CONTROL ENGINEERING

21ME72

Module - 5
System Compensation & State Variable Characteristics of Linear Systems

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OBJECTIVES:

- To study different system compensators and variable characteristics of linear systems

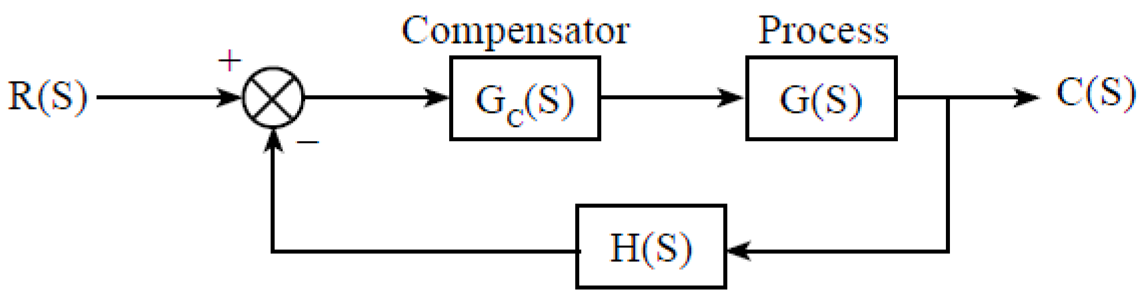
Compensator:

The additional component or device that compensates the performance deficiency is called as compensator

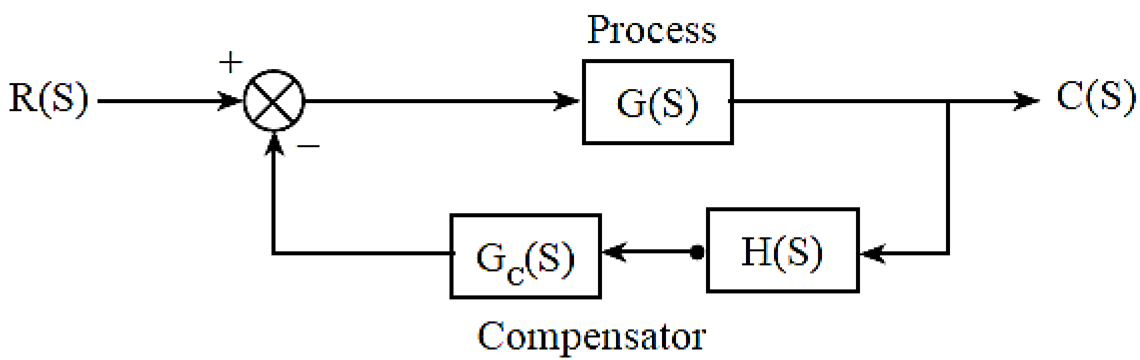
There are four types of system compensators,

1. Series or cascade compensator
2. Parallel or feedback compensator
3. Input compensator
4. Output compensator

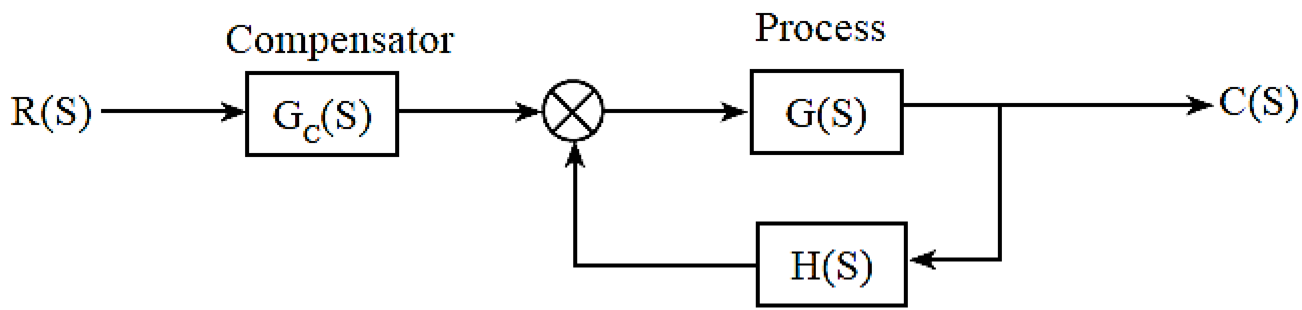
Series compensator :



