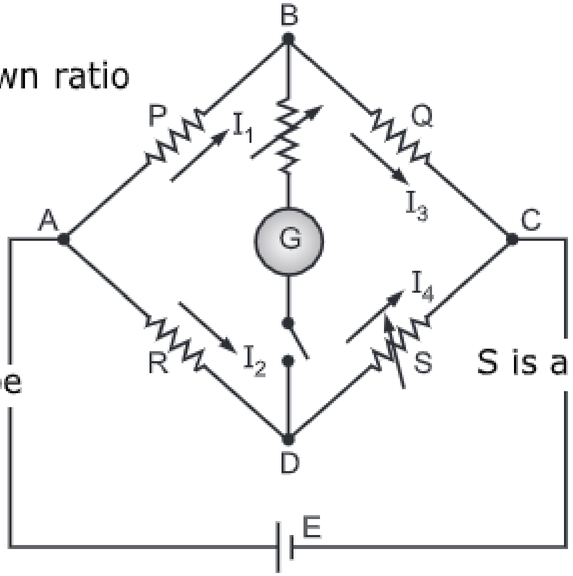
CONSTRUCTION

consists of four resistance arms P, Q, R and S

P and Q are of known value and give a known ratio

R is an unknown resistance whose value is to be determined

S is a standard variable resistance whose value can be easily measured



Wheatstone's bridge

WORKING PRINCIPAL

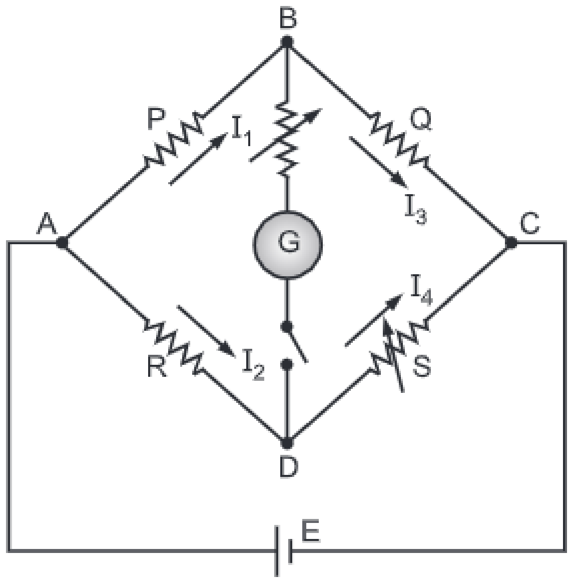
A source of e.m.f. is connected between point AC

galvanometer G is connected between BD

which is used as a null detector

The current flowing through the galvanometer will depend upon

the potential difference between B and D when there is no current through the galvanometer, which will be obtained by adjusting the value of standard resistance



Wheatstone's bridge

Under this condition, voltage from point B to A will be equal to the voltage from point D to point A

or

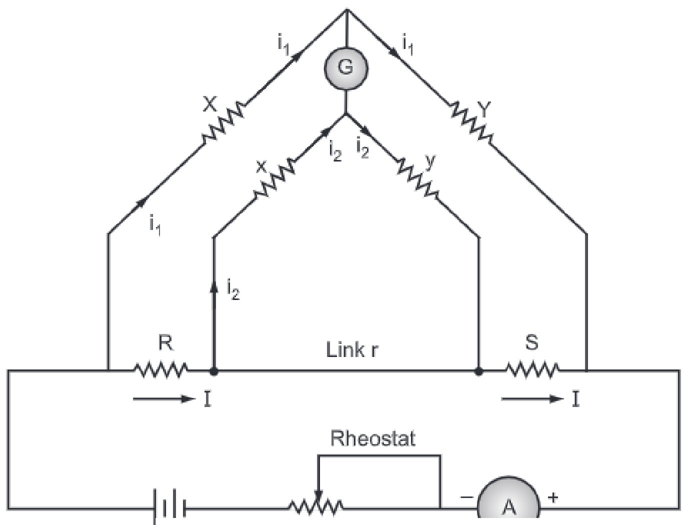
when the voltage from point D to C is equal to the voltage from point B to C.

At zero deflection,

$$\frac{R}{S} = \frac{X}{Y} = \frac{x}{y}$$

or

$$R = \frac{X}{Y} S$$



Kelvin's Double Bridge

using Kirchhoff's laws

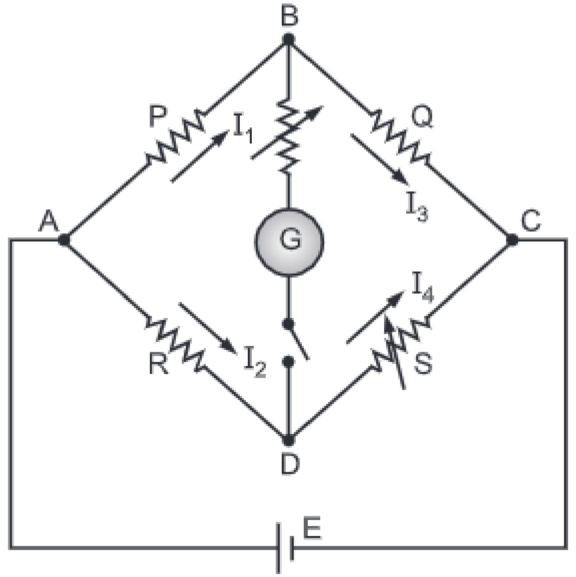
- Let the value of current flowing through branch X = current flowing through branch Y be equal to i_1 amperes; at zero deflection of galvanometer.
- Current through R = Current through S = I. Also from voltage drop across X = voltage drop across R + voltage drop across x.

$$i_1 X = IR + i_2 x \qquad \ldots(1)$$

and

$$i_1 Y = IS + i_2 y \qquad \ldots(2)$$

For the balanced condition of bridge, we can write the equations if the assumed current in different branches of Wheatstone's bridge are as shown in Fig



$$I_1 P = I_2 R \qquad \dots (1)$$

$$I_1 = I_3$$

$$I_1 = \frac{E}{P + Q} = I_3 \qquad \dots (2)$$

$$I_2 = I_4 = \frac{E}{R + S} \qquad \dots (3)$$

where, E is e.m.f. of the battery

Combining (1), (2) and (3) and simplifying we get

$$\frac{P}{P + Q} = \frac{R}{R + S}$$

or

$$QR = PS$$

or

$$R = \frac{P}{Q} \times S$$

The unknown can be determined

Errors in a Wheatstone Bridge

A Wheatstone bridge is a fairly convenient and accurate method for measuring resistance. However, it is not free from errors as listed below:

1. Discrepancies between the true and marked values of resistances of the three known arms can introduce errors in measurement.

2. Inaccuracy of the balance point due to insufficient sensitivity of the galvanometer may result in false null points.

3. Bridge resistances may change due to self-heating (I^2R) resulting in error in measurement calculations.

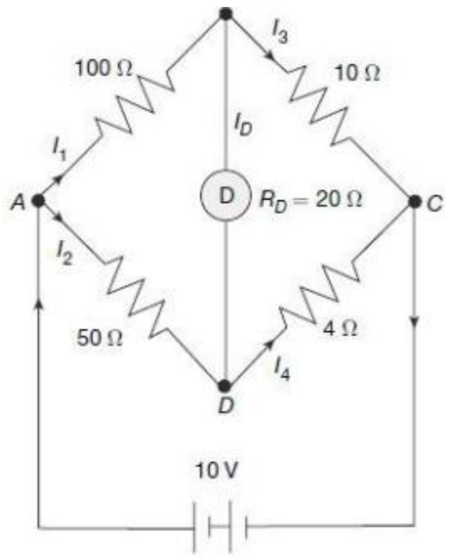
4. Thermal emfs generated in the bridge circuit or in the galvanometer in the connection points may lead to error in measurement.

5. Errors may creep into measurement due to resistances of leads and contacts. This effect is however, negligible unless the unknown resistance is of very low value.

6. There may also be personal errors in finding the proper null point, taking readings, or during calculations.

Problems

Four arms of a Wheatstone bridge are as follows: AB = 100 Ω, BC = 10 Ω, CD = 4 Ω, DA = 50 Ω. A galvanometer with internal resistance of 20 Ω is connected between BD, while a battery of 10-V dc is connected between AC. Find the current through the galvanometer. Find the value of the resistance to be put on the arm DA so that the bridge is balanced.



Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out Thevenin equivalent voltage across nodes *BD* and also the Thevenin equivalent resistance between terminals *BD*.

To find out Thevenin's equivalent voltage across *BD*, the galvanometer is open circuited, and the circuit then looks like the figure given below.

At this condition, voltage drop across the arm BC is given by

$$V_{BC} = 10 \times \frac{10}{100+10} = 0.91 \text{ V}$$

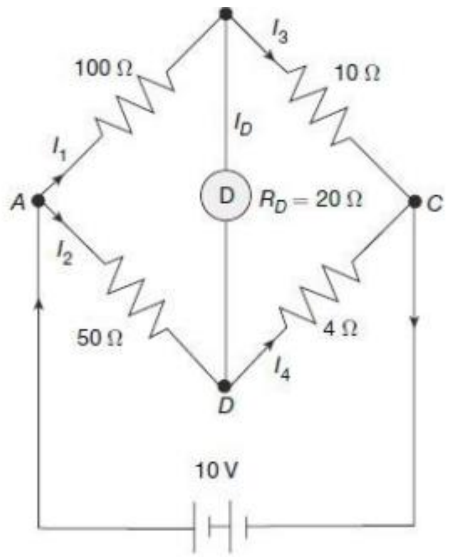
Voltage drop across the arm DC is given by:

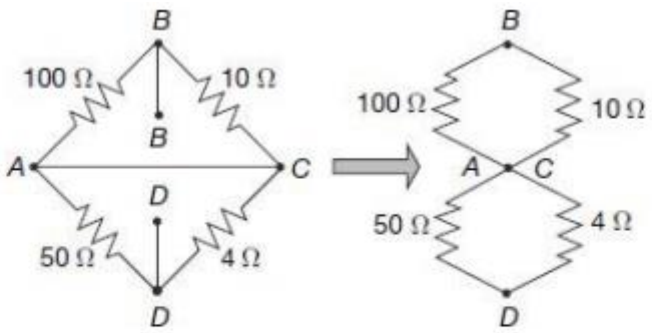
$$V_{DC} = 10 \times \frac{4}{50+4} = 0.74 \text{ V}$$

Hence, voltage difference between the nodes *B* and *D*, or the Thevenin equivalent voltage between nodes *B* and *D* is

$$V_{TH} = V_{BD} = V_B - V_D = V_{BC} - V_{DC} = 0.91 - 0.74 = 0.17 \text{ V}$$

To obtain the Thevenin equivalent resistance between nodes *B* and D, the 10 V source need to be shorted, and the circuit looks like the figure given below.





The Thevenin equivalent resistance between the nodes B and D is thus

$$R_{Th} = \frac{100\times 10}{100+10} + \frac{50\times 4}{50+4} = 12.79\ \Omega$$

Hence, current through galvanometer is

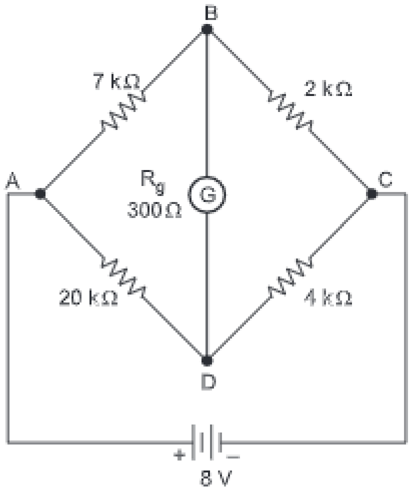
$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.17}{20+12.79} = 5.18\ \text{mA}$$

In order to balance the bridge, there should be no current through the galvanometer, or in other words, nodes *B* and *D* must be at the same potential.

Balance equation is thus

$$\frac{100}{10} = \frac{R_{DA}}{4} \quad \text{or, } R_{DA} = 40\ \Omega$$

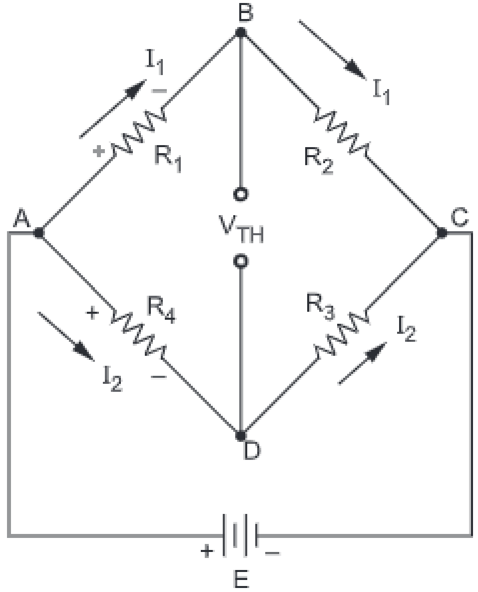
Calculate the current through the galvanometer for the bridge shown in the Fig.



Solution : From the Fig.

$R_1 = 7\ \text{k}\Omega$, $R_2 = 2\ \text{k}\Omega$, $R_3 = 4\ \text{k}\Omega$,

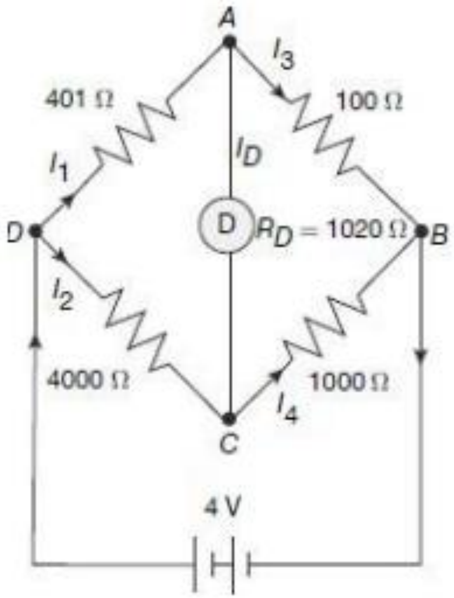
$R_4 = 20\ \text{k}\Omega$, $E = 8\ \text{V}$.



Use Thevenin's equivalent for I_g.

$$V_{TH} = V_{BD} = V_{AD} - V_{AB} = I_2 R_4 - I_1 R_1$$

The four arms of a Wheatstone bridge are as follows: AB = 100 Ω, BC = 1000 Ω, CD = 4000 Ω, DA = 400 Ω. A galvanometer with internal resistance of 100 Ω and sensitivity of 10 mm/μA is connected between AC, while a battery of 4 V dc is connected between BD. Calculate the current through the galvanometer and its deflection if the resistance of arm DA is changed from 400 Ω to 401 Ω.



Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out the Thevenin equivalent voltage across nodes *AC* and also the Thevenin equivalent resistance between terminals AC.

To find out Thevenin equivalent voltage across AC, the galvanometer is open circuited. At this condition, voltage drop across the arm *AB* is given by

$$V_{AB} = 4 \times \frac{100}{100+401} = 0.798 \text{ V}$$

Hence, voltage difference between the nodes *A* and *C*, or the Thevenin equivalent voltage between nodes *A* and *C* is

$$V_{TH} = V_{AC}$$
$$= V_A - V_C$$
$$= V_{AB} - V_{CB}$$
$$= 0.798 - 0.8 = -0.002 \text{ V}$$

To obtain the Thevenin equivalent resistance between nodes *A* and C, the 10 V source need to be shorted. Under this condition, the Thevenin equivalent resistance between the nodes *A* and *C* is thus

$$R_{Th} = \frac{100 \times 401}{100 + 401} + \frac{1000 \times 4000}{1000 + 4000} = 880.04\ \Omega$$

Hence, current through the galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.002}{100 + 880.04} = 2.04\ \mu\text{A}$$

Deflection of the galvanometer = Sensitivity × Current

10 mm/μA 2.04 μA

= 20.4 mm

Kelvin's Double - Bridge Method

A galvanometer is connected between the dividing points XY and xy

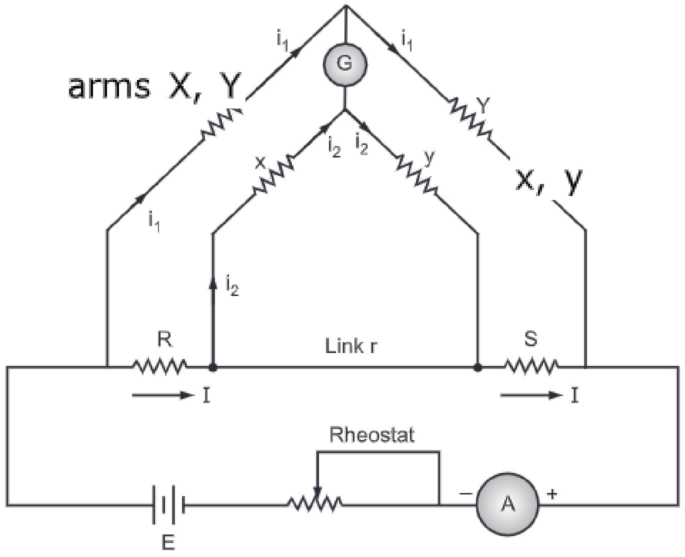
selected that $\frac{X}{Y} = \frac{x}{y}$ under this condition the galvanometer will give zero deflection.

arms X, Y

x, y

resistance 'R' is an unknown resistance whose value is to calculated

consists of a standard resistance 'S' the value of which can be chang



Kelvin's Double Bridge

As x and y are in parallel with resistance r,

$$i_2 = \frac{r}{(r + x + y)} I \qquad \ldots(3)$$

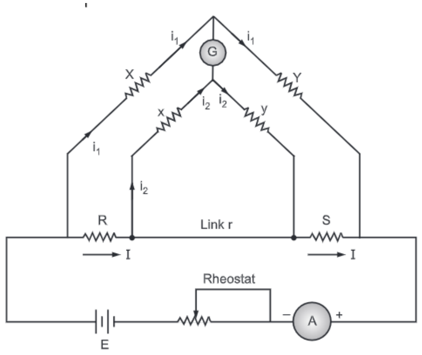
Substituting the value of i_2 in equations (1) and (2), we get

$$i_1 X = IR + \frac{r}{(r + x + y)} I \qquad \ldots(4)$$

$$i_1 Y = IS + \frac{r}{(r + x + y)} \times I \qquad \ldots(5)$$

Taking ratio of (4) and (5), we get

$$\frac{X}{Y} = \frac{R + \frac{rx}{r + x + y}}{S + \frac{ry}{r + x + y}} \qquad \ldots (6)$$



Cross multiplying (6),

$$XS + X\frac{ry}{r + x + y} = RY + Y\frac{rx}{r + x + y}$$

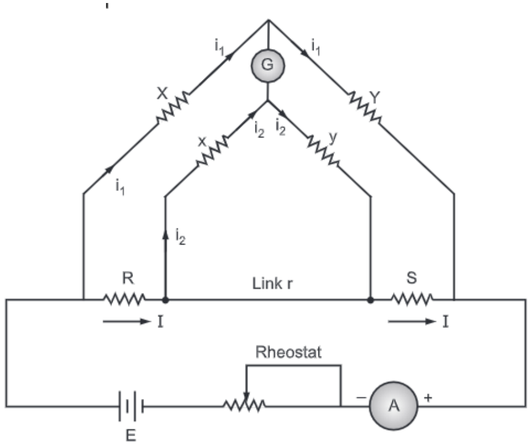
$$RY = XS + \frac{Xry}{(r + x + y)} - \frac{Yrx}{(r + x + y)}$$

$$= XS + \frac{r}{(r + x + y)}(Xy - Yx)$$

unknown resistance,

$$R = \frac{XS}{Y} + \frac{r}{(r + x + y)}\left(\frac{Xy}{Y} - x\right)$$

$$R = \frac{XS}{Y} + \frac{ry}{(r + x + y)}\left(\frac{X}{Y} - \frac{x}{y}\right)$$



As r is having a very small value

$$\frac{X}{Y} = \frac{x}{y} \text{ the value } \frac{ry}{(r + x + y)} \left(\frac{X}{Y} - \frac{x}{y}\right) \quad \text{will be equal to zero}$$

$$\therefore \qquad R = \frac{X}{Y} \times S$$

- The second reading is taken by reversing the direction of flow of current so as to avoid the error that can be introduced by thermoelectric effects of junction. The mean of the two readings will then give the correct value of R.

A 4-terminal resistor was measured with the help of a Kelvin's double bridge having the following components: Standard resistor = 98.02 nW, inner ratio arms = 98.022 Ω and 202 W, outer ratio arms = 98.025 Ω and 201.96 W, resistance of the link connecting the standard resistance and the unknown resistance = 600 nW. Calculate the value of the unknown resistance.

