Mechanics of Materials
BME301
Module-2
Bi-axial Stress system, thick and thin cylinders

1. Objectives.

Derive the equations for principal stress and maximum in-plane shear stress and calculate their magnitude and direction. Draw Mohr circle for plane stress system and interpret this circle.

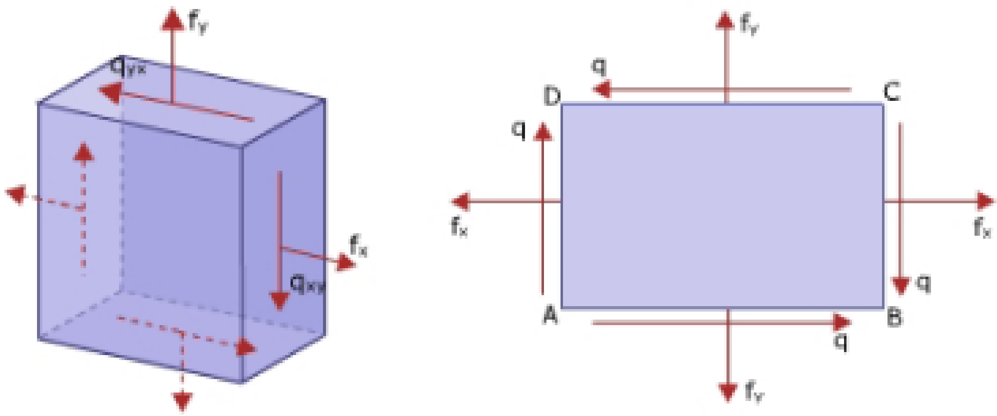
Learning Structure.

2.1 Introduction

Structural members are subjected to various kinds of loads. This results in a combination of different stresses which changes from point to point. When an element (considered at any point) in a body is subjected to a combination of normal stresses (tensile and/or compressive) and shear stresses over its various planes, the stress system is known as compound stress system. In a compound stress system, the magnitude of normal stress may be maximum o n some plane and minimum on some plane, when compared with those acting on the element. Similarly, the magnitude of shear stresses may also be maximum on two planes when compared with those acting on the element. Hence, for the considered compound stress system it is important to find the magnitudes of maximum and minimum normal stresses, maximum shear stresses and the inclination of planes on which they act.

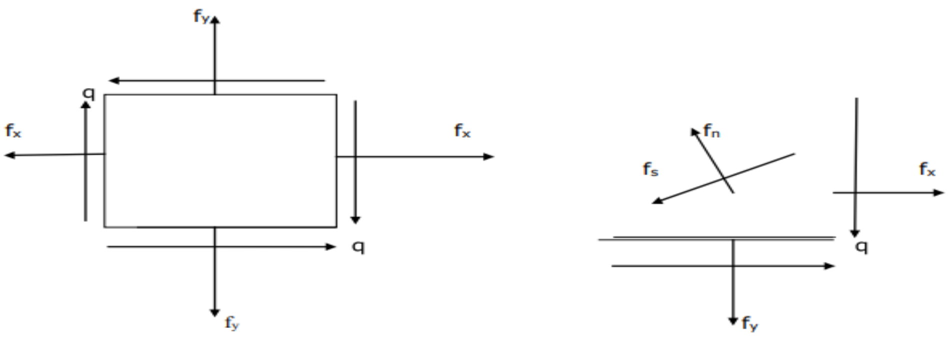
2.2 PLANE STRESS OR 2–D STRESS SYSTEM OR BIAXIAL STRESS SYSTEM.

