CONTROL ENGINEERING

21ME72

Module - 4
Frequency Domain Analysis

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OBJECTIVES:

- To study the Frequency response analysis using polar plot, Nyquist plot and Bode plot.

Introduction :

Why Frequency Response?

1. Weakness of root locus method relies on the existence of open-loop transfer function

2. Weakness of time-domain analysis method is that time response is very difficult to obtain
   - Computational complex
   - Difficult for higher order system
   - Difficult to partition into main parts
   - Not easy to show the effects by graphical method

Advantages of Frequency Response Analysis:

- Frequency response (mathematical modeling) can be obtained directly by experimental approaches.
- Easy to analyze effects of the system with sinusoidal signals.
- Convenient to measure system sensitivity to noise and parameter variations.

Note: *Frequency domain analysis* is a kind of (indirect method) engineering method. It studies the system based on frequency response which is also a kind of mathematical model.

Frequency response (or characteristic) is the ratio of the complex vector of the steady-state output versus sinusoidal input for a linear system.

$$G(j\omega) = \frac{1}{1 + j\omega T} = A(\omega)e^{j\varphi(\omega)}$$

$$A(\omega) = \left|\frac{1}{1 + j\omega T}\right| = \frac{1}{\sqrt{1 + \omega^2 T^2}};$$ **Magnitude response**

$$\phi(\omega) = \angle\left(\frac{1}{1 + j\omega T}\right) = -\mathrm{arctg}\,\omega T$$ **Phase response**

$\omega = 0$ The output has same magnitude and phase with input

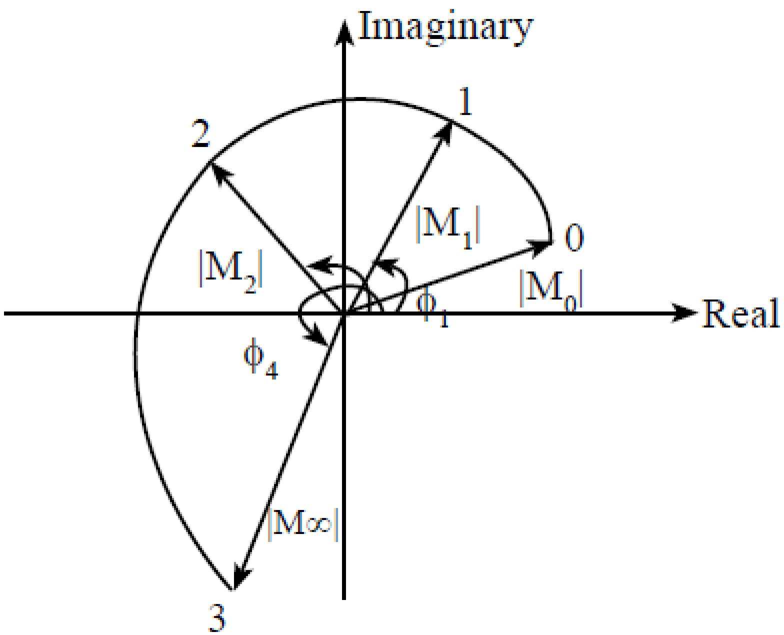
$\omega \uparrow$ Magnitude will be attenuated and phase lag is increased

Polar plot :

Plotting the variations of magnitude and phase angle v/s input frequency

$$Magnitude, M = |G(j\omega)H(j\omega)|$$

$$Phase\ angle, \emptyset = \angle G(j\omega)H(j\omega)$$



It can be calculated the value of Magnitude (M) and Phase angle (Ø) by varying the value of "ω" from "0" to "∞"

Example 1: Obtain polar plot for a open loop transfer function. $G(s)H(s) = \frac{20}{s}$

Solution:

Replace 'S,, by 'jω'

