ATME College of Engineering

Mechanics of Materials BME301

Module-1

Simple stress and strain

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1. Objectives

Classify the stresses into various categories and define elastic properties of materials and compute stress and strain intensities caused by applied loads in simple and compound sections and temperature changes.

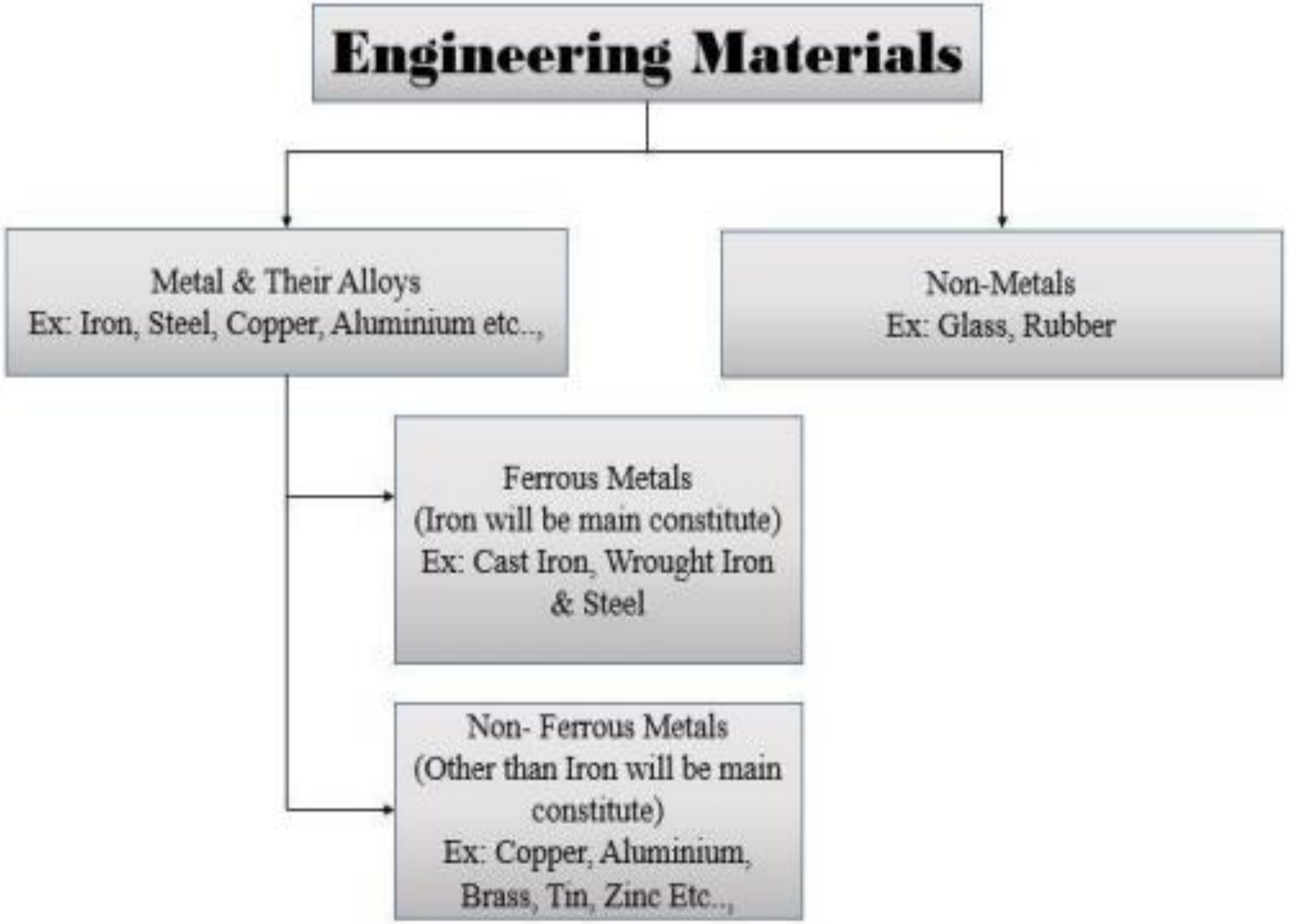
Learning Structure

- Classification Of Engineering Materials
- Choice Of Selection Of Engineering Materials
- Physical Properties Of Materials
- Mechanical Properties
- Stress, Strain And Hook’s Law
- Stress – Strain Relation Or Diagram For Ductile Material
- Stress – Strain Relation Or Diagram For Brittle Material
- Problems

Learning Structure

- Elongation Of Tapering Bars Of Circular Cross Section
- Elongation Of Tapering Bars Of Rectangular Cross Section
- Elongation In Bar Due To Self-Weight
- Compound Or Composite Bars
- Temperature Stresses In A Single Bar
- Temperature Stresses In A Composite Bar
- Simple Shear Stress And Shear Strain
- Complementary Shear Stresses
- Volumetric Strain , Bulk Modulus, Relation Between Elastic Constants

1.1 Classification of Engineering Materials



1.2 Choice of Selection of Engineering Materials

- Availability of materials.
- Sustainability of materials for the working conditions in service.
- Cost of materials.
- Mechanical properties of the materials.

