MODULE –5a

Unbalanced Three Phase Systems

Content:

Unbalanced Three Phase Systems:

1. Analysis of three phase systems (3-wire and 4 wire systems)
2. Calculation of real and reactive Powers.

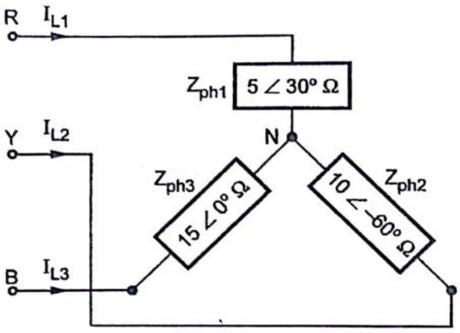
5.1 Introduction

- Industrial and residential consumers use equipment that can be single phase and / or three phase.
- In general, the distribution networks offer the two types of voltage, and the single phase is obtained from the three-phase using only the neutral and one of the phases.
- As the load on the three phases changes constantly, a four-wire (neutral + 3 phase) system is used to maintain stable voltage and provide a return path for the neutral current due to load unbalance.
- This system is used by energy distribution companies electrical, when in low voltage, for homes, commercial rooms and small industries.

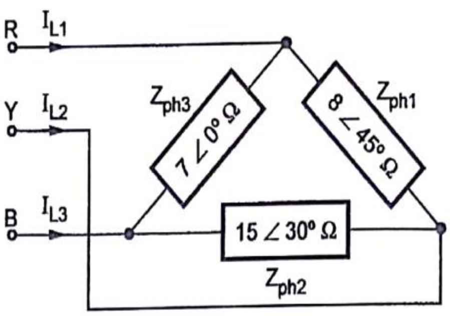
5.1.1 For a three-phase system to be considered unbalanced if:

- The voltages of the source are not equal in modulus and / or differ in phase (different angles);
- The load impedances are unequal (at least one of them). This is the most likely situation to happen.

Example:



(a) Star connected unbalanced load



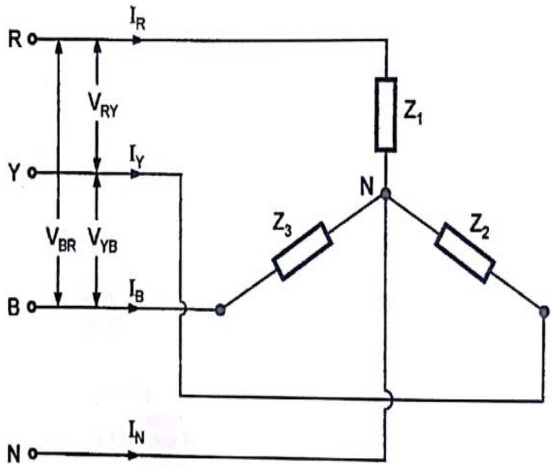
(b) Delta connected unbalanced load

5.2 Types of Unbalanced Loads

- The unbalanced load is the load in which the load impedances are not same but having different values.

- The values of voltages and currents are also different in each phase.
- The unbalanced loads may also be connected either in star or in delta.
- With unbalanced loads, the power in each phase has to be calculated separately.

Consider three phase star connected four wire unbalanced load system as shown in Fig.



I_R, I_Y, I_B -line currents which are same as the phase currents.

V_{RY}, V_{YB} and V_{BR} -line voltages.

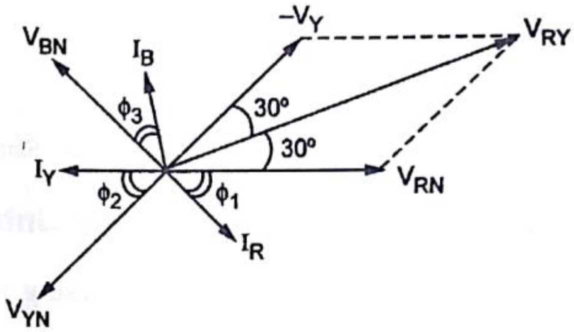
$I_N = I_R + I_Y + I_B$

The phase voltage is lagging by 30^0 from the line voltage.

Let V_{RN} - reference phase voltage

Phase voltages are given by,

$$V_{RN} = V_{ph}\angle 0^\circ \text{ volts}$$
$$V_{YN} = V_{ph}\angle -120^\circ \text{ volts}$$
$$V_{BN} = V_{ph}\angle 120^\circ \text{ volts}$$



If Z_1, Z_2 and Z_3 are load impedances then the currents are given by,

$$I_R = \frac{V_{ph}}{Z_1} = \frac{V_{ph}\angle 0^\circ}{Z_1\angle\phi_1} = \frac{V_{ph}}{Z_1}\angle -\phi_1 \text{ Amp.}$$
$$I_Y = \frac{V_{YN}}{Z_2} = \frac{V_{ph}\angle -120^\circ}{Z_2\angle\phi_2} = \frac{V_{ph}}{Z_2}\angle -120^\circ-\phi_2 \text{ Amp.}$$
$$I_B = \frac{V_{BN}}{Z_3} = \frac{V_{ph}\angle 120^\circ}{Z_3\angle\phi_3} = \frac{V_{ph}}{Z_3}\angle +120^\circ-\phi_3 \text{ Amp.}$$

Thus in general following types of unbalanced loads are possible in practice

i) Unbalanced three wire star connected load.

ii) Unbalanced four wire star connected load.

iii) Unbalanced delta connected load.