$$G(\mathrm{j\omega})H(\mathrm{j\omega}) = \frac{20}{\mathrm{j\omega}}$$

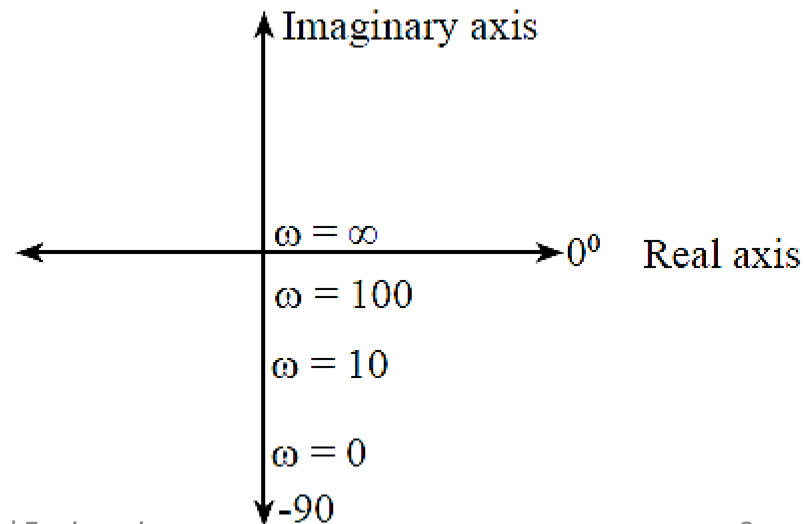
$$Magnitude, M = |G(j\omega)H(j\omega)\,| = \frac{20}{\omega}$$

$$Phase\ angle, \emptyset = \measuredangle G(j\omega)H(j\omega) = \frac{\tan^{-1}\left(\frac{0}{20}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)} = \frac{0^0}{\tan^{-1}\left(\frac{\omega}{0}\right)}$$

$$Magnitude, M = |G(j\omega)H(j\omega)| = \frac{20}{\omega}$$

$$Phase\ angle, \emptyset = \angle G(j\omega)H(j\omega) = \frac{0^0}{\tan^{-1}\left(\frac{\omega}{0}\right)} = \frac{0^0}{90^0}$$

ω	Magnitude (M)	Phase angle (Ø)
0	∞	-90^0
10	2	-90^0
∞	0	-90^0



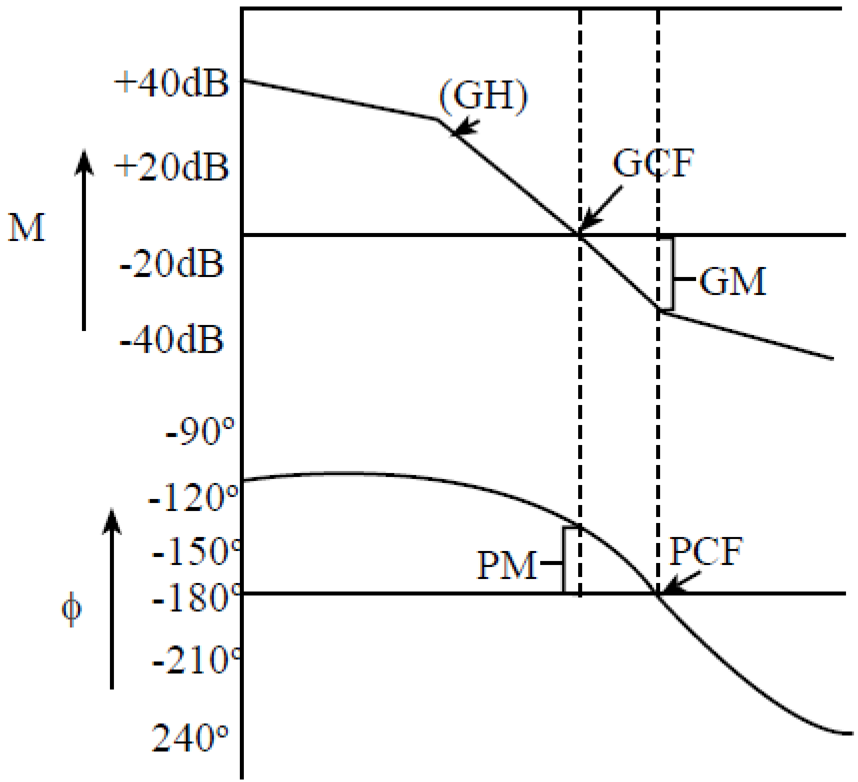
Bode plots

H.W Bode suggested the method in which logarithmic values of magnitude is to be plotted against logarithmic values of frequency, such plots are called "***logarithmic plots***".

