ATME College of Engineering

Department of Physics

ATME College of Engineering, Mysuru

Course: Applied Physics for EE Stream

Course Code: BPHYE102/202

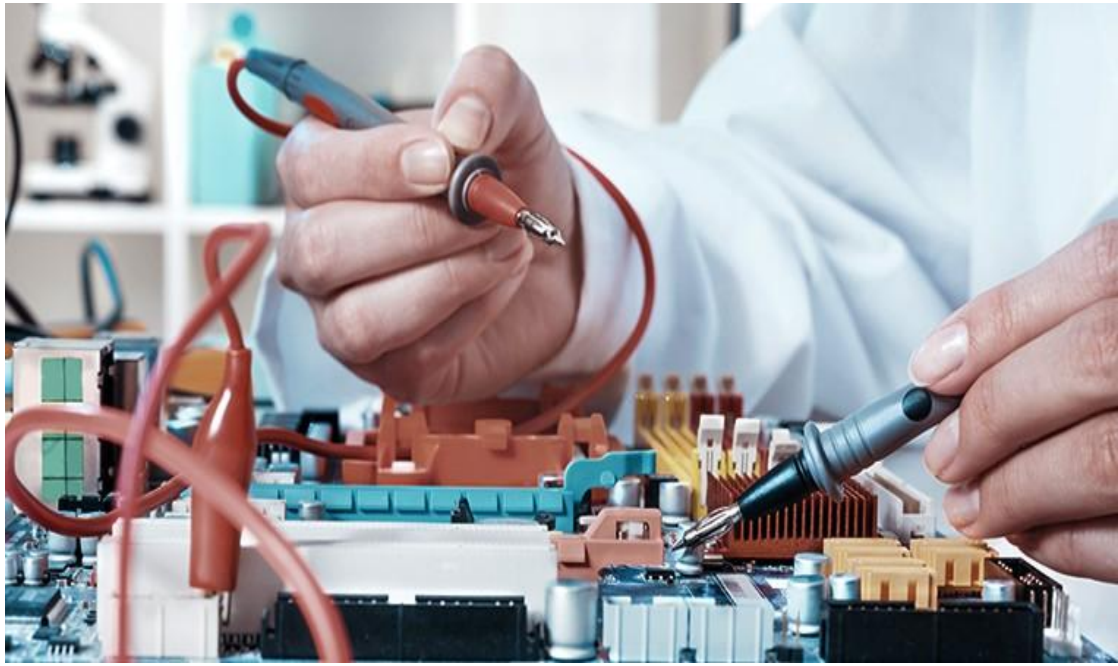
Engineers are involved in the design, evaluation, development, testing, modification, inspection and maintaining of a wide range of products, structures and systems. This involves everything from the recommending of materials and processes, overseeing manufacturing and construction processes, and conducting failure analysis and investigation, to providing consultancy services and teaching engineering to students and trainees.

The word 'engine' itself comes from the Latin word 'ingenium, which means '<u>innate quality</u>, especially mental power, hence a clever invention.'


Scientists dream about doing great things. Engineers do them.
James A. Michener

Importance of Physics in Electrical Engineering

Electrical engineering involves designing electrical circuits including motors, electronic appliances, optical fiber networks, computers, and communication links. Electrical engineers often need to convert electrical energy to other forms of energy, with the understanding of mechanics and thermodynamics. Knowing the fundamentals of Electrical Engineering, in addition to, how small-scale components like integrated circuits and various types of transistor logic, all functions require at least an intermediate understanding of Electromagnetism, which you learn from Physics.



Importance of Physics in Electronics Engineering

Electronics include the workings of transistors, diodes and semiconductors. Integrated circuit uses physics to study how various tiny transistors are connected in circuits. Electromagnetism is used for antennae design, RF signals, wireless communications, etc. The field of robotics relies on a lot of things physics such as dynamics, chaos, mechanics, motors, etc. as well as optics (for cameras for computer vision).

Course Title:	**Physics for Electrical & Electronics Engineering Stream**		
Course Code:	**22PHYE12/22**	CIE Marks	50
Course Type (Theory/Practical/Integrated)	Integrated	SEE Marks	50
		Total Marks	100
Teaching Hours/Week (L:T:P: S)	2:2:2:0	Exam Hours	03+02
Total Hours of Pedagogy	40 hours+10-12 Lab Slots	Credits	04

Course objectives

- To study the principles of quantum mechanics
- To understand the properties of dielectrics and superconductors
- To study the essentials of photonics for engineering applications.
- To understand the fundamentals of vector calculus and EM waves.
- To study the knowledge about semiconductors and devices.

Teaching-Learning Process

These are sample Strategies, which teachers can use to accelerate the attainment of the various course outcomes and make Teaching –Learning more effective

1. Flipped Class
2. Chalk and Talk
3. Blended Mode of Learning
4. Simulations, Interactive Simulations and Animations
5. NPTEL and Other Videos for theory topics
6. Smart Class Room
7. Lab Experiment Videos

Course outcome:

CO1: Describe the fundamental principles of Quantum Mechanics and the essentials of Photonics

CO2: Elucidate the concepts of dielectrics and superconductivity

CO3: Discuss the fundamentals of vector calculus and their applications in Maxwell's Equations and EM Waves

CO4: Summarize the properties of semiconductors and the working principles of semiconductor devices

CO5: Practice working in groups to conduct experiments in physics and perform precise and honest measurements

Suggested Learning Resources:

Books (Title of the Book/Name of the author/Name of the publisher/Edition and Year)

1. A Textbook of Engineering Physics- M.N. Avadhanulu and P.G. Kshirsagar, 10th revised Ed, S. Chand. & Company Ltd, New Delhi.
2. An Introduction to Lasers theory and applications by M.N.Avadhanulu and P.S.Hemne revised Edition 2012. S. Chand and Company Ltd -New Delhi.
3. Engineering Physics-Gaur and Gupta-Dhanpat Rai Publications-2017.
4. Concepts of Modern Physics-Arthur Beiser: 6th Ed;Tata McGraw Hill Edu Pvt Ltd- New Delhi 2006.
5. Fundamentals of Fibre Optics in Telecommunication & Sensor Systems, B.P. Pal, New Age International Publishers.
6. Introduction to Electrodynamics, David Griffith, 4th Edition, Cambridge University Press 2017.
7. Lasers and Non-Linear Optics – B.B. Laud, 3rd Ed, New Age International Publishers 2011.
8. LASERS Principles, Types and Applications by K.R. Nambiar-New Age International Publishers.
9. Solid State Physics-S O Pillai, 8th Ed- New Age International Publishers-2018.

List of Experiments:

1. Wavelength of LASER using Grating
2. Numerical Aperture using optical fiber
3. Four Probe Method
4. Charging and Discharging of a Capacitor
5. Transistor Characteristics
6. Photo-Diode Characteristics
7. Series and Parallel LCR Circuits
8. Magnetic Field at any point along the axis of a circular coil
9. Plank's Constant using LEDs.
10. Fermi Energy
11. Black Box
12. Energy Gap of the given Semiconductor
13. Dielectric Constant
14. PHET Interactive Simulations (https://phet.colorado.edu/en/simulations/filter?subjects=physics&type=html,prototype)
15. Online Circuit Simulator (https://www.partsim.com/simulator)
16. Study of Electrical quantities using spreadsheet

BPHYE202 - Electrical & Electronics Engineering

Module 1	Quantum Mechanics
Module 2	Electrical Properties of Solids
Module 3	Lasers and Optical Fibers
Module 4	Maxwell's Equations and EM waves
Module 5	Semiconductor and Devices

Module – 1: Quantum Mechanics:

de Broglie Hypothesis and Matter Waves, de Broglie wavelength and derivation of expression by analogy, Phase Velocity and Group Velocity, Heisenberg's Uncertainty Principle and its application (Non-existence of electron inside the nucleus-Non Relativistic), Principle of Complementarity, Wave Function, Time independent Schrodinger wave equation, Physical Significance of a wave function and Born Interpretation, Expectation value, Eigen functions and Eigen Values, Particle inside one-dimensional infinite potential well, Waveforms and Probabilities. Numerical Problems.

Pre-requisite: Wave-Particle dualism

Self-learning: de Broglie Hypothesis

Module – 2: Electrical Properties of Solids:

Dielectric Properties: Polar and non-polar dielectrics, Types of Polarization, internal fields in solid, Clausius-Mossottiequation (Derivation), solid, liquid and gaseous dielectrics. Application of dielectrics in transformers, Capacitors, and Electrical Insulation. Numerical problems.

Superconductivity: Introduction to Superconductors, Temperature dependence of resistivity, Meissner's Effect, Silsbee Effect, Types of Super Conductors, Temperature dependence of Critical field, BCS theory (Qualitative), HighTemperature superconductivity, SQUID, MAGLEV, Numerical problems.

Pre-requisites: Difference between Insulators & Dielectrics.

Self-learning: Dielectrics Basics

Module – 3: Lasers and Optical Fibers:

Lasers: Characteristics of LASER, Interaction of radiation with matter, Expression for energy density equation and its significance. Requisites of a Laser system. Conditions for Laser action. Principle, Construction and working of carbon dioxide laser. Application of Lasers in Defence (Laser range finder) and Laser Printing. Numerical problems.

Optical Fibers: Propagation mechanism, TIR, angle of acceptance, Numerical aperture, fractional index change, Modes of propagation, Number of modes and V parameter, Types of optical fibers. Attenuation and Mention of expression for attenuation coefficient, Attenuation spectrum of an optical fiber with optical windows. Discussion of the block diagram of point-to-point communication, Intensity-based fiber optic displacement sensor, Merits and demerits. Numerical problems.

Pre-requisite: Properties of light Self-learning: Propagation Mechanism &TIR in optical fiber

Module – 4: Maxwell's Equations and EM waves:

Maxwell's Equations: Fundamentals of vector calculus. Divergence and curl of electric field and magnetic field (static), Gauss' divergence theorem and Stokes' theorem. Description of laws of electrostatics, magnetism and Faraday's laws of EMI. Current density & equation of Continuity; displacement current (with derivation) Maxwell's equations in vacuum.

EM Waves: The wave equation in differential form in free space (Derivation of the equation using Maxwell's equations), Plane electromagnetic waves in vacuum, and their transverse nature. Numerical problems.

Pre-requisite:Electricity & Magnetism Self-learning: Fundamentals of vector calculus

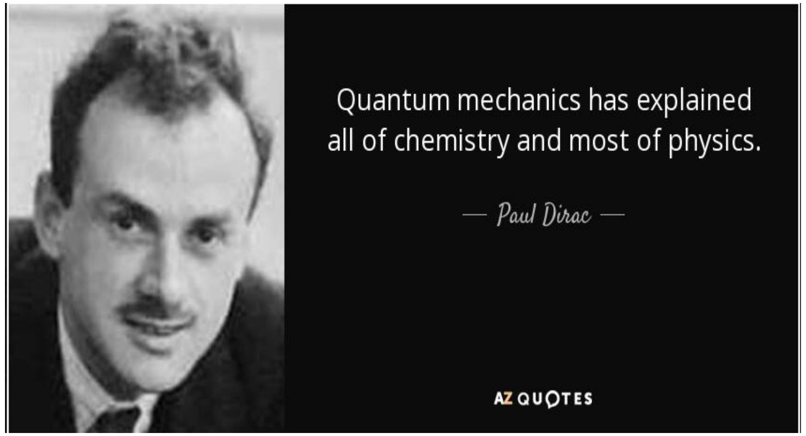
Module – 5: Semiconductor and Devices:

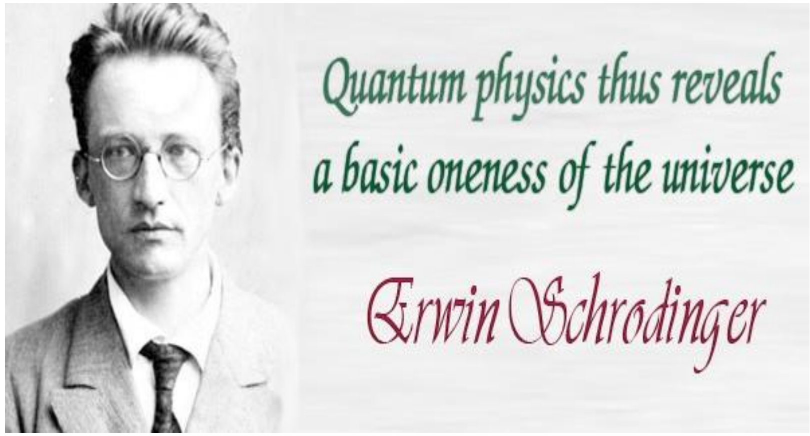
Fermi energy and Fermi level, Fermi level in intrinsic semiconductors, Expression for concentration of electrons in conduction band & holes concentration in valance band (only mention the expression), Law of mass action, Electrical conductivity of a semiconductor (derivation), Hall effect, Expression for Hall coefficient (derivation) and its application. Photodiode and Power responsivity, Construction and working of Semiconducting Laser,Four probe method to determine resistivity, Phototransistor. Numerical problems.

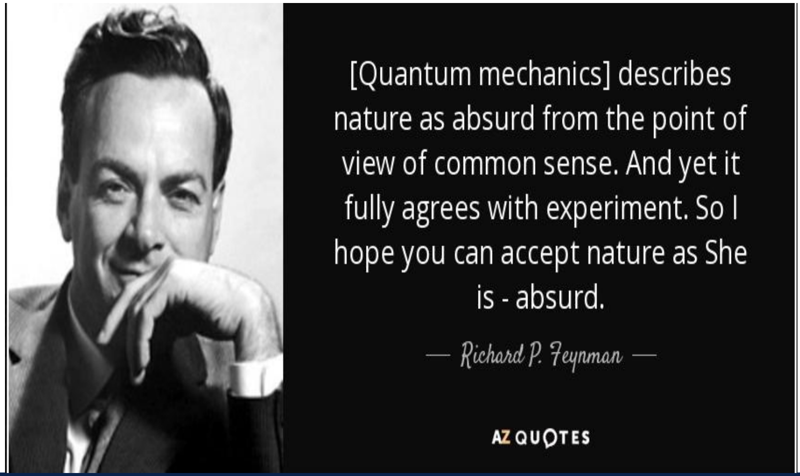
Pre-requisite: Basics of Semiconductors Self-learning: Photodiode

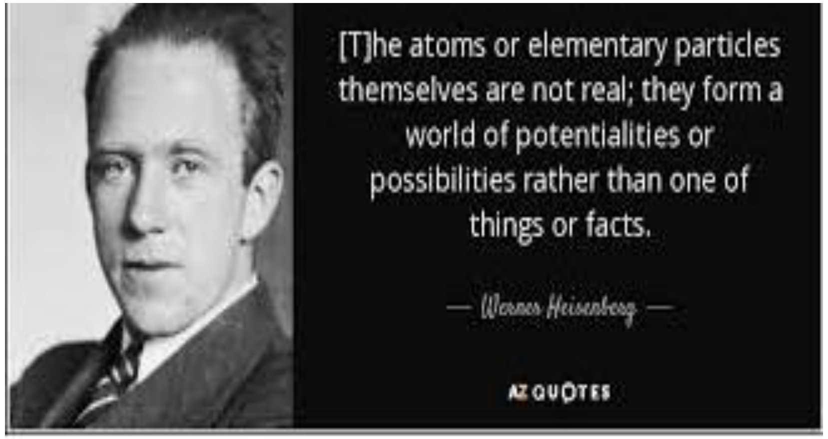
Module 1: Quantum Mechanics

Famous Quotes..







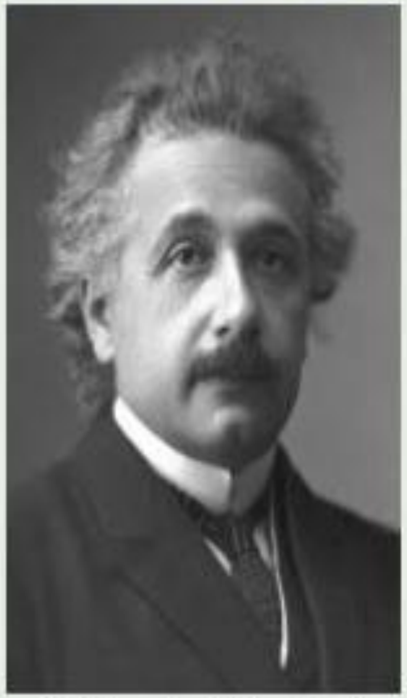


Contributors..

Black-body radiation

Max Planck

Nature of Light



Albert Einstein

Structure of the atom

Neils Bohr

Birth of Quantum Mechanics..

At the end of 19th century Physics was essentially consist of

1. Classical Mechanics
2. Electrodynamics
3. Thermodynamics

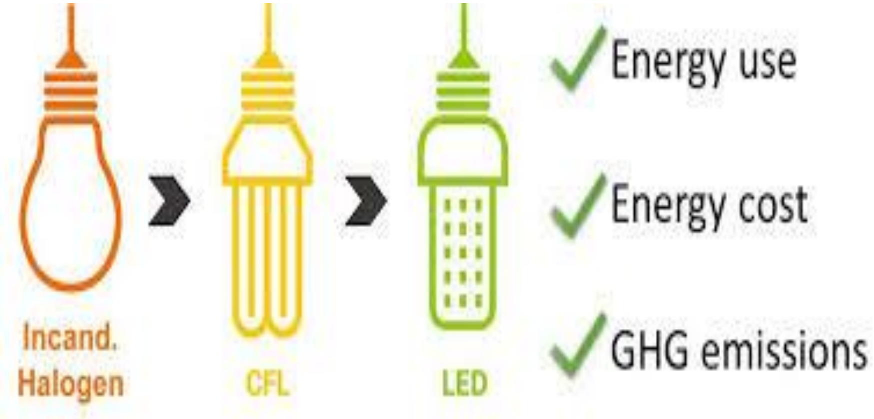
Historical review..

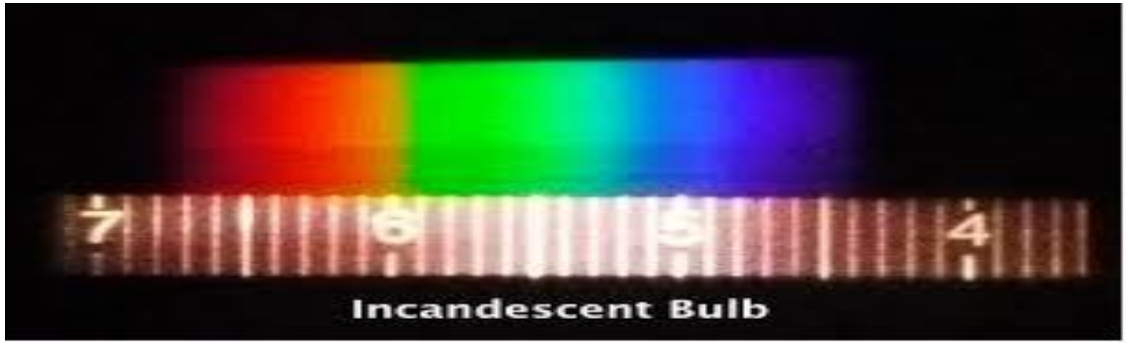
1859	Gustav Robert Kirchhoff reported the coincidence of the wavelengths of spectrally resolved lines of absorption and of emission of visible light.
1864	John Tyndall presented measurements of the infrared emission by a platinum filament and the corresponding color of the filament
1879	Josef Stefan was deduced the fourth power law of the absolute temperature on the basis of Tyndall's experimental measurements
1893	Wilhelm Wien based on a thermodynamic argument derived expression for energy distribution in black body radiation spectrum.
1900	Rayleigh–Jeans law based on classical physical arguments and empirical facts
1900	German physicist Max Planck successfully explained the energy distribution in the black body radiation

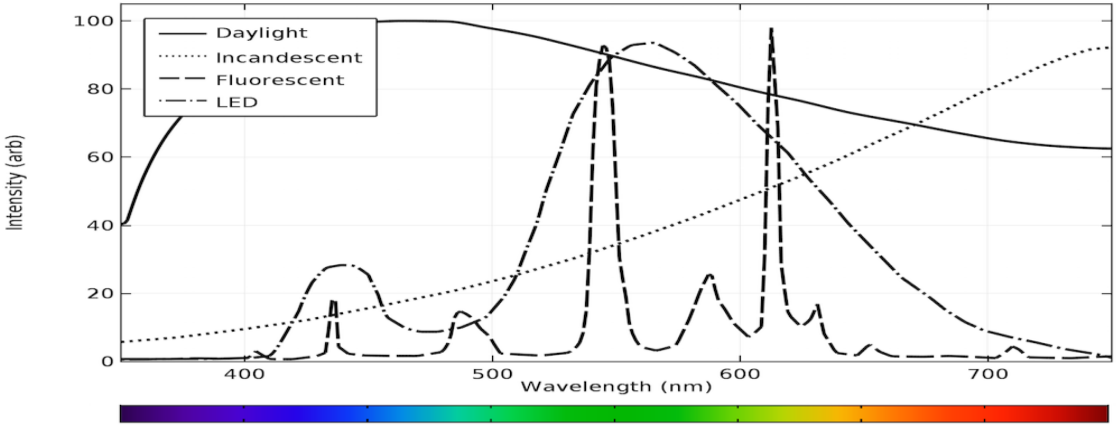
Definition..

The branch of mechanics that deals with the mathematical description of the motion and interaction of subatomic particles, incorporating the concepts of quantization of energy, wave–particle duality, the uncertainty principle, and the correspondence principle.

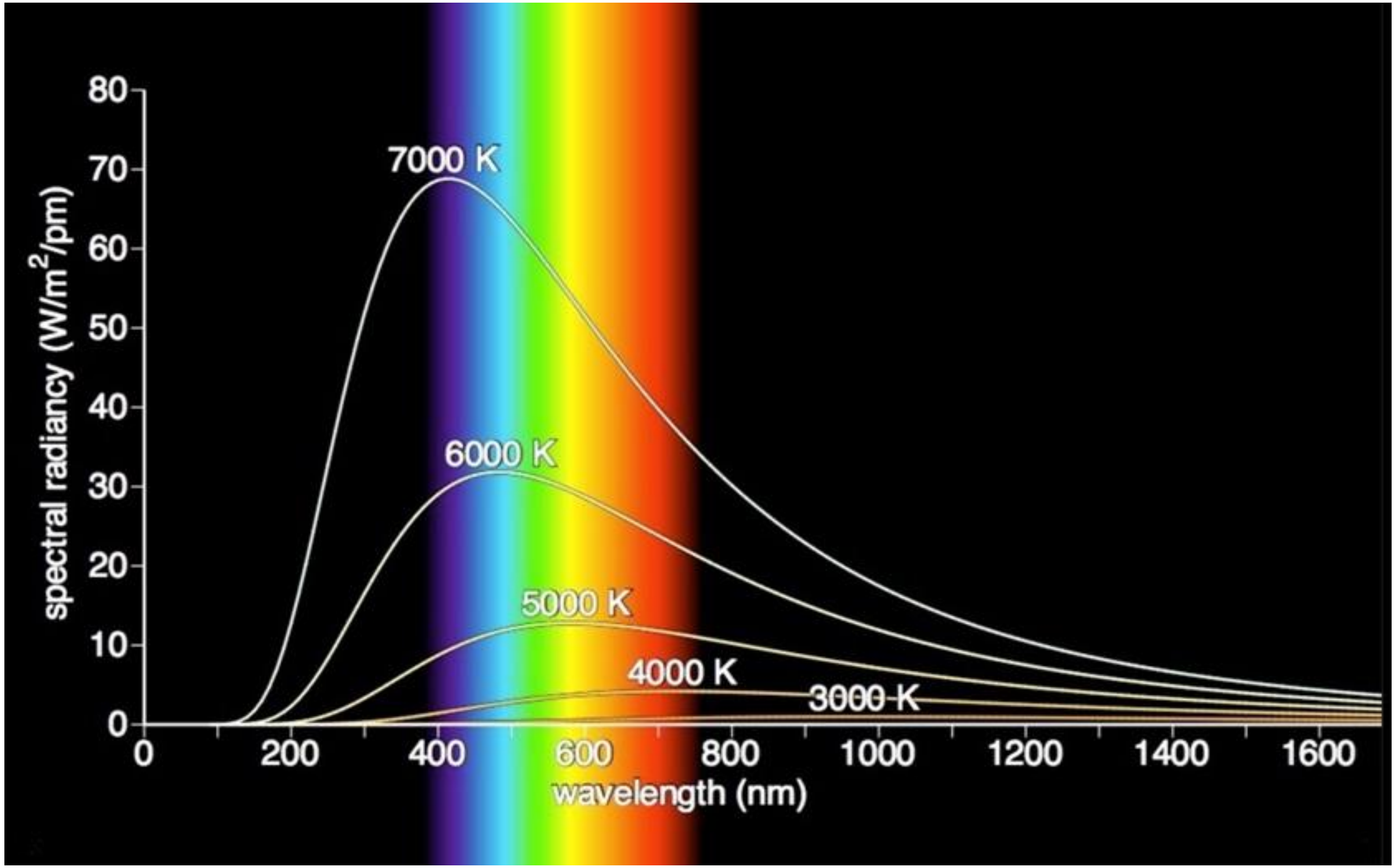
We know that any heated substance gives out energy in the form of radiation belonging to different region of the complete electromagnetic spectrum.

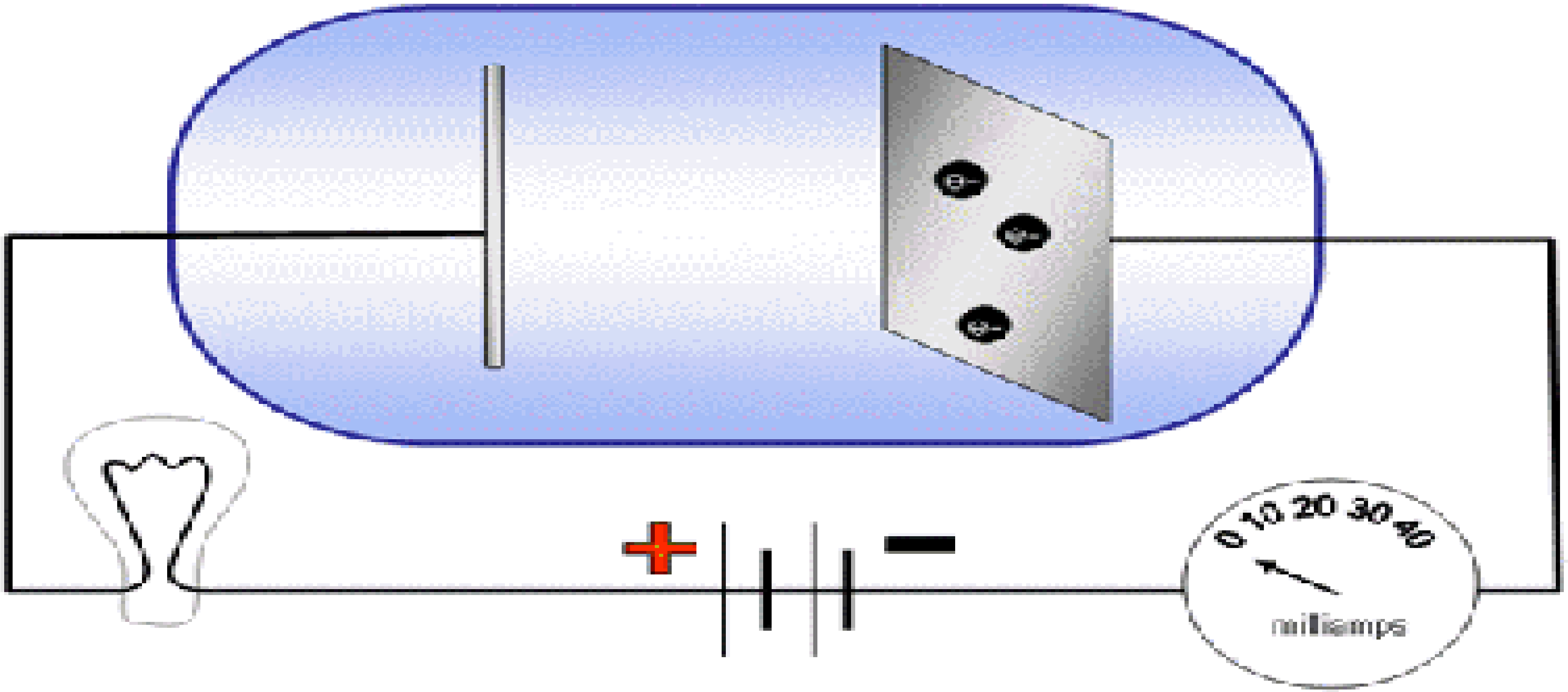




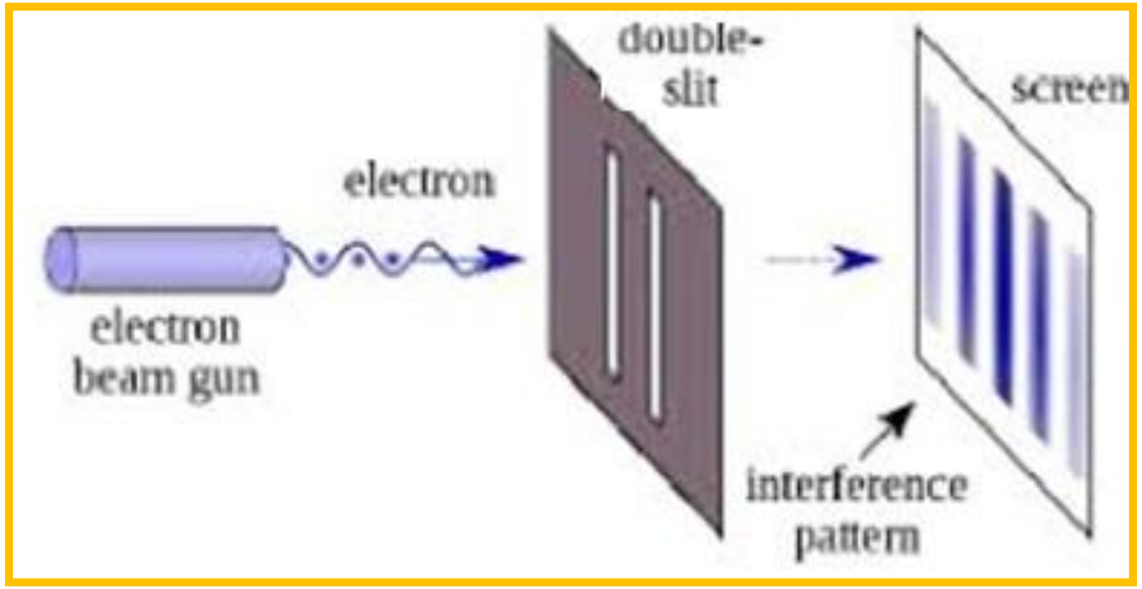


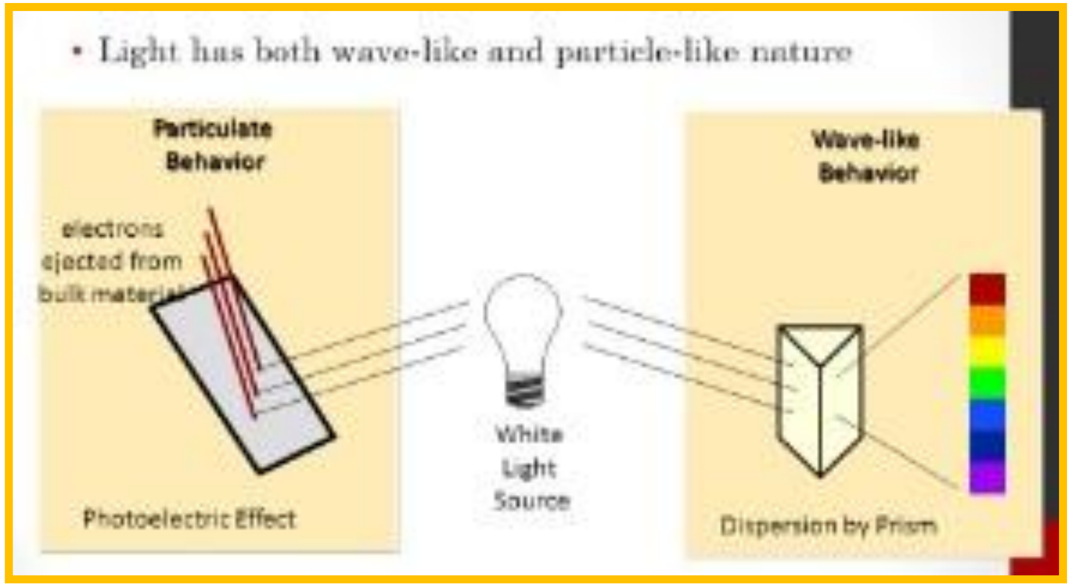
Black body radiation curve..

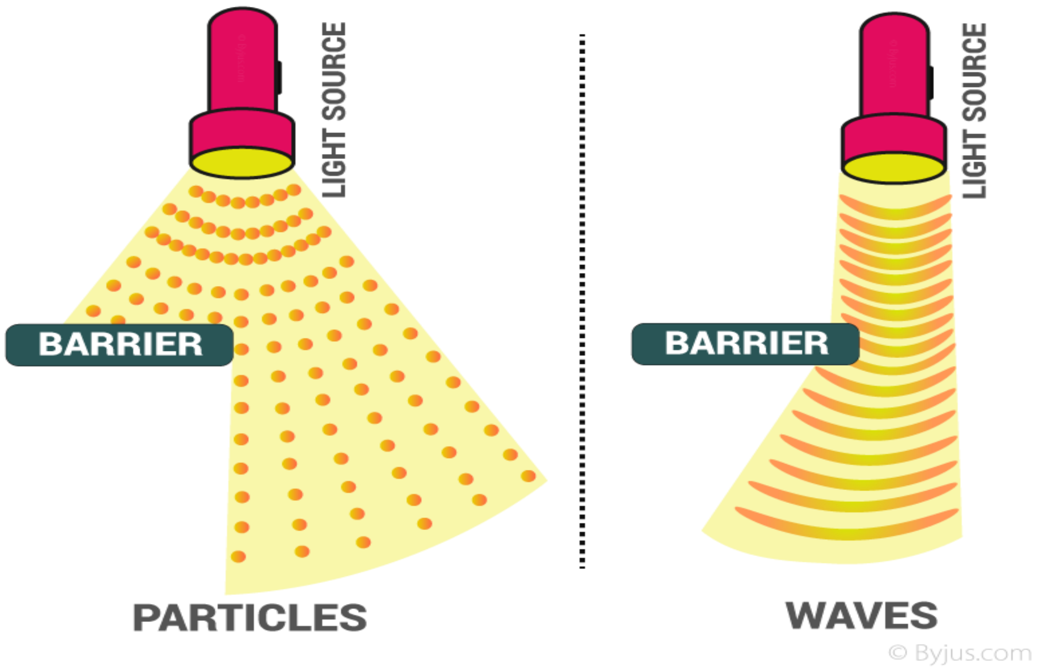


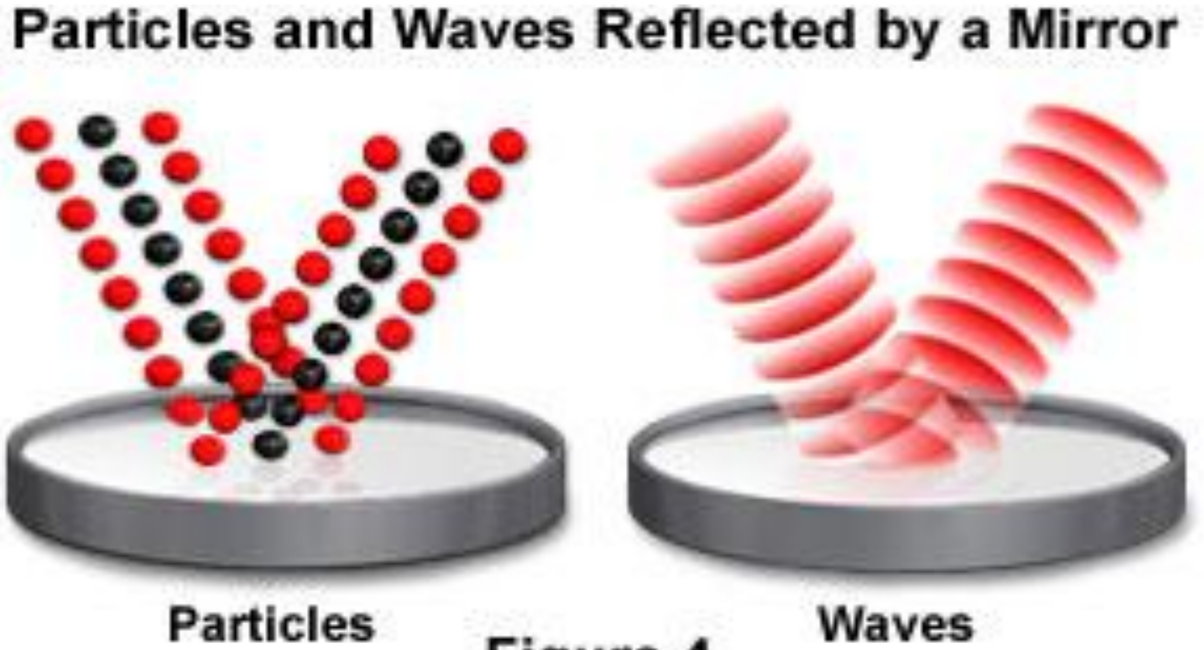


Photoelectric Effect


double-
slit
screen
electron
electron
beam gun
interference
pattern


• Light has both wave-like and particle-like nature
Particulate
Behavior
electrons
ejected from
bulk material
Photoelectric Effect
White
Light
Source
Wave-like
Behavior
Dispersion by Prism


LIGHT SOURCE
BARRIER
PARTICLES
LIGHT SOURCE
BARRIER
WAVES
© Byjus.com


Particles and Waves Reflected by a Mirror
Particles
Waves
Figure 4

Dual nature of radiation..

Since Nature loves symmetry, if radiation has particle nature then, particle should have wave nature.

Light has a dual nature,..

1. Sometimes it behaves like a particle (called a photon), which explains how light travels in straight lines – Photoelectric effect, Blackbody radiation, scattering, .
2. Sometimes it behaves like a wave, which explains how light bends (or diffracts) around an object – reflection, refraction, interference, diffraction, polarization,..

All microscopic **particles**, whether massless, like photons, or having mass, like electrons, have **wave properties**. The relationship between momentum and wavelength is fundamental for all **particles**.

De Broglie Hypothesis..

- In the year 1924 French physicist Louis de Broglie extended wave-particle dualism through a hypothesis stating If radiant energy could behave like waves in some experiments and particles or photons in others and since nature loves symmetry, the one can expect the particles like protons and electrons to exhibit wave nature under suitable circumstances. This is well known as de Broglie's hypothesis.
- Therefore some sort of waves can be even associated with moving material particles called Matter waves or de Broglie waves and the wavelength associated with matter waves is called de Broglie wavelength.

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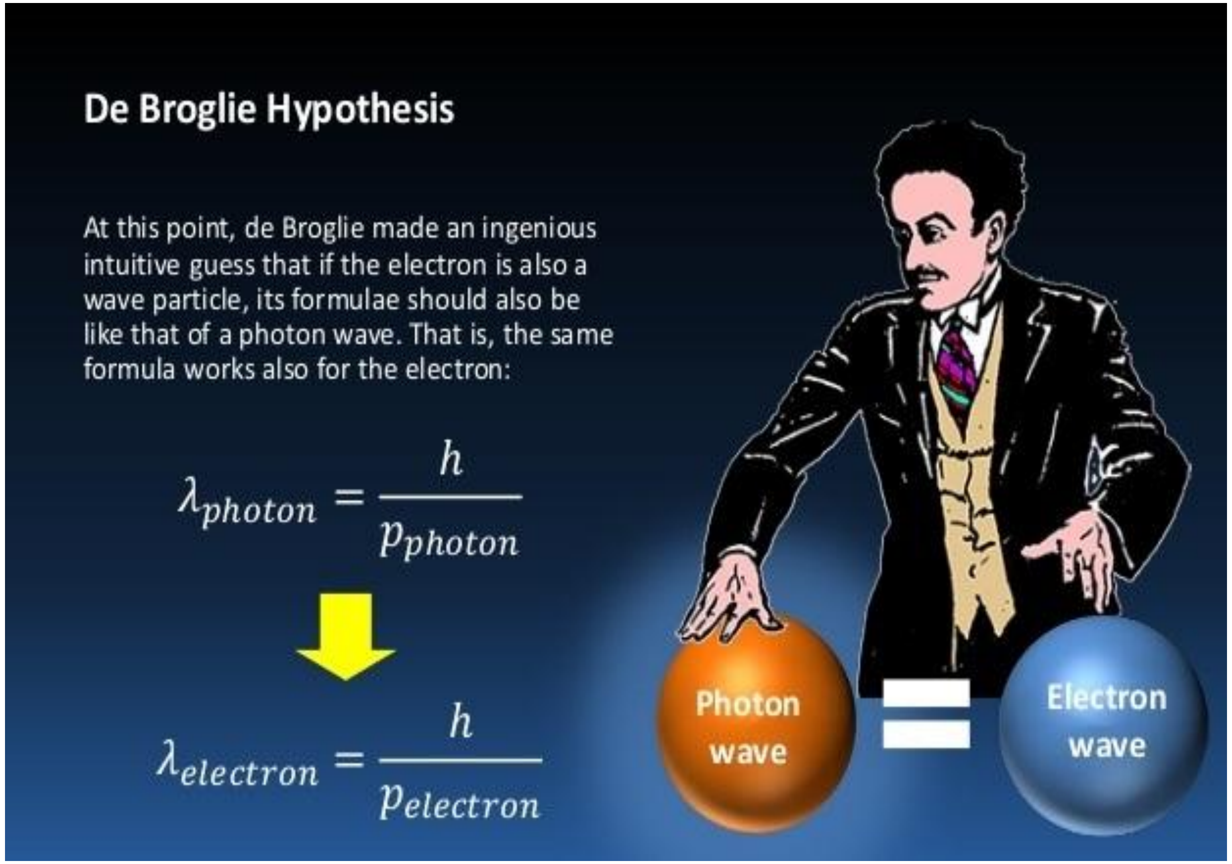
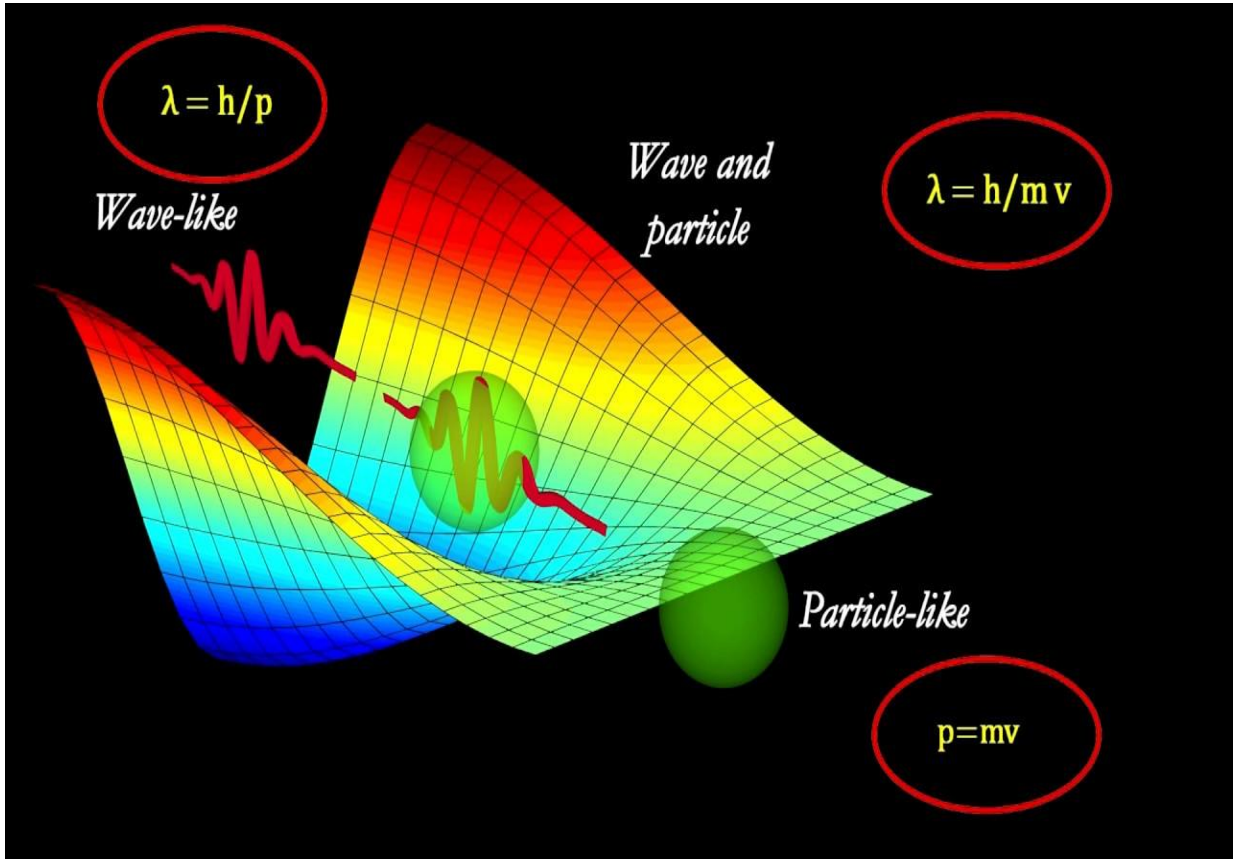
De Broglie Hypothesis..





Expression for De Broglie Wavelength..

W.k.t $E = mc^2$

And $E = h\nu = \frac{hc}{\lambda}$

$\therefore mc^2 = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{mc^2} = \frac{h}{mc}$

For materialistic particle

$c = v$

$\therefore \lambda = \frac{h}{mv} = \frac{h}{p}$

The wavelength of a photon in terms of its momentum is given by $\boxed{\lambda = \frac{h}{p}}$

Hence the de Broglie wavelength of matter waves is given by $\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$

Here m is the mass of the moving particle and v is its velocity.

For a particle, charged or uncharged, moving with kinetic energy E the de Broglie wavelength is given by

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

The kinetic energy is

$E = \frac{1}{2}mv^2$

$E = \frac{1}{2}\frac{m^2v^2}{m}$

$2mE = m^2v^2$

$2mE = p^2$

$p = \sqrt{2mE}$

$\therefore \lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

Expression for De Broglie Wavelength..

w.k.t

$$K.E = \frac{3}{2}kT$$

$$K.E = \frac{1}{2}mv^2$$

$$\therefore \frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$m^2v^2 = 3mkT$$

$$p = \sqrt{3mkT}$$

$$\therefore \lambda = \frac{h}{\sqrt{3mkT}}$$

For a charged particle accelerated with a Potential V volt, the de Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mqV}}$

For an electron accelerated through a potential difference of V volt, the de Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2meV}}$

Further substituting the values of h,m and e, de Broglie wavelength is given by $\lambda = \frac{12.26}{\sqrt{V}}$ Å

The de Broglie waves depends on temperature

Then, $\lambda = \frac{h}{\sqrt{3mkT}}$

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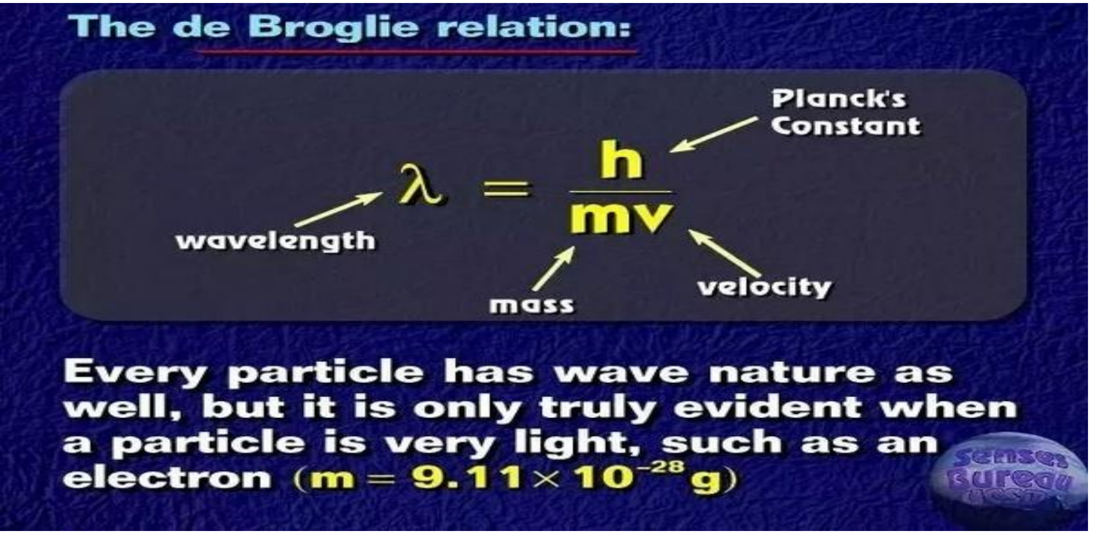
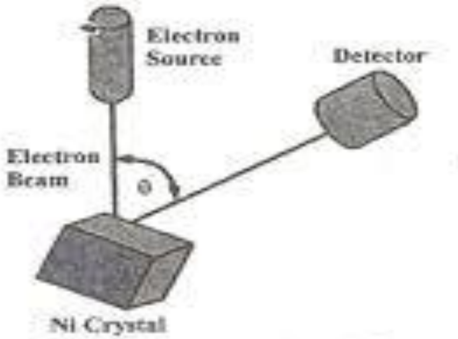
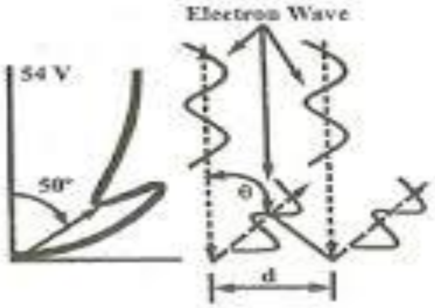
Explanation for De Broglie Wavelength..