5.3 Analysis of Three Phase Unbalanced Loads

a. Unbalanced Four Wire Star Connected Load

In such loads a fourth wire is used to connect the neutral of the load to the neutral of supply system. There are two cases in such a load.

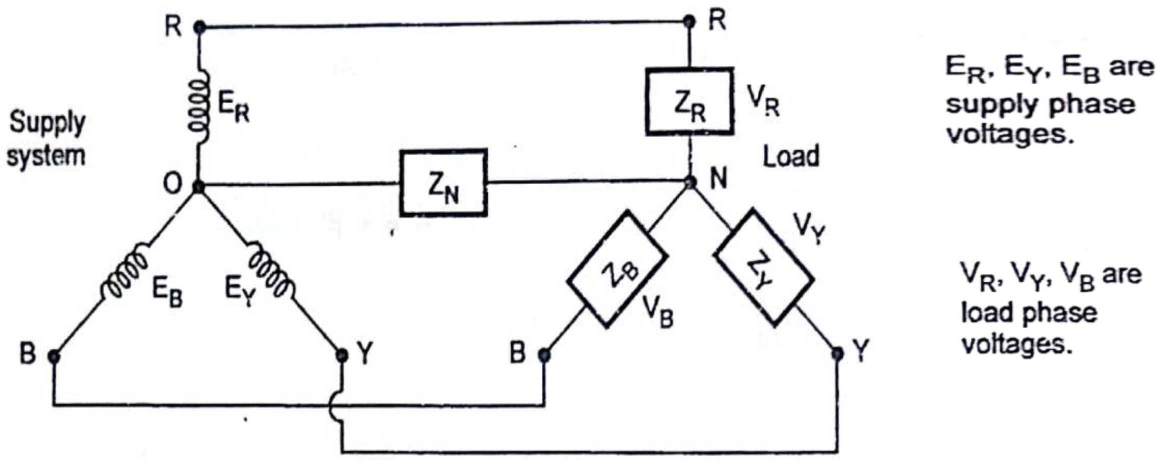
1. Neutral wire with zero impedance

If neutral wire has negligible impedance, it is assumed to have zero impedance. The star point of load and generator supply system get directly connected and are at the same potential. Thus in this case if line voltages are balanced & then each phase voltage is a forced voltage such &tat the free phase voltages are balanced. The voltage across the three load impedances are equalized. Here neutral current is given by,

$$\overline{I}_N = \overline{I}_R + \overline{I}_Y + \overline{I}_B$$

2. Neutral wire with Impedance Z_N

In some case a neutral of load is connected to neutral of the supply through an impedance Z_N. This is shown in the Fig.



In this method, the voltage $\overline{V}_{NO}$ is given by,

$$V_{NO} = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B + Y_N}$$

Where Y_R, Y_Y and Y_B are the load phase admittances. Due to V_{NO}, the load phase voltages are not equal to one another.

$$\overline{V}_R = \overline{E}_R - \overline{V}_{NO} \qquad \overline{V}_Y = \overline{E}_Y - \overline{V}_{NO} \qquad \overline{V}_B = \overline{E}_B - \overline{V}_{NO}$$

b. Unbalanced Delta Connected Load

In unbalanced delta connected load, the phase voltages are same as the line voltages hence the phase currents can be obtained by dividing the phase voltages by the respective phase impedances. From the phase the line currents can be obtained.

5.4 Power Factor of Unbalanced Load

- The power factor is defined as the ratio of active power to the apparent power. But
- in unbalanced system, each phase has different power factor hence overall factor can not
- be precisely defined.
- Still the ratio of total active power $\sqrt{3}*V_L*I_L*\cos\phi$ to the total apparent power $\sqrt{3}*V_L*I_L$ gives approximate power factor of the unbalanced load.
- In unbalanced loads a vector power factor is defined which is defined as,

$$\text{Power factor} = \frac{\sum V I \cos\phi}{\sum V I}$$

The numerator is the algebraic sum of all the active powers of all individual phases.

$$\therefore \quad \sum V I \cos\phi = V_{ph1} I_{ph1} \cos\phi_1 + V_{ph2} I_{ph2} \cos\phi_2 + V_{ph3} I_{ph3} \cos\phi_3$$

While, $$\sum V I = \sqrt{[\sum V I \cos\phi]^2 + [\sum V I \sin\phi]^2}$$

Where $\sum V I \sin\phi$ is the algebraic sum of the reactive volt-amperes of all individual phases.

$$\therefore \quad \sum VI \sin\phi = V_{ph1} I_{ph1} \sin\phi_1 + V_{ph2} I_{ph2} \sin\phi_2 + V_{ph3} I_{ph3} \sin\phi_3$$

Numericals:

1. A 3-phase, 3-wire, 400V delta-connected load has impedances Zab=10∠0 ∘ Ω, Zbc=10∠−30 ∘ Ω and Zca=10∠30 ∘ Ω. The phase sequence is abc. Determine the phase currents and line currents.