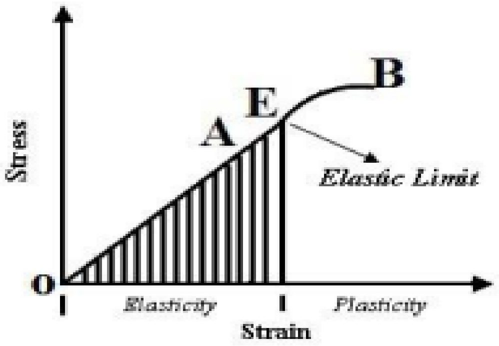
1.3 Physical Properties of Materials

- Luster
- Color
- Size
- Density
- Shape

1.4 Mechanical Properties

• Load (F or P) It is defined as any external force acting on a body

• **Elasticity;-** It is the property by virtue of which a material deformed under the load is enable to return to its original dimension when load is removed. If the body regains completely its original shape, it is said to be perfectly elastic.

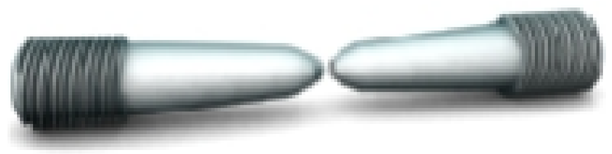


In the above figure, the specimen is loaded up to point A, well within the elastic limit E. When load corresponding to point A is gradually removed the curve follows the same path AO and Strain completely disappears. Such a behaviour is known as Elastic behavior. Steel is more elastic than rubber.

1.4 Mechanical Properties

- **Plasticity;-** It is the converse of Elasticity. It is the property of a material which retains the deformation produced under the load permanently.

- **Ductility;-** It is the property of a material which exhibits large deformations in longitudinal direction under the application of tensile force before failure. A ductile material must be strong and plastic. The ductility is measured in terms of % elongation or % reduction in cross-sectional area of test specimen.

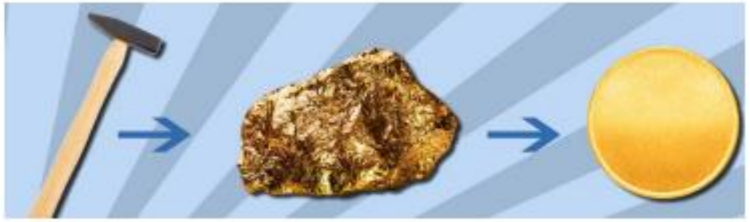


Ex: Mild steel, Brass, Aluminium, Nickel, Zinc, Tin, Lead etc..

1.4 Mechanical Properties

- **Brittleness;-**It is the property of a material which exhibits little or no yielding before failure. Generally brittle materials are higher strength in compression than in tension.

- **Malleability;-** It is the property of a material which permits the material to be extended in all directions without rupture. A malleable material possesses a high degree of plasticity but not necessarily great strength



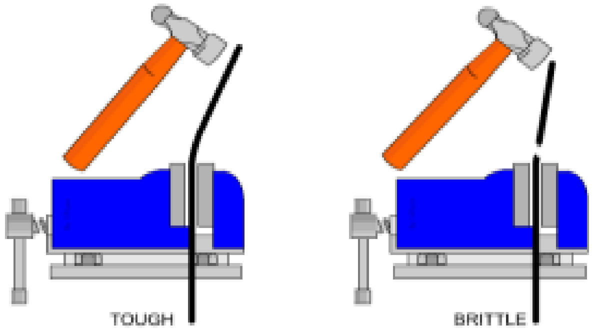
Ex: Gold, Lead, Soft steel, wrought iron, Copper, Aluminium,

1.4 Mechanical Properties

- **Strength;-**It is the ability of a material to resist the externally applied forces without breaking or yielding. The load required to cause fracture divided by the area of the test specimen is termed as ultimate strength of the material.

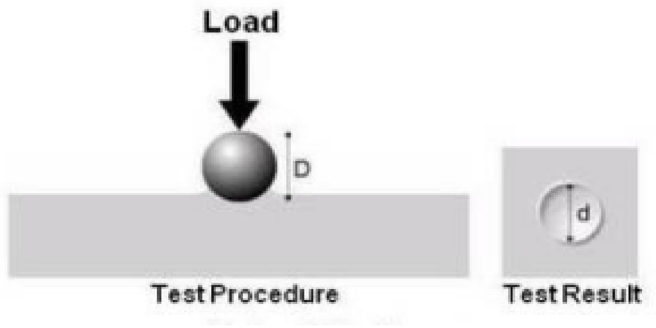
- **Toughness;-** It is the property of a material which enables it to absorb energy without fracture.