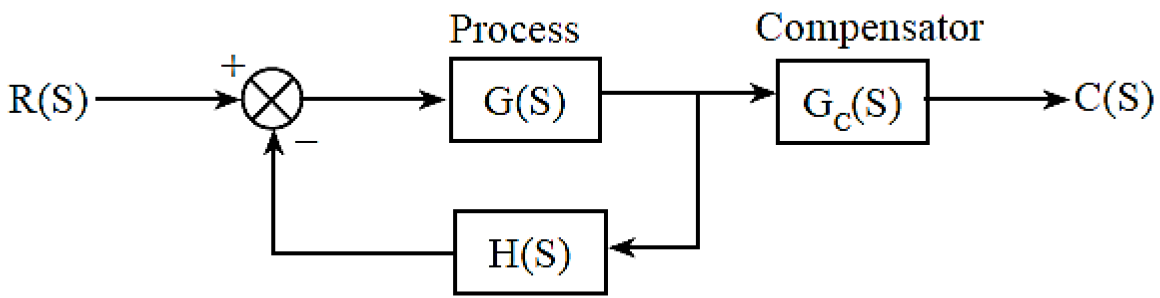
Parallel compensator :



Input compensator :

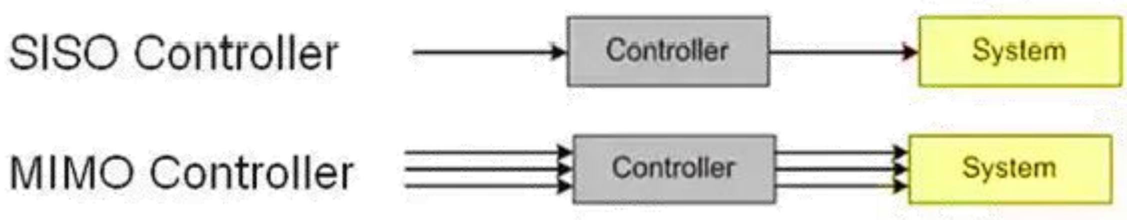


Output compensator :



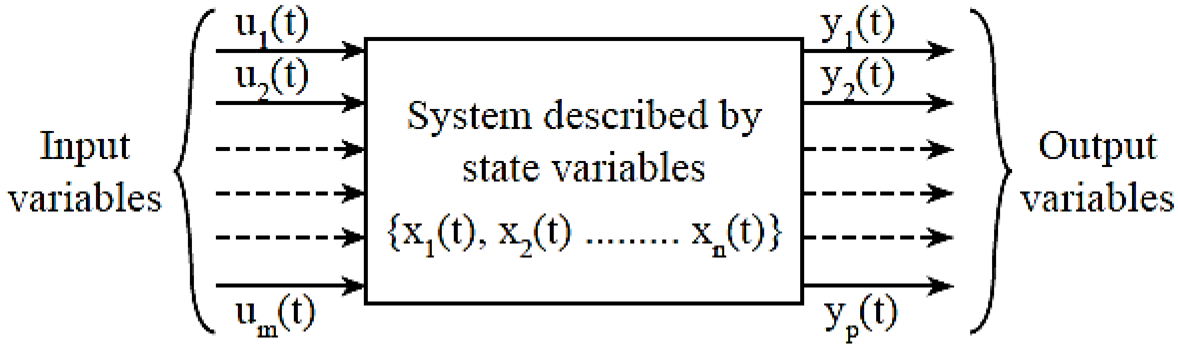
Introduction to state concepts :

State variable models are basically time domain models which involves the analysis and study of linear and nonlinear, time invariant or time varying multi input multi output Control system.