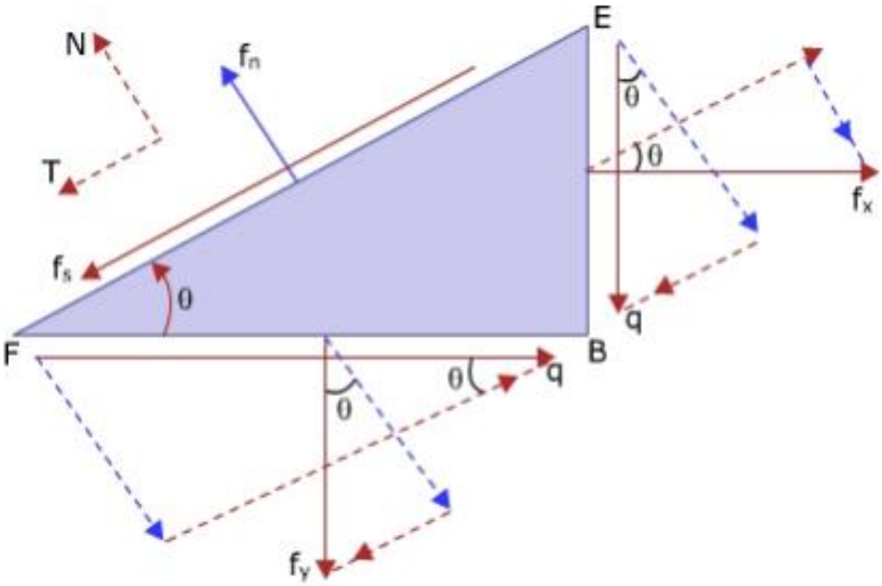
Generally, a body is subjected to a 3–D state of stress system with both normal and shear stresses acting in all the three directions. However, for convenience, in most problems, variation of stresses along a particular direction can be neglected and the remaining stresses are assumed to act in a plane. Such a system is called 2–D stress system and the body is called plane stress body.



In a general two-dimensional stress system, a body consists of two normal stresses (fx and fy), which are mutually perpendicular to each other, with a state of shear (q) as shown in figure. Further, since planes AD and BC carry normal stress fx they are called planes of fx these planes are parallel to Y–axis. Similarly, planes AB and CD represent planes of fy, which are parallel to X–axis.

2.3 EXPRESSIONS FOR NORMAL AND TANGENTIAL COMPONENTS OF STRESS ON A GIVEN PLANE





Applying equilibrium along N-direction, we have

$$\Sigma F_N = 0 \quad [\ \nwarrow \ +ve]$$

$$f_n\,(EF.1) - f_x\,(BE.1)\sin\theta - q\,(BE.1)\cos\theta - f_y\,(BF.1)\cos\theta - q\,(BF.1)\sin\theta = 0$$

$$f_n = f_x\,\frac{BE}{EF}\sin\theta + q\,\frac{BE}{EF}\cos\theta + f_y\,\frac{BF}{EF}\cos\theta + q\,\frac{BF}{EF}\sin\theta$$

Since $\frac{BE}{EF} = \sin\theta$ and $\frac{BF}{EF} = \cos\theta$

$$\therefore\ f_n = f_x \sin^2\theta + 2q\sin\theta\cos\theta + f_y\cos^2\theta$$

But $\cos 2\theta = 2\cos^2\theta - 1$ Hence $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

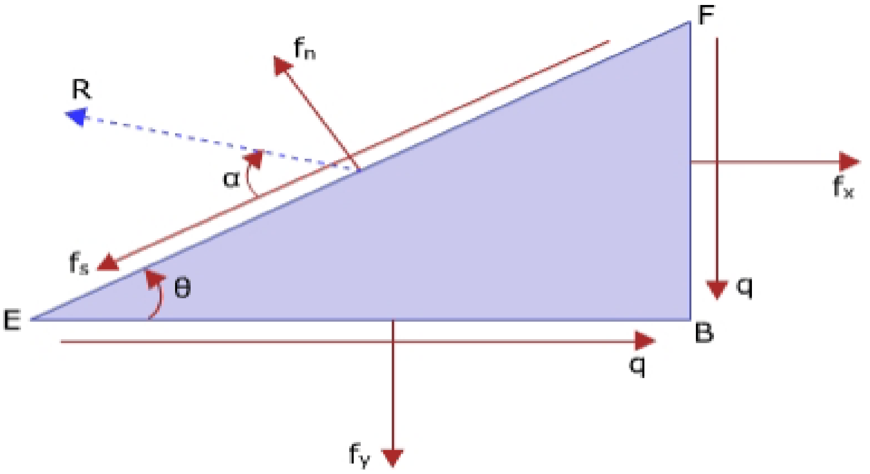
Also $\cos 2\theta = 1 - 2\sin^2\theta$ Hence $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$f_x = f_x\,\frac{1}{2}(1-\cos 2\theta) + f_y\,\frac{1}{2}(1+\cos 2\theta) + q\sin 2\theta$$