This property is desirable in parts subjected to impact and shock loads. Toughness is measured in terms of energy required per unit volume of the material to cause rupture under the action of gradually increasing tensile load.



1.4 Mechanical Properties

- **Hardness;-** It is the ability of the material to resist indentation or surface abrasion. It embraces many different properties such as resistance to wear, scratching, deformation, machinability etc..



- **Stiffness:-** It is the ability of a material to resist deformation under stress. The stiffness is measured by the modulus of elasticity in case of axially loaded members
- **Creep;-**Whenever a member or part of a machine subjected to a constant stress at high temperature for a longer period, it will undergo a slow and permanent deformation called creep.
- **Resilience;-** It is the property of the material to absorb energy and to resist shock and Impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit.

1.5 Stress, Strain and Hook's law

The most fundamental concepts in mechanics of materials are stress and strain. These concepts can be illustrated in their most elementary form by considering a prismatic bar subjected to axial forces. A prismatic bar is a straight structural member having the same cross section throughout its length, and an axial force is a load directed along the axis of the member, resulting in either tension or compression in the bar.

1.5.1 Stress.

When a body is acted upon by external force F, or Load P, internal resisting force is setup in the body such a body is said to be in state of stress, hence the resistance offered by the body against deformation due to the application of load is called as stress. Or The Internal resisting force per unit area at any section of the body is known as Stress It is denoted by σ (Sigma),

$$\text{Stress } \sigma = \frac{\text{Applied Load or Force}}{\text{Cross-sectional Area}} = \frac{F \text{ or } P}{A} \frac{N}{mm^2}$$

In general, the stresses s acting on a plane surface may be uniform throughout the area or may vary in intensity from one point to another.

1.5.1.1 Types of Stresses

1) Normal Stress.

a) Tensile Stress

b) Compressive Stress

2) Shear Stress.

3) Bearing Stress.

1. **Normal Stress;-** A normal stress is a stress that occurs when a member is loaded by an axial force. (Axial force is the force acting along the axis of the specimen). Normal stress can be either tensile or compressive in nature.

a) Tensile stress;- When a load is acting in such a way that it tends to extend the material in the direction of application of load is called tensile load and the corresponding stress is called tensile stress.

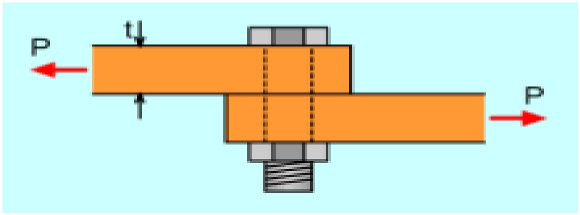
$$\text{Tensile stress, } \sigma = \frac{P \text{ or } F}{A} \; \frac{N}{mm^2}$$

1.5.1.1 Types of Stresses

2. Shear Stress;- Shearing stress is a force that causes two contacting parts or layers to slide upon each other in opposite directions. The stress developed at the contacting surfaces is known as shear stress.

$$\text{Shear Stress, } \tau = \frac{\text{Shearing Force}}{\text{Shearing Area}} = \frac{P \text{ or } F}{A} \frac{N}{mm^2}$$

3. Bearing Stress;-A Localized compressive stress at the surface of contact between two members of a machine part that are relatively at rest is known as Bearing stress or crushing stress.



Where, t = Thickness of Plate
d = Diameter of the bolt

$$\text{Bearing Stress} = \frac{P}{A} = \frac{P}{td} \frac{N}{mm^2}$$

1.5.2 Strain

When a body is subjected to some external force there is some change in dimensions of the body. The ratio of change in dimensions of the body to the original dimensions is known as Strain (ε)

Strain is dimensionless.

$$\text{Strain } \varepsilon = \frac{\text{Change in Dimension}}{\text{Original Dimension}}$$

1.5.2.1 Types of Strain

1)Linear Strain

a) Tensile Strain

b) Compressive Strain

2) Lateral Strain

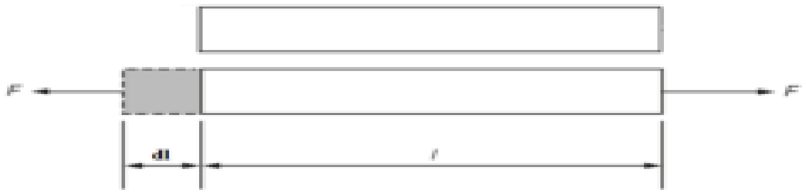
3) Shear Strain

4) Volumetric Strain

1. Linear Strain

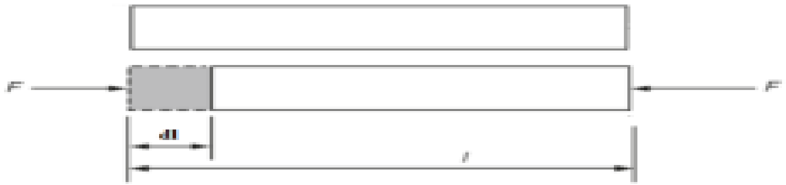
A straight bar will change in length when loaded axially, becoming longer when in tension and shorter when in compression. This change in dimensions in axial direction is known as Linear Strain.