Experimental Electron Wavelength from D-G expt.: Path length difference : $d \sin\theta = 2.15 \sin 50° = \lambda$ = 1.65 Å for constructive interference (Ni lattice spacing, d = 2.15 Å)



Theoretical electron wavelength from de-Broglie hypothesis : $\lambda = h/mv = 1.67$ Å for 54 V

Consider a particle of mass 1 kg

Moving with velocity 1 ms^{-1}

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1 \times 1}$$

$$\therefore \lambda = 6.626 \times 10^{-34} m$$

Now consider, electron moving with

Speed 2.2×10^{6} ms^{-1}

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.2 \times 10^{6}}$$

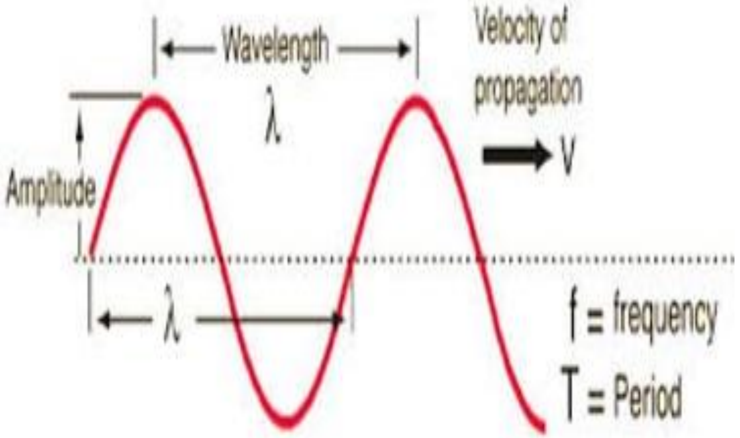
$$\therefore \lambda = 3.31 \times 10^{-10} m$$

Phase Velocity..

- The velocity with which a wave travels is called phase velocity (v_p) and is also called wave velocity. If a point is marked on the wave representing the phase of the particle then the velocity with which the phase propagates from one point to another is called phase velocity.

- The phase velocity of a wave is given by

$$v_p = \frac{\omega}{k} = f\lambda$$



- The wave velocity of a matter wave is given by

$$V_p = \frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \frac{\frac{E}{h}}{\frac{P}{h}} = \frac{E}{P} = \frac{c^2}{v}$$

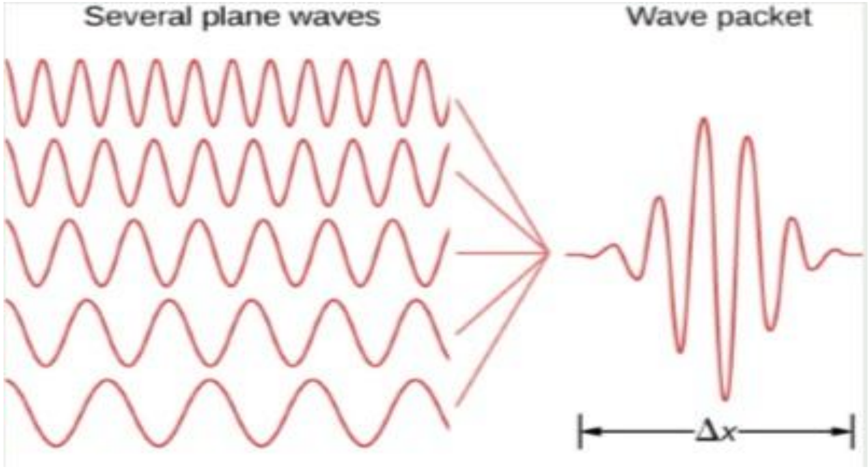
Group Velocity..

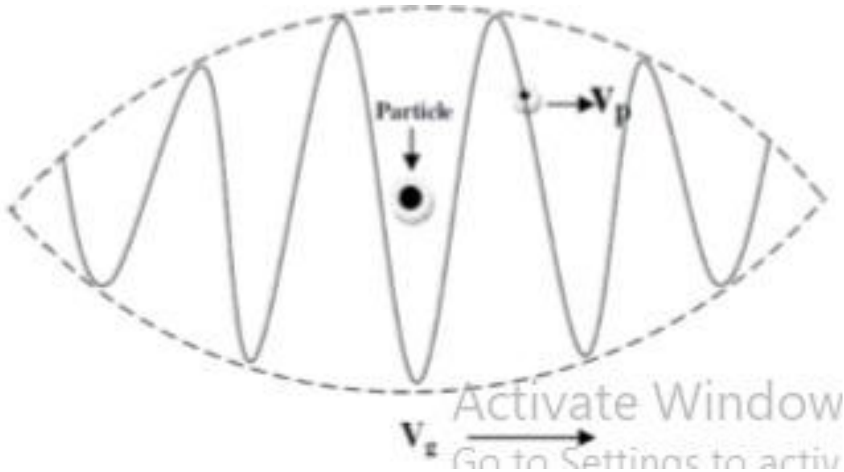
- Phase velocity has no physical meaning for matter waves' Thus the concept of Group velocity is introduced.

- Matter wave can be considered as a resultant wave due to the superposition of many component waves whose velocities differ slightly called Wave Packet.

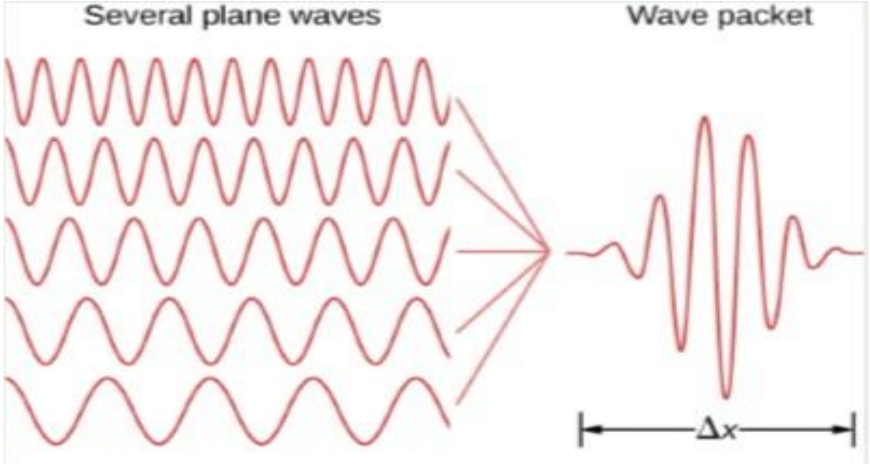
- The velocity with which the wave group travels is called Group Velocity and is same as Particle Velocity. Wave Velocity is given by

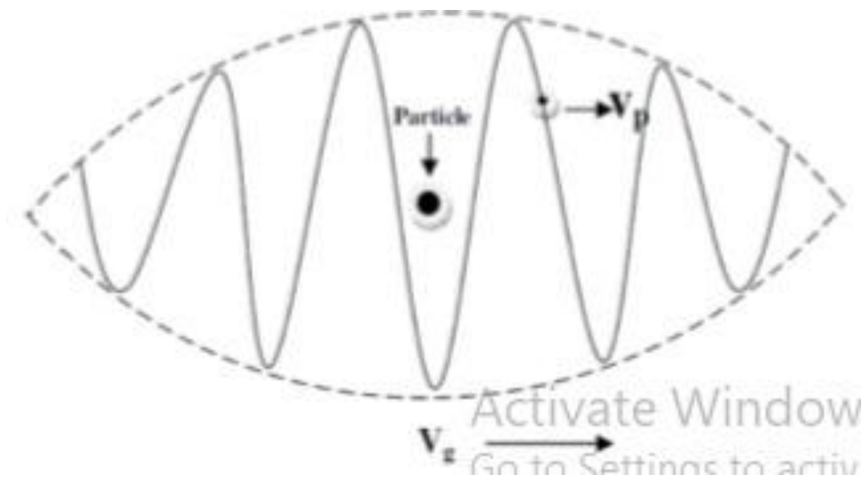
$$V_g = \frac{d\omega}{dk}$$





Activate Window
Go to Settings to activ


Several plane waves
Wave packet
Δx


Particle
Vp
Vg

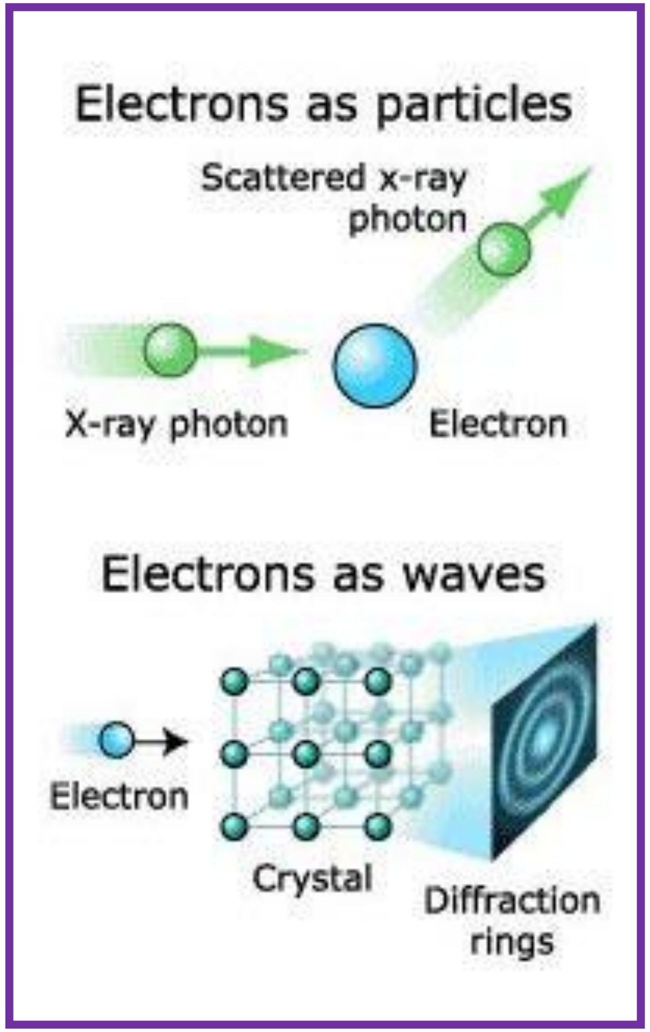
Properties of matter waves..

1. Matter Waves are associated with only moving particles.
2. Matter waves are non Electromagnetic in nature.
3. Larger the momentum of the particle smaller will be the wavelength of the matter wave.
4. More massive the particle, smaller will be the wave length.
5. Moving particle can be associated with wave packets, some kind of localized wave travelling with same speed as that of the particle. The speed of the wave envelop is called as Group velocity which is same as the particle velocity.
6. The amplitude of the matter wave at a point is associated with the probability of finding the particle at that point.

Complementarity principle..

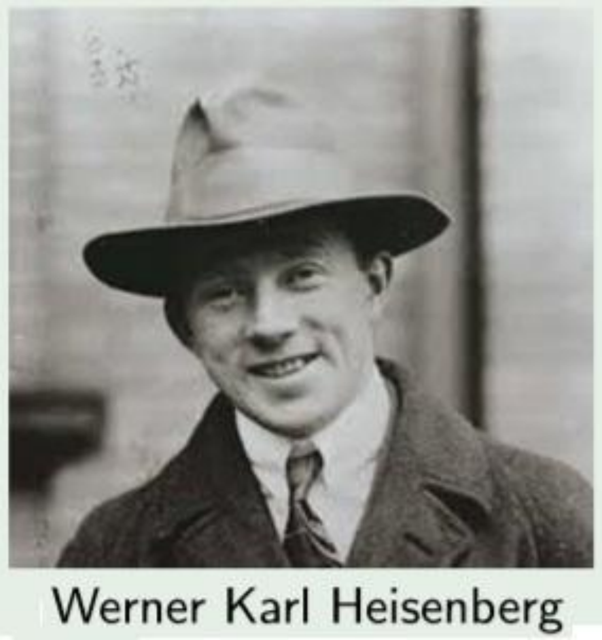
Neils Bohr introduced the complementarity principle in the year 1928.

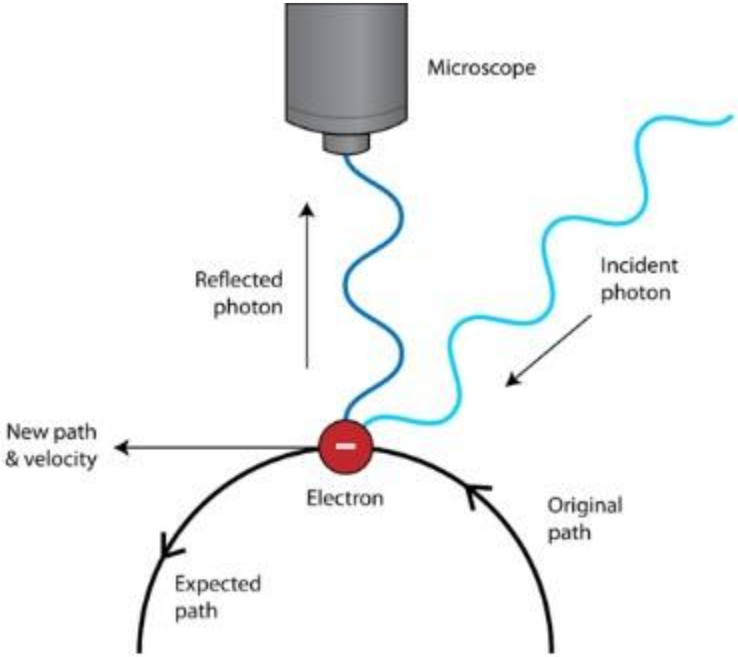
The principle of complementarity definition states that, when the particle nature of the matter (for example light) is measured or displayed, the wave nature of the matter is necessarily suppressed and at the same time if the wave nature of the matter is displayed then the particle nature will be suppressed. *The inability to observe the wave nature and the particle nature of the matter simultaneously is known as the complementarity principle.*



Heisenberge's Uncertainty Principle..

- In the Year 1927 German Physicist Werner Karl Heisenberg Proposed the uncertainty principle.
- "The simultaneous determination of the exact position and momentum of a moving particle is impossible".





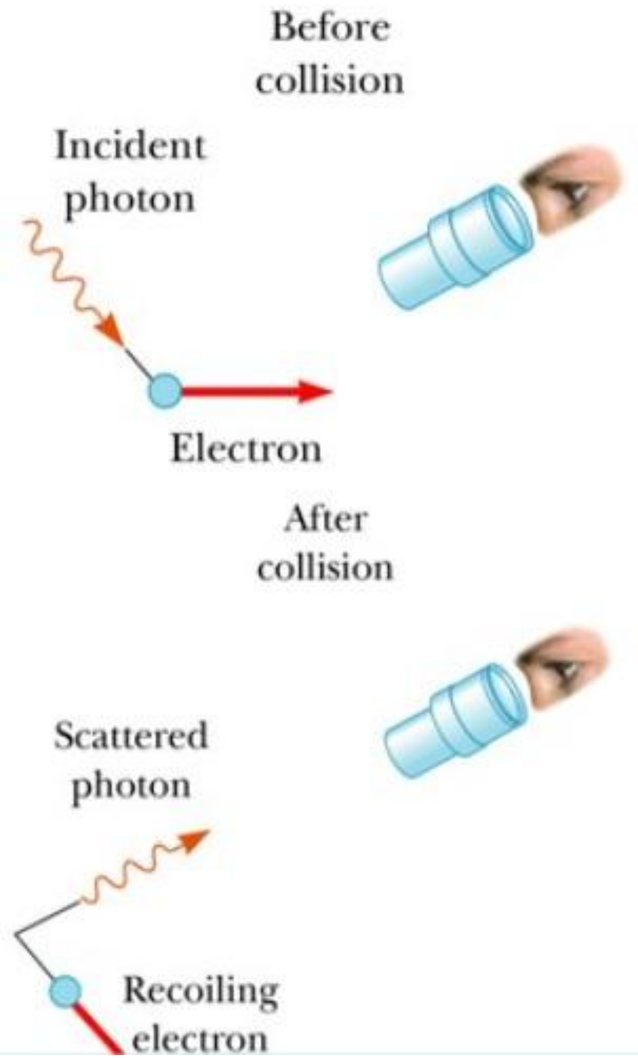
Heisenberge’s Uncertainty Principle..

- According to this principle if Δx is the error involved in the measurement of position and Δpx is the error involved in the measurement of momentum during their simultaneous measurement, then the product of the corresponding uncertainties is given by

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$\Delta \theta \Delta L \geq \frac{h}{4\pi}$$



- The product of the errors is of the order of Planck’s constant. If one quantity is measured with high accuracy then the simultaneous measurement of the other quantity becomes less accurate.

Heisenberge's Uncertainty Principle..

- According to Newtonian physics the simultaneous measurement of position and momentum are exact.

- The existence of matter waves induces serious problems due to the limit to accuracy associated with the simultaneous measurement.

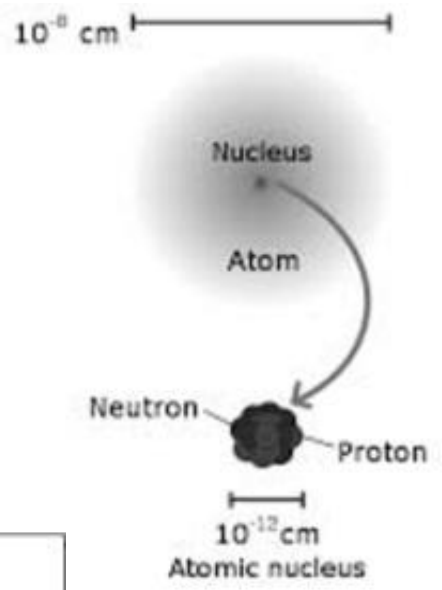
- Hence the exactness in Newtonian physics is replaced with probability in quantum mechanics.

Application of Uncertainty principle: Non existence of electron inside the nucleus

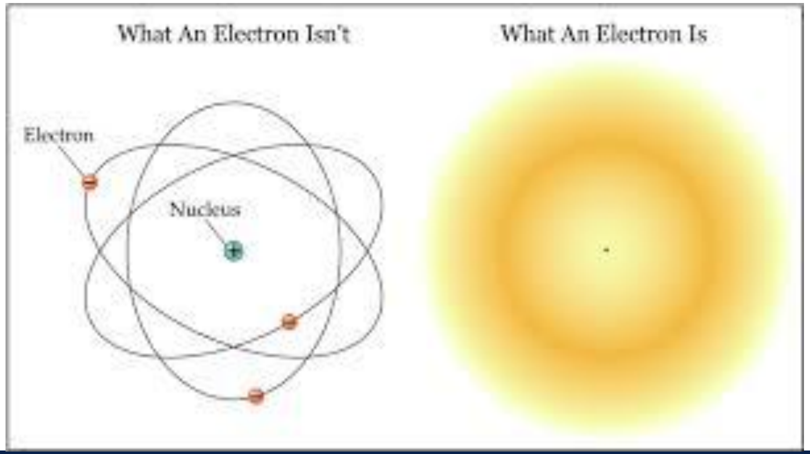
- Beta rays are emitted by the nucleus It was believed that electrons exist inside the nucleus and are emitted at certain instant.

- If the electron can exist inside the atomic nucleus then uncertainty in its position must not exceed the diameter of the nucleus.

- The diameter of the nucleus is of the order of



$$\Delta x_{max} \text{ is } 10^{-14} m.$$



Application of Uncertainty principle: Non existence of electron inside the nucleus

Applying Heisenberg's uncertainty principle for an electron expected to be inside the nucleus we get

$$\Delta x_{max} \Delta p_{min} \geq \frac{h}{4\pi}$$

$$\Delta p_{min} \geq \frac{h}{4\pi \Delta x_{max}}$$

$$\Delta p_{min} \geq \frac{6.625 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}}$$

$$\Delta p_{min} \geq 5.276 \times 10^{-21} kgms^{-1}$$

Thus the momentum of the electron is give by

$$p_{min} \simeq \Delta p_{min} = 5.276 \times 10^{-21} kgms^{-1}$$

Application of Uncertainty principle: Non existence of electron inside the nucleus

Applying Heisenberg's uncertainty principle for an electron expected to be inside the nucleus we get

Relativistic equation for the energy of the electron is given by

$$E = \sqrt{p^2c^2 + m_0{}^2c^4}$$

Since, $m_0{}^2c^4 \ll p^2c^2$, we get $E = pc$.

Therefore

$$E_{min} = p_{min}c = 5.276 \times 10^{-21} \times 3 \times 10^8$$

$$E_{min} = 1.58 \times 10^{-12} J$$

$$E_{min} = \frac{1.58 \times 10^{-12}}{1.6 \times 10^{-19}} = 9.9\text{MeV}$$

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Application of Uncertainty principle: Non existence of electron inside the nucleus

- According to the Experiments the energy associated with beta decay is around 3 MeV.
- The energy of the electron expected to be inside the nucleus is 9.9 MeV calculated using Heisenberg's uncertainty Principle.
- Thus the electron does not exist inside the nucleus.
- Electron is emitted instantaneously during the decay of the neutron inside the nucleus.

Time Independent Schrodinger wave equation..

- The wave function for a one dimensional classical wave traveling along x-axis with the oscillations of the particle along Y axis is given by $y = A\sin(\omega t - kx)$
- The wave equation for a classical wave traveling along x axis is given by

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

- Erwin Schrodinger was a Austrian - Irish Physicist.
- Awarded Nobel Prize in the Year 1933 for the formulation of the Schrodinger equation.
- Schrodinger equation provides a way to calculate the wave function of a system and how it changes dynamically in time.

Time Independent Schrodinger wave equation..

- The wave equation which has variations only with respect to position and describes the steady state is called Time Independent Schrodinger wave equation.
- Consider a particle of mass m moving with velocity v along +ve x–axis. The de Broglie wave length λ is given by

$$\lambda = \frac{h}{mv}$$

- The wave equation for one dimensional propagation of waves is given by $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$
- The wave function is given by $$\psi = \psi_0 \, e^{i(kx - \omega t)}$$
- here ψ_0 is the amplitude at the point of consideration, ω is angular frequency and k is the wave number.

Time Independent Schrodinger wave equation..

Time Independent Schrodinger wave equation..

Wave function..

- According to the de Broglie's hypothesis the relation between momentum and wavelength is found to be experimentally valid for both photons and particles.
- The quanta of matter or radiation can be represented in agreement with uncertainty principle by wave packets.
- Wave function, in quantum mechanics, variable quantity that mathematically describes the wave characteristics of a particle.
- The variations of which make up the matter wave is called Wave Function

Wave function..

A wave function that depends on space (x, y and z) and time(t) and is denoted by $\psi(r, t)$.
The wave function for a wave packet moving along +ve x axis is given by

$$\psi = \psi_0 \, e^{i(kx - \omega t)}$$

The quantity ψ is assumed to have the following three basic properties

1. It can interfere with itself so that it can account for diffraction experiments.
2. It is large in magnitude where is particle or photon is likely to be found and small else where.
3. It will be regarded as describing the behavior of single particle or photon and not statistical distribution of number of quanta.

Wave function..

- The wave function ψ just as itself has no direct physical meaning.
- The physical interpretation of the amplitude of the wave is more difficult.
- The amplitude of the wave function ψ is certainly not like displacement of particles in a classical wave.
- The quantity squared absolute value of the amplitude gives the probability density of the particle.
- Probability density is the probability of finding the particle per unit volume at the given location in space and is referred to as probability density.
- This is also referred to as Max Born's interpretation of the wave function of a system and how it changes dynamically in time.

The Probability Density is given by

$$P(x) = |\psi|^2$$

Thus, in one dimension the probability of finding a particle in the width dx of length x

$$P(x)dx = |\psi|^2 dx$$

Similarly, in three dimension, the probability (P) of finding a particle in a given small volume $dV = dx\ dy\ d$ of volume V is given by

$$P\ \ dv = |\psi|^2 dv$$

Normalization of wave function..

- Consider a space of volume ‘V’. Let ‘P’ be the probability density of finding a particle at a point inside ‘V’.
- Consider a small volume ‘dV’ around that point.
- Since the particle exists somewhere in volume V then the probability of finding the particle in the given volume V is equal 1.
- Thus $$\int_0^V |\psi|^2 dV = \int_0^V P dV = 1$$
- If we are unable to locate the particle in volume V then the notion can be extended to the whole space with $$\int_{-\infty}^{+\infty} |\psi|^2 dx\, dy\, dz = 1$$
- But, normally, the value of the above integral will not be unity but contains an indefinite constant which can be determined along with sign using above considerations. The process is called Normalization and the wave function which satisfies the above condition is called Normalized wave function.

Max Born's interpretation..

if the wave function defining the quantum mechanical state is real, we can use ψ^2 as the probability density;

if the wave function does contain the imaginary part (like $\psi = a + ib$), $\psi\psi^*$ must be used to yield real values.

$$\psi = a + ib; \qquad \psi^* = a - ib$$

$$\psi\psi^* = (a + ib) \times (a - ib)$$

$$\psi\psi^* = a^2 + b^2$$

if ψ is real, $\psi = \psi^*$,

$\psi\psi^*$ becomes ψ^2

Eigen values and eigen function

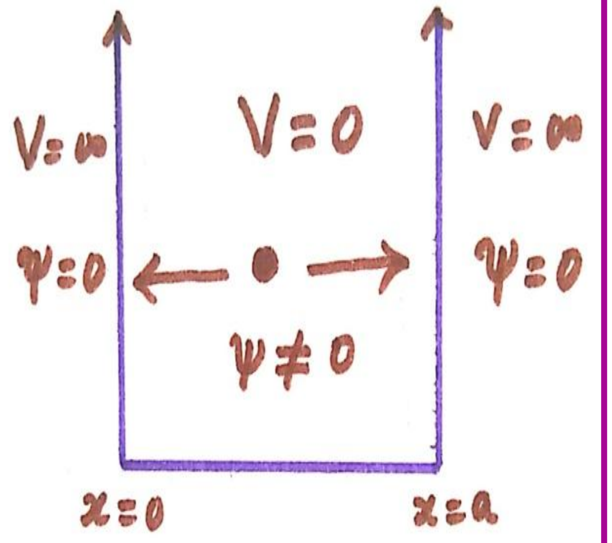
- Eigen values and eigen functions The Schrodinger wave equation is a second order differential equation. Thus solving the Schrodinger wave equation to a particular system we get many expressions for wave function(ψ).
- However, all wave functions are not acceptable.

Only those wave functions which satisfy certain conditions are acceptable. Such wave functions are called Eigen functions for the system.

- The energy values corresponding to the Eigen functions are called Eigen values. The wave functions are acceptable if they satisfy the some conditions.

Application of Schrodinger wave equation: Particle in one dimensional potential well of infinite height

- A particle of mass m bouncing back and forth between the walls of one dimensional potential well of Infinite Height.
- Motion of the particle is confined along x axis between two infinitely hard walls at $x = 0$ and $x = a$.
- In between walls i.e. $0 < x < a$, the potential $V = 0$. Beyond the walls i.e. $x \leq 0$ and $x \geq a$, the potential $V = \infty$.
- The particle has to gain infinite energy to come out of the well and its impossible.
- The particle is unable to penetrate the hard walls it exists only inside the potential well.
- Hence $\psi = 0$ and the probability of finding the particle outside the potential well is also zero



For a free particle, in one dimensional motion

$$F = \frac{dV}{dx}$$

if $F=0$ then, V is constant

∴ choose $V=0$

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Application of Schrodinger wave equation: Particle in one dimensional potential well of infinite height

The Schrodinger wave equation is given by

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-U)}{h^2}\psi = 0 \qquad (1)$$

Since the potential inside the well $V=0$, hence potential energy $U=0$, the Schrodinger wave equation becomes

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-0)}{h^2}\psi = 0 \qquad (2)$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\psi = 0 \qquad (3)$$

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0 \qquad (4)$$

$$k^2 = \frac{8\pi^2 mE}{h^2} \qquad (5)$$

here k is a constant for a given value of energy E. The general solution for equation 4 is given by

$$\psi(x) = A\sin kx + B\cos kx \qquad (6)$$

Here in the above equation A and B are arbitrary constants which can be evaluated by applying boundary conditions.

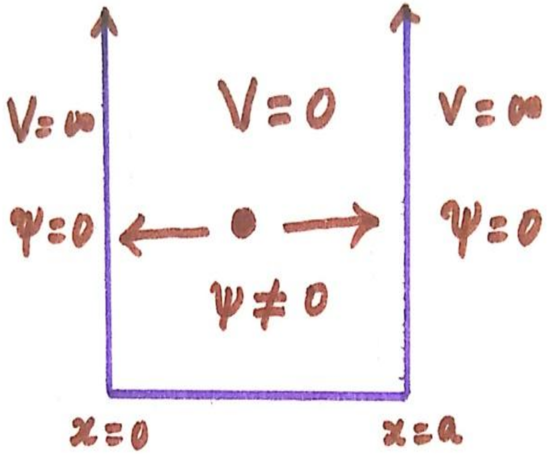
Application of Schrodinger wave equation: Particle in one dimensional potential well of infinite height

1. The first boundary condition is, at $x = 0$, $\psi(x) = 0$. Applying this to equation 6, we get $0 = A\sin 0 + B\cos 0 \implies B = 0$ hence equation 6 reduces to

$$\psi(x) = A\sin kx \quad (7)$$

2. The second boundary condition is,at $x = a$, $\psi(x) = 0$. Applying this to equation 7, we get $0 = A\sin ka$. Since $A \neq 0$ then $\sin ka = 0$. This results in $ka = n\pi$ which further could be written as $k = \frac{n\pi}{a}$. n can take integer values. Hence equation 7 could be written as

$$\psi(x) = A\sin\left(\frac{n\pi x}{a}\right). \quad (8)$$



For a free particle, in one dimensional motion

$F = \frac{dV}{dx}$

if $F=0$ then, V is constant

∴ choose $V=0$

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Application of Schrodinger wave equation: Particle in one dimensional potential well of infinite height

also from equation 5

$$k^2 = \frac{n^2\pi^2}{a^2} = \frac{8\pi^2 mE_n}{h^2} \quad (9)$$

$$E_n = \frac{n^2h^2}{8ma^2} \quad (10)$$

- Substituting for $n = 1, 2, 3, 4, ..$ in the above equation **Energy Eigen Values** are obtained.
- The lowest energy state corresponds to lowest integral value of $n = 1$ which is also called as **Zero Point Energy** is given by $E_1 = \frac{h^2}{8ma^2}$.
- The energy values of a bound particle in one dimensional potential well are quantized (discrete) and are represented by the equation $E_n = n^2E_1$.

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Normalization of wave equation..
Expression for eigen function..

The wave function for a particle in one dimensional potential well of infinite height is given by the equation 8 $\psi(x) = A\sin\left(\frac{n\pi x}{a}\right)$. In this equation A is an arbitrary constant and it can take any value. The process of determination of value of the arbitrary constant is called *Normalization of wave function*. The particle has to exist some where inside the potential the probability of the finding the particle inside the potential well is given by

$$\int_0^a |\psi(x)|^2 dx = \int_0^a P dx = 1 \tag{11}$$

Substituting for the wave function in the integral

$$\int_0^a A^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1 \tag{12}$$

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Normalization of wave equation..
Expression for eigen function..

from trigonometry $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$. There fore the above equation could be written as

$$\int_0^a \frac{A^2}{2}\left[1-\cos\left(\frac{2n\pi x}{a}\right)\right] dx = 1 \tag{13}$$

integrating the above equation we get

$$\frac{A^2}{2}\left[x-\frac{a}{2n\pi}\sin\left(\frac{2n\pi x}{a}\right)\right]_0^a = 1 \tag{14}$$

$$\Longrightarrow A = \sqrt{\frac{2}{a}} \tag{15}$$

Normalization of wave equation..
Expression for eigen function..

Substituting this in equation 8 the normalized wave function or eigen function for a particle in one dimensional potential well of infinite height is given by

$$\psi(x)_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right). \qquad (16)$$

The wave functions and the probability densities for the first three values of n are as shown in fig

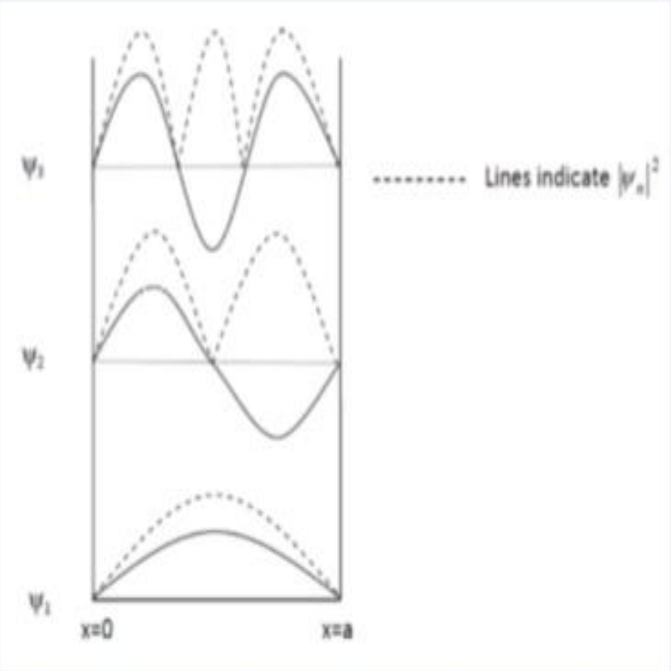

FIGURE: Wave function and Probability density

Application of Schrodinger wave equation.. Particle in one dimensional potential well of infinite height

$$\psi(x)_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right).$$

Substituting for $x = 0$, $x = \frac{a}{2}$ and $x = a$ in the above equation we get $\psi(0)_1 = 0$, $\psi(\frac{a}{2})_1 = 0$ and $\psi(a)_1 = 0$

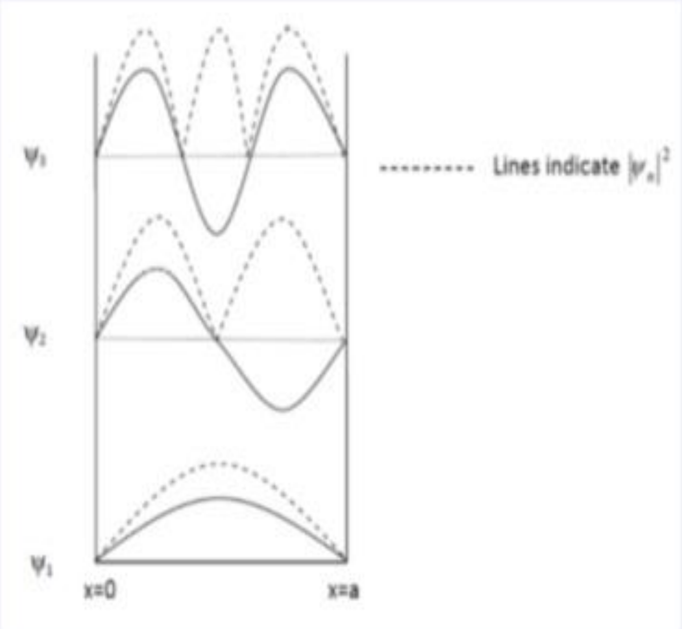

FIGURE: Wave function and Probability density

Numerical problems on Quantum Mechanics

- Collect Data Given
- Write all the Physical quantities in SI units
- Suitable Formula
- Substitution and Calculation
- Result with SI unit

Formulae

de Broglie wavelength

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Uncertainty eqn

$$\Delta x\, \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E\, \Delta t \geq \frac{h}{4\pi}$$

$$\Delta p_x = m\, \Delta v$$

$$\Delta x\, m\, \Delta v \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi\, m\, \Delta v}$$

Energy eigen values

$$E_n = \frac{n^2 h^2}{8ma^2}$$

$$E_1 = \frac{h^2}{8ma^2}$$ ground state

$$E_2 = \frac{4h^2}{8ma^2}$$ 1st excited state

$$E_3 = \frac{9h^2}{8ma^2}$$ 2nd excited state

1. The velocity of an electron of a Hydrogen atom in the ground state is $2.19 \times 10^{6} ms^{-1}$. Calculate the wavelength of the de Broglie waves associated with its motion.

Given

$v = 2.19 \times 10^{6} m/s, \quad \lambda = ?$

Solution

The de Broglie wavelength is given by

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 2.19 \times 10^{6}}$$

$$\lambda = 3.321 \times 10^{-10} \text{ m}$$

2. Compute the de Broglie wavelength for a neutron moving with one tenth part of the velocity of light, given mass of neutron 1.674×10^{-27}kg.

Given

$\lambda = ?\quad v = \frac{1}{10}c = v = \frac{1}{10}\times 3\times10^{8} = 3\times10^{7}\ \mathrm{m/s},\ m_{\mathrm{n}} = 1.674\times10^{-27}\mathrm{kg}$

Solution

The de Broglie wavelength is given by

$$\lambda = \frac{h}{m_{\mathrm{n}}v}$$

$$\lambda = \frac{6.63\times10^{-34}}{1.674\times10^{-27}\times3\times10^{7}}$$

$$\lambda = 1.314\times10^{-14}\mathrm{m}$$

3. Calculate de Broglie wavelength associated with electrons whose speed is 0.01 part of the speed of light.

Given

$$\lambda = ? \quad v = 0.01\,c = 0.01 \times 3 \times 10^{8} = 3 \times 10^{6} ms^{-1}$$

Solution

The de Broglie wavelength is given by

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{6}}$$

$$\lambda = 2.427 \times 10^{-10}\ \text{m}$$

$$\lambda = 2.427\ \text{Å}$$

4. Calculate the wavelength associated with an electron having kinetic energy 100eV

Given

$$\lambda = ? \quad E = 100\ eV = 100 \times 1.602 \times 10^{-19}\ J = 1.602 \times 10^{-17} J$$

Solution

The de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.602 \times 10^{-17}}}$$

$$\lambda = 1.227 \times 10^{-10}\ \text{m}$$

$$\lambda = 1.227 \text{Å}$$

5. Calculate de Broglie wavelength associated with electron carrying energy 2000eV

Given

$\lambda = ?\quad E = 2000\ eV = 2000 \times 1.602 \times 10^{-19} J = 3.204 \times 10^{-16} J$

Solution

The de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.204 \times 10^{-16}}}$$

$$\lambda = 2.743 \times 10^{-11}\ \text{m}$$

$$\lambda = 0.274\ \text{Å}$$

6. Calculate the momentum of the particle and the de Broglie wavelength associated with an electron with a kinetic energy of 1.5 keV.

Given

$$p = ?,\ \lambda = ?\ \ E = 1.5\ keV = 1.5 \times 10^{3} \times 1.602 \times 10^{-19} J = 2.403 \times 10^{-16} J$$

Solution

The momentum is given by

$$p = \sqrt{2mE}$$

$$p = \sqrt{2 \times 9.1 \times 10^{-31} \times 2.403 \times 10^{-16}}$$

$$p = 2.091 \times 10^{-23} kgms^{-1}$$

The de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2.403 \times 10^{-16}}}$$

$$\lambda = 3.168 \times 10^{-11}\ \text{m}$$

7. Find the energy of the neutron in eV whose de Broglie wavelength is 1Å.

Given

$E = ?\ in\ eV, \quad \lambda = 1Å, \quad w.k.t\ m_n = 1.674 \times 10^{-27} kg$

Solution

The de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda^2 = \frac{h^2}{2mE}$$

$$E = \frac{h^2}{2m\lambda^2}$$

$$E = \frac{(6.626 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

$$E = 13.113 \times 10^{-18} J$$

Energy in eV

$$E = \frac{13.113 \times 10^{-18}}{1.602 \times 10^{-19}}$$

$$E = 81.85\ eV$$

8. A particle of mass 0.5 MeV/C² has kinetic energy of 100 eV. Find the de Broglie wavelength where c is the speed of light.

Given $m = 0.5\ \text{MeV}/C^2,\ E = 100eV = 100 \times 1.6 \times 10^{-19}\ \text{J},\ \lambda = ?$

Solution We have $m = 0.5\ \frac{\text{MeV}}{c^2} = \frac{0.5 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2} = 8.9 \times 10^{-31}\text{kg}$

The de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 8.9 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.24 \times 10^{-10}\text{m}$$

9. A particle of mass 0.65 MeV/C^2 has kinetic energy of 80 eV. Find the de Broglie wavelength where c is the speed of light.

Given

$m = 0.65\,\text{MeV}/C^2$, $E = 80\,eV = 80 \times 1.6 \times 10^{-19}\,\text{J}$, $\lambda = ?$

Solution

We have $m = 0.65\frac{\text{MeV}}{c^2} = \frac{0.65\times10^{6}\times1.6\times10^{-19}}{(3\times10^{8})^2} = 11.55 \times 10^{-31}\text{kg}$

The de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times 11.55\times10^{-31}\times100\times1.6\times10^{-19}}}$$

$$\lambda = 1.09\times 10^{-10}\text{m}$$

10. Find the de Broglie wavelength of an electron accelerated through a potential difference of 182V and object of mass 1kg moving with a speed of $1ms^{-1}$. Compare the results and comment.

Given

$\lambda = ?$, $V = 182\ V$, $m = 1kg$, $v = 1ms^{-1}$. Compare result

Solution

For electron

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{6.626\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.602\times10^{-19}\times182}}$$

$$\lambda = 9.096\times10^{-11}\ m$$

For object

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626\times10^{-34}}{1\times1}$$

$$\lambda = 6.626\times10^{-34}\ m$$

de Broglie wavelength associated with object is too small, hence could not observe.

11. The position and momentum of an electron with energy 0.5 keV are determined. What is the minimum percentage uncertainty in its momentum if the uncertainty in the measurement of its position is 0.5Å.

Given

The energy of the electron, $E = 0.5 \text{ keV} = 0.5 \times 10^3 \times 1.6 \times 10^{-19}$ J

Minimum Uncertainty in position, $\Delta x = 0.5 \times 10^{-10}$m

Minimum percentage uncertainty in momentum ?

Solution

$$\text{We have, } \Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta p_x \geq \frac{h}{4\pi \, \Delta x}$$

$$\Delta p_x \geq \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 0.5 \times 10^{-10}}$$

$$\Delta p_x \geq 1.06 \times 10^{-24} \text{ kgm/s}$$

11. The position and momentum of an electron with energy 0.5 keV are determined. What is the minimum percentage uncertainty in its momentum if the uncertainty in the measurement of its position is 0.5Å.

Solution

We have the equation for Momentum,

$$p_x = \sqrt{2mE}$$

$$p_x = \sqrt{2 \times 9.11 \times 10^{-31} \times 0.5 \times 10^3 \times 1.6 \times 10^{-19}}$$

$$p_x = 1.207 \times 10^{-23} \text{kgm/s}$$

The percentage uncertainty in momentum is $\frac{\Delta p_x}{p_x} \times 100$

$$\text{Percentage Uncertainty in Momenutm} = \frac{1.06 \times 10^{-24}}{1.207 \times 10^{-23}} \times 100$$

$$\text{Percentage Uncertainty in Momenutm} = 8.8$$

12. The speed of electron is measured to with in an uncertainty of $2.2 \times 10^4\ ms^{-1}$ in one dimension. What is the minimum width required by the electron to be confined in an atom?

Given $\Delta v = 2.2 \times 10^4\ ms^{-1},\ \Delta x = ?$

Solution Using Heisenberg's uncertainty principle we have,

$$\Delta x\, m \Delta v = \frac{h}{4\pi}$$

$$\Delta x = \frac{h}{4\pi m\, \Delta v}$$

$$\Delta x = \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 2.2 \times 10^4}$$

$$\Delta x = 2.635 \times 10^{-9}\ m$$

$$\Delta x = 2.635\ nm$$

13. An electron is confined to a box of length 10^{-9} m, calculate the minimum uncertainty in its velocity.

Given

$$\Delta x = 10^{-9} m, \quad \Delta v = ?$$

Solution

Using Heisenberg's uncertainty principle we have,

$$\Delta x \, m \Delta v = \frac{h}{4\pi}$$

$$\Delta v = \frac{h}{4\pi m \, \Delta x}$$

$$\Delta v = \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 10^{-9}}$$

$$\Delta v = 57.97 \; kms^{-1}$$

14. A spectral line of wavelength 5461Å has a width of 10^{-4} Å . Evaluate the minimum time spent by the electron in the upper energy level.

Given $\lambda = 5461 \times 10^{-10}$m, $\Delta\lambda = 10^{-4}$Å $= 10^{-4} \times 10^{-10} = 10^{-14}$ m , $\Delta t = ?$

Solution Using Heisenberg's uncertainty principle we have,

$$\Delta E\, \Delta t \geq \frac{h}{4\pi}$$

$$E = h\nu = \frac{hc}{\lambda}$$

$$\Delta E = hc\ \Delta\left(\frac{1}{\lambda}\right)$$

$$\Delta E = hc\ \frac{\Delta\lambda}{\lambda^2}$$ (On ignoring the negative sign)

$$\Delta t \geq \frac{h}{4\pi\Delta E} = \frac{h}{4\pi hc\left(\frac{\Delta\lambda}{\lambda^2}\right)}$$

$$\Delta t \geq \frac{\lambda^2}{4\pi c\ \Delta\lambda}$$

$$\Delta t \geq \frac{(5461 \times 10^{-10})^2}{(4 \times 3.14 \times 3 \times 10^8 \times 10^{-14})}$$

$$\Delta t \geq 0.8 \times 10^{-8}\ \text{s.}$$

15. An electron is bound in one dimensional potential well of width 0.18nm. Find the energy value in eV of the second excited state.

Given

$a = 0.18\ nm,\ \ Second\ excited\ state\ E_3 = ?$

Solution

Energy eigen value is given by,

$$E_n = \frac{n^2 h^2}{8ma^2}$$

For second excited state, n=3

$$E_3 = \frac{3^2 h^2}{8ma^2}$$

$$E_3 = \frac{3^2 \times (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.18 \times 10^{-9})^2}$$

$$E_3 = 1.675 \times 10^{-17}\ \mathrm{J}$$

In eV

$$E_3 = \frac{1.675 \times 10^{-17}}{1.602 \times 10^{-19}}$$

$$E_3 = 104.56\ eV$$

16. An electron is bound in one dimensional potential well of width 0.18nm. Find the energy value in eV of the second excited state.

Given

$a = 0.18\ nm,\ \ Second\ excited\ state\ E_3 = ?$

Solution

Energy eigen value is given by,

$$E_n = \frac{n^2 h^2}{8ma^2}$$

For second excited state, n=3

$$E_3 = \frac{3^2 h^2}{8ma^2}$$

$$E_3 = \frac{3^2 \times (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.18 \times 10^{-9})^2}$$

$$E_3 = 1.675 \times 10^{-17}\ J$$

In eV

$$E_3 = \frac{1.675 \times 10^{-17}}{1.602 \times 10^{-19}}$$

$$E_3 = 104.56\ eV$$

17. The first excited state energy of an electron in an infinite well is 240eV. What will be its ground state energy when the width of the potential well is doubled?

Given

$E_2 = 240\ eV, \quad E_1 = ?\ \ when\ a = 2a$

Solution

Energy eigen value is given by,

$$E_n = \frac{n^2 h^2}{8ma^2}$$

Put n = 2

$$E_2 = \frac{2^2 h^2}{8ma^2}$$

When a = 2a, eqn for E1 is,

$$E_1 = \frac{1^2 h^2}{8m(2a)^2}$$

$$\frac{E_1}{E_2} = \frac{1^2 h^2}{8m(2a)^2} \times \frac{8ma^2}{2^2 h^2}$$

$$\frac{E_1}{E_2} = \frac{1}{16}$$

$$E_1 = \frac{E_2}{16}$$

$$E_1 = \frac{240}{16}$$

$$E_1 = 15\ eV$$

18. An electron is bound in an one dimensional potential well of width 1 Å, but of infinite wall height. Find its energy in the ground state and also in first two excited states.

Given

$a = 1Å = 1 \times 10^{-10}\text{m}$ $E_1 = ?$ $E_2 = ?$ $E_3 = ?$

Solution

Energy eigen value is given by,

$$E_n = \frac{n^2h^2}{8ma^2}$$

For E_1, put n = 1

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (1 \times 10^{-10})^2}$$

$$E_1 = 6.0314 \times 10^{-18}\text{J}$$

$$E_1 = \frac{6.0314 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$E_1 = 37.64\text{ eV}$$

Consider

$$E_n = n^2 E_1$$

For E_2, put n = 2

$$E_2 = 2^2 E_1$$

$$E_2 = 4 \times 37.64$$

$$E_2 = 150.54\text{ eV}$$

For E_3, put n = 3

$$E_3 = 3^2 E_1$$

$$E_3 = 9 \times 37.64$$

$$E_3 = 338.7\text{ eV}$$

Module – 2: Electrical Properties of Solids

Module – 2: Electrical Properties of Solids:

Dielectric Properties: Polar and non-polar dielectrics, Types of Polarization, internal fields in solid, Clausius-Mossottiequation (Derivation), solid, liquid and gaseous dielectrics. Application of dielectrics in transformers, Capacitors, and Electrical Insulation. Numerical problems.

Superconductivity: Introduction to Superconductors, Temperature dependence of resistivity, Meissner's Effect, Silsbee Effect, Types of Super Conductors, Temperature dependence of Critical field, BCS theory (Qualitative), HighTemperature superconductivity, SQUID, MAGLEV, Numerical problems.

Pre-requisites: Difference between Insulators & Dielectrics.

Self-learning: Dielectrics Basics

Fermi –Dirac Statistics and Fermi factor

- A metal contains large number free of electrons. The occupation of energy levels in the valence band is according to Pauli's exclusion principle.
- This distribution of electrons follows a certain universal rule called Fermi-Dirac Distribution.
- This statistics is applicable for the distribution of particles of spin half.
- A distribution statistics provides the probability of occupation of an energy level with energy (E) at a given temperature T under thermal equilibrium.
- The probability of occupation of an energy level of energy (E) at temperature (T) under thermal equilibrium is given by *Fermi Factor* $(f(E))$.

