ATME College of Engineering

Mechanics of Materials BME301

Module-2

Bi-axial Stress system ,Thick and Thin cylinders

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1. Objectives

Derive the equations for principal stress and maximum in-plane shear stress and calculate their magnitude and direction. Draw Mohr circle for plane stress system and interpret this circle.

Learning Structure

2.1 Introduction

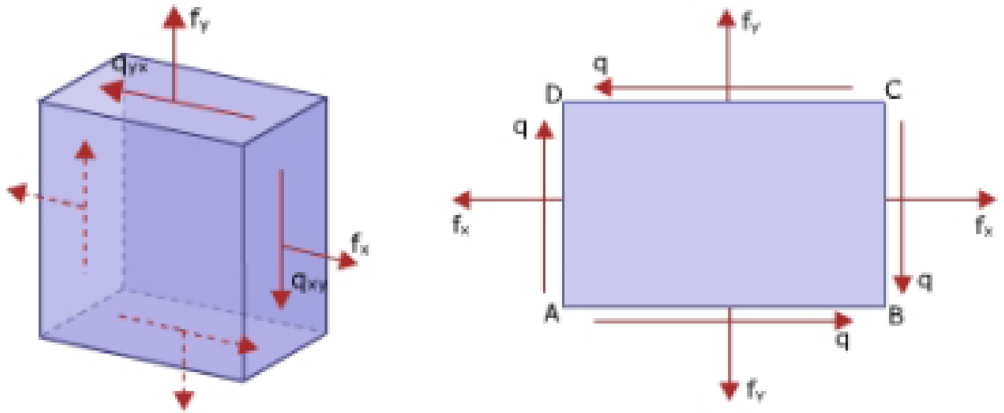
- 2.2 Plane Stress Or 2–D Stress System Or Biaxial Stress System
- 2.3 Expressions For Normal And Tangential Components Of Stress On A Given Plane
- 2.4 Mohr’s Circle
- 2.5 Problems
- 2.6 Thick Cylinders
- 2.7 Thin Cylinders • Outcomes
- Further Reading

2.1 Introduction

Structural members are subjected to various kinds of loads. This results in combination of different stresses which changes from point to point. When an element (considered at any point) in a body is subjected to a combination of normal stresses (tensile and/or compressive) and shear stresses over its various planes, the stress system is known as compound stress system. In a compound stress system, the magnitude of normal stress may be maximum o n some plane and minimum on some plane, when compared with those acting on the element. Similarly, the magnitude of shear stresses may also be maximum on two planes when compared with those acting on the element. Hence, for the considered compound stress system it is important to find the magnitudes of maximum and minimum normal stresses, maximum shear stresses and the inclination of planes on which they act.

2.2 PLANE STRESS OR 2–D STRESS SYSTEM OR BIAXIAL STRESS SYSTEM

Generally, a body is subjected to 3–D state of stress system with both normal and shear stresses acting in all the three directions. However, for convenience, in most problems, variation of stresses along a particular direction can be neglected and the remaining stresses are assumed to act in a plane. Such a system is called 2–D stress system and the body is called plane stress body



In a general two-dimensional stress system, a body consists of two normal stresses (fx and fy), which are mutually perpendicular to each other, with a state of shear (q) as shown in figure. Further, since planes AD and BC carry normal stress fx they are called planes of fx these planes are parallel to Y–axis. Similarly, planes AB and CD represent planes of fy, which are parallel to X–axis.

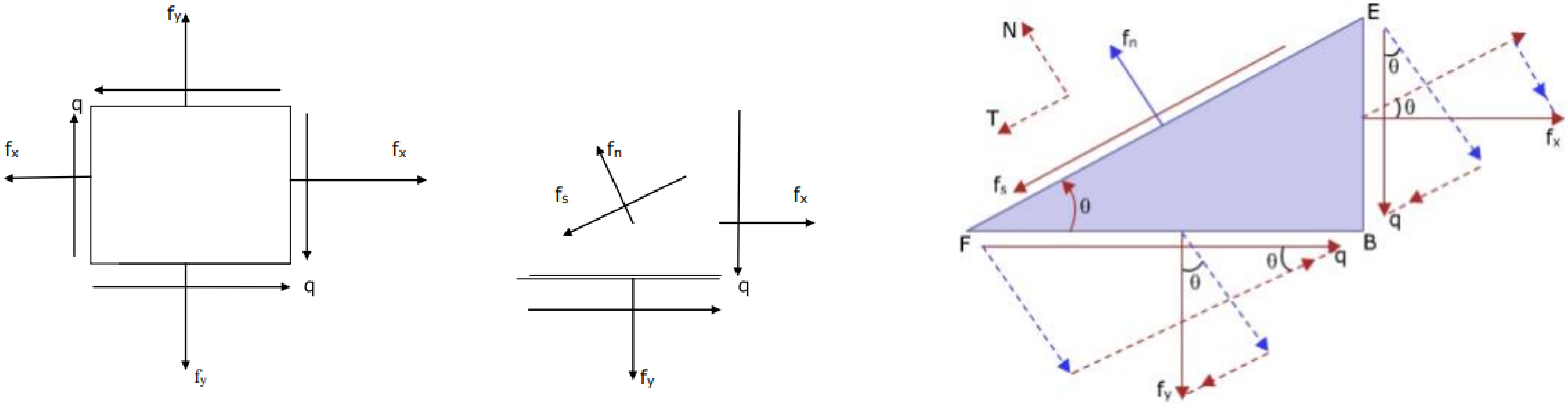
PRINCIPAL STRESSES AND PRINCIPAL PLANES

- **2.2.1 PRINCIPAL STRESSES AND PRINCIPAL PLANES** For a given compound stress system, there exists a maximum normal stress and a minimum normal stress which are called the Principal stresses. The planes on which these Principal stresses act are called Principal planes. In a general 2-D stress system, there are two Principal planes which are always mutually perpendicular to each other. Principal planes are free from shear stresses. In other words Principal planes carry only normal stresses.