Tensile Strain,



$$\text{Tensile Strain } \varepsilon = \frac{\text{Change in length (Extension)}}{\text{Original length}} = \frac{dl}{l} = \frac{\delta l}{l}$$

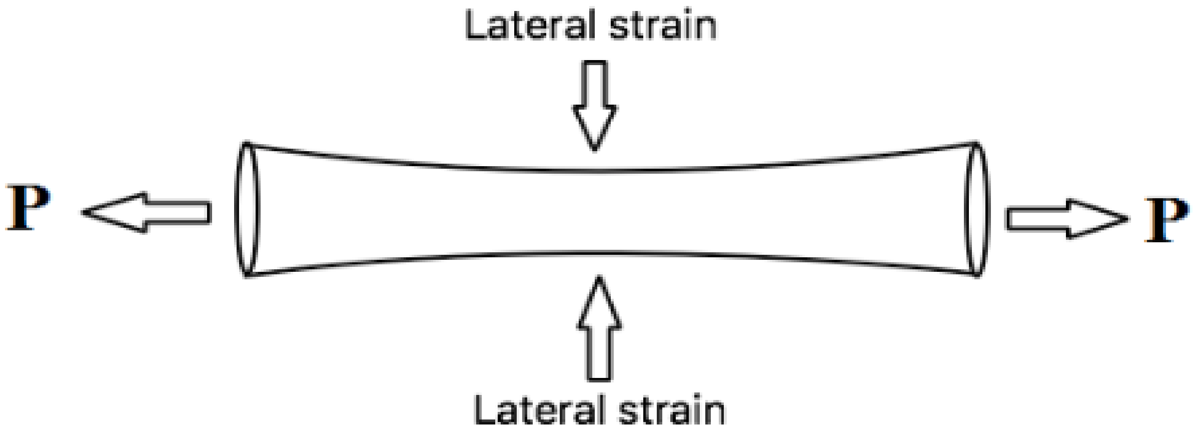
Compressive Strain,



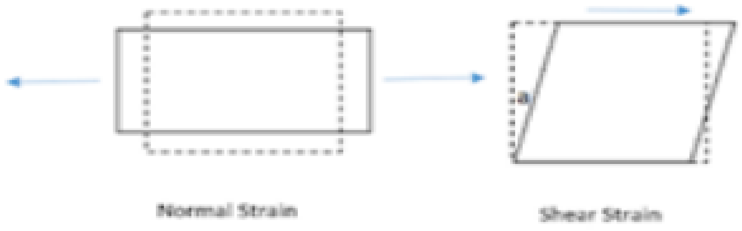
$$\text{Compressive Strain } \varepsilon = \frac{\text{Change in Length (Reduction)}}{\text{Original Length}} = \frac{dl}{l} = \frac{\delta l}{l}$$

2. Lateral Strain

Lateral strain, also known as transverse strain, which takes place at right angles to the direction of applied load is known as lateral strain.



3. Shear Strain;- Shear strain is the ratio of deformation to original dimensions. In the case of shear strain, it is the amount of deformation perpendicular to a given line rather than parallel to it.



4. Volumetric Strain

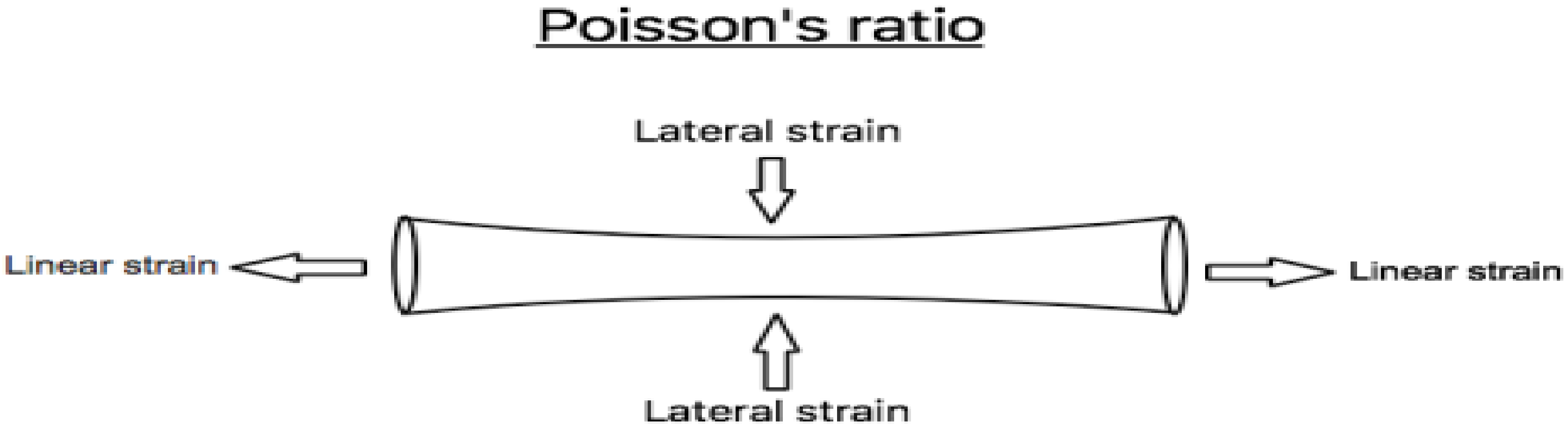
It is the ratio of change in volume to its original volume