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \tag{2}$$

Number Density of Free Electrons

The number density of electrons defined as the number of free electrons per unit volume the material in the energy range E and $E + dE$ in the valence band of the material is given by $n(E)dE$. Mathematically

$$N(E)dE = g(E)dE \ f(E) \tag{3}$$

- The number density of electrons is the product of the number of available energy states and their occupation probability. This is called Fermi-Dirac Distribution
- As per this rule at temperature **0 K** all levels below the Fermi level are completely filled and above the Fermi level are empty.
- At higher temperatures the probability of occupation of energy levels above the Fermi level increases and below the Fermi level decreases due to the increase in thermal energy of the valence electrons.

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Dependence of Fermi factor on energy and temperature

Case 1 : Probability of occupation of levels with Energy $E < E_F$ and at T = 0K

Fermi factor is a function of energy and temperature. The Fermi factor or Fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \qquad (1)$$

Here $E - E_f$ is negative. Substituting the value for $T = 0$

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{k*0}\right)} + 1}$$

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0+1}$$

There fore $f(E) = 1$.

Hence, at $T = 0K$, all energy levels below the Fermi level are completely filled.

Dependence of Fermi factor on energy and temperature

Case 2 : Probability of occupation of levels with Energy $E > E_F$ and at T = 0K

The Fermi factor or fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \qquad (2)$$

Here $E - E_f$ is positive. Substituting the value for $T = 0$

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{k*0}\right)} + 1}$$

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty + 1}$$

There fore $f(E) = 0$.

Hence, at $T = 0K$, all energy levels above the Fermi level are empty.

Dependence of Fermi factor on energy and temperature

Case 3 :Probability of occupation of levels with Eneregy $E = E_F$ and at $T > 0K$

The Fermi factor or Fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \tag{3}$$

Here $E - E_f = 0$. Substituting the values

$$f(E) = \frac{1}{e^{\left(\frac{0}{kT}\right)} + 1} \tag{4}$$

$$f(E) = \frac{1}{e^{\left(\frac{0}{kT}\right)} + 1} \tag{5}$$

$$f(E) = \frac{1}{1+1} = \frac{1}{2} = 0.5 \tag{6}$$

- Thus for all temperatures above 0 K the probability of occupation of Fermi level is $\frac{1}{2}$.
- Thus the variation of Fermi factor with temperature is as shown in the graph 1

Graphical Representation

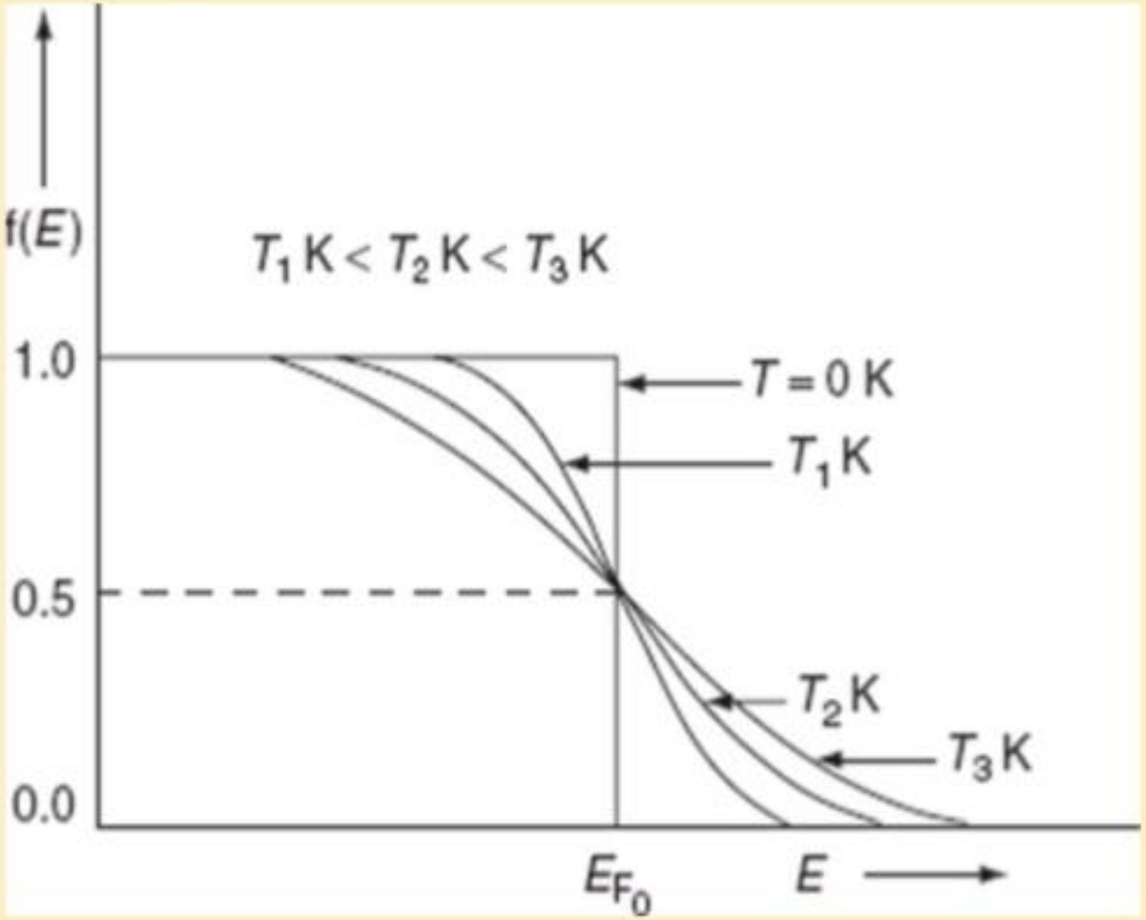

Figure: Variation of $f(E)$ as a function of Temperature and Energy

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Outline

Dielectrics

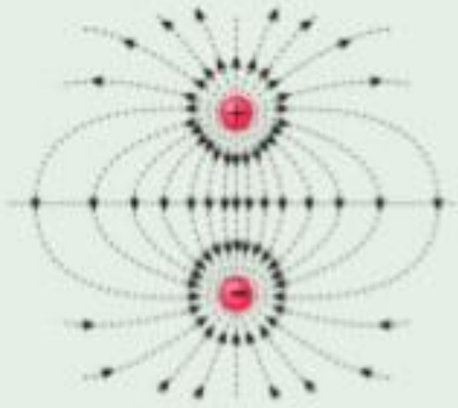
- Fundamentals of Dielectrics
  - Introduction
  - Dielectric Constant
- Polarization of Dielectric
  - Polarization
  - Relation between Polarization and Dielectric Constant
  - Types of Polarization or Polarization Mechanisms
  - Polar and Non Polar Dielectrics
- Internal Fields in Solids
  - Expression for Internal field in case of one dimension
  - Expression for Internal field in case of three dimension- Lorentz Field

Dielectrics

- Internal Fields in Solids
  - Expression for Internal field in case of one dimension
  - Expression for Internal field in case of three dimension & Lorentz Field
- Clausius-Mossotti Relation
- Examples of Dielectrics Materials
  - Solid Dielectrics
  - Liquid Dielectric
  - Gas Dielectrics
- Application of Dielectrics in Transformers
  - Application in Transformers
  - Corona Discharge

Introduction

1. Dielectrics are electrically non-conducting materials such as glass, porcelain etc, which exhibit remarkable behavior because of the ability of the electric field to polarize the material creating electric dipoles.
2. Electric dipole is an entity which contains two equal and opposite charges separated by a small distance.
3. The dipole moment of a dipole is given by $\vec{\mu} = q\vec{d}$. here $\vec{\mu}$ is the dipole moment, q is the magnitude of either of the charges and $\vec{d}$ is the separation of magnitude d.



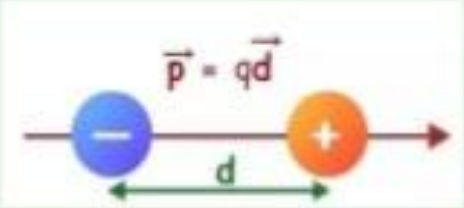
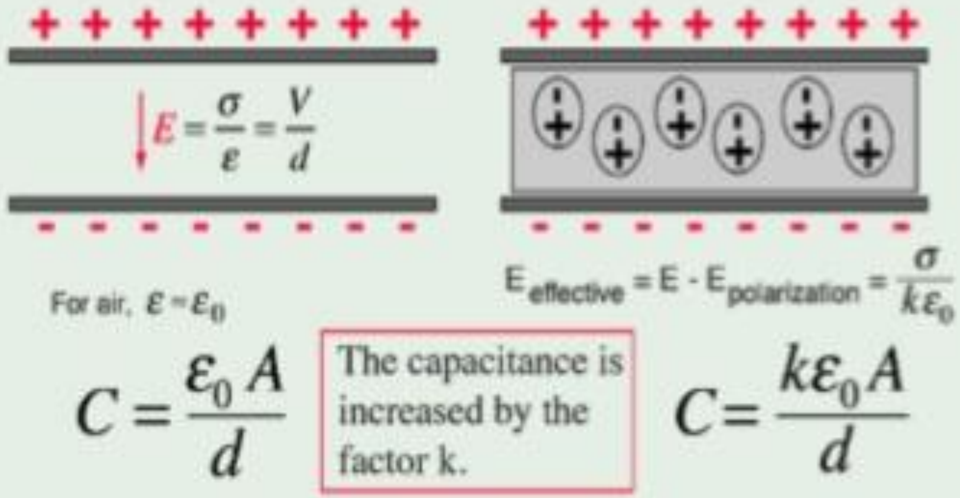


FIGURE: Electronic Polarization

Dielectric Constant

1. Faraday discovered that, If C_0 is the capacitance of the capacitor without dielectric and C is the capacitance with dielectric then the ratio $\frac{C}{C_0}$ gives ϵ_r called Relative Permittivity or Dielectric constant.
2. For a given isotropic material the electric flux density is related to the applied field strength by the equation $D = \epsilon E$, Here ϵ is Absolute permittivity.
3. $\epsilon = \epsilon_0 \epsilon_r$, Here ϵ_0 is permittivity of free space. ϵ_r or k is relative permittivity or dielectric constant.

1. For an isotropic material, under static field conditions, the Relative permittivity is called Static Dielectric Constant. It depends on the structure of the atoms of which the material is composed.



Polarization of Dielectric

The displacement of charged particles in atoms or molecules of dielectric material so that net dipole moment is developed in the material along the applied field direction is called Polarization of dielectric. Polarization is measured as net dipole moment per unit volume and is a vector quantity.

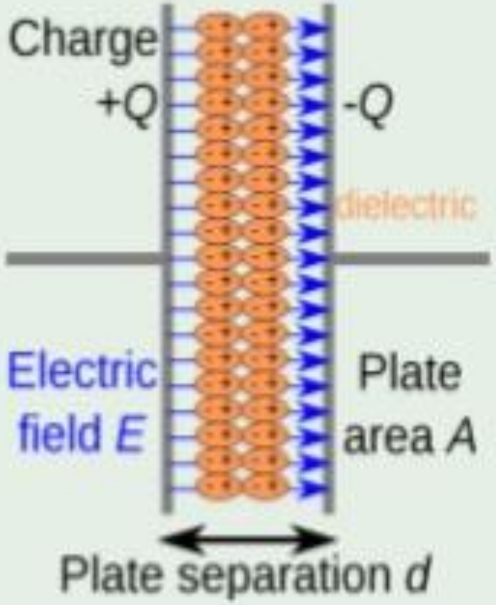
$$\vec{p} = N\vec{\mu} \tag{1}$$

Here $\vec{p}$ is polarization, N number of dipoles per unit volume, $\vec{\mu}$ is average dipole moment per molecule.

The Dipole Moment $vec\mu$ is given by

$$\vec{\mu} = \alpha\vec{E} \tag{2}$$

Here α is defined as Polarizability. Link



Polarization of Dielectric

As the polarization P measures the additional flux density arising from the presence of the material as compared to free space it has the same unit as D and is related to it as

$$\vec{P} = \epsilon_0 (\epsilon_r - 1) \vec{E} \tag{3}$$

$$\chi = \frac{\vec{P}}{\epsilon_0 \vec{E}} \tag{4}$$

here χ is Dielectric Susceptibility
ϵ_0 is Permittivity of Free Space
ϵ_r is Relative Permittivity or Dielectric Constant.

Types of Dielectric Polarization or Polarization Mechanisms

The electrical polarization in dielectrics occurs place through four different mechanisms. They are

1. Electronic polarization P_e
2. Ionic polarization P_i
3. Orientation polarization P_o
4. Space charge polarization P_s

The net polarization of the material is due to the contribution of all four polarization mechanism. The net polarization is given by the vector sum of individual polarization mechanisms.

$$\vec{P} = \vec{P_e} + \vec{P_i} + \vec{P_o} + \vec{P_s} \tag{5}$$

Electronic Polarization

1. This involves the separation of the center of the electron cloud around an atom with respect to the center of its nucleus under the application of electric field.
2. Hence dipoles are induced within the material.
3. This leads to the development of net dipole moment in the material and is the vector sum of dipole moments of individual dipoles.

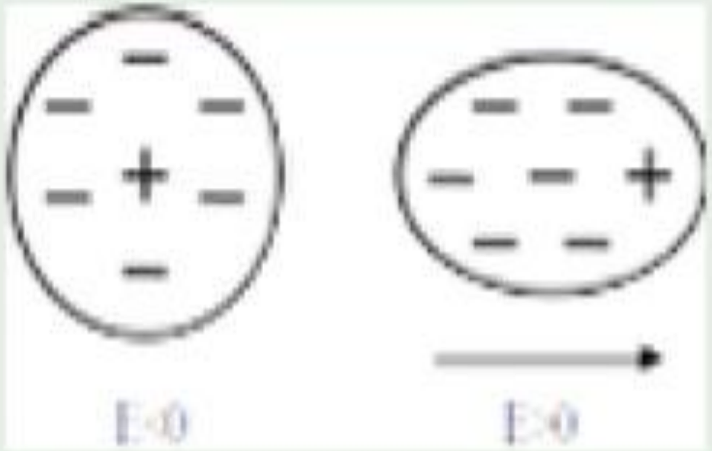
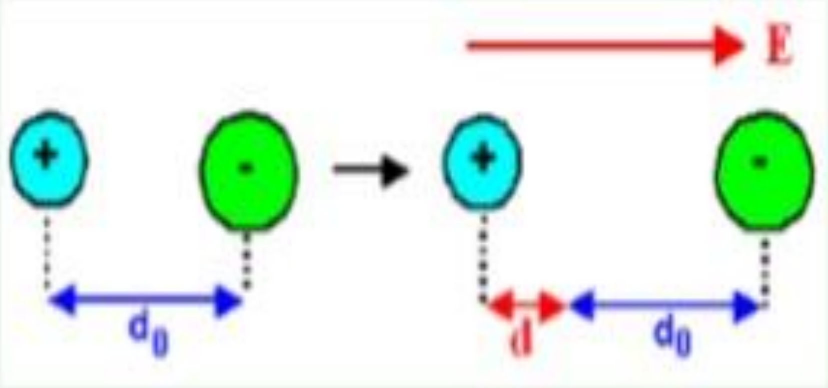


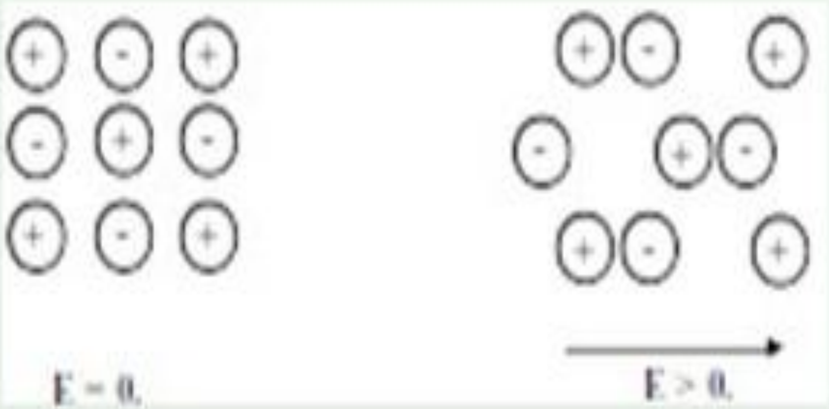
FIGURE: Electronic Polarization

Link

Ionic Polarization

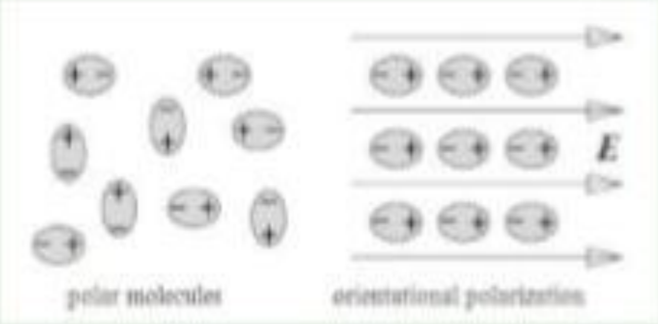
1. This occurs in ionic solids such as sodium chloride etc.
2. Ionic solids possess net dipole moment even in the absence of external electric field.
3. But when the external electric field is applied the separation between the ions further increases.
4. Hence the net dipole moment of the material also increases

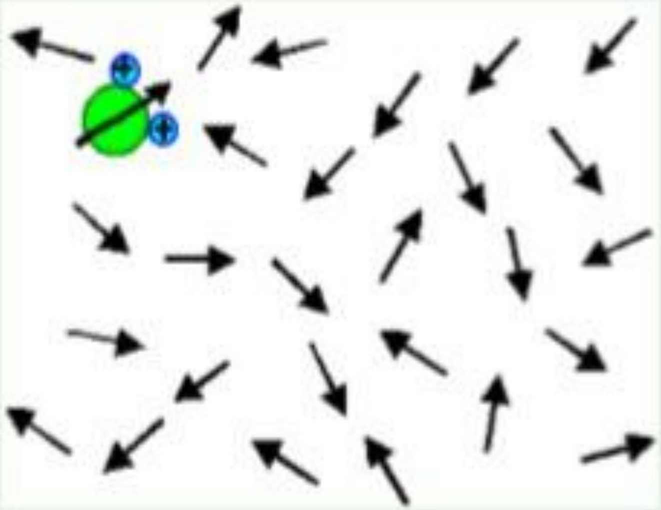




Orientation Polarization

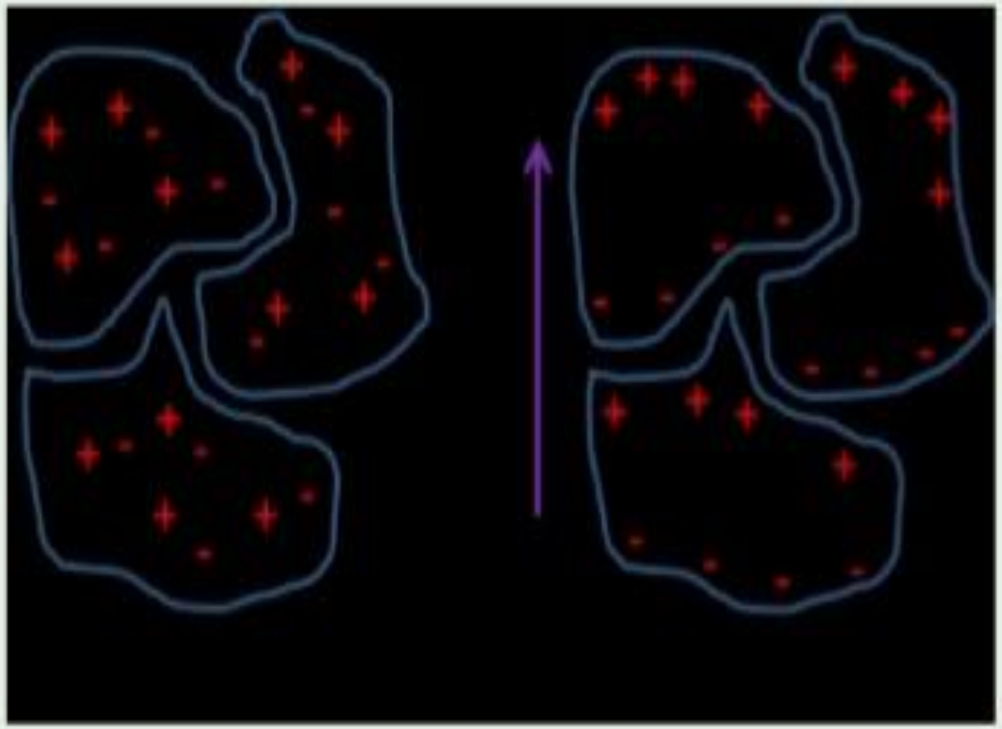
1. In polar dielectrics the permanent dipoles are randomly oriented due thermal agitation.
2. Therefore net dipole moment of the material is zero.
3. But when the external electric field is applied all dipoles tend to align in the field direction.
4. There fore dipole moment develops across the material. It is temperature dependent.





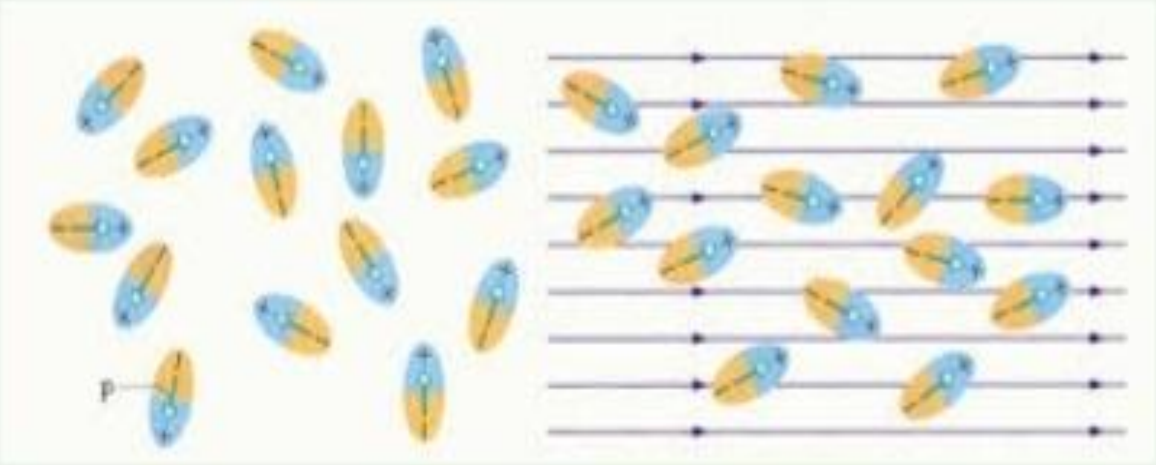
Space Charge Polarization

1. This involves limited movement of charges resulting in alignment of charge dipoles under applied field.
2. This usually happens at the grain boundaries or lattice defects and localized charge is set up.
3. The contribution of Space Charge Polarization to the net polarization is very small and hence it can be neglected.



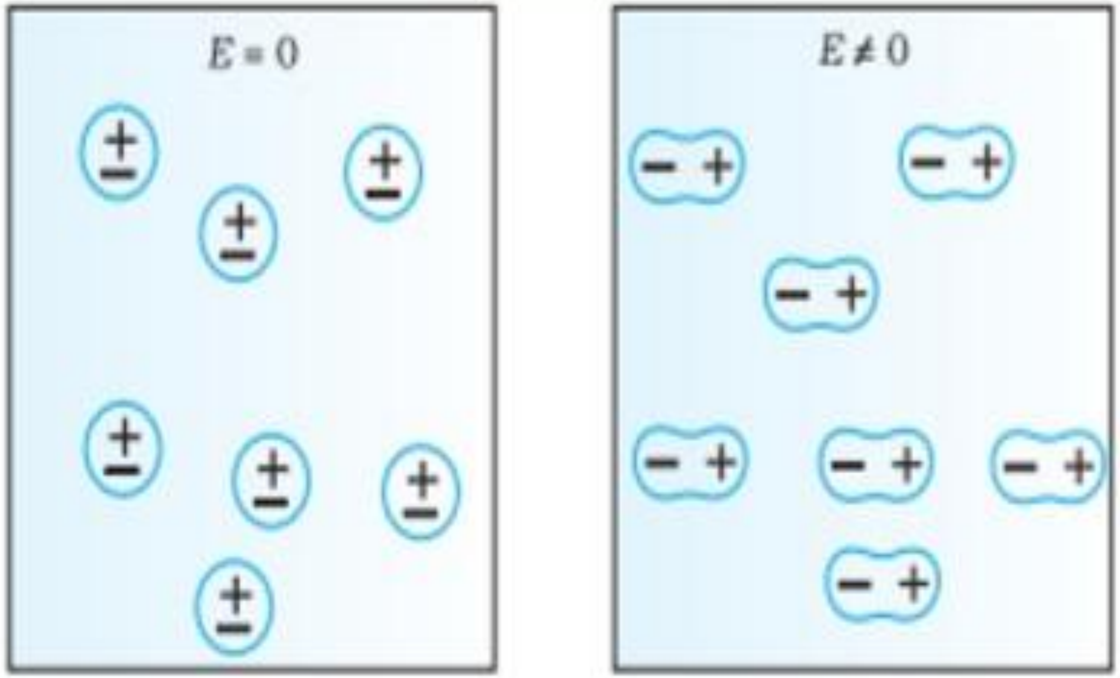
Classification of Dielectrics : Polar Dielectrics

Polar dielectric materials possess permanent electric dipoles in the material and are oriented in random directions so that the net dipole moment of the material is zero in the absence of applied electric field. If polar dielectric materials are placed in the external electric field then all dipoles tend to align in the field direction and hence net dipole moment develops cross dielectric material. This is the polarization of polar dielectric materials. Materials like H_2O, NH_3 are the examples for polar dielectrics.



Classification of Dielectrics : Non-Polar Dielectrics

Non polar dielectric materials do not possess permanent electric dipoles. Thus the net dipole moment across the material is zero in the absence of external electric field. In non polar dielectric materials dipoles are induced due to the applied electric field which results in the net dipole moment in the dielectric material in the direction of the applied field. This is the polarization of non-polar dielectric materials.Elementary gasses like He, H_2 are the examples for non polar dielectrics.



(a) Non-polar molecules

Internal Field in Dielectrics

When a dielectric material is placed in the external electric field polarization occurs. Hence the net electric field at any point within the dielectric material is given by The sum of external field and the field due to all dipoles surrounding that point. This net field is called internal field or Local field.

Expression for Internal field in case of one dimension

Consider a dielectric material placed in the external electric field E. Consider an array of dipoles in the polarized dielectric material as shown in the figure 13. Let a be the distance between to successive dipoles in the array.

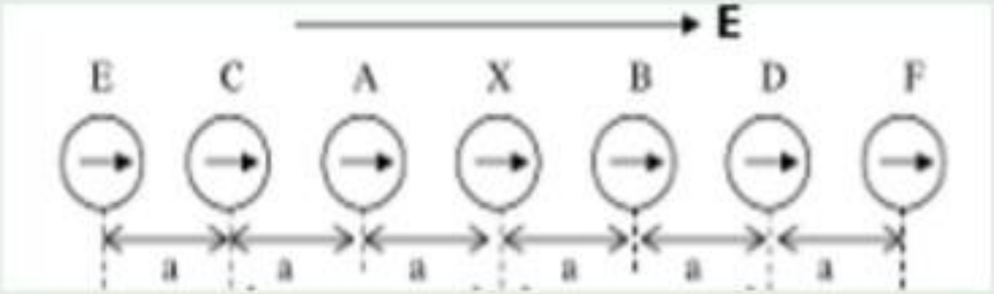


FIGURE: One Dimensional Array of Dipoles

Expression for Internal field in case of one dimension

The internal field at a dipole 'X' due to all dipoles in the array is given by

$$E_i = E + \frac{1.2\mu}{\pi\epsilon_0 a^3} \qquad (1)$$

Here *mu* is the dipole moment, ϵ_0 is the permittivity of free space, a is the inter dipole distance in the array.

$$E_i = E + \frac{1.2\alpha E}{\pi\epsilon_0 a^3} \qquad (2)$$

here α is polarizability. Since α, ϵ_0 and a are positive quantities the local field $E_i > E$.

Expression for Internal field in case of three dimension and Lorentz Field

For three dimension $\frac{1}{a^3}$ could be replaced with number of dipoles per unit volume N and $\frac{1.2}{\pi}$ could be replaced with γ, in equation 2. Thus we get

$$E_i = E + \frac{\gamma N \alpha E}{\epsilon_0} \tag{3}$$

$$E_i = E + \frac{\gamma P}{\epsilon_0} \tag{4}$$

Here the polarization $P = N\alpha E$. For an elemental solid dielectric material $\gamma = \frac{1}{3}$, Thus equation 4 becomes

$$E_L = E + \frac{P}{3\epsilon_0} \tag{5}$$

Thus the Lorentz field is given by equation 5.

Clausius - Mossotti Relation

Consider an Elemental solid dielectric material. Since they don't posses permanent dipoles, for such materials, the ionic and orientation polarizabilities are zero. Hence the polarization P is given by

$$P = N\alpha_e E_L \tag{6}$$

Here $E_L = E + \frac{P}{3\epsilon_0}$ is the Lorentz field. Substituting for Lorentz Field in 6, we get

$$P = N\alpha_e \left[E + \frac{P}{3\epsilon_0} \right]$$

$$P = N\alpha_e E + N\alpha_e \frac{P}{3\epsilon_0}$$

$$P - N\alpha_e \frac{P}{3\epsilon_0} = N\alpha_e E$$

Clausius - Mossotti Relation

$$P\left[1-\frac{N\alpha_e}{3\epsilon_0}\right]=N\alpha_e E$$

$$P=\frac{N\alpha_e E}{\left[1-\frac{N\alpha_e}{3\epsilon_0}\right]} \tag{7}$$

The relation between the Polarization and Electric field strength is given by

$$P=\epsilon_0\left(\epsilon_r-1\right)E \tag{8}$$

equating equations 7 and 8

$$\frac{N\alpha_e E}{\left[1-\frac{N\alpha_e}{3\epsilon_0}\right]}=\epsilon_0\left(\epsilon_r-1\right)E$$

Clausius - Mossotti Relation

$$\frac{N\alpha_e}{\epsilon_0(\epsilon_r-1)} = \left[1-\frac{N\alpha_e}{3\epsilon_0}\right]$$

$$\frac{N\alpha_e}{\epsilon_0(\epsilon_r-1)} + \frac{N\alpha_e}{3\epsilon_0} = 1$$

$$\frac{N\alpha_e}{3\epsilon_0}\left[\frac{3}{(\epsilon_r-1)}+1\right] = 1$$

$$\frac{N\alpha_e}{3\epsilon_0}\left[\frac{\epsilon_r+2}{\epsilon_r-1}\right] = 1$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r-1}{\epsilon_r+2} \tag{9}$$

Equation 9 is called Clausius-Mossotti relation.

Solid Dielectrics

Solid dielectrics are the solids which exhibit polarization and are commonly used. Some examples include porcelain, glass, and most plastics. Some applications of solid dielectrics are

1. Specially processed dielectrics called electrets exhibit spontaneous polarization. They are the electric equivalent of magnets.
2. Some dielectrics can generate a potential difference when subjected to mechanical stress. This property is called piezo-electricity.
3. Some solid dielectrics exhibit ferro-electricity by retaining dipole moment which switches accordingly with the direction of applied field.
4. Paper impregnated with electrolyte is a dielectric used in electrolytic capacitor.
5. Synthetic material filled by natural substances, such as glass and rubber posses specific properties like good strength, hardness, resistance to chemical attack.
6. Industrial coatings such as parylene provide a dielectric barrier between the substrate and its environment.

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Liquid Dielectrics

A liquid dielectric is a dielectric material in liquid state. Its main purpose is to prevent or rapidly quench electric discharges and to keep the system cool. Liquid dielectrics perform a number of functions simultaneously, namely

1. Dielectric liquids are used as electrical insulators in high voltage applications.
2. Insulating oils are used in power instruments, transformers, power cables, circuit breakers and power capacitors.
3. Liquid impregnated in thin layers of paper or other materials are used in transformers and capacitors. For eg. wax or varnish.
4. Mineral oils circulate in transformers and oil filled cables and keep it cool through convection.
5. Filling up of the voids to form an electrically stronger integral part of a composite dielectric
6. They are used for arc extinction in circuit breaker

Gas Dielectrics

A dielectric gas, or insulating gas, is a dielectric material in gaseous state. Its main purpose is to prevent or rapidly quench electric discharges.

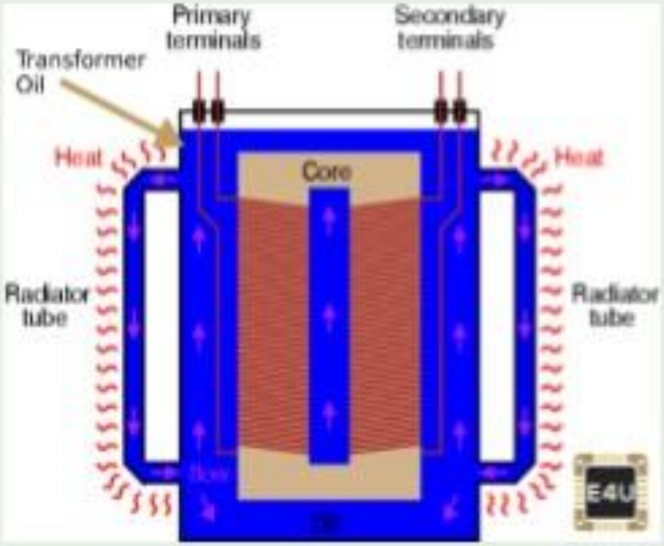
1. Dielectric gases are used as electrical insulators in high voltage applications.
2. Three most commonly used gas dielectrics are air, nitrogen and sulfur hexafluoride.
3. Dielectric gases can also serve as coolants. Vacuum is an alternative for gas in some applications.

Dielectric constants of some useful dielectrics are as given in the table 12.

Dielectric	ϵ_r
Dry Air	1.006
Water	80
Glass	3-7
Mica	2.5 - 7

Application of Dielectrics in Transformers

- A transformer posses two insulated conducting coils wound on a core. Insulation is also provided to the core. Insulation is provided between wingdings, coils and also between core and coils. Mica, paper and cloth are used for insulation purposes. Paper is impregnated with wax or varnish to fill the air gaps.
- Transformer oil or insulating oil is stable at high temperatures and has excellent electrical insulating properties. It is used in oil-filled transformers.
- Either its Natural (Mineral oil) or Synthetic substance (silicone oil or organic esters). Its functions are to insulate (both Thermally and electrically), suppresses corona discharge, arcing and to serve as a coolant.



Problem No. 1

The electron and hole mobilities of silicon are $0.14 m^2 V^{-1} s^{-1}$ and $0.05 m^2 V^{-1} s^{-1}$ respectively at a certain temperature. If the electron concentration in silicon is $1.5 \times 10^{16} electrons/m^3$ calculate the resistivity of silicon.

DATA:

Silicon is intrinsic Semiconductor Electron Mobility $= 0.14 m^2 V^{-1} s^{-1}$

Hole Mobility $= 0.05 m^2 V^{-1} s^{-1}$

Electron concentration $=$ $1.5 \times 10^{16} electrons/m^3$

for Intrinsic semiconductor $N_e = N_h = N_i$

CALCULATE :

Resistivity of Silicon $=$?

FORMULA :

$$\sigma = \frac{1}{\rho} = n_i\ e(\mu_e + \mu_h)$$

$$\rho = \frac{1}{n_i\ e(\mu_e + \mu_h)}$$

CALCULATION :

$$\rho = \frac{1}{1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.14 + 0.05)}$$

$$\rho = 2193 \Omega m$$

Problem No. 2

The resistivity of silicon at $27C$ is $3000\Omega m$. Assuming electron and hole mobilities of 0.17 and 0.35 $m^2V^{-1}s^{-1}$ respectively, calculate the intrinsic carrier concentration at $27C$.

DATA:

Silicon is intrinsic Semiconductor

resistivity of silicon at $27^0C = 3000\Omega m$

Electron Mobility $= 0.17m^2V^{-1}s^{-1}$

Hole Mobility $= 0.35m^2V^{-1}s^{-1}$

CALCULATE :

intrinsic carrier concentration at 27^0C =?

FORMULA :

$$\sigma = \frac{1}{\rho} = n_i\, e(\mu_e + \mu_h)$$

$$n_i = \frac{1}{\rho\, e(\mu_e+\mu_h)}$$

CALCULATION :

$$n_i = \frac{1}{3000\times1.6\times10^{-19}(0.17+0.35)}$$

$$n_i = 4.006 \times 10^{15}\ m^{-3}$$

Problem No. 3

The mobilities of holes and electrons in an extrinsic germanium at 300K is given by 0.2 $m^2V^{-1}s^{-1}$ and 0.36 $m^2V^{-1}s^{-1}$. If the electron concentration and hole concentrations are $1.8 \times 10^{20} electrons/cm^3$ and $1.6 \times 10^5 electrons/cm^3$ estimate the resistivity of germanium at room temperature.Is it p-type or n-type?

DATA:

Extrinsic Germanium

Electron Mobility $= 0.36\ m^2V^{-1}s^{-1}$

Hole Mobility $= 0.2\ m^2V^{-1}s^{-1}$

Electron concentration $=$ $1.8 \times 10^{20} electrons/cm^3$

Hole concentration $= 1.6 \times 10^5 electrons/cm^3$

CALCULATE :

Resistivity of Germanium $= \rho = ?\ \Omega m$

P-Type or N-Type $= ?$

FORMULA :

$$\sigma = \frac{1}{\rho} = e(N_e\mu_e + N_h\mu_h)$$

$$\rho = \frac{1}{e(N_e\mu_e + N_h\mu_h)}$$

CALCULATION :

$$\rho = \frac{1}{1.6\times10^{-19}(0.36\times1.8\times10^{26}+0.2\times1.6\times10^{11})}$$

$$\rho = 1.73 \times 10^{-7}\ \Omega m$$

Since $N_e > N_h$ it is N-Type Germanium.

Problem No. 4

Calculate the concentrations at which the acceptor atoms must be added to a germanium sample to get a p-type semiconductor with conductivity 0.15 per ohm-m. Given the mobility of holes $0.17 m^2 V^{-1} s^{-1}$.

DATA:

Its P-type Germanium,

Electrical Conductivity $= 0.15\ \Omega^{-1} m^{-1}$

Mobility of holes $= 0.17 m^2 V^{-1} s^{-1}$

CALCULATE :

Concentration of Acceptor atom $= N_a = ?$

For each acceptor atom one hole is created

hence $N_a = N_h$

$N_h = ?$

FORMULA :

$\sigma = \frac{1}{\rho} = e(N_e \mu_e + N_h \mu_h)$

Here $N_h \mu_h \gg N_e \mu_e$ and hence $N_e \mu_e$ can be neglected.

$\sigma = e(N_h \mu_h)$

$N_h = \frac{\sigma}{e \mu_h}$

CALCULATION :

$N_a = N_h = \frac{0.15}{1.6 \times 10^{-19} \times 0.17} = 5.52 \times 10^{18} m^{-3}$

PROBLEM NO. 5

The conductivity and Hall co-efficient of an n-type silicon specimen are 112/ohm-m and $1.25 \times 10^{-3} m^3/C$, respectively. Calculate the charge carrier concentration and electron mobility.Ans. $N_e = 5 \times 10^{21} m^{-3}$, $\mu_e = 0.14$.

DATA:

Conductivity of n-type silicon $= 112\ \Omega^{-1} m^{-1}$

Hall Co-efficient $= 1.25 \times 10^{-3} m^3/C$

CALCULATE :

Charge Carrier Concentration= $N_e = ?$

Mobility of Electrons $= \mu_h = ?$

FORMULA :

$R_H = \frac{1}{N_e e}$ Therefore $N_e = \frac{1}{R_H\ e}$

$\sigma = e(N_e \mu_e)$ Therefore $\mu_e = \frac{\sigma}{e N_e}$

CALCULATION :

$$N_e = \frac{1}{1.25\times10^{-3}\times1.6\times10^{-19}} = 5 \times 10^{21} m^{-3}$$

$$\mu_e = \frac{112}{1.6\times10^{-19}\times5\times10^{21}} = 0.14 m^2 V^{-1} s^{-1}$$

Numerical Problems

Problem No. 6

A rectangular plane sheet of semiconductor material has dimensions 2cm along Y-direction and 1mm along Z-direction. Hall probes are attached on its surfaces parallel to X-Y plane and a magnetic field of flux density $1 weber/m^2$ is applied along Z-direction. A current of 3 mA is flowing in it in the X-direction. Calculate the hall voltage measured by the probe, if the hall co-efficient of the material is $3.66 \times 10^{-4}/coulomb$.

DATA:

$B = 1 weber\ m^2$, Thickness d (y) $= 2 \times 10^{-2} m$,

Breadth b (Z)$= 1 \times 10^{-3} m$,

$R_H = 3.66 \times 10^{-4} coulomb^{-1}$, $I = 3 \times 10^{-3} A$

CALCULATE :

$V_H = ?$ V

FORMULA :

$V_H = E_H d = R_H B J d$, $J = \frac{I}{A}$, $A = bd$

CALCULATION :

$A = 2 \times 10^{-2} \times 1 \times 10^{-3} = 2 \times 10^{-5}$

$J = \frac{3\times10^{-3}}{2\times10^{-5}} = 150$ A m^{-2}

$V_H = 3.66 \times 10^{-4} \times 1 \times 150 \times 2 \times 10^{-2} = 1.098 \times 10^{-3}$ V

Activate

Problem No. 7

Determine the polarization produced in a crystal by an electric field of strength 6000 V/cm if it has a dielectric constant of 5.

DATA:

Electric field Strength E = 6000 V/cm = 6000 × 100 V/m

Dielectric $\epsilon_r = 5$

Permittivity of Free Space = ϵ_0 = $8.85 \times 10^{-12}\ Fm^{-1}$

CALCULATE :

Polarization = P = ? Coulomb m^2

FORMULA :

$P = \epsilon_0 (\epsilon_r - 1) E$

CALCULATION :

$P = 8.85 \times 10^{-12} \times (5-1) \times 6 \times 10^5$

$P = 2.124 \times 10^{-5} Cm^{-2}$

PROBLEM NO. 8

An elemental solid dielectric material has polarizability $7 \times 10^{-40} Fm^{-2}$. Assuming the internal field to be Lorentz, calculate the dielectric constant for the material if the material has $3 \times 10^{28} atoms/m^3$.

DATA:

Polarizability $\alpha_e = 7 \times 10^{-40} Fm^{-2}$

No. of Dipoles per unit volume $=$ $3 \times 10^{28} atoms/m^3$

Assume Lorentz Field

CALCULATE :

Dielectric Constant $= \epsilon_r = ?$

FORMULA :

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

CALCULATION :

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{7 \times 10^{-40} \times 3 \times 10^{28}}{3 \times 8.85 \times 10^{-12}}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = 0.791$$

$$\epsilon_r = \frac{2.582}{0.209} = 12.35$$

Numerical Problems

PROBLEM NO. 9

The dielectric constant of Sulphur is 3.4. Assuming a cubic lattice for its structure, calculate the electronic polarizability of Sulphur. Given: for sulphur density = 2.07 gm/cc, and atomic weight 32.07.

DATA:

Dielectric Constant of Sulphur = $\epsilon_r = 3.4$

Density of Sulphur = 2.07 gm/cc

$= 2.07 \times \frac{10^{-3}}{(10^{-2})^3} = 2070\ kg/m^3$

Atomic Weight = A = 32.07

CALCULATE :

Number of Dipoles per unit Volume = N = m^{-3}

Polarizability of Sulphur $\alpha_e = ?$

FORMULA :

For Cubic Lattice elemental solid Dielectric

$$N = \frac{N_A D}{A}$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\alpha_e = \frac{3\epsilon_0}{N}\left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)$$

CALCULATION :

$$N = \frac{6.023 \times 10^{26} \times 2070}{32.07} = 3.89 \times 10^{28}\ m^{-3}$$

$$\alpha_e = \frac{3 \times 8.85 \times 10^{-12}}{3.89 \times 10^{28}}\left(\frac{3.4-1}{3.4+2}\right)$$

$$\alpha_e = 3.035 \times 10^{-40}\ Fm^{-2}$$

Activate

Problem No. 10

A solid contains $5 \times 10^{28} atoms/m^3$ each with a polarizability of $2 \times 10^{-40} Fm^2$. Assuming that the internal field is given by Lorentz formula. Calculate the ratio of internal field to the external field. Given $\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$.

DATA:

No. of atomic dipoles per unit volume =
$5 \times 10^{28} atoms/m^3$

Polarizability $= \alpha = 2 \times 10^{-40} Fm^2$.

Permittivity of Free Space =
$\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$.

CALCULATE :

$\frac{E_L}{E} = ?$

FORMULA :

Lorentz Formula $= E_L = E + \frac{1.2N\alpha E}{\pi\epsilon_0}$

$$\frac{E_L}{E} = 1 + \frac{1.2N\alpha}{\pi\epsilon_0}$$

CALCULATION :

$$\frac{E_L}{E} = 1 + \frac{1.2 \times 5 \times 10^{28} \times 2 \times 10^{-40}}{3.142 \times 8.854 \times 10^{-12}}$$

$$\frac{E_L}{E} = 1 + 0.432 = 1.432$$

Superconductivity

Introduction

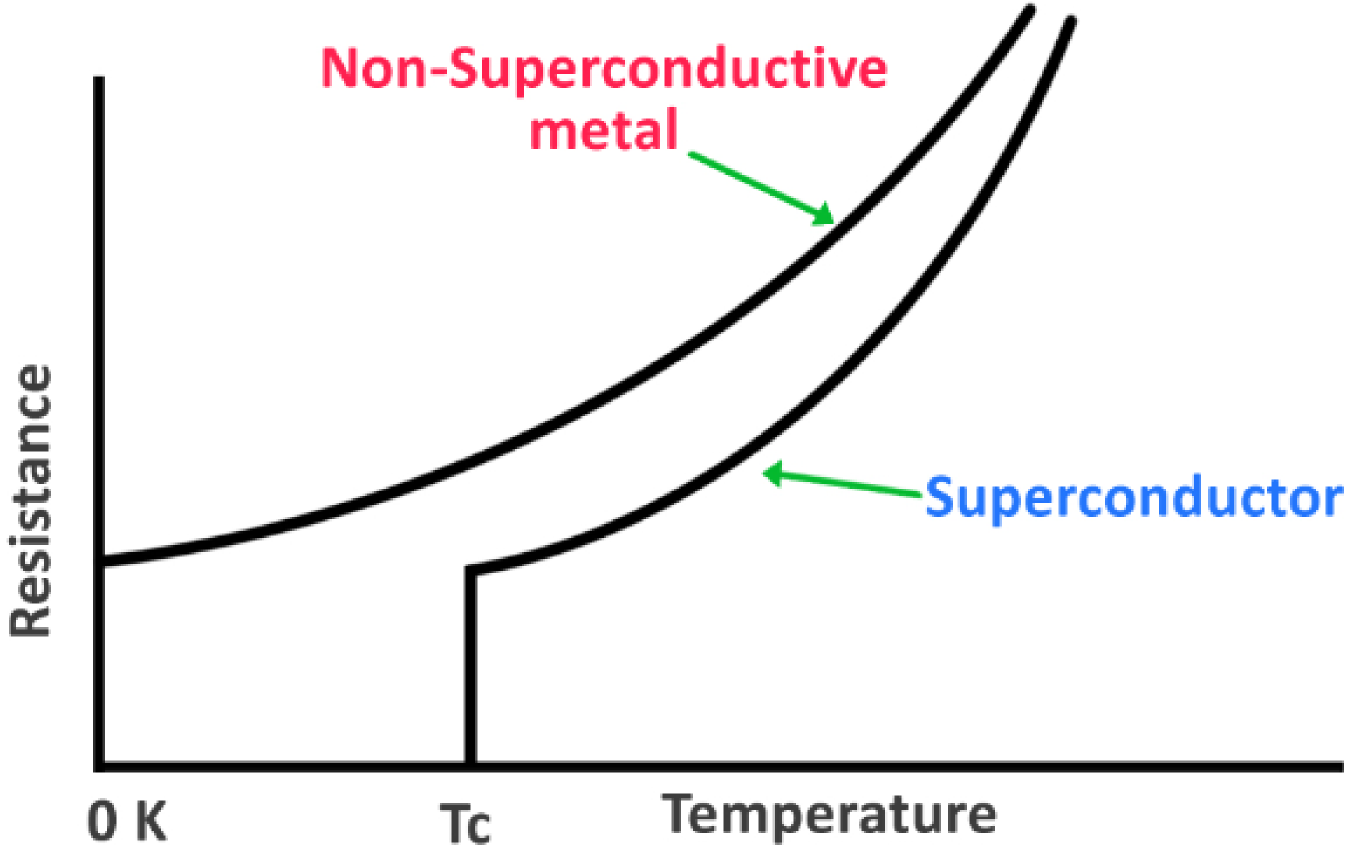
- Lord Kamerlingh Onnes discovered the phenomenon of superconductivity in the year 1911.
- he found that resistance of Mercury decreases with temperature with the decrease in temperature up to a particular temperature Tc = 4.15K .
- Below this temperature the resistance of mercury abruptly drops to zero. Between 4.15K and 0 *K* Mercury offered no resistance for the flow of electric current.
- This phenomenon is reversible and material becomes normal once again when temperature was increased above 4.15K.

Superconductivity

"The phenomenon in which resistance of certain metals, alloys and compounds drops to zero abruptly, below certain temperature is called superconductivity

Examples of superconductors include aluminium, niobium, magnesium diboride, yttrium barium and copper oxide.