Solution From Eq. value of the unknown resistance is

$$X = \frac{P}{Q} \times S + \frac{qr}{p+q+r}\left(\frac{P}{Q} - \frac{p}{q}\right)$$

$$X = \frac{98.025}{201.96} \times 98.02 \times 10^{-6} + \frac{202 \times 600 \times 10^{-6}}{98.022 + 202 + 600 \times 10^{-6}}\left(\frac{98.025}{201.96} - \frac{98.022}{202}\right)$$

$$X = 47.62\ \mu\Omega$$

Numerical: In a Kelvin's double bridge, there is error due to mismatch between the ratios of outer and inner arm resistances. The bridge uses,

Standard resistance=100.03Ω

Inner ratio arms=100.31Ω and 200 Ω

Outer ratio arms=100.24 Ω and 200 Ω

The resistance of the connecting leads from standard to unknown resistance is 700u Ω. Calculate the unknown resistance under this condition.

$$R_3 = 100.03\ \mu\Omega\ ,\ R_2 = 100.24\ \Omega\ ,\ R_1 = 200\ \Omega$$

$$b = 100.31\ \Omega\ ,\ a = 200\ \Omega\ ,\ R_y = 700\ \mu\Omega$$

Thus unknown resistance is,

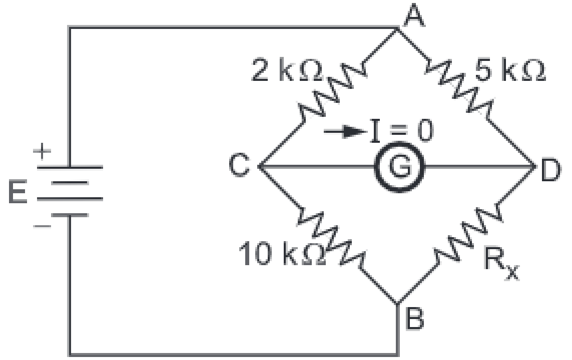
$$R_x = \frac{R_1\ R_3}{R_2} + \frac{b\ R_y}{[R_y + a + b]}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$

$$= \frac{200 \times 100.03 \times 10^{-6}}{100.24} + \frac{100.31 \times 700 \times 10^{-6}}{[700 \times 10^{-6} + 200 + 100.31]}\left\{\frac{200}{100.24} - \frac{200}{100.31}\right\}$$

$$= 1.9958 \times 10^{-4} + (2.3381 \times 10^{-4})(1.3923 \times 10^{-3})$$

$$= 1.999 \times 10^{-4}\ \Omega = \mathbf{199.905\ \mu\Omega}$$

The Wheatstone bridge is shown in the Fig. Calculate the value of unknown resistance, assuming the bridge to be in balanced condition.



Solution : As per the bridge shown in the Fig.

$$R_1 = 10\ k\Omega,\ \ R_2 = 2\ k\Omega,\ \ R_3 = 5\ k\Omega \text{ and } R_4 = R_x$$

Under balanced condition,

$$R_4 = R_x = \frac{R_1}{R_2} R_3 = \frac{10}{2} \times 5 = 5 \times 5 = \mathbf{25\ k\Omega}$$

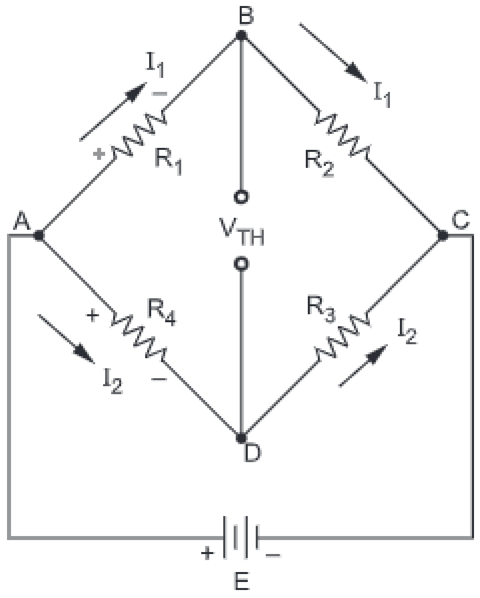
Thus unknown resistance is **25 kΩ .**

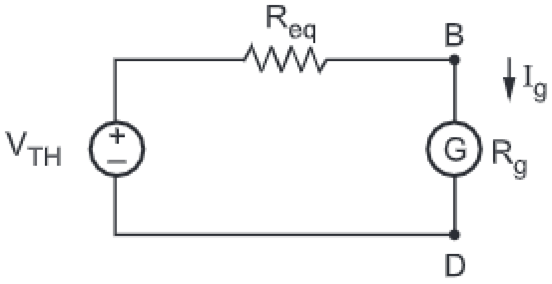
$$V_{TH} = V_{BD} = V_{AD} - V_{AB} = I_2 R_4 - I_1 R_1$$

$$= \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$

$$= 8 \left\{ \frac{20}{20+4} - \frac{7}{7+2} \right\}$$

$$= 0.444 \text{ V}$$





Now $R_{eq} = [R_1 || R_2] + [R_3 || R_4]$ ... with E shorted

$$= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 4.888 \text{ k}\Omega$$

$$I_g = \frac{V_{TH}}{R_{eq} + R_g}$$

$$= \frac{0.444}{4.888 \times 10^3 + 300} = \mathbf{85.62\ \mu A}$$

Purpose of earthing......

- Safety of the living beings around the vicinity of the substation
- Proper functioning of the protection system under fault condition
- To limit the touch and step potential within tolerable limits

Human Safety

Current Range	Effects on Humans
1 mA	Threshold of perception
1-6 mA	Let go currents
9-25 mA	Pain full, hard to let go
25-60 mA	Muscular contractions
60-100 mA	Ventricular fibrillation

Maximum Body Current: $Ik = \frac{0.116}{\sqrt{t}}$ for $t = .03s$ to $3s$

Earth resistance Measurement

The Provision of an earth electrode for an electrical system is necessitated by the following reasons.

- All the parts of electrical equipment, like casing of machines , Switches and circuit breakers, lead sheathing and tanks of transformer etc.., which have to be at earth potential, must be connected to an electrode. The purpose of this is to protect the various parts of the installations, as well as the persons working against damage in case the insulation of a system fails at any points.
- The earth electrode ensure that in the event of over voltage on the system due to lighting discharges or other system faults, those parts of equipment which are normally dead as far as voltage are concerned, do not attain dangerously high potential. In a 3phase circuit the neutral of the system is earthed in order to stabilize the potential of the circuit wrt earth.
- All the electrical equipment's are earthed through an electrode to avoid shocks to the person who touch the body of the equipment. The earthing provides a bypass to the leakage currents and hence should have a very small resistance (3 to 5 ohms)
- The main factors on which the resistance of any earthing system depends are:
- 1) Shape and material of the earth electrodes.
- 2) Depth of the electrodes at which they are buried in the soil.
- 3) Specific resistance of the soil surrounding the electrode.
- The specific resistance of the soil varies from one type of the soil to another. The amount of moisture present in the soil affects the specific resistance. Depending on the moisture content , the specific resistance of the soil varies, hence it is necessary to check the earth resistance regularly and resistance of earth electrode is not a constant factor but suffers seasonal variations.

Method of Measuring earth resistance

There are two methods : 1) Fall of Potential method 2) By using megger

Method 1: Fall of Potential method

1. Fig 1a shows the circuit for measurement of earth resistance with fall of potential method.
2. A current is passed through earth electrode E and an auxiliary electrode B and a Second auxiliary electrode A is inserted in earth between E and B.
3. The potential difference V between E and A is measured for a given current I.
4. The flow of ground current is shown in fig 1c. The lines of the 1st electrode current diverge and those of the 2nd electrode current converge..
5. The potential distribution between the electrodes is shown in fig 1b. It is apparent from this curve that the potential rises in the proximity of electrodes E and B and is constant along the middle section. The resistance of the earth is $R_E = V/I$ or V_{EA}/I.

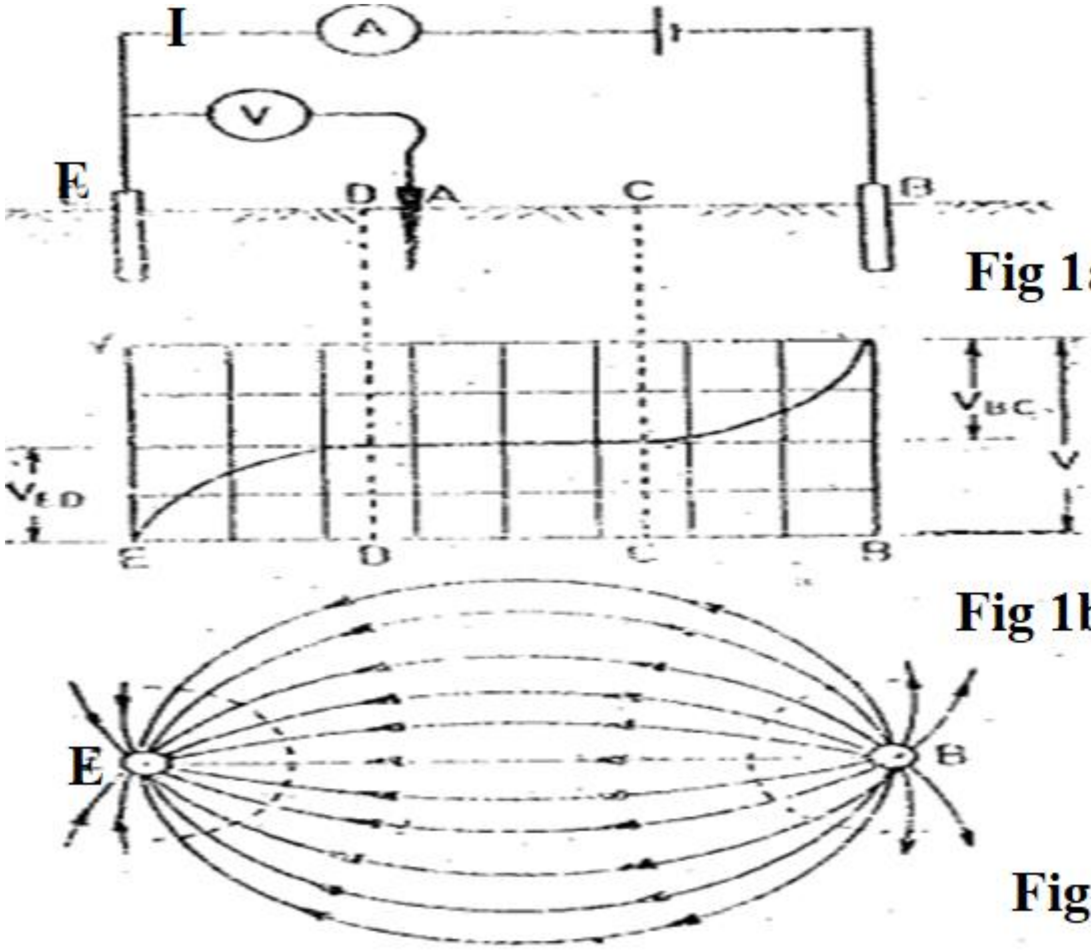


Fig 1a

Fig 1b

Fig 1c

Distribution of potentials between two earthing electrodes.

5) The position of electrode E and B is fixed and the position of electrode A is changed and resistance measurements are done for various position of electrode A. It is clear that the measured value of earth resistance depends on the position of the auxiliary electrode A.

6) The earth resistance rises rapidly initially. When the distance between earth electrode E and auxiliary electrode A is increased, it then becomes constant and when the auxiliary electrode A approaches the auxiliary electrode B and the resistance rises again.

7)The placing of electrode is thus very important and serious error may be caused by incorrect placing of the electrodes.

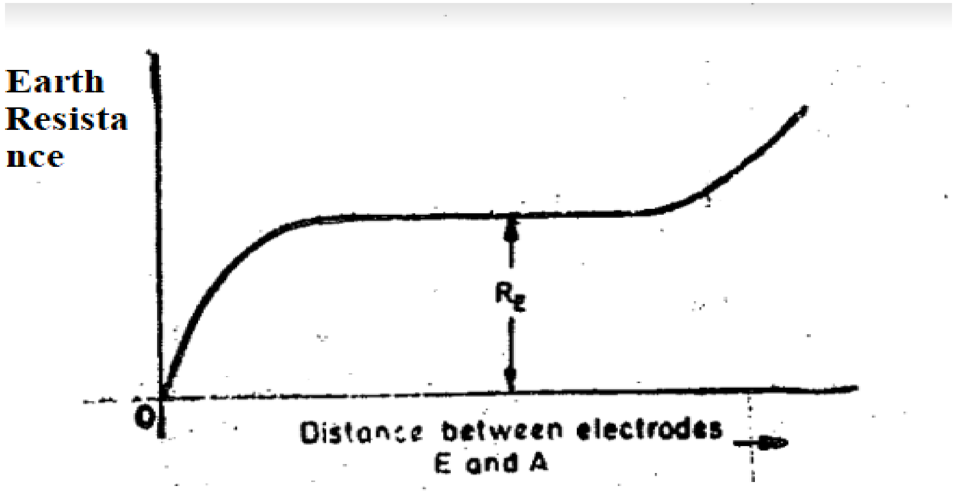


Fig. Variation of Earth resistance with distance between Electrode E and A

Method 2 : By using megger

What is megger

- Megger is a measuring device
- Megger is expert in ground earth testing
- It gives high accuracy
- Megger tester are easy to use and having lightweight

Why use megger & uses of megger

- Insulation of any electrical system decreased
- To minimize the error we use megger
- To determine the Leakage current
- It is used for verifying the Insulation level

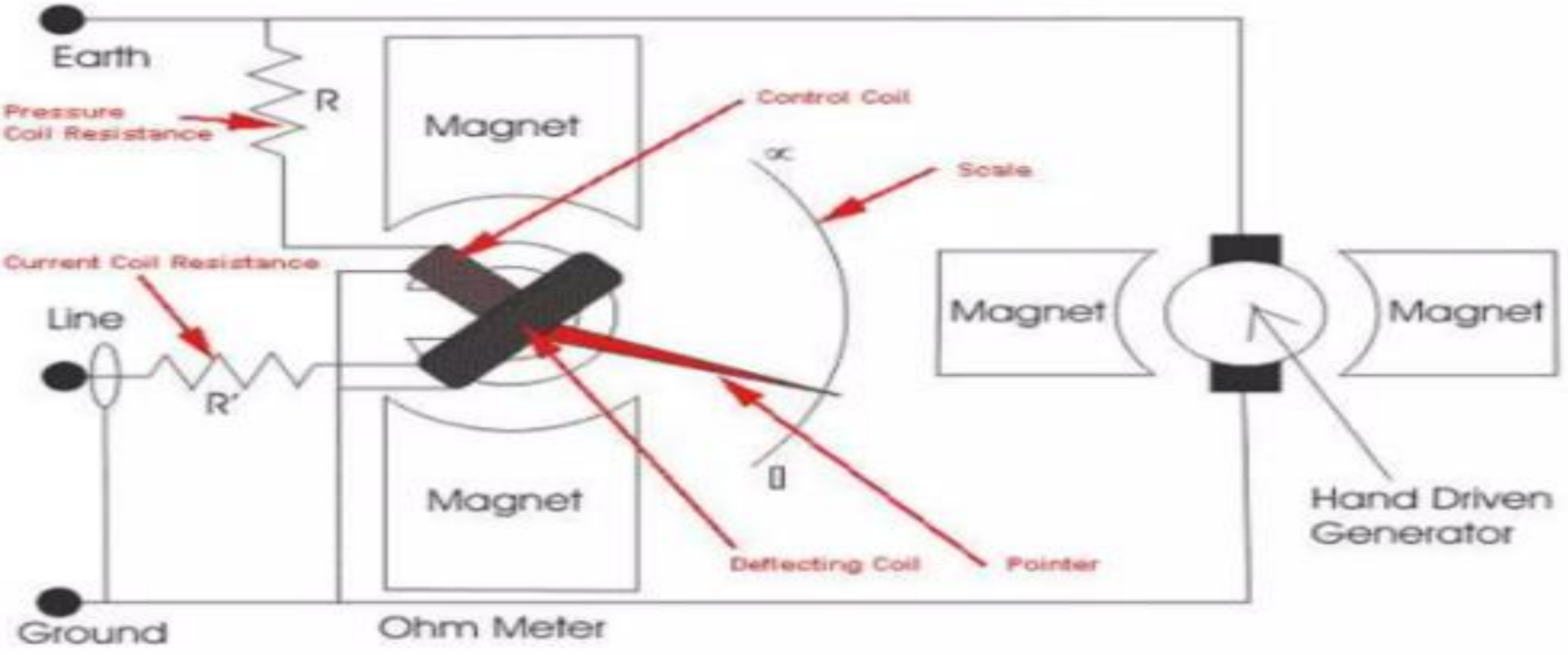
Megger is an instrument used to measure very high resistance of the order of mega ohms, Such as the insulation resistance of cable.

Resistances of the order of 0.1 MΩ

These high resistances are measured by portable, instrument known as **megger**.

It is based on the principle of electromagnetic induction

Construction of megger



Construction of Megger

It consists of hand driven DC generator and a direct reading ohm meter.

ohmmeter consist of three coil
- deflection coil,
- pressure or control coil
- compensating coil

1. There is a potential coil PC and a current coil CC, which are fixed to the Spindle and which are free to rotate about a vertical axis between the N and S of permanent magnet.
2. The coils are connected through a flexible leads called ligaments. The CC is connected in series with a resistance R1 connected between the generator terminals and the supply line marked L, which limits the current through CC.
3. The potential coil is connected in series with a compensating coil and protection resistance R2 and across the generator terminals.
4. A guard ring is provided to shunt leakage current over the test terminals. Terminals G is guard terminals, used to connect the guard ring to the insulation under test. The test voltage generated by the generator is usually 500V or 1000V.

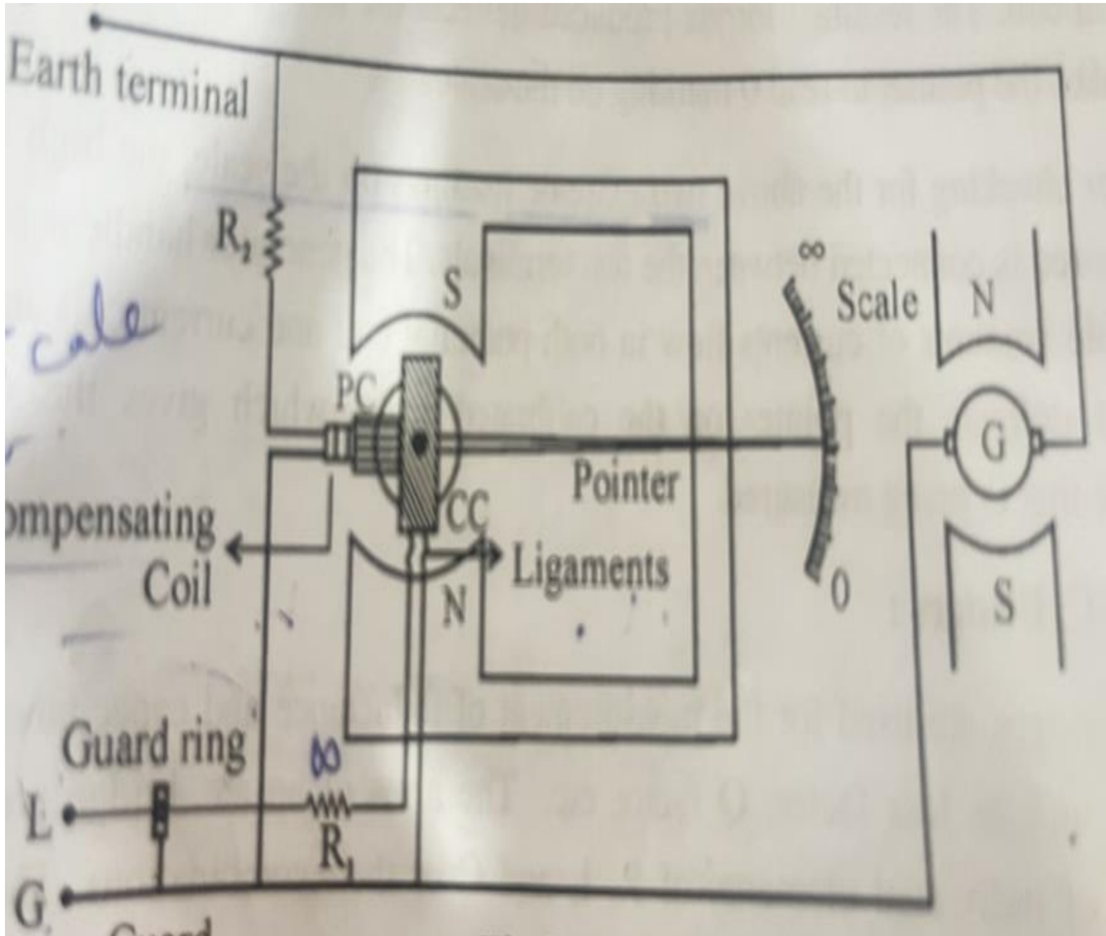


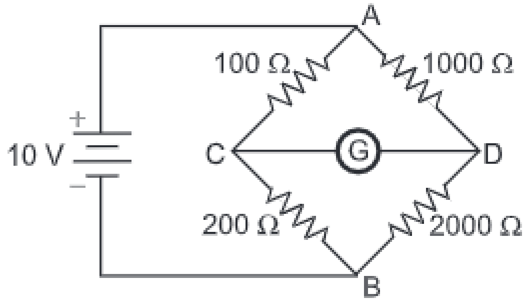
Fig 2.19

Working Principle of Megger

1. The high resistance to be measure is connected between the test terminals L and G. The generator handle is then steadily rotated at a uniform speed till the pointer gives steady reading which gives the values of resistance.
2. The satisfactory working of the megger may be tested as follows.
3. The test terminals L and G are **kept open**, So the resistance across L and G is infinite. The generator handle is rotated due to which, current flows through PC and there is no current through CC. therefore the pointer shows infinity on scale.
4. The test terminals are **short circuited** and the generator handle is rotated. Now a very large current flows through CC and a very small current flows through PC . The resultant toque deflects pointer in opposite direction and shows O on scale.
5. After checking above 2 extreme conditions, high resistance to be measured is connected between test terminals and generator is rotated so that reasonable amount of current flows in CC and PC and the torque produced deflects pointer and gives the value of resistance.