- **2.2.2 MAXIMUM SHEAR STRESSES ANDITS PLANES** For a given 2–D stress system, there will be two maximum shear stresses (of equal magnitude) which act on two planes. These planes are called planes of maximum shear. These planes are mutually perpendicular. Further, these planes may or may not carry normal stress. The planes of maximum shear are always inclined at 450 with Principal planes.

2.3 EXPRESSIONS FOR NORMAL AND TANGENTIAL COMPONENTS OF STRESS ON A GIVEN PLANE

Consider a rectangular element ABCD of unit thickness subjected to a general 2-D stress system as shown in figure. Let f n and f s represent the normal and tangential components of resultant stress ‘R' on any plane EF which is inclined at an angle ‘?' measured counter clockwise with respect to the plane of f y or X–axis.



2.3 EXPRESSIONS FOR NORMAL AND TANGENTIAL COMPONENTS OF STRESS ON A GIVEN PLANE

Applying equilibrium along N-direction, we have

$$\Sigma F_N = 0 \quad [\ \nwarrow\ +ve]$$

$$f_n(EF.1) - f_x(BE.1)\sin\theta - q(BE.1)\cos\theta - f_y(BF.1)\cos\theta - q(BF.1)\sin\theta = 0$$

$$f_n = f_x\frac{BE}{EF}\sin\theta + q\frac{BE}{EF}\cos\theta + f_y\frac{BF}{EF}\cos\theta + q\frac{BF}{EF}\sin\theta$$

Since $\frac{BE}{EF} = \sin\theta$ and $\frac{BF}{EF} = \cos\theta$

$$\therefore f_n = f_x\sin^2\theta + 2q\sin\theta\cos\theta + f_y\cos^2\theta$$

But $\cos 2\theta = 2\cos^2\theta - 1$ Hence $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

Also $\cos 2\theta = 1 - 2\sin^2\theta$ Hence $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

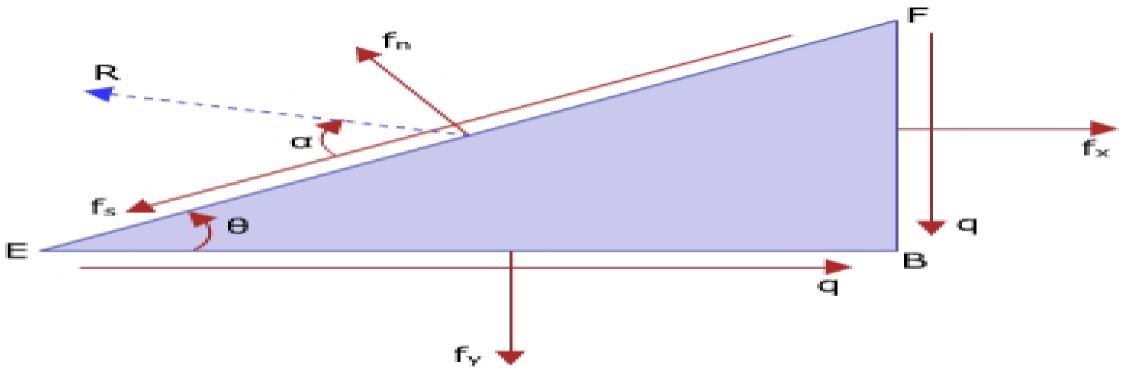
$$f_x = f_x\frac{1}{2}(1-\cos 2\theta) + f_y\frac{1}{2}(1+\cos 2\theta) + q\sin 2\theta$$

$$f_n = \left(\frac{f_x+f_y}{2}\right) - \left(\frac{f_x-f_y}{2}\right)\cos 2\theta + q\sin 2\theta \qquad \text{-------- (1)}$$

Equation (1) is the desired expression for normal component of stress on a given plane, inclined at an angle ‘ ' measured counter clockwise with respect to the plane of fY or X–axis.

2.3 EXPRESSIONS FOR NORMAL AND TANGENTIAL COMPONENTS OF STRESS ON A GIVEN PLANE

To derive expression for fs Consider the Free Body Diagram of portion FBE shown infigure above. For equilibrium along T direction, we have.



$$\Sigma F_T = 0 \; [\leftarrow +ve]$$

$$f_s(EF.1) - f_x(BE.1)\cos\theta + q(BE.1)\sin\theta + f_y(BF.1)\sin\theta - q(BF.1)\cos\theta = 0$$

$$f_s = f_x\frac{BE}{EF}\cos\theta - q\frac{BE}{EF}\sin\theta - f_y\frac{BF}{EF}\sin\theta + q\frac{BF}{EF}\cos\theta$$

Since $\frac{BE}{EF} = \sin\theta \qquad \frac{BF}{EF} = \cos\theta$

$$\therefore f_s = f_x\sin\theta\cos\theta - q\sin^2\theta - f_y\cos\theta\sin\theta + q\cos^2\theta$$

$$\therefore f_s = (f_x - f_y)\sin\theta\cos\theta + q(\cos^2\theta - \sin^2\theta)$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$f_s = \left(\frac{fx - fy}{2}\right)\sin 2\theta + q\cos 2\theta \qquad \text{-------- (2)}$$