Variation of Resistivity with Temperature

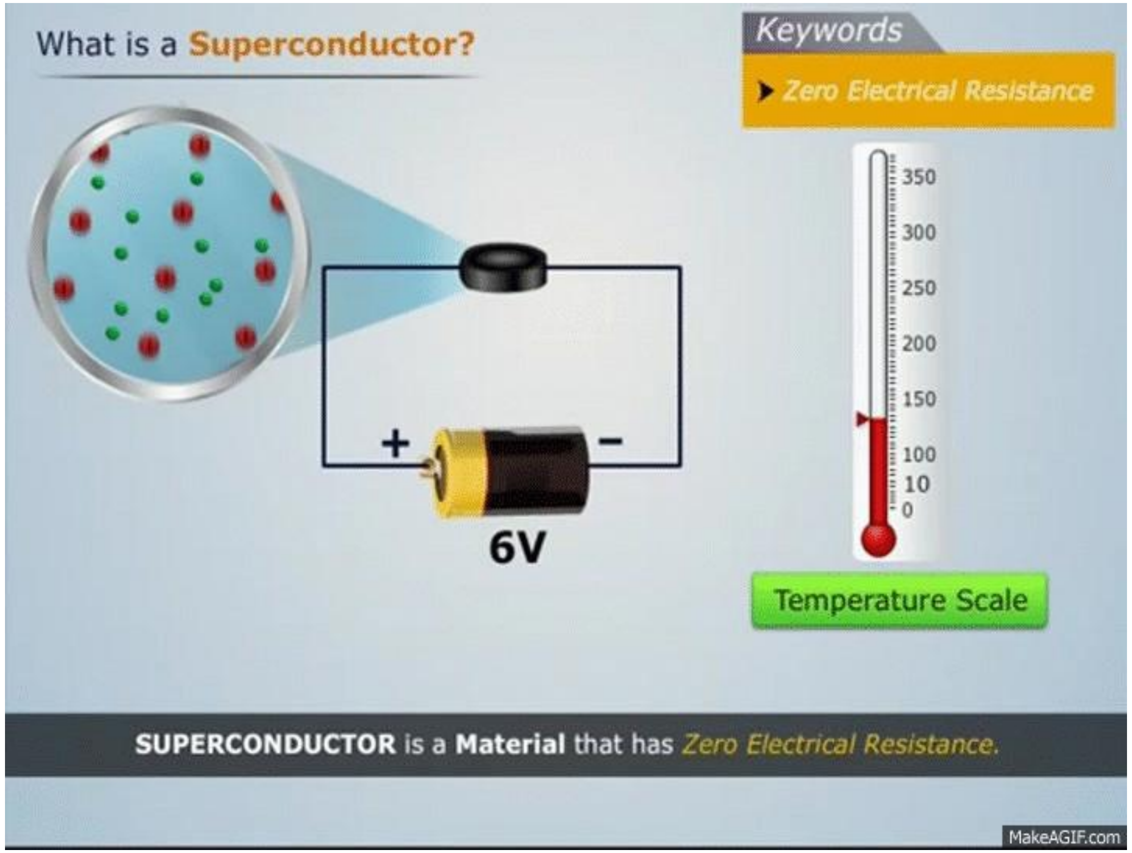


Critical Temperature :

The temperature, below which materials exhibit superconducting property is called critical temperature, denoted by *Tc*. Critical temperature *Tc* is different for different substances.

Above critical temperature material is said to be in normal state and offers resistance for the flow of electric current.

Below critical temperature material is said to be in superconducting state. Thus *Tc* is also called as transition temperature.



Critical field (Hc)

When superconductor is placed in a magnetic field it expels magnetic lines of force completely out of the body and becomes a perfect diamagnet.

If the strength of the magnetic field is further increased, it was found that for a particular value of the magnetic field, material looses its superconducting property and becomes a normal conductor.

The value of the magnetic field at which superconductivity is destroyed is called the Critical magnetic field, denoted by Hc .

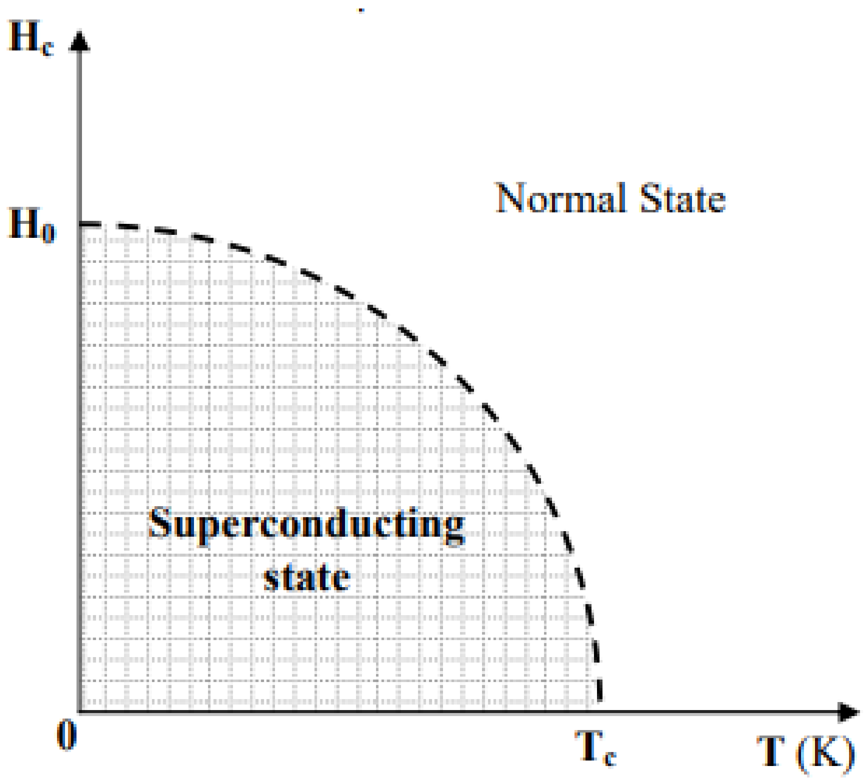
It was found that by reducing the temperature of the material further superconducting property of the material could be restored. Thus, critical field doesn’t destroy the superconducting property of the material completely but only reduces the critical temperature of the material.

Critical field

Critical magnetic field Hc depends on the temperature of the material. The relationship between the two is given by

$$H_c = H_0\left[1 - \frac{T^2}{T_c^2}\right]$$

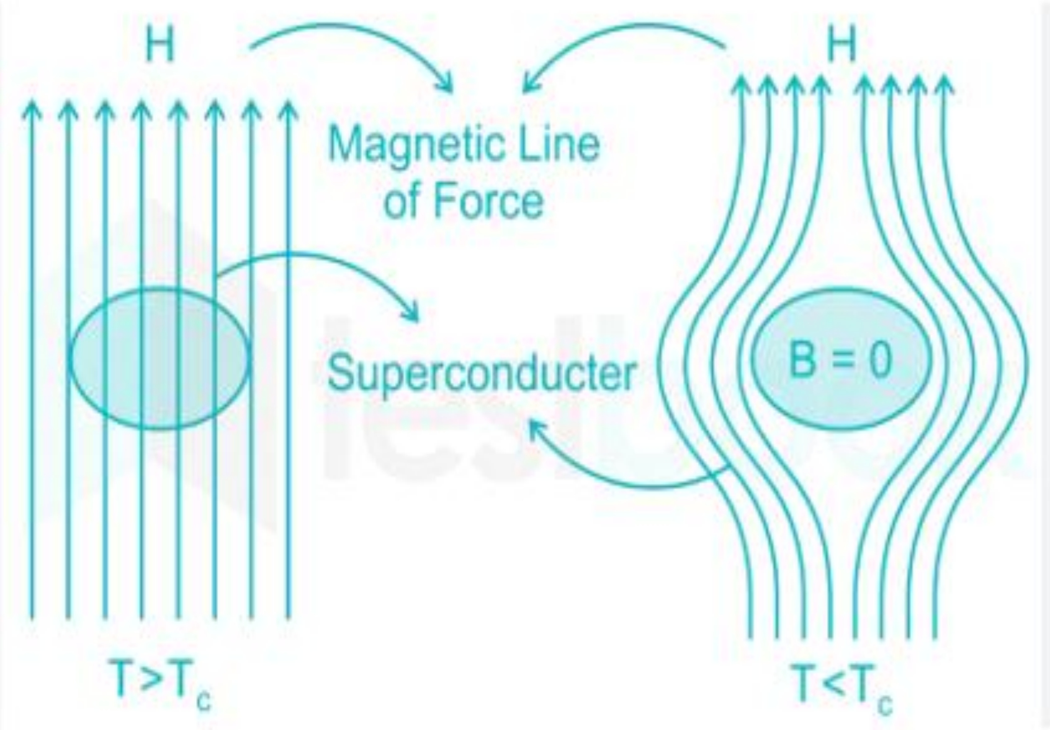
Here H_c is the Critical field at any temperature T less than T_c, H_0 is the Critical field at $T = 0K$.



Meissner's Effect

In 1933, Meissner and Ochsenfeld showed that when a superconducting material is placed in a magnetic field it allows magnetic lines of force to pass through, if it's temperature is above *Tc*.

If the temperature is reduced below the critical temperature Tc then it expels all the flux lines completely out of the specimen and exhibits perfect diamagnetism. This is known as Meissner's effect. Since superconductor exhibits perfect diamagnetism below the critical temperature Tc, magnetic flux density inside the material is zero.



Meissner's Effect

The expression for magnetic flux density is given by

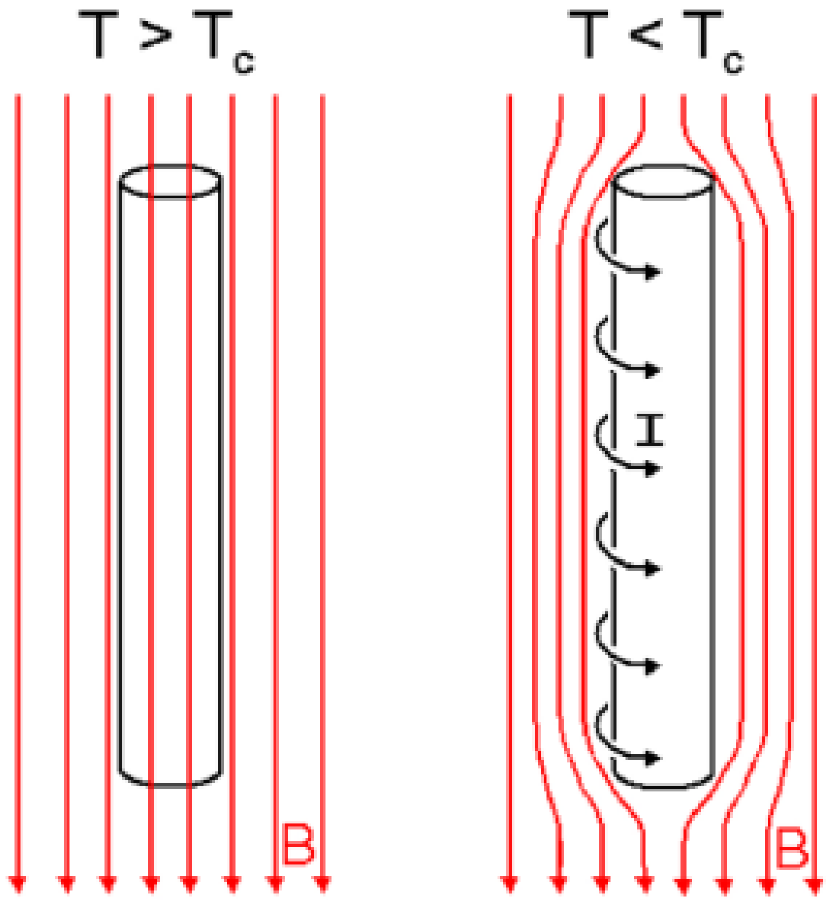
$$B = \mu_0 \, (M + H)$$

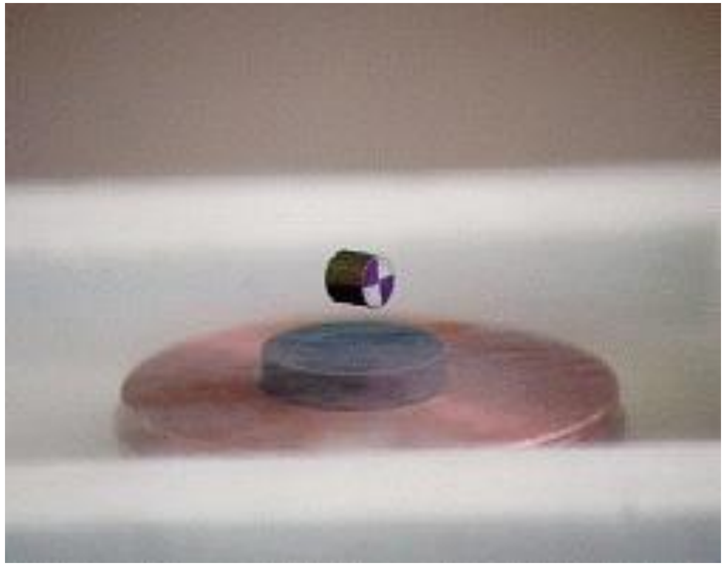
Here B is Magnetic Flux Density, M is Magnetization and H is the applied magnetic field strength.

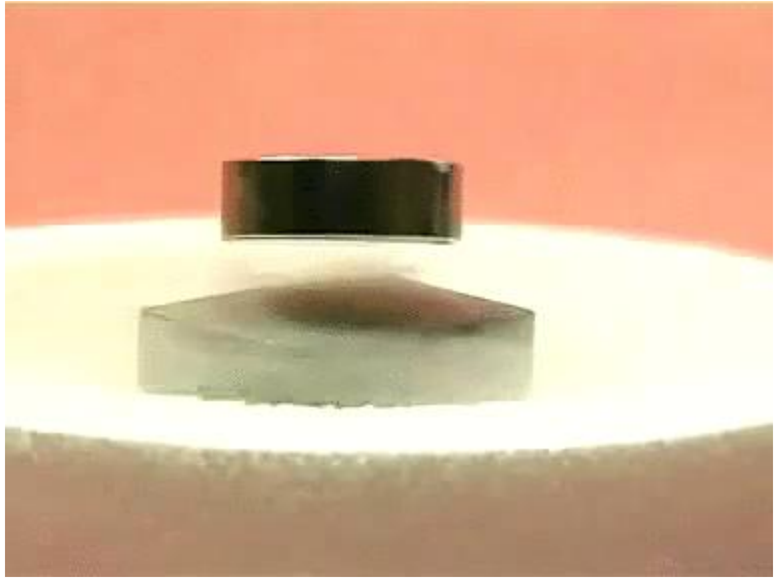
For a superconductor, $B = 0$ at $T < Tc$.

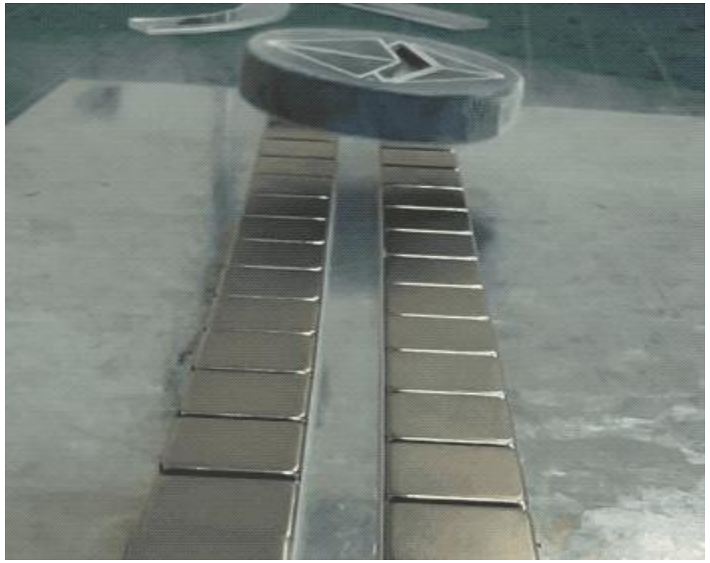
Thus we get $M = -H$

Thus Meissner's Effect signifies the negative magnetic moment associated with superconductors.







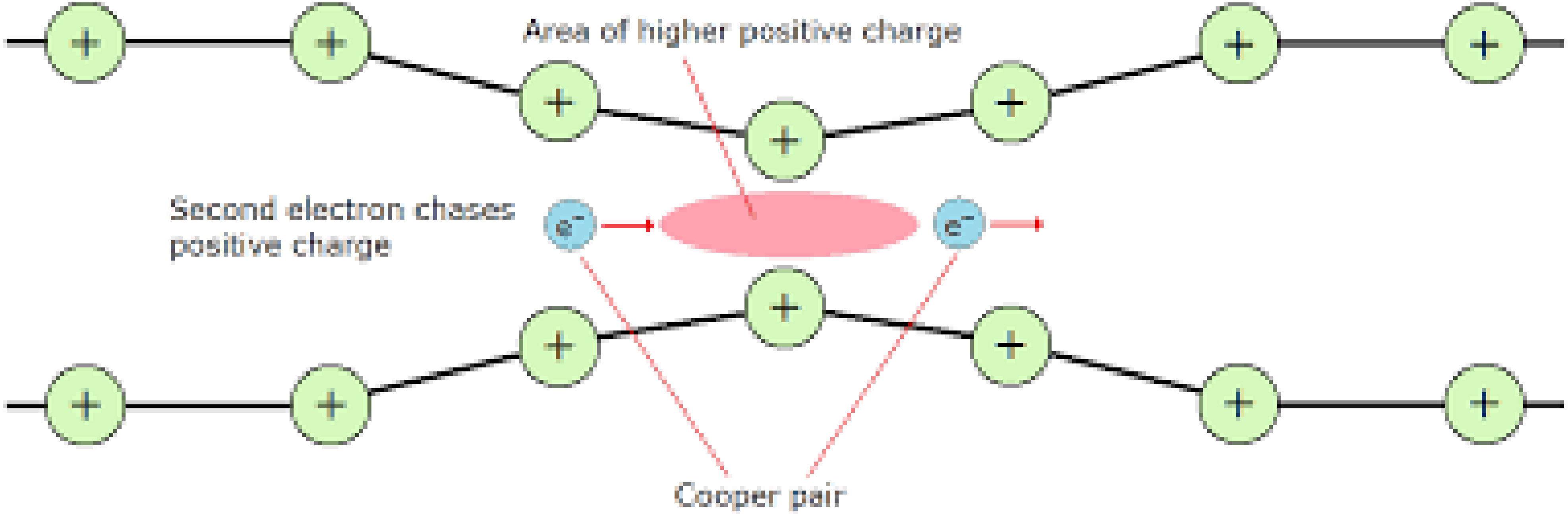


BCS theory of Superconductivity

- Bardeen, Cooper and Schrieffer explained the phenomenon of superconductivity in 1957.
- Resistance of the conductor is due to the scattering of electrons from the lattice ions.
- Consider an electron moving very close to a lattice ion. Due to coulomb interaction between electron and ion, the ion core gets distorted from its mean position-called lattice distortion.
- Now another electron moving close to this lattice ion interacts with it. This results in the reduction in the energy of the electron.
- This interaction can be looked upon as equivalent to the interaction between two electrons via lattice.

BCS theory of Superconductivity

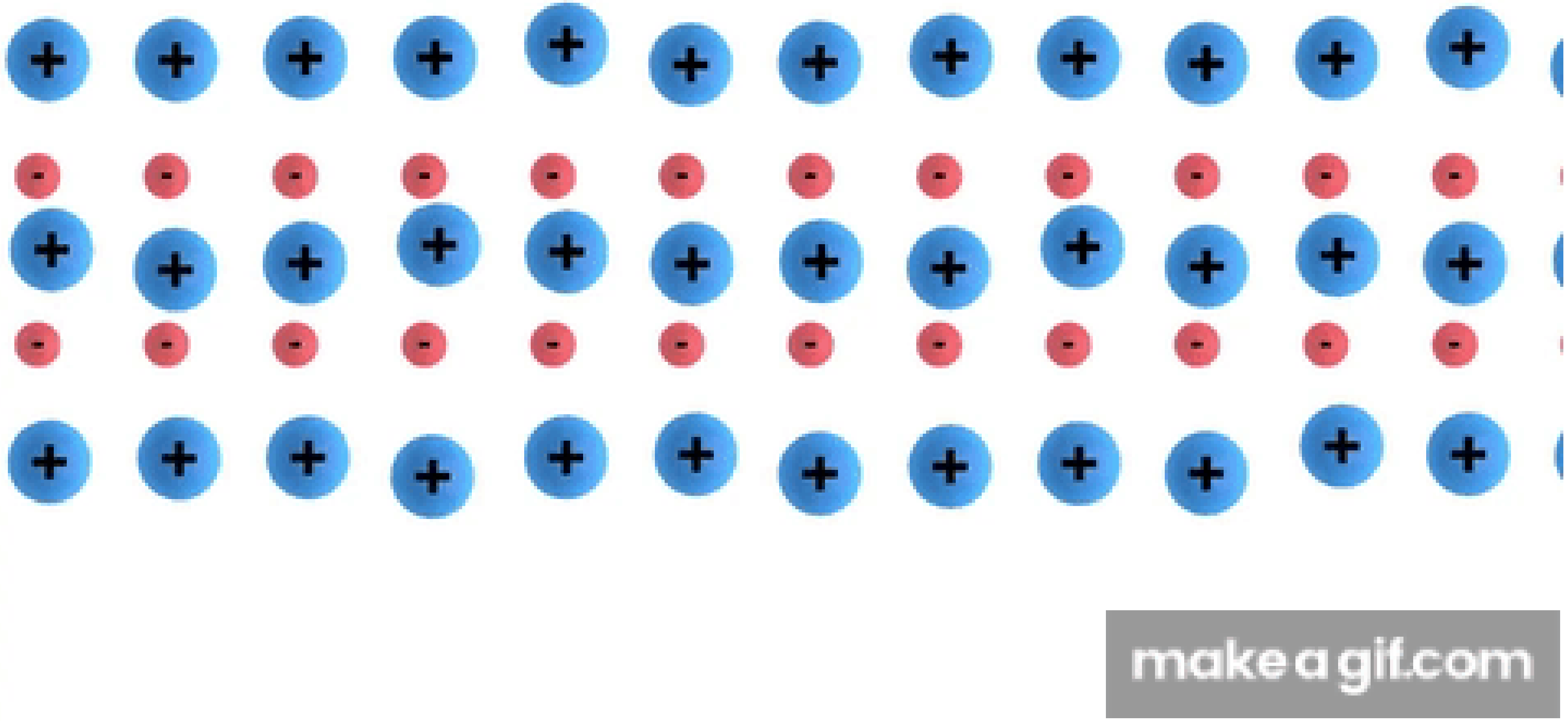
- During the interaction exchange of phonon takes place between electron and the lattice. This interaction is called electron-lattice –electron interaction via the phonon field.
- Because of the reduction in energy between the two electrons, an attractive force comes into effect between two electrons.
- **It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum.**
- The attractive force may exceed coulombs repulsive force between the two electrons below the critical temperature, results in the formation of bound pair of electrons called cooper pairs.


Area of higher positive charge
Second electron chases
positive charge
e⁻
e⁻
Cooper pair

BCS theory of Superconductivity

- At temperatures below the critical temperature large number of electron-Lattice-electron interaction takes place and all electrons form a cloud of cooper pairs.
- Cooper pairs in turn move in a cohesive manner through the crystal, which results in an ordered state of the conduction electrons without any scattering on the lattice ions.

This results in a state of zero resistance in the material.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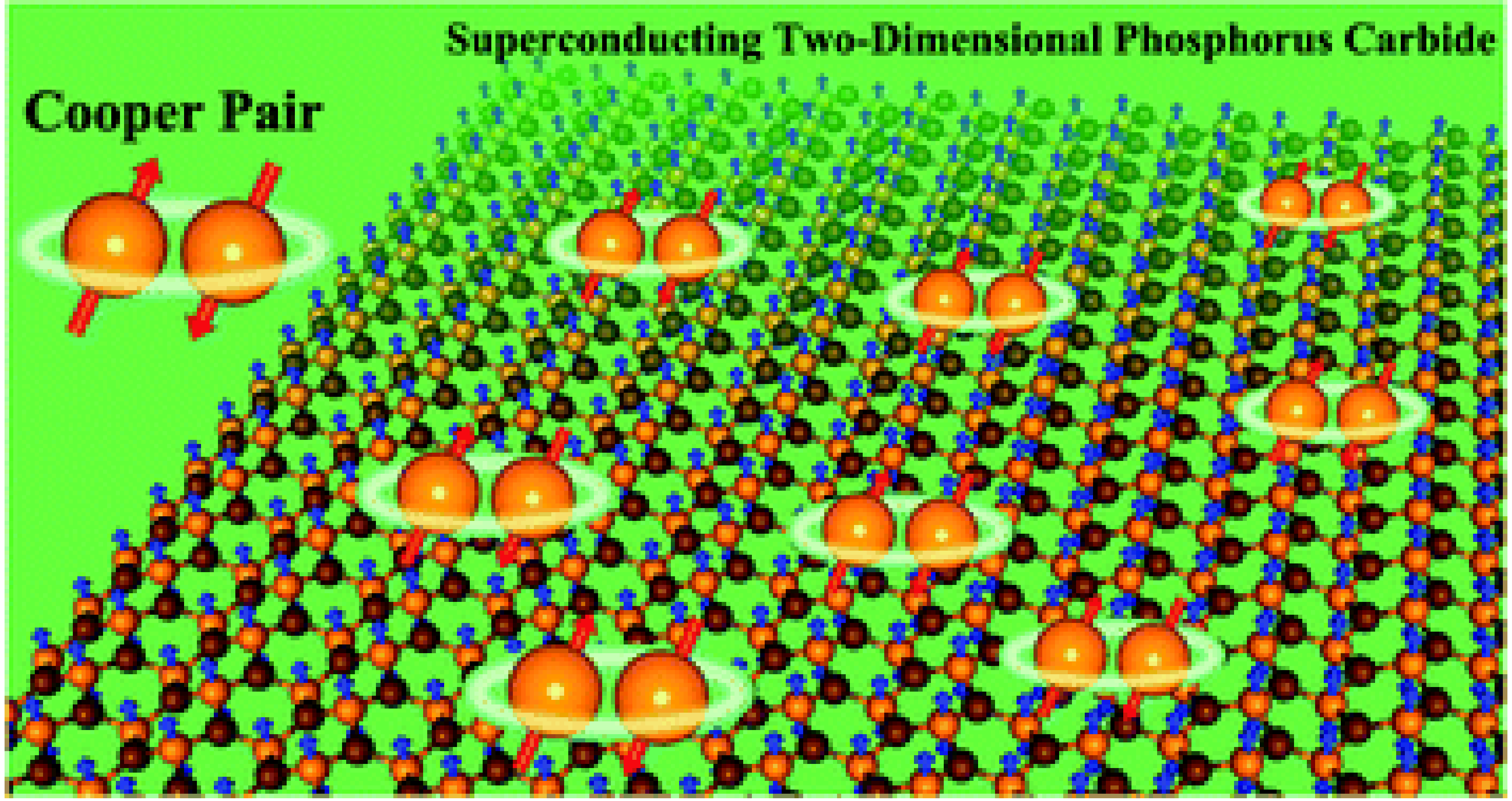
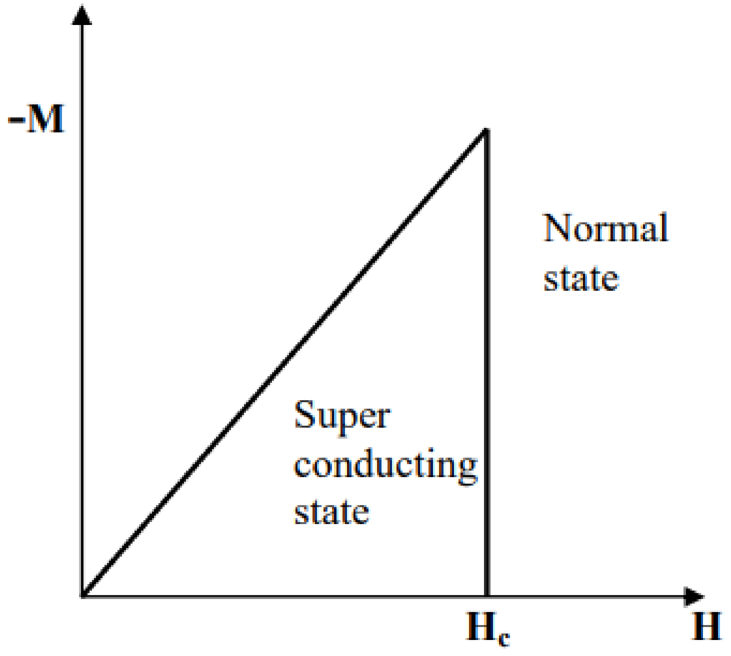

Superconducting Two-Dimensional Phosphorus Carbide
Cooper Pair

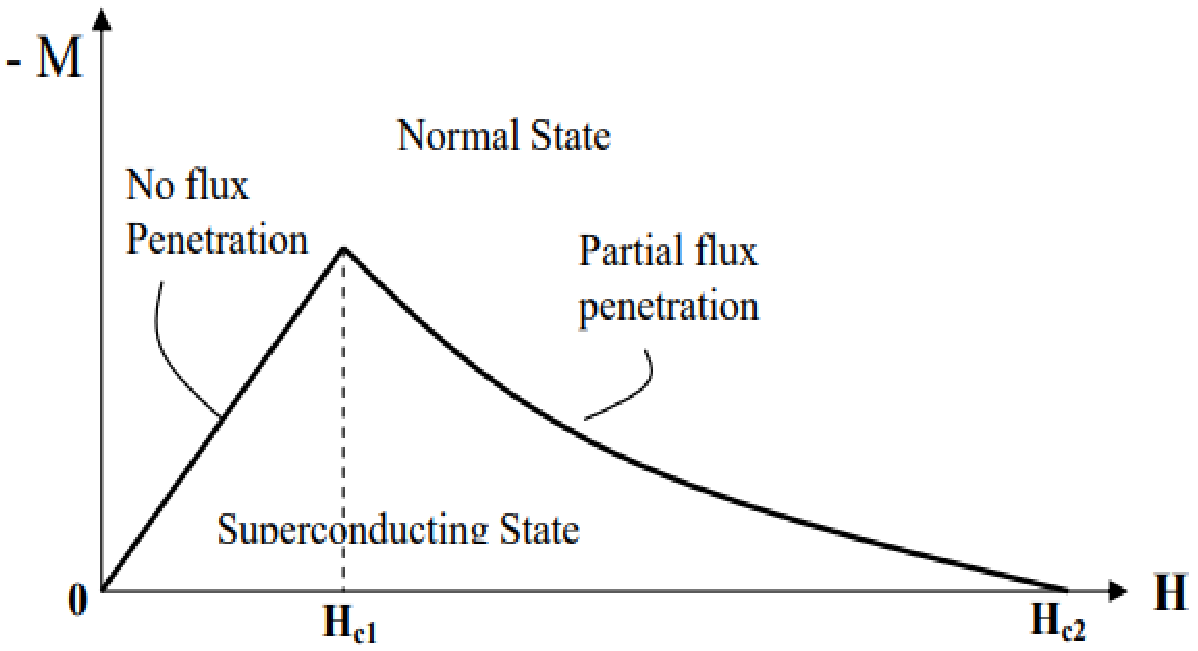
Type I Superconductors or Soft Superconductors

- ❑ Exhibit complete Meissner effect
- ❑ exhibit perfect diamagnetism. Therefore they possess negative magnetic moment.
- ❑ The graph of magnetic moment Vs magnetic field is as shown in the Fig.
- ❑ As field strength increases material becomes more and more diamagnetic until H becomes equal to Hc.
- ❑ At Hc, material losses both diamagnetic and superconducting properties to become normal conductor.
- ❑ It allows magnetic flux to penetrate through its body.
- ❑ soft superconductors cannot withstand high magnetic fields.



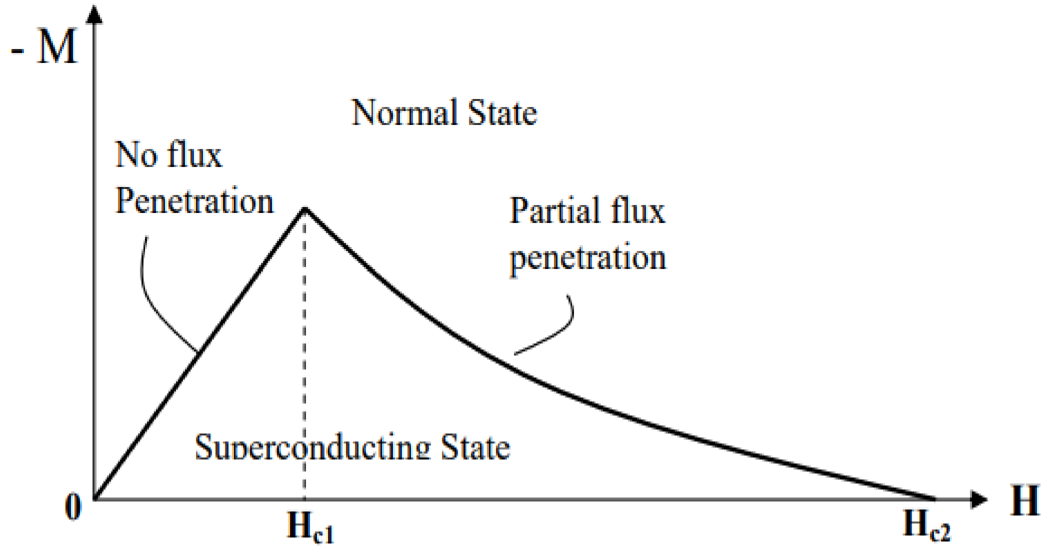
Type II Superconductors or Hard Superconductors

- Superconducting materials, which can withstand high value of critical magnetic fields, are called Hard Superconductors.
- The graph of magnetic moment Vs magnetic field is as shown in the Fig.
- Hard superconductors are characterized by two critical fields Hc1 and Hc2 .



Type II Superconductors or Hard Superconductors

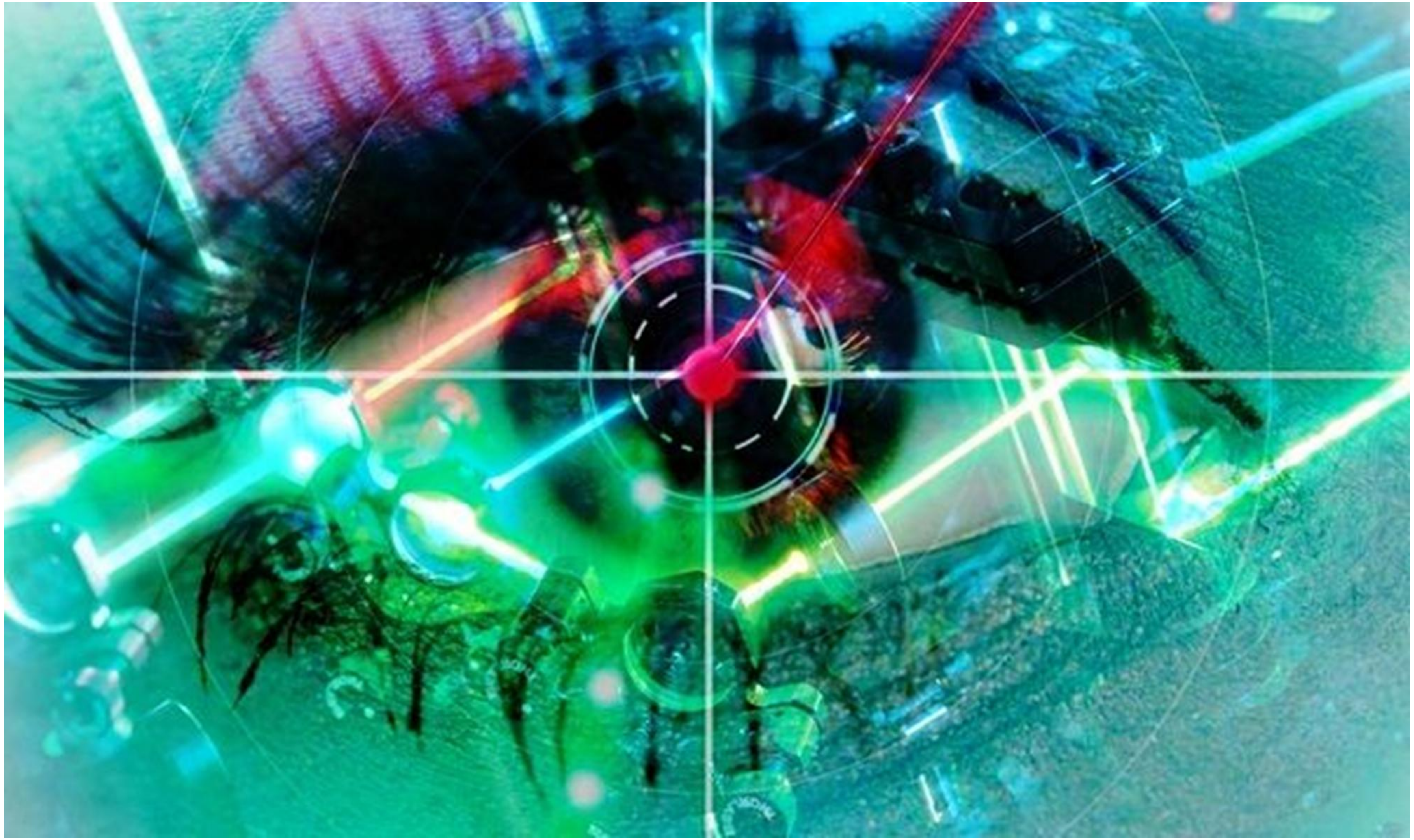
- When applied magnetic field is less than Hc1 material exhibits perfect diamagnetism.
- Beyond Hc1 flux penetrates and fills the body partially.
- At Hc2 flux fills the body completely and material losses its diamagnetic property as well as superconducting property completely.
- Between Hc1 and Hc2 material is said to be in vortex state
- The value of Hc2 is hundreds of times greater than Hc of soft superconductors. Therefore they are used for making powerful superconducting magnets.



TYPE I SUPERCONDUCTOR	TYPE II SUPERCONDUCTOR
They exhibit complete Meissner effect	They exhibit partial Meissner effect.
These are perfect diamagnetics	These are not perfect diamagnetics.
These are known as soft superconductors.	These are known as hard superconductors.
They have only one critical magnetic field.	They have two critical magnetic fields.
. These materials undergoes a sharp transition from the superconducting state of the normal state at the critical magnetic field.	These materials undergoes a gradual transition from the superconducting state to the normal state between the two critical magnetic fields.
Low Critical magnetic field	High Critical magnetic field
Applications are very limited.	They are used to generate very high magnetic field.
No mixed state exists	A mixed state exists
Examples:- lead, tin, mercury , etc.	Examples:- alloys like Nb-Sn, Nb-Ti, Nb- Zr, etc.

Module 3 (8 Hours)
Lasers and Optical Fibers: **Lasers**: Characteristics of LASER, Interaction of radiation with matter, Expression for Energy Density and its significance. Requisites of a Laser System. Conditions for Laser action. Principle, Construction and Working of Carbon Dioxide Laser. Application of Lasers in Defense (Laser range finder) and Laser Printing. Numerical Problems. **Optical Fibers:** Total Internal Reflection**,** Propagation mechanism, Angle of Acceptance, Numerical Aperture, Fractional Index Change, Modes of Propagation, Number of Modes and V Number, Types of Optical Fibers. Attenuation and Mention of Expression for Attenuation coefficient, Attenuation Spectrum of an Optical Fiber with Optical Windows. Discussion of Block Diagram of Point to Point Communication, Intensity based Fiber Optic Displacement Sensor, Merits and Demerits, Numerical problems. **Pre-requisite: Properties of light** **Self-learning: Total Internal Reflection**

LASERs..



L Light

A Amplification by

S Stimulated

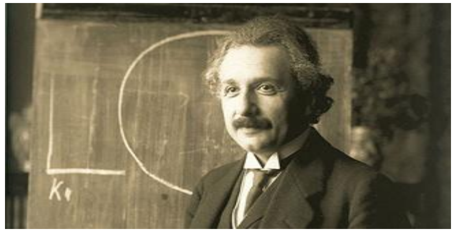
E Emission of

R Radiation

LASERs..

- Stimulated emission was first predicted by

  Albert Einstein

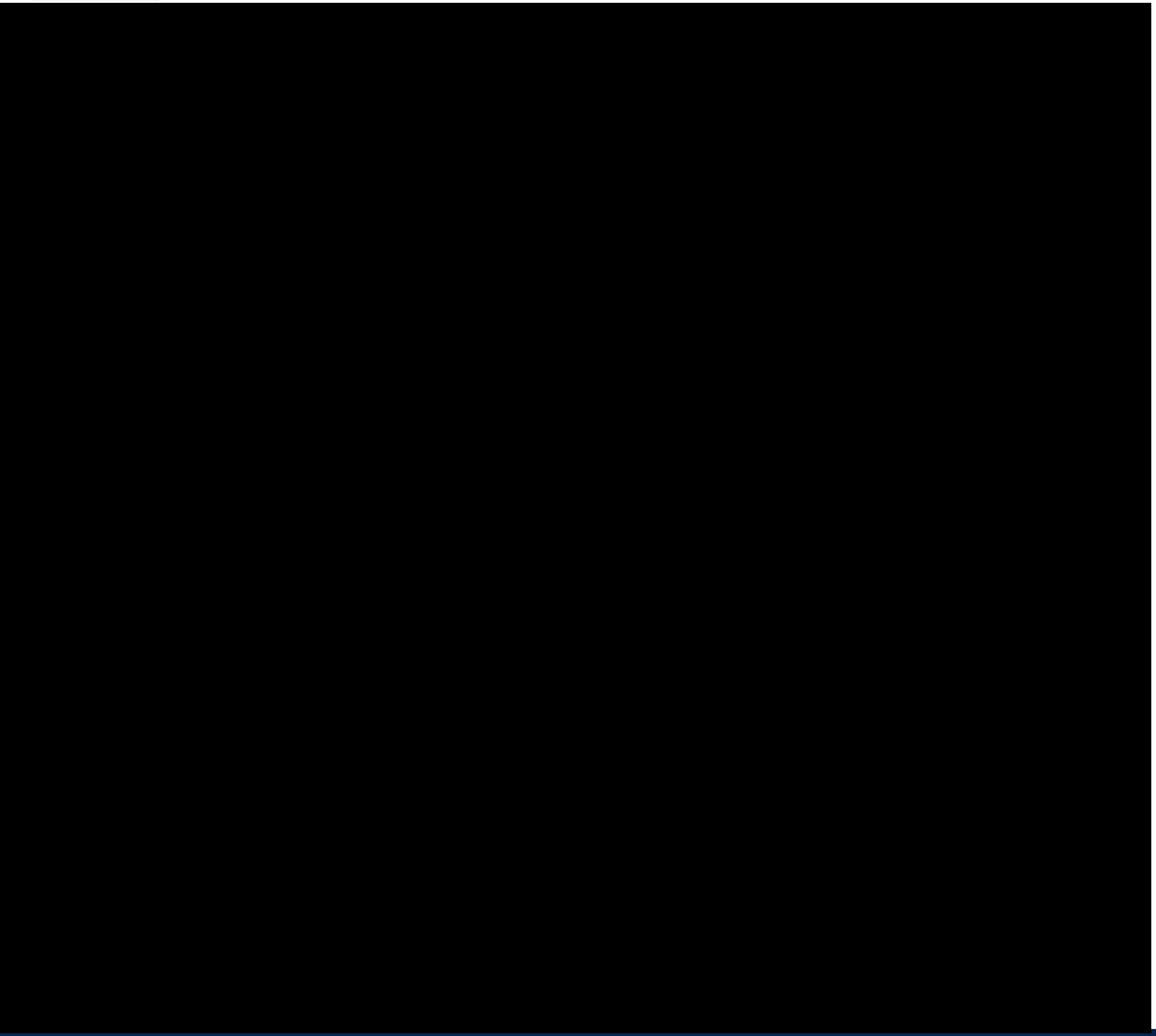


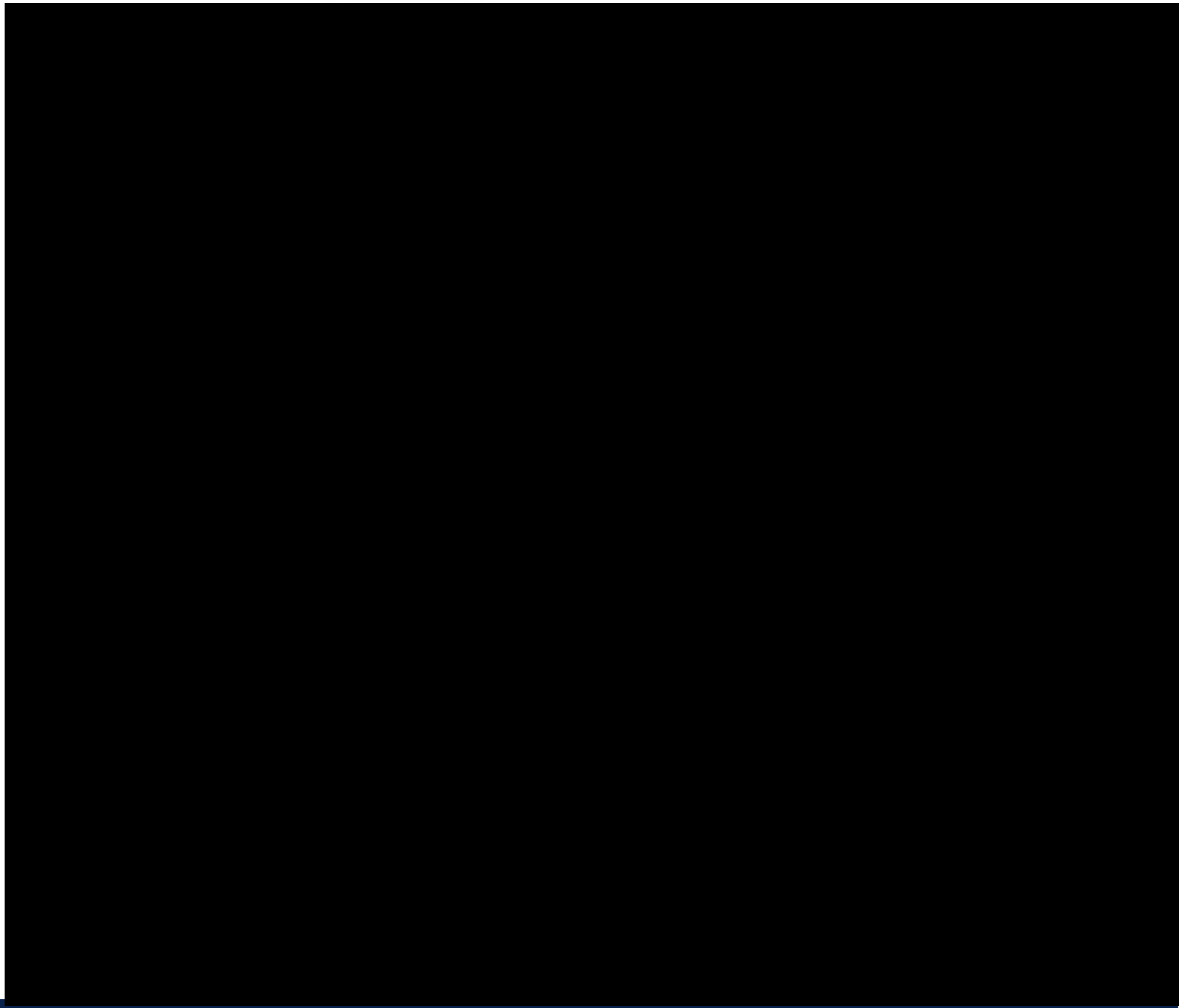
- The term LASER was used by

  Gordon Gould

- First LASER was developed in 1960

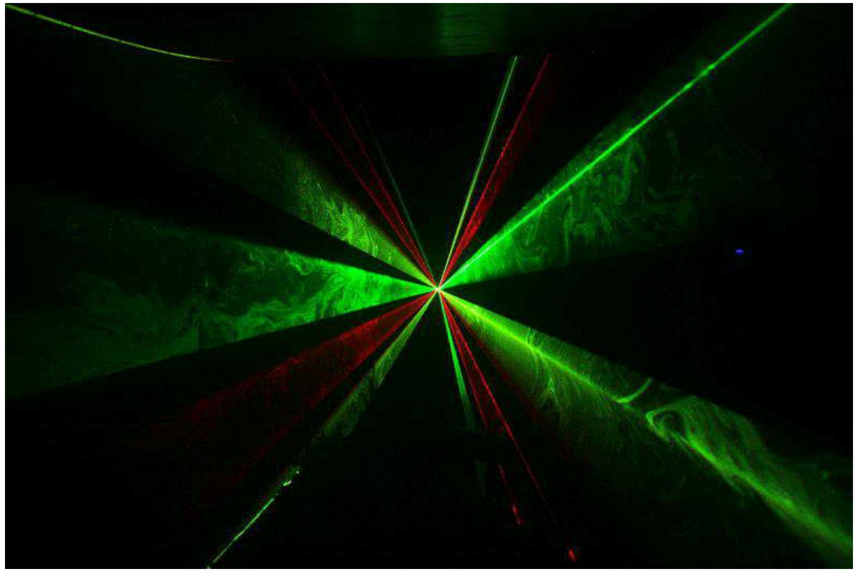
  Theodore Maiman





LASERs..

❖LASER is an optical device to produce Highly monochromatic, Highly coherent, Highly directed, Highly intense and Sharp focussing beam of light.