The Wheatstone bridge is shown in the Fig. The galvanometer has a current sensitivity of 12 mm / μA. The internal resistance of galvanometer is 200 Ω. Calculate the deflection of the galvanometer caused due to 5 Ω unbalance in the arm BD.

Solution : From the given bridge,



$$R_1 = 100\ \Omega \quad R_2 = 1000\ \Omega$$

$$R_3 = 200\ \Omega \quad R_4 = 2000\ \Omega$$

Now $R_1 R_4 = 100 \times 2000 = 200000$

$R_2 R_3 = 200 \times 1000 = 200000.$

For $R_4 = 2000\ \Omega$, the bridge is balanced. But there is unbalance of 5 Ω in the resistance of arm BD i.e. R_4.

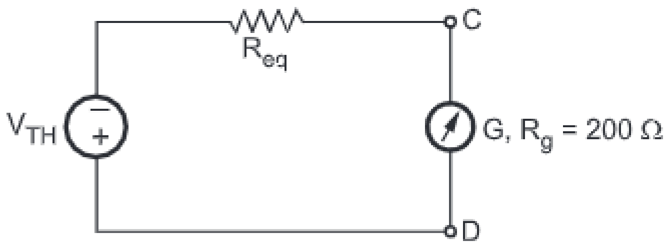
$$R_4 = 2000 + 5 = 2005\ \Omega$$

By Thevenin's equivalent, $V_{TH} = E\left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right] = 10\left[\frac{200}{100+200} - \frac{2005}{1000+2005}\right] = 10\ [0.6667 - 0.6672] = \mathbf{-5.213\ mV}$

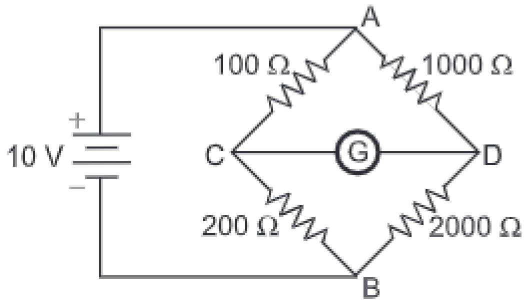
$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{100 \times 200}{(100+200)} + \frac{1000 \times 2005}{(1000+2005)} = \mathbf{733.888\ \Omega}$$

Hence Thevenin's equivalent is,



$$I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{5.213 \times 10^{-3}}{733.888 + 200} = \mathbf{5.582\ \mu A}$$



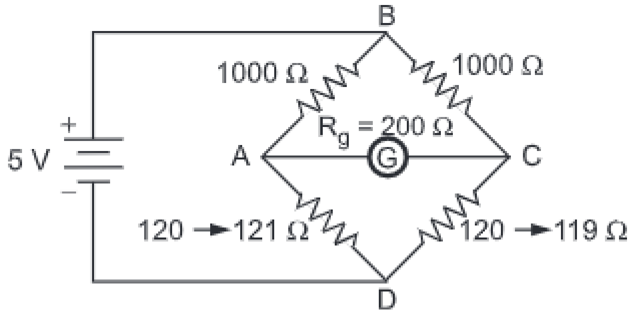
Now deflection of galvanometer is proportional to its sensitivity.

$$S = \frac{D}{I}$$

$$D = S \times I$$

$$= 12\ \text{mm}/\mu\text{A} \times 5.582\ \mu\text{A} = \mathbf{66.98\ mm}$$

The four arms of the Wheatstone bridge have the following resistances, AB = 1000 Ω, BC = 1000 Ω, CD = 120 Ω, DA = 120 Ω. The bridge is used for strain measurement and supplied from 5 V ideal battery. The galvanometer has sensitivity of 1 mm/μA with internal resistance of 200 Ω. Determine the deflection of the galvano arm DA increases to 121 Ω and arm CD decreases to 119 Ω.



Solution : The bridge given is shown in the Fig.

Now $R_1 = 1000\ \Omega \quad R_2 = 1000\ \Omega$

$R_3 = 121\ \Omega \quad R_4 = 119\ \Omega$

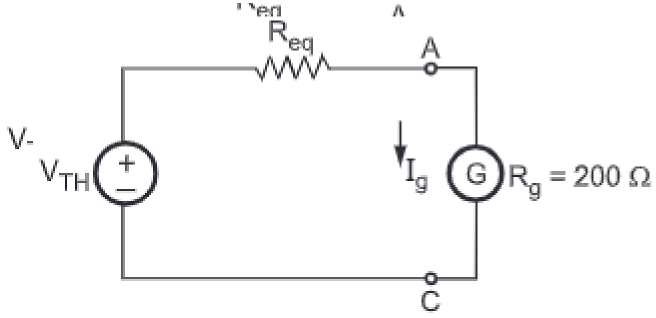
Let us calculate Thevenin's equivalent due to change in R_3 and R_4.

$$V_{TH} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right] = 5\left[\frac{121}{1000+121} - \frac{119}{1000+119}\right] = 5\,[0.1079 - 0.1063] = \mathbf{7.975\ mV}$$

$$R_{eq} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4} = \mathbf{214.2842\ \Omega}$$

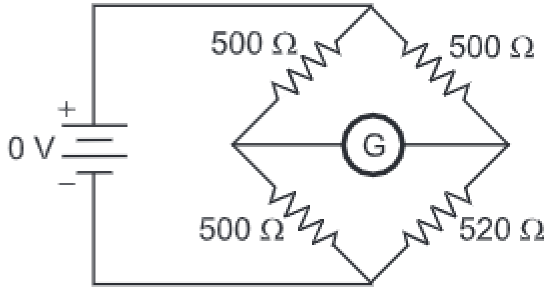
$$= \frac{121\times1000}{121+1000} + \frac{119\times1000}{119+1000} = 107.9393 + 106.3449$$

Thevenin's equivalent circuit is,



$$I_g = \frac{V_{TH}}{R_{eq}+R_g} = \mathbf{19.24\ \mu A} \qquad S = \frac{D}{I} = \mathbf{19.24\ mm}$$

Using the approximation of slightly unbalanced bridge, calculate the current through the galvanometer having internal resistance of 125 Ω, for the bridge shown in the Fig.



Solution : For the bridge shown,

$$R = 500\ \Omega \quad \text{and} \quad \Delta r = 20\ \Omega$$

Using approximate result,

$$V_{TH} = \frac{E\,\Delta r}{4\,R} = \frac{10 \times 20}{4 \times 500} = \mathbf{0.1\ V}$$

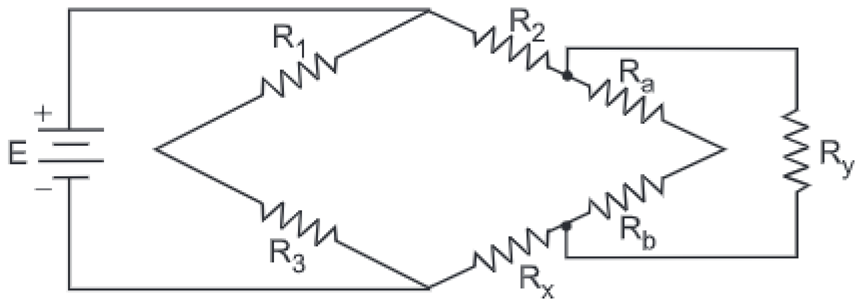
while $R_{eq} = R = \mathbf{500\ \Omega}$

$R_g = 125\ \Omega$ give

$$I_g = \frac{V_{TH}}{R_{eq} + R_g}$$

$$= \frac{0.1}{500 + 125} = \mathbf{160\ \mu A}$$

In the Fig. the Kelvin's double bridge is shown. The ratio of R_a *to* R_b *is 1200 Ω while* R_1 *is 10 Ω and* $R_1 = 0.5 R_2$*. Calculate the value of unknown resistance* R_x*.*



From the Fig. $$\frac{R_x}{R_2} = \frac{R_3}{R_1}$$

For Kelvin's double bridge,

$$\frac{R_3}{R_1} = \text{Ratio of resistances of ratio arms}$$

$$\frac{R_b}{R_a} = \text{Ratio of resistances of second ratio arms}$$

$$\frac{R_3}{R_1} = \frac{R_b}{R_a}$$

Now

$$R_1 = 10\ \Omega$$

$$R_1 = 0.5R_2$$

$$\frac{R_x}{R_2} = \frac{R_b}{R_a} = \frac{1}{1200}$$

$$R_2 = \frac{R_1}{0.5} = \frac{10}{0.5} = \mathbf{20\ \Omega}$$

$$\frac{R_x}{20} = \frac{1}{1200} = \mathbf{0.0167\ \Omega}$$

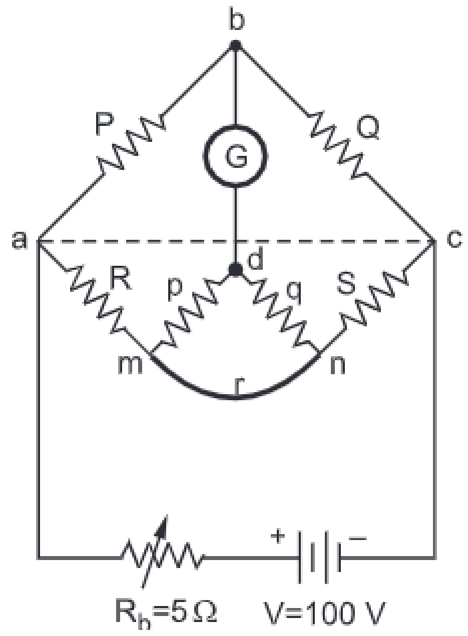
A Kelvin double bridge has each of the ratio arms $P = Q = p = q = 1000\ \Omega$ The e.m.f. of the battery is 100 V and resistance of 5 Ω is included in the circuit. The galvanometer has a resistance of 500 Ω and the resistance of the link connecting the unknown resistance to the standard resistance may be neglected. The bridge is balanced when standard resistance $S = 0.001\ \Omega$.

a) Determine the value of unknown resistance.

b) Determine the current (approximate value) through the unknown resistance and at balance.

c) Determine the deflection of the galvanometer when the unknown resistance R is changed by 0.1 % from its value at balance.

The galvanometer has a sensitivity of 200 mm / μA.



Solution : A Kelvin double bridge is as shown in the Fig.

a) At balance, $R = \frac{P}{Q} \cdot S = \frac{1000}{1000} (0.001) = \mathbf{0.001\ \Omega}$

b) Current under balance condition,

$$I = \frac{V}{R_b + R + S} = \frac{100}{S + 0.001 + 0.001} = \mathbf{19.99\ A}$$

c) The value of R is changed by 0.1 %.

$\therefore$ New value of R = $0.001 \times 0.1 = 0.0001\ \Omega$

$$V_{ac} = \left[\frac{R + r + S}{R_b + R + r + S}\right] V$$

Neglecting r,

$$V_{ac} = \frac{R + S}{R_b + R + S} V = \frac{0.0001 + 0.001}{5 + 0.0001 + 0.001} \times 100 = 29.995\ \text{mV}$$

$$V_{ab} = \frac{P}{P + Q} V_{ac} = \frac{1000}{1000 + 1000} (29.995 \times 10^{-3}) = 14.9978\ \text{mV}$$

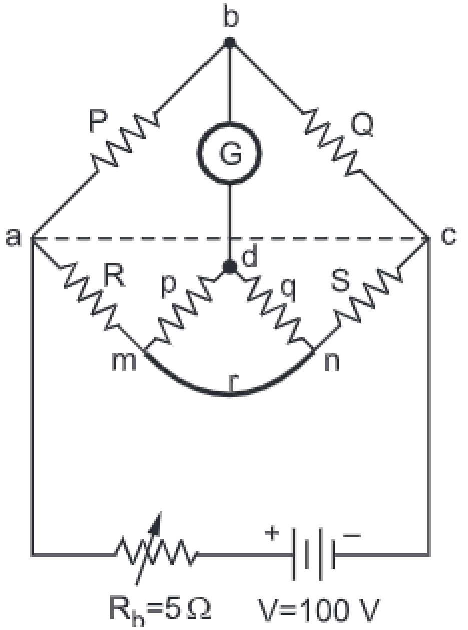
$$V_{amd} = \left[\frac{R + \dfrac{P_r}{p + q + r}}{R + S + \dfrac{(p + q)\, r}{p + q + r}}\right] V_{ab}$$

Neglecting r,

$$V_{amd} = \frac{R}{R + S} V_{ab}$$

$$= \frac{0.0001}{0.0001 + 0.001} (14.9978 \times 10^{-3})$$

$$= 1.3634\ \text{mV}$$



Hence output voltage is given as,

$$V_{out} = V_{ab} - V_{amd}$$

$$= 14.9978\ \text{mV} - 1.3634\ \text{mV}$$

$$= \mathbf{0.01362\ V}$$

Types of Bridges

The two types of d.c. bridges are,

1) D.C. bridges used to measure the **resistances**

2) A.C. bridges used to measure the **impedances** consisting capacitances and inductances.

D.C. bridges

1. Wheatstone bridge
2. Kelvin bridge

A.C. bridges

1. Capacitance comparison bridge.
2. Inductance comparison bridge.
3. Maxwell's bridge.
4. Hay's bridge.
5. Anderson bridge.
6. Schering bridge.
7. Wien bridge.

Sources and Detectors

For bridge measurements

at very low frequencies,

the power line itself may act as a source of supply to the bridge circuit.

at higher frequencies

electronic oscillators are used as a source of supply to the bridge circuit.

i) The output waveform is very close to sine wave.

ii) The output frequency is very stable.

iii) The output frequency is easily determinable with accuracy and also it is easily adjustable.

iv) The output power is sufficient to drive the bridge circuits.

40 Hz to 125 kHz with power output of 7 W.

For the a.c. bridges commonly used detectors are as follows.

i) Headphones :

250 Hz upto 3 to 4 kHz.

ii) Vibration galvanometers :

5 Hz to 1000 Hz.

These detectors can be effectively used below 200 Hz with greater sensitivity than the headphones.

iii) Tuneable amplifier detectors :

The frequency range for these detectors is 10 Hz to 100 kHz.

Shielding of bridges / Grounding of bridges:

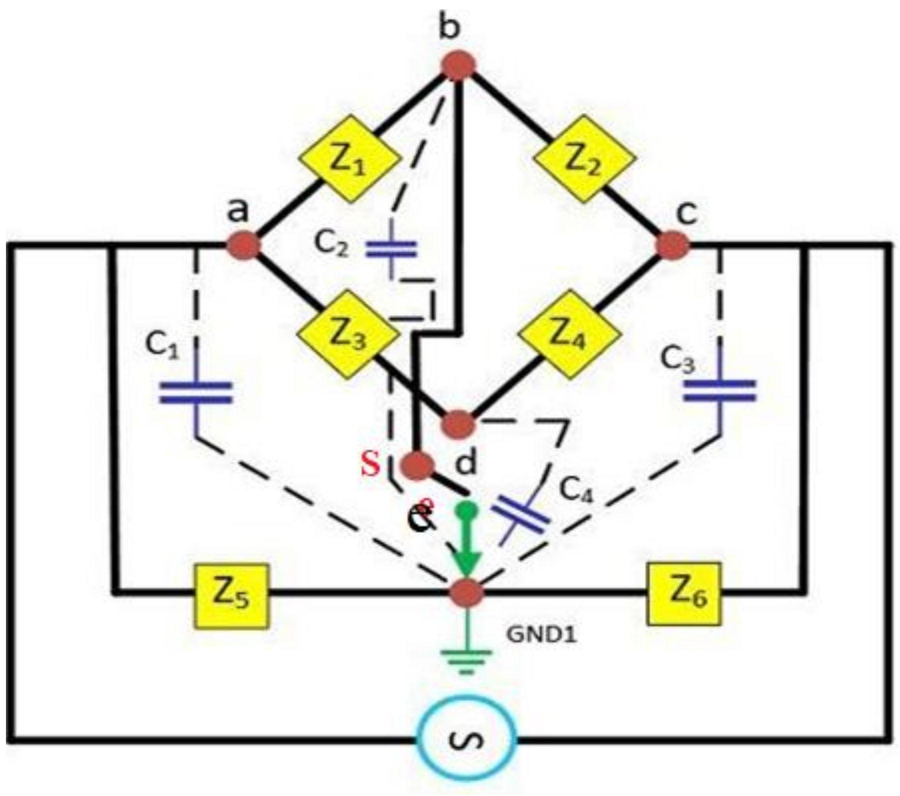
A **Wagner Earth device** is generally used for the **Schering bridge** for **grounding purpose**.

Method 1: The shielding and grounding of bridge is the one way of reducing **the effect of the stray capacitance**. **But this technique does not eliminate the stray capacitance but makes them constant in value** and hence they can be computed.

Method 2: One very effective and popular method of eliminating the stray capacitance and the capacitance between the bridge arms is using a ground connection called Wagner ground connection.

Used to eliminate the effect of stray capacitance between null detector and the earth.

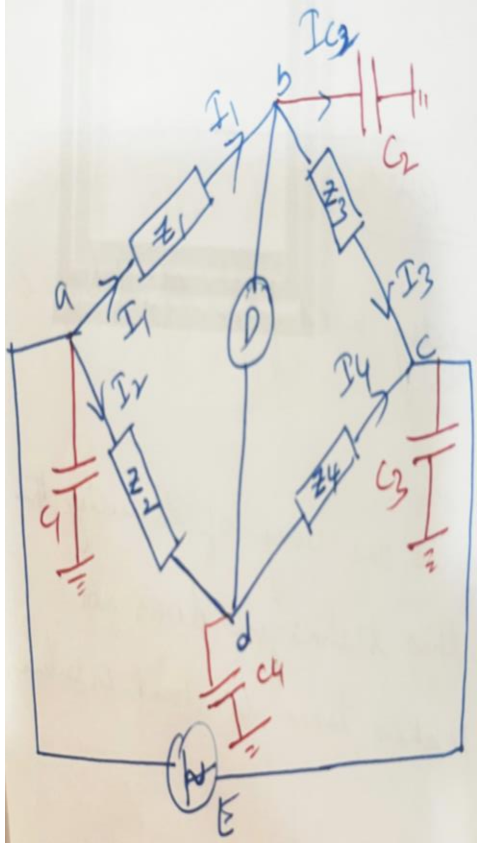
stray capacitance is nothing but unavoidable, unintended and unwanted capacitance that exist between the parts of a circuit because of their proximity to each other with in the circuit.



b arm can be connected either to d arm or to ground potential (SPST Switch is used)

Method 1 Explanation :

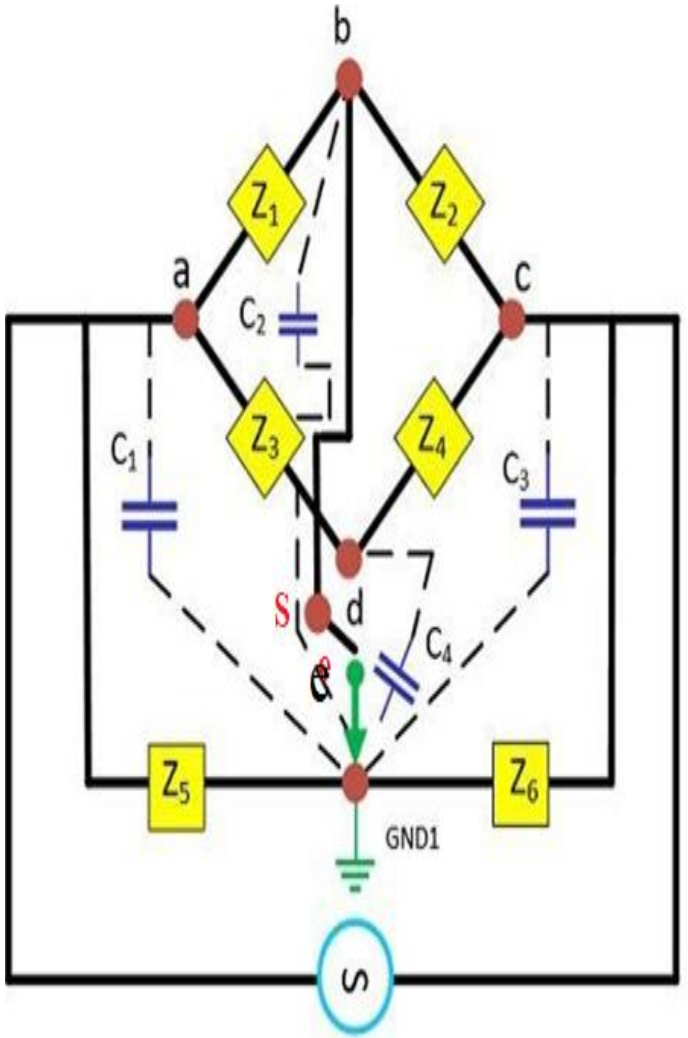
- E is sinusoidal, **potential "a" will be Positive** wrt to **potential "C"** and other half cycle point "C" will be Positive wrt "a" (either point a or point C will be ground potential.
- **Capacitance** is **formed between two conductor** which are placed at different potential.
- We chose **component between ab,bc,cd and ad** such that **no stray capacitance will be present**.
- **C1 and C3 does not effect the bridge balance** and **it is not part of the bridge**(C1 is at point "a**", I1 and I2 is not affected** due to the placement of capapcitance C1) similarly neither I3 and I4 is not affected due to C3 presence.
- Only **supply current** will be **distributed** due to **C1 and C3** but **bridge balance is not affected**.
- **C2 and C4 affect the bridge balance** as I3 not equal to I1 and I4 not equal to I2 due to charging current of C2 and C3.
- Therefore I1 enters "b" point and I3 leaves "b" point but some current are present in C2 **[I1= I3+Ic2]** and so we cannot write balance equation as Z1Z4=Z2Z3, Similarly at point "d" also.
- So **to avoid this problem b and d should be at ground potential under balance**
- To make bridge balance I1=I2=I3=I4(Therefore if potential b and d itself are ground potential , obviously IC2 ad IC4=0 , then I1=I3 and I2= I4.
- Therefore initially IC2= Potential difference between b and ground / impedance of C2 (Z = R – jXC)
- = Vbd/ -JxC2= Vbd /-j *1/2Pi fC = wc2.
- This is done by Wagner earthing device.