Bode diagram consists of two plots,

1. Magnitude expressed in logarithmic values of frequency called as **Magnitude plot.**
2. Phase angle in degrees against logarithmic values of frequency called as **Phase plot.**

	Magnitude (dB)	Slope (dB/ decade)	Phase angle (degrees)	Corner (rad/sec) frequency
k	$M = 20\log_{10} k$	0	$\phi = 0^\circ$ if $k = +ve$ OR $\phi = 180^\circ$ if $k = -ve$	Nil
$S^{\pm n}$	$M = \pm n20\log_{10}\omega$	±n20	$\phi = \pm n90^\circ$	Nil
$(1+ST)^{\pm n}$	$M = \pm n20\log_{10}\sqrt{1+(\omega T)^2}$	±n20	$\phi = \pm n\tan^{-1}\left(\frac{\omega T}{1}\right)$	$\omega_{CT} = \frac{1}{T}$
$(s^2 + 2\varepsilon\omega_n s + w^2_n)^{\pm n}$	$M = \pm n\,40\log_{10}\left(\frac{\omega}{\omega_n}\right)$	±n40	$\phi = \pm n\tan^{-1}\left(\frac{2\xi\frac{\omega}{\omega n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$	$\omega_{CF} = \omega_n$



GM = Gain Margin

PM = Phase Margin

GCF = Gain crossover frequency

PCF = Phase cross over frequency

Example 1: The open loop transfer function of a system is given by $\boldsymbol{G(s)H(s) = \frac{10(s+10)}{s(s+2)(s+5)}}$

Draw bode diagram. Find gain cross - over frequency (GCF), phase cross-over frequency (PCF), gain margin (GM) phase margin (PM). Find stability of the system.

Solution

Step1:

$$G(s)H(s) = \frac{10 \times 10(1 + 0.1s)}{s \times 2(1 + 0.5s) \times 5 \times (1 + 0.2s)}$$

$$G(s)H(s) = \frac{10(1 + 0.1s)}{s(1 + 0.5s)(1 + 0.2s)}$$

$$G(s)H(s) = \frac{10(1+0.1s)}{s(1+0.5s)(1+0.2s)}$$

Step 2: Replace „S‟ by "jω"

$$G(j\omega)H(j\omega) = \frac{10(1+0.1j\omega)}{(j\omega)(1+0.5j\omega)(1+0.2j\omega)}$$

$$G(j\omega)H(j\omega) = \frac{10(1+0.1j\omega)}{(j\omega)(1+0.5j\omega)(1+0.2j\omega)}$$

Step3: Find Corner Frequency

Factor	CF
10	NIL
$(j\omega)^{-1}$	NIL
$(1+0.1j\omega)$	$\omega = \frac{1}{T} = \frac{1}{0.1} = 10\, rad/s$
$(1+0.5j\omega)^{-1}$	$\omega = \frac{1}{T} = \frac{1}{0.5} = 2\, rad/s$
$(1+0.2j\omega)^{-1}$	$\omega = \frac{1}{T} = \frac{1}{0.2} = 5\, rad/s$

Step 4 : Arrange CF in increasing order

Factor	CF (rad/s)	Slope (dB/decade)	Net slope (dB/decade)	Magnitude (dB)	Phase angle (Ø)
10	NIL	0	0	$20\log_{10}(10)$	0^0
$(j\omega)^{-1}$	NIL	-20	-20	$-20\log_{10}(\omega)$	-90^0
$(1+0.5j\omega)^{-1}$	2	-20	-40	$-20\log_{10}\sqrt{1+(0.5\omega)^2}$	$-\tan^{-1}\left(\frac{0.5\omega}{1}\right)$
$(1+0.2j\omega)^{-1}$	5	-20	-60	$-20\log_{10}\sqrt{1+(0.2\omega)^2}$	$-\tan^{-1}\left(\frac{0.2\omega}{1}\right)$
$(1+0.1j\omega)$	10	20	-40	$20\log_{10}\sqrt{1+(0.1\omega)^2}$	$+\tan^{-1}\left(\frac{0.1\omega}{1}\right)$

Step 5 : Find magnitude

$$M = 20\log_{10}(10) \quad -20\log_{10}(\omega) \quad -20\log_{10}\sqrt{1+(0.5\omega)^2} - 20\log_{10}\sqrt{1+(0.2\omega)^2}$$