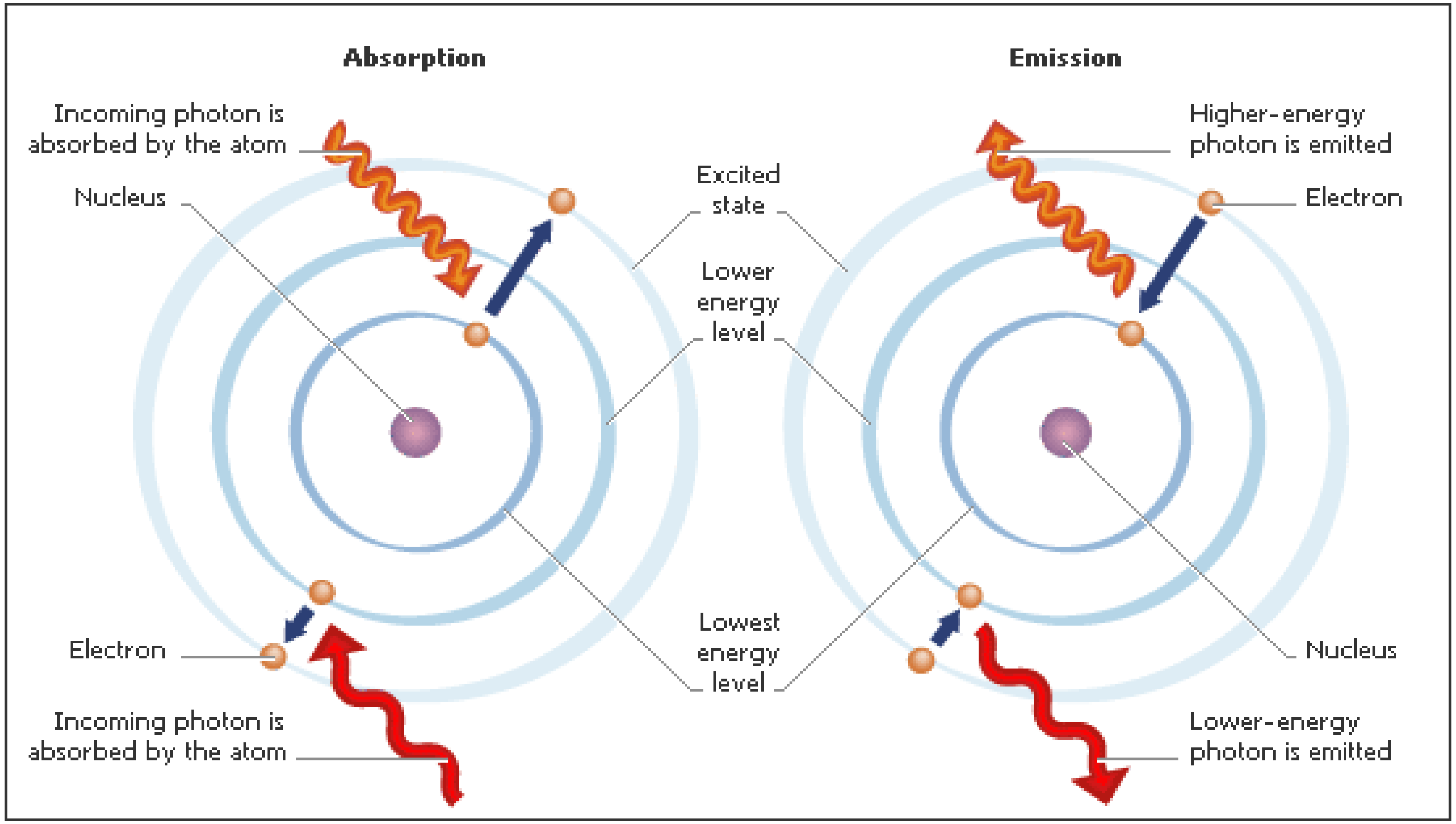
❖ ***LASER***

Acronym for ***L***ight ***A***mplification by ***S***timulated ***E***mission of ***R***adiation

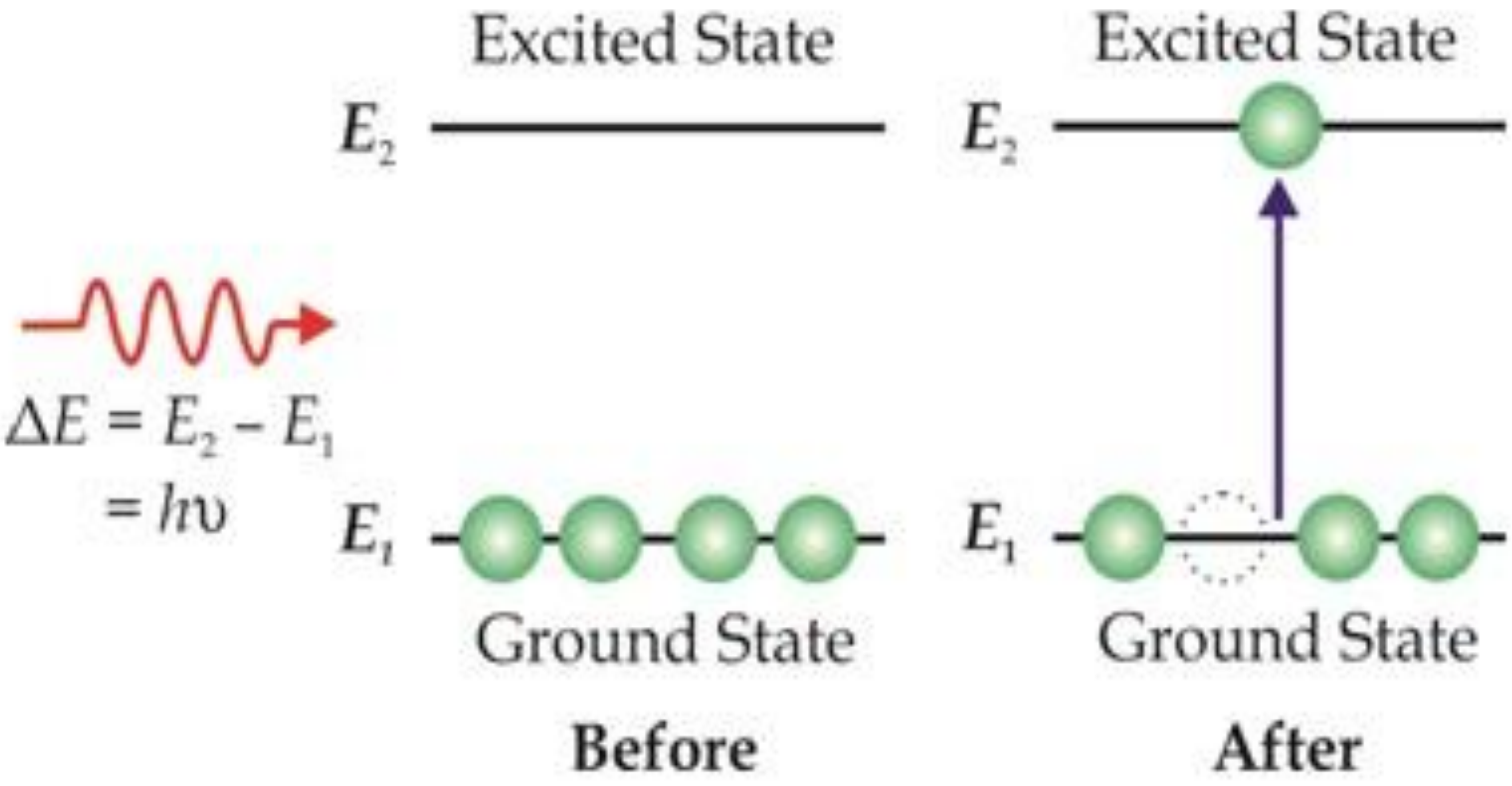
Interaction of Radiation with Matter..

They include

- Induced Absorption
- Spontaneous Emission
- Stimulated Emission



Induced Absorption..



Induced Absorption..

It is the process in which an atom in the ground state absorbs energy from the photon whose energy precisely equal to the energy between two states, thereby atom making transition to higher energy state.

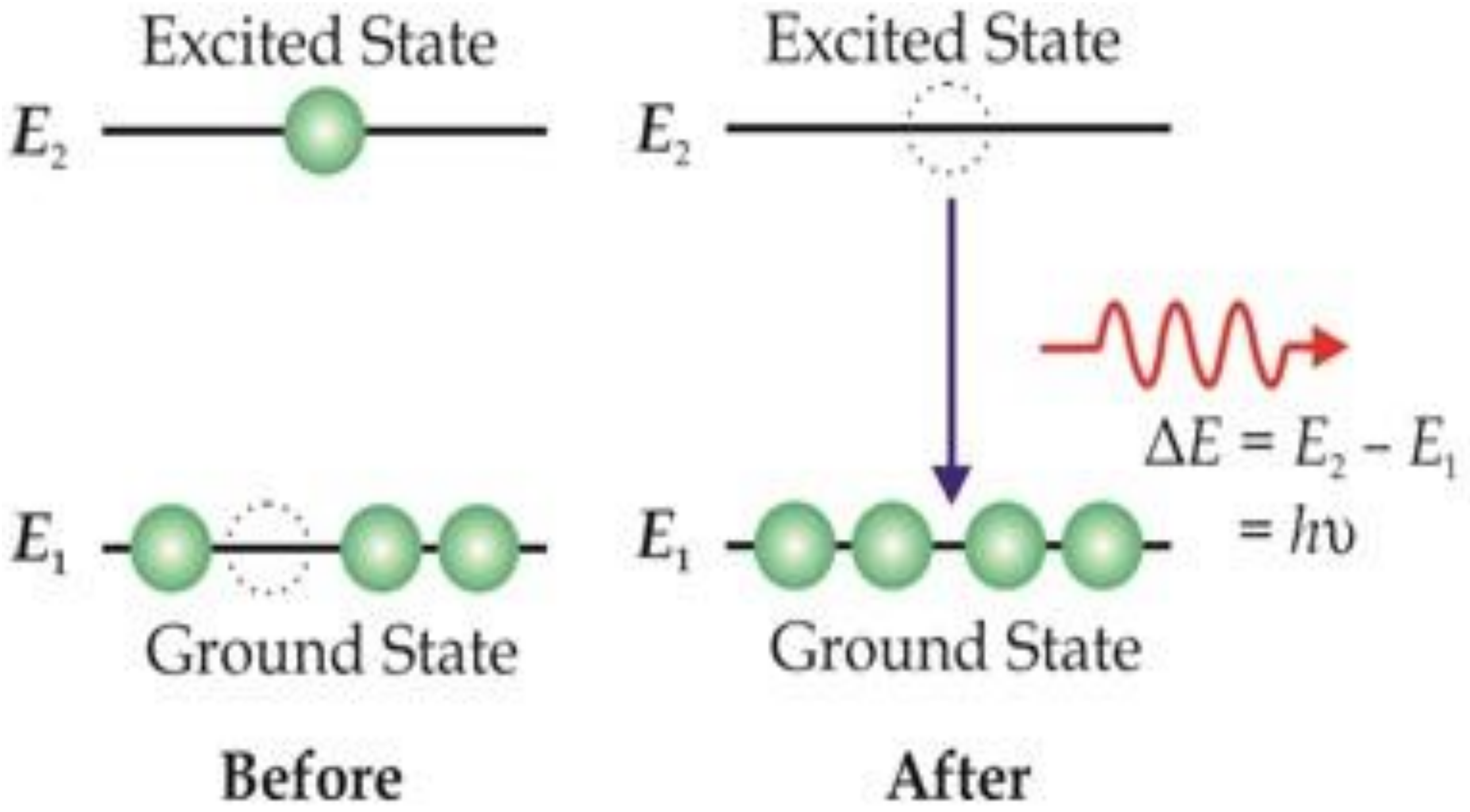
Represented as

$$\text{Atom} + \text{Photon} \longrightarrow \text{Atom}^*$$

here Atom* represents the atom in the excited state

Atom represents the atom in the ground state

Spontaneous Emission..



Spontaneous Emission..

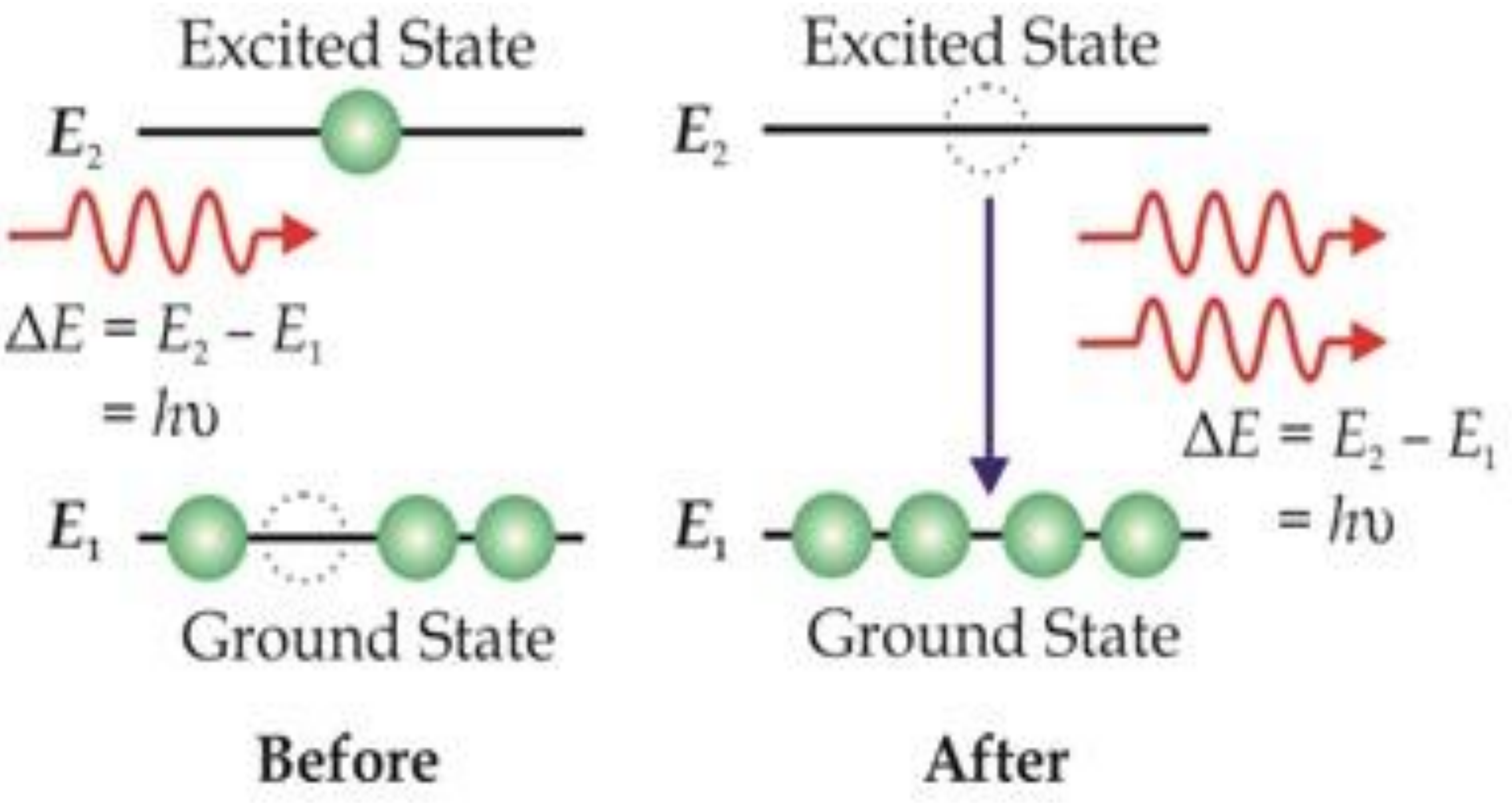
Spontaneous emission is the process of emission of photon, when an atom transits from higher energy level to lower energy level without the influence of any external energy.

Represented as,

$$\text{Atom}^* \longrightarrow \text{Atom} + \text{Photon}$$

Spontaneous emission is ultimately responsible for most of the light we see all around us.

Stimulated Emission..



Stimulated Emission..

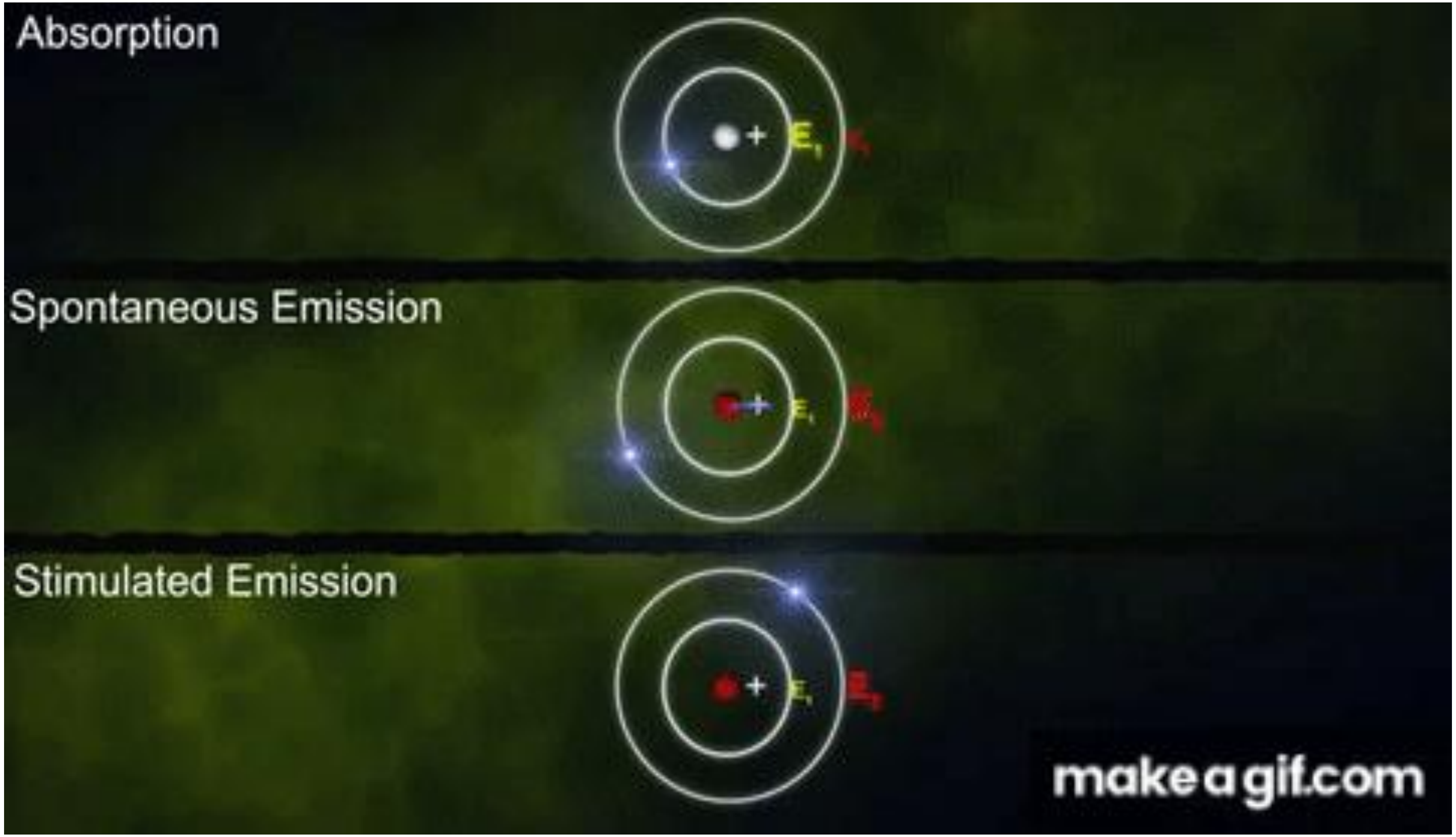
Stimulated emission is the emission of a photon by an atom under the influence of a stimulating photon of right energy, due to which the atom makes transition from a higher energy state to a lower energy state.

Represented as,

$$\text{Atom}^* + \text{Photon} \longrightarrow \text{Atom} + \text{Photon} + \text{Photon}$$

Stimulated emission could be used to generate a highly coherent directional beam of light.

Radiation – Matter interaction..



Boltzmann equation- Equation for ratio of population of two energy levels

Consider a system kept under thermally equilibrium condition at temperature.

Let N_i is the number of atoms presents in E_i energy level.

The population of the atom in given energy level is given by Boltzmann equation

$$N_i = N_0\, e^{\frac{-E_i}{kT}}$$

Where N_o is number atom in unit volume.

If N_1 and N_2 are the population of energy levels and E_1 and E_2 respectively then,

The ratio of population could be written as

$$\frac{N_2}{N_1} = e^{\frac{-(E_2 - E_1)}{kT}}$$

$$\frac{N_2}{N_1} = e^{\frac{-h\nu}{kT}}$$

But $c = \nu\lambda$ or $\nu = \frac{c}{\lambda}$

$$\boxed{\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}}$$

Since

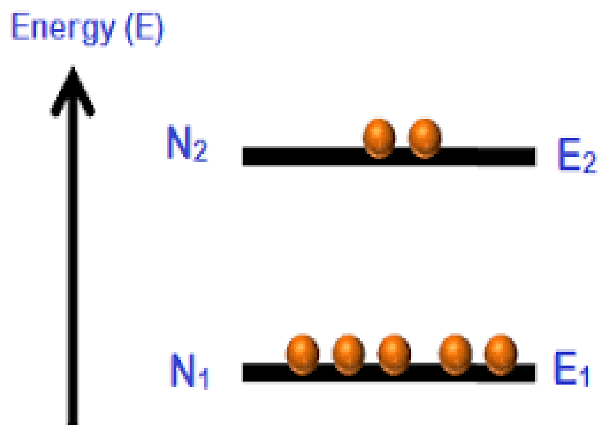
$$E_2 > E_1$$

$$e^{\frac{-(E_2 - E_1)}{kT}} > 1$$

Then we can prove that

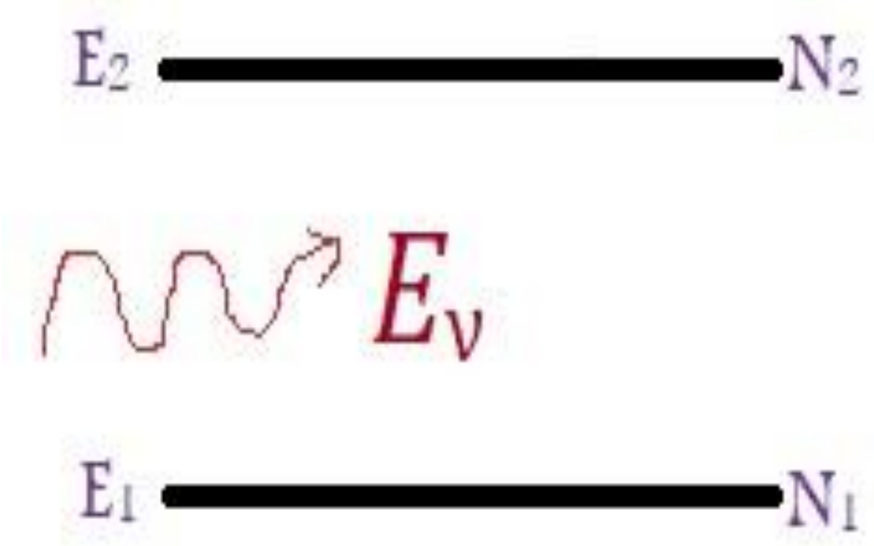
$$\boxed{N_2 < N_1}$$

Therefore, at equilibrium condition there is more number of Atoms in lower energy level compared to higher energy level.



Expression for Energy density in terms of Einstein's A & B coefficients..

- Consider a system under thermal equilibrium.
- E_1 and E_2 be the lower and higher energy levels that contain N_1 and N_2 number of atoms respectively.
- The incident energy density of the radiation be E_ν.
- The system absorbs and emits the energy
- The energy of the photons absorbed and emitted by the atoms is $E = h\nu = (E_2 - E_1)$



Expression for Energy density in terms of Einstein's A & B coefficients..

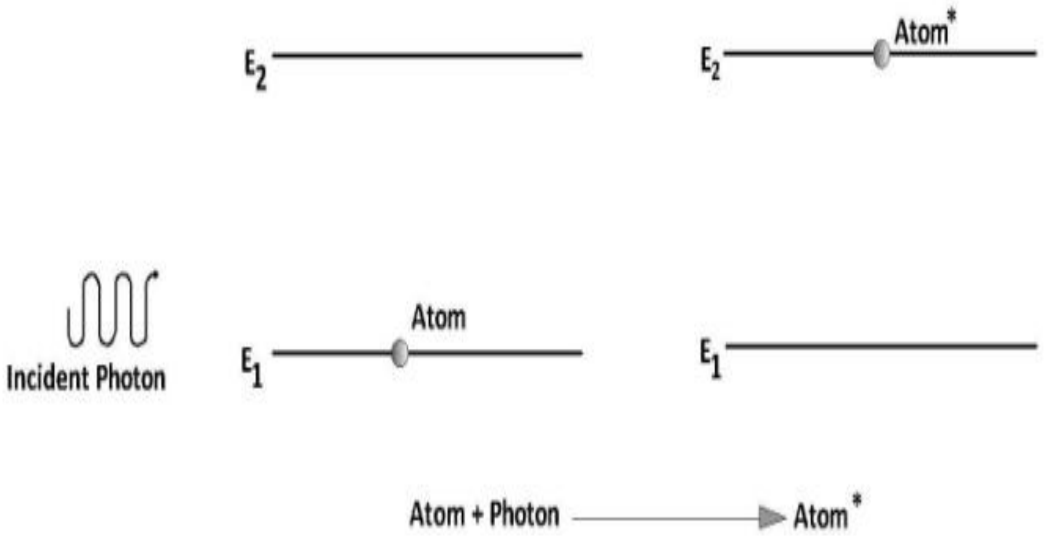
According to Planck's radiation law the equation for energy density in the frequency domain is given by

$$E_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right]$$

Expression for Energy density in terms of Einstein's A & B coefficients..

Rate of induced absorption

Number of atoms in the lower energy state N_1 & the incident energy density E_ν.



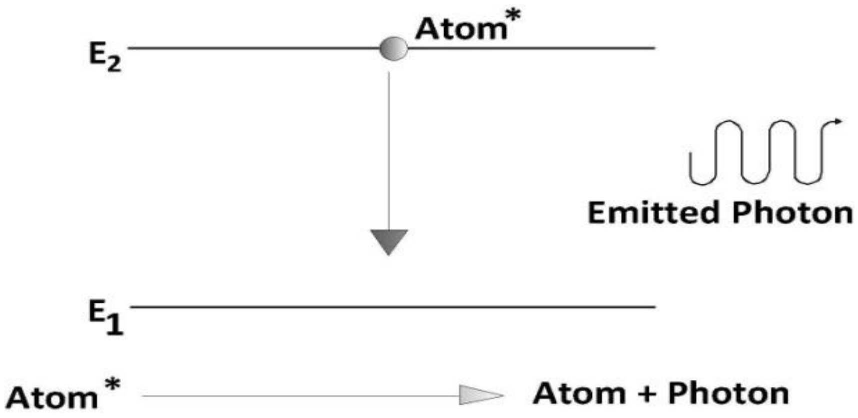
- Rate of Induced absorption $\propto N_1 E_\nu$
- Rate of Induced absorption= $B_{12} N_1 E_\nu$

Here B_{12} is proportionality constant called Einsteins co-efficient of Induced absorption.

Expression for Energy density in terms of Einstein's A & B coefficients..

Rate of spontaneous emission

- Spontaneous emission is independent of energy density E_ν.



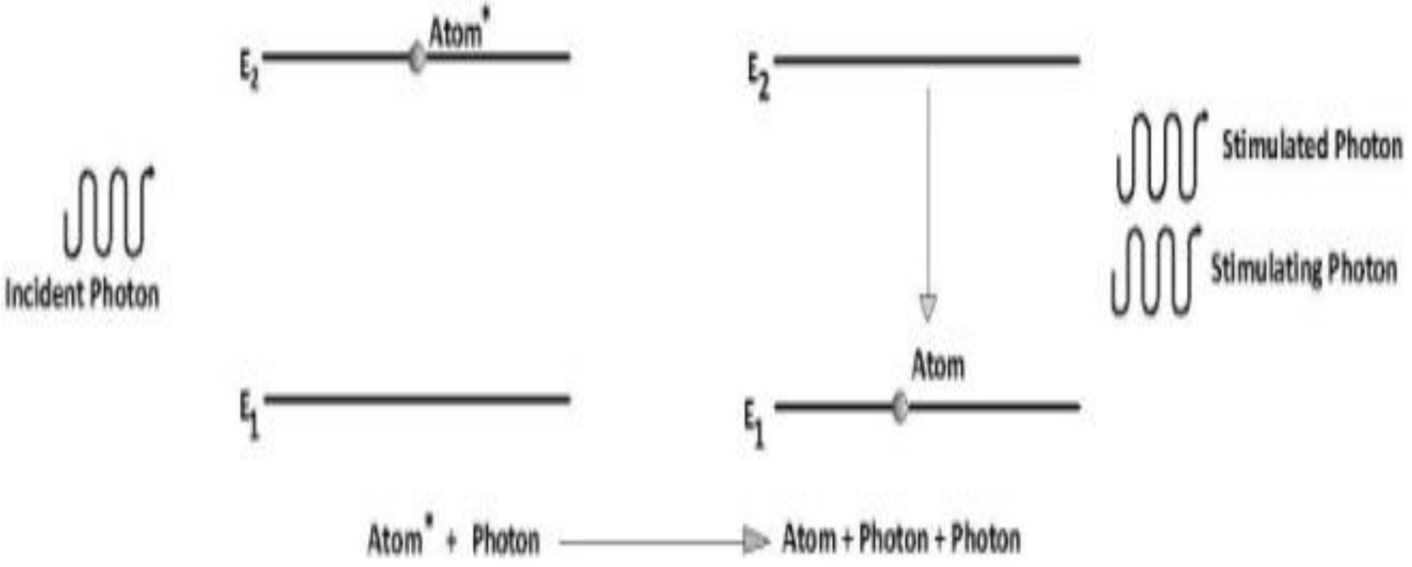
- Rate of spontaneous emission $\propto N_2$
- Rate of Spontaneous emission = $A_{21} N_2$

Here A_{21} is the proportionality constant called Einstein's co-efficient of spontaneous emission

Expression for Energy density in terms of Einstein's A & B coefficients..

Rate of stimulated emission

- Number of atoms in the higher energy state (N_2) & The energy density (E_v).
- The Rate of stimulated emission ∝ $N_2 E_v$
- Rate of stimulated emission = $B_{21} N_2 E_v$



Here the proportionality constant called B_{21} is Einstein's coefficient of stimulated emission.

Expression for Energy density in terms of Einstein's A & B coefficients..

Under Thermal Equilibrium the total Energy of the System remains unchanged. Hence Rate of Absorption is equal to rate of emission.

∴ Rate of Induced Absorption = [Rate of Spontaneous emission + Rate of Stimulated Emission]

∴

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu$$

$$(B_{12} N_1 - B_{21} N_2) E_\nu = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$E_\nu = \frac{A_{21}}{B_{12} \frac{N_1}{N_2} - B_{21}}$$

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1} \right]$$

According to Boltzmann relation the we have

$$\frac{N_2}{N_1} = e^{\frac{-h\nu}{kT}}$$

or we can re-write as,

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$$

Here h is the Planck's constant, c is the speed of light in vacuum, λ is the wavelength of the photon, k is the Boltzmann constant and T is the absolute temperature. Substituting for $\frac{N_1}{N_2}$ in equation

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \left(e^{\frac{h\nu}{kT}}\right) - 1} \right]$$

$$E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}\left(e^{\frac{h\nu}{kT}}\right)-1}\right]$$

Expression for Energy density in terms of Einstein's A & B coefficients..

According to Planck's radiation law, the equation for energy density in the frequency domain is given by

$$E_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{kT}}-1}\right]$$

on comparing equations we can get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

and

$$\frac{B_{12}}{B_{21}} = 1$$

or $B_{12} = B_{21}$

This means that Probability of Induced absorption is equal to Probability of Stimulated emission. Hence A_{21} & B_{21} can be replaced by A & B. Thus

$$E_\nu = \frac{A}{B}\left[\frac{1}{e^{\frac{h\nu}{kT}}-1}\right]$$

Hence the expression for energy density in terms of Einstein's co-efficient A and B.

Condition for LASER..

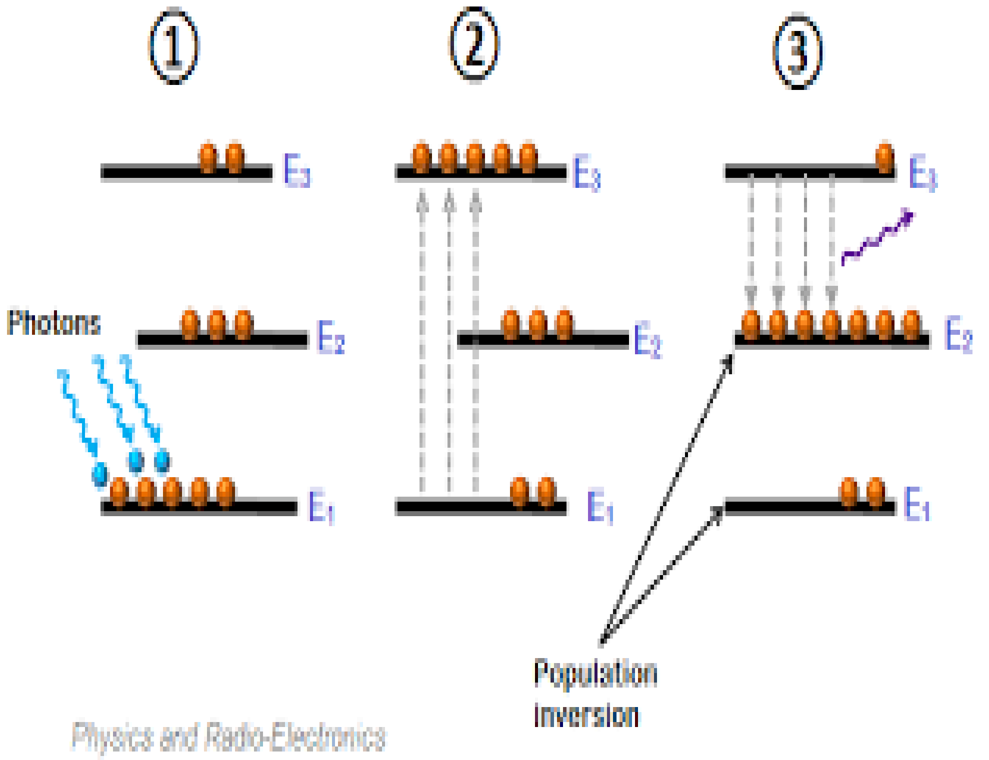
- Condition
  - Population inversion
  - Metastable state

Population Inversion..

Population inversion is the state of the system at which the population of a particular higher energy state is more than that of specified lower energy state.

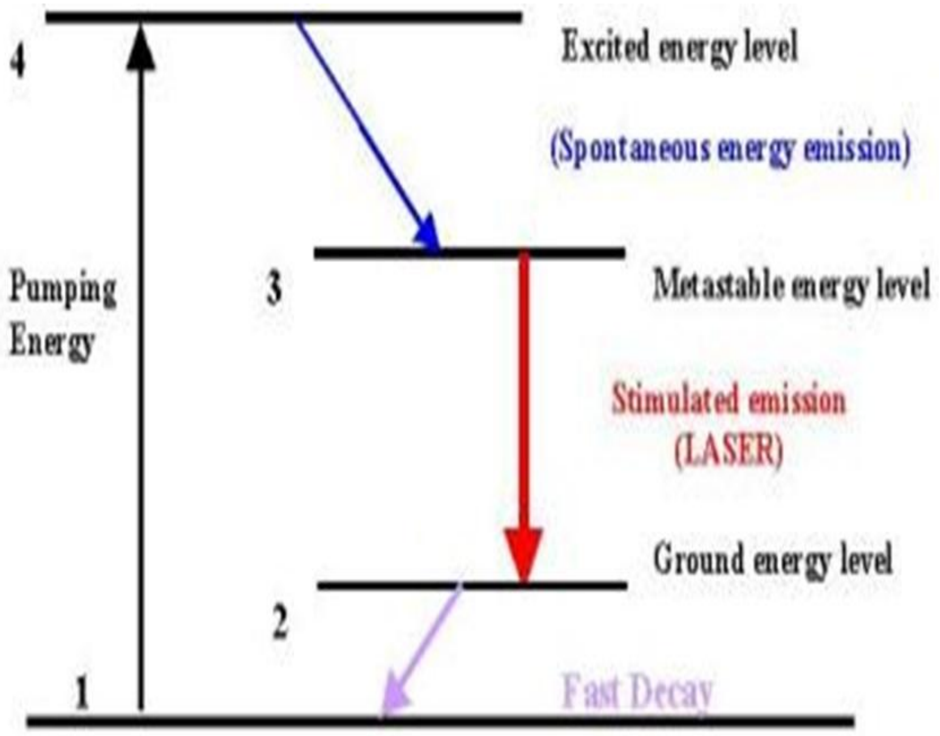
To achieve…

- Higher energy state should posses a longer life time.
- The number of atoms in the higher energy state must be greater than the number of atoms in the lower energy state

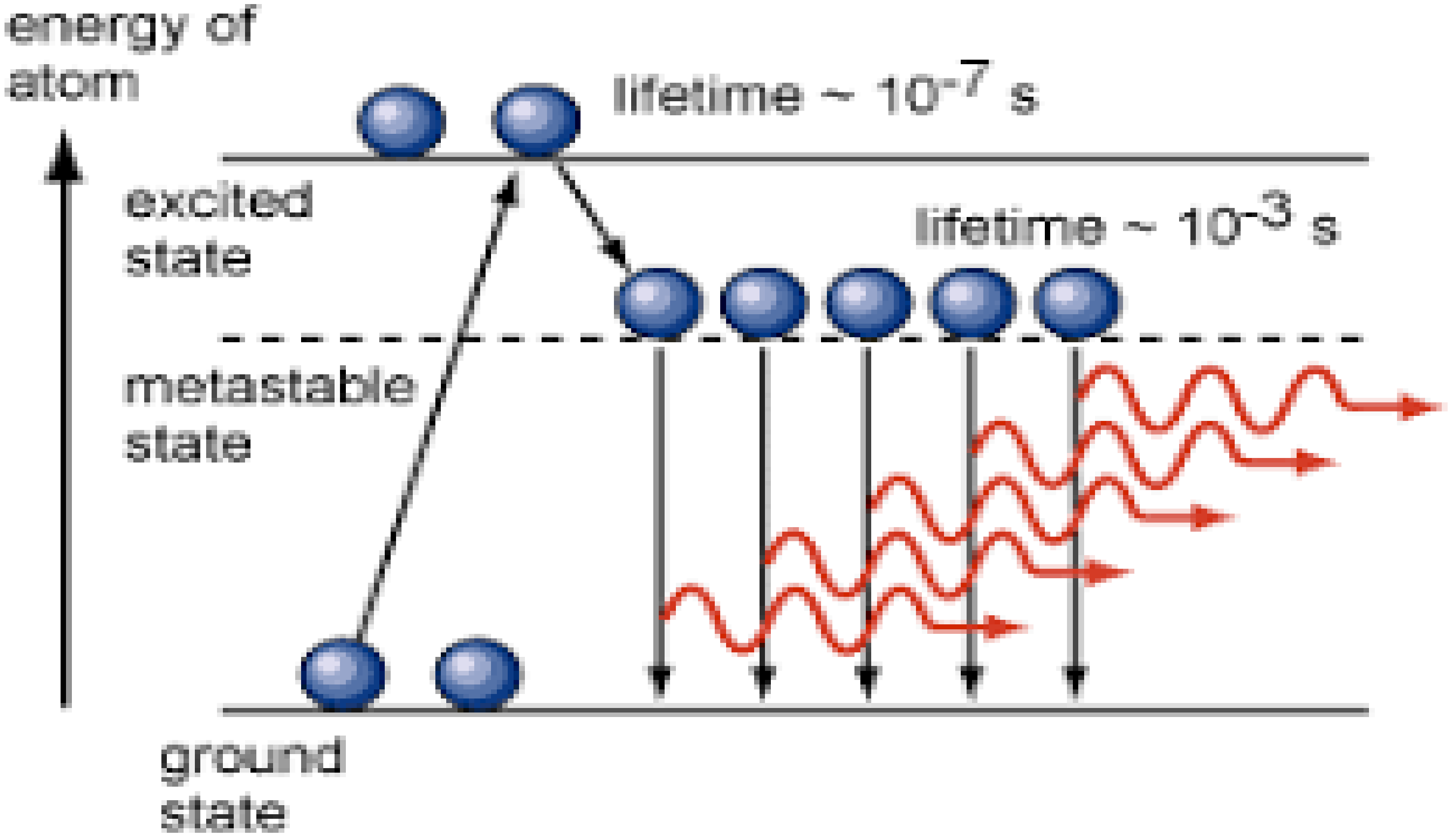


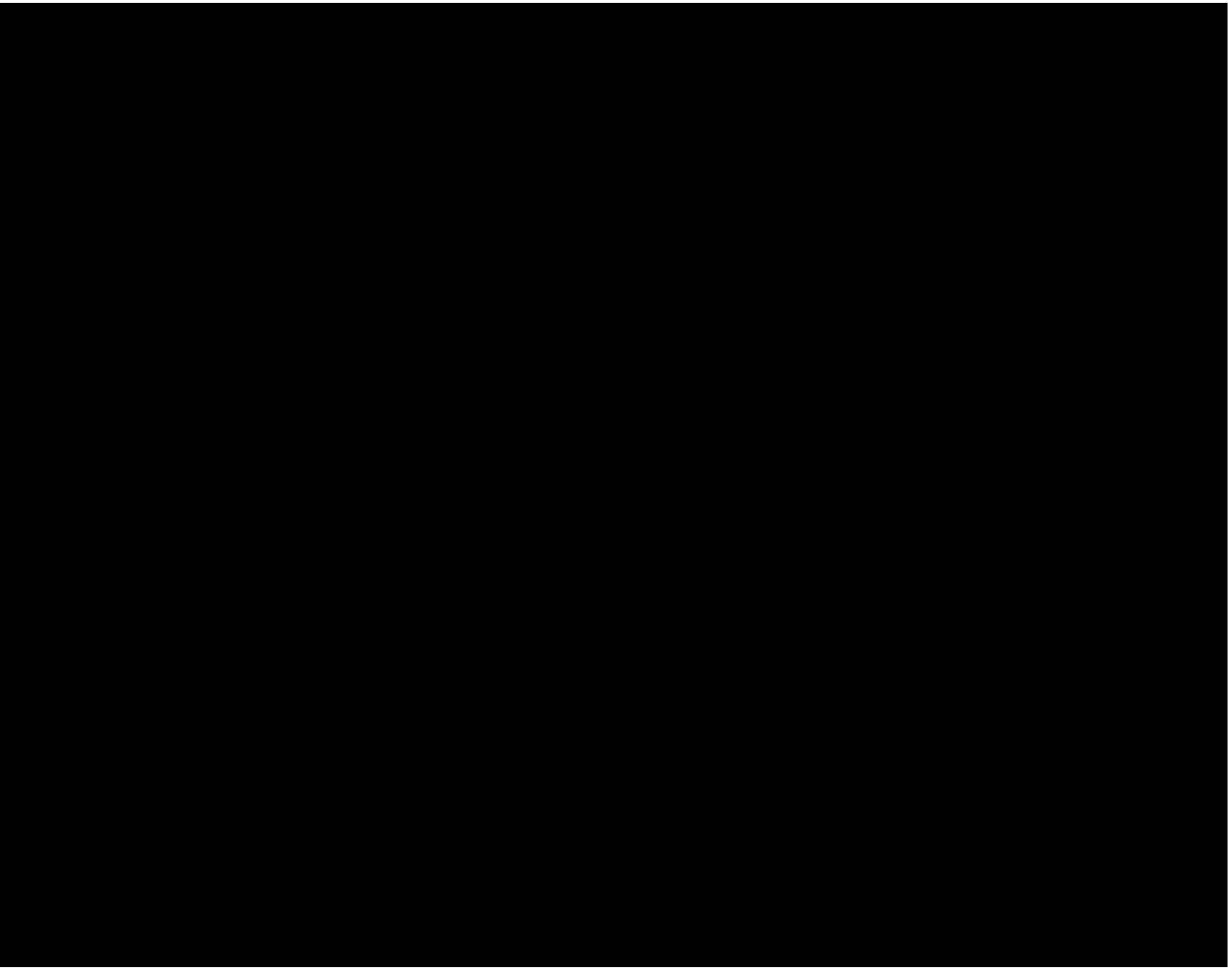
Metastable State..

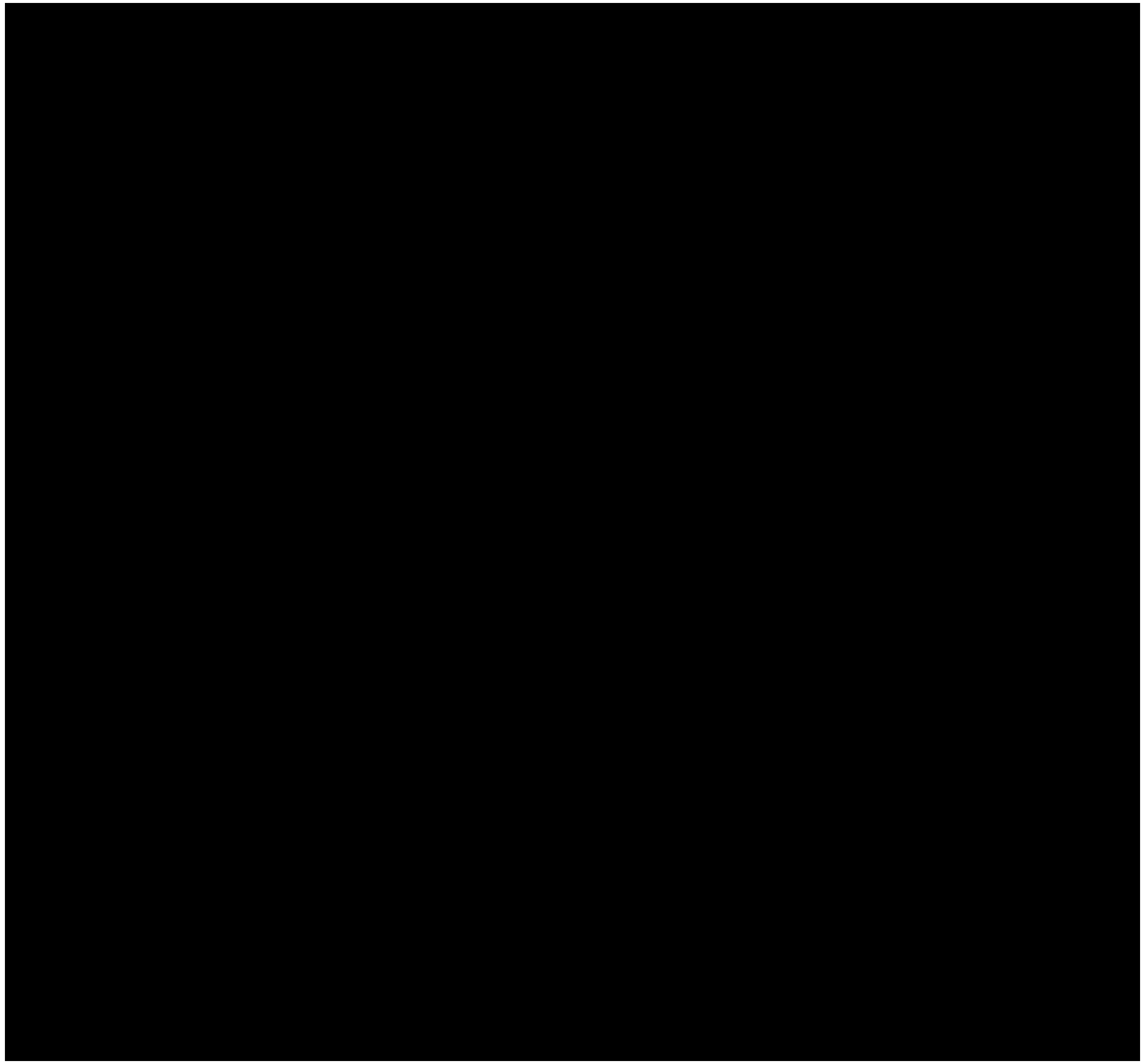
- The intermediate State in which the average life time of atoms is of the order of few milli second.
- The transition of atoms from higher excited state to this particular state is high enough.



Condition for Laser..







Requisites for LASER..

Requisites

- Excitation source (for Pumping action)
- Active medium (supports Population inversion)
- Laser cavity / Resonant cavity

Requisites for LASER

Excitation source

The excitation source used to supply appropriate energy for Pumping the atoms from lower energy state to higher energy state.

Pumping mechanism

Energy supply to bring the atoms from lower energy state to higher energy state.

Example: Electrical discharge pumping, forward bias pumping, optical pumping, chemical pumping…..

Active medium

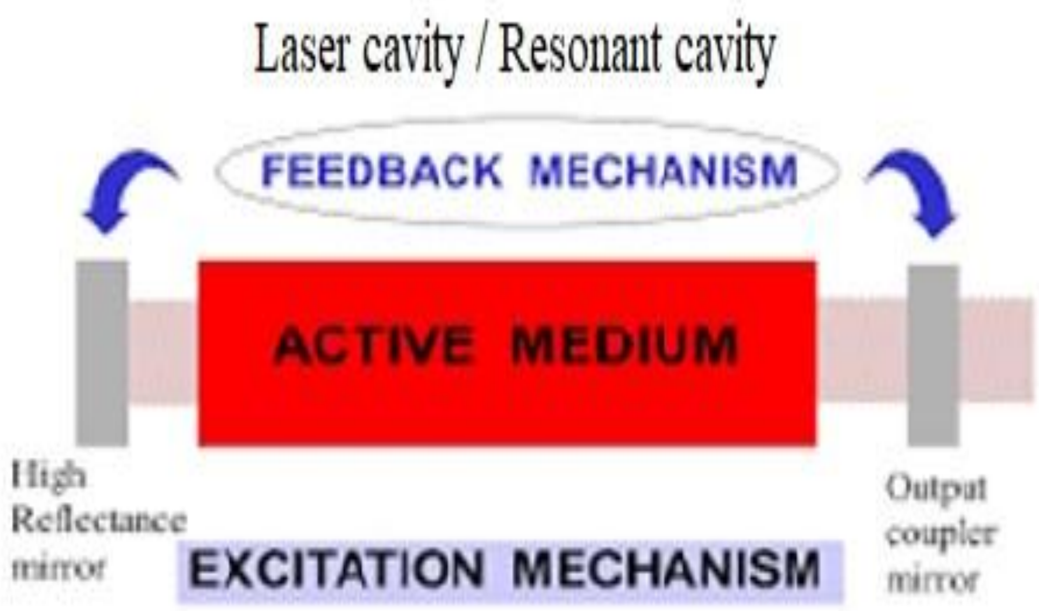
Is a solid / liquid / gas medium in which stimulated emission and amplification of radiation could achieved.