ATME College of Engineering

Method 2 Explanation :

- This **device removes all the earth capacitances from the bridge network**.
- It contains a **voltage divider circuit (Z5- Z6)** with **Centre point** connected to ground ie.., **Point "e"**.
- **Z1,Z2,Z3,Z4 are the impedance of the bridge arms C1, C2, C3 and C4 are the stray earth capacitances** appearing at the apexes (suspension)of the bridge. D is the detector.
- **Z5 and Z6 can be R,L, C any component such that they can be capable of forming a balanced bridge with the bridge arms Z1-Z3 or Z2- Z4**
- **Steps:**
- Connect "**S" to "d"** and adjust **Z2 and Z4** to get balance (it is like normal bridge it will not affect z5 and Z6) until headphone or detector shows the **minimum sound.**
- Connect "**s" to "e**" and **Adjust Z5 and Z4** to get **minimum sound in headphone**.
- Repeat Step1,2 until no sound is audible when "s" is connected to " d" and again minimum or no sound when "S" is connected to "e" that means the bridge is balanced. This means potential **d and e are at same potential** it will not affect the bridge
- Under balance condition **potential difference across the detector =0** "bd" are at same potential "de" are at same potential therefore both points **b and d are at ground potential**. Under these condition **No current flows in the earth capacitance C2 and C4** and since **C1 and C3 shunt the Wagner arms z5 and Z6,** these **capacitance are eliminated from the bridge network Z1, Z2, Z3 and Z4.**



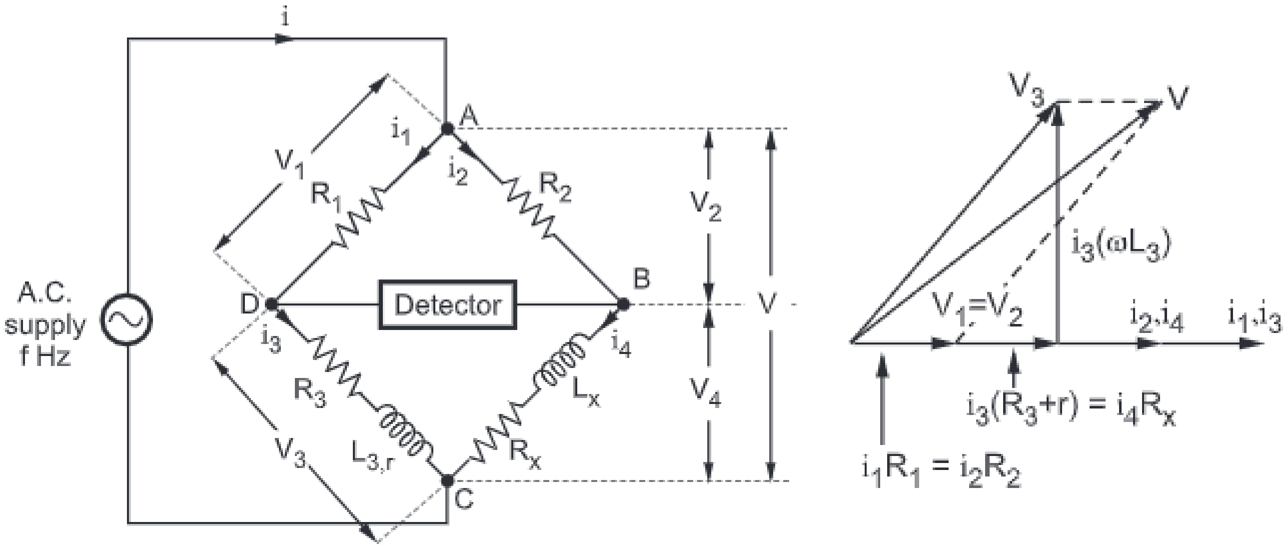
Maxwell's Inductance Bridge

Using this bridge, we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit as shown in Fig.

Two branches consist of non-inductive resistances R_1 and R_2.

One of the arms consists variable inductance with series resistance r.

The remaining arm consists unknown inductance L_x.



(a) Circuit diagram

At balance, we get condition as

$$\frac{R_1}{[(R_3+r)+j\omega L_3]} = \frac{R_2}{R_x+j\omega L_x} \quad \text{... (1)}$$

$$\therefore \quad R_1[R_x+j\omega L_x] = R_2[(R_3+r)+j\omega L_3]$$

$$\therefore \quad R_1R_x+j\omega R_1L_x = R_2(R_3+r)+j\omega R_2L_3$$

Equating imaginary terms, we can write

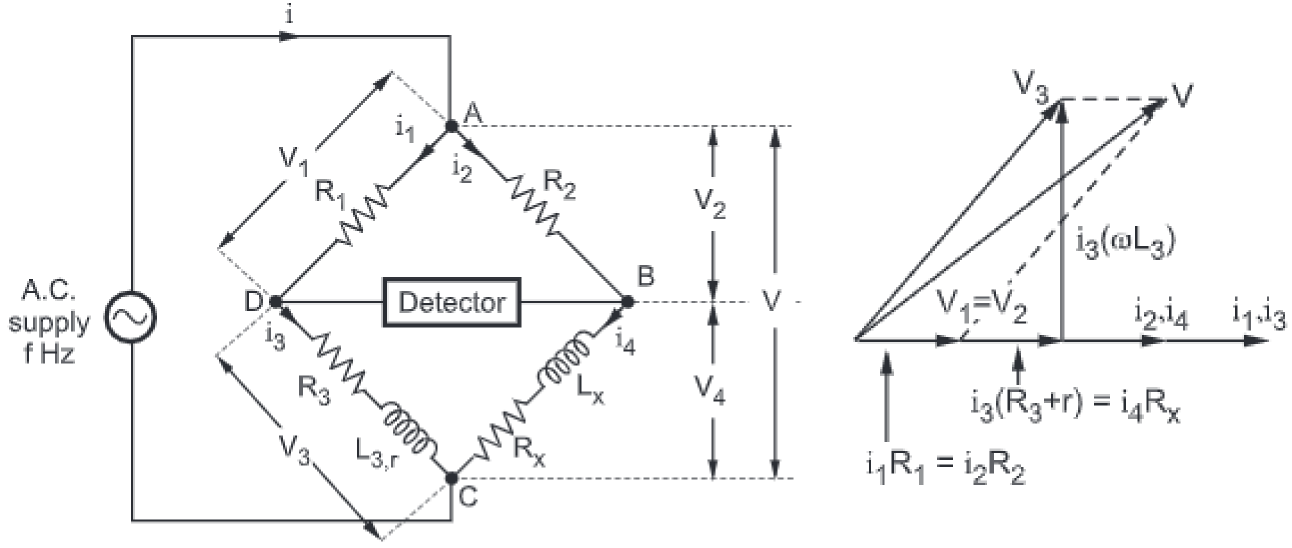
$$R_1 L_x = R_2 L_3$$

$$\boxed{L_x = \frac{R_3}{R_1} L_3} \qquad \dots (2)$$

Equating real terms, we can write,

$$R_1 R_x = R_2 (R_3 + r)$$

$$\boxed{R_x = \frac{R_2}{R_1}(R_3 + r)} \qquad \dots (3)$$



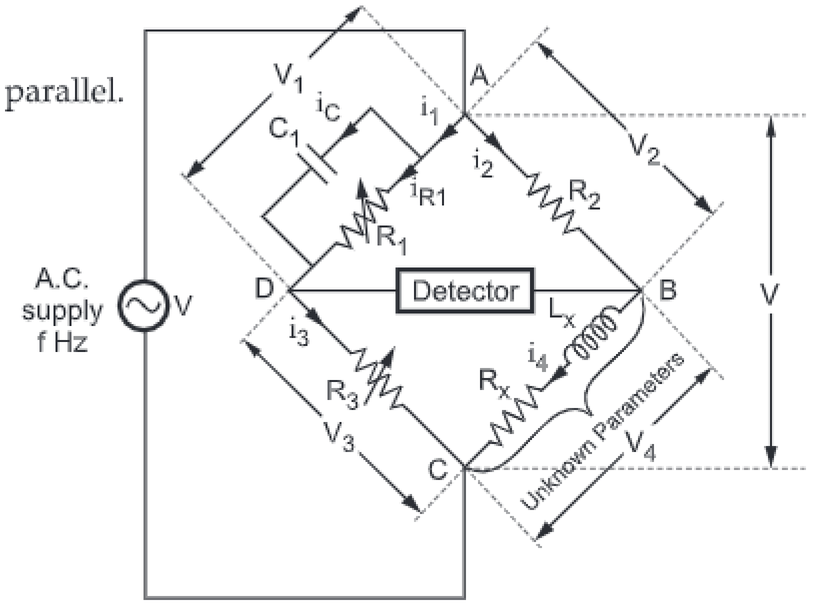
(a) Circuit diagram

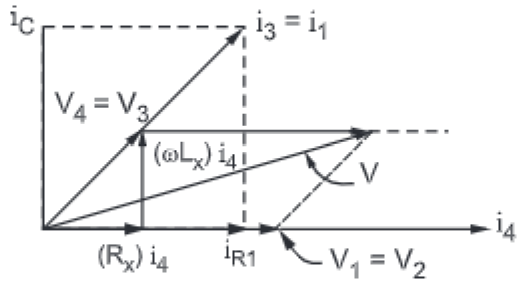
Maxwell's Inductance Capacitance Bridge

Using this bridge, we can measure inductance by comparing with a variable standard capacitor.

The bridge circuit diagram is as shown in the Fig.

One of the ratio arms consists of resistance and capacitance in parallel.





The general bridge balance equation is,

$$\overline{Z_1\, Z_x} = \overline{Z_2\, Z_3}$$

$$\overline{Z_x} = \frac{\overline{Z_2\, Z_3}}{\overline{Z_1}} = \overline{Z_2\, Z_3}\ \overline{Y_1} \qquad \text{... (1)}$$

where $\overline{Y_1} = \dfrac{1}{\overline{Z_1}}$ i.e. R_1 in parallel with C_1

$$\overline{Z_2} = R_2$$

$$\overline{Z_3} = R_3$$

$\overline{Z_x} = R_x + j\,\omega L_x$, as L_x in series with R_x

Now $\overline{Y_1} = \frac{1}{R_1} + j\,\omega C_1$

$\overline{Z_1} = R_1 \,||\, j\left(\frac{1}{\omega C_1}\right)$ as $\frac{1}{j} = -j$

Substituting all the values in equation (1)

we get,

$$R_x + j\,\omega L_x = R_2\, R_3 \left[\frac{1}{R_1} + j\omega C_1\right]$$

$$R_x + j\,\omega L_x = \frac{R_2\, R_3}{R_1} + j\, R_2\, R_3\, \omega C_1$$

Equating real parts,

$$\boxed{R_x = \frac{R_2\, R_3}{R_1}}$$

Equating imaginary parts,

$$\omega L_x = R_2\, R_3\, \omega C_1$$

$$\boxed{L_x = R_2\, R_3\, C_1}$$

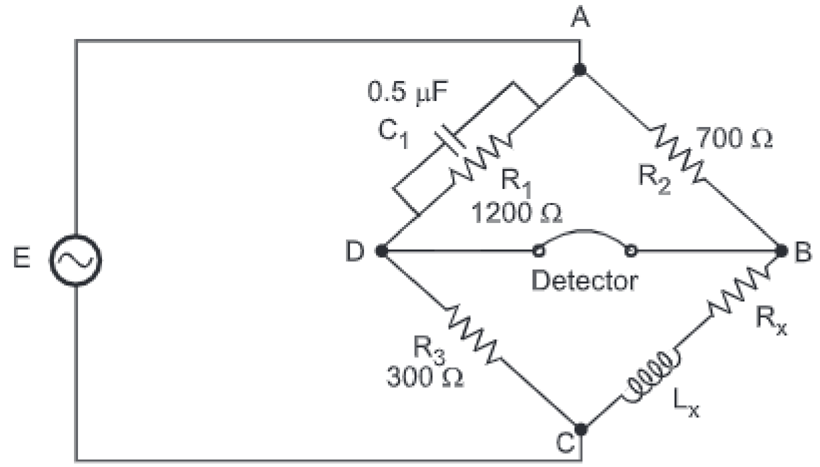
The quality factor of the coil is given by,

$$Q = \frac{\omega L_x}{R_x}$$

$$= \frac{\omega R_2\, R_3\, C_1}{\left(\frac{R_2\, R_3}{R_1}\right)}$$

$$\boxed{Q = \omega R_1\, C_1}$$

The arms of an a.c. Maxwell's bridge are adjusted as :
Arm AB : non-reactive resistance of 700 Ω
Arm CD : non-reactive resistance of 300 Ω
Arm AD : non-reactive resistance of 1200 Ω in parallel with capacitor of 0.5 μF.
If the bridge is balanced under this condition, find the components of the branch BC.



Solution : The bridge is shown in the Fig.

From the bridge,

$$C_1 = 0.5\ \mu F, \quad R_1 = 1200\ \Omega$$

$$R_2 = 700\ \Omega, \quad R_3 = 300\ \Omega$$

From bridge balance equation,

$$R_x = \frac{R_2\ R_3}{R_1} = \frac{700 \times 300}{1200} = \mathbf{175\ \Omega}$$

And

$$L_x = R_2\ R_3\ C_1 = 700 \times 300 \times 0.5 \times 10^{-6} = \mathbf{105\ mH}$$

Anderson Bridge

It is another important a.c. bridge used for the measurement of self inductance interms of a standard capacitor.

Actually this bridge is modified Maxwell's bridge in

the value of self inductance is obtained by comparing it with a standard capacitor.

The Anderson bridge is as shown in the Fig.

One arm of the bridge consists of unknown inductor L_x with known resistance in series with L_x.

This resistance R_1 includes resistance of the inductor.

C is the standard capacitor with r, R_2, R_3 and R_4 are non-inductive known resistances.

The bridge balance equations are,

$$i_1 = i_3,$$

$$i_2 = i_4 + i_C,$$

$$V_2 = i_2 R_2,$$

$$V_3 = i_3 R_3$$

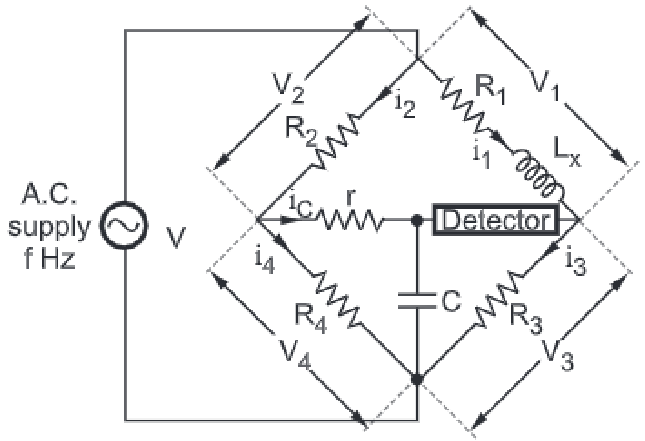
$$V_1 = V_2 + i_C r$$

$$V_4 = V_3 + i_C r,$$

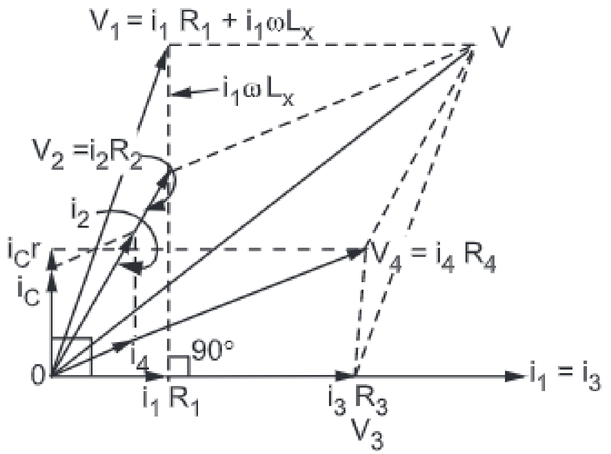
$$V_1 = i_1 R_1 + i_1 \omega L_1,$$

$$V_4 = i_4 R_4$$

$$V = \overline{V}_2 + \overline{V}_4 = \overline{V}_1 + \overline{V}_3$$



(a) Circuit diagram



(b) Phasor diagram

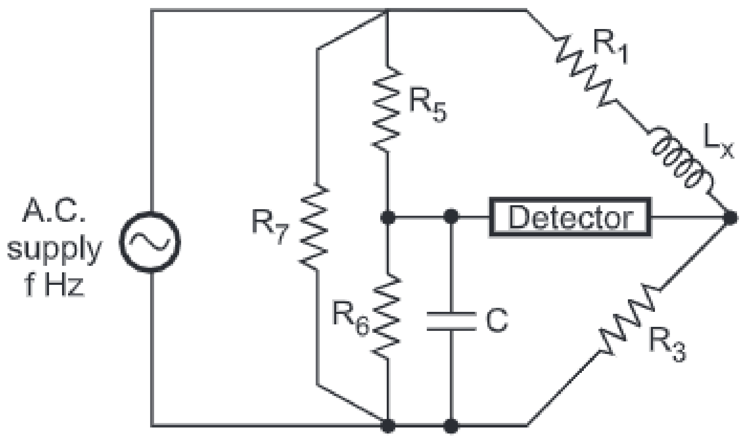
To find balance equations transforming a star formed by R_2, R_4 and r into its equivalent delta as shown in the Fig. (a) and (b).

The elements in equivalent delta are given by,

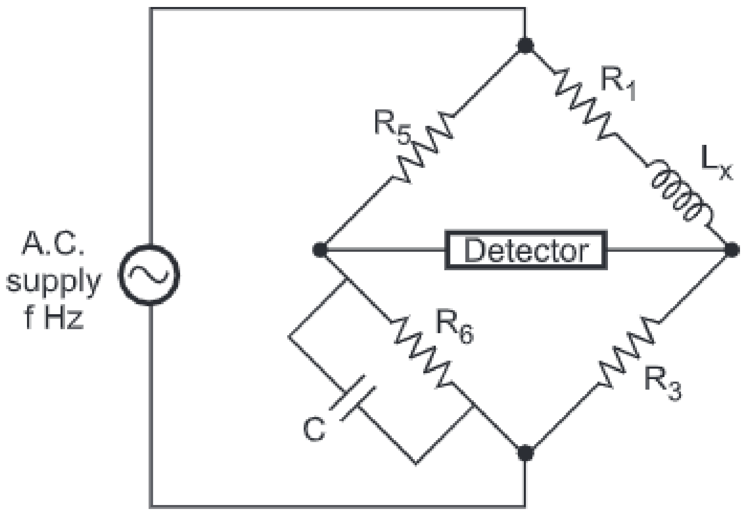
$$R_5 = \frac{R_2 r + R_4 r + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 r + R_4 r + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 r + R_4 r + R_2 R_4}{r}$$



(a)



(b)

Thus, balance equations are given by,

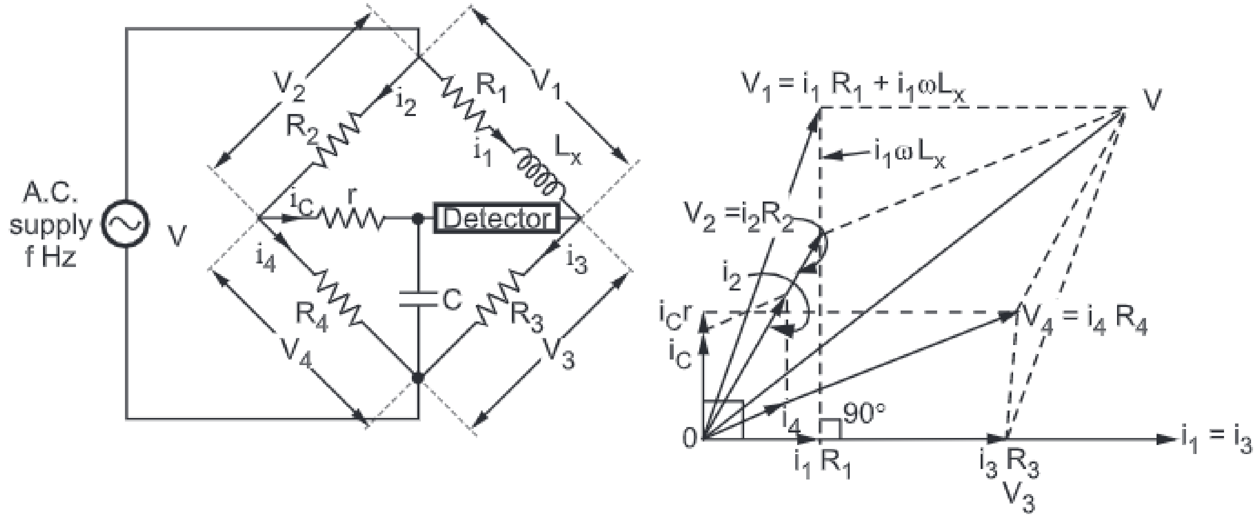
$$L_x = CR_3R_5$$

$$R_1 = R_3\frac{R_5}{R_6}$$

Substituting values of R_5 and R_6, we can write,

$$\boxed{L_x = \frac{CR_3}{R_4}[R_2r + R_4r + R_2R_4] \quad \text{and}}$$

$$\boxed{R_1 = \frac{R_2R_3}{R_4}}$$



(a) Circuit diagram (b) Phasor diagram

If the capacitor used is not perfect, the value of inductance remains unchanged, but the value of R_1 changes.