Advantages of state variables analysis are

- It can be applied to non linear system
- It can be applied to time invariant system
- It can be applied to multiple input multiple output system
- It gives the idea about the internal state of the system.



$$\frac{d}{dt}X_1(t) = X_1 = f_1(X_1, X_2 .. X_{n,} U_1, U_2 .. U_m)$$

$$\frac{d}{dt}X_2(t) = X_2 = f_2(X_1, X_2 .. X_{n,} U_1, U_2 .. U_m)$$

$$\frac{d}{dt}X_n(t) = X_n = f_n(X_1, X_2 .. X_{n,} U_1, U_2 .. U_m)$$

On integrating above equation

$$X_i(t) = X_i(t_0) + \int_{t_0} f_n(X_1, X_2 .. X_{n,} U_1, U_2 .. U_m)\ dt$$

Thus , 'n' differential equation can be represented in vector form as

$$X(t) = f\ (X(t), U(t))$$

$$X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \\ \cdot \\ \cdot \\ X_n(t) \end{bmatrix} \qquad U(t) = \begin{bmatrix} U_1(t) \\ U_2(t) \\ \cdot \\ \cdot \\ U_m(t) \end{bmatrix}$$

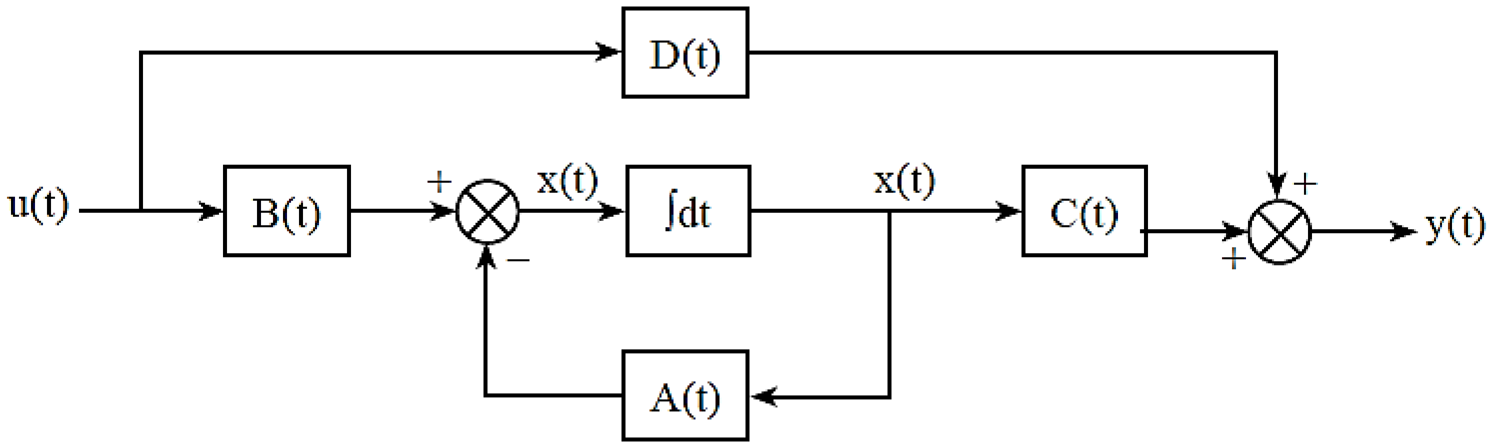
For, Time varying systems

$$X(t) = f\ (X(t), U(t), t)$$

The output vector Y(t) can be generally expressed in terms of state vector X(t), as follows

$$Y(t) = g\ (X(t), U(t))$$

Matrix representation of state equations



Thus the derivative of each state variable can be expressed in terms of linear combination of system states and input as

$$\left.\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \ldots + b_{1m}u_m \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + b_{21}x_n + b_{21}u_1 + b_{22}u_2 + \ldots + b_{2m}u_m \\
&\;\cdot \\
&\;\cdot \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + \ldots = b_{nm}u_m
\end{aligned}\right\} \rightarrow 1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ . \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ ... & & & \\ a_{n1} & a_{n2} & .... & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & .... & b_{1m} \\ b_{21} & b_{22} & .... & b_{2m} \\ ... & & & \\ b_{n1} & b_{n2} & .... & b_{n}m \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_m \end{bmatrix} \quad ...(2)$$

The above equation can be reduced in matrix from known as "***State equation***"

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{AX}(\boldsymbol{t}) + \boldsymbol{BU}(\boldsymbol{t}) \qquad \text{----Eq. 3}$$