LASER cavity or Resonant cavity..

The LASER Cavity consists of an active medium bound between two highly parallel mirrors.

The reflection of photons from the mirrors makes the multiple traverse of photons through the active medium inducing more and more stimulated emissions.

Thus amplification of light is achieved.



LASER cavity or Resonant cavity..

The cavity resonates when the distance 'L' between the mirrors is equal to an integral multiple of $\frac{\lambda}{2}$, where λ is the wavelength of radiation in the active system

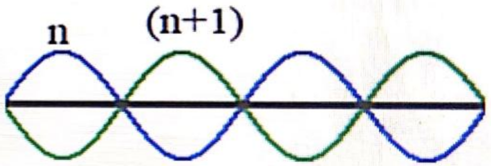
$$L = \frac{n\lambda}{2}$$

but $v = f\lambda$

$$\therefore \quad L = \frac{nv}{2f}$$

$$\text{Or } f = \frac{nv}{2L}$$

Laser cavity / Resonant cavity



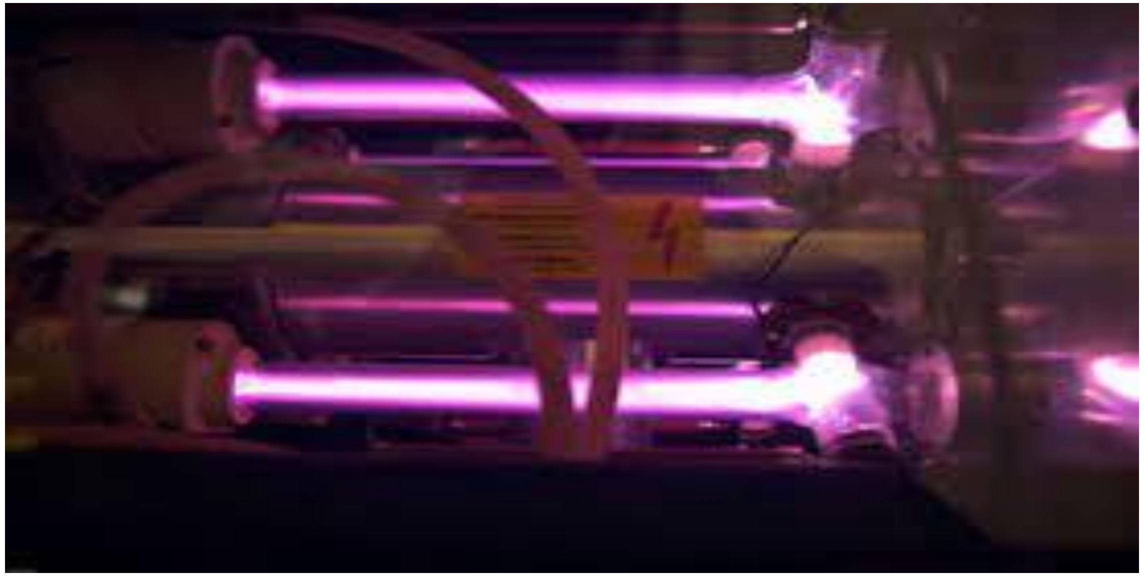
The frequency difference between two adjacent modes of vibration is,

$$\Delta f = (n+1)\frac{v}{2L} - \frac{nv}{2L}$$

$$\therefore \Delta f = \frac{v}{2L}$$

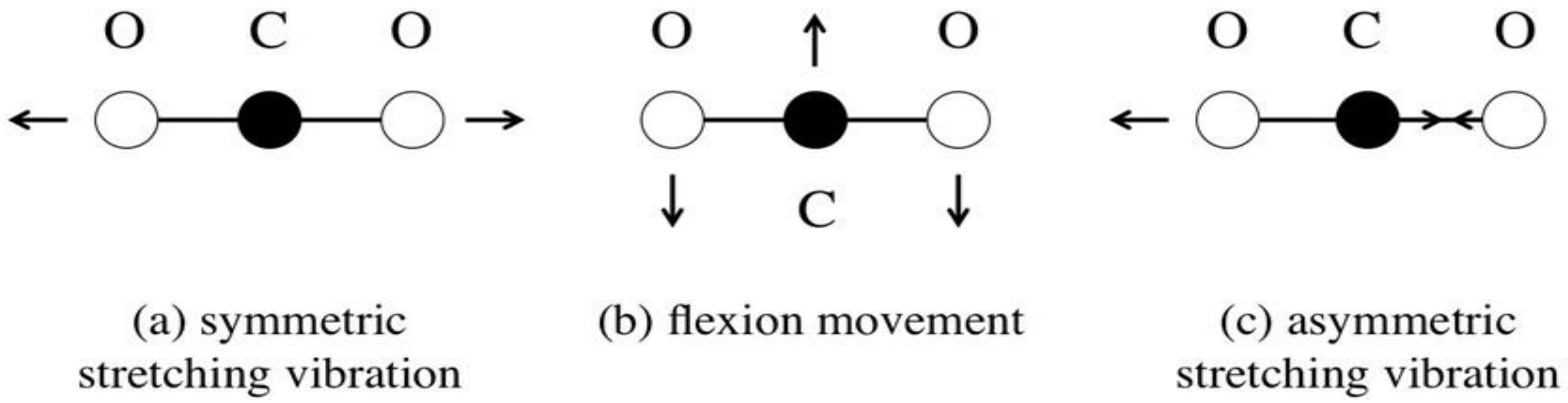
Construction and Working of CO_2 Laser..

- It is a molecular gas Laser, designed by C.K.N. Patel in 1963.
- The Lasing Action occurs between Vibration energy levels of the molecules.
- High efficiency (up to 30%) and is widely used in industry & medical applications.
- It operates in the IR region
- Continuous & pulsed output waveforms.
- Pumping: Electrical discharge pumping
- Active medium: Carbon dioxide

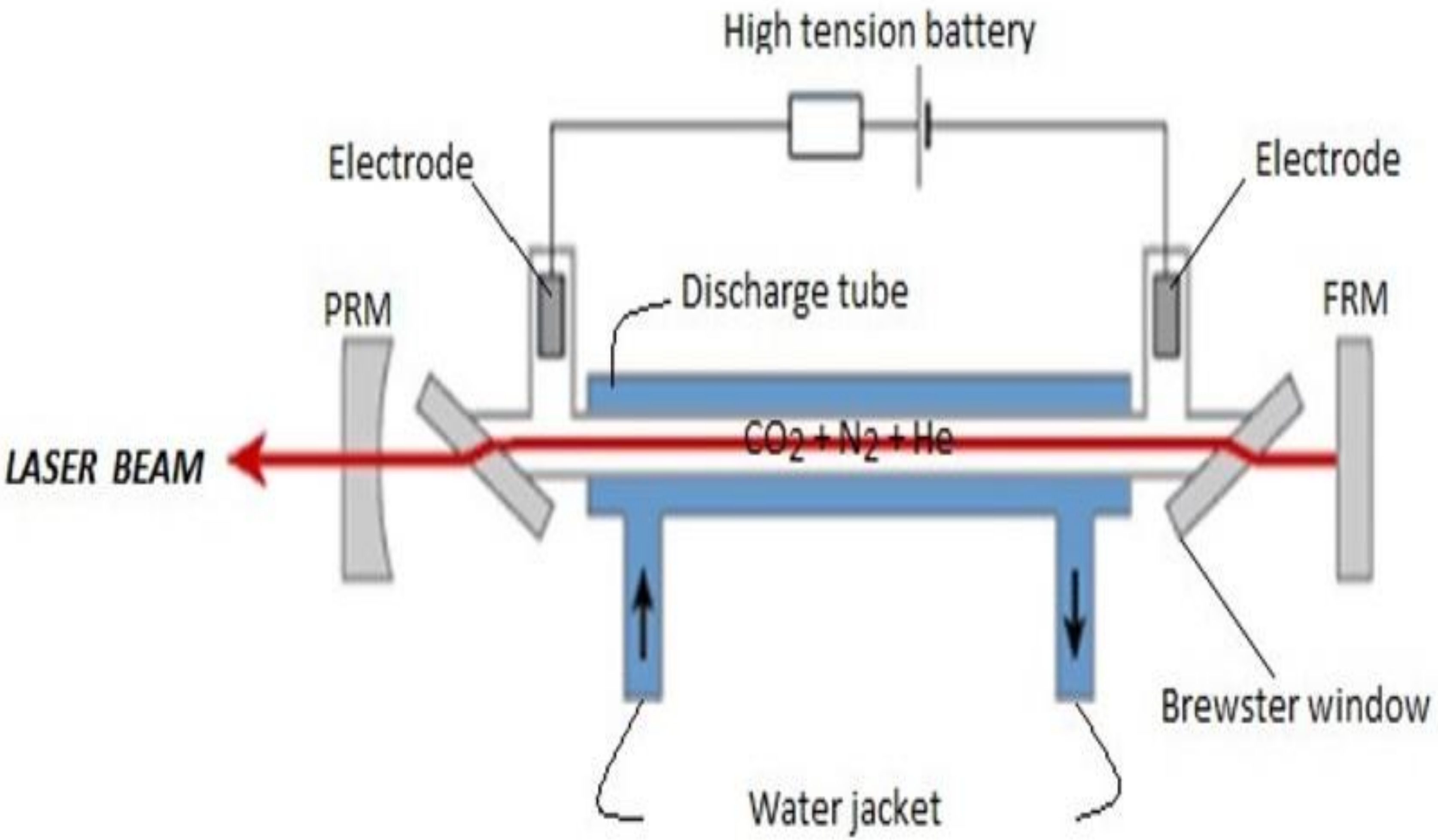


CO2 Laser..

- ❖CO2 molecule has one carbon atom about which two oxygen atoms are symmetrically located.
- ❖CO2 molecule can vibrate in 3 different modes. In each mode, the centre of gravity remains same.



Construction of CO2 Laser..



Tube length around 5 m

Tube diameter around 2.5 cm

CO_2:N_2: He is 1:2:3

Working of CO2 Laser..

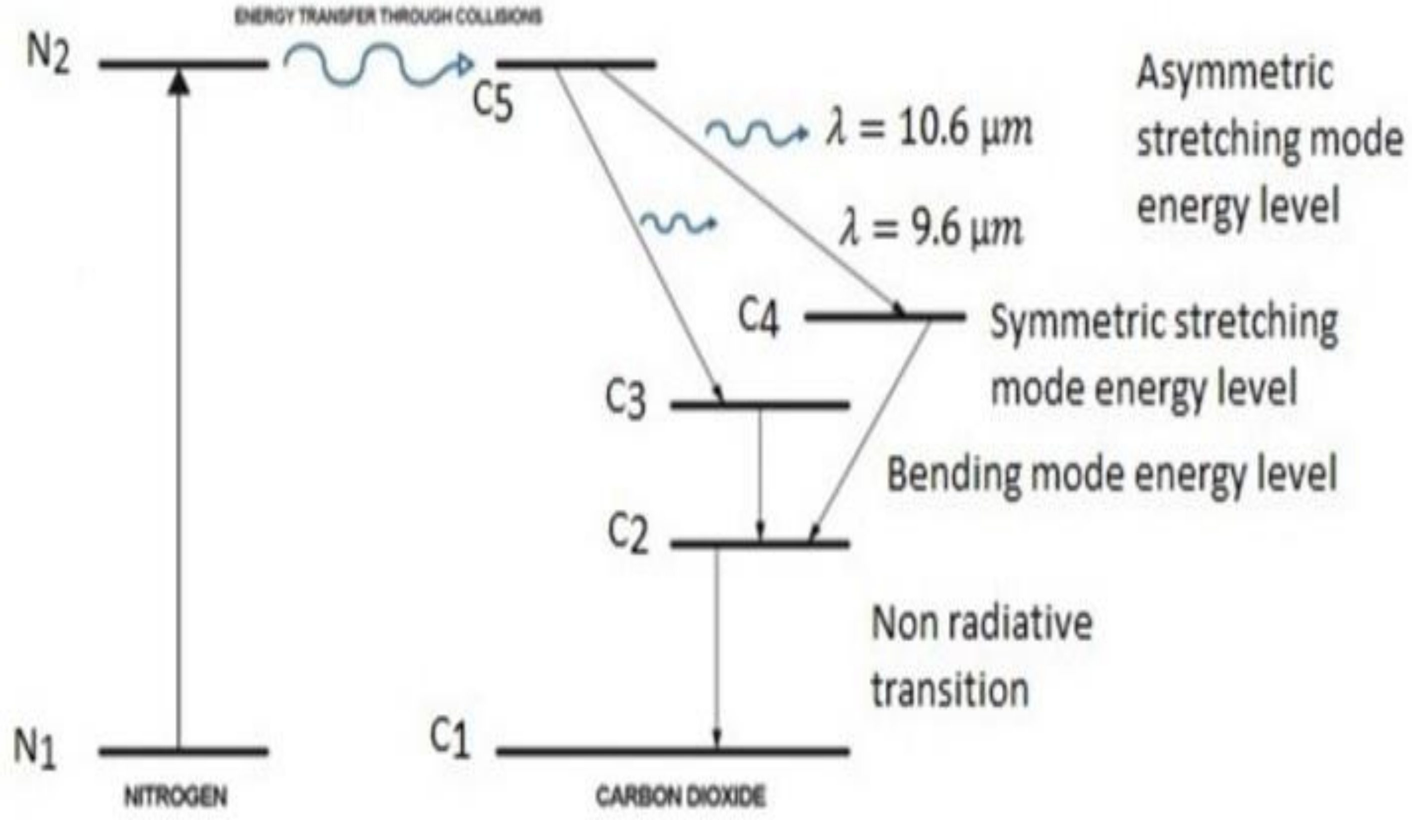
Collision of First kind

$$e^1 + N_2 \rightarrow N_2^* + e^2$$

$$e^3 + CO_2 \rightarrow CO_2^* + e^4$$

Collision of Second kind

$$N_2^* + CO_2 \rightarrow N_2 + CO_2^*$$



CO2 Laser..

❑ Advantages

- It has given continuous & pulsating output
- It is high directional & high monochromatic
- High efficient compared to other Lasers

❑ Applications

- It is used in industrial applications like welding, cutting, drilling
- It is used in LIDAR due to minimum atmospheric attenuation.
- Used in communication systems.

Expression for Energy density in terms of Einstein's A & B coefficients..

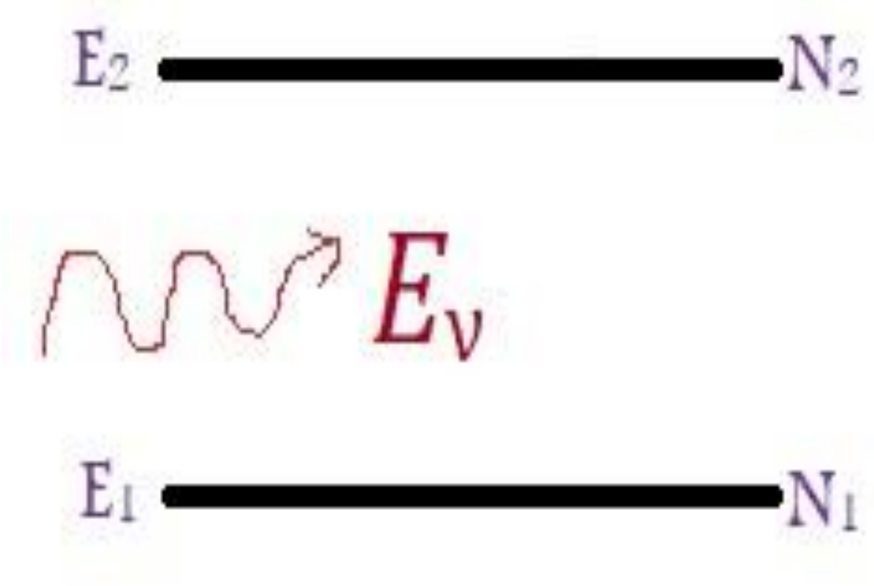
- Consider a system under thermal equilibrium.

- E_1 and E_2 be the lower and higher energy levels that contain N_1 and N_2 number of atoms respectively.

- The incident energy density of the radiation be E_ν.

- The system absorbs and emits the energy

- The energy of the photons absorbed and emitted by the atoms is $E = h\nu = (E_2 - E_1)$



Expression for Energy density in terms of Einstein's A & B coefficients..

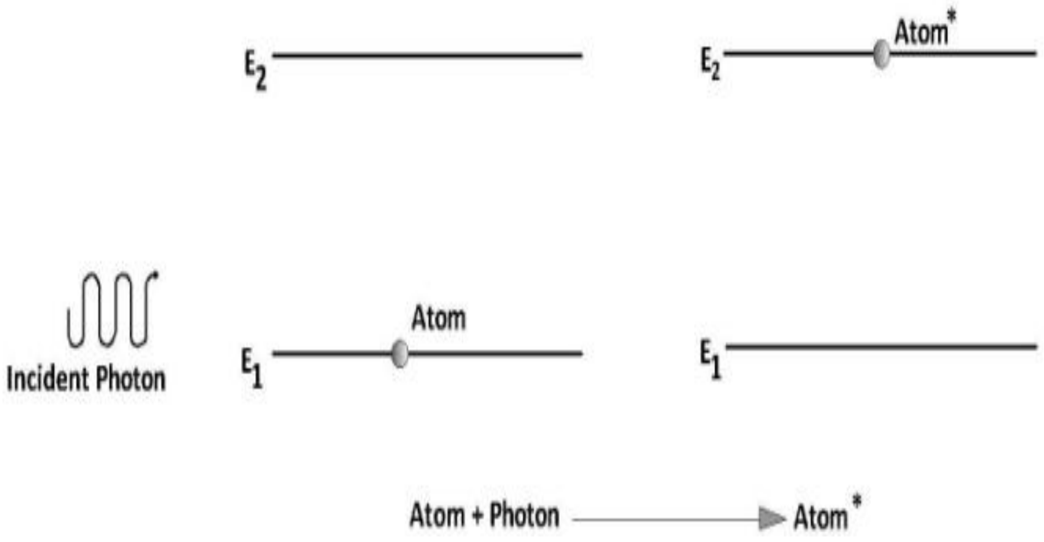
According to Planck's radiation law the equation for energy density in the frequency domain is given by

$$E_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right]$$

Expression for Energy density in terms of Einstein's A & B coefficients..

Rate of induced absorption

Number of atoms in the lower energy state N_1 & the incident energy density E_ν.



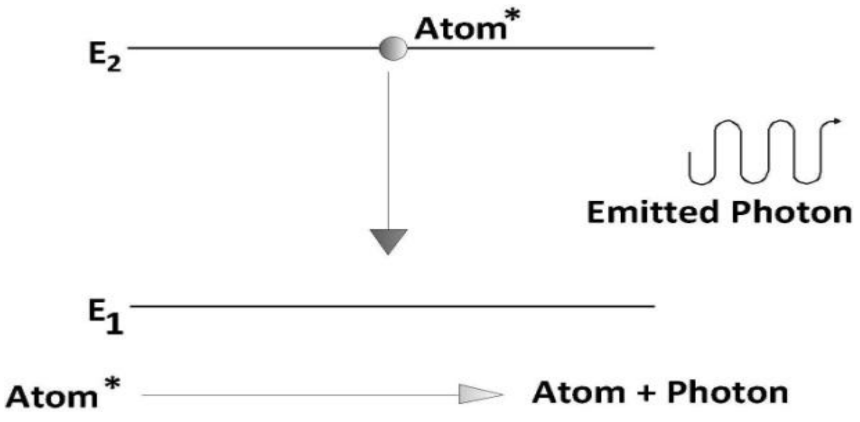
- Rate of Induced absorption $\propto N_1 E_\nu$
- Rate of Induced absorption= $B_{12} N_1 E_\nu$

Here B_{12} is proportionality constant called Einsteins coef- ficient of Induced absorption.

Expression for Energy density in terms of Einstein's A & B coefficients..

Rate of spontaneous emission

- Spontaneous emission is independent of energy density E_ν.



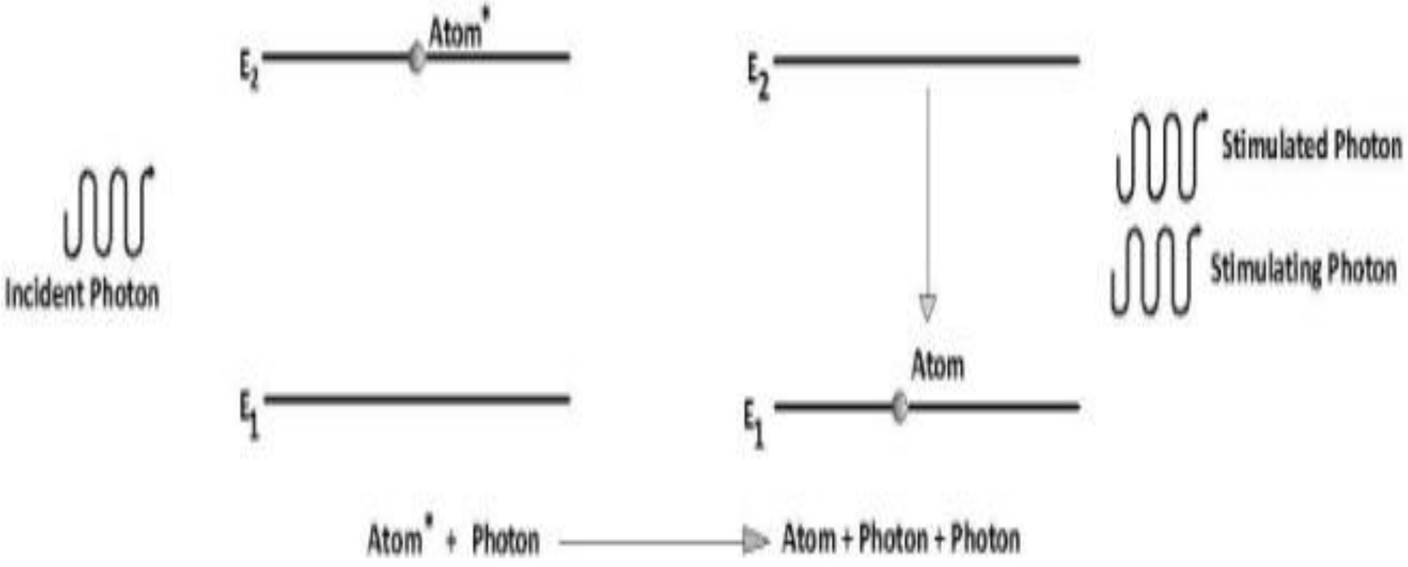
- Rate of spontaneous emission $\propto N_2$
- Rate of Spontaneous emission = $A_{21} N_2$

Here A_{21} is the proportionality constant called Einstein's co-efficient of spontaneous emission

Expression for Energy density in terms of Einstein's A & B coefficients..

Rate of stimulated emission

- Number of atoms in the higher energy state (N_2) & The energy density (E_ν).



- The Rate of stimulated emission $\propto N_2 E_\nu$
- Rate of stimulated emission = $B_{21} N_2 E_\nu$

Here the proportionality constant called B_{21} is Einstein's coefficient of stimulated emission.

Expression for Energy density in terms of Einstein's A & B coefficients..

Under Thermal Equilibrium the total Energy of the System remains unchanged. Hence Rate of Absorption is equal to rate of emission.

∴ Rate of Induced Absorption = [Rate of Spontaneous emission + Rate of Stimulated Emission]

∴

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu$$

$$(B_{12} N_1 - B_{21} N_2) E_\nu = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$E_\nu = \frac{A_{21}}{B_{12} \frac{N_1}{N_2} - B_{21}}$$

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1} \right]$$

According to Boltzmann relation the we have

$$\frac{N_2}{N_1} = e^{\frac{-h\nu}{kT}}$$

or we can re-write as,

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$$

Here h is the Planck's constant, c is the speed of light in vacuum, λ is the wavelength of the photon, k is the Boltzmann constant and T is the absolute temperature. Substituting for $\frac{N_1}{N_2}$ in equation

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \left(e^{\frac{h\nu}{kT}} \right) - 1} \right]$$

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \left(e^{\frac{h\nu}{kT}} \right) - 1} \right]$$

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$$E_\nu = \frac{A}{B} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right]$$

Hence the expression for energy density in terms of Einstein's co-efficient A and B.

Requisites for LASER..

Requisites

- Excitation source (for Pumping action)
- Active medium (supports Population inversion)
- Laser cavity / Resonant cavity

Requisites for LASER

Excitation source

The excitation source used to supply appropriate energy for Pumping the atoms from lower energy state to higher energy state.

Pumping mechanism

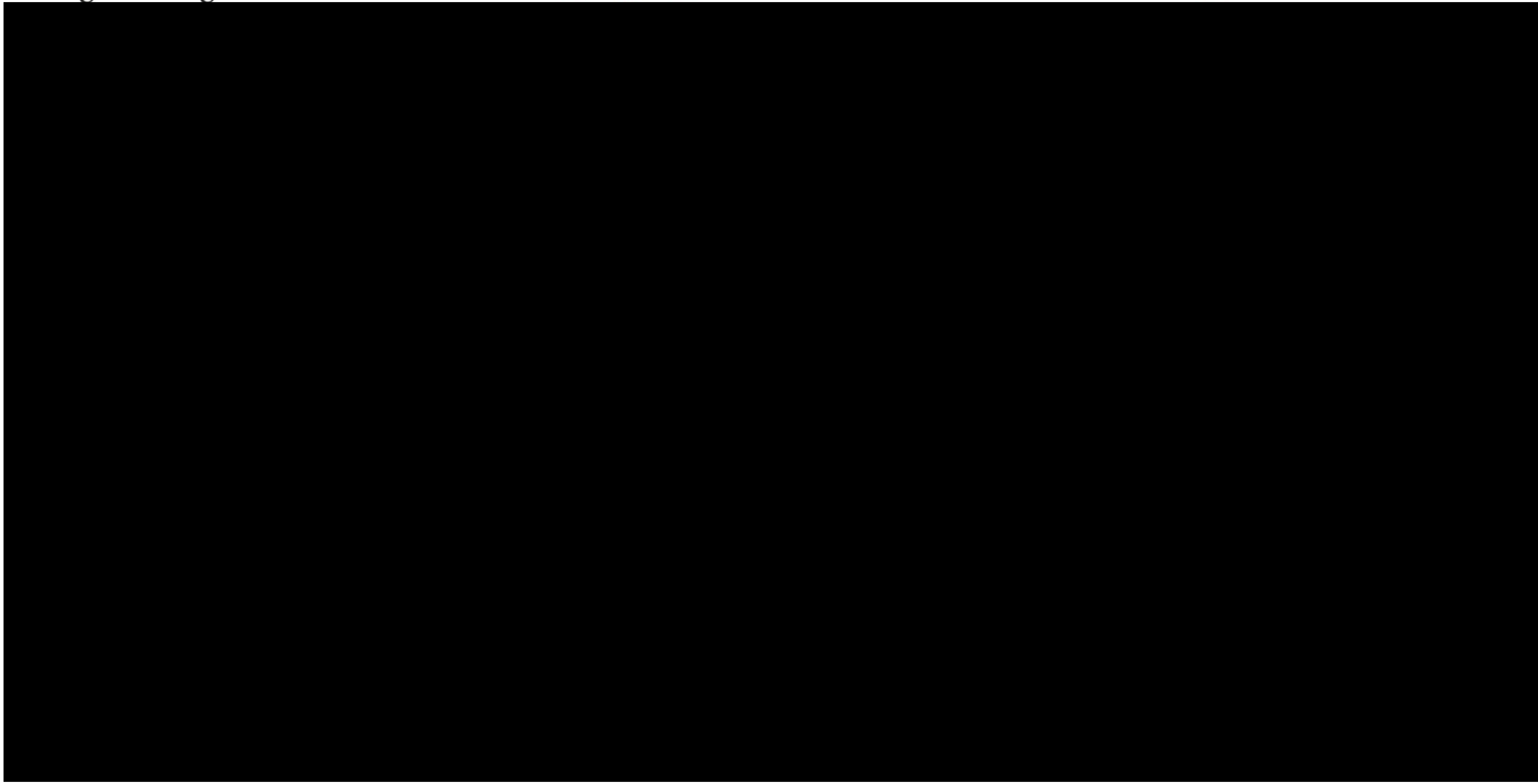
Energy supply to bring the atoms from lower energy state to higher energy state.

Example: Electrical discharge pumping, forward bias pumping, optical pumping, chemical pumping…..

Active medium

Is a solid / liquid / gas medium in which stimulated emission and amplification of radiation could achieved.

Lasing action..



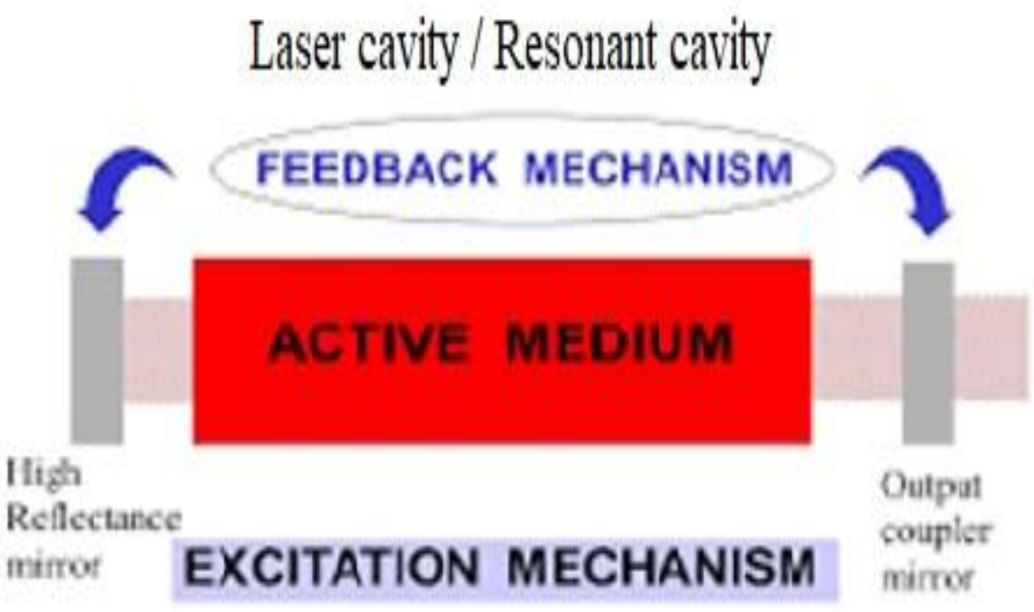
Production of LASERs..

LASER cavity or Resonant cavity..

The LASER Cavity consists of an active medium bound between two highly parallel mirrors.

The reflection of photons from the mirrors makes the multiple traverse of photons through the active medium inducing more and more stimulated emissions.

Thus amplification of light is achieved.



LASER cavity or Resonant cavity..

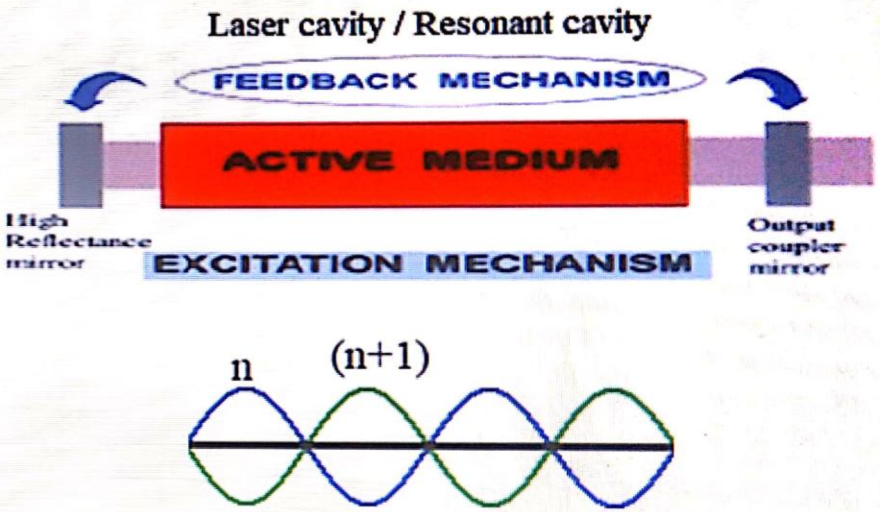
The cavity resonates when the distance 'L' between the mirrors is equal to an integral multiple of $\frac{\lambda}{2}$, where λ is the wavelength of radiation in the active system

$$L = \frac{n\lambda}{2}$$

but $v = f\lambda$

$$\therefore \quad L = \frac{nv}{2f}$$

$$\text{Or } f = \frac{nv}{2L}$$



The frequency difference between two adjacent modes of vibration is,

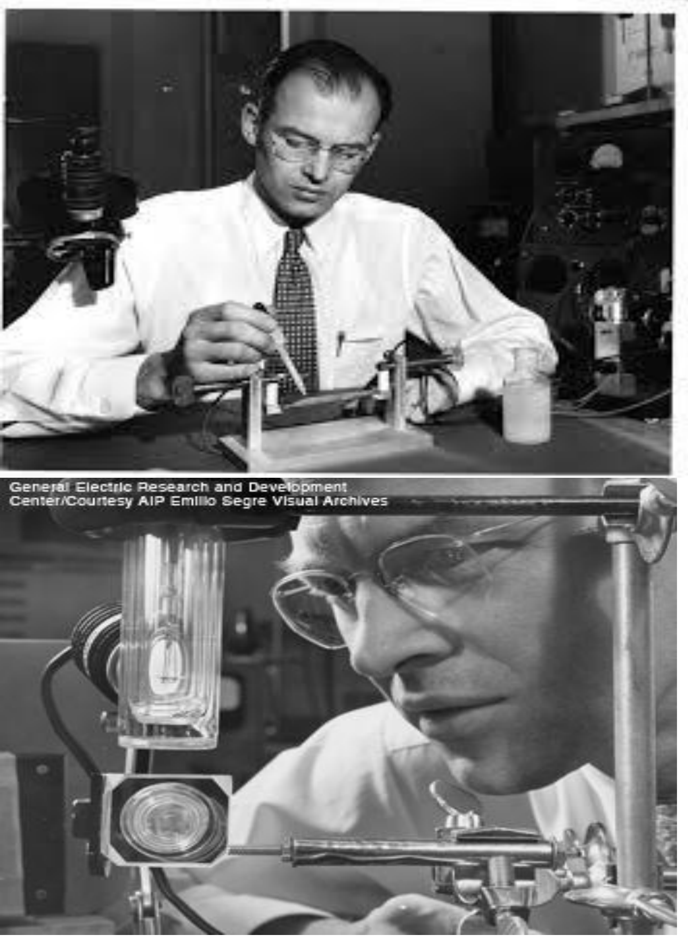
$$\Delta f = (n+1)\frac{v}{2L} - \frac{nv}{2L}$$

$$\therefore \Delta f = \frac{v}{2L}$$

SD Laser..

- First semiconductor LASER was fabricated in 1962 by Hall with the help of his coworkers. He used Gallium Arsenide (GaAs) for this purpose.
- The GaAs LASER diode belongs to direct band gap semiconductors.
- Heavily doped semiconductor (Degenerate semiconductor).
- The **n-section** is derived by doping the substrate with **Tellurium** and **p-section** is derived by doping the substrate with **Zinc.**
- It is a low cost and high efficiency LASER.

SD Laser..


General Electric Research and Development Center/Courtesy AIP Emilio Segre Visual Archives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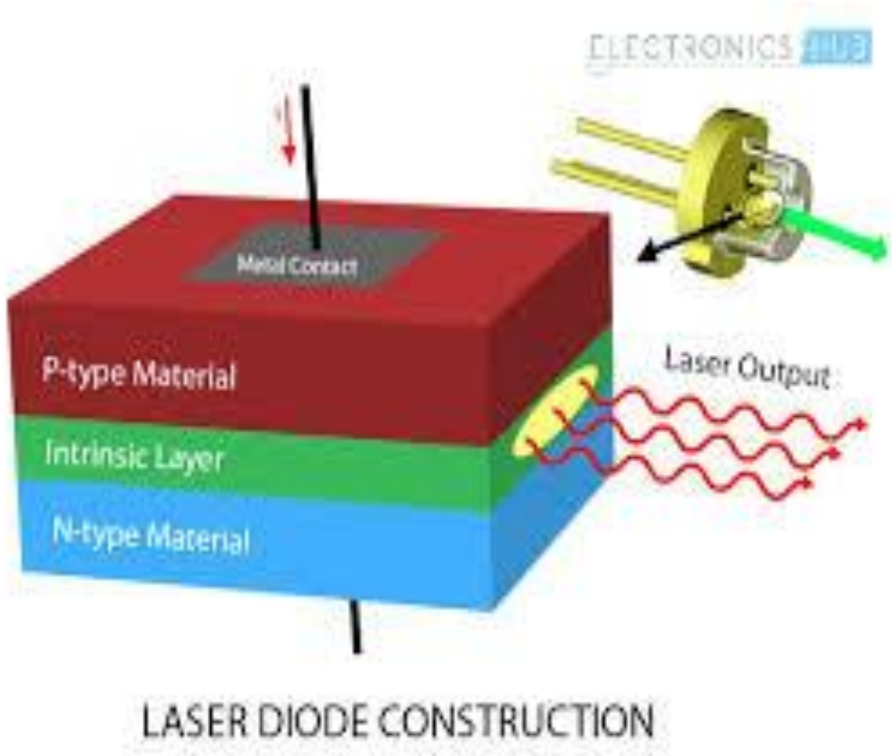
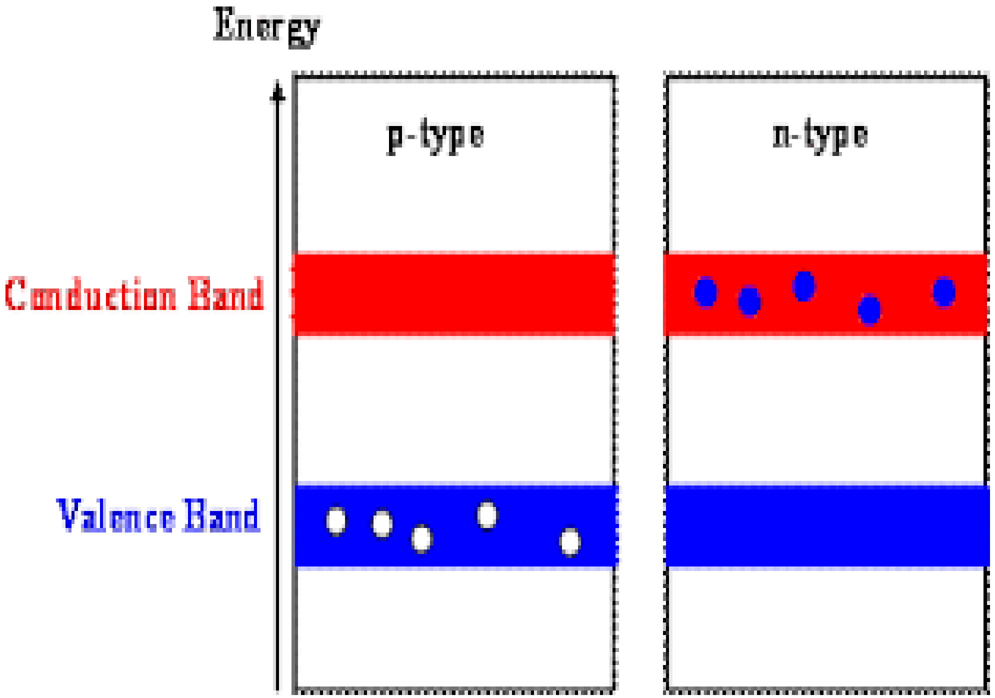
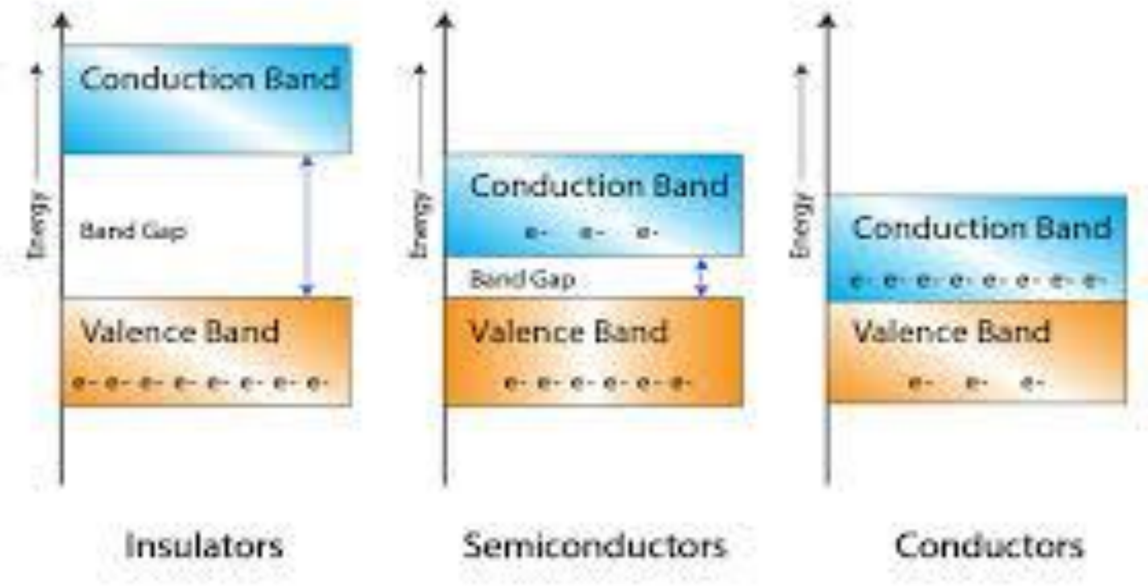

ELECTRONICS HUB
Metal Contact
P-type Material
Intrinsic Layer
N-type Material
Laser Output
LASER DIODE CONSTRUCTION

SD Laser..


Energy
p-type
n-type
Conduction Band
Valence Band


Conduction Band
Band Gap
Valence Band
Insulators
Conduction Band
Band Gap
Valence Band
Semiconductors
Conduction Band
Valence Band
Conductors

SD Laser..

- In the case of Semiconductor diode LASER the "Active medium" will be formulated by Semiconducting material.

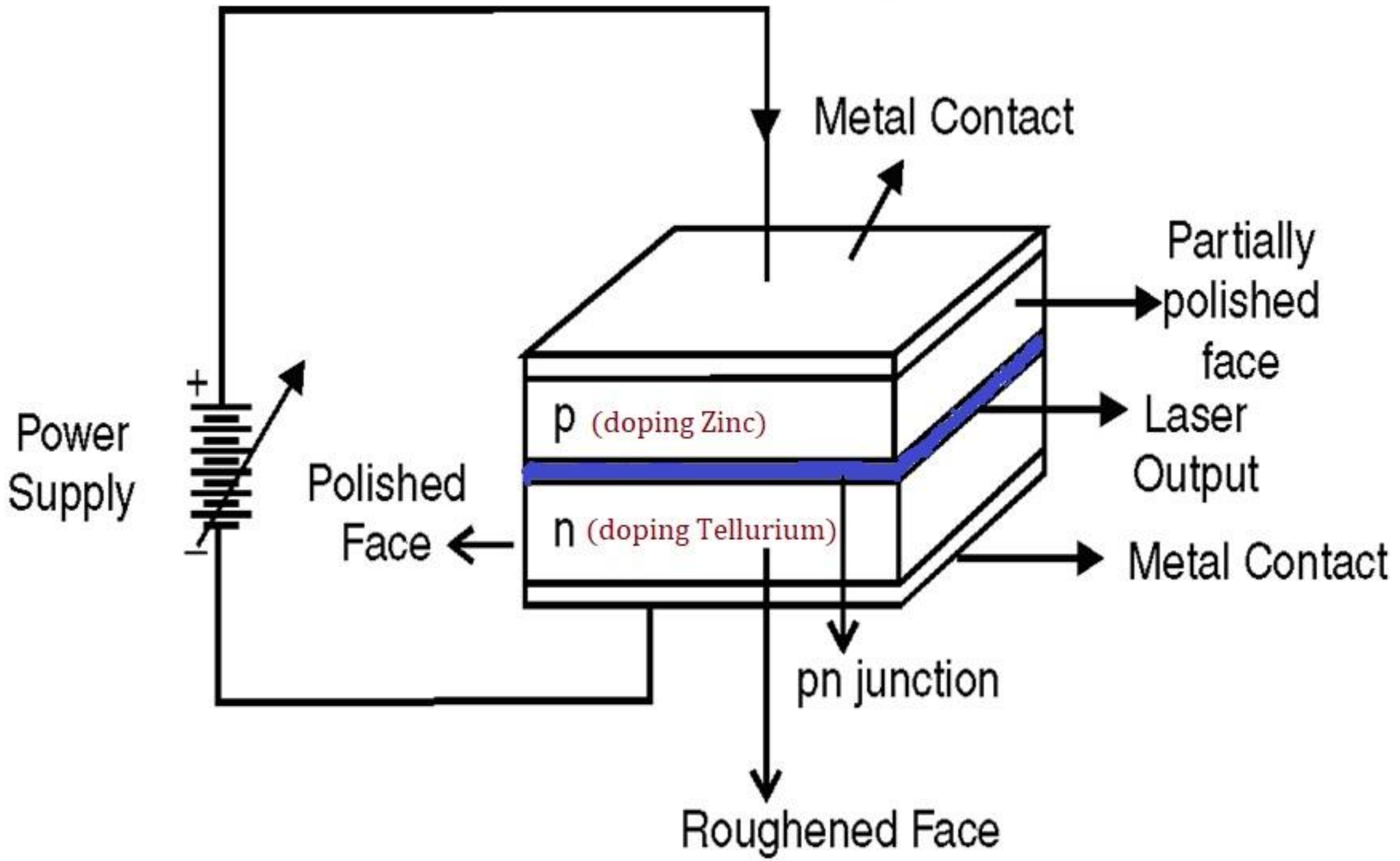
Principle RECOMBINATION OF ELECTRON AND HOLE

- Whenever electron recombines with hole it results in the emission of photon. This is due to the transition of electron from conduction band to valance band.

Recombination of e and holes

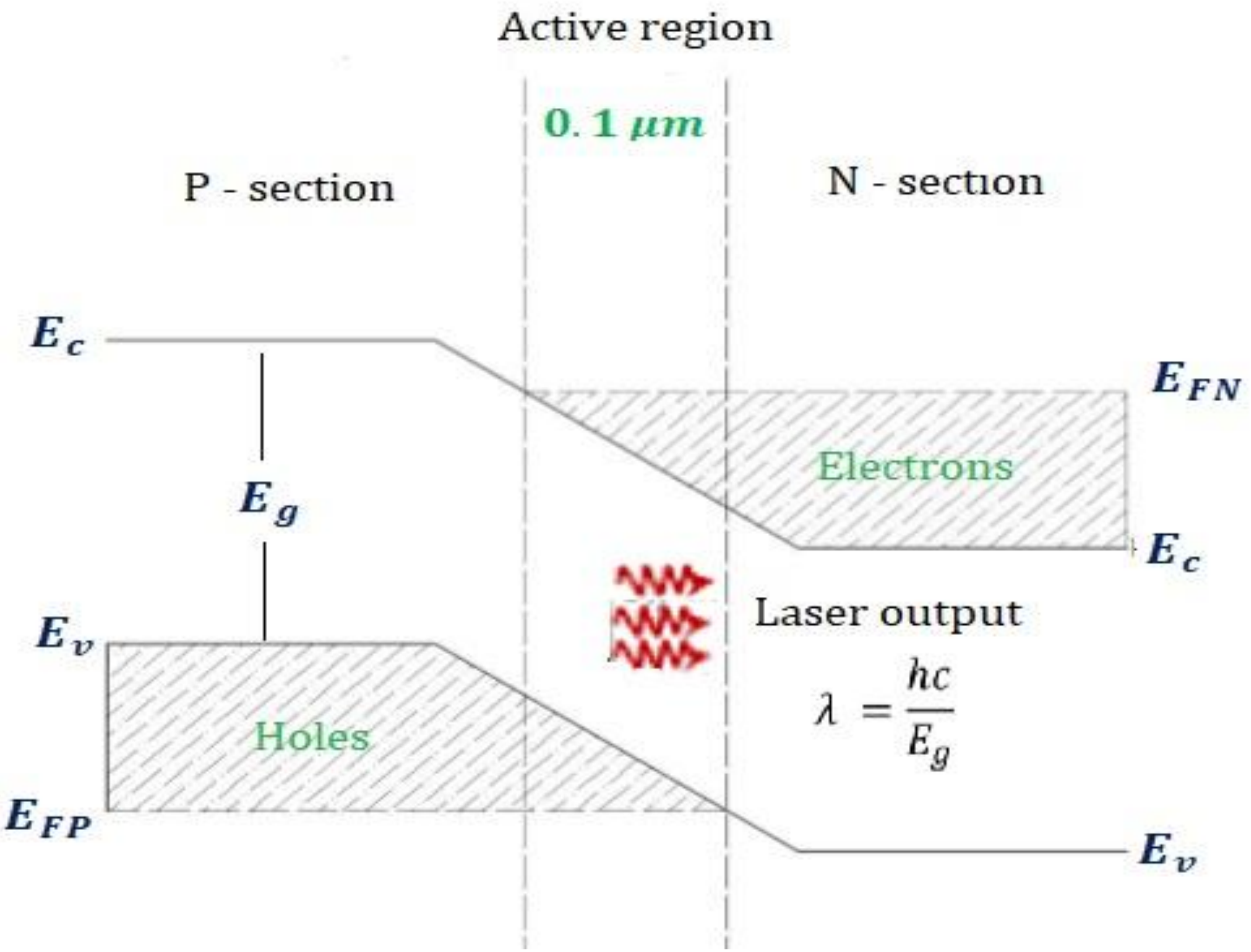
SD Laser..

Construction



SD Laser..

Working



SD Laser..

The energy gap E_g of GaAs Semiconductor is 1.4 eV.