$$f_n = \left(\frac{f_x + f_y}{2}\right) - \left(\frac{f_x - f_y}{2}\right)\cos 2\theta + q\sin 2\theta \qquad \text{-------- (1)}$$

Equation (1) is the desired expression for normal component of stress on a given plane, inclined at an angle ‘ ' measured counter clockwise with respect to the plane of fY or X– axis.

To derive expression for fs Consider the Free Body Diagram of portion FBE shown in figure above. For equilibrium along T direction, we have.



$$\Sigma F_T = 0 \; [\swarrow +ve]$$

$$f_s(EF.1) - f_x(BE.1)\cos\theta + q\,(BE.1)\sin\theta + f_y\,(BF.1)\sin\theta - q(BF.1)\cos\theta = 0$$

$$f_s = f_x\frac{BE}{EF}\cos\theta - q\frac{BE}{EF}\sin\theta - f_y\frac{BF}{EF}\sin\theta + q\frac{BF}{EF}\cos\theta$$

Since $\frac{BE}{EF} = \sin\theta \qquad \frac{BF}{EF} = \cos\theta$

$$\therefore f_s = f_x\sin\theta\cos\theta - q\sin^2\theta - f_y\cos\theta\sin\theta + q\cos^2\theta$$

$$\therefore f_s = (f_x - f_y)\sin\theta\cos\theta + q\,(\cos^2\theta - \sin^2\theta)$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$f_s = \left(\frac{fx - fy}{2}\right)\sin 2\theta + q\cos 2\theta \qquad \text{-------- (2)}$$

Equation (2) is the desired expression for tangential component of stress on a given plane, inclined at an angle ‘ ' measured counterclockwise with respect to the plane of fy or X– axis.
Note: The resultant stress ‘R’, and its inclination ‘? ’ on the given plane EF which is inclined at an angle ‘ ’ measured counterclockwise with respect to the plane of fy or X–axis, can be determined from the normal (fn) and tangential (fs) components obtained from eqns. (1) and

$$R = \sqrt{f_n^2 + f_s^2}$$

$$\alpha = \tan^{-1}\left(\frac{f_n}{f_s}\right)$$

(2).

2.4 Mohr's Circle

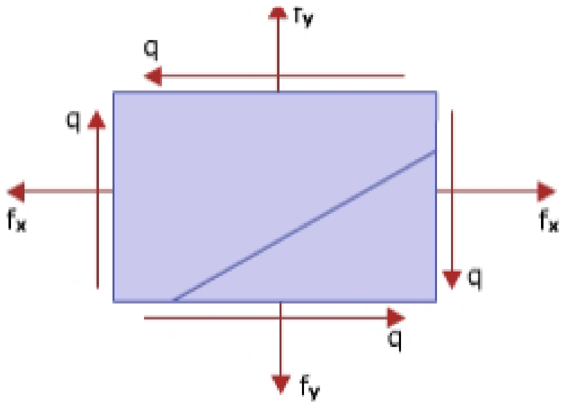
The formulae developed so far (to find fn, fs, fn-max , fn-min , θp1 , θp2 , fs max, θs1 , θs2) may be used for any case of plane stress. A visual interpretation of these relations, devised by the German Engineer Christian Otto Mohr in 1882, eliminates the necessity of remembering them. In this interpretation a circle is used; accordingly, the construction is called Mohr's Circle. If this construction is plotted to scale the results can be obtained graphically; usually, however, only a rough sketch is drawn, and results are obtained from it analytically.