Where

$\dot{X}(t)$ = Derivative of state vector of order (n x 1)
X(t) = State vector matrix of order (n x 1)
U(t) = Input vector matrix of order (m x 1)
A = System matrix or evolution matrix of order (n x n)
B = Input matrix or control matrix of order (n x m)

Similarly the output variables can be expressed as linear combinations of the state variables and input variables at time „t" can be expressed as

$$\left.\begin{array}{l} Y_1(t)=C_{11}x_1(t)+C_{12}x_2(t)+.+C_{1n}x_n(t)+d_{11}U_1(t)+d_{12}U_2(t)+...d_{1m}U_m(t) \\ . \\ . \\ y_p(t)=C_{p1}x_1(t)+c_{p2}x_2(t)+...C_{pn}x_n(t)+d_{p1}U_1(t)+d_{p2}U_2(t)+...+d_{pm}U_m(t)...(a) \end{array}\right\} ....(4)$$

In matrix form

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ y_p \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & .... & c_{1n} \\ c_{21} & c_{22} & .... & c_{2n} \\ ... & & & \\ c_{p1} & c_{p2} & .... & c_{pn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ x_n(t) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & .... & d_{1m} \\ d_{21} & d_{22} & .... & d_{2m} \\ ... & & & \\ d_{p1} & d_{p2} & .... & d_{pm} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ . \\ u_m(t) \end{bmatrix} \quad ...(5)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ y_p \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & .... & c_{1n} \\ c_{21} & c_{22} & .... & c_{2n} \\ ... & & & \\ c_{p1} & c_{p2} & .... & c_{pn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ x_n(t) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & .... & d_{1m} \\ d_{21} & d_{22} & .... & d_{2m} \\ ... & & & \\ d_{p1} & d_{p2} & .... & d_{pm} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ . \\ . \\ u_m(t) \end{bmatrix} \quad ...(5)$$

$$\boldsymbol{Y(t) = CX(t) + DU(t)} \qquad \text{----Eq. 6}$$

Where,

Y (t)= Output vector matrix of order (p×1)
C = Output matrix of order (p×n)
D = Transmission matrix of order (p×n)

Example 1: Obtain the state model for the equation $\boldsymbol{y + 3y + 2y + y = r(t)}$

Solution.

Choose the state variation

$$X_1 = y$$
$$X_2 = y$$
$$X_3 = y$$

$$\boldsymbol{X(t) = AX(t) + BU(t)}$$
$$\boldsymbol{Y(t) = CX(t) + DU(t)}$$

Differentiating above equation

$$X_1 = y \quad = X_2$$
$$X_2 = y \quad = X_3$$
$$X_3 = y$$

$$\boldsymbol{y = -3y - 2y - y + r(t)}$$

Differentiating above equation

$$X_1 = y \quad = X_2$$

$$X_2 = y \quad = X_3$$

$$X_3 = y$$

$$\boldsymbol{y = -3y - 2y - y + r(t)}$$

$$X_3 = -\mathbf{3}X_3 - \mathbf{2}X_2 - X_1 + \boldsymbol{r(t)}$$

Therefore the above equation can be written in the matrix form

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \boldsymbol{r(t)}$$

$$\boldsymbol{X(t) = AX(t) + BU(t)}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \boldsymbol{r}(\boldsymbol{t})$$

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{U}(\boldsymbol{t})$$

where, $$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Output equation can written as

$$\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{CX}(\boldsymbol{t}) + \boldsymbol{DU}(\boldsymbol{t})$$

$$X_1 = y$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{CX}(\boldsymbol{t})$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

State controllability and observability

1. Is it possible to transfer the system under consideration from any initial state to desired state by the application of suitable control force with the specified time?

2. Is it possible to determine the initial stats of the system if the output vector is known for a finite length of time.

The system is said to be completely controllable if it is possible to transfer the system state from any initial state $x(t_0)$ to any other desired state $x(t_f)$ in a specified finite time interval $(t_0 \leq t \leq t_p)$ by unconstrained control vector $U(T)$.