We know that, $E_g = h\nu$.

Where h is the Planck's constant and ν is the frequency of the emitted photon.

But, the speed of light is $c = \nu\lambda$.

Therefore expression for λ becomes

$$\lambda = \frac{hc}{E_g}$$

On using the values $h = 6.63 \times 10^{-34}$ Js, $c = 3 \times 10^{8}$ ms^{-1}

$E_g = 1.4 \times 1.602 \times 10^{-19}$ J

We get, the wavelength as $\lambda = 8400$ Å

This shows that the Laser belongs to IR region of the spectrum.

SD Laser..

Advantages

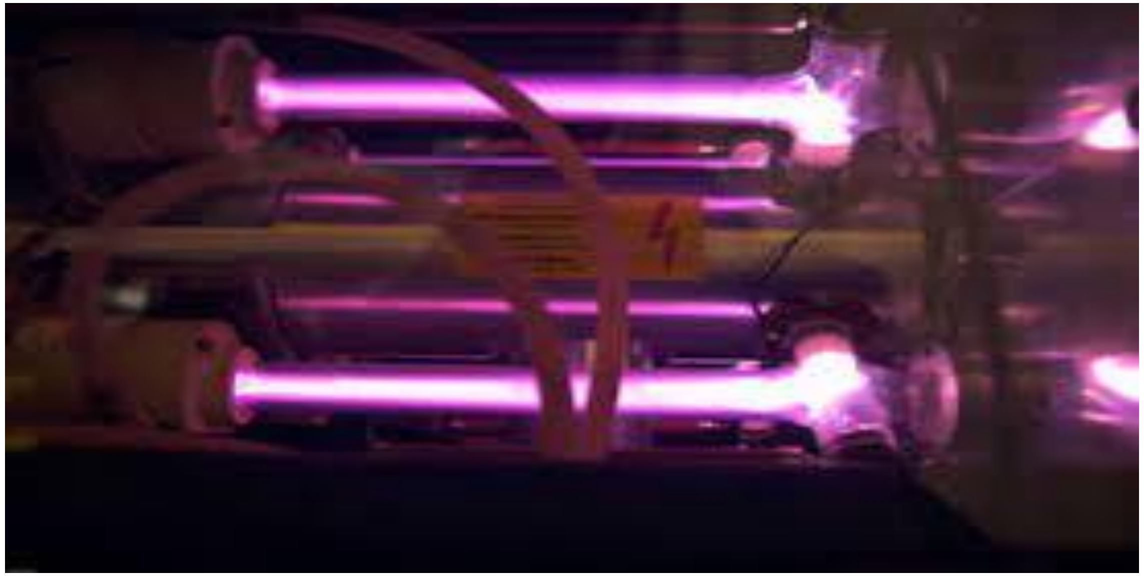
- It has excellent efficiency
- The output can be modulated
- It produces both continuous wave output or pulsed output.
- It is highly economical

Applications

- It is used in optical fiber communication.
- It is used in commercial CD recording and reading.

CO_2 Laser..

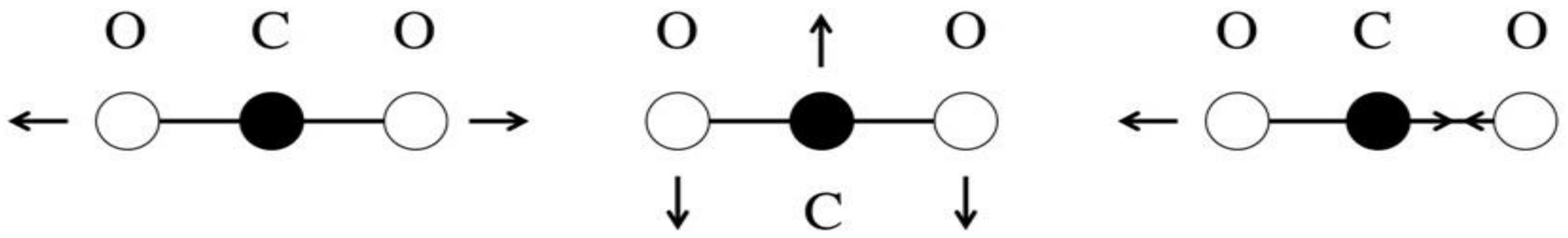
- It is a molecular gas Laser, designed by C.K.N. Patel in 1963.
- The Lasing Action occurs between Vibration energy levels of the molecules.
- High efficiency (up to 30%) and is widely used in industry & medical applications.
- It operates in the IR region
- Continuous & pulsed output waveforms.
- Pumping: Electrical discharge pumping
- Active medium: Carbon dioxide



CO2 Laser..

❖ CO_2 molecule has one carbon atom about which two oxygen atoms are symmetrically located.

❖ CO_2 molecule can vibrate in 3 different modes. In each mode, the centre of gravity remains same.

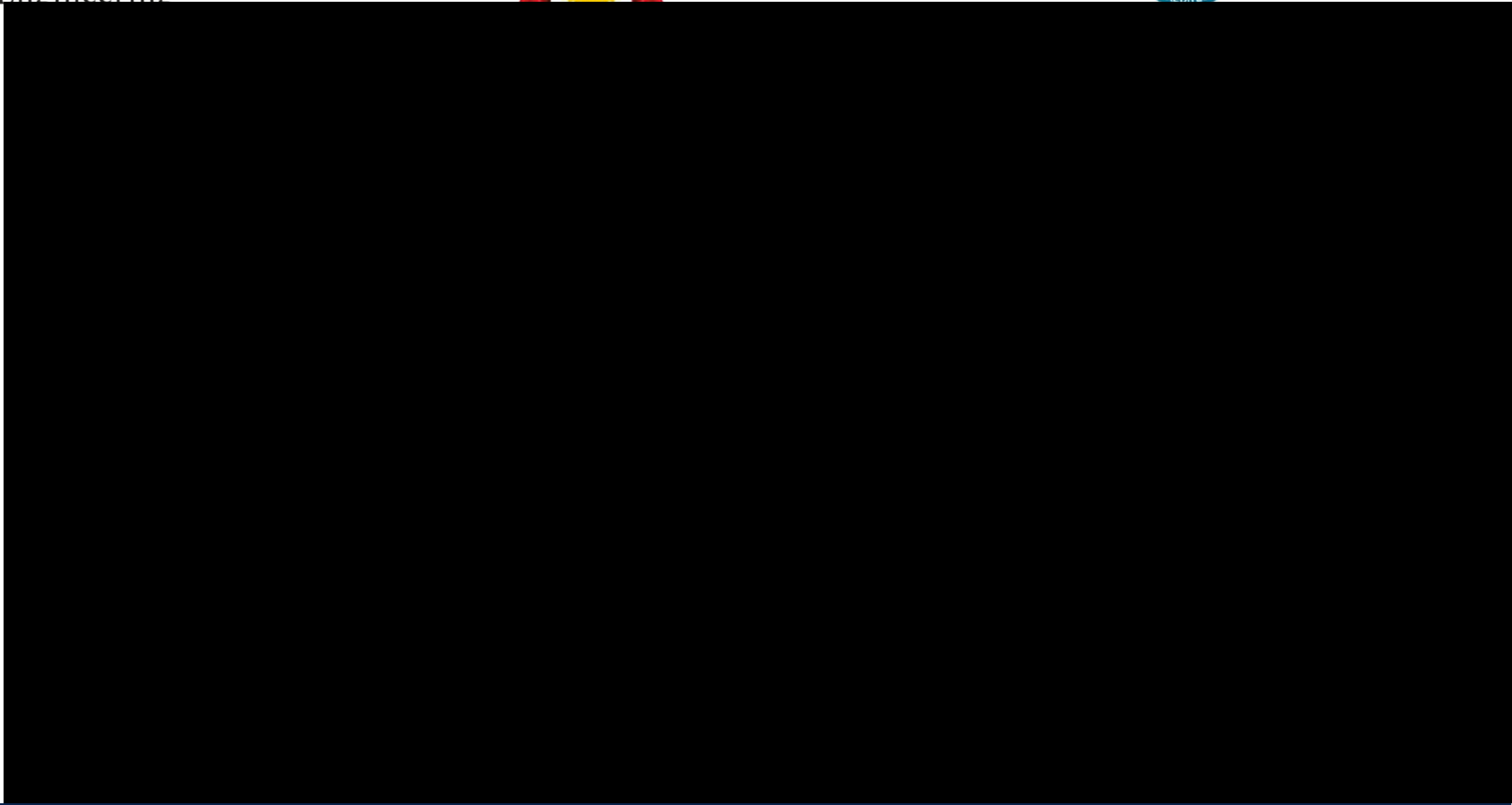


(a) symmetric stretching vibration

(b) flexion movement

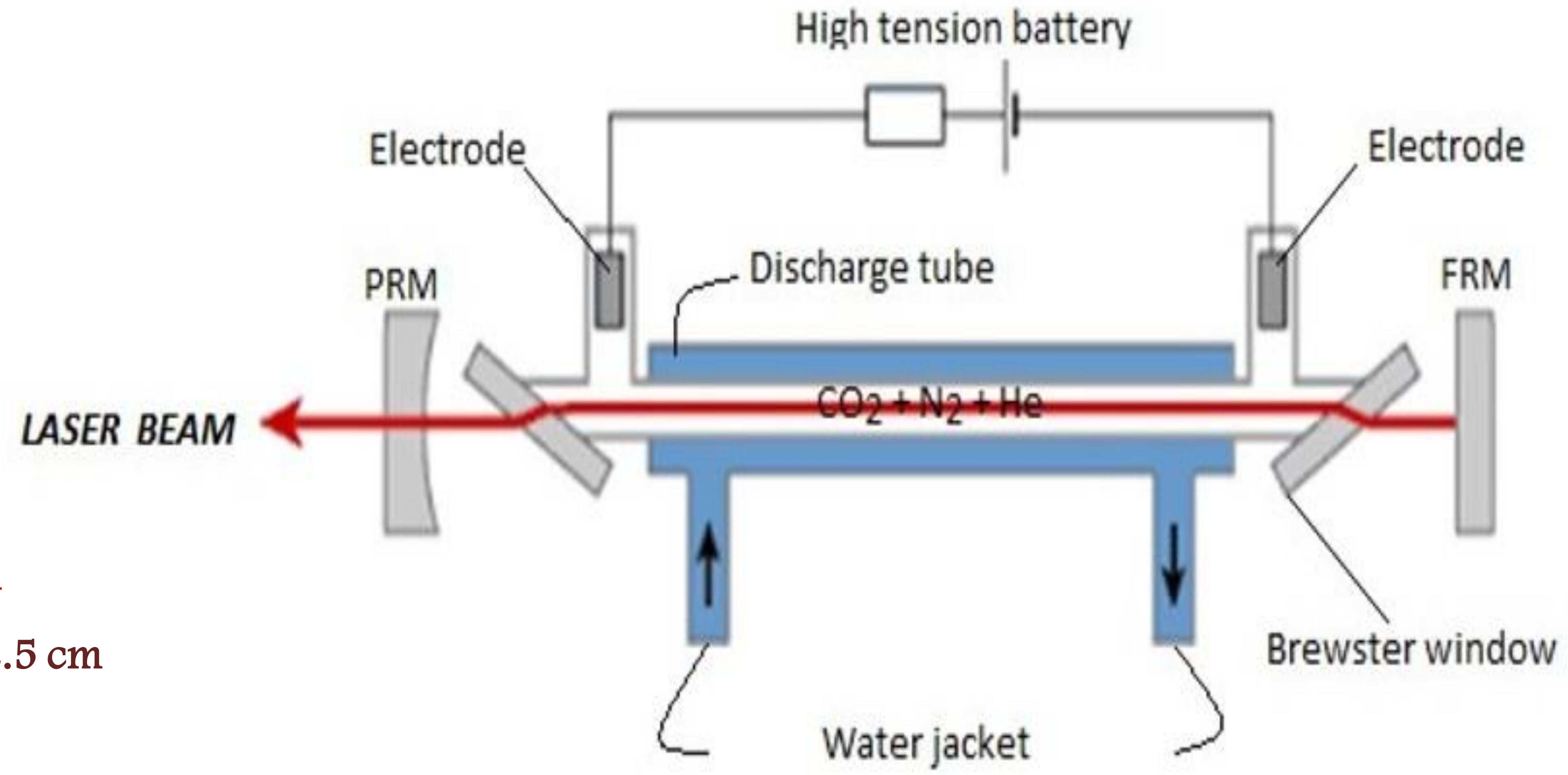
(c) asymmetric stretching vibration

CO_2 Laser..



CO2 Laser..

Construction



Tube length around 5 m

Tube diameter around 2.5 cm

CO_2:N_2: He is 1:2:3

CO_2 Laser..

Working

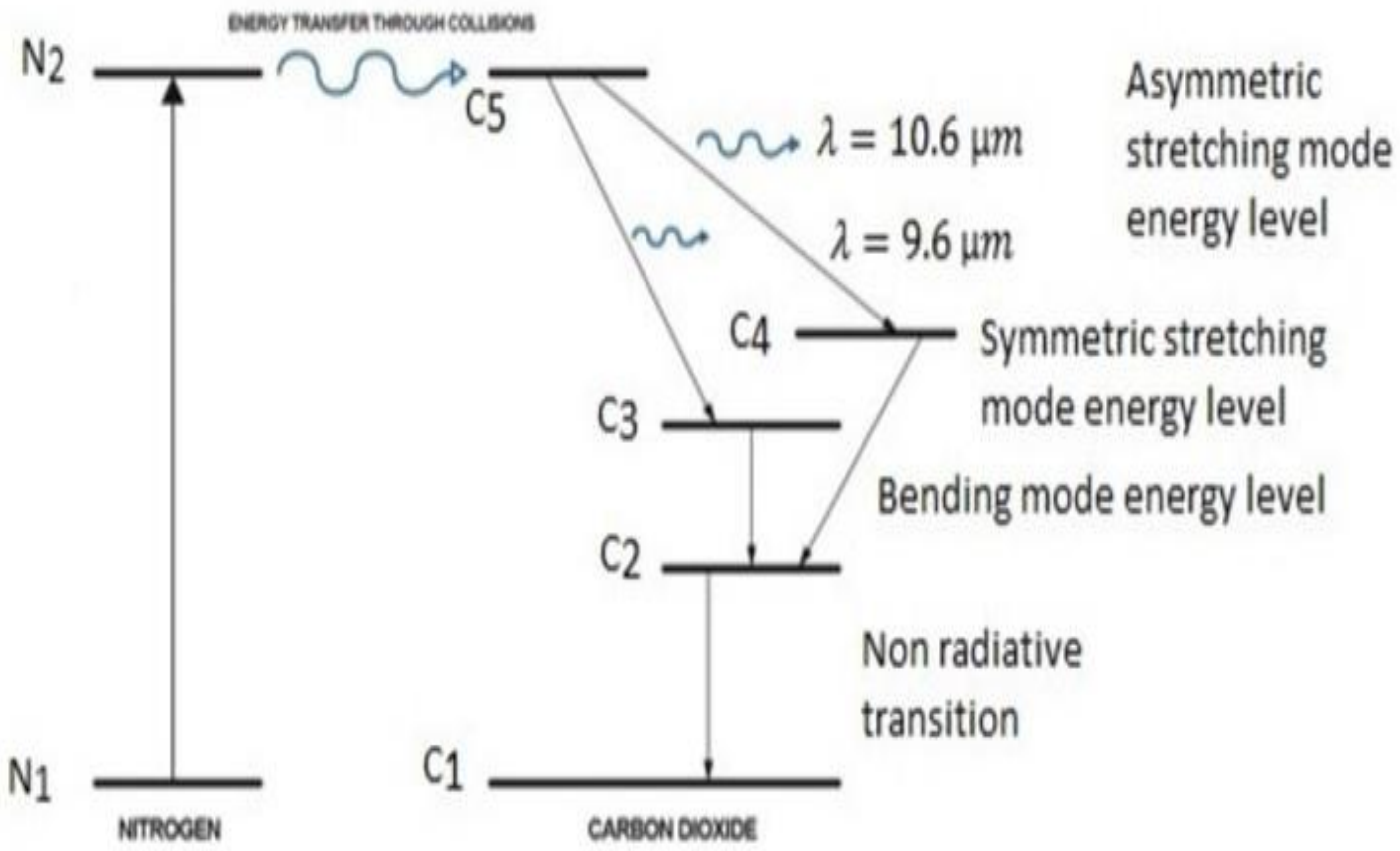
Collision of First kind

$$e^1 + N_2 \rightarrow N_2^* + e^2$$

$$e^3 + CO_2 \rightarrow CO_2^* + e^4$$

Collision of Second kind

$$N_2^* + CO_2 \rightarrow N_2 + CO_2^*$$



CO2 Laser..

Advantages

- It has given continuous & pulsating output
- It is high directional & high monochromatic
- High efficient compared to other Lasers

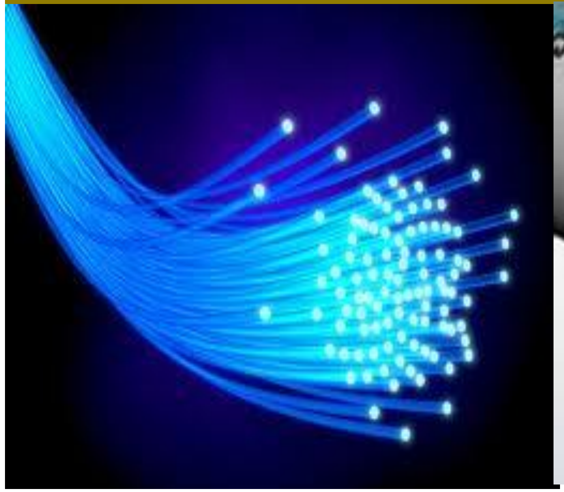
Applications

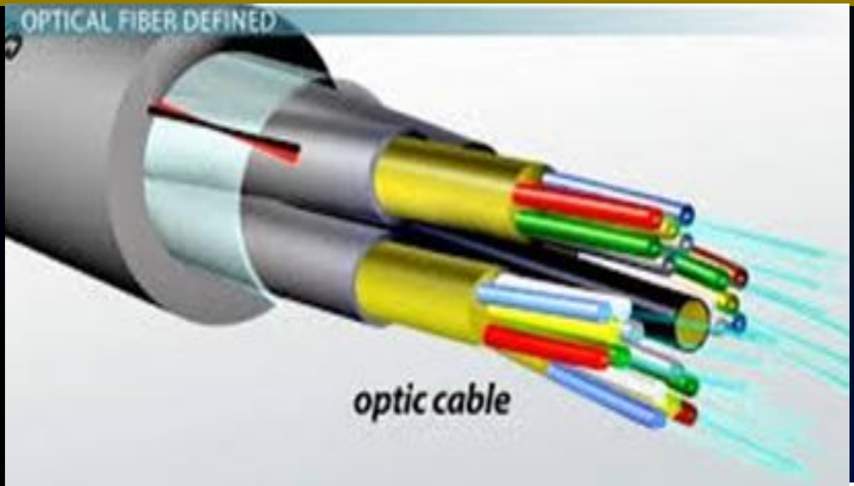
- It is used in industrial applications like welding, cutting, drilling
- It is used in LIDAR due to minimum atmospheric attenuation.
- Used in communication systems.

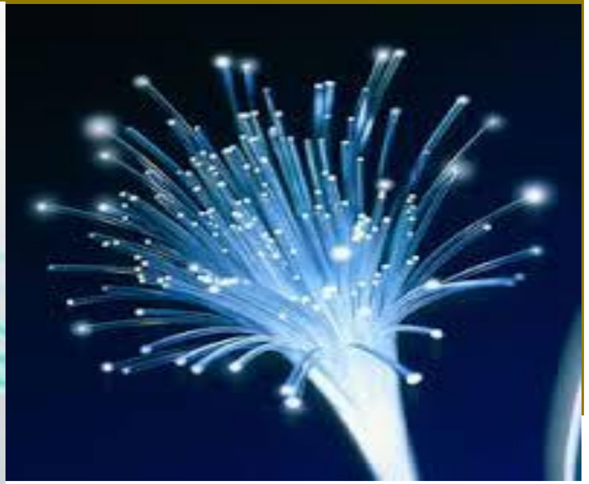
Optical fiber..

Introduction

- Optical fibers are the wires and strands made of transparent dielectrics which guide light over longer distances.
- Many optical fibers are bundled together and are given a protective layer of covering using an insulating material.

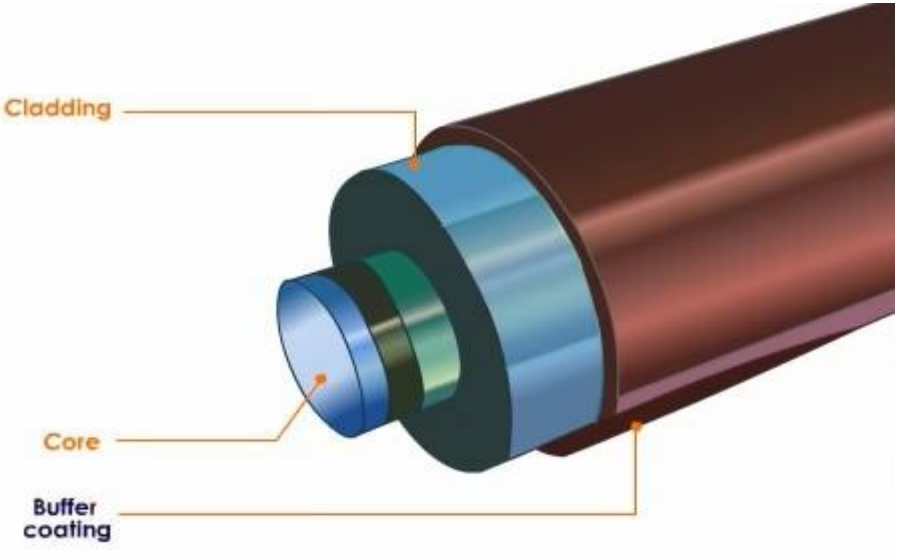


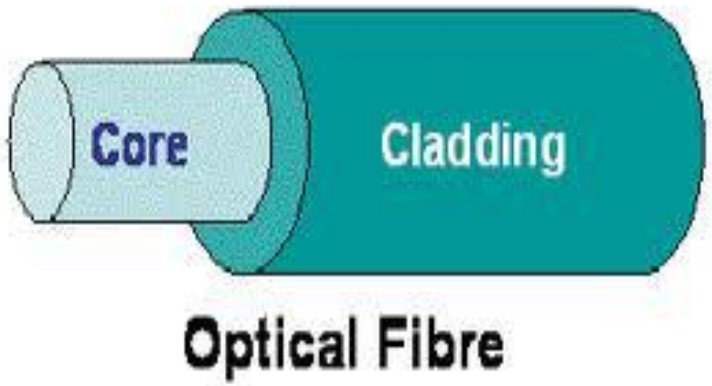




Optical fiber..

Construction

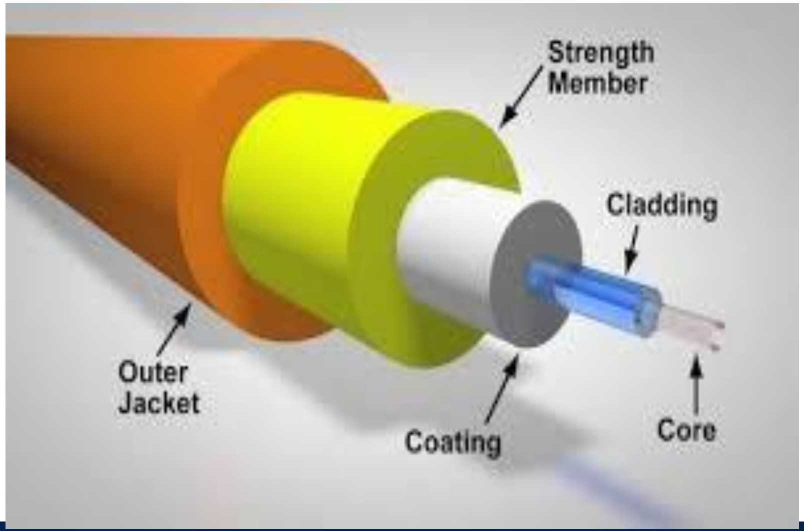


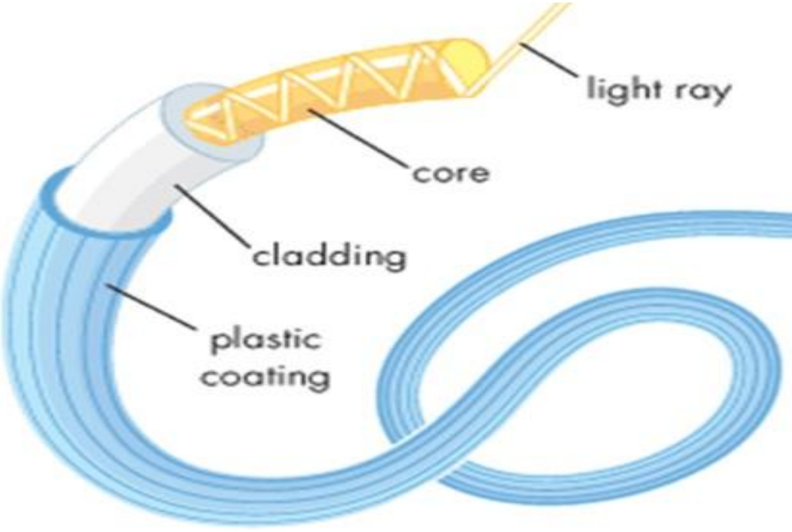


Refractive Index of Core n_1

Refractive Index of Cladding n_2

$(n_1 > n_2)$

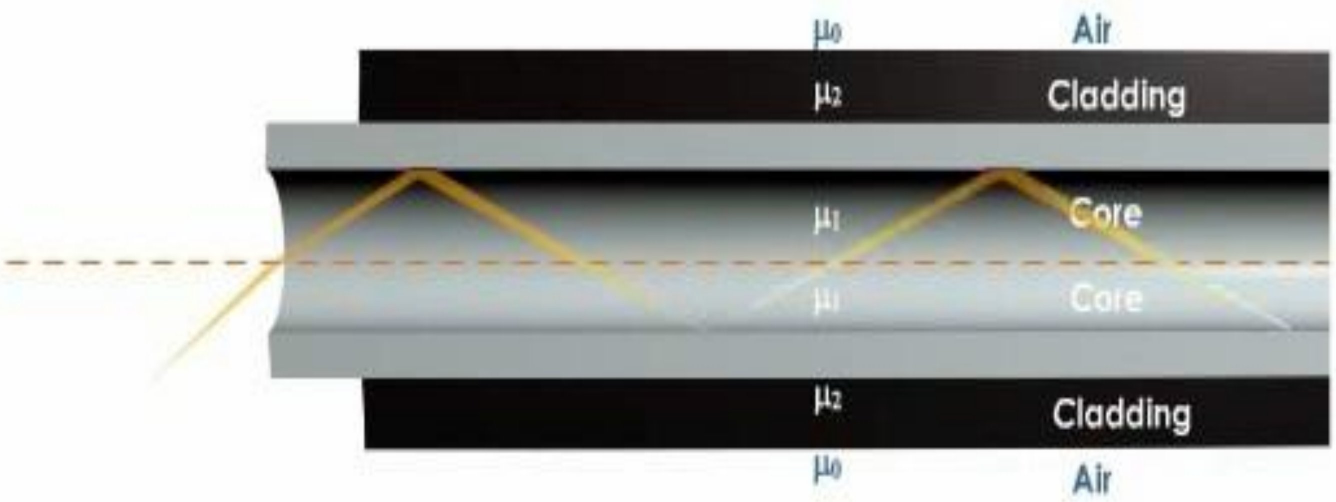


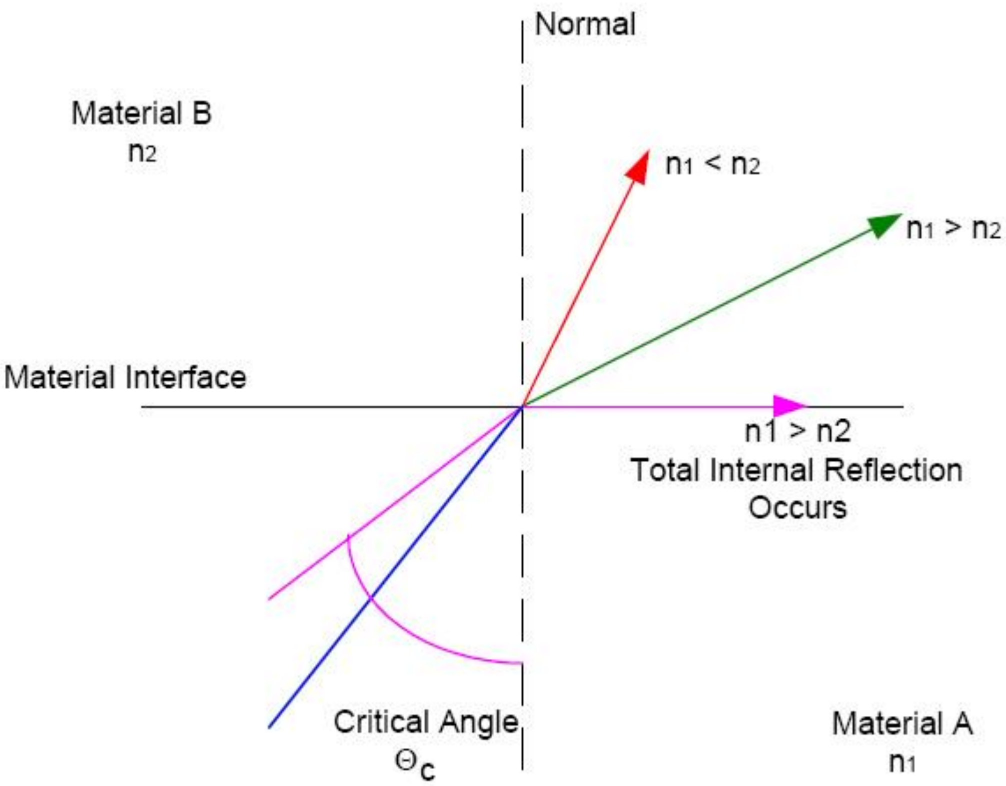


Optical fiber..

Total Internal Reflection (TIR)

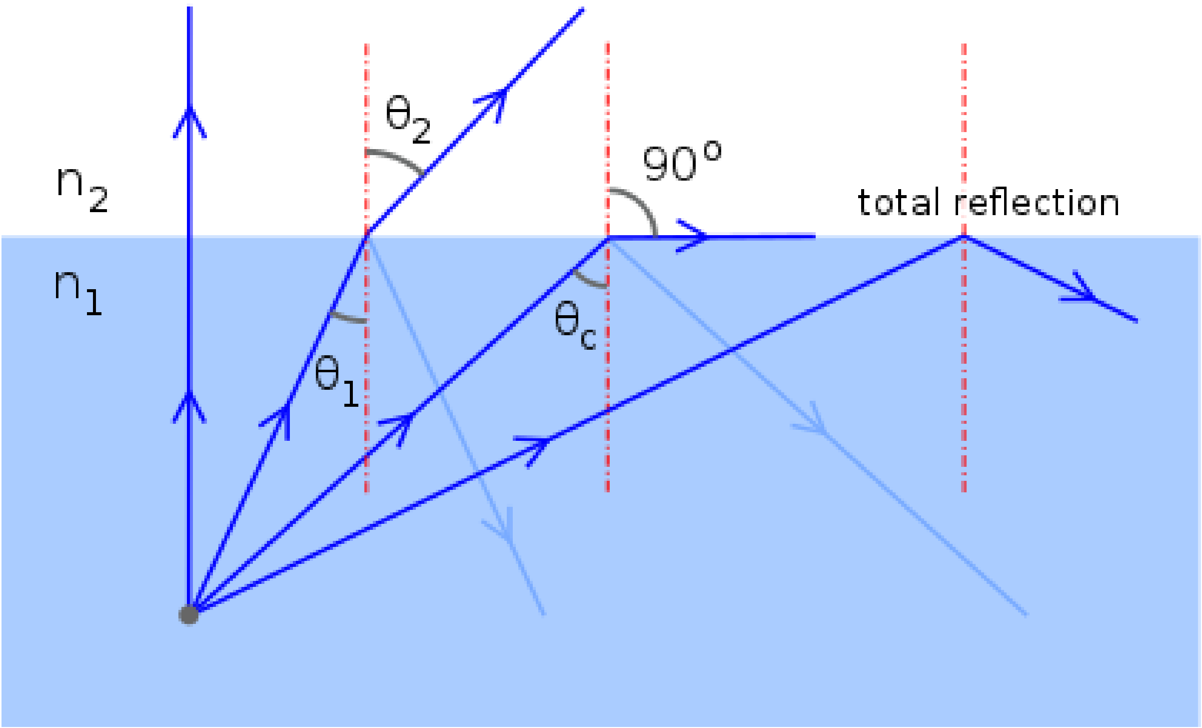
When the angle of incidence is greater than the critical angle, the incident ray is reflected back to the medium. We call this phenomenon total internal reflection





Optical fiber..

TIR



According to Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$when\ \theta_1 = \theta_c\ then,\ \theta_2 = 90^0$$

$$n_1 \sin\theta_c = n_2 \sin 90^0$$

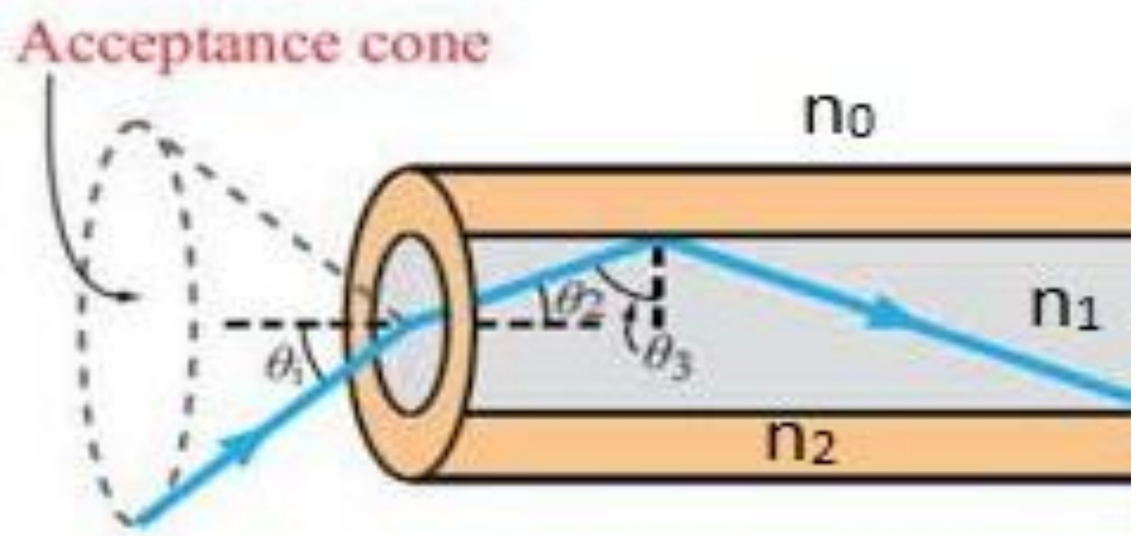
$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Optical fiber..

Acceptance angle (θ_0)

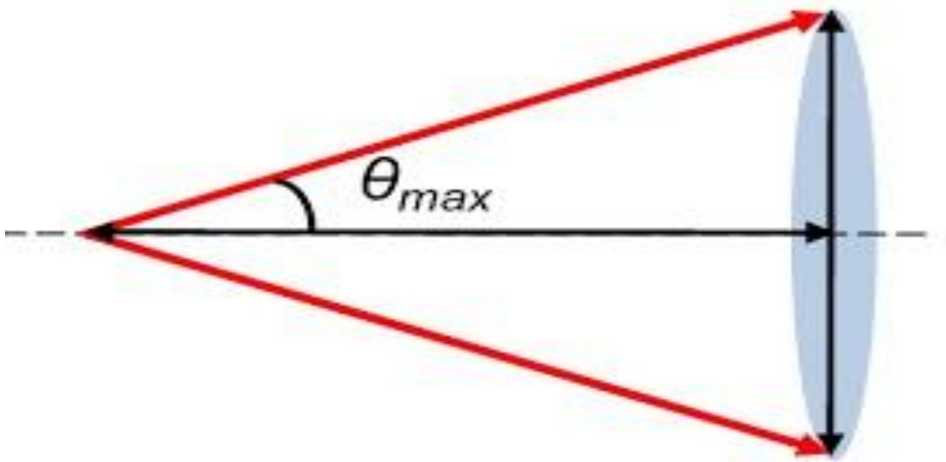
- Acceptance angle is the maximum angle of incidence with which the ray is sent into the fiber core which allows the incident light to be guided by the core.
- It is also called as waveguide acceptance angle or acceptance cone half angle. Denoted by θ_0



Optical fiber..

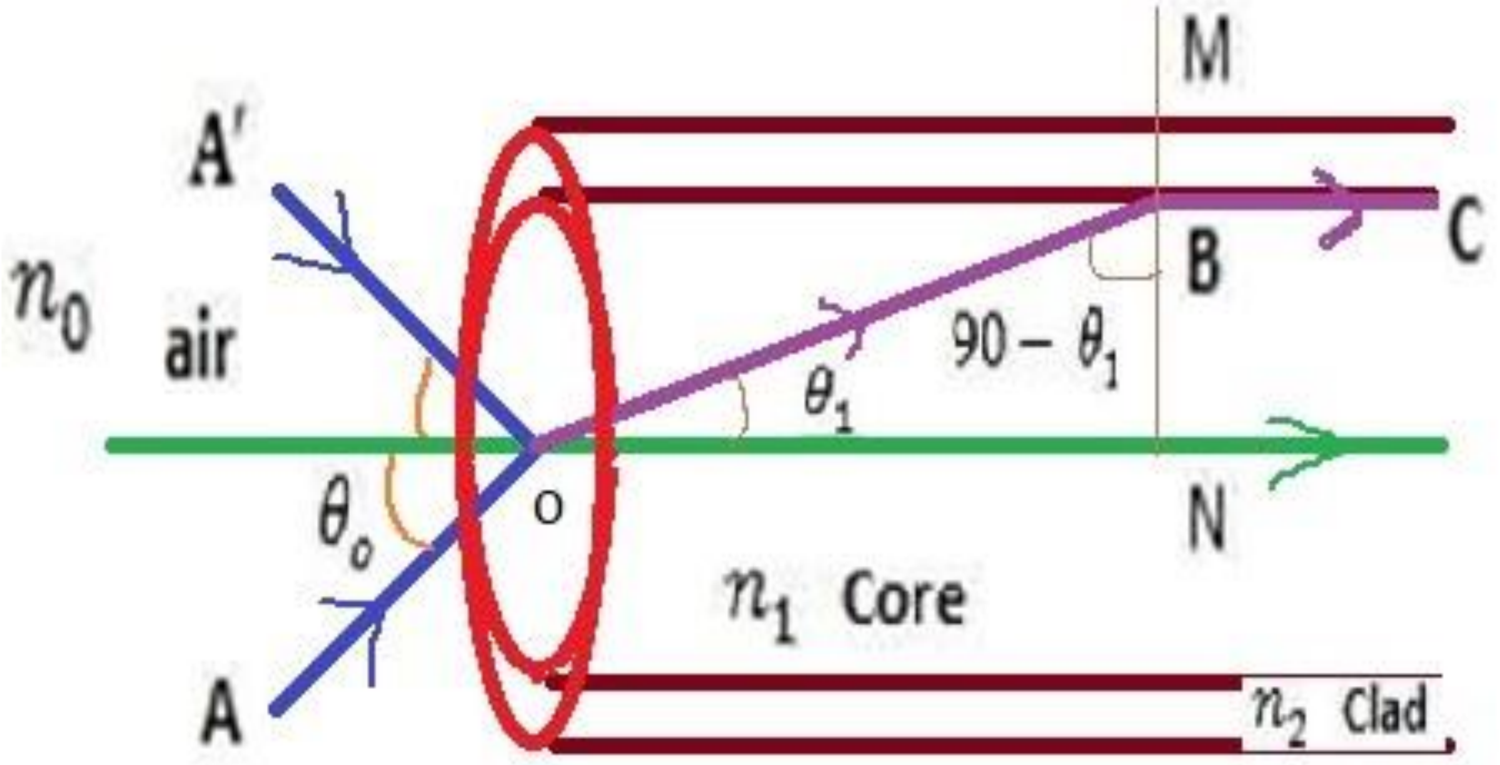
Numerical Aperture (N.A)

- The numerical aperture (N.A) of an optical fiber is a dimensionless number that characterizes the range of angles over which the fiber can accept light.
- Numerical aperture represents the light gathering capability of optical fiber.
- **N.A = sinθo**



Optical fiber..

Condition for propagation



Apply Snell's law at point 'O'

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\therefore \ \sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1$$

Apply Snell's law at point 'B'

$$n_1 \sin(90^0 - \theta_1) = n_2 \sin 90^0$$

$$n_1 \cos\theta_1 = n_2$$

$$\therefore \ \cos\theta_1 = \frac{n_2}{n_1}$$

Optical fiber..

Condition for propagation

We know that,

$$sin^2\theta_1 + cos^2\theta_1 = 1$$

$$sin\theta_1 = \sqrt{1 - cos^2\theta_1}$$

$$sin\theta_1 = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$sin\theta_1 = \sqrt{\frac{{n_1}^2 - {n_2}^2}{{n_1}^2}}$$

$$sin\theta_1 = \frac{1}{n_1}\sqrt{{n_1}^2 - {n_2}^2}$$

We know that,

$$\therefore \; sin\theta_0 = \frac{n_1}{n_0} sin\theta_1$$

$$sin\theta_0 = \frac{n_1}{n_0}\frac{1}{n_1}\sqrt{{n_1}^2 - {n_2}^2}$$

$$sin\theta_0 = \frac{1}{n_0}\sqrt{{n_1}^2 - {n_2}^2}$$

Numerical aperture $N.A = sin\theta_0$

$$N.A = \frac{1}{n_0}\sqrt{{n_1}^2 - {n_2}^2}$$

If the fiber is in air $n_0 = 1$ then,

$$N.A = sin\theta_0 = \sqrt{{n_1}^2 - {n_2}^2}$$

Optical fiber..

Condition for propagation

Light transmitted through the fiber only when

$$\theta_i \le \theta_0$$

$$\sin\theta_i \le \sin\theta_0$$

$$\sin\theta_i \le \sqrt{n_1^2 - n_2^2}$$

$$\sin\theta_i \le N.A$$

Optical fiber..

Fractional Index Change (Δ)

Fractional Index Change / Relative Refractive Index (Δ) is defined as the ratio of difference in refractive indices of core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Optical fiber..

Fractional Index Change (Δ)

Fractional Index Change / Relative Refractive Index (Δ) is defined as the ratio of difference in refractive indices of core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$n_1 \Delta = n_1 - n_2$$

Relation between NA & Δ

consider the equation

$$N.A = sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$N.A = \sqrt{(n_1 + n_2)\ (n_1 - n_2)}$$

For small difference of n_1 & n_2

$$n_1 \approx n_2$$

$$\therefore\ n_1 + n_2 \approx 2n_1$$

$$\therefore\ N.A = \sqrt{(2\, n_1)\ (n_1\, \Delta)}$$

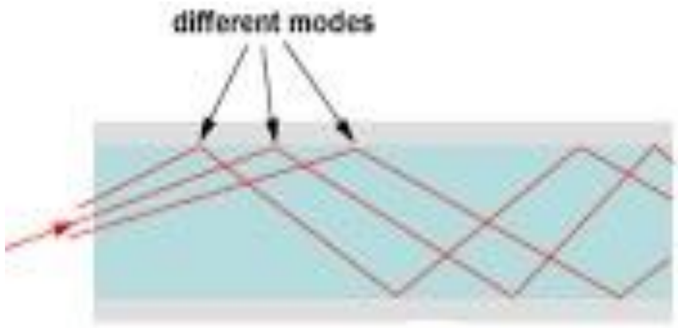
$$N.A = \sqrt{2\, n_1^2\, \Delta}$$

$$N.A = n_1 \sqrt{2\, \Delta}$$

Optical fiber..

V number

This determines the Number of modes supported by an optical fiber for the propagation.



$$V = \frac{\pi\, d}{\lambda}\sqrt{n_1^{\,2} - n_2^{\,2}}$$

$$V = \frac{\pi\, d}{\lambda}\, N.A$$

$$V = \frac{\pi\, d}{\lambda}\,\frac{\sqrt{n_1^{\,2} - n_2^{\,2}}}{n_0}$$

If $V \gg 1$, the number of modes supported by fiber can be determined using the formula

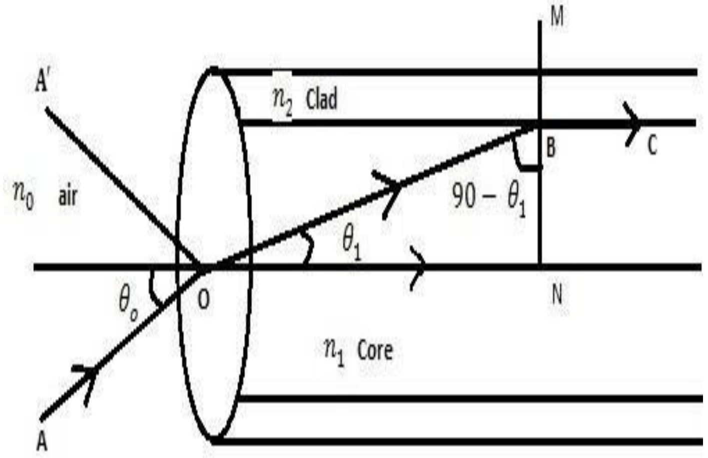
$$N \cong \frac{V^2}{2}$$

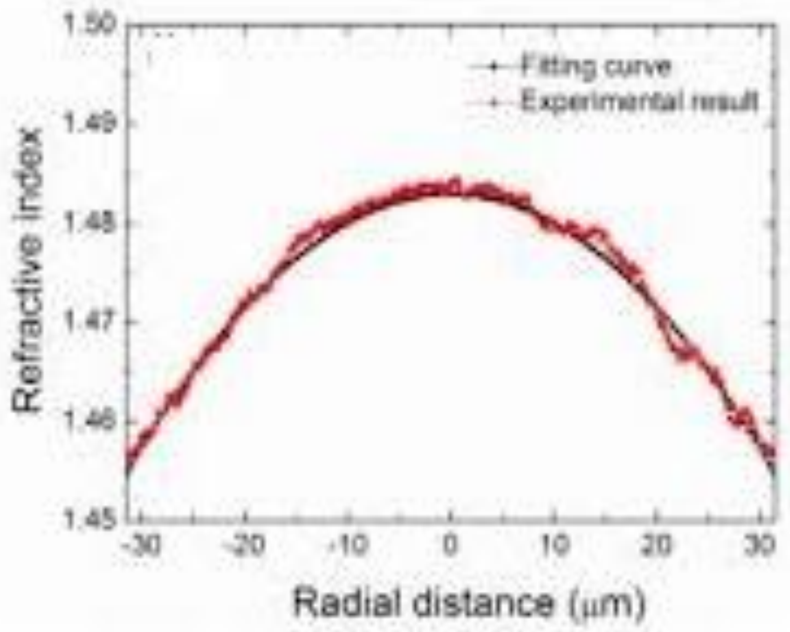
$$N \cong \frac{\pi^2\, d^2}{2\,\lambda^2}\left(\frac{n_1^{\,2} - n_2^{\,2}}{n_0^2}\right)$$

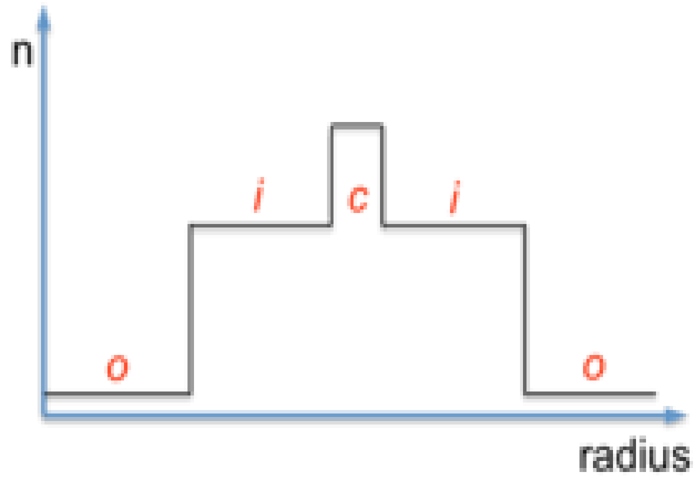
Optical fiber..