$$\text{Volumetric Strain, } \varepsilon_v = \frac{\delta v}{v}$$

1.5.3 Poison's ratio

It is the ratio of lateral strain to linear strain

$$\text{Poison's ratio } \mu = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$$



1.5.4 Hook's Law

- It states that "When a material is loaded within its elastic limit, stress is directly proportional to the strain"

Stress α Strain

$$\text{i.e } \frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

$$\text{i.e } \frac{\sigma}{\varepsilon} = E$$

Where,

E = A constant of proportionality known as Modulus of Elasticity I

σ = Stress & ε = Strain

Hook's law holds good for tension as well as compression.

1.5.5 Modulus of Elasticity or Young's Modulus (E)

Modulus of Elasticity or Young's Modulus (E) is the constant of proportionality and is defined as the ratio of linear stress to linear strain within elastic limit

$$\text{Modulus of Elasticity, } E = \frac{\text{Linear stress (Tensile or Compressive)}}{\text{Linear Strain (Tensile or Compressive)}} = \frac{\sigma}{\varepsilon}$$

$$\therefore E = \frac{\sigma}{\varepsilon} \text{ MPa or GPa}$$

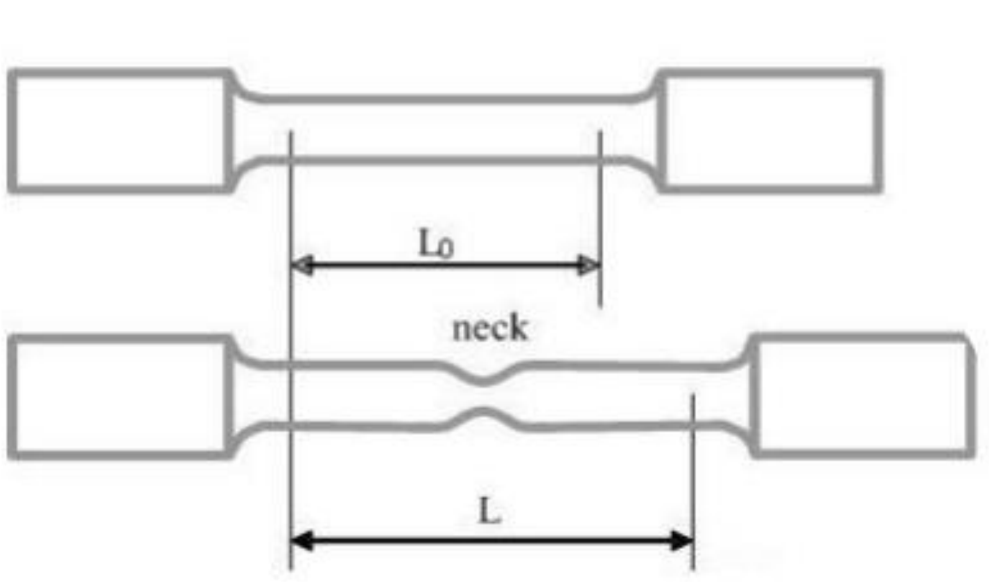
1.5.6 Factor of Safety (FOS);- It is defined as the ratio of ultimate stress or yield stress to the working or allowable or design stress.

$$FOS = \frac{\text{Ultimate or Yield Stress}}{\text{Working or Allowable or Design Stress}}$$