This method can also be used to measure the capacitance of the capacitor C if a calibrated self inductance is available.

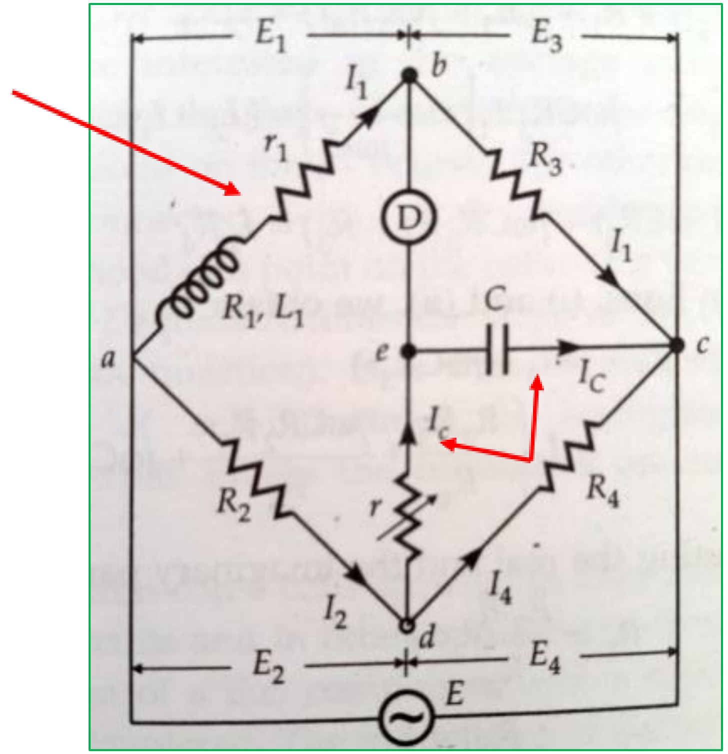
Advantages of Anderson Bridge

1. Can be used for accurate measurement of capacitance interms of inductance.
2. Other bridges require variable capacitor but a fixed capacitor can be used for Anderson's bridge.
3. The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q.

Disadvantages of Anderson Bridge

1. It is more complicated than other bridges.
2. Uses more number of components.
3. Balance equations are also complicated to derive.
4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

Anderson's Bridge Phasor diagram



At Balance, Ig = 0

Vbc = Vec

-----➐i.e E3 = Ic/ jwC

i1R3 = Ic/ jwC

Similarly,

Vab = Vade

------➐i.e E1 = I2R2 + Ic*r

but, E1 = I1 (r1+R1)+ I1(jwL1)

Therefore, I1 (r1+R1)+ I1(jwL1) = I2R2 + Ic*r

And also,

Vced = Vcd

--------➐but, Vced = Ic/jwc + Ic*r & Vcd = I4R4 = E4

At Balance, Ig = 0

Vbc = Vec

-----❼i.e E3 = Ic/ jwC

$\boxed{\text{i1R3 = Ic/ jwC}}$

Vab = Vade

------❼i.e E1 = I2R2 + Ic*r

$\boxed{\text{E1 = I1 (r1+R1)+ I1(jwL1}}$ $\boxed{\text{I2R2 + Ic*r}}$

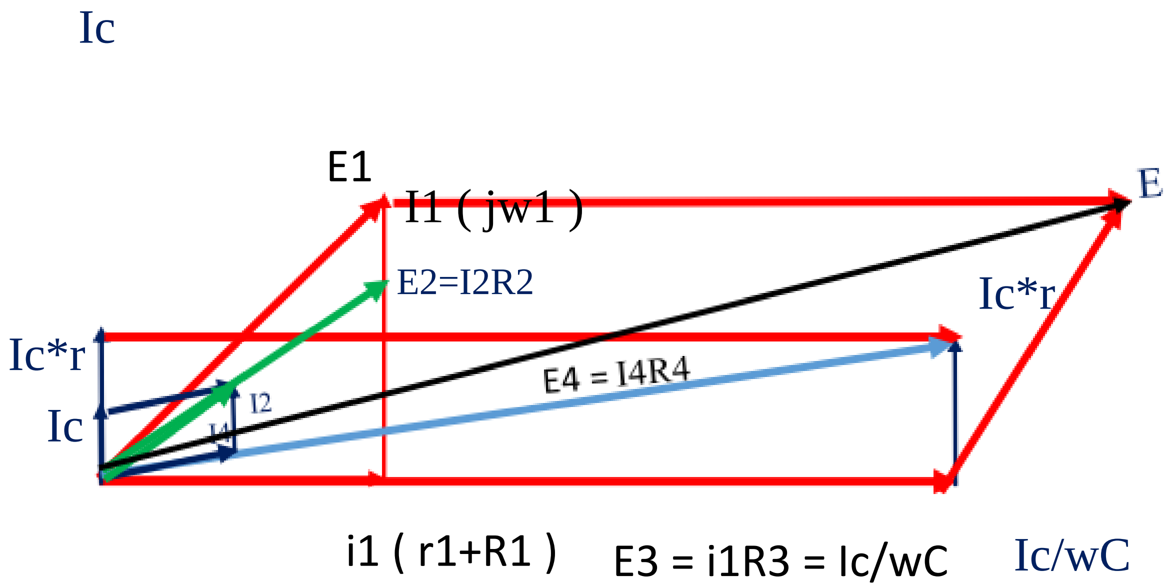
Therefore, I1 (r1+R1)+ I1(jwL1) = I2R2 + Ic*r

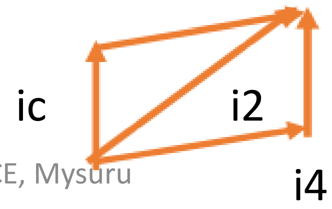
I1 (r1+R1)+ I1(jwL1) - Ic*r = I2R2

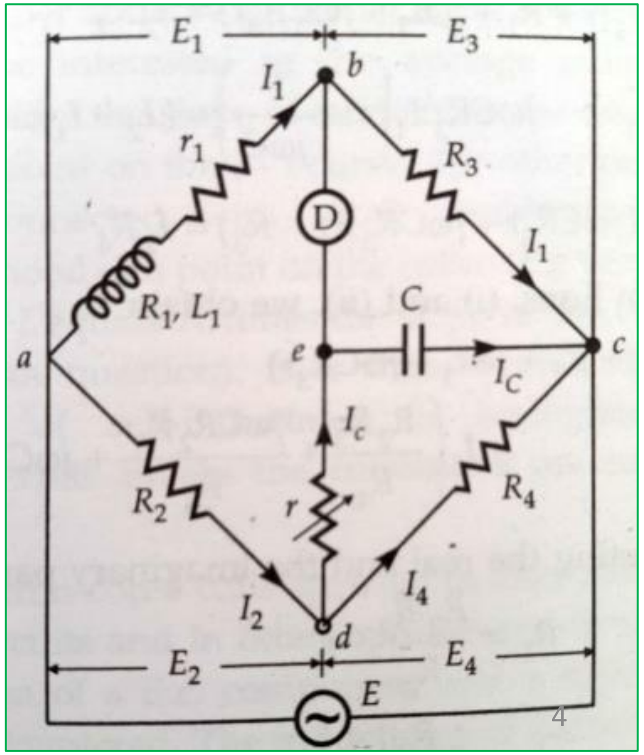
$\boxed{\text{Since I2 = Ic+I4}}$

Vced = Vcd

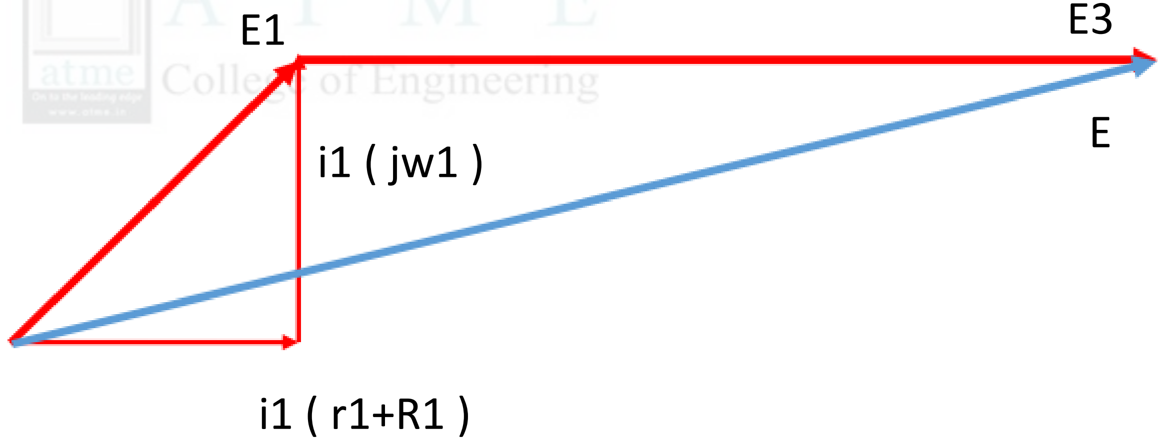
--------❼but, Vced = Ic/jwc + Ic*r & Vcd = I4R4 = E4