Profile Curve

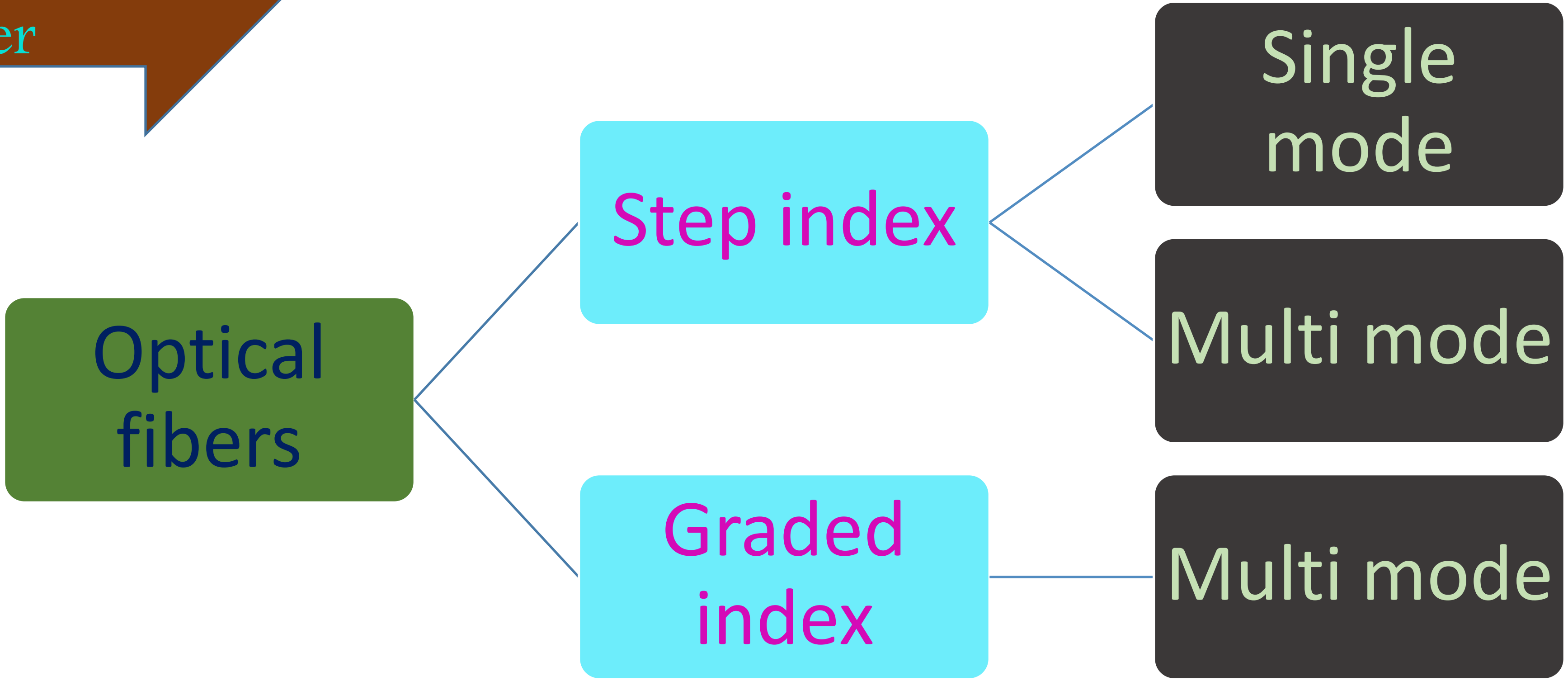
- The curve which represents the variation of refractive index with respect the radial distance from the axis of the fiber is called the refractive index profile.







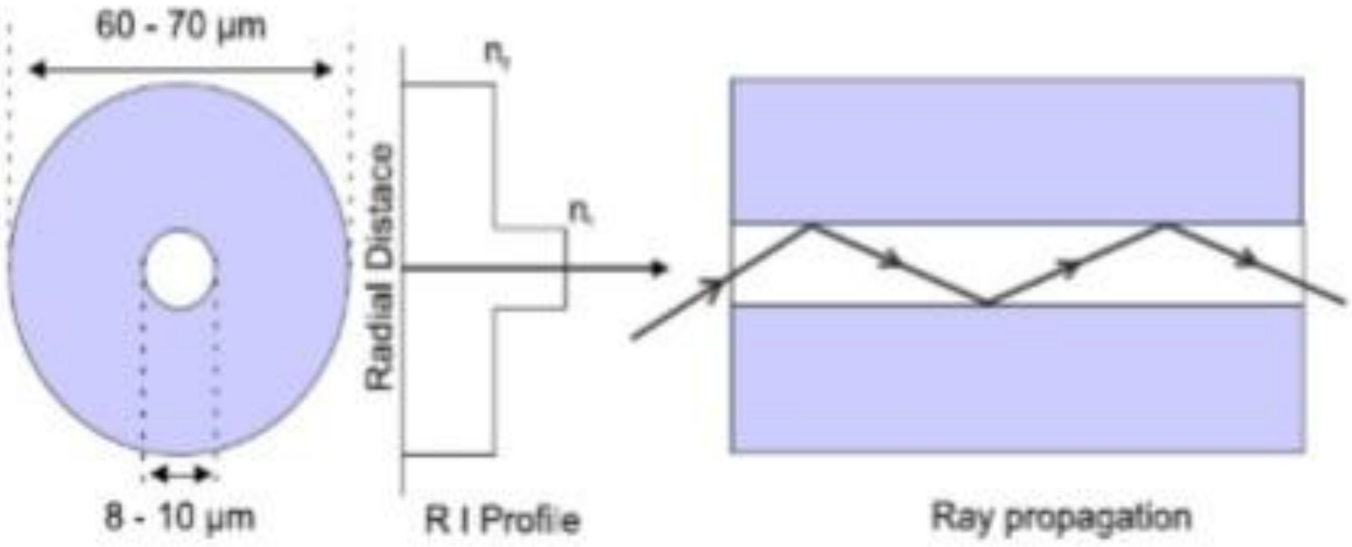
Types of Optical Fiber



Optical fiber..

Step Index Single mode Optical fiber

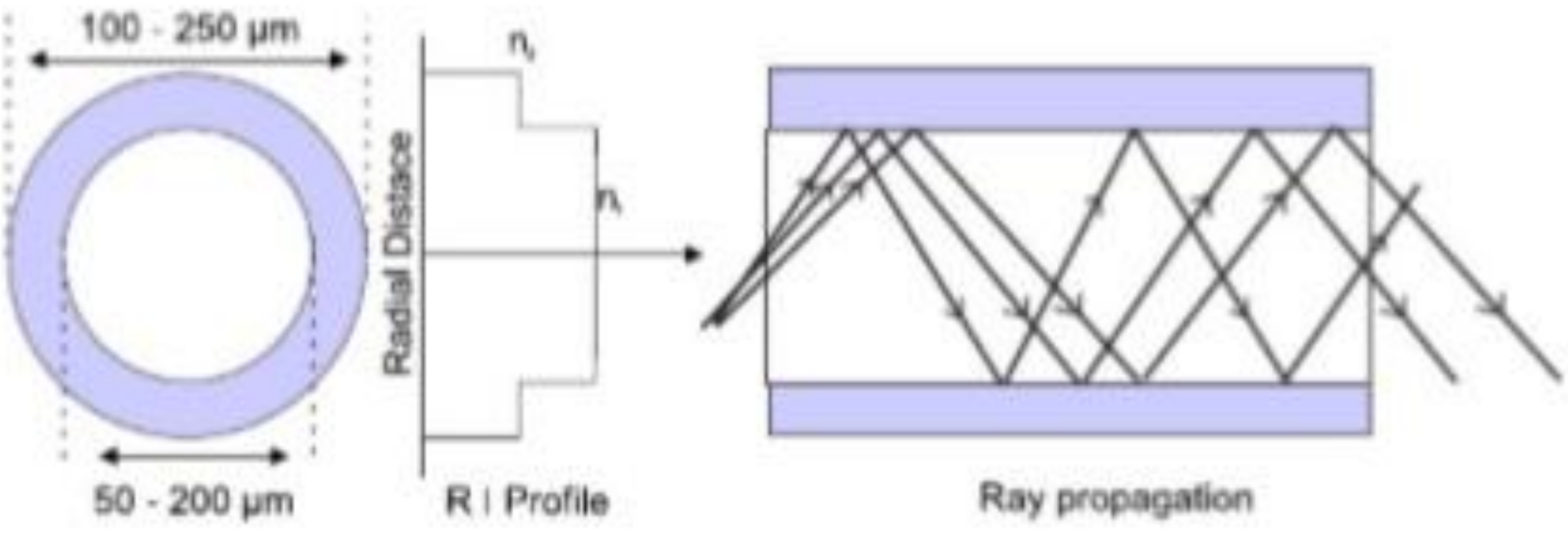
- A single mode step index fiber consists of a very fine thin core of uniform RI surrounded by Cladding of RI lower than that of Core. Since there is abrupt change in the RI of Core and Cladding at the interface it is called step index fiber. Since the Core size is small the Numerical aperture is also small and hence support single mode. They accept light from LASER source. Splicing is difficult. They are used in submarine cables.



Optical fiber..

Step Index
Multi mode
Optical fiber

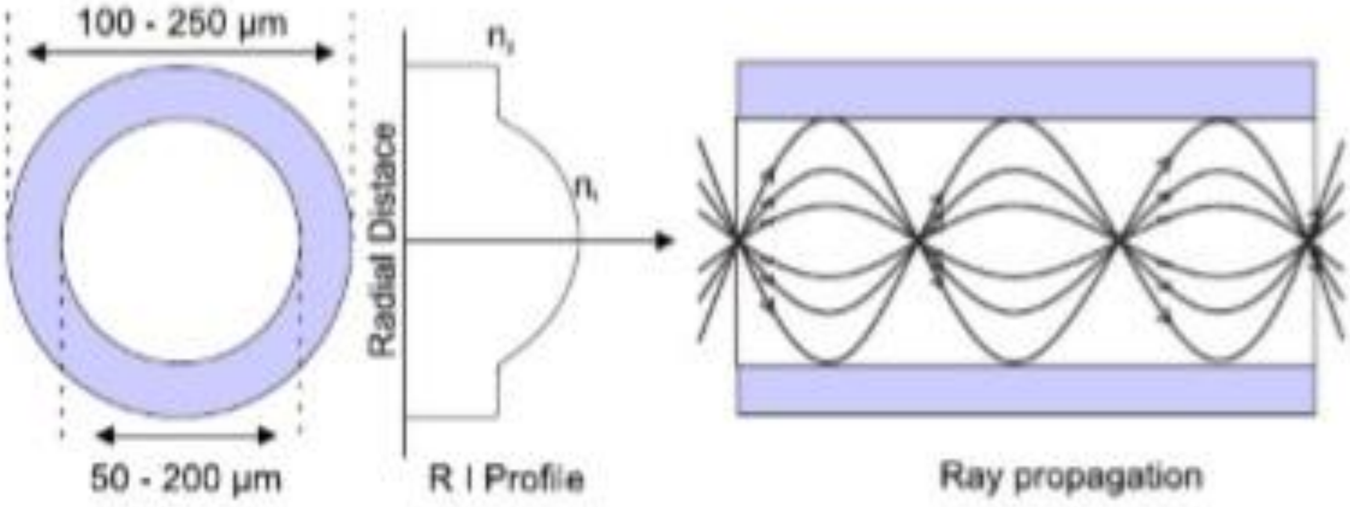
- This is similar to single mode step index fiber but has a larger core diameter. A typical multimode step index fiber is as shown in figure. The numerical aperture is large because of large core size and thus support multi modes. They accept light from both LASER as well as from LED. They are used in data links.



Optical fiber..

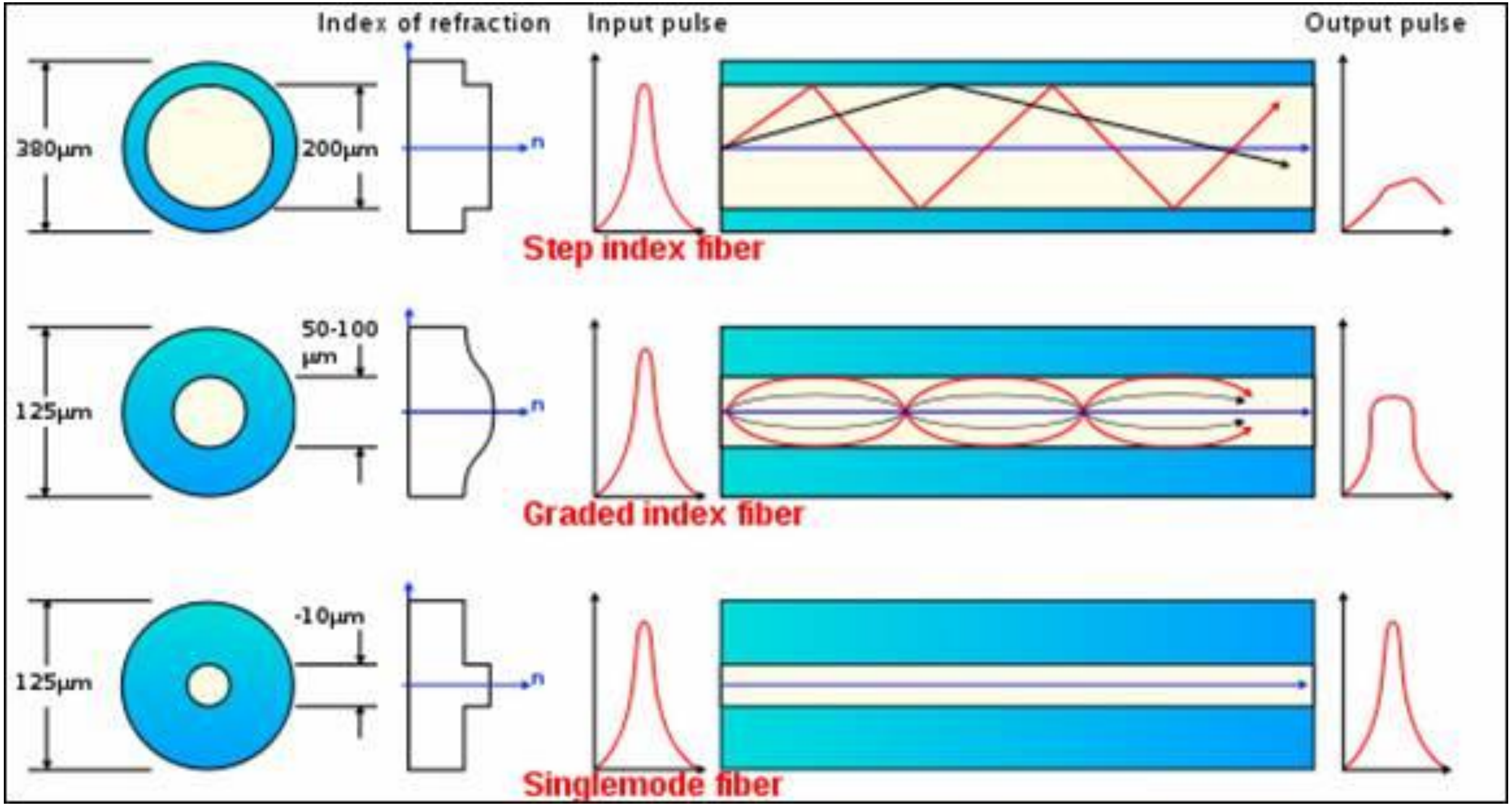
Graded Index Multi mode Optical fiber

- A multimode fiber has concentric layers of RI is called GRIN fiber. It means the R I of the Core varies with distance from the fiber axis. The RI is maximum at the center and decreases with radial distance towards to core cladding interface. In GRIN fibers the acceptance angle and numerical aperture diminish with radial distance. They accept light from both LASER &LED. They are used for telephone link between central offices.



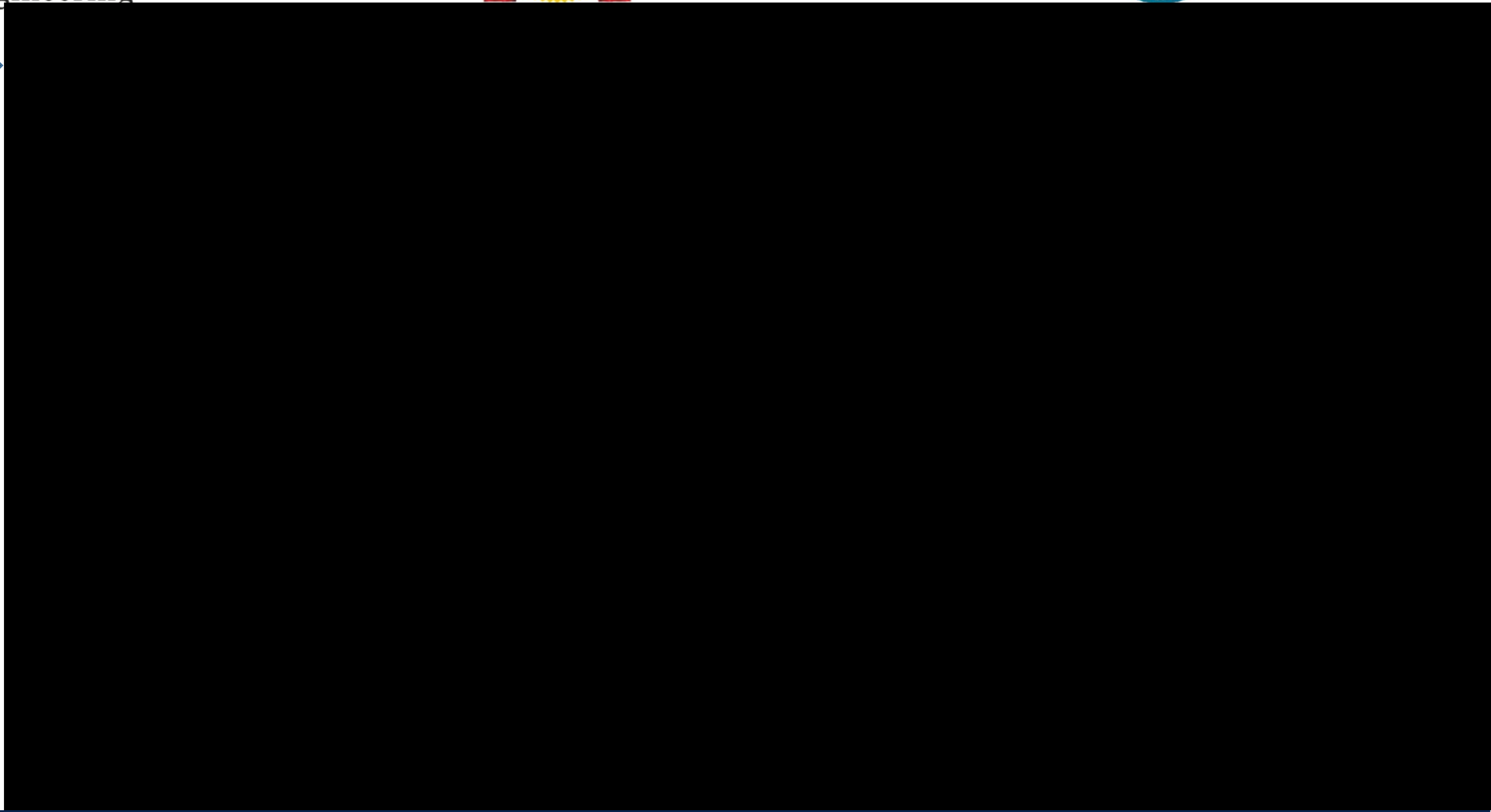
Optical fiber..

Types



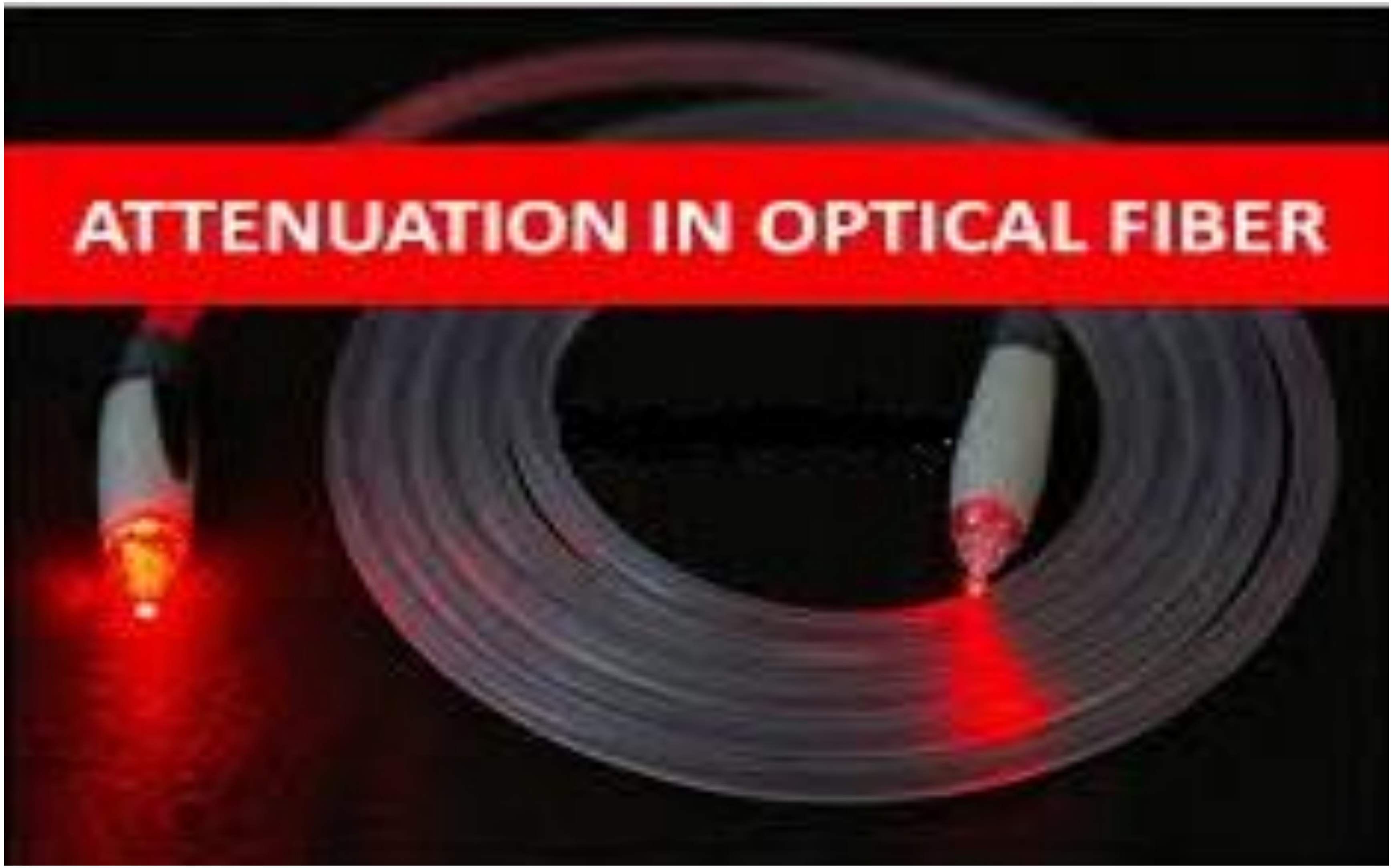
Optical fiber..

Types



Optical fiber..

Attenuation



Optical fiber..

Attenuation

- Attenuation is the loss of power suffered by the optical signal as it propagates through the fiber.
- It is also called the fiber loss or significant loss.
- The power loss in is given by,

$$P_L = -10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

Attenuation co-efficient (α)

- The attenuation is measured in terms of attenuation co-efficient.
- The attenuation co-efficient α is defined as the ratio of optical power output to the optical power input for a fiber of length L and for a given wavelength of propagating light.
- Attenuation co-efficient is given by

$$\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad dB/km$$

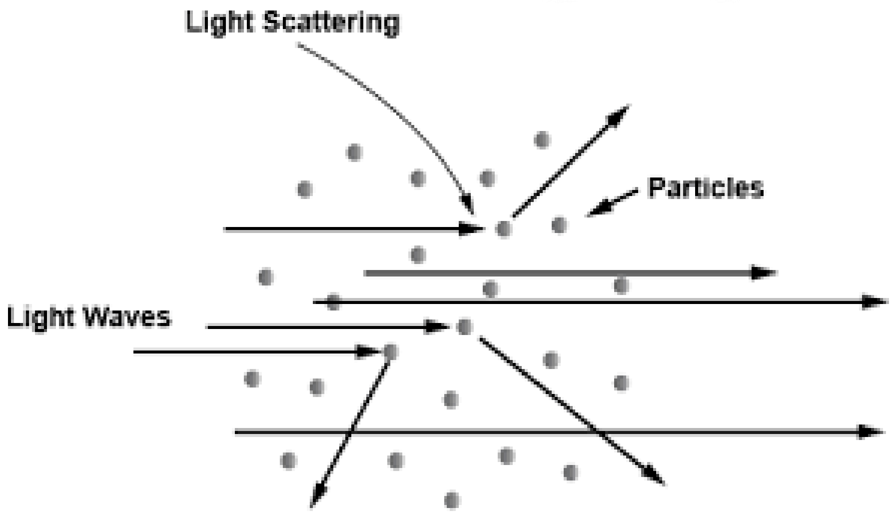
- It is expressed in dB/km.

Optical fiber..

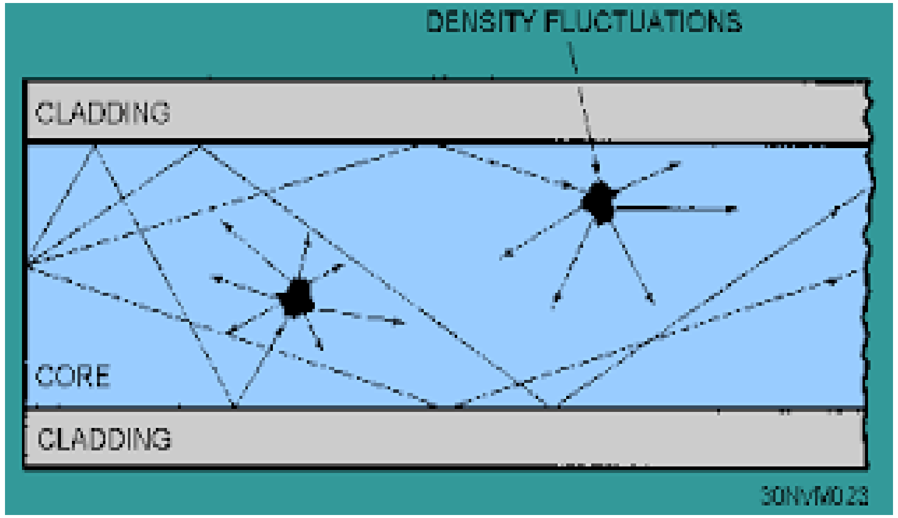
Types of Attenuation

- Attenuation
  - Absorption losses
  - Scattering loss
  - Radiation losses

Optical fiber..



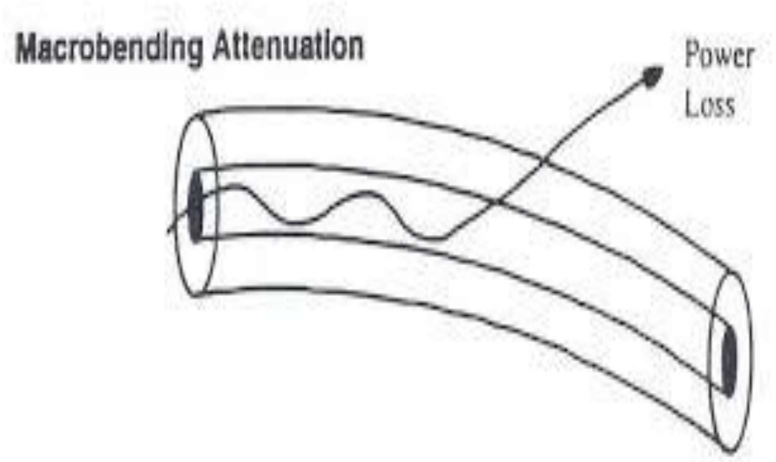
Absorption Loss

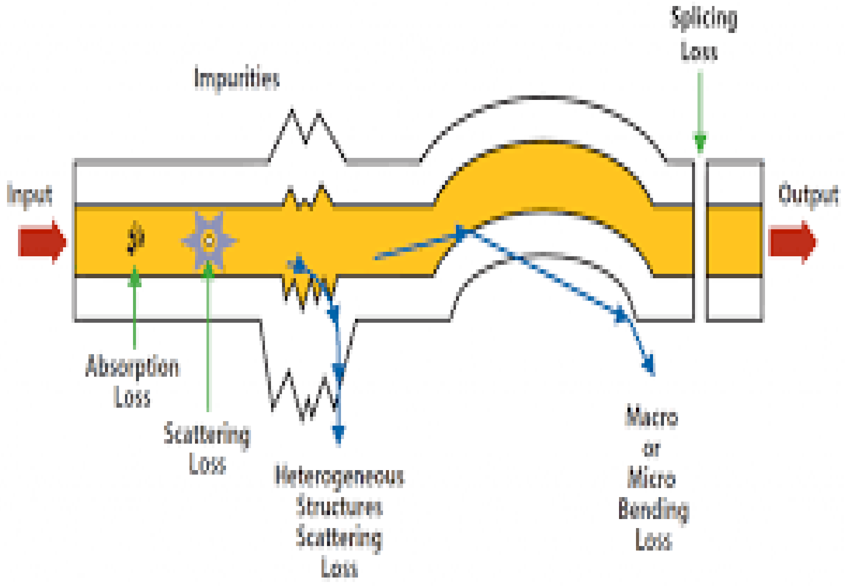


Scattering Loss

Optical fiber..

Radiation Loss





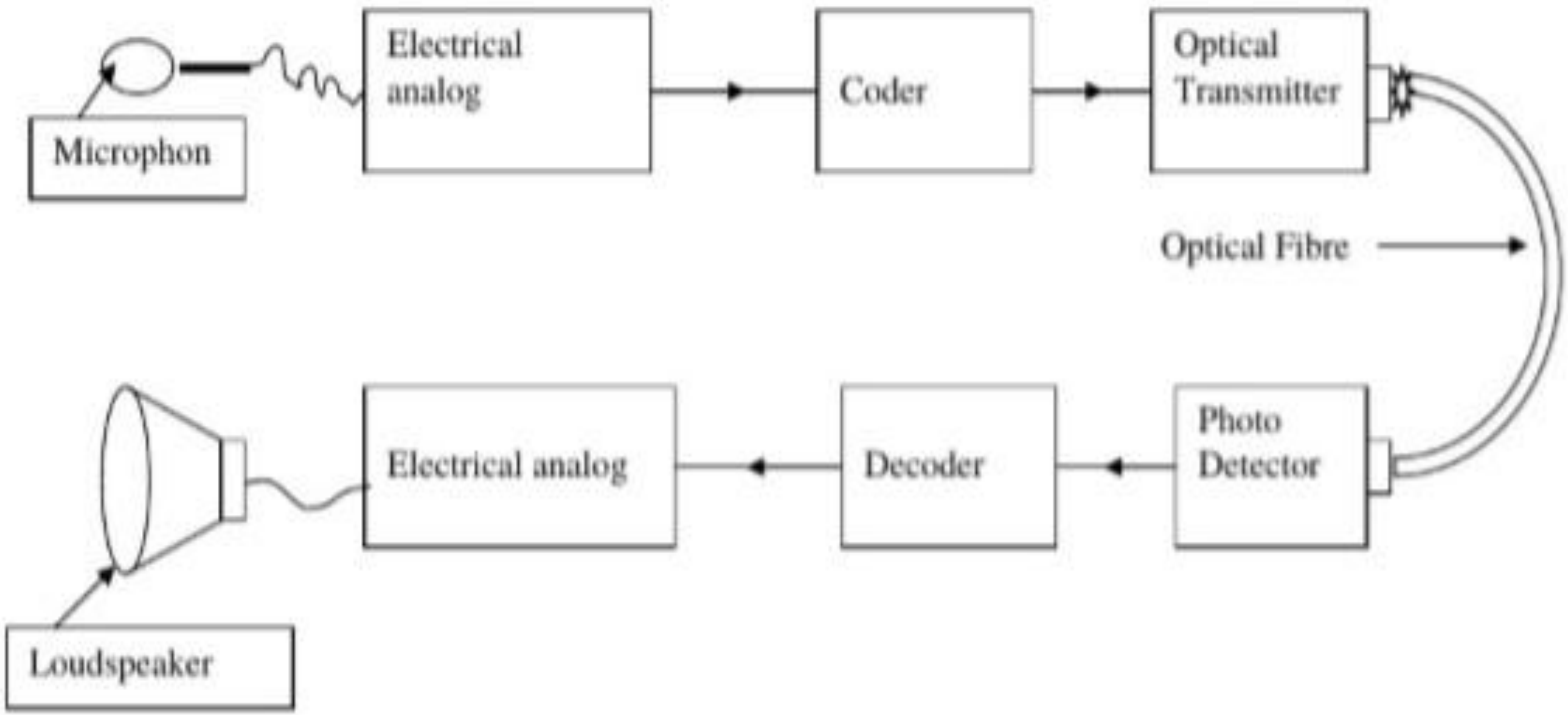
Optical fiber..

Point to point communication

- If data transfer takes place between only two devices then, it is called point to point communication.
- Optical fibers are used in communication to transmit signals for long distances.
- An optical signal derived from electrical analog signal is transmitted through the optical fiber. At the other end again the optical signal is converted into electrical signal.
- In a conventional optical fibre communication system, the input signals, audio, video or other digital data, are used to modulate light from a source like a LED or a semiconductor laser and is transmitted through optical fiber.
- At the receiving end, the signal is demodulated to reproduce the input signal.

Optical fiber..

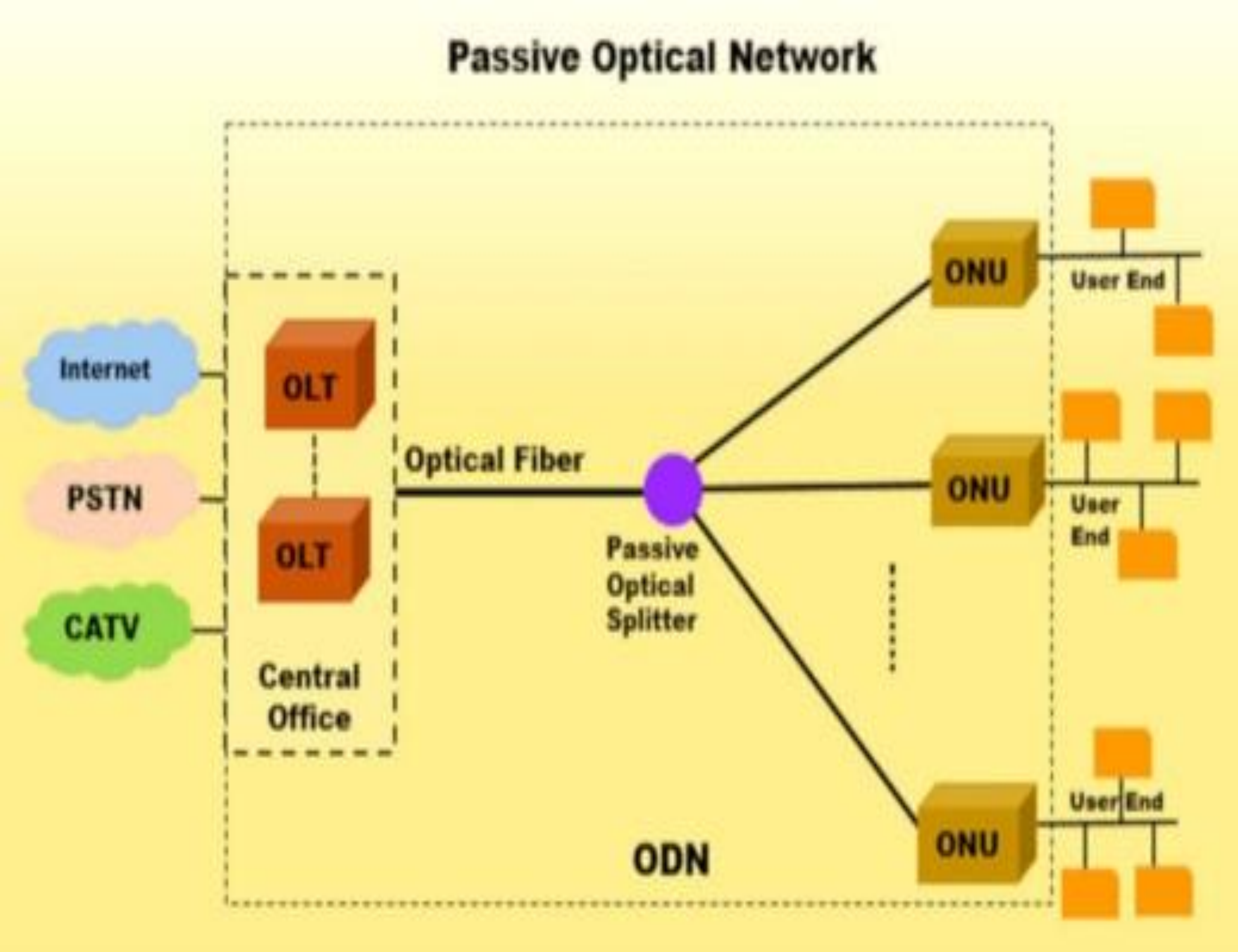
Point to point communication



Fiber Optic Networking
Local Area Network (LAN)

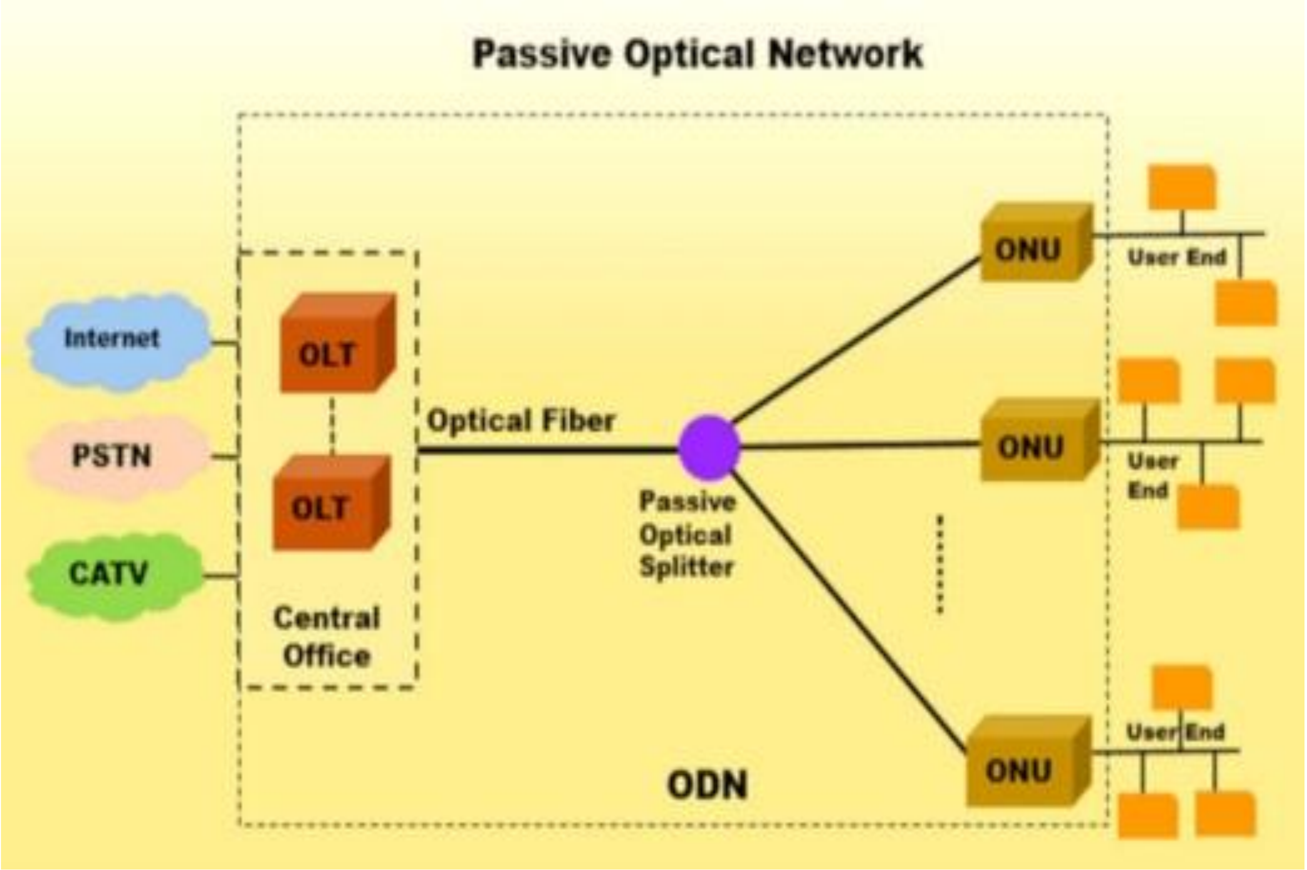
- A Fiber Optic Local Area Network (LAN) is a type of computer network that interconnects multiple computers and computer-driven devices in a particular physical location using optical fibers and devices.

- Traditionlly copper coaxial cables are used for for LAN.



Fiber Optic Networking
Local Area Network (LAN)

1. PON - Passive Optical Network
2. ONT - Optical Network Terminal
3. ODN - Optical Distribution Network
4. OLT - Optical Line Terminal
5. ONU - Optical Network Unit



Passive Optical LAN

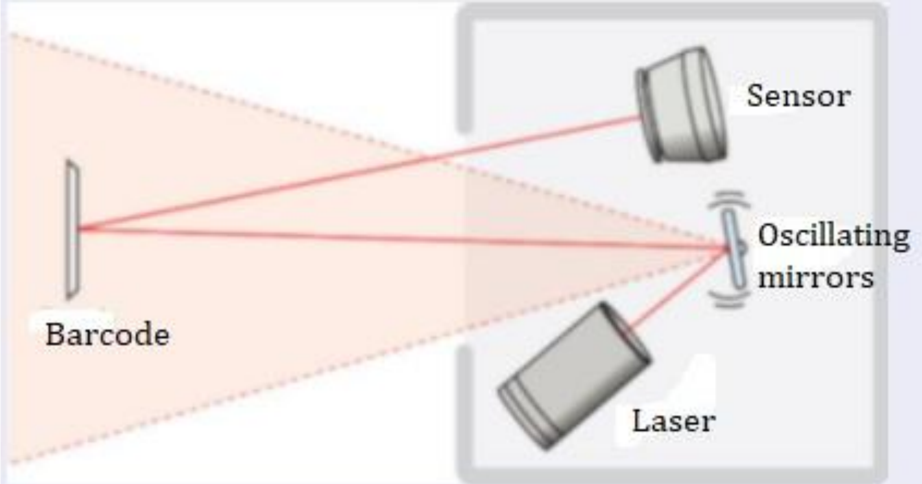
- A passive optical network refers to a fiber-optic network utilizing a point-to-multipoint topology and optical splitters to deliver data from a single transmission point to multiple user endpoints.

- Passive here refers to the unpowered condition of the opotical fiber and splitting/combining components.

- The passive optical LAN works on the concept of optical network terminals (ONT) and passive optical splitters.

- Network switches act as passive splitters and the commercial media converters act as optical network terminals in a real-time application of passive optical LAN.

LASER Barcode Scanner/Reader



- A barcode is a printed series of parallel bars or lines of varying width that is used for entering data into a computer system.
- A barcode scanner uses Light, lenses, and a sensor that decodes and captures the information contained in barcodes.
- Laser scanners use LASER and employ oscillating mirrors to scan the laser beam back and forth across the barcode.
- A photodiode then measures the reflected light from the barcode. Anolog signal created is digitized.

Numerical problems on Optical fiber

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$N.A = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$N.A = \sin\theta_0 = \frac{1}{n_m}\sqrt{n_1^2 - n_2^2}$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$V = \frac{\pi d}{\lambda}\frac{\sqrt{n_1{}^2 - n_2{}^2}}{n_0}$$

$$N \cong \frac{V^2}{2} \text{ (for V>>1)}$$

$$N = \frac{\pi^2 d^2}{2\lambda^2}(n_1{}^2 - n_2{}^2)$$

$$\alpha = \frac{-10}{L}\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\frac{P_{out}}{P_{in}} = 10^{\left(\frac{-\alpha L}{10}\right)}$$

Numerical problems on Optical fiber

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$N.A = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$N.A = \sin\theta_0 = \frac{1}{n_m}\sqrt{n_1^2 - n_2^2}$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\alpha = \frac{-10}{L}\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\frac{P_{out}}{P_{in}} = 10^{\left(\frac{-\alpha L}{10}\right)}$$

1. The Refractive indices of Core and Cladding are 1.50 and 1.48 respectively in an Optical fiber. Find the Numerical aperture and angle of acceptance.

Given

$n_1 = 1.50,\ n_2 = 1.48.$

$n_0 = 1$ (for air medium)

$N.A = ?\quad \theta_0 = ?$

Solution

The Numerical aperture of the given fiber is

$$N.A = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0}$$

$$N.A = \frac{\sqrt{1.50^2 - 1.48^2}}{1}$$

$$N.A = 0.244$$

The acceptance angle θ_0 is related to $N.A$ given by

$$N.A = \sin\theta_0$$

$$\theta_0 = sin^{-1}(N.A)$$

$$\theta_0 = sin^{-1}(0.244)$$

$$\theta_0 = 14.1^0$$

2. Calculate the numerical aperture, acceptance angle and critical angle for a fiber with core and cladding of refractive indices 1.50 and 1.45 respectively.

Given

$n_1 = 1.50,\ n_2 = 1.45.\ n_0 = 1,\ N.A = ?,\quad \theta_0 = ?\quad \theta_c = ?$

Solution

The numerical aperture is given by

$$N.A = \frac{\sqrt{n_1{}^2 - n_2{}^2}}{n_0}$$

$$N.A = \frac{\sqrt{1.5^2 - 1.45^2}}{1}$$

$$N.A = 0.384$$

The acceptance angle is given by,

$$\theta_0 = sin^{-1}(N.A)$$

$$\theta_0 = sin^{-1}(0.384)$$

$$\theta_0 = 22.58^0$$

The critical angle is given by,

$$\theta_c = sin^{-1}(\frac{n_2}{n_1})$$

$$\theta_c = sin^{-1}(\frac{1.45}{1.5})$$

$$\theta_c = 75.16^0$$

3. An optical fiber has a numerical aperture of 0.32. The refractive index of cladding is 1.48. Calculate the refractive index of the core and the fractional index change.

Given

$N.A = 0.32,\ n_2 = 1.48,\ n_1 = ?\ \Delta = ?$

Solution

For n_1 we have

$$N.A = \sqrt{n_1^2 - n_2^2}$$

$$N.A^2 = n_1^2 - n_2^2$$

$$n_1^2 = N.A^2 + n_2^2$$

$$n_1 = \sqrt{N.A^2 + n_2^2}$$

$$n_1 = \sqrt{0.32^2 + 1.48^2}$$

$$n_1 = 1.514$$

For Δ we have

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\Delta = \frac{1.514 - 1.48}{1.514}$$

$$\Delta = 0.022$$

4. An optical fiber has core of RI 1.5 and has cladding of RI which is 3 % less than RI of core index. Calculate numerical aperture, angle of acceptance and critical angle.

Given

$n_1 = 1.5, n_0 = 1, n_2 = n_1 - \frac{3}{100}n_1, \ N.A = ?, \quad \theta_0 = ? \quad \theta_c = ?$

Solution

The RI of Cladding,

$$n_2 = n_1 - \frac{3}{100}n_1$$

$$n_2 = 1.5 - \frac{3}{100}1.5$$

$$n_2 = 1.455$$

The numerical aperture is given by

$$N.A = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$N.A = \frac{\sqrt{1.5^2 - 1.455^2}}{1}$$

$$N.A = 0.3646$$

The acceptance angle is given by,

$$\theta_0 = sin^{-1}(N.A)$$

$$\theta_0 = sin^{-1}(0.3646)$$

$$\theta_0 = \dots\dots$$

The critical angle is given by,

$$\theta_c = sin^{-1}(\frac{n_2}{n_1})$$

$$\theta_c = sin^{-1}(\frac{1.455}{1.5})$$

$$\theta_c = \dots\dots\dots\dots$$

5. Calculate the Numerical aperture and the acceptance angle for an Optical fiber having refractive indices 1.563 and 1.498 for core and cladding respectively.

Given

$n_1 = 1.563,\ n_2 = 1.498.$

$n_0 = 1$ (for air medium)

$N.A = ?\quad \theta_0 = ?$

Solution

The Numerical aperture of the given fiber is

$$N.A = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0}$$

$$N.A = \frac{\sqrt{1.563^2 - 1.498^2}}{1}$$

$$N.A =$$

The acceptance angle θ_0 is related to $N.A$ given by

$$N.A = \sin\theta_0$$

$$\theta_0 = sin^{-1}(N.A)$$

$$\theta_0 = sin^{-1}(\dots\dots\dots.)$$

$$\theta_0 =$$

6. The numerical aperture of the give optical fiber is 0.2 when surrounded by air medium. Determine the Refractive index of its core given the RI of the Cladding 1.59. Also find the Acceptance angle when the fiber is in water. Assume the RI of the water to be 1.33.

Given

$N.A_a = 0.2,\ n_2 = 1.59.$ For water medium, $n_0 = 1.33.\ n_1 = ?\qquad \theta_0 = ?$

Solution

When the Optical fiber placed in air medium, the Numerical aperture is

$$N.A_a = \sqrt{{n_1}^2 - {n_2}^2}$$

$$\sqrt{{n_1}^2 - 1.59^2} = 0.2 \text{ .... (1)}$$

$${n_1}^2 - 1.59^2 = 0.04$$

$${n_1}^2 = 0.04 + 1.59^2$$

$$n_1 = 1.6$$

When the Optical fiber placed in water medium, the Numerical aperture is

$$N.A_w = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0}$$

$$N.A_w = \frac{0.2}{1.33} \quad \text{(From equation 1)}$$

$$N.A_w = 0.01504$$

Also we have,

$$N.A_w = \sin(\theta)$$

$$\theta = sin^{-1}(N.A_w)$$

$$\theta = sin^{-1}(0.01504)$$

$$\theta = 8.65^0.$$

7. The angle of acceptance of an optical fiber is 30° when kept in air. Find the angle of acceptance when it is in a medium of Refractive index 1.33.

Given

$\theta_0 = 30^0, \quad \theta_0' = ? \quad n_0' = 1.33.$

Solution

Consider

$$N.A = \sin\theta_0 = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

For air medium, $n_0 = 1, \theta_0 = 30^0$

$$\sin 30^0 = \sqrt{n_1^2 - n_2^2}$$

$$0.5 = \sqrt{n_1^2 - n_2^2} \quad ..... (1)$$

When the Optical fiber is kept in water medium, $n_0' = 1.33.$

$$\sin\theta_0' = \frac{\sqrt{n_1^2 - n_2^2}}{n_0'}$$

$$\sin\theta_0' = \frac{\sqrt{n_1^2 - n_2^2}}{1.33} \quad .... (2)$$

On using equation (1) in (2)

$$\sin\theta_0' = \frac{0.5}{1.33}$$

$$\theta_0' = \sin^{-1}\left(\frac{0.5}{1.33}\right)$$

$$\boldsymbol{\theta_0' = 22^0}$$

9. Calculate N.A, the