Soln. Given Data:

VLL=400 V

Zab=10∠0∘ Ω

Zbc=10∠−30∘ Ω

Zca=10∠30∘ Ω

Calculate the Phase Voltages $$V_{ph} = \frac{V_{LL}}{\sqrt{3}}$$

$$V_{ab} = V_{bc} = V_{ca} = V_{LL} = 400\text{ V}$$

Calculate the Phase Currents

$$I_{ab} = \frac{V_{ab}}{Z_{ab}}, \quad I_{bc} = \frac{V_{bc}}{Z_{bc}}, \quad I_{ca} = \frac{V_{ca}}{Z_{ca}}$$

$$I_{ab} = \frac{400\angle 0^\circ}{10\angle 0^\circ} = 40\angle 0^\circ\text{ A}$$

$$I_{bc} = \frac{400\angle -120^\circ}{10\angle -30^\circ} = 40\angle(-120^\circ + 30^\circ) = 40\angle -90^\circ\text{ A}$$

$$I_{ca} = \frac{400\angle 120^\circ}{10\angle 30^\circ} = 40\angle(120^\circ - 30^\circ) = 40\angle 90^\circ\text{ A}$$

Calculate the Line Currents

$$I_a = I_{ab} - I_{ca}, \quad I_b = I_{bc} - I_{ab}, \quad I_c = I_{ca} - I_{bc}$$

$$I_a = 40\angle 0^\circ - 40\angle 90^\circ$$

$$|I_a| = \sqrt{40^2 + 40^2} = \sqrt{3200} \approx 56.57\text{ A},$$

$$\angle I_a = \tan^{-1}\left(\frac{-40}{40}\right) = -45^\circ$$

$$I_b = 40\angle-90^\circ - 40\angle 0^\circ = -j40 - 40 = -40 - j40 \text{ A}$$

$$|I_b| = \sqrt{40^2 + 40^2} \approx 56.57 \text{ A}, \qquad \angle I_b = \tan^{-1}\left(\frac{-40}{-40}\right) = 225^\circ \text{ (or } -135^\circ)$$

$$I_c = 40\angle 90^\circ - 40\angle-90^\circ = j40 + j40 = j80 \text{ A} \qquad |I_c| = 80 \text{ A}, \quad \angle I_c = 90^\circ$$

Summary of Results

- Phase Currents:

$$I_{ab} = 40\angle 0^\circ \text{ A}, \quad I_{bc} = 40\angle-90^\circ \text{ A}, \quad I_{ca} = 40\angle 90^\circ \text{ A}$$

- Line Currents:

$$I_a \approx 56.57\angle-45^\circ \text{ A}, \quad I_b \approx 56.57\angle-135^\circ \text{ A}, \quad I_c = 80\angle 90^\circ \text{ A}$$

2. A three-phase, 4-wire system with a 150V supply has a Y-connected load with the following impedances: ZA=6∠0° ΩZB=6∠30° Ω, ZC=5∠45° Ω. The phase sequence is CBA. Obtain all line currents and draw a phasor diagram. (10 Marks).

Soln. Given Data:
Voltage (VL): 150V
Phase Sequence: CBA
Load Impedances

$$Z_A = 6\angle 0^\circ\ \Omega$$

$$Z_B = 6\angle 30^\circ\ \Omega$$

$$Z_C = 5\angle 45^\circ\ \Omega$$

Calculate the Phase Voltages
The line voltage is given as 150V.
For a Y-connected system:

$$V_{ph} = \frac{V_{line}}{\sqrt{3}} = \frac{150V}{\sqrt{3}} = 86.6V$$

The phase voltages for the CBA sequence are:

$$V_A = 86.6\angle 0^\circ\ V$$

$$V_B = 86.6\angle-120^\circ\ V$$

$$V_C = 86.6\angle 120^\circ\ V$$

Calculate the Line Currents

Using Ohm's Law $I = \frac{V}{Z}$ for each phase:

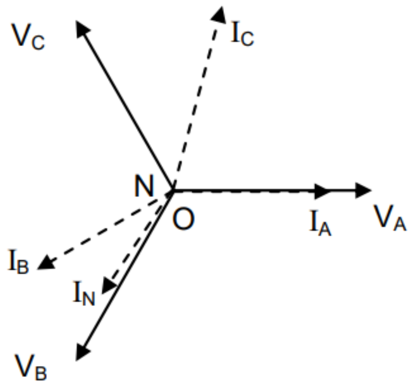
For Phase A: $$I_A = \frac{V_A}{Z_A} = \frac{86.6\angle 0^\circ}{6\angle 0^\circ} = 14.43\angle 0^\circ\ A$$

For Phase B: $$I_B = \frac{V_B}{Z_B} = \frac{86.6\angle -120^\circ}{6\angle 30^\circ} = 14.43\angle(-120^\circ - 30^\circ) = 14.43\angle -150^\circ\ A$$

For Phase C: $$I_C = \frac{V_C}{Z_C} = \frac{86.6\angle 120^\circ}{5\angle 45^\circ} = 17.32\angle(120^\circ - 45^\circ) = 17.32\angle 75^\circ\ A$$

Draw the Phasor Diagram

- To draw the phasor diagram:
- Start with IA=14.43∠0° on the positive real axis.
- Draw IB=14.43∠−150 °, which will be at 150 degrees counterclockwise from the positive real axis.
- Draw IC=17.32∠75 ° , which will be at 75 degrees counterclockwise from the positive real axis.



3. **Determine the line current and total power supplied to a delta-connected load with the following impedances:*Zab*=10∠60° Ω, *Zbc*=20 Ω∠90°, *Zca*=25∠30° Ω. Assume a 3-phase, 400V, ABC system**

Solution:

Determine the Phase Voltages

In a delta-connected system: VL=Vph $$V_{ph} = V_{line} = 400V$$

Calculate the Phase Currents

Using Ohm's Law $I=V/Z$, we can calculate the phase currents for each impedance.

For Zab: $$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{400\angle 0^\circ}{10\angle 60^\circ} = 40\angle -60^\circ\ A$$

For Zbc: $$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{400\angle -120^\circ}{20\angle 90^\circ} = 20\angle -210^\circ\ A$$

For Zca $$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{400\angle 120^\circ}{25\angle 30^\circ} = 16\angle 90^\circ\ A$$

Determine the Line Currents

$$I_a = I_{ab} - I_{ca}$$
$$I_b = I_{bc} - I_{ab}$$
$$I_c = I_{ca} - I_{bc}$$

$$I_a = I_{ab} - I_{ca} = 40\angle -60^\circ - 16\angle 90^\circ$$
$$I_a = 44.49\angle -77.47^\circ\ A$$

$$I_b = I_{bc} - I_{ab} = 20\angle 150^\circ - 40\angle -60^\circ$$
$$I_b = 50.63\angle 154.17^\circ\ A$$

$$I_c = I_{ca} - I_{bc} = 16\angle 90^\circ - 20\angle 150^\circ$$
$$I_c = 21.48\angle 108.43^\circ\ A$$

Phase-wise Power Calculations:

The power in phase AB is: $$P_{AB} = \frac{|V_{AB}|^2}{|Z_{AB}|} \cdot \cos(\theta_{AB})$$

$$P_{AB} = \frac{(400)^2}{10} \times \cos(60^\circ) = 16000 \times 0.5 = 8000\ \mathrm{W}$$

The power in phase BC is: $$P_{BC} = \frac{|V_{BC}|^2}{|Z_{BC}|} \cdot \cos(\theta_{BC})$$

$$P_{BC} = \frac{(400)^2}{20} \times 0 = 0\ \mathrm{W}$$

The power in phase CA is: $$P_{CA} = \frac{|V_{CA}|^2}{|Z_{CA}|} \cdot \cos(\theta_{CA})$$

$$P_{CA} = \frac{(400)^2}{25} \times \cos(30^\circ) = 6400 \times \frac{\sqrt{3}}{2} = 5542.5\ \mathrm{W}$$

Total Power Supplied:

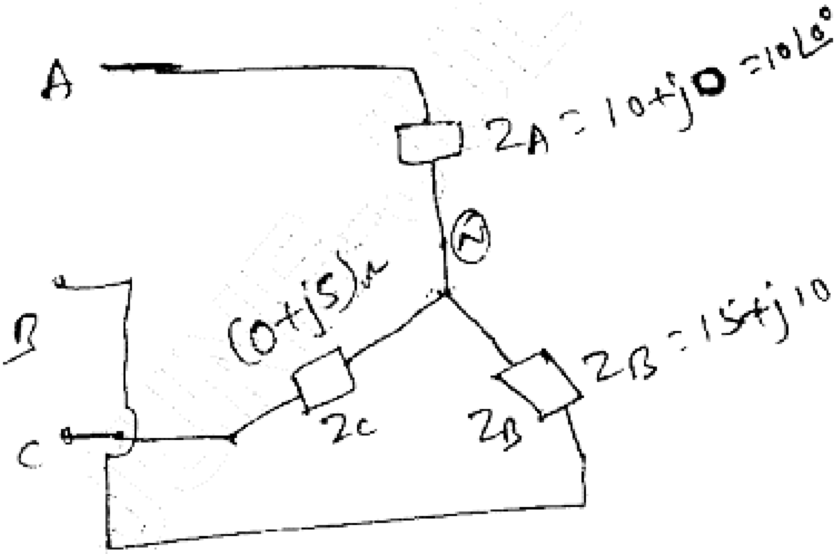
The total power supplied is the sum of the power in each phase:

$$P_{\text{total}} = P_{AB} + P_{BC} + P_{CA}$$

$$P_{\text{total}} = 8000 + 0 + 5542.5 = 13542.5\,\text{W}$$

The total power supplied to the delta-connected load is **13.5425 kW**.

4. A 3-phase, 400 V, 4-wire system has a star-connected load with ZA=10∠0° Ω, ZB=(15+j10) Ω, ZC=(0+j5) Ω. Find the line current and current through the neutral conductor for phase sequence ABC for Fig.



Solution:

Given:

- $Z_A = 10\angle 0^\circ\,\Omega$
- $Z_B = 15 + j10\,\Omega$
- $Z_C = 0 + j5\,\Omega$
- Line-to-neutral voltage $V_{ph} = \frac{400}{\sqrt{3}}\,\text{V}$

Calculate the Phase Voltages

The line-to-neutral voltage for a star-connected system is:

$$V_{ph} = \frac{V_{LL}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\,\mathrm{V}$$

Calculate the Phase Currents

For a star-connected system, the phase current III in each branch is given by:

$$I = \frac{V_{ph}}{Z}$$

Calculate I_A, I_B, I_C:

$$I_A = \frac{230.94\angle 0^\circ}{10\angle 0^\circ} = 23.094\angle 0^\circ\ \mathrm{A}$$

Given $Z_B = 15 + j10\,\Omega$, convert to polar form: $Z_B = 18.03\angle 33.69^\circ\ \Omega$

$$I_B = \frac{230.94\angle -120^\circ}{18.03\angle 33.69^\circ} = 12.8\angle -153.69^\circ\ \mathrm{A}$$

$Z_C = 0 + j5\,\Omega$, convert to polar form: $Z_C = 5\angle 90^\circ\ \Omega$

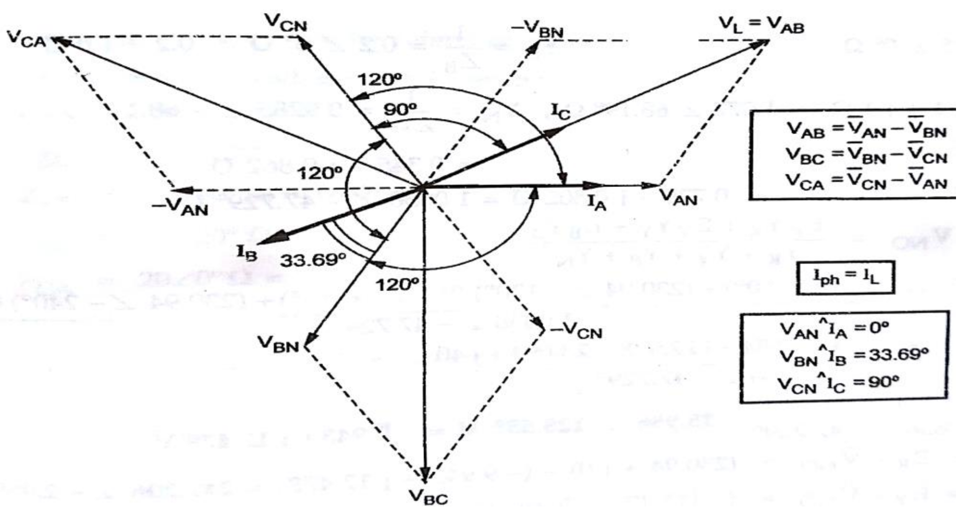
$$I_C = \frac{230.94\angle 120^\circ}{5\angle 90^\circ} = 46.188\angle 30^\circ\ \mathrm{A}$$

$$I_N = I_A + I_B + I_C$$

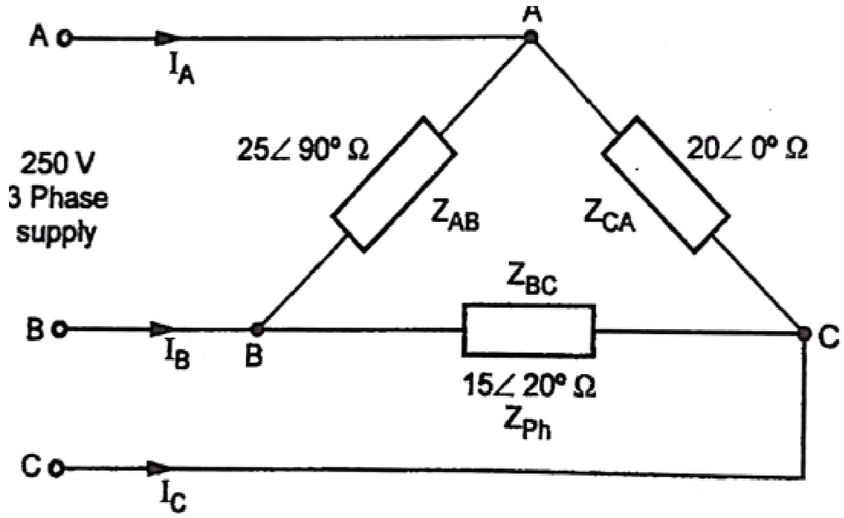
$$I_N = 23.094\angle 0^\circ + 12.8\angle -153.69^\circ + 46.188\angle 30^\circ$$

$$I_N = 52.014 + j16.344\,\mathrm{A}$$

$$I_N = 54.58\angle 17.55^\circ\ \mathrm{A}$$



5. A 3 phase supply with the line voltage of 250 V has an unbalanced delta connected load as shown in the Fig. Determine line currents, total active and reactive power if phase sequence is ABC.



Solution:

$\bar{V}_{AB} = 250\angle 0^\circ$ V Reference phasor

$$\bar{V}_{BC} = 250\angle{-120^\circ} \text{ V}$$
$$\bar{V}_{CA} = 250\angle 120^\circ \text{ V}$$
$$Z_{AB} = 25\angle 90^\circ \ \Omega = 0+j25 \ \Omega$$
$$Z_{BC} = 15\angle 20^\circ \ \Omega = 14.09 + j5.13 \ \Omega$$
$$Z_{CA} = 20\angle 0^\circ \ \Omega = 20+j0 \ \Omega$$

Phase currents:

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z_{AB}} = \frac{250\angle 0^\circ}{25\angle 90^\circ} = 10\angle{-90^\circ} \text{A} = (0 - j10) \text{ A}$$

$$\bar{I}_{BC} = \frac{\bar{V}_{BC}}{Z_{BC}} = \frac{250\angle{-120^\circ}}{15\angle 20^\circ} = 16.67\angle{-140^\circ} \text{A} = (-12.76 - j10.71) \text{ A}$$

$$\bar{I}_{CA} = \frac{\bar{V}_{CA}}{Z_{CA}} = \frac{250\angle 120^\circ}{20\angle 0^\circ} = 12.5\angle 120^\circ \text{A} = (-6.25 + j10.82) \text{ A}$$

Line currents:

$$\bar{I}_A = \bar{I}_{AB} + \bar{I}_{CA} = (0 - j10) + (-6.25 + j10.82) = -6.25 + j0.82$$

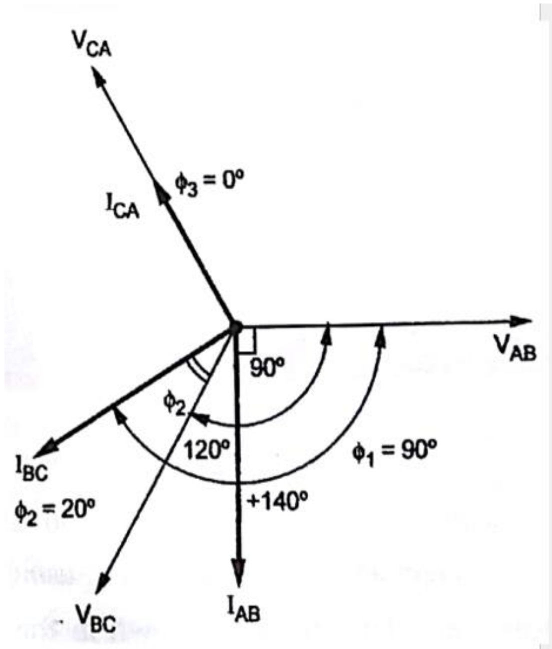
$$= \mathbf{6.30 \angle 172.52^\circ \text{ A}}$$

$$\bar{I}_B = \bar{I}_{BC} + \bar{I}_{AB} = (-12.76 - j10.71) + (0 - j10) = -12.76 - j20.71$$

$$= \mathbf{24.32 \angle -121.63^\circ \text{ A}}$$

$$\bar{I}_C = \bar{I}_{CA} + \bar{I}_{BC} = (-6.25 + j10.82) + (-12.76 - j10.71) = -19.01 + j0.11$$

$$= \mathbf{19.01 \angle 179.66 \text{ A}^\circ}$$



$$\text{Total active power} = V_{AB} \cdot I_{AB} \cos(V_{AB} \wedge I_{AB}) + V_{BC} \cdot I_{BC} \cos(V_{BC} \wedge I_{BC}) + V_{CA} \cdot I_{CA} \cos(V_{CA} \wedge I_{CA})$$

$$= (250)(10)\cos(+90^\circ) + (250)(16.67)\cos(+20^\circ) + (250)(12.5)\cos(0^\circ) = 0 + 3916.16 + 3125$$

$$= 7041.16 \text{ W} = \mathbf{7.04 \text{ kW}}$$

$$\text{Total reactive power} = V_{AB} \cdot I_{AB} \sin(V_{AB} \wedge I_{AB}) + V_{BC} \cdot I_{BC} \sin(V_{BC} \wedge I_{BC}) + V_{CA} \cdot I_{CA} \sin(V_{CA} \wedge I_{CA})$$

$$= (250)(10)\sin(+90^\circ) + (250)(16.67)\sin(20^\circ) + (250)(12.5)\sin(0^\circ)$$

$$= 2500 + 1425.36$$

$$= \mathbf{3925.36 \text{ VAR} = 3.925 \text{ kVAR}}$$

6. **A 3 phase4-wire CBA system of phase sequence, with effective line voltage of 100 V has a star - connected impedances given by : ZA=3<$0^0\Omega$, ZB = 4.5<$56.31^0\Omega$, Z c = 2.24<$26.57^0\Omega$. Obtain the line currents and current in neutral wire. Draw the phasor diagram.**

Solution : The given $V_L = 100$ V and star connection.

$$|V_{ph}| = \frac{V_L}{\sqrt{3}} = \frac{100}{\sqrt{3}} = 57.7350 \text{ V}$$

The phase sequence is CBA. Let phase voltage V_{CN} be the reference.

$\therefore V_{CN} = 57.735\angle 0° \text{ V}, \; V_{BN} = 57.735\angle -120° \text{ V}, V_{AN} = 57.735\angle +120° \text{ V}$

$\therefore$ The various phase currents are,

$$I_C = \frac{V_{CN}}{Z_C} = \frac{57.735\angle 0°}{2.24\angle -26.57°} = 25.7745\angle +26.57° = 23.0524 + j11.5286 \text{ A}$$

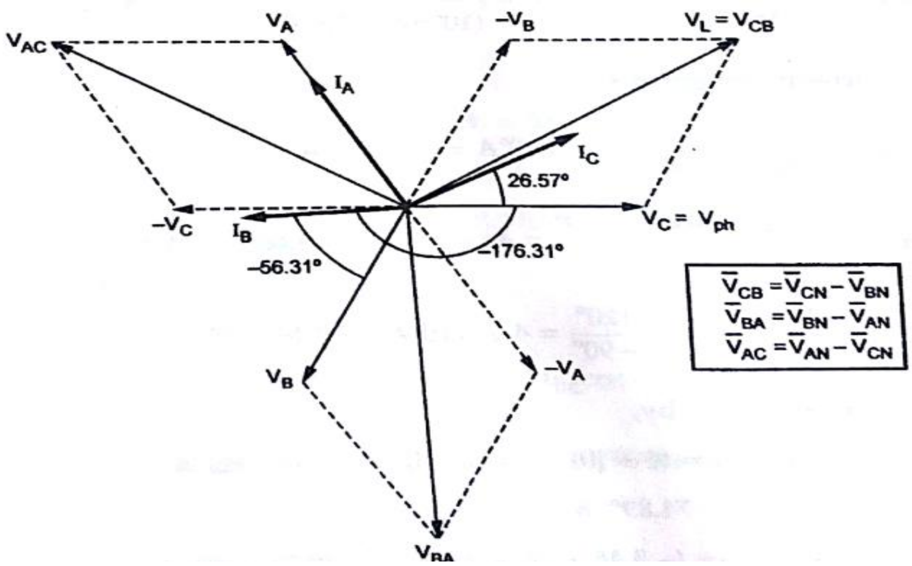
$$I_B = \frac{V_{BN}}{Z_B} = \frac{57.735\angle -120°}{4.5\angle +56.31°} = 12.83\angle -176.31° = -12.8 - j\,0.8257 \text{ A}$$

$$I_A = \frac{V_{AN}}{Z_A} = \frac{57.735\angle +120°}{3\angle 0°} = 19.245\angle +120° = -9.6225 + j\,16.667 \text{ A}$$

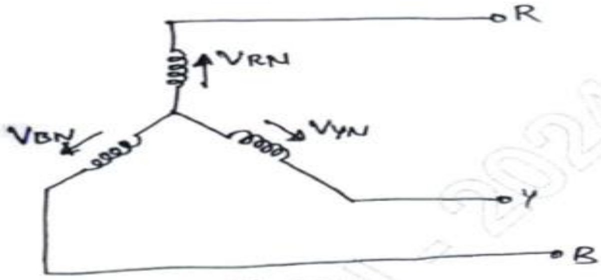
For star, $I_L = I_{ph}$

Hence I_C, I_B and I_A calculated are the respective line currents.

$$\bar{I}_N = \bar{I}_C + \bar{I}_B + \bar{I}_A = 23.0524 + j11.5286 - 12.8 - j0.8257 - 9.6225 + j16.667 \text{A}$$



7. **A symmetrical star-connected system is shown in Fig. Calculate the three-phase line voltage and power given *VRN*=230∠0∘ Volts. Assume phase sequence is RYB.**



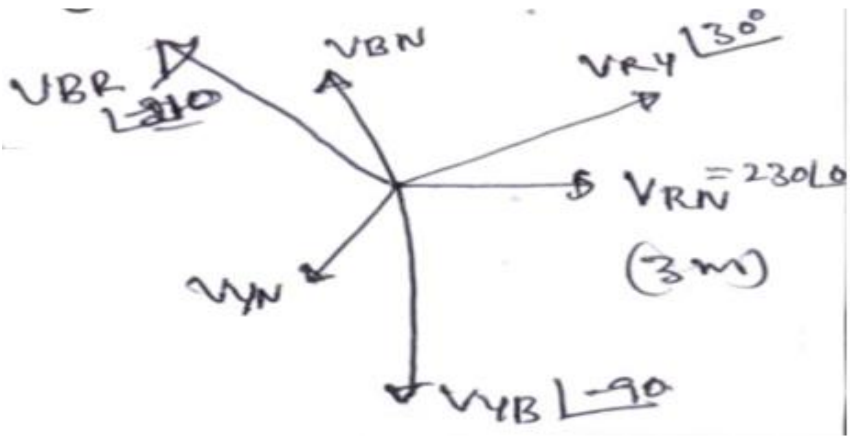
Given:
VRN=230∠0∘V
VYN=230∠−120∘V
VBN=230∠120∘V

Line Voltage:

$$V_{RY} = \sqrt{3} \times 230\angle 30^\circ = 398.37\angle 30^\circ \text{ V}$$
$$V_{YB} = \sqrt{3} \times 230\angle -90^\circ = 398.37\angle -90^\circ \text{ V}$$
$$V_{BR} = \sqrt{3} \times 230\angle 150^\circ = 398.37\angle 150^\circ \text{ V}$$

Total Power:

$$P = P_{RY} + P_{YB} + P_{BR}$$



8. Determine the line currents in an unbalanced star-connected load supplied from a symmetrical 3-phase, 4-wire, 440V system. The branch impedances of the load are: ZA =5∠30 ∘ Ω, ZB =10∠45 ∘ Ω, ZC =10∠60 ∘ Ω. The phase sequence is ABC.

Given:

$$Z_A = 5\angle 30^\circ\,\Omega,\ Z_B = 10\angle 45^\circ\,\Omega,\ Z_C = 10\angle 60^\circ\,\Omega$$
$$V_L = 440\,\text{volts}$$

Phase sequence is ABC

$$V_{AN} = \frac{440}{\sqrt{3}}\angle 0^\circ = 254.034\angle 0^\circ\,\text{volts}$$
$$V_{BN} = 254.034\angle -120^\circ\,\text{volts},\ V_{CN} = 254.034\angle 120^\circ\,\text{volts}$$

Phase Currents:

$$I_A(\text{phase}) = \frac{V_{AN}}{Z_A} = \frac{254.034\angle 0^\circ}{5\angle 30^\circ} = 50.81\angle -30^\circ \text{ A}$$

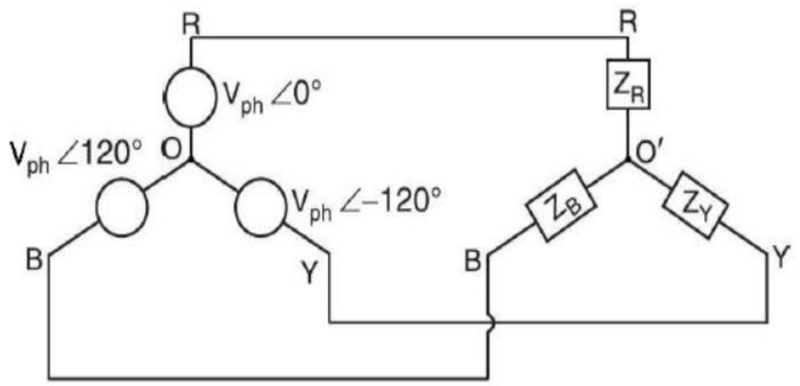
$$I_B(\text{phase}) = \frac{V_{BN}}{Z_B} = \frac{254.034\angle -120^\circ}{10\angle 45^\circ} = 25.4\angle -165^\circ \text{ A}$$

$$I_C(\text{phase}) = \frac{V_{CN}}{Z_C} = \frac{254.034\angle 120^\circ}{10\angle 60^\circ} = 25.4\angle 60^\circ \text{ A}$$

The line current = phase current.
The line currents are 50.81 A, 25.4 A, and 25.4 A in magnitude.

9. Using Millman's theorem, find the phase voltages of a 3-phase, 3-wire unbalanced star-connected load.

Consider an unbalanced Y load connected to a balanced 3-phase supply as shown



O - star point of the supply (normally at zero potential)

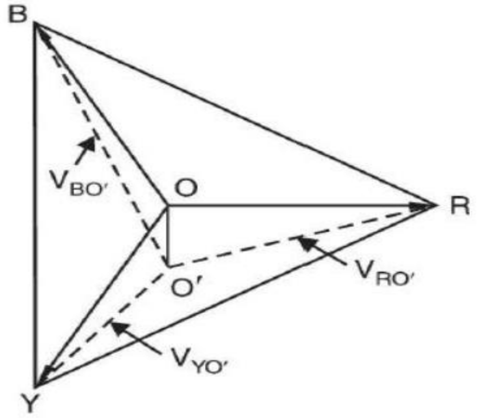
O' -load star point.

Due to load unbalance, the potential of O' is different from that of O.

According to *Millman's theorem, the voltage $V_{0'0}$ is given by

$$V_{O'O} = \frac{V_{RO}\,Y_R + V_{YO}\,Y_Y + V_{BO}Y_B}{Y_R + Y_Y + Y_B}$$

where V_{R0}, V_{Y0} and V_{B0} are the phase voltages of the supply and are equal in magnitude but 120° apart in phase. The quantities Y_R, Y_Y. and Y_B are the admittances of the branches of the unbalanced Y load.



The voltage across load phase R is $V_{RO'}$. Using double-subscript notation, this is given by ;

$$V_{RO'} = V_{RO} + V_{OO'}$$
$$= V_{RO} - V_{O'O} \quad \ldots$$

Thus load phase voltage $V_{RO'}$ is obtained by subtracting $V_{O'O}$ from the supply phase voltage V_{RO} (phasor difference).

Similarly, $$V_{YO'} = V_{YO} - V_{O'O} \quad \ldots(ii)$$

and $$V_{BO'} = V_{BO} - V_{O'O} \quad \ldots(iii)$$

The line currents in the unbalanced Y load are :

$$I_{RO'} = (V_{RO} - V_{O'O})\, Y_R \;;\; I_{YO'} = (V_{YO} - V_{O'O})\, Y_Y \;;\; I_{BO'} = (V_{BO} - V_{O'O})\, Y_B$$

The point 0' is the load star point. Due to load unbalance, 0' has some potential w.r.t. 0 and is shifted away from the centre of the triangle. Such a diagram is very useful in analysing an unbalanced 3-wire star load