The state controllability tests can be performed by two methods

1. **Kalman's test for controllability**
2. **Gilbert's test for controllability**

Kalman test for state controllability

If the n[th] order multiple input linear time invariant system represented by state equation as

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{U}(\boldsymbol{t})$$

Then, $Q_C = [B \quad AB \quad A^2B \quad A^3B \quad .. \quad .. \quad A^{n-1}B]$

The system is said to be controllable if the rank of the controllability matrix (Q_c) is „n"then the determined of order (n x n) of any sub matrix of Q_c has non zero value.

If the rank of the controllability matrix (Q_c) is less than (n), then the system is not completely state controllable.

Example 1 : From the controllability of the system by Kalman's test of order 2 which is given by

$$X(t) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(t)$$

Solution:

Comparing with standard equation $\boldsymbol{X(t) = AX(t) + BU(t)}$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Controllability / composite matrix $Q_C = [B \quad AB]$

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad AB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad = 0$$

$$|Q_C| = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

Since the $|Q_C| = 0 \neq n$ then the system is ***not completely controllable.***

Gilbert's test for state controllability

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{U}(\boldsymbol{t})$$

$$\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{C}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{D}\boldsymbol{U}(\boldsymbol{t})$$

Case 1 :

If matrix A is an diagonal canonical form, then the transformation matrix is the identity matrix (T = I)

$$B_t = T^{-1}B$$

If no elements of the matrix are zero, the system is completely state controllable.

Case 2 : If matrix A is a not in diagonal canonical form following steps are followed.

Step 1 : Find Eigen value of matrix A

$$|\lambda I - A| = 0$$

Step 2 : Find the transformation matrix

Develop Vander monde matrix of A which will be used as transformation matrix.

$$T = V = \begin{bmatrix} 1 & 1 & 1 & \dots. & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \dots. & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \dots & \lambda_n^2 \\ .. & .. & \dots & & .. \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & \dots & \lambda_n^{n-1} \end{bmatrix}$$

Step 3 : Find $B_t = T^{-1}B$

If no elements of the matrix are zero, the system is completely state controllable.

Example 1 : Determine the controllability of a system for the given state equation by Gilbert's test.

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

Solution:

Considering with standard equation $\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{AX}(\boldsymbol{t}) + \boldsymbol{BU}(\boldsymbol{t})$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Since matrix A is not canonical, find Eigen values of A.

$$|\lambda I - A| = 0$$

$$\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = 0$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda + 6 \end{vmatrix} = 0$$

$$\lambda[\lambda^2 + 6\lambda - (-11)] - 1(-6 + 0) + 0 = 0$$

$$\boldsymbol{\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0}$$

Roots of above equation

$$\boldsymbol{\lambda}_1 = -1 \quad \boldsymbol{\lambda}_2 = -2 \quad \boldsymbol{\lambda}_3 = -3$$

$$\lambda_1 = -1 \quad \lambda_2 = -2 \quad \lambda_3 = -3$$

Find Vander monde matrix

$$T = V = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

Find inverse of Vander monde matrix (T)

$$T^{-1} = \frac{adj(T)}{|T|}$$

Cofactor matrix $C(T) = \begin{bmatrix} -6 & 6 & -2 \\ -5 & 8 & -3 \\ -1 & 2 & -1 \end{bmatrix}$

Adjugate matrix $adj(T) = C^T = \begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$

$$|T| = \begin{vmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{vmatrix} = -2 \qquad adj(T) = \begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$$

$$T^{-1} = \frac{adj(T)}{|T|} = \frac{1}{-2}\begin{bmatrix} -6 & -5 & -1 \\ 6 & 8 & 2 \\ -2 & -3 & -1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 3 & \frac{5}{2} & \frac{1}{2} \\ -3 & -4 & -1 \\ 1 & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

$$B_t = T^{-1}B$$

$$T^{-1} = \begin{bmatrix} 3 & \frac{5}{2} & \frac{1}{2} \\ -3 & -4 & -1 \\ 1 & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$B_t = \begin{bmatrix} 3 & \frac{5}{2} & \frac{1}{2} \\ -3 & -4 & -1 \\ 1 & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$B_t = \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix}$$

Since no elements of the Matrix $\boldsymbol{B_t}$ are ***zero***, the system is completely ***state controllable***.

State Observability

The system is said to be completely observable if every $X(t_0)$ can be completely observable, if every state $X(t_0)$ can be completely identified by measurement of the Output $Y(t)$ over a finite time interval $(t \geq t_0)$ assuming that the control signal $U(T)$ is also available.