1.6 Stress – Strain Relation or Diagram for Ductile Material (Mild Steel or Low carbon steel)

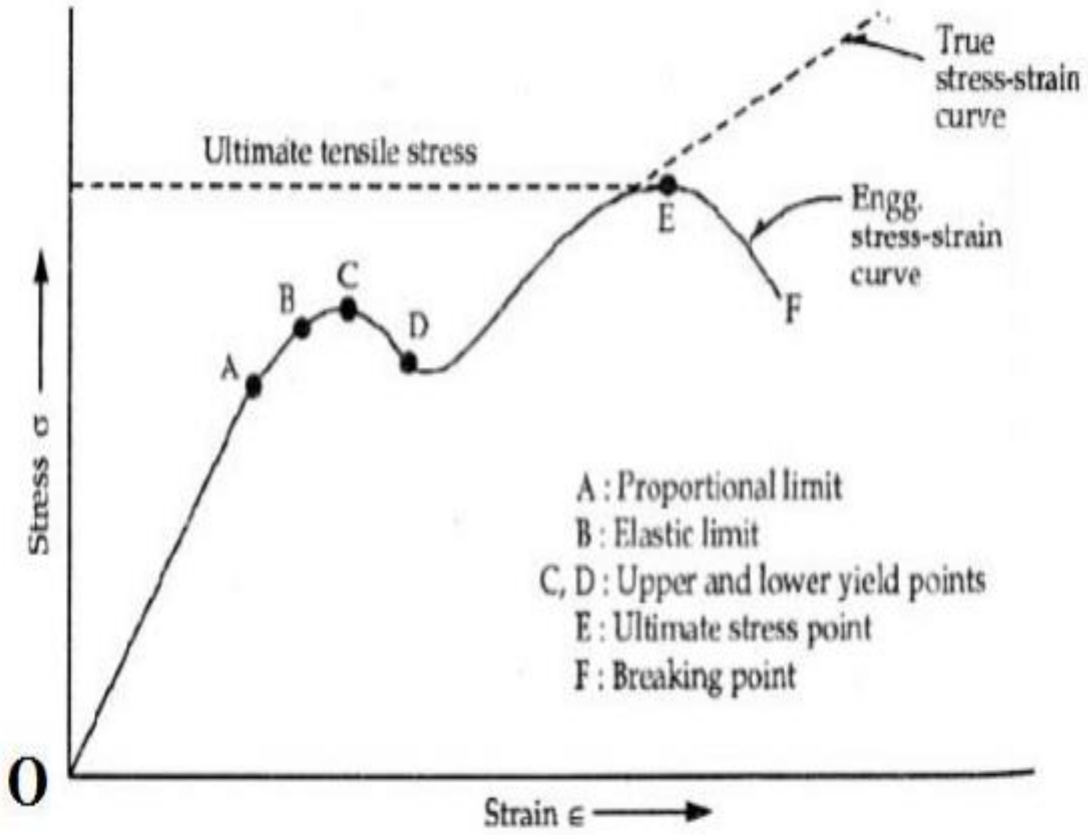
- A stress-strain diagram for a typical structural steel as a specimen in tension is shown in Figure. Strains are plotted on the horizontal axis and stresses on the vertical axis.

Standard tensile test specimen



The load on the test specimen is increased gradually from zero in suitable increments till the specimen fails and the corresponding graph will be computed as shown in the figure below.

Stress-Strain diagram



Proportional Limit (A) From O to A the curve is straight and linear and hence proportional limit is the limiting value of stress up to which stress is directly proportional to strain and hence Hooke's law holds good up to point A.

Stress α Strain.

Elastic Limit (B) The point B is slightly beyond point A and is known as Elastic limit. Up to point B, the material will regain its original size and shape when load is removed. This indicates that the material has elastic properties up to point B.

Stress-Strain diagram

- **Upper Yield Point (C);-** If the material is stressed beyond point B, plastic deformation starts and the material does not regain its original size and shape upon unload and this phenomenon is called as Yielding. A point at which Maximum load or stress required to initiate the plastic deformation or yielding of the material is called as Upper yield point “C”. At this point the dislocations or slip in the crystalline structure starts moving.
- **Lower Yield Point (D);-** As the dislocations or slip is taking place in the material, it offers less resistance to the material and hence curve falls slightly. A point at which minimum load or stress required to maintain the plastic deformation or yielding of the material is called as Lower yield point “D” and this point depicts the end of plastic deformation of the material. Dislocations or slip become too much in number and they restrict each other’s movement
- **Ultimate Stress point (E);-** After Lower Yield point D, Strain Hardening in the materials takes place. Strain hardening, also known as work hardening, is the strengthening of a metal occurs because of dislocation movements within the crystal structure of the material and hence there is a positive rise in curve from D to E. In this region as stress increases strain also increases At point E the specimen takes maximum load, and the corresponding stress at point E is called the ultimate stress point “E”.
- **Breaking Point (F);-** Beyond the ultimate stress point is reached Necking takes place and the cross sectional area considerably decreases, the load carrying capacity of the specimen reduces and hence in the portion E to F the strain increases with decrease in stress. At point F the specimen breaks. The stress at this point is called breaking stress or fracture stress

1.7 True Stress - Strain and Engineering Stress - Strain

Let P be the load, A_o be the original area of Cross-section, A be the area of cross-section at any instant.