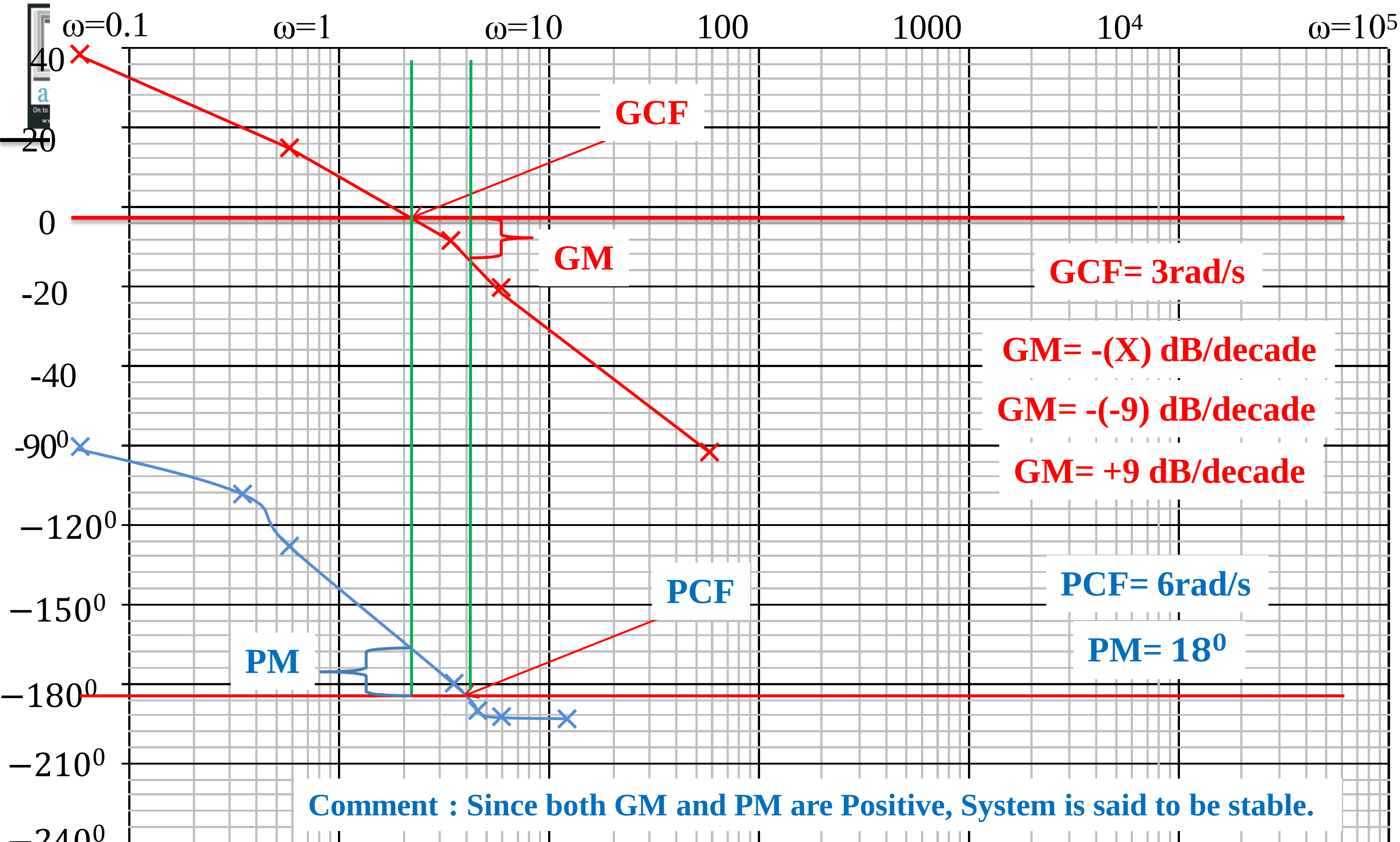
$$+20\log_{10}\sqrt{1+(0.1\omega)^2}$$

ω	Magnitude M (dB)
0.1	40
1	17.56
5	-4.62
10	-18.13
100	-59.96

Step 6 : Find phase angle

$$\varnothing = 0^0 - 90^0 - \tan^{-1}\left(\frac{0.5\omega}{1}\right) - \tan^{-1}\left(\frac{0.2\omega}{1}\right) + \tan^{-1}\left(\frac{0.1\omega}{1}\right)$$

ω	**Phase Angle Ø (deg)**
0.1	-93.43^0
0.5	-107^0
1	-122.16^0
5	-176.63^0
6	-180.79^0
8	-185.29^0
10	-187.66^0
20	-186.81^0



ω=0.1
ω=1
ω=10
100
1000
10⁴
ω=10⁵
40
20
0
-20
-40
-90⁰
−120⁰
−150⁰
−180⁰
−210⁰
−240⁰
GCF
GM
GCF= 3rad/s
GM= -(X) dB/decade
GM= -(-9) dB/decade
GM= +9 dB/decade
PCF
PCF= 6rad/s
PM
PM= 18⁰
Comment : Since both GM and PM are Positive, System is said to be stable.