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{U}(\boldsymbol{t})$$

$$\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{C}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{D}\boldsymbol{U}(\boldsymbol{t})$$

The state observability tests can be performed by two methods

1. **Kalman's test for observability**
2. **Gilbert's test for observability**

Kalman test for state observability

If the n^{th} order multiple input linear time invariant system represented by state equation as

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{AX}(\boldsymbol{t}) + \boldsymbol{BU}(\boldsymbol{t})$$

Then, $Q_0 = [C^T \quad A^T C^T \quad .. \quad .. \quad (A^T)^{n-1} C^T \,]$

The system is said to be observable if the order of the rank of the observability matrix (Q_0) is „n" and value of the determinant, $|Q_0| \neq 0$.

Example 1 : Find the observability of the system by Kalman's for the state model.

$$X(t) = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(t) \qquad Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Solution:

Comparing with standard equation $\boldsymbol{X(t) = AX(t) + BU(t)}$

$$\boldsymbol{Y(t) = CX(t) + DU(t)}$$

$$A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

Composite matrix of Observability, $Q_0 = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Q_0 = [C^T \quad A^T C^T]$$

$$Q_O = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$|Q_O| = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \quad \neq 0$$

Since the rank of $\boldsymbol{Q_O}$**=2** , and also $|\boldsymbol{Q_C}| = \mathbf{1} \neq \mathbf{0}$ then the system is ***completely Observable.***

Gilbert's test for state Observability

$$\boldsymbol{X(t) = AX(t) + BU(t)}$$

$$\boldsymbol{Y(t) = CX(t) + DU(t)}$$

Case 1 :

If matrix A is an diagonal canonical form, then the transformation matrix is the identity matrix (T = I)

$$A_t = A \qquad B_t = B \qquad C_t = C \qquad D_t = D$$

Then,

$$A_t = T^{-1}AT$$

$$B_t = T^{-1}B$$

$$C_t = CT$$

$$D_t = D$$

If $C_t = CT$ matrix and does not contain any zero element, the system is completely state Observable.

Case 2 : If matrix A is a not in diagonal canonical form following steps are followed.

Step 1 : Find Eigen value of matrix A

$$|\lambda I - A| = 0$$

Step 2 : Find the transformation matrix

Develop Vander monde matrix of A which will be used as transformation matrix.

$$T = V = \begin{bmatrix} 1 & 1 & 1 & .... & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & .... & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & ... & \lambda_n^2 \\ .. & .. & ... & & .. \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & ... & \lambda_n^{n-1} \end{bmatrix}$$

Step 3 : Find $C_t = \mathrm{CT}$

If $C_t = CT$ matrix and does not contain any zero element, the system is completely state Observable.

Example 1 : Determine the observability of a state model by Gilbert's test.

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U \qquad Y = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Solution:

Considering with standard equation $\boldsymbol{X(t) = AX(t) + BU(t)}$

$$\boldsymbol{Y(t) = CX(t) + DU(t)}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} \quad D = 0$$

Since matrix A is not canonical, find Eigen values of A.

$$|\lambda I - A| = 0$$

$$\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} = 0$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & 2 & \lambda + 3 \end{vmatrix} = 0$$

$$\lambda[\lambda^2 + 3\lambda - (-2)] + 1(0) + 0 = 0$$

$$\boldsymbol{\lambda^3 + 3\lambda^2 + 2\lambda = 0}$$

Roots of above equation

$$\boldsymbol{\lambda}_1 = 0 \qquad \boldsymbol{\lambda}_2 = -1 \quad \boldsymbol{\lambda}_3 = -2$$

$$\lambda_1 = 0 \qquad \lambda_2 = -1 \quad \lambda_3 = -2$$

Find Vander monde matrix

$$T = V = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

Transform the model into canonical form

$$C_t = CT$$

$$C_t == \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$C_t == \begin{bmatrix} 3 & 0 & -1 \end{bmatrix}$$

Since one elements of the Matrix C_t are ***zero***, the system is ***not Completely Observable***.

Model Question Paper

Paper 1

Paper 2

Any Questions …?

thank you