Engineering stress is the applied load divided by the original cross-sectional area of a material. Also known as nominal stress.

$$\text{Engineering Stress } \sigma = \frac{\text{Load}}{\text{Original Area of Cross-section}} = \frac{P}{A_o} \frac{N}{mm^2}$$

True stress is the applied load divided by the actual cross-sectional area (the changing area with respect to time) of the specimen at that load

$$\text{True Stress } \sigma = \frac{\text{Load}}{\text{Actual Area of Cross-section at any instant}} = \frac{P}{A} \frac{N}{mm^2}$$

Engineering strain is the change in length to its original length in a tensile test. Also known as nominal strain.

$$\text{Engineering Strain } \varepsilon = \frac{\delta l}{l}$$

True strain is the sum of all the strains over the original length. True Strain $\varepsilon = \sum \frac{\delta l}{l}$

1.8 Problems

1. The following data refer to a mild steel specimen tested in a laboratory

- Diameter of the specimen - 25mm
- Length of the specimen – 300 mm
- Extension under a load of 15kN – 0.045 mm
- Load at yield point – 127.65kN
- Maximum load – 208.60kN
- Length of the specimen after failure – 375mm
- Neck diameter – 17.75mm

Determine Young's modulus, Yield stress, Ultimate stress, % Elongation, %Reduction in area, safe or permissible stress adopting a factor of safety 2

1.8 Problems

Given Data:

$d_o = 25mm$, $d_f = 17.75mm$, $L_o = 300mm$, $L_f = 375mm$, $\delta l = 0.045mm$, $F_{max} = 208.60kN$

Area of the specimen, $A = \frac{\pi d_o^2}{4} = \frac{\pi x 25^2}{4} = \mathbf{490.87\ mm^2}$

$\sigma = \frac{F}{A} = \frac{15x10^3}{490.87} = \mathbf{30.55\ \frac{N}{mm^2}}$

$\varepsilon = \frac{\delta l}{l} = \frac{0.045}{300} = \mathbf{1.5x10^{-4}}$

$\rightarrow$Young's Modulus $E = \frac{\sigma}{\varepsilon} = \frac{30.55}{1.5x10^{-4}} = \mathbf{203.66x10^3\ \frac{N}{mm^2}}$

$\rightarrow$ Yield stress $\sigma_y = \frac{F}{A} = \frac{127.65x10^3}{490.87} = \mathbf{260.04\ \frac{N}{mm^2}}$

$\rightarrow$ Ultimate stress $\sigma_u = \frac{F}{A} = \frac{208.60x10^3}{490.87} = \mathbf{424.95\ \frac{N}{mm^2}}$

$\rightarrow$ % Elongation $= \frac{L_f - L_i}{L_i}$ x 100 $= \frac{375-300}{300}$ x100 = **25%**

$\rightarrow$% Reduction $= \frac{A_i - A_f}{A_i}$ x 100

$A_f = \frac{\pi d_f^2}{4} = \frac{\pi x 17.75^2}{4} = \mathbf{247.44\ mm^2}$

$\therefore \quad \frac{490.87-247.44}{490.87}$ x100 = **49.59 %**

1.8 Problems

2. A rod 150 cm long and a diameter 2 cm is subjected to an axial pull of 20kN. If the modulus of elasticity of material is 200 GPa. Determine stress, strain and Elongation of rod.

Given data:

$l = 150\ \text{cm} = 1500\ \text{mm}$

$d = 2\ \text{cm} = 20\ \text{mm}$

$P = 20\text{kN} = 20\text{x}10^3\ \text{N}$

$E = 200\ \text{GPa} = 200\text{x}10^3\ \frac{\text{N}}{\text{mm}^2}$

$$\text{Area} = \frac{\pi d^2}{4} = \frac{\pi 20^2}{4} = \mathbf{314.15\ mm^2}$$

$$\rightarrow \sigma = \frac{F}{A} = \frac{20x10^3}{314.15} = \mathbf{63.66}\ \frac{\mathbf{N}}{\mathbf{mm^2}}$$

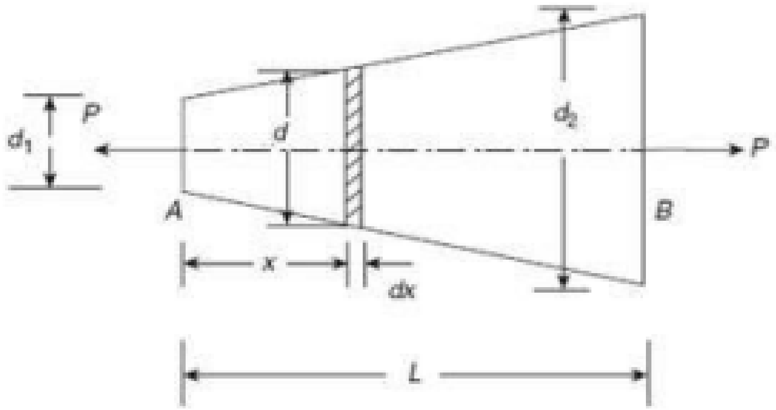
$$\rightarrow E = \frac{\sigma}{\varepsilon} \longrightarrow \varepsilon = \frac{\sigma}{E} = \frac{63.66}{200\text{x}10^3} = \mathbf{3.183x10^{-4}}$$

$$\rightarrow \varepsilon = \frac{\delta l}{l} \longrightarrow \delta l = \varepsilon \text{ x } l = 3.183\text{x}10^{-4} \text{ x } 1500$$

$$\mathbf{\delta l = 0.477\ mm}$$

1.9 Elongation of tapering bars of circular cross section

- Consider a circular bar uniformly tapered from diameter d1 at one end and gradually increasing to diameter d2 at the other end over an axial length L as shown in the figure below.