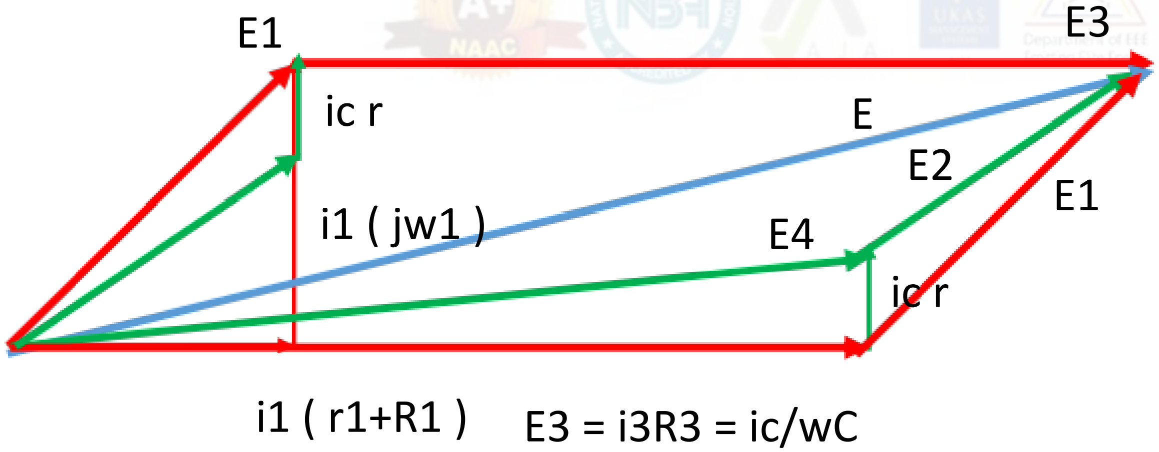


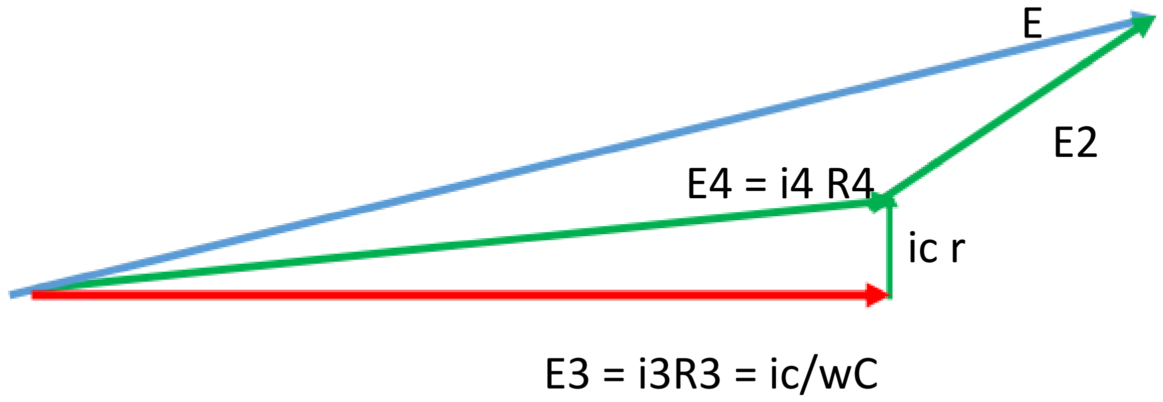


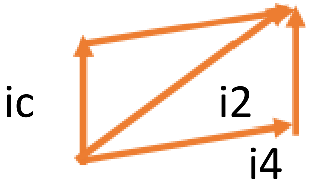


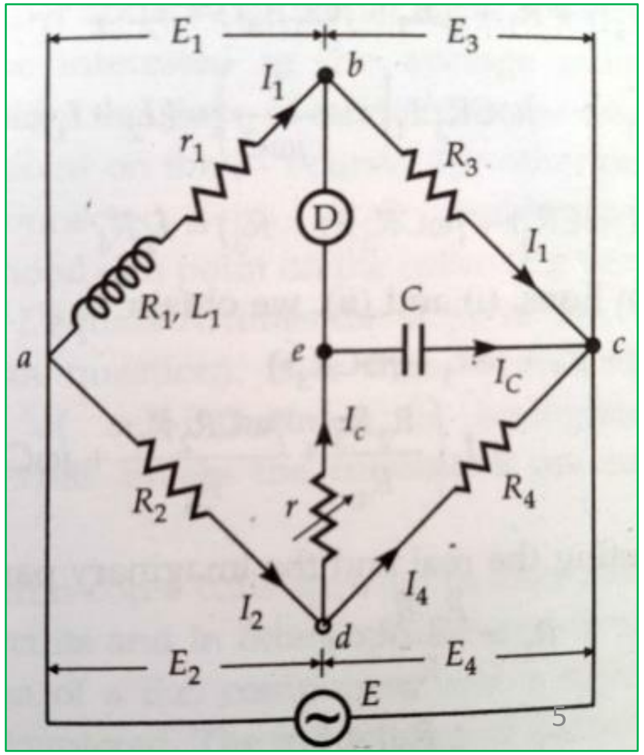
Anderson’s Bridge Phasor diagram

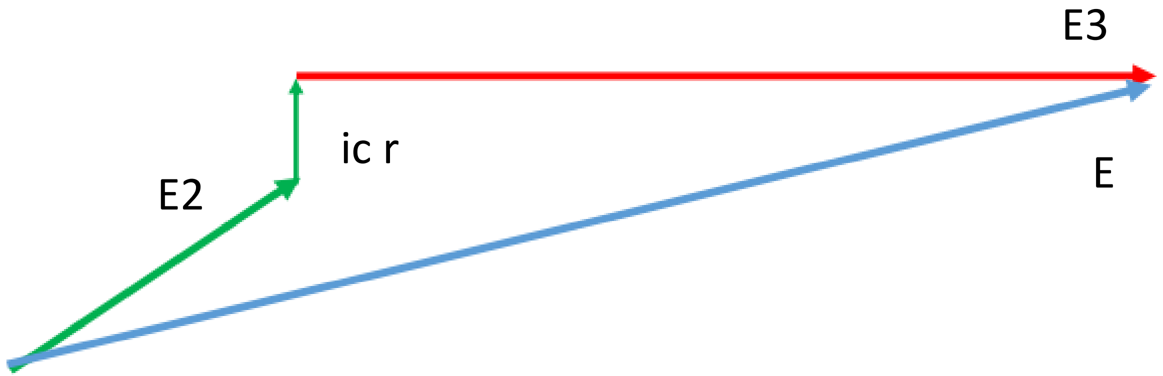




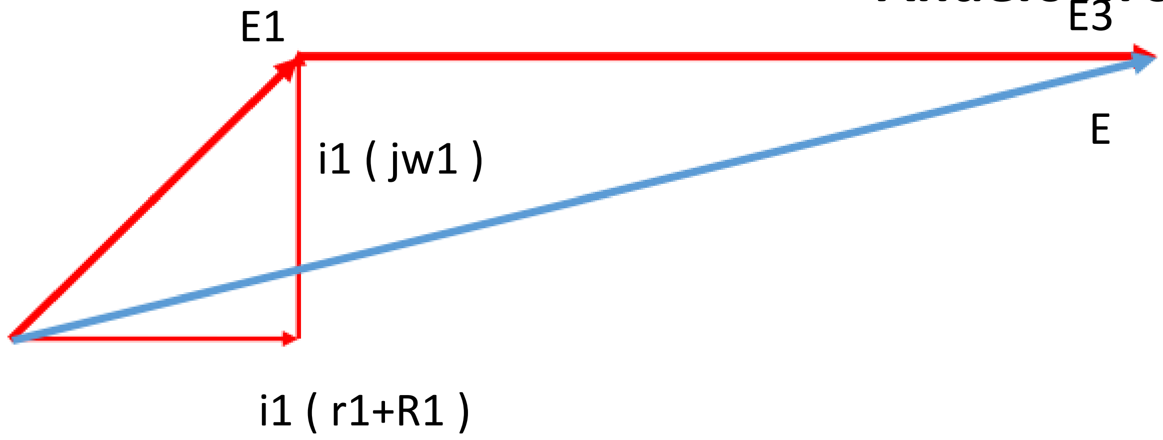


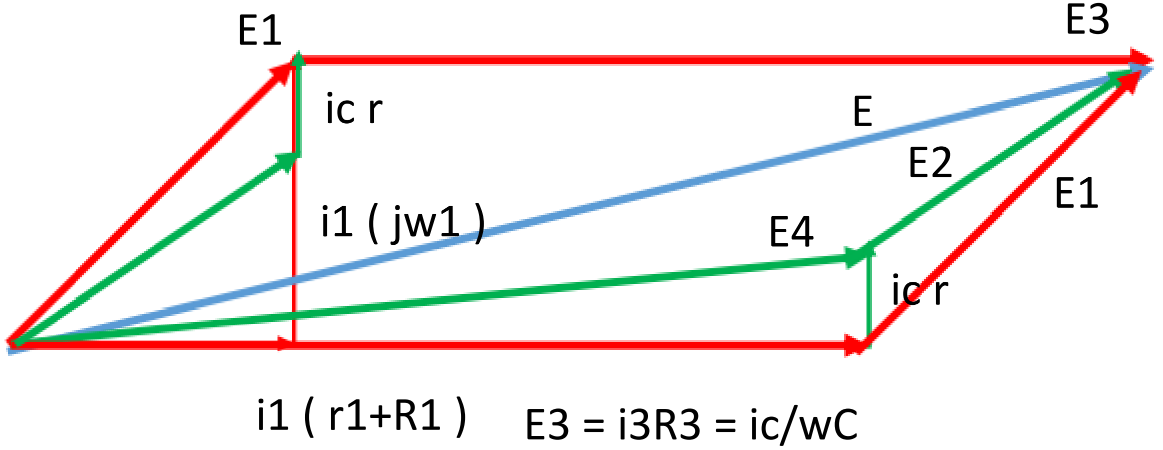


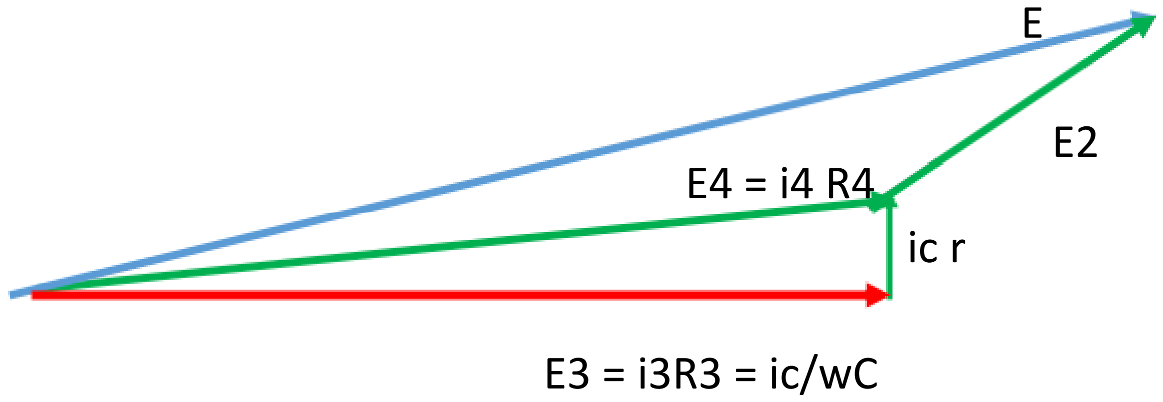


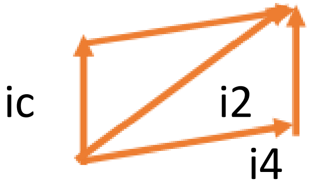


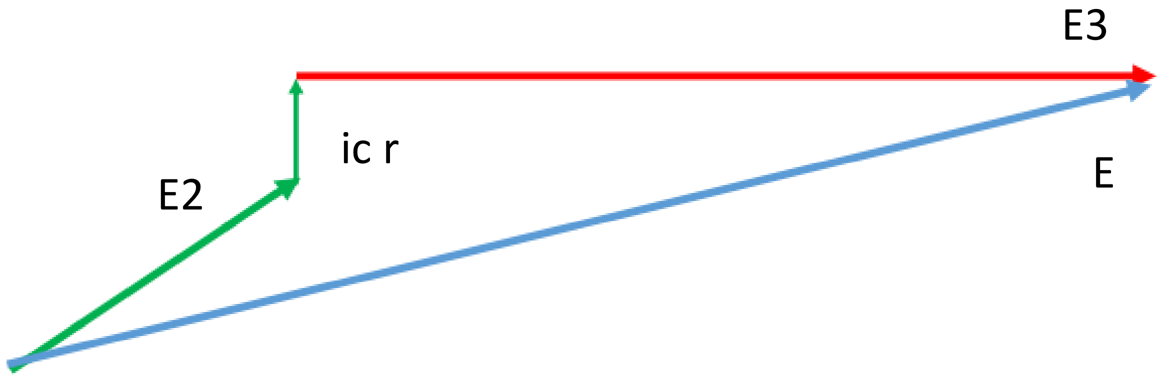
Anderson's Bridge Phasor diagram

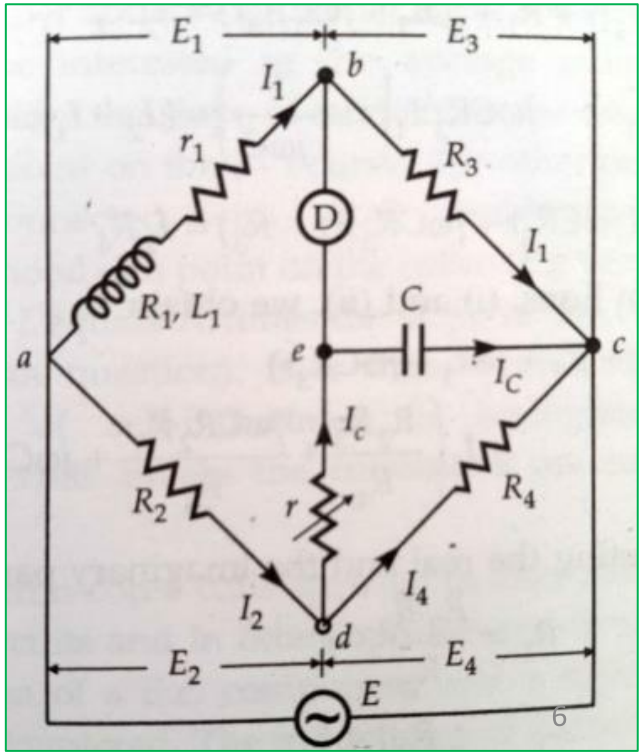


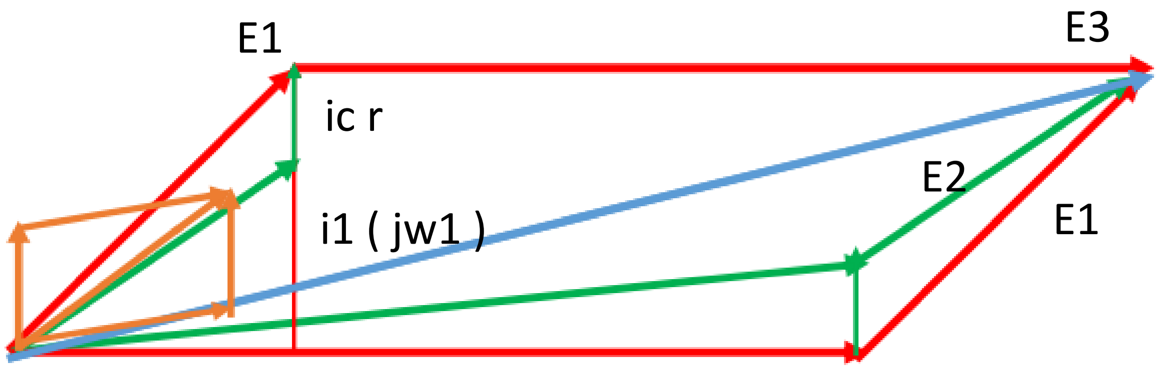




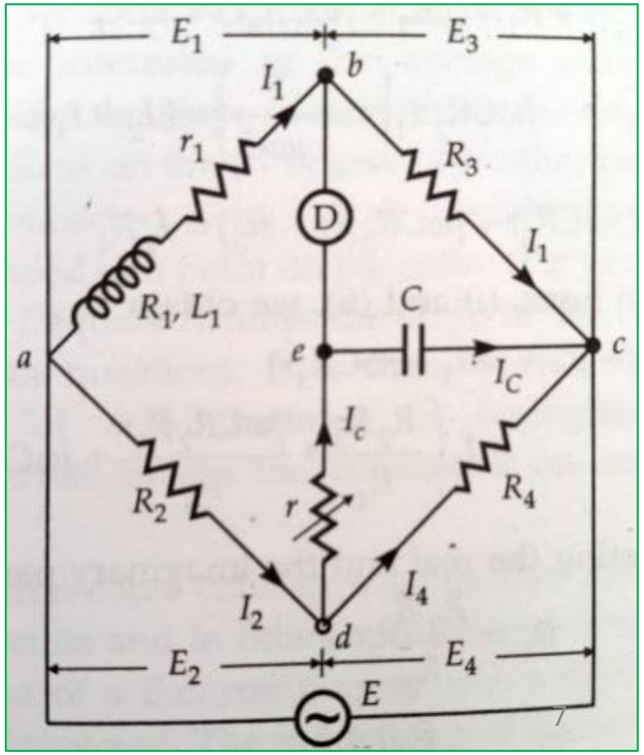







E1
E3
ic r
E2
E1
i1 (jw1)

E3 = i3R3 = ic/wC


E_1
E_3
b
I_1
r_1
R_3
D
R_1, L_1
I_1
C
a
e
c
I_C
I_c
R_2
r
R_4
I_2
I_4
d
E_2
E_4
E

An Anderson a.c. bridge is as follows :

Arm AB : Unknown inductance R_x and L_x ,

Arm BC : Non-reactive resistance $R_2 = 1000\ \Omega$

Arm CD : Non-reactive resistance $R_4 = 1000\ \Omega$

Arm DA : Non-reactive resistance $R_3 = 500\ \Omega$

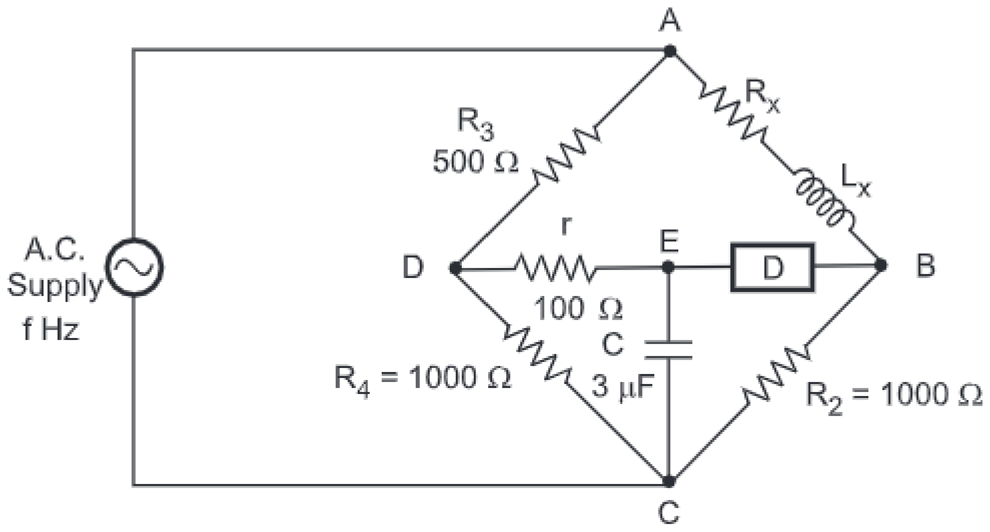
Arm DE : Resistance $r = 100\ \Omega$

Arm EB : Detector and a.c. supply between AC

Arm EC : Capacitor $C = 3\ \mu F$

State the expressions for L_x and R_x and find the values of them for given values of elements.

Solution : The bridge is shown in the Fig.

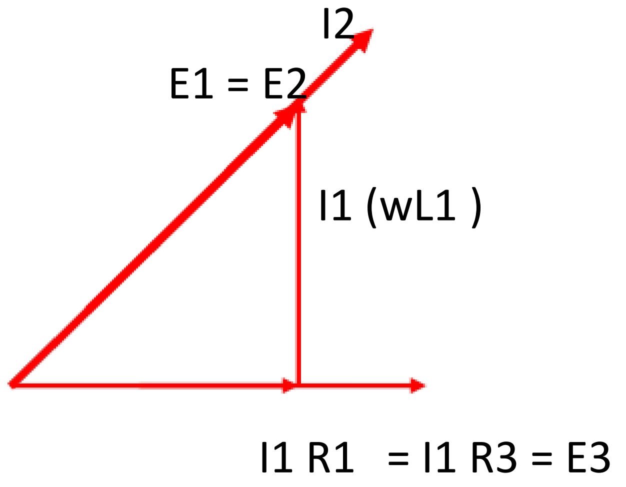


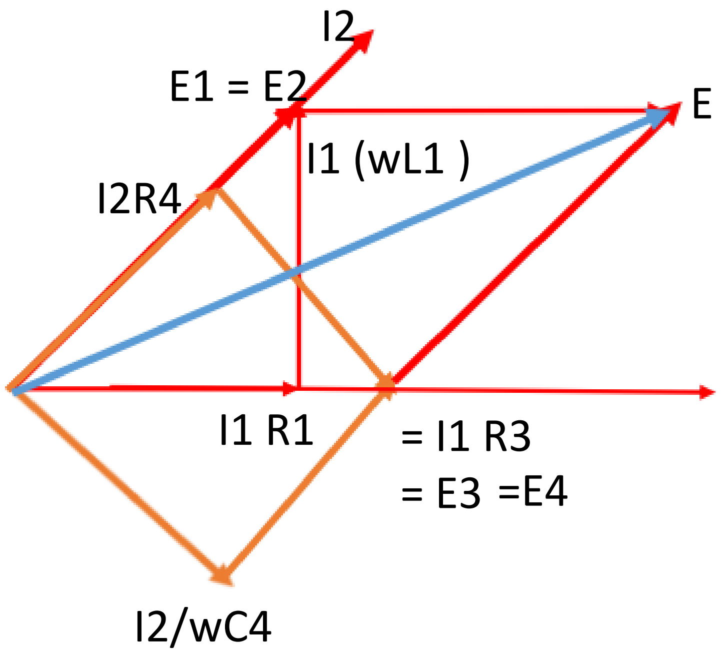
$$R_x = \frac{R_2\ R_3}{R_4} = \frac{1000 \times 500}{1000} = \mathbf{500\ \Omega}$$

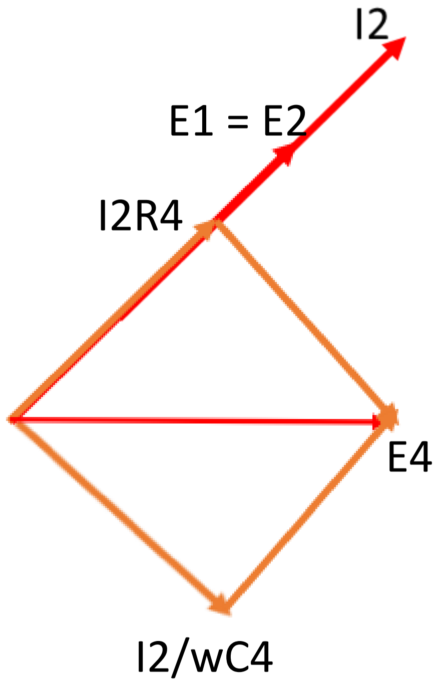
$$L_x = \frac{C\,R_2}{R_4}\left[R_3\ r + R_4\ r + R_3\ R_4\right]$$

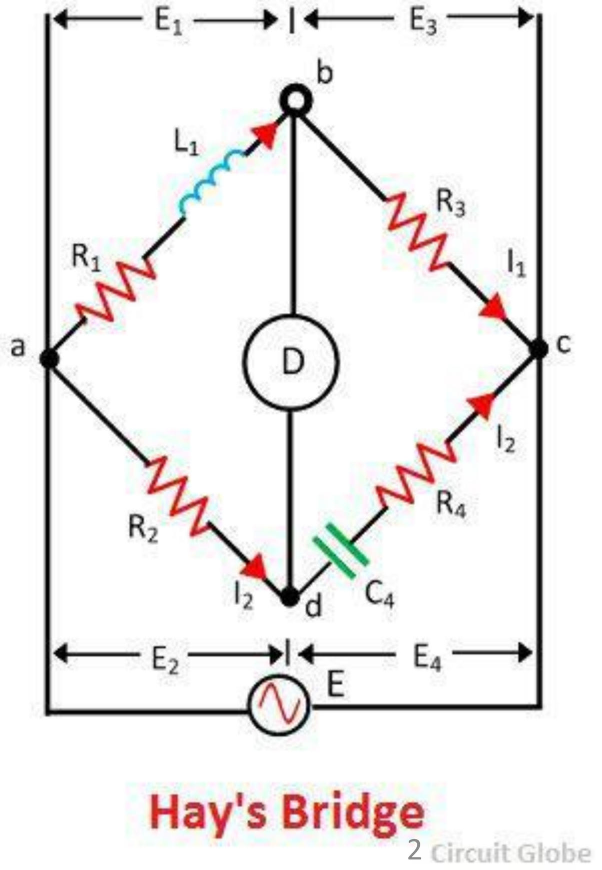
$$= \frac{3 \times 10^{-6} \times 1000}{1000}\left[500 \times 100 + 1000 \times 100 + 500 \times 1000\right] = \mathbf{1.95\ H}$$

Hay's Bridge


I2
E1 = E2
I1 (wL1)
I1 R1 = I1 R3 = E3


I2
E1 = E2
E
I1 (wL1)
I2R4
I1 R1
= I1 R3
= E3 =E4
I2/wC4


I2
E1 = E2
I2R4
E4
I2/wC4


E_1
E_3
b
L_1
R_3
R_1
I_1
a
D
c
I_2
R_4
R_2
I_2
d
C_4
E_2
E_4
E
Hay's Bridge
Circuit Globe

Hay's Bridge

The limitation of Maxwell's bridge is that it cannot be used for high Q values.

Hay's bridge is suitable for the coils having high Q values.

Hay's bridge consists of resistance R_1 in series with the standard capacitor C_1 in one of the ratio arms.

larger phase angles R_1 needed is very low, which is practicable.

Hence bridge can be used for the coils with high Q values.

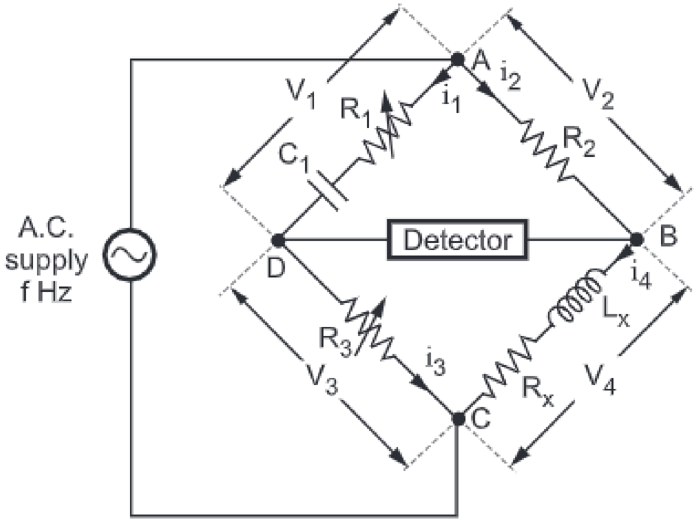
Under balanced condition,

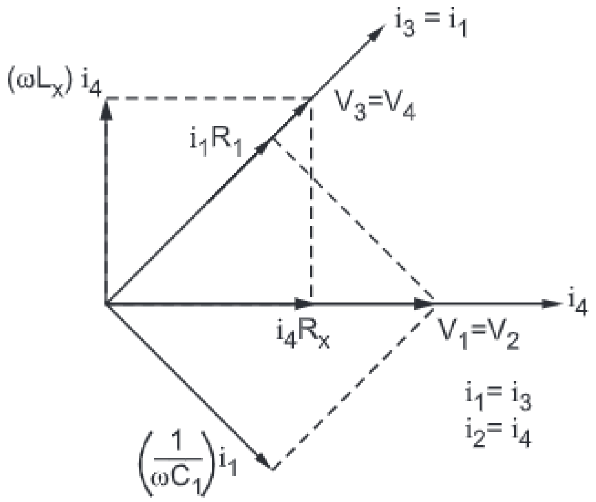
The various constants of the bridge are :

$$Z_1 = R_1 - j X_{C_1} = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2 \quad \text{and} \quad Z_3 = R_3$$

$$Z_4 = Z_x = R_x + j\,(\omega L_x)$$





At the balance condition,

$$\overline{Z_1\ Z_x} = \overline{Z_2\ Z_3}$$

$$\therefore \qquad \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right]\left[R_x + j\,(\omega L_x)\right] = R_2\ R_3$$

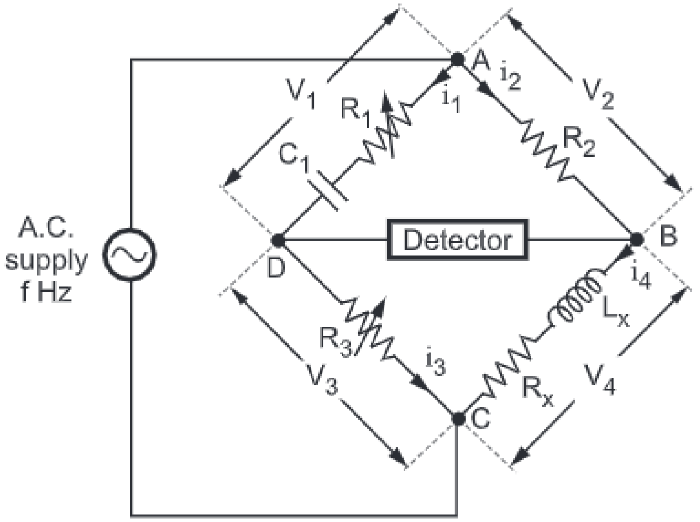
$$\therefore \qquad \left[R_x\ R_1 + \frac{L_x}{C_1}\right] + j\left[\omega R_1\ L_x - \frac{R_x}{\omega C_1}\right] = R_2\ R_3 \qquad \dots (1)$$

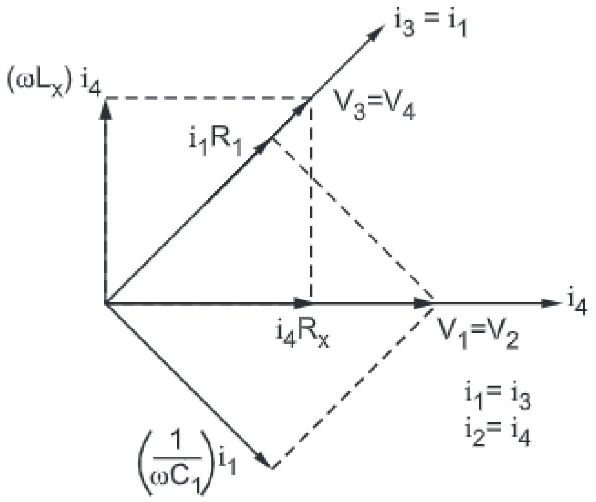
Equating the real parts of both sides,

$$R_x\ R_1 + \frac{L_x}{C_1} = R_2\ R_3 \qquad \dots (2)$$

Equating the imaginary parts of both sides of equation (1),

$$\omega R_1\ L_x - \frac{R_x}{\omega C_1} = 0 \qquad \dots (3)$$





To obtain R_x and L_x, solve equation (2) and equation (3) simultaneously.

From equation (3),

$$\omega R_1 L_x = \frac{R_x}{\omega C_1}$$

$$L_x = \frac{R_x}{\omega^2 R_1 C_1} \quad \dots (4)$$

Substituting in equation (2),

$$R_x R_1 + \frac{R_x}{\omega^2 R_1 C_1{}^2} = R_2 R_3$$

$$R_x \left[R_1 + \frac{1}{\omega^2 R_1 C_1{}^2} \right] = R_2 R_3$$

$$R_x \left[\frac{1+\omega^2 R_1^2 C_1{}^2}{\omega^2 R_1 C_1{}^2} \right] = R_2 R_3$$

$$\boxed{R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1+\omega^2 R_1^2 C_1{}^2}} \quad \dots (5)$$

Substituting equation (5) in equation (4) we get,

$$L_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{(1+\omega^2 R_1^2 C_1{}^2)\, \omega^2 R_1 C_1}$$

$$\boxed{L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1{}^2}} \quad \dots (6)$$

This is same as Maxwell's bridge equation.