Rules for applying Mohr's Circle to compound stresses

1. The normal stresses fx and fy are plotted along X-axis. Tensile stresses are treated as positive and compressive stresses are treated as negative.
2. The shear stress q is plotted along Y-axis. It is considered positive when its moment of the center of the element is clockwise and negative when its moment about the center of the element is anti-clockwise.
3. Positive angles in the circle are obtained when measured in counterclockwise sense. Further, an angle of '2θ' in the circle corresponds to an angle θ in the element.
4. A plane in the given element corresponds to a point on the Mohr's circle. Further, the coordinates of the point on the Mohr's circle represent the stresses acting on the plane

Procedure to construct Mohr's circle

Consider an element subjected to normal stresses fx and fy accompanied by shear stress q as shown in figure. Let fx be greater than fy.



1. In the rectangular coordinate system, locate point A which will be should be a point on the circle representing the stress condition on the plane fx of the element. The coordinates of point A are (fx , q).

2. Similarly locate point B, representing stress conditions on plane fy of the element. The coordinates of point B are (f y – q).

3. Join AB to cut X-axis at point C. Point C corresponds to the center of Mohr's circle.

4. With C as center and CA as radius, draw a circle.

2.5 Thick Cylinders

1) 2.5.1 Difference in treatment between thin and thick cylinders - basic assumptions: The theoretical treatment of thin cylinders assumes that the hoop stress is constant across the thickness of the cylinder wall (Fig. 6.1), and also that there is no pressure gradient across the wall. Neither of these assumptions can be used for thick cylinders for which the variation of hoop and radial stresses Shown in (Fig. 6.2), their values being given by the Lame equations

$$\sigma_H = A + \frac{B}{r^2}$$

$$\sigma_r = A - \frac{B}{r^2}$$

Where: -

σ_H= Hoop stress ($\frac{N}{m^2} = Pa$).

σ_r= Radial stress ($\frac{N}{m^2} = Pa$).

r= Radius (m). A and B are Constants.

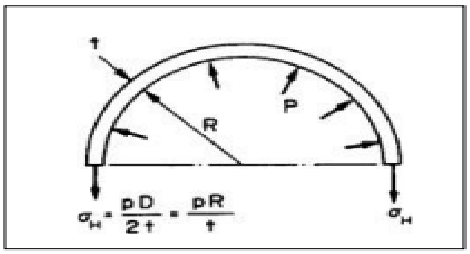
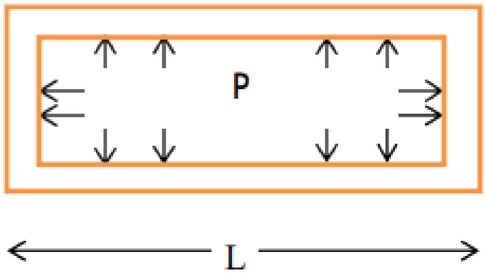
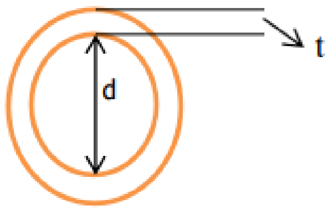


Figure 6.1: - Thin cylinder subjected to internal pressure.

2.6 Thin Cylinders

- 2.6.1 Introduction When the thickness of the wall of the cylinder is less than 1/1 of the diameter of cylinder then the cylinder is considered as thin cylinder. Otherwise it is termed as thick cylinder.





L=Length of the cylinder, d= Diameter of cylinder, t = thickness of cylinder P= Internal Pressure due to fluid.