Since the diameter of the bar is continuously changing, the elongation is first computed over an elementary length and then integrated over the entire length. Consider an elementary strip of diameter d and length dx at a distance of x from end A.

1.9 Elongation of tapering bars of circular cross section

Using the principle of similar triangles the following equation for d can be obtained

$$d = d_1 + \frac{d_2 - d_1}{L} x = d_1 + kx \text{ , where } k = \frac{d_2 - d_1}{L}$$

Cross–sectional area of the bar at x : $A_x = \frac{\pi (d_1 + kx)^2}{4}$

Axial stress at x:$\sigma_x = \frac{P}{A_x} = \frac{4P}{\pi (d_1 + kx)^2}$

Change in length over dx :$\delta dx = \frac{\sigma_x dx}{E} = \frac{4P\, dx}{\pi E (d_1 + kx)^2}$

Total change in length: $\delta L = \int_0^L \frac{4P\, dx}{\pi E (d_1 + kx)^2} = \frac{4P}{\pi E}\left[\frac{(d_1 + kx)^{-1}}{-k}\right]_0^L$

After rearranging the terms: $\delta L = -\frac{4P}{\pi E k}\left[\frac{1}{(d_1 + kx)}\right]_0^L$

Upon substituting the limits : $\delta L = -\frac{4P}{\pi E k}\left[\frac{1}{(d_1 + kL)} - \frac{1}{d_1}\right]$

After rearranging the terms: $\delta L = \frac{4P}{\pi E k}\left[\frac{1}{d_1} - \frac{1}{(d_1 + kL)}\right]$

But $(d_1 + kL) = d_1 + \frac{d_2 - d_1}{L} L = d_2$

With the above substitution: $\delta L = \frac{4P}{\pi E k}\left[\frac{1}{d_1} - \frac{1}{d_2}\right] = \frac{4P}{\pi E k}\left[\frac{d_2 - d_1}{d_1 d_2}\right]$

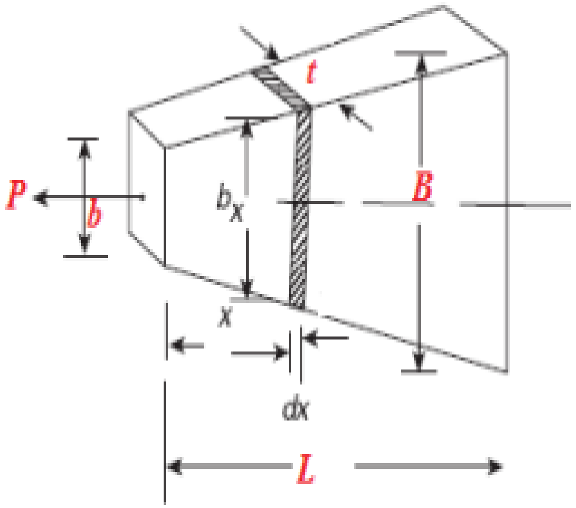
Substituting for $k = \frac{d_2 - d_1}{L}$ in the above expression, following equation for elongation of tapering bar of circular section can be obtained

Total change in length: $\delta L = \frac{4P\, L}{\pi E\, d_1 d_2}$

1.10 Elongation of tapering bars of rectangular cross section

- Consider a bar of same thickness t throughout its length, tapering uniformly from a breadth B at one end to a breadth b at the other end over an axial length L. The flat is subjected to an axial force P as shown in the Since the breadth of the bar is continuously changing the elongation is first computed over an elementary length and then integrated over the entire length. Consider an elementary strip of breadth bx and length dx at x from left end. Using the principle of similar triangular the following equation for bx can be obtained

$$b_x = b + \frac{B-b}{L}x = b + kx \text{ , where } k = \frac{B-b}{L}$$



Cross–sectional area of the bar at x : $A_x = b_x t = (b+kx)t$

Axial stress at x: $\sigma_x = \frac{P}{A_x} = \frac{P}{(b+kx)t}$

Change in length over dx : $\delta dx = \frac{\sigma_x\, dx}{E} = \frac{P\,dx}{Et(b+kx)}$

Total change in length: $\delta L = \int_0^L \frac{P\,dx}{Et(b+kx)} = \frac{P}{Etk}[ln(b+kx)]_0^L$

Upon substituting the limits : $\delta L = \frac{P}{Etk}[ln(b+kL) - \ln(b)]$

$But\ (b+kL) = b + \frac{B-b}{L}L = B$

With the above substitution: $\delta L = \frac{P}{Etk}[ln(B) - \ln(b)] = \frac{P}{Etk}\ln(B/b)$

Substituting for $k = \frac{B-b}{L}$ in the above expression, following equation for elongation of tapering bar of rectangular section can be obtained

$$\delta L = \frac{P\,L}{Et(B-b)}\ln(B/b)$$