De Sauty Bridge

De Sauty bridge is to compare two capacitance.

The circuit diagram of De Sauty bridge is as shown in the Fig.

The De Sauty bridge consists a capacitor C_1 which is capacitance under test in branch AB.

The branch AD consists a known, standard capacitor.

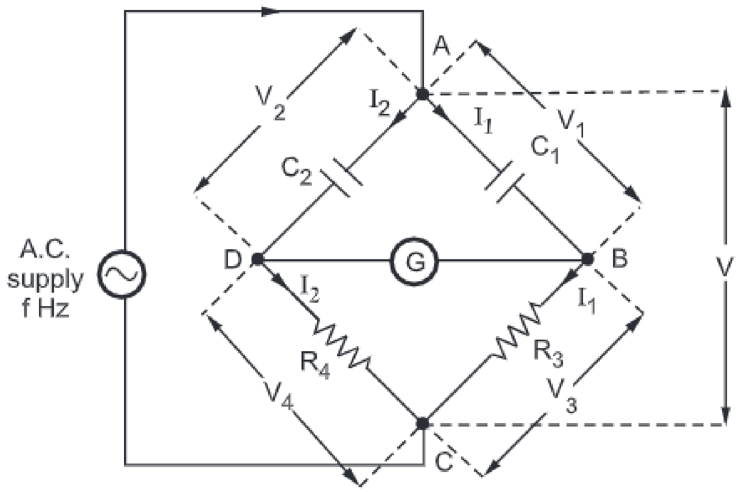
The remaining branches BC and CD consists non-inductive resistances R_3 and R_4 respectively.

At balance, we get condition as,

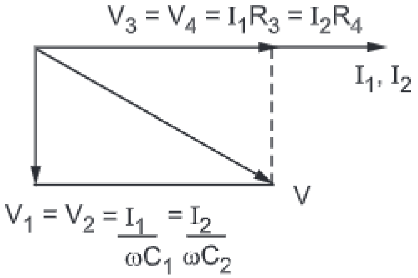
$$\frac{-j X_{C_1}}{R_3} = \frac{-j X_{C_2}}{R_4} \qquad \dots (1)$$

$$\left(\frac{-j}{\omega C_1}\right) R_4 = \left(\frac{-j}{\omega C_2}\right) R_3$$

$$\boxed{C_1 = \frac{C_2 \cdot R_4}{R_1}} \qquad \dots (2)$$



(a) Circuit diagram



(b) Phasor diagram

The balance in the bridge can be achieved by varying either R_3 or R_4. Under the balanced condition the vector diagram for the De Sauty bridge is as shown in the Fig.

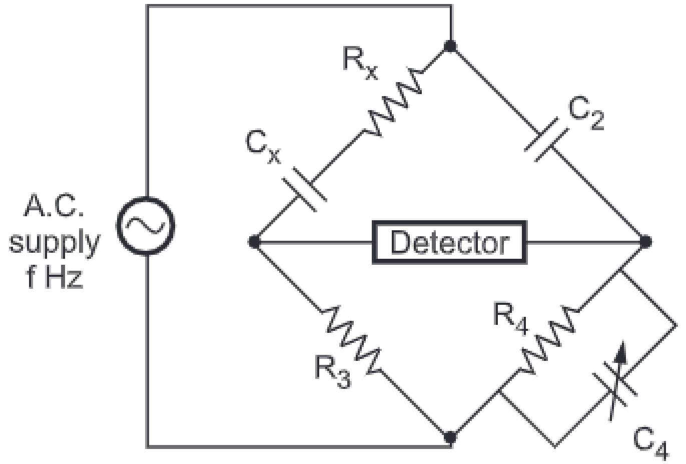
The main advantage of De Sauty bridge is that it is very simple in construction and the measurement of the capacitor is done with simplicity. Inspite of the simplicity of the measurement, it has one drawback that when the capacitors are with dielectric losses, then it is highly impossible to achieve balance condition. Thus practically De Sauty can be used effectively for the capacitors with very low dielectric losses such as air capacitors.

Schering Bridge

It is one of the most widely used a.c. bridges for the measurement of unknown capacitors, dielectric loss and power factor.

It can be used for low voltages.

The C_x is perfect capacitor to be measured. R_x is series resistance. C_2 is standard air capacitor having very stable value. R_3 and R_4 are non-inductive resistances while C_4 is variable capacitor.



From the general balance equation,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$Z_1 = R_x - j\frac{1}{\omega C_x}$$

$$Z_4 = R_4 || \frac{-j}{\omega C_4} = \frac{R_4\left(-\frac{j}{\omega C_4}\right)}{\left(R_4 - j\frac{1}{\omega C_4}\right)}$$

$$Z_2 = -\frac{j}{\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = \frac{-jR_4}{\omega R_4 C_4 - j} = \frac{-jR_4(\omega R_4 C_4 + j)}{(\omega R_4 C_4 - j)(\omega R_4 C_4 + j)} = \frac{R_4 - j\omega R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1}$$

$$Z_1 = \frac{Z_2 Z_3}{Z_4} = \frac{\left(-\frac{j}{\omega C_2}\right)(R_3)}{\left(\frac{R_4 - j\omega R_4^2 C_4}{1 + \omega^2 R_4^2 C_4^2}\right)} = \frac{(1 + \omega^2 R_4^2 C_4^2) R_3 \left(-\frac{j}{\omega C_2}\right)}{(R_4 - j\omega R_4^2 C_4)}$$

$$\text{Rationalising, } Z_1 = R_3(1 + \omega^2 R_4^2 C_4^2)\left\{\frac{-\frac{j}{\omega C_2}(R_4 + j\omega R_4^2 C_4)}{R_4^2 + \omega^2 R_4^4 C_4^2}\right\}$$

$$\therefore \quad R_x - j\frac{1}{\omega C_x} = \frac{R_3(1 + \omega^2 R_4^2 C_4^2)}{R_4^2(1 + \omega^2 R_4^2 C_4^2)}\left\{\frac{R_4^2 C_4}{C_2} - \frac{jR_4}{\omega C_2}\right\}$$

Equating real and imaginary parts,

$$\boxed{R_x = \frac{R_3}{R_4^2} \times \frac{R_4^2\, C_4}{C_2} = \frac{R_3\, C_4}{C_2}} \qquad \text{... (1)}$$

$$-j\frac{1}{\omega C_x} = -j\frac{R_3}{R_4^2} \times \frac{R_4}{\omega C_2} = -j\left[\frac{1}{\frac{R_4}{R_3}\,\omega C_2}\right]$$

$$\omega C_x = \frac{R_4}{R_3}\,\omega C_2$$

$$\boxed{C_x = \frac{R_4}{R_3}\, C_2} \qquad \text{... (2)}$$

The equations (1) and (2) gives the required values of C_x and R_x.

Power Factor and Loss Angle

i) **Power factor (p.f.) :** The power factor of the series RC combination is defined as the cosine of the phase angle of the circuit. Thus,

$$\text{p.f.} = \cos \phi_x = \frac{R_x}{Z_x}$$

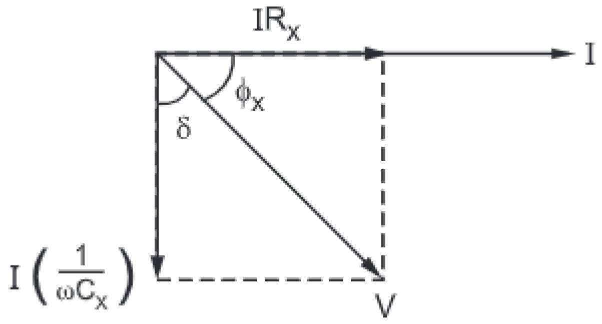
For phase angles very close to 90°, the reactance is almost equal to the impedance,

$$\therefore \qquad \text{p.f.} = \frac{R_x}{X_x} = \frac{R_x}{\left(\frac{1}{\omega C_x}\right)}$$

$$\boxed{\text{p.f.} = \omega R_x C_x}$$

ii) Loss angle (δ) : For a series combination of R_x and C_x, the angle between the voltage across the series combination and voltage across the capacitor C_x is called **loss angle** δ.

$$\text{Now } \tan\delta = \frac{I R_x}{I\left(\frac{1}{\omega C_x}\right)} = \omega R_x C_x$$



$$\therefore \quad \tan\delta = \omega\left(\frac{R_3 C_4}{C_2}\right)\left(\frac{R_4}{R_3} C_2\right) = \omega R_4 C_4$$

Thus loss angle can be measured, knowing the values of ω, R_4 and C_4.

iii) Dissipation factor (D) : For R_x - C_x series circuit, it is cotangent of the phase angle ϕ_x.

$$\boxed{D = \cot\phi_x = \frac{1}{\tan\phi_x} = \frac{1}{\left[\frac{I\left(\frac{1}{\omega C_x}\right)}{I R_x}\right]} = \omega R_x C_x = \omega R_4 C_4}$$

The Schering bridge has the following constants :

Arm AB - capacitor of 1 μF in parallel with 1.2 kΩ resistance
Arm AD - resistance of 4.7 kΩ
Arm BC - capacitor of 1 μF
Arm CD - unknown capacitor C_x and R_x.
The frequency of supply is 0.5 kHz. Calculate the unknown capacitance and its dissipation factor.

Solution : From the given information,

$$R_1 = 1.2\ \text{k}\Omega \qquad C_1 = 1\ \mu\text{F}$$

$$R_2 = 4.7\ \text{k}\Omega \qquad C_3 = 1\ \mu\text{F}$$

From the balance equations,

$$R_x = \frac{R_2\, C_1}{C_3} = \frac{4.7\times 10^3 \times 1\times 10^{-6}}{1\times 10^{-6}} = \mathbf{4.7\ k\Omega}$$

$$C_x = \frac{R_1\, C_3}{R_2} = \frac{1.2\times 10^3 \times 1\times 10^{-6}}{4.7\times 10^3}$$

$$= \mathbf{0.255\ \mu F}$$

The dissipation factor,

$$D = \omega C_x R_x = 2\pi f C_x R_x$$

$$= 2\pi \times 0.5\times 10^3 \times 0.255\times 10^{-6} \times 4.7\times 10^3$$

$$= \mathbf{3.765}$$

Given the Maxwell bridge as shown in the Fig. find the equivalent series resistance and inductance of R_x *and* L_x *at balance.*

$$Z_1 = R_1 \,||\, X_{C1}$$

$$Y_1 = \frac{1}{R_1} + j\,\omega C_1$$

$$= \frac{1}{600} + j\left(2\pi \times 1000 \times 1 \times 10^{-6}\right)$$

$$= 1.66 \times 10^{-3} + j\,6.283 \times 10^{-3}$$

$$Z_2 = 100\ \Omega$$

$$Z_3 = 1000\ \Omega$$

$$Z_4 = R_x + j\,X_L = Z_x$$

From the basic balance equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1$$

$$= 100 \times 1000 \times [1.66 \times 10^{-3} + j\ 6.283 \times 10^{-3}]$$

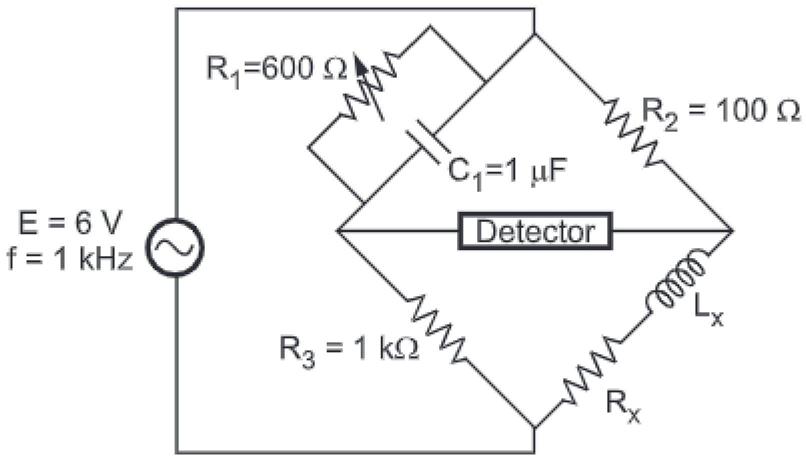
$$= 166 + j\ 628.3\ \Omega = R_x + j\,X_L\ \Omega$$

$$R_x = \mathbf{166\ \Omega}$$

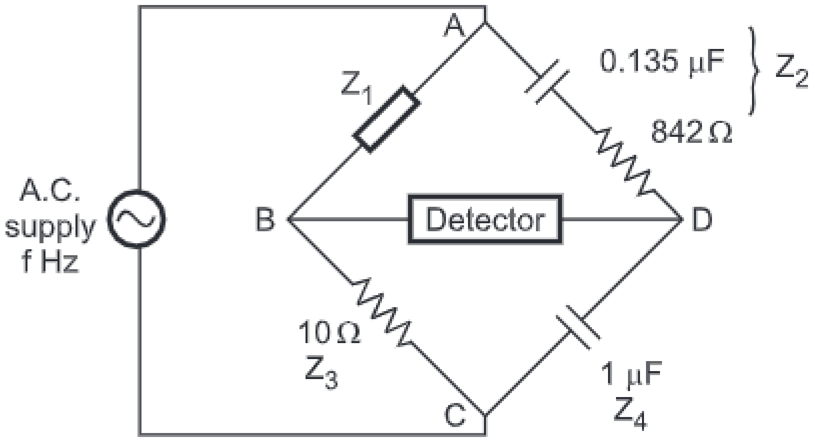
$$X_L = 628.3 = 2\pi f L_x$$

$$L_x = \frac{628.3}{2\pi \times 1000}$$

$$\mathbf{= 0.099\ H}$$



An a.c. bridge circuit for measurement of effective inductance and capacitance of an iron cored coil is as follows : Arm AB : the unknown impedance, Arm BC : a pure resistance of 10 Ω, Arm CB : a loss free capacitance of 1 μF and Arm AD : a capacitance of 0.135 μF in series with 842 Ω resistance. Obtain the balance equations of the bridge and determine the unknown parameters in the arm AB.



The general balance equation is,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \quad \boxed{Z_1 = \frac{Z_2 Z_3}{Z_4}}$$

Let the frequency is ω rad/sec.

$$Z_2 = \left[842 - j\frac{1}{\omega\, 0.135\times 10^{-6}}\right] = 842 - j\frac{7.4074\times 10^{6}}{\omega}\ \Omega$$

$$Z_3 = 10 + j\,0 = 10\ \angle 0°\ \Omega$$

$$Z_4 = 0 - j\frac{1}{\omega \times 1 \times 10^{-6}} = -j\frac{10^6}{\omega} = \frac{10^6}{\omega}\angle{-90^\circ}\ \Omega$$

$$Z_1 = \frac{\left[842 - j\frac{7.4074 \times 10^6}{\omega}\right][10]}{\frac{10^6}{\omega}\angle{-90^\circ}} = \left[8420 - j\frac{7.4076 \times 10^7}{\omega}\right]\frac{1}{10^6}\angle{+90^\circ}$$

$$= \left[8420 - j\frac{7.4076 \times 10^7}{\omega}\right]\left[+j\frac{\omega}{10^6}\right] = j\omega\frac{8420}{10^6} + \frac{7.4076 \times 10^7}{10^6}$$

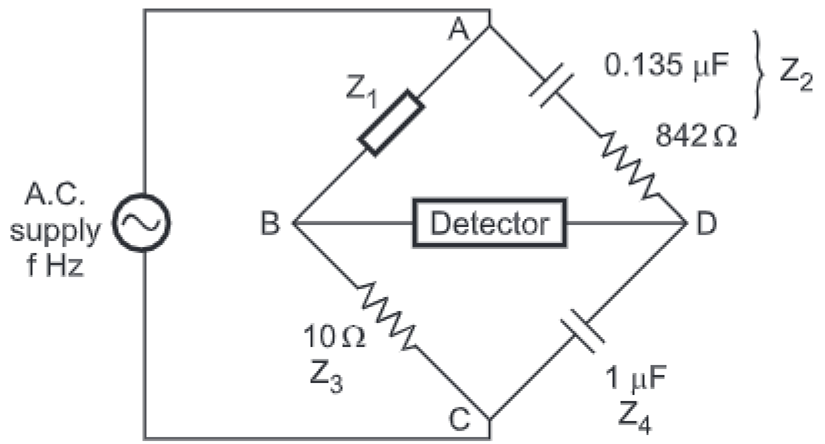
$$= 74.076 + j\omega\, 8420 \times 10^{-6}$$

Comparing Z_1 with $R_1 + j\omega L_1$

$\therefore \quad R_1 = \mathbf{74.076}\ \Omega$

$L_1 = 8420\ \mu H = \mathbf{8.42\ mH.}$

These are the unknown parameters of arm AB.



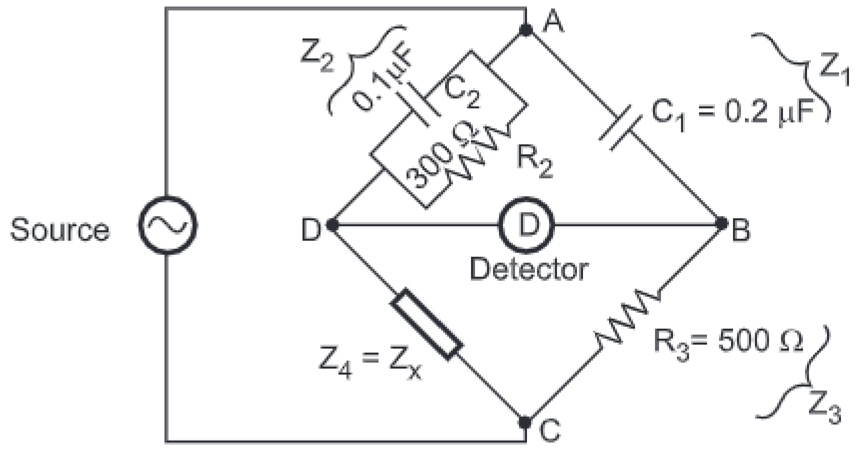
An a.c. bridge circuit working at 1 kHz have its arms as follows.

Arm AB : 0.2 μF capacitance *Arm BC : 500 Ω resistance*

Arm CD : unknown impedance

Arm DA : 300 Ω resistance in parallel with 0.1 μF capacitor.

Find R and L or C constants of the arm CD considering it as a series circuit.



Consider the basic a.c. bridge as shown in the Fig.

The branch AB consists $C_1 = 0.2$ μF. Hence impedance of branch AB i.e. Z_1 is given by,

$$Z_1 = -jX_{C_1} = -j\frac{1}{\omega C_1} = \frac{-j}{2\pi f C_1} = \frac{-j}{2\times\pi\times 1\times 10^3\times 0.2\times 10^{-6}}$$

$$Z_1 = -j\,795.77\,\Omega = 795.77\angle -90^\circ\ \Omega$$

Similarly, branch AD consists parallel combination of $R_2 = 300\ \Omega$ and $C_2 = 0.1\mu F$. Hence, impedance of branch AD i.e. Z_2 is given bv.

$$Z_2 = R_2 || -jX_{C_2} = R_2 \ || \left(\frac{-j}{\omega C_2}\right) = \frac{R_2\left(\frac{-j}{\omega C_2}\right)}{R_2 - \frac{j}{\omega C_2}}$$

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2} = \frac{R_2}{1 + j(2\times\pi\times f\times C_2\times R_2)} = \frac{300}{1 + j\left(2\times\pi\times 1\times 10^3\times 0.1\times 10^{-6}\times 300\right)} = \frac{300}{1 + j\,0.1885}$$

$$= \frac{300}{1.0176\angle + 10.695^\circ}$$

$$Z_2 = 294.81\ \angle - 10.67^\circ\ \Omega$$

For balance, the condition is given by,

$$Z_4 = \frac{Z_2 \cdot Z_3}{Z_1}$$

Substituting values of Z_1, Z_2 and Z_3, we get,

$$Z_4 = \frac{(294.81 \angle -10.67°)\ (500)}{795.77 \angle -90°}$$

$$Z_4 = 185.2356 \angle 79.33° \ \Omega$$

$$Z_4 = R_4 + j X_{L_4} = (34.2967 + j\,182.03)\ \Omega$$

Thus inductive reactance can be written as,

$$X_{L_4} = \omega L_4 = 2\pi f L_4 = 182.03$$

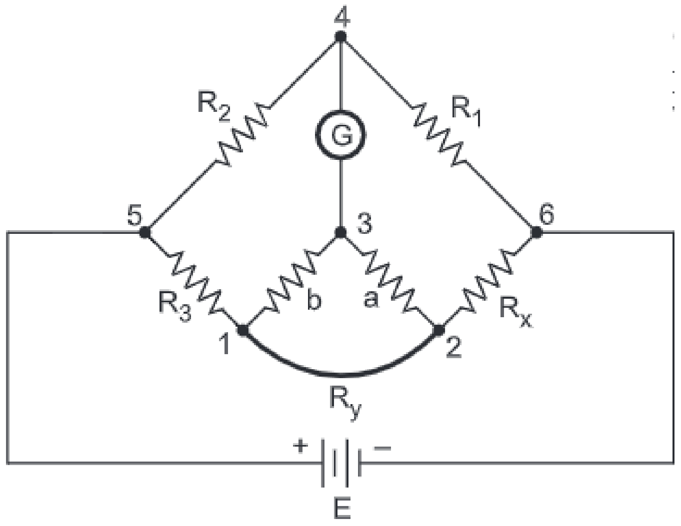
$$\therefore \quad 2 \times \pi \times 1 \times 10^3 \times L_4 = 182.03$$

$L_4 =$ **28.97 mH**

$R_4 =$ **34.2967 Ω**

Thus branch CD is a series R-L circuit consisting R_4 = 34.2967 Ω and L_4 = 28.97 mH.