Nyquist stability criteria :

Nyquist plot are the continuation of polar plot for finding stability of a closed loop control system by varying the values of 'ω' from "–∞" to " ∞"

Nyquist stability criteria is based on Cauchy's theorem of complex variable, popularly known as "***Principle of argument***".

The importance of Nyquist stability lies in the fact that it can also be used to determine the relative degree of system stability by producing the so-called phase and gain stability margins.

Nyquist stability criteria is based on Cauchy's theorem

For a SISO feedback system the closed-loop transfer function is given by

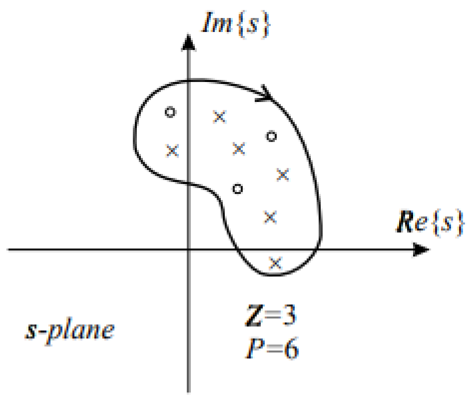
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \Rightarrow F(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$D(s) = 1 + G(s)H(s)$$

The Nyquist stability test is obtained by applying the Cauchy principle of argument to the complex function $\boldsymbol{D(s)}$.

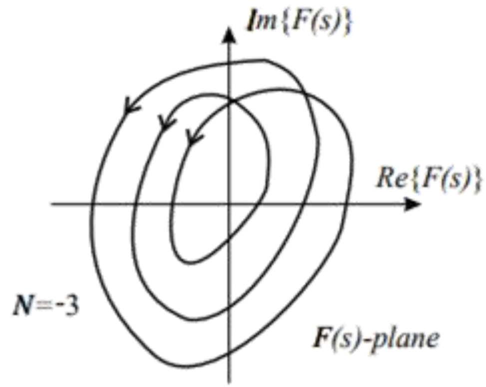
Cauchy‟s Principle of Argument

Let *„F(s)'* be an analytic function in a closed region of the complex plane given in Figure, except at a finite number of points.

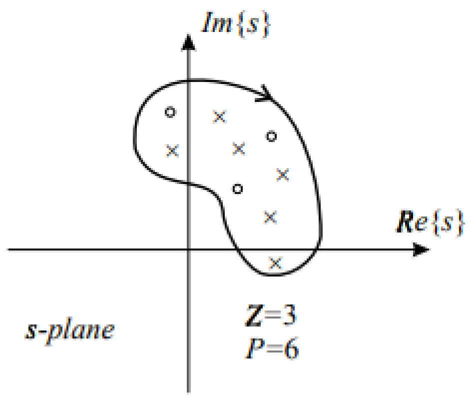


Number of encirclements, $N = P - Z$

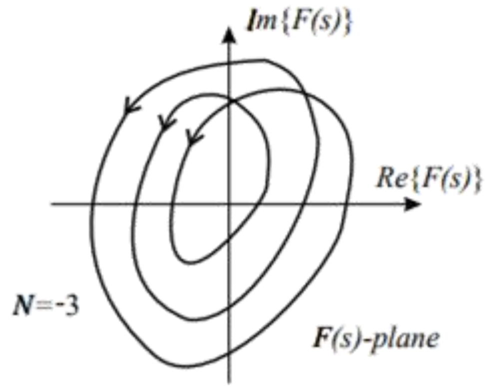
Then, as 's' travels around the contour in the ***s-plane*** *in the clockwise direction, the function* ***F(s)*** *encircles the origin in the* ***(Re{F(s)}, Im{F(s)})*** *– Plane in the same direction N times*



If the enclosed „*S-plane*" closed path contains only poles, then the direction the encirclement in the G(s) H(s) plane will be opposite to the direction of the enclosed path in the "***S plane***".



If the enclosed „*S-plane*" closed path contains only zero's, then the direction of encirclement in the G(s) H(S) plane will be in the same direction as that of enclosed closed path in the "***S-plane***".



Previous year VTU Question paper

Dec-Jan 2019

Model QP 1

Model QP 2

Any Questions …?

thank you