ATME COLLEGE OF ENGINEERING

13th KM Stone, Bannur Road, Mysore - 570 028

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

NOTES

Course Title : Digital Logic Circuits

Course CODE: BEE306A

SEMESTER: III

Academic Year - 2024-25

INSTITUTIONAL VISION AND MISSION

VISION:

- Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

MISSION:

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torchbearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence.

Department Vision and Mission

Vision:

To create Electrical & Electronics Engineers who excel to be technically competent and fulfill the cultural and social aspirations of the society.

Mission:

- To provide knowledge to students that builds a strong foundation in the basic principles of electrical engineering, problem solving abilities, analytical skills, soft skills and communication skills for their overall development.
- To offer outcome based technical education.
- To encourage faculty in training & development and to offer consultancy through research & industry interaction.

Program Educational Objectives (PEOs)

PEO1:

To produce competent and ethical Electrical and Electronics Engineers who will exhibit the necessary technical and managerial skills to perform their duties in society

PEO2:

To make students continuously acquire and enhance their technical and socio-economic skills

PEO3:

To allow students to embark on R&D activities leading to offering solutions and excel in various career paths.

PEO4:

To produce quality engineers who have the capability to work in teams and contribute to real time projects

Program Outcomes (POs)

Engineering Graduates will be able to:

PO1: Engineering Knowledge: Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3: Design / Development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4: Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6: The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9: Individual and team work: Function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11: Project management and finance: Demonstrate knowledge and understanding of the engineering management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12: Life-long learning: Recognize the need for and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

The students will develop an ability to produce the following engineering traits:

PSO1:Apply the concepts of Electrical & Electronics Engineering to evaluate the performance of power systems and also to control industrial drives using power electronics

PSO2: Demonstrate the concepts of process control for Industrial Automation, design models for environmental and social concerns and also exhibit continuous self- learning

Digital Logic Circuit

	Academic Year: 2024-2025
Department: Electrical and Electronics Engineering	

Course Code	Course Title	Core/Elective	Prerequisite	Contact Hours			Total Hrs/ Sessions
				L	T	P	
BEE306A	**Digital Logic Circuit**	**Elective**	**Basic Electronics**	**3**	**-**	**2**	**40 theory**

Topics Covered as per Syllabus

Module-1

Definition of combinational logic, canonical forms, Generation of switching equations from truth tables, Karnaugh maps-3,4,5 variables, Incompletely specified functions (Don‘t care terms) Simplifying Max term equations, Quine-McCluskey minimization technique, Quine-McCluskey using don‘t care terms, Reduced prime implicants Tables.

Module-2

General approach to combinational logic design, Decoders, BCD decoders, Encoders, digital multiplexers, Using multiplexers as Boolean function generators, Adders and subtractors, Cascading full adders, Look ahead carry, Binary comparators

Module-3

Basic Bistable elements, Latches, Timing considerations, The master-slave flip-flops (pulsetriggered flip-flops): SR flip-flops, JK flip-flops, Edge triggered flip- flops, Characteristic equations

Module-4

Registers, binary ripple counters, synchronous binary counters, Counters based on shift registers, Design of a synchronous counter, Design of a synchronous mod-n counter using clocked T, JK, D and SR flip-flops.

Module-5

Mealy and Moore models, State machine notation, Synchronous Sequential circuit analysis, Construction of state diagrams, counter design.

Memories: Read only and Read/Write Memories, Programmable ROM, EPROM, Flash memory.

List of Text Books

“Digital Logic Applications and Design” by John M Yarbrough, 2011 edition.
“HDL Programming (VHDL and Verilog)” by Nazeih M. Botros, 1 st Edition
“Digital Principles and Design “, Donald D Givone, Tata McGraw Hill Edition,2002.

List of Reference Books

“Logic Design” by RD Sudhaker Samuel

List of URLs, Text Books, Notes, Multimedia Content, etc: https://www.youtube.com/watch?v=VnZLRrJYa2I

MODULE 1

PrinciplesofCombinationallogic

Structure

- Objective
- Introduction
- ReviewofBooleanAlgebra.

- Definitionofcombinational
- Canonicalforms
- Generationofswitchingequationsfromtruthtables
- Karnaughmaps-3,4and5variables.Incompletelyspecifiedfunctions(Don'tcareterms).
- Simplifyingmax-termequations.
- Quine-McCluskyminimizationtechnique, Quine- McCluskyusingdon'tcareterms,
- ReducedPrimeImplicanttables,
- Mapentered variables.
- Outcome
- FutureReadings

Objevtive

- **Studentattheendwillbeablerepresenta expressionfromTTorviceversa**
- **Different types of reducing a expression so that the Boolean expression obtained will have minimum variables which in turn help in reducing the component size.**
- **Advantagesand disadvantagesod reduction technique.**

Introduction

LogicCircuitsarecategorizedinto2types(basedonwhethertheycontain memoryornot):

- CombinationalLogicCircuits-Circuitswithout memory
- SequentialLogicCircuits-Circuitswith memory

ReviewofBooleanAlgebra.

Example 1:

xy+xy+xy= x+y

LHS x(y+y)+ xy usingdistributivelaw

x •1+ xy usingy+y=1theorem
x(1+ y) +xy (1+y) =1theorem x+
xy+ xyusingdistributivelawx + y(x+
x)
x+y• 1=x+yRHS

Example 2:
(x+y)(x+z) = xz+xy usingdistributivelaw
x x + x z + x y + y z using P5 postulate
x z + x y +(x+x) yz usingdistributivelaw
xz+ x y + xyz+xyz usingdistributivelaw
xz(1+y) +x y(1+z) using T2 theorem
xz•1 +xy • 1 = xz + xyRHS

PRINCIPLEOFDUALITY
One can transform the given expression by interchanging theoperation(+)and(•)as wellastheidentityelements0and1.Thentheexpressionwillbereferredasdual of each other. This is known as the principle ofduality.
Example x + x = 1thenthedualexpressionis x • x = 0

BOOLEANFORMULASAND FUNCTIONS

BooleanexpressionsorformulasareconstructedbyusingBooleanconstants and variables with the Boolean operations like (+) , (•) and‘not’
Example:(x+y) z
f(x,y.z)=(x+y)zorf=(x+y)z

xyz	xy	xz	xyz	f
000	0	0	0	0
001	0	0	0	0
010	0	0	0	0
011	0	0	0	0
100	0	1	0	1
101	0	0	0	0
110	1	1	1	1
111	1	0	0	1

Truthtableforthe aboveBooleanexpressionis

Example2: Writeatruthtableforfollowingfunction

f=xyz+xy+xz

xyz	xy	xz	xyz	f
000	0	0	0	0
001	0	0	0	0
010	0	0	0	0
011	0	0	0	0
100	0	1	0	1
101	0	0	0	0
110	1	1	1	1
111	1	0	0	1

Definitionofcombinational

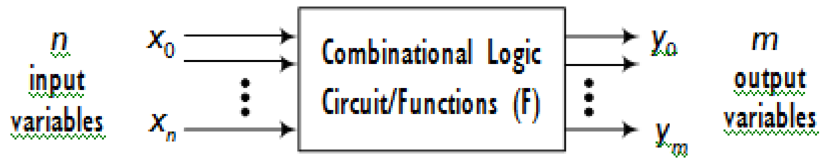
1.4.1COMBINATIONALNETWORK

Theinterconnections0fgatesresultinagatenetwork.Ifthenetworkhasthe property thatits outputs atany timearedeterminedstrictlybytheinputsatthattime then the network is said to be a combinational network. Ex. Adders. Multiplexers.etc

Letusconsideransetofnsignalsatanytimeiscalledinputstateorinputvector of the network. While a set of resulting signals appearing atthem output terminalsiscalled the output state or output vector. The network can be expressedas

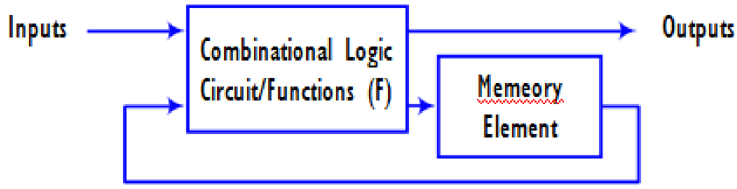
z1, z2,……zm as Boolean function thenZi

= fi(x1,x2,….xn) for i= 1,2,…m.



1.4.2.SEQUENTIALNETWORKS

A second type of logic network is the sequentialnetworks.Sequentialnetwork have memory property, so that the the outputs fromthesenetworksare dependentnotonlyupon the currentinputsbutuponpreviousinputaswell.Feedbackpathareusedin the sequential circuits. Ex. Counters. Shift registersetc.



1.4.3ANALYSISPROCEDURE

Analysisprocedureforacombinationalnetworkisasfollows

1. Eachgateoutputthatisonlyafunctionoftheinput variablesislabeled.

2. Booleanalgebraicexpressionforthe outputsofeachofthese gates arethenwritten.

3. Next these gates outputs that are a function of just inputs variablesandpreviously labeled gate outputs.

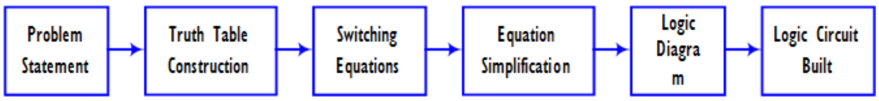
4. Theneachof thepreviouslydefinedlabelsisreplacedbythealreadywritten Boolean expression and this process is continued untiltheoutputofthenetworkislabeled and till final expression isobtained.

f(w,x,y,z) =w•(y+z)+wxy

=w•G1 + G2

f(w,x,y,z) =G2+G3

•GeneralProcedure



NORMALFORMULAS

- **Booleanexpressioncanberepresentedby followingstructures**

1. Sumofproducts(SOPordisjunctivenormalform

2. Productofsum(POSorConjunctiveform

•In **SOP** normal formisaBooleanformulathatiswrittenas a singleproductterm or as asum (alsocalleddisjunctive)ofproducttermsissaidtobeinthesumof product form or disjunctive normalform.

Example:

f(w,x,y,z)=x+w y+w yz

InthePOSnormalformisaBooleanformulawhichiswrittenasasinglesum termorasaproductofsum (alsocalledconjunctive)termsissaidtobein product of sums form or conjunctive normalform.

Example:

f(w,x,y,z)=z(x+y)(w+y+z)

Canonicalforms

Aprocedurewhichwill beusedto writeBooleanexpressions formtruthtable is known as canonical formula. The canonical formulas are of twotypes

- **Mintermcanonicalformulas**
- **Maxtermcanonicalformulas**

1.5.1MINTERMCANONICALFORMULAS

Mintermsareproduct oftermswhichrepresentsthefunctionalvaluesofthevariables appear either in complemented or un complementedform.

Ex:**f(x,y,z)=xyz+xyz+xyz**

The Boolean expressionwhichisrepresentedaboveis alsoknownasSOPor disjunctive formula

Thetruthtableis

x y z	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	0
1 1 1	0

m- NOTATION

Tosimplifythewritingofamintermincanonicalformulafora function is performedusingthesymbol**mi.**Where**i**standsfortherownumberfor which the functionevaluatesto1.

Them-notationfor3-variableanfunctionBooleanfunction

f(x,y,z)=xyz+xyz+ xyzis writtenas

f(x,y,z)=m1+ m3+m4 or

f(x,y,z)=∑m(1,3,4)

Athreevariablem- notationtruthvariable

xyz	Decimal designator of row	Minterm	m-notation
0 0 0	0	x y z	m0
0 0 1	1	x y z	m1
0 1 0	2	x y z	m2
0 1 1	3	x y z	m3
1 0 0	4	x y z	m4
1 0 1	5	x y z	m5
1 1 0	6	x y z	m6
1 1 1	7	x y z	m7

1.5.2.MAXTERMCANONICALFORM

Maxterm are sum terms where the variable appear once either in complement or un-complement
formsand thesetermscorrespondstoafunctionalvaluerepresenting0.

Ex.f(x,y,z)=(x+y+z)(x+y+z)(x+y+z)
= ∏M(0,2, 5)
=M0,M2,M5

M-NOTATION

Amaxterm in a canonicalform canberepresentedas Mi.Whereistandsforrow numberforwhichthethefunctionevaluatesto0.Aproductofmaxtermsare represented asΠM.

MINTERMCANONICALFORMULA

1. ApplyDeMorgan’s lawasufficient no.oftimesuntilalltheNOToperationsappear only with the singlevariables.

2. ApplydistributiveofANDover OR (•) over (+) i.e.x•(y+z)=xy+xzinorderto manipulate the formula into disjunctive normalformula.

3. Removeduplicateliterals andturnsbyidempotentlawwellasanytermthat are identically zero (x•x = 0)

4. Ifanyproductinthedisjunctivenormalformuladoesnotcontainallthevariables of the Boolean function thenthese missingvariablesareintroducedbyANDingthetermswithlogic1intheformofxi+xiwherexiisthemissingvariablebeing introduced. This process is repeated for eachmissingvariablesineachofthe productturns of the disjunctive normal form

5. Apply distributiveof(•)over(+)againsothateachvariableappearsexactlyonce in eachterm.

6. Remove duplicatetermifany.

Example : (x + y) + (y + x z)(x + y)

MAXTERMCANONICALFORMULA

1. ApplyDeMorgan’slawuntilalltheNOT operationsappearwithsinglevariables.

2. Applydistributivelawof(+)over(•)i.e.(x+yz)=(x+y)(x+z)andbringthe expressions into its conjugate normal form (POS).

3. The missing variables are introduced into the sumtermsby OR inglogic 0’s inthe form xi •xi = 0 where xi is missingvariable.

4. Distributivelawof(+)over(•)isagainapplied.

5. Duplicateliteralsaredeleted.

Ex: f(x,y,z)= (x+ y)+(y+ xz)(x+y)

1.5.6.COMPEMENTSOFCANONICALFORMULAS

Evenbytakingcomplementsmintermexpression mayresultdifferentexpressions.

Ex:f(x,y,z)=∑m(0,2,4,6)itscomplement expressionis f(x,y,z) =

∑m(1, 3, 5,7)

Similarly

AMaxtermcanonicalexpressionmayberepresentedincompletedfor asfollows Ex :

f(x,y,z) = Π M (1, 2, 4, 7) its complement expressionis

f(x,y,z)=ΠM(0 3,5,6)

Generationofswitchingequationsfromtruth tables,

Karnaughmaps-3,4and5variables.Incompletelyspecifiedfunctions(Don’t care terms).

A method for graphically determiningimplicants and implicates of a Booleanfunction was developed by Veitch and modified by Karnaugh . The methodinvolvesadiagrammaticrepresentationofaBooleanalgebra. This graphicrepresentation is calledmap.

It is seen that the truth table can be used to representcompletefunctionofn-variables.Sinceeach variablecanhavevalueof0or1.Thetruth tablehas2n rows.Eachrowsof the truthtableconsistoftwoparts(1)ann-tuplewhich corresponds to an assignment to the n-variables and (2) a functionalvalue.

A Karnaugh map (K-map) isa geometrical configuration of2ncellssuch that each of the n-tuplescorresponds toa row of a truth table uniquelylocatesacellonthemap. Thefunctional values assigned tothen- tuples are placedas entriesinthecells,i.e.0 or 1 are placed in the associatedcell.

2–VariableKarnaughMap

ConsidertheVenndiagramforthetwovariablesA andB.

Onevariable:Onevariableneedsamapof2^1= 2cellsmapasshownbelow xf(x)

0 f(0)

1 f(1)

TWOVARIABLE: Twovariableneedsamapof2^2=4cells

x	y	f(x,y)
0	0	f(0,0)
0	1	f(0,1)
1	0	f(1,0)
1	1	f(1,1)

THREEVARIABLE: Threevariableneedsamapof2^3= 8cells.Thearrangementofcellsareas follows

xyz	f(x,y,z)
000	f(0,0,0)
001	f(0,0,1)

010	f(0,1,0)
011	f(0,1,1)
100	f(1,0,0)
101	f(1,0,1)
110	f(1,1,0)
111	f(1,1,1)

FOURVARIABLE:Four variableneedsamapof2^4=16 cells.Thearrangementofcellsareas follows

wxyz	f(w,x,y,z)	wxyz	f(w,x,y,z)
0 0 00	f(0,0,0,0)	1010	f(1,0,1,0)
0 0 01	f(0,0,0,1)	1011	f(1,0,1,1)
0 0 10	f(0,0,1,0)	1100	f(1,1,0,0)
0 0 11	f(0,0,1,1)	1101	f(1,1,0,1)
0 1 00	f(0,1,0,0)	1110	f(1,1,10)
0 1 01	f(0,1,0,1)	1111	f(1,1,1,1)
0 1 10	f(0,1,1,0)		
0 1 11	f(0,1,1,1)		
1 0 00	f(1,0,0,0)		
1 0 01	f(1,0,0,1)		

0000	0001	0011	0010
0100	0101	0111	1010
1100	1101	1111	1110
1000	1001	1011	1010

Obtainthemintermcanonicalformulaofthethreevariableproblemgivenbelow f(x,y,z)=xy z+xyz+xyz+xyz f(x,y,z) =

$\sum$m(0,2,4,5)

00	01	11	11
1	0	0	1
1	1	0	0

PRODUCTANDSUMTERMREPRESENTATIONOF K

–MAP

1. The importance of K-map lies in the fact thatitispossibletodeterminetheimplicantsand implicates of a functionfromthepattern of0’sand1’sappearinginthemap. Thecellof a K-maphas entry of 1’s is refereed as 1-cell and that of 0,s is referred as0-cell.

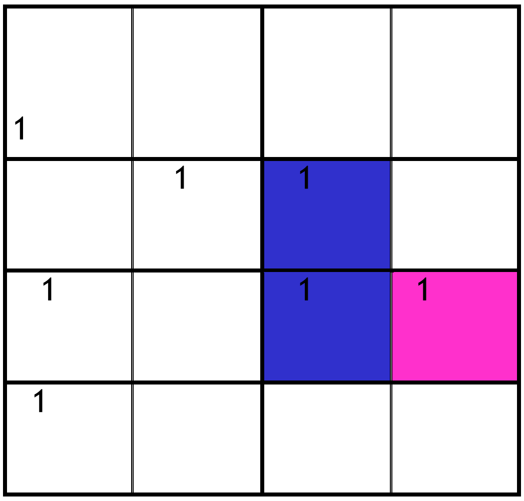
2. The construction of ann-variablemapissuchthatanysetof 1-cellsor0-cellswhichforma2^ax2^brectangulargroupingdescribingaproductorsumtermwithn-a-b variables,wherea and b are non-negative no.s

3. Therectangulargrouping ofthesedimensionsreferredasSubcubes.Thesubcubesmust be the power of 2 i.e. 2^{a+b} equals to 1,2,4,8 etc.

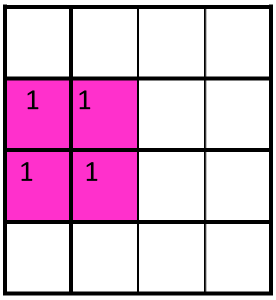
4. Forthreevariableandfourvariable K-mapitmustberememberedthattheedgesare also adjacent cells or subcubes hence they will be groupedtogether.

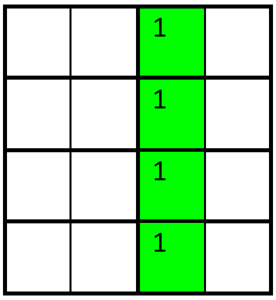
5. Givenann-variablemapwithapair ofadjacent 1-cellsor0-cellscanresultn-1variable.Where as if agroupoffour adjacentsubcubesareformedthanitcanresultn-2 variables.Finallyif we have eight adjacent cells are grouped may result n-3 variable product or sumterm.

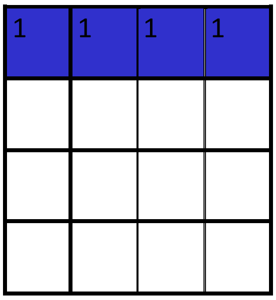
Typicalpairofsubcubes



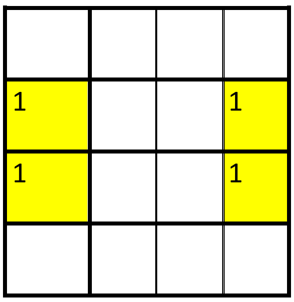
Typicalgroupoffouradjacentsubcubes



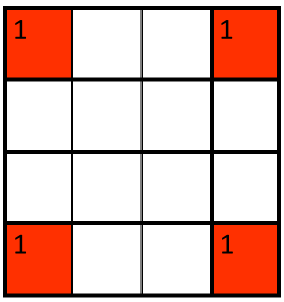




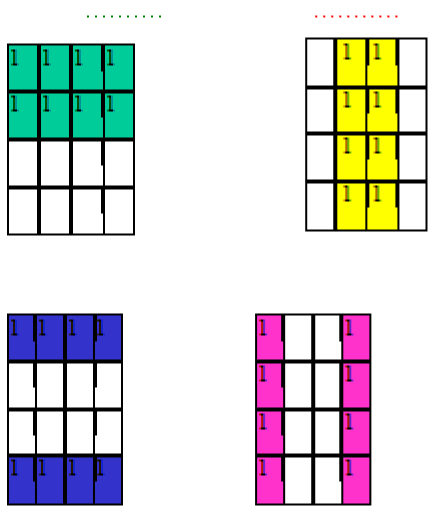
Typicalgroupoffouradjacentsubcubes.



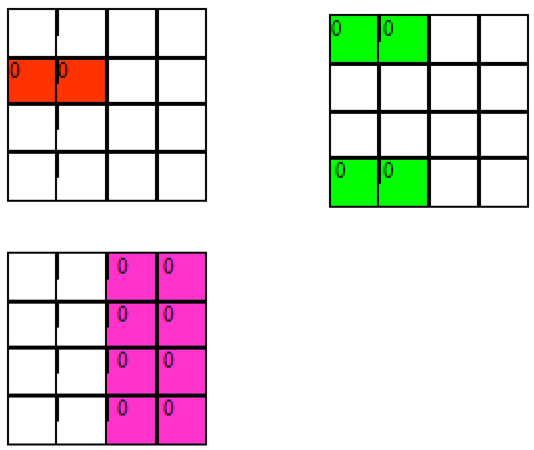
TypicalgroupofEightadjacent



subcubes.



Typical map subcubes describing sum terms



USINGK-MAPTOOBTAINMINIMALEXPRESSION FOR COMPLETEBOOLEANFUNCTIONS:

HowtoobtainaminimalexpressionofSOPorPOSofgivenfunctionisdiscussed. PRIME IMPLICANTS and K-MAPS :

CONCEPTOFESSENTIALPRIMEIMPLICANT

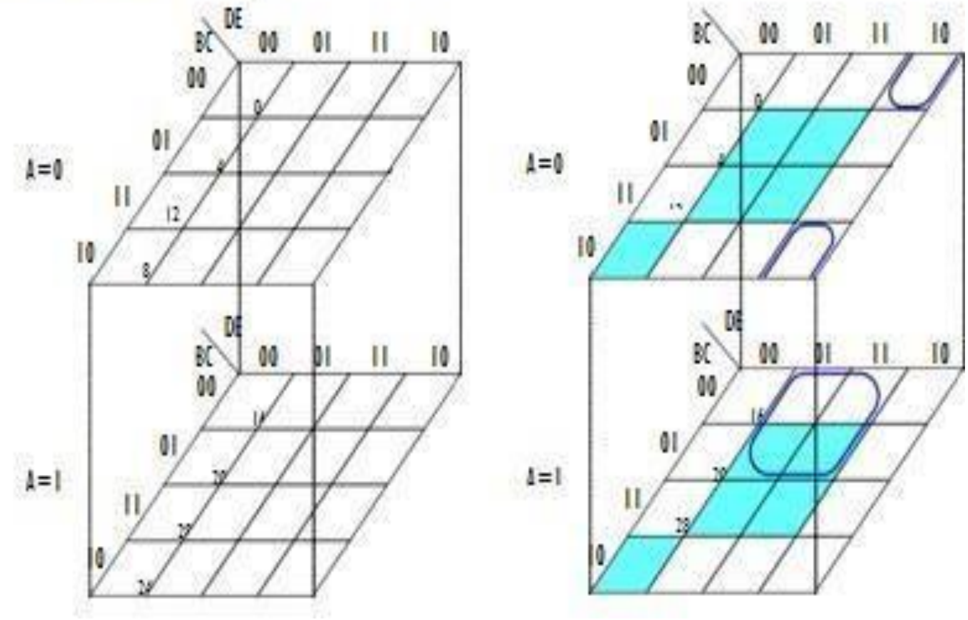
00 01 11 10

0	0	0	1
0	0	1	1

f(x,y,z)=xy+yz

(I) K-Maps Revisited: 5- and 6-Variable Functions

- Five-Variable K-Map



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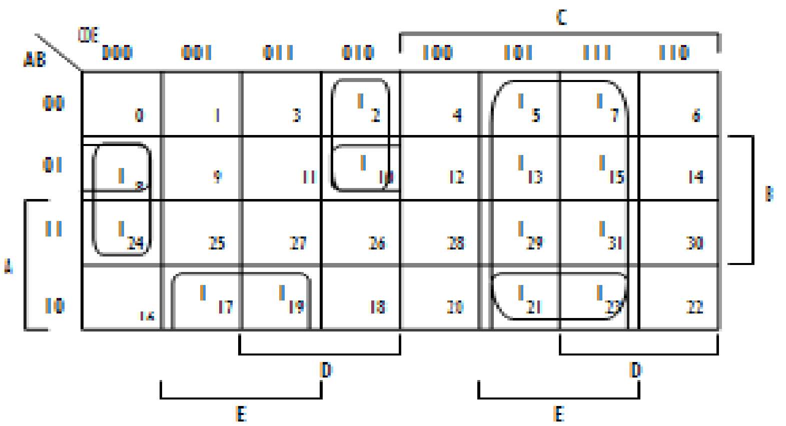
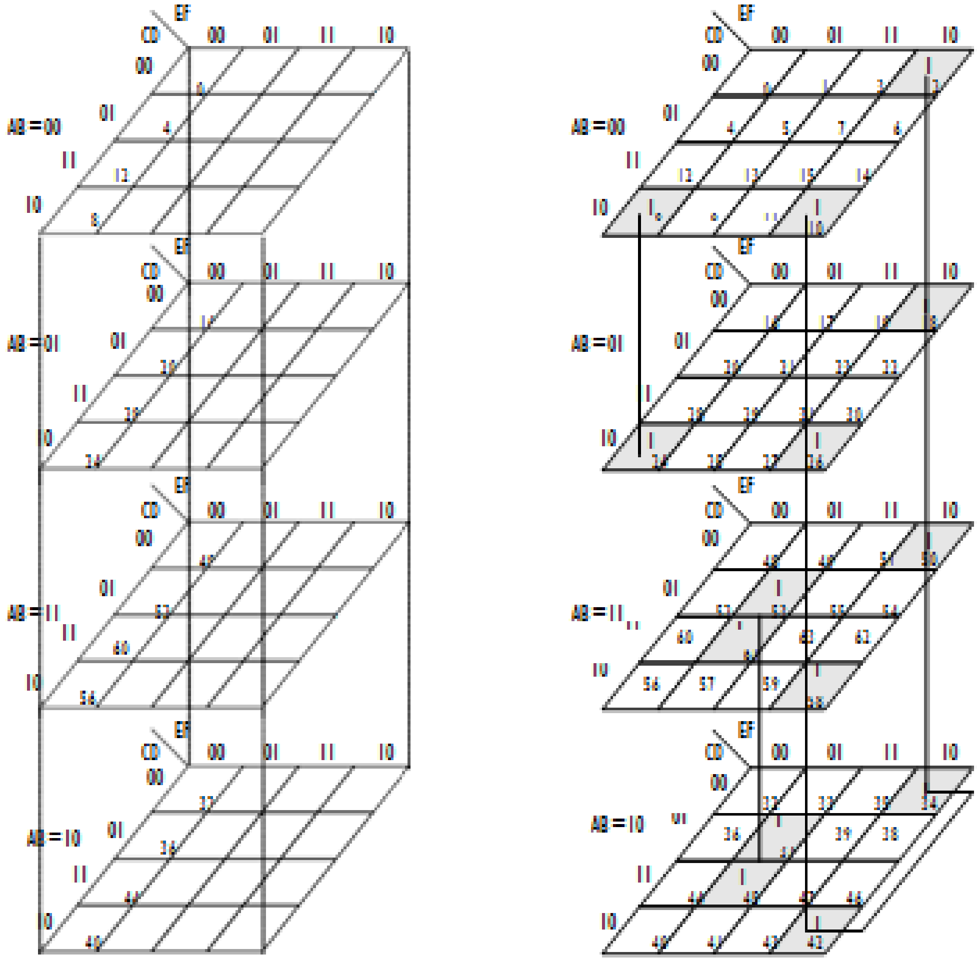
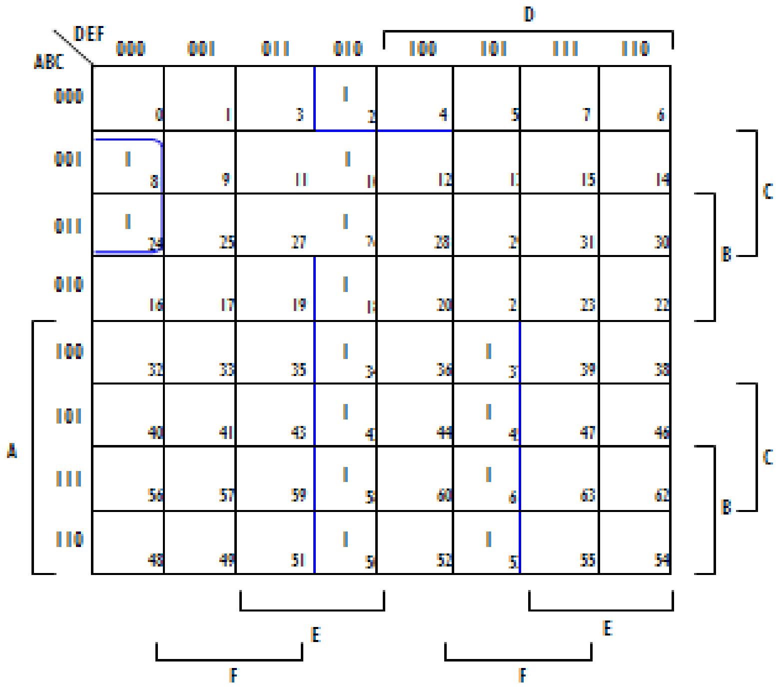


$$f(A,B,C,D,E) = \Sigma m(2,5,7,8,10,13,15,17,19,21,23,24,29,31)$$
$$= CE + BC'D'E' + AB'E + A'C'DE'$$

- Six-Variable K-Map



$$f(A,B,C,D,E,F) = \Sigma m(2,8,10,18,24,26,34,37,42,45,50,53,58,61)$$
$$= D'EF' + ADE'F + A'CD'F'$$



INCOMPLETEBOOLEANFUNCTIONWITHDONOTCARECONDITIONS

InthistypeofBooleanfunctionshavingn-variablessothatitmayhave2ncombinations of subsets and if all values are not specified such functions are called incompletely specified functions.

Ex.Letusconsiderathreevariablefunctionwithfollowingtruthtable Wecan describe the incomplete function in the SOP form as

f(x,y,z)=∑m(0,1,7)+dc(3,5)

Alsowecanrepresentthefunctionintheposformas f(x,y,z) =

ΠM(2,4,6) +dc(3,5)

Theoddparitygenerationresultoutputexpression

f(w,x,y,z) = ∑m(0,3,5,6,9) +dc(10,11,12,13,14,15)

f(w,x,y,z)=wx yz +x yz+x y z+xyz+w z

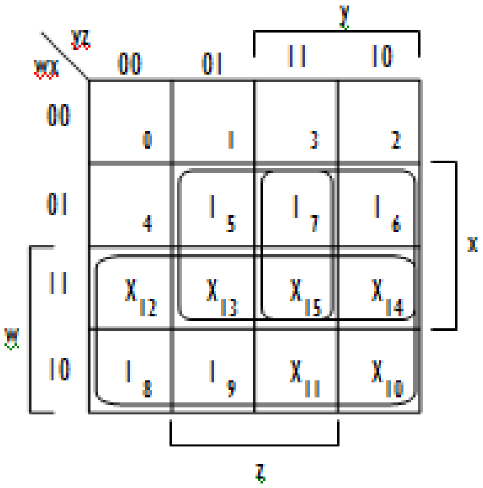
(1) IncompletelySpecifiedFunctions(Don’tCareTerms)

- Don’tcare:mintermsormaxtermsthatarenotusedaspartoftheoutput ⍰

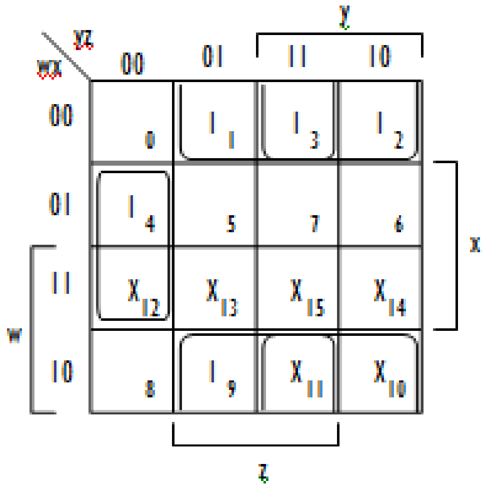
Ex: Binary to EX-3 BCD codeconversion

Binary				EX-3BCD			
W	X	Y	Z	A	B	C	D
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

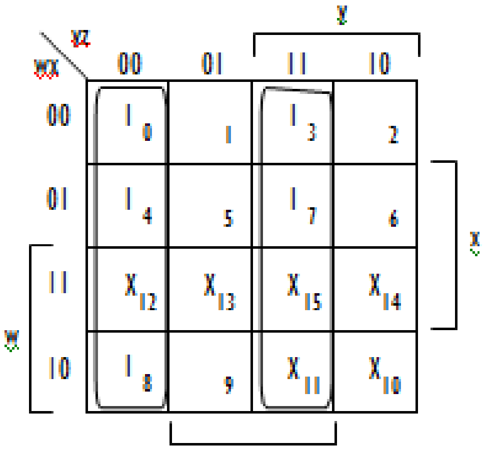
$$A = f(W,X,Y,Z) = \Sigma m(5,6,7,8,9) + \Sigma d(10,11,12,13,14,15)$$
$$B = f(W,X,Y,Z) = \Sigma m(1,2,3,4,9) + \Sigma d(10,11,12,13,14,15)$$
$$C = f(W,X,Y,Z) = \Sigma m(0,3,4,7,8) + \Sigma d(10,11,12,13,14,15)$$
$$D = f(W,X,Y,Z) = \Sigma m(0,2,4,6,8) + \Sigma d(10,11,12,13,14,15)$$



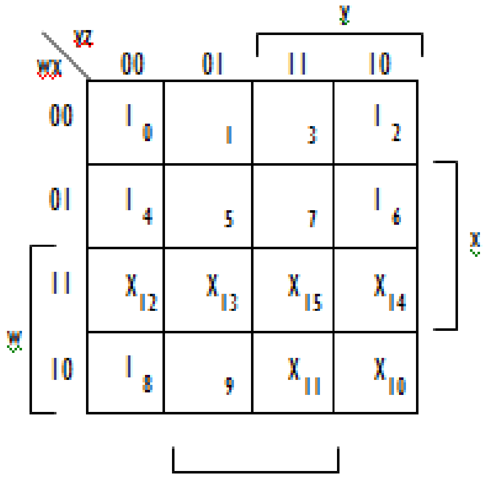
$A = W + XZ + XY$



$B = X'Y + X'Z + XY'Z'$



$C = Y'Z' + YZ'$



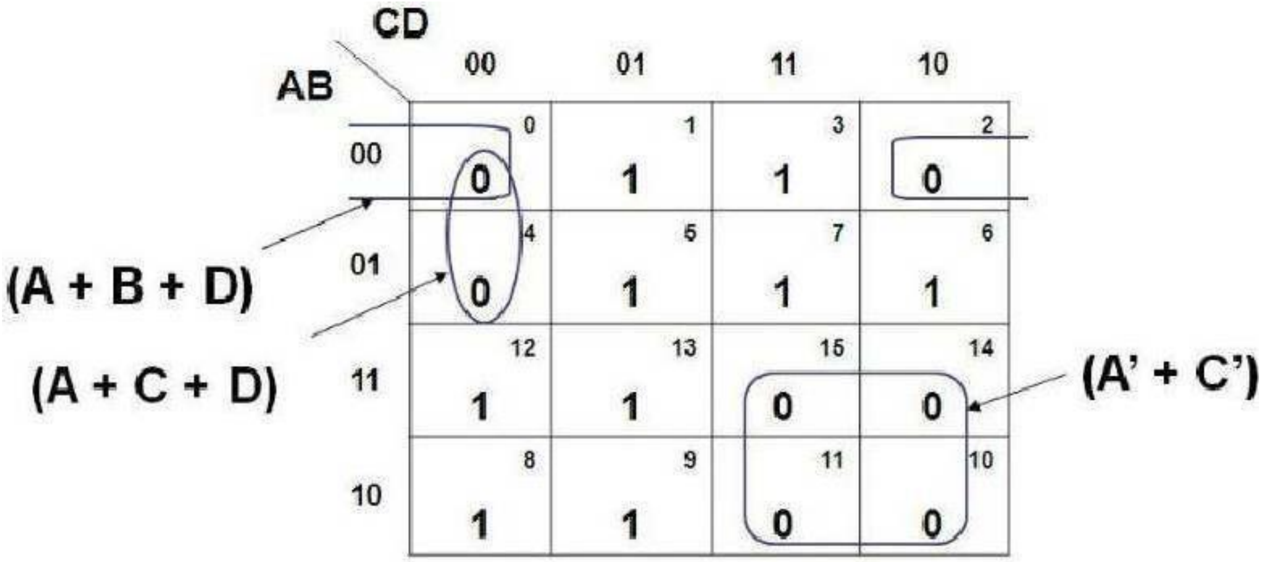
$D = Z'$

Simplifying max-termequations.

ReducethefollowingfunctionusingKarnaughmaptechnique f

(A, B, C, D) = πM(0, 2, 4, 10, 11, 14, 15)

TheK-mapforthegivenlogicfunctionisdrawnasshown below:



ThesimplifiedlogicequationinPOSformisY=(A+B+D).(A+C+D).(A'+C').

Quine-McCluskyminimizationtechnique,Quine-McCluskyusingdon't care terms,

Developedinthemid 1950s.

A systematicprocedureforgeneratingall primeimplicantsandextracting aminimumset of primes covering the on-set.

Ex. $F = f(A,B,C,D) = \Sigma(0,1,2,8,10,11,14,15)$

Step1:Groupbinaryrepresentationofthemintermsaccordingtothenumberof1's contained.

Step2:Anytwomintermswhichdifferfromeachotherbyonlyonevariablecanbe combined,andtheunmatchedvariableremoved.Themintermsinonesectionare compared with those of the next section down only, because two terms differing by more than one bit cannot match.

Step3:Repeatstep2.

Step4:Theuncheckedterms inthetableformtheprime-implicants.Step5: Prime-implicant table

- Determinationofprime-implicant

Step1		Step 2		Step2	
	ABCD		*ABCD*		*ABCD*
0	0000✓	0,1	000-	0,2,8,10	-0-0
1	0001✓	0,2	00-0✓	0,8,2,10	-0-0
2	0010✓	0,8	-000✓		
				10,11,14,15	1-1-
8	1000✓	2,10	-010✓	10,14,11,15	1-1-
10	1010✓	8,10	10-0✓		
11	1011✓	10,11	101-✓		
14	1110✓	10,14	1-10✓		
15	1111✓	11,15	1-11✓		
		14,15	111-✓		

- Prime-implicanttable

	Minterms							
	0	1	2	8	10	11	14	15
✓0,1(000-)🕓	X	X						
✓0,2,8,10(-0-0)🕓	X		X	X	X			
✓10,11,14,15(1-1-)🕓					X	X	X	X
	✓⅄	✓⅄	✓⅄	✓⅄	✓⅄	✓⅄	✓⅄	✓⅄

$$F = A'B'C' + AC + B'D'$$

Ex.

$$F(A,B,C,D) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,5)$$

- Determinationofprime-implicant

Step1		Step2	Step2	
	ABCD			*ABC D*
0	0000 ✓	0,1(1) ✓	0,1,2,3(1,2)	00- -
1	0001 ✓	0,2(2) ✓	0,1,2,3(1,2)	00- -
2	0010 ✓			
		1,3(2) ✓	1,3,5,7(2,4)	0-- 1
3	0011 ✓	1,5(4) ✓	1,3,5,7(2,4)	0-- 1
5	0101 ✓			
		2,3(1) ✓	3,7,11,15(4,8)	--1 1
7	0111 ✓	3,7(4) ✓	3,7,11,15(4,8)	--1 1
11	1011 ✓	3,11(8) ✓		
15	1111 ✓	5,7(2) ✓		
		7,15(8) ✓		
		11,15(4) ✓		

- Prime-implicanttable

	Minterms				
	1	3	7	11	15
✓ 0,1,2,3(00--)✹	X	X			
1,3,5,7(0--1)	X	X	X		
✓3,7,11,15(--11)		X	X	X	X
	✓	✓	✓	✓	✓

$F = A'B' + CD$ or $F = A'D + C$

Ex.

$F(A,B,C,D) = \sum m(1,4,6,7,8,9,10,11,15)$

- Determinationofprime-implicant

Step1		Step2	Step2	
	ABCD			ABCD
1	0 0 0 1 ✓	1,9(8)	8,9,10,11(1,2)	0 0 --
4	0 1 0 0 ✓	4,6(2)	8,9,10,11(1,2)	0 0 --
8	1 0 0 0 ✓	8,9(1) ✓		
		8,10(2) ✓		
6	0 1 1 0 ✓	6,7(1)		
9	1 0 0 1 ✓	9,11(2)		
10	10 1 0 ✓	10,11(1)		
7	0 1 1 1	7,15(8)		
11	10 1 1	11,15(4)		
15	1 1 1 1✓			

- Prime-implicanttable

	Minterms								
	1	4	6	7	8	9	10	11	15
1,9(-001)	X					X			
4,6(01-0)		X	X						
6,7(011-)			X	X					
7,15(-111)				X					X

11,15(1-11)								X	X
□8,9,10,11(10--)					X	X	X	X	
	□□	□□	□□	□□	□□	□□	□□	□□	□□

F□B□CD□ □A□BD□ □BCD□AB□

ReducedPrimeImplicanttables,

PRIMEIMPLICATE:Iftheimplicatedoesnotsubsumesanyother implicatewithfewer literals of the same function. In other words if we remove prime implicate term from the expression the remaining sum terms no longer implies thefunction

Ex.xand(y+z)areprimeimplicates

x y z	f
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	1

Ex1:Minimizef(A, B, C,D) =Σ (0, 1,2, 8,9, 15, 17,21, 24,25,27, 31).

Step 1: Say we have obtained the following prime implicants:

P1: BC'D'	P4: ABDE	P7: ABC'D'	P10: A'B'C'E'
P2: C'D'E	P5: BCDE	P8: A'B'D'E	P11: A'B'C'D'
P3: A'C'D'	P6: ABC'E	P9: B'C'D'E	

Step 2: Prime Implicant Chart

	0	1	2	8	9	15	17	21	24	25	27	31
P1				X	X				X	X		
P2		X			X		X			X		
P3	X	X		X	X							
P4											X	X
* P5						X						X
P6										X	X	
P7									X	X		
* P8							X	X				
P9		X					X					
* P10	X		X									
P11	X	X										

The prime implicants P5, P7, and P10 are essential. They are included in the solution. Theydo not cover all the minterms. So secondary essential prime implicants have to be found by using the reduced prime implicant chart.

ReducedPrimeImplicantChart(Essentialprimeimplicantsremoved)

Wearenotabletofindcolumnswithsingle'X'.Nowwefindthedominancerelations. Column Dominance:

9 >8

25 >24

RowDominance:

P1 > P7

P6 >P4

P2 > P9

P2 >P11

	1	8	9	24	25	27
P1		X	X	X	X	
P2	X		X		X	
P3	X	X	X			
P4						X
P6					X	X
P7				X	X	
P9	X					
P11	X					

PrimeImplicantChartReductionSteps:

- Allthedominatingcolumnsanddominatedrowsofaprime-implicantchartcanbe removed without affecting the table for obtaining a minimal solution.
- Dominatingcolumnisguaranteedtobecoveredbytherowthatcoversitsdominated column.
- The columns ofthedominated row areguaranteed tobecovered by its dominatingrow.

FindingSecondaryEssentialPIs:

PIChartafterthedominatingcolumnsandthedominatedrowsaredeleted:

PIs \ MTs	1	√ 8	√ 24	√ 27
** P1		X	X	
P2	X			
P3	X	X		
** P6				X

FinalSolution

PIs \ Min term	√ 1
P2	X
P3	X

Minterm1canbecoveredbyP2orP3. Ifwe selectP2,wehavethesolution: Y = P1 + P2 + P5 + P6 + P8 + P10

Mapenteredvariables.

In entered variable map one ofthe input variablesis placed inside Karnaugh map. Thisis done separatelynotinghowtheinputvariableisrelatedwithoutput.ThisreducestheKarnaughmapsize by one degree. Thistechnique is particularly useful formapping problems with more than four input variables.

Example:

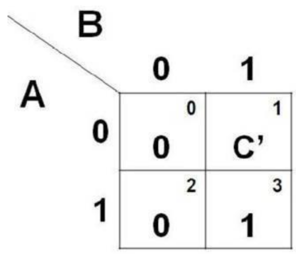
Considerthe3-variabletruthtableasshownbelow.TheoutputYisrewrittenintermsof variable C.

A	B	C	Y	Y
0	0	0	0	0
0	0	1	0	
0	1	0	1	C'
0	1	1	0	
1	0	0	0	0
1	0	1	0	
1	1	0	1	1
1	1	1	1	

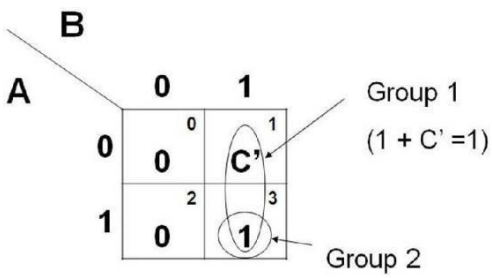
The3–variabletruthtablereducesto2–variabletruthtableasshownbelow:

A	B	Y
0	0	0
0	1	C'
1	0	0
1	1	1

The2– variableKarnaughmapisdrawnasshownbelow:



TheKarnaughmapisnowcalledanentered variablemap.Thesimplificationofentered variable map is as illustrated next:



The product term representing each group is obtained by including map entered variable in the group as an additional ANDed term. Group 1 gives B.(C') and group 2 gives AB.1. Therefore, the simplified expression is obtained as Y = BC' + AB.

Outcome

- **RepresentationofBooleanexpressionincanonicalforms.**
- **Reducegateswithminimumnumberofvariableusingdifferenttechniques.**

Future Readings

http://nptel.ac.in/courses/117105080/

https://www.youtube.com/watch?v=VnZLRrJYa2I
"LogicDesign"byRDSudhaker Samuel

"DigitalLogicApplicationsandDesign"byJohnMYarbrough,2011edition.