A 4 terminal resistance of approximately 50 μΩ was measured with the help of Kelvin double bridge under the following conditions
Value of standard resistance 100.03 μΩ ;
Resistance of inner ratio arms 100.31 Ω and 200 Ω
Resistance of outer ratio arms 100.24 Ω and 200 Ω ;
Value of low resistance link 700 μΩ
Calculate the magnitude of error in the measurements.



A Kelvin double bridge can be drawn as shown in the Fig.

By using the expression of unknown resistance we can write,

$$R_x = \frac{R_1}{R_2} R_3 + \frac{bR_y}{a+b+R_y}\left[\frac{R_1}{R_2} - \frac{a}{b}\right]$$

$$R_x = \frac{100.24}{200}\left(100.03\times10^{-6}\right)+\frac{(200)\left(700\times10^{-6}\right)}{100.31+200+700\times10^{-6}}\left[\frac{100.24}{200} - \frac{100.31}{200}\right]$$

$$R_x = 49.97\times10^{-6}\ \Omega = \mathbf{49.97}\ \mu\Omega$$

Hence the magnitude of error in the measurement is given by,

$$|\text{ error }| = |\text{ Actual value of resistance} - \text{Measured value of resistance }|$$

$$= \left|50 \times 10^{-6} - 49.97 \times 10^{-6}\right| = 0.03 \times 10^{-6}$$

The four impedances of an a.c. bridge are

$Z_{AB} = 400\angle 50^\circ\ \Omega,\qquad Z_{AD} = 200\angle 40^\circ\ \Omega,$

$Z_{BC} = 800\angle -50^\circ\ \Omega,\qquad Z_{CD} = 400\angle 20^\circ\ \Omega$

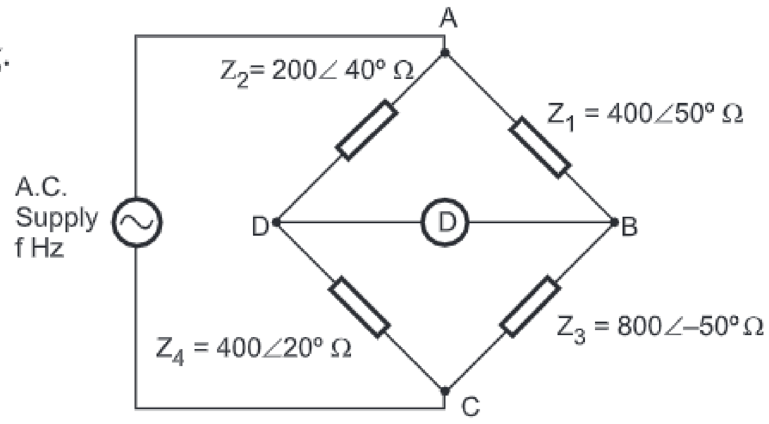
Find out whether the bridge is balanced under these conditions are not.

Solution : For an a.c. bridge, the balance conditions are given by,

$$Z_1 Z_4 = Z_2 Z_3 \qquad \text{... condition of balance for magnitudes}$$

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \qquad \text{... condition of balance for phases.}$$

Consider the basic a.c. bridge with four impedances as shown in the Fig.



Applying the condition of balance for the magnitudes, we get,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$(400)\ (400) = (200)\ (800)$$

That means condition of balance for magnitudes is satisfied.

Applying the condition of balance for phases, we get

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

$$\text{L.H.S.} = \angle\theta_1 + \angle\theta_4 = [50^\circ + 20^\circ] = 70^\circ$$

$$\text{R.H.S.} = \angle\theta_2 + \angle\theta_3 = [40^\circ - 50^\circ] = -10^\circ$$

As the values on L.H.S. and R.H.S. of equation (b) are not equal, **the condition of balance for phases is not satisfied.**

Thus for above given conditions, the bridge is in unbalanced condition because eventhough condition of balance for magnitudes is satisfied; condition of balance for phases is not satisfied.

A four arm a.c. bridge a-b-c-d has following impedances.

Arm ab : $Z_1 = 200\angle 60^\circ\ \Omega$, *Arm ad* : $Z_2 = 400\angle -60^\circ\ \Omega$

Arm bc : $Z_3 = 300\angle 0^\circ\ \Omega$, *Arm cd* : $Z_4 = 600\angle 30^\circ\ \Omega$

Determine whether it is possible to balance the bridge under above conditions.

The arms of five node bridge are as follows :

Arm ab : an unknown impedance (R_1, L_1) in series with a non-variable resistor r_1.

Arm bc : a non-inductive resistor $R_3 = 100\ \Omega$.

Arm cd : a non-inductive resistor $R_4 = 200\ \Omega$.

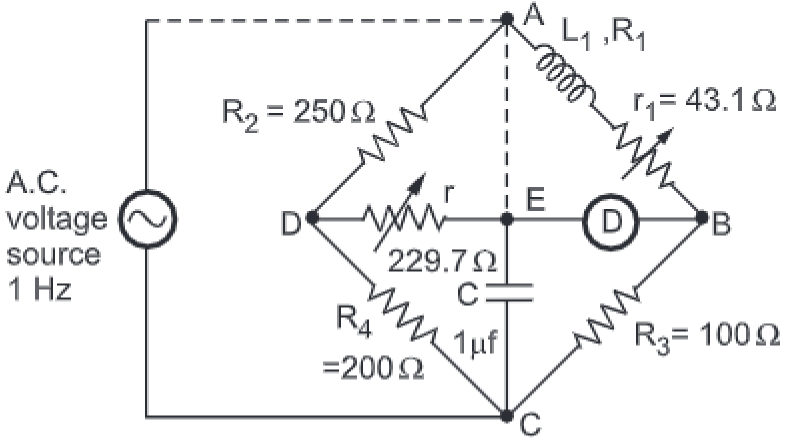
Arm da : a non-inductive resistor $R_2 = 250\ \Omega$.

Arm de : a variable non-inductive resistor r.

Arm ec : a lossless capacitor $C = 1\ \mu F$.

An a.c. supply is connected between a and c. Detector is between b and e. Calculate the resistance R_1 and inductance L_1 when under balance condition $r_1 = 43.1\ \Omega$ and $r = 229.7\ \Omega$.

Solution : The a.c. bridge is as shown in the Fig.



From circuit arrangement, it is Anderson's bridge. The value of unknown resistor is given by,

$$R_1 = \frac{R_2 R_3}{R_4} - r_1 = \frac{(250)(100)}{200} - 43.1 = \mathbf{81.9\ \Omega}$$

Unknown inductance is given by,

$$L_1 = \frac{CR_3}{R_4}\left[\left(R_2 + R_4\right) r + R_2 R_4\right]$$

$$= \frac{1 \times 10^{-6} \times 100}{200}\ [(250 + 200)\ (229.7) + (250)\ (200)]$$

$$= \mathbf{0.0766\ H = 76.6825\ mH}$$

The four arms of the bridge are as follows :

Arm ab : An imperfect capacitor C_1 with an equivalent series resistance of r_1

Arm bc : A non-inductive resistance R_3

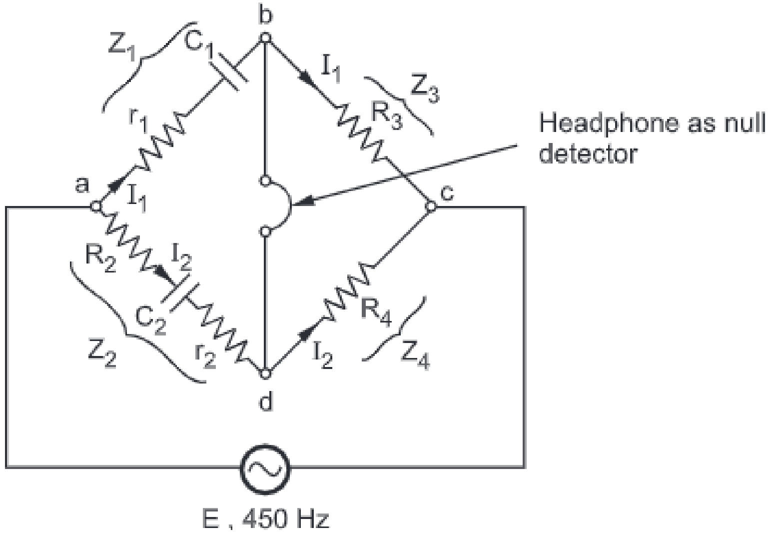
Arm cd : A non-inductive resistance R_4

Arm da : An imperfect capacitor C_2 with an equivalent resistance of r_2 in series with resistance R_2.

A supply at 450 Hz is connected between terminals a and c and the detector is connected between b and d. At the balance condition :

$R_2 = 4.8\ \Omega$, $R_3 = 200\ \Omega$, $R_4 = 2850\ \Omega$, and $C_2 = 0.5\ \mu F$, $r_2 = 0.4\ \Omega$

Calculate values of C_1 and r_1 and also of the dissipating factor for the capacitor.



Solution : The bridge is as shown in the Fig.

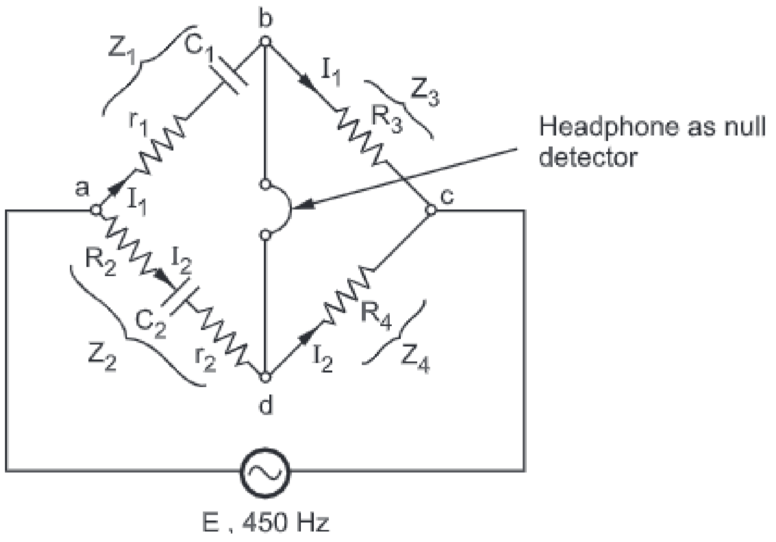
$$Z_1 = r_1 - j\frac{1}{\omega C_1}\ \Omega$$

$$Z_2 = (R_2 + r_2) - j\frac{1}{\omega C_2} = (4.8 + 0.4) - j\frac{1}{2\pi \times 450 \times 0.5 \times 10^{-6}} = 5.2 - j\ 707.3553\ \Omega = 707.3744 \angle -89.5788^\circ\ \Omega$$

$$Z_3 = 200 + j\,0\ \Omega = 200 \angle\ 0^\circ\ \Omega$$

$$Z_4 = 2850 + j\,0\ \Omega = 2850 \angle\ 0^\circ\ \Omega$$

At a bridge balance, no current flows through the detector.



$$\therefore \qquad I_1 = \frac{E}{Z_1 + Z_3}$$

$$I_2 = \frac{E}{Z_2 + Z_4}$$

Now $\qquad I_1 Z_1 = I_2 Z_2 \quad$ for null deflection of detector

$$\therefore \qquad \frac{E\,Z_1}{Z_1 + Z_3} = \frac{E\,Z_2}{Z_2 + Z_4}$$

$$\therefore \qquad Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \quad 2850\left[r_1 - j\frac{1}{\omega C_1}\right] = 200 \angle 0^\circ \times 707.3744 \angle -89.5788^\circ$$

$$\therefore \quad r_1 - j\frac{1}{\omega C_1} = 49.6403 \angle -89.5788^\circ = 0.3649 - j\,49.6389\ \Omega$$

Comparing both sides,

$$r_1 = \mathbf{0.3649\ \Omega} \qquad \text{and} \qquad \frac{1}{\omega C_1} = 49.6389$$

$$C_1 = \frac{1}{2\pi \times 450 \times 49.6389} = \mathbf{7.125}\ \mu\text{F}$$

$$\text{Dissipating factor} = \omega r_1 C_1 = 2\pi \times 450 \times 0.3649 \times 7.125 \times 10^{-6}$$

$$= \mathbf{0.007351}$$

The four arms of the Maxwell's capacitance bridge at balance are :

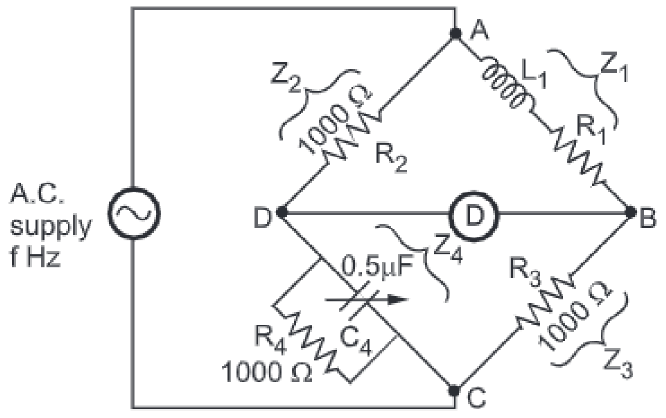
Arm ab : unknown inductance L_1 having an inherent resistance R_1

Arm bc : A non-inductive resistance of 1000 Ω,

Arm cd : A capacitor of 0.5 μF in parallel with a resistance of 1000 Ω .

Arm da : A resistance of 1000 Ω

Determine the values of R_1 and L_1. Draw the phasor diagram of the bridge.



Solution : From the given information, the Maxwell's capacitance bridge is as shown

The equation for balance is,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$\left[R_1 + j\omega L_1\right]\left[\frac{R_4}{1+j\omega C_4 R_4}\right] = (R_2)\,(R_3)$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 R_4 C_4$$

Equating real terms, we get,

$$R_1 R_4 = R_2 R_3$$

$$\therefore \quad R_1 = \frac{R_2 R_3}{R_4} = \frac{(1000)(1000)}{1000} = \mathbf{1000\ \Omega}$$

Equating imaginary terms, we get

$$\omega L_1 R_4 = \omega R_2 R_3 R_4 C_4$$

$$L_1 = R_2 R_3 C_4 = (1000)(1000)\ (0.5 \times 10^{-6}) = \mathbf{0.5\ H}$$

The four arms of Hay's bridge are arranged as follows :

AB : coil of unknown impedance.

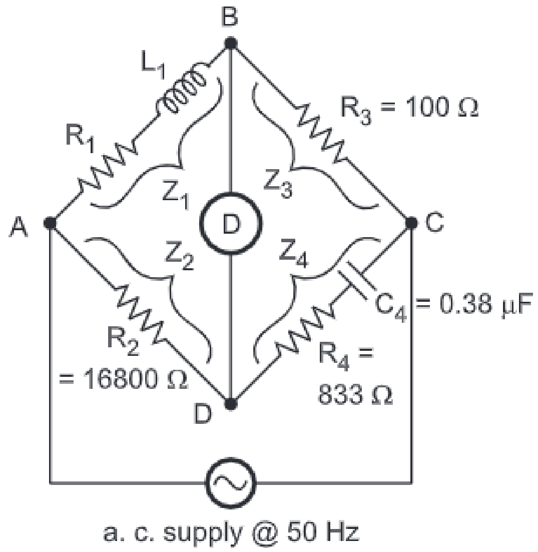
BC : non-reactive resistance of 100 Ω.

CD : non-reactive resistance of 833 Ω in series with 0.38 μF capacitor.

DA : non-reactive resistor of 16800 Ω .

If the supply frequency is 50 Hz, determine the inductance and resistance at the balance condition.

From given data, Hay's bridge can be drawn as shown in the Fig.



At f = 50 Hz, reactance offered by C_4 is,

$$-jX_{C_4} = \frac{-j}{\omega C_4} = \frac{-j}{2\pi f C_4}$$

$$= \frac{-j}{2\times\pi\times 50\times 0.38\times 10^{-6}}$$

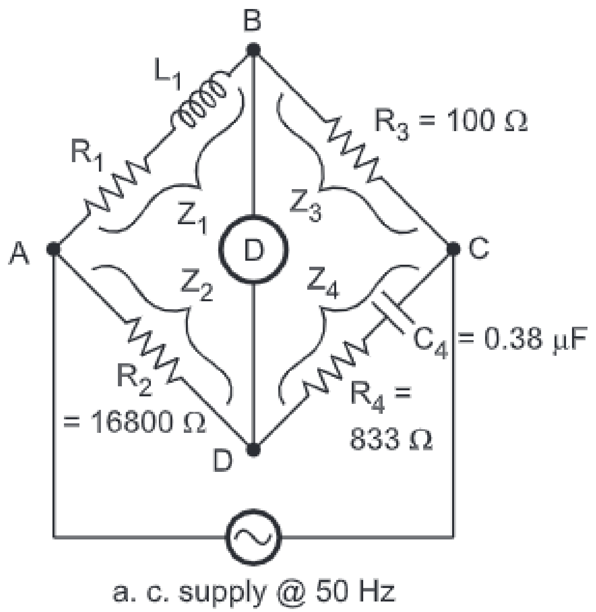
$$\therefore \ -jX_{C_4} = -j\ 8376.5759\ \Omega$$

$\therefore\ Z_4 = R_4 - j\,X_{C_4}$

$= 833 - j\ 8376.5759$

$\therefore\ Z_4 = 8417.8924\,\angle -84.32^\circ$

Now in general for a.c. bridge, the condition of balance is given by,



$$Z_1\ Z_4 = Z_2\ Z_3$$

$$Z_1 = \frac{Z_2\ Z_3}{Z_4} = \frac{(16800)\ (100)}{8417.8929\,\angle -84.32^\circ} = 199.5749\,\angle +84.32^\circ$$

$$Z_1 = R_1 + jX_{L_1} = R_1 + j\,\omega\,L_1 = R_1 + j\,2\pi\,f\,L_1 = 19.7524 + j\ 198.595\ \Omega$$

Comparing imaginary terms on both the sides,

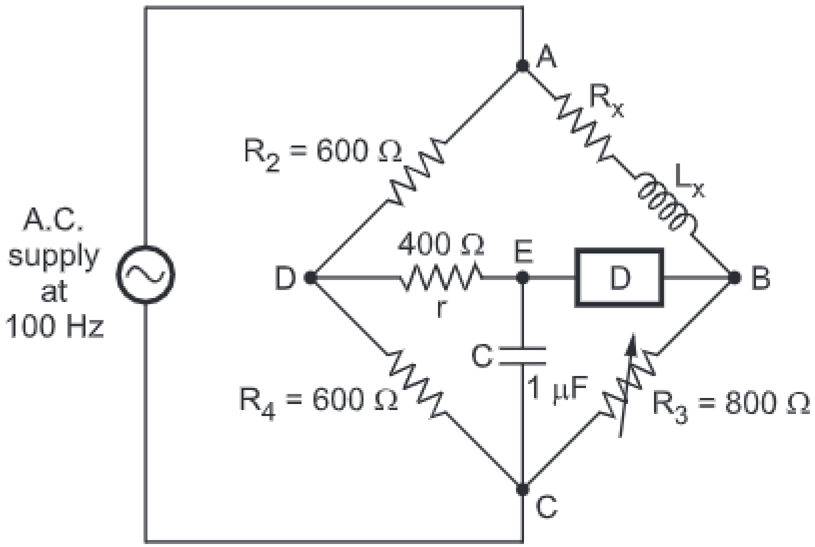
$$2\pi f L_1 = 198.595$$

$$L_1 = \frac{198.595}{2\pi f} = \frac{198.595}{2 \times \pi \times 50} = \mathbf{0.6321\ H}$$

Comparing real terms on both the sides,

$$R_1 = \mathbf{19.7524\ \Omega}$$

In an Anderson's bridge for the measurement of inductance, the arm AB consists of an unknown impedance with inductance L and R, a unknown variable resistance in arm BC, fixed resistance of 600 Ω each in arms CD and DA, a unknown variable resistance in arm DE and a capacitor with fixed capacitance of 1 μF in the arm CE. The a.c. supply of 100 Hz is connected across A and C and the detector is connected between B and E. If the balance is obtained with a resistance of 400 Ω in the arm DE and a resistance of 800 Ω in the arm BC, calculate the values of R and L.



By the standard formula, the unknown resistance R_x in branch AB is given by,

$$R_x = R_1 = \frac{R_2 R_3}{R_4} = \frac{(600)(800)}{(600)} = \mathbf{800\,\Omega}$$

Similarly the unknown inductance in branch AB is given by,

$$L_x = \frac{C\,R_3}{R_4}\left[R_2 r + R_4 r + R_2 R_4\right] = \frac{(1\times10^{-6})(800)}{600}\left[(600)(400)+(600\times400)+(600)(600)\right] = \mathbf{1.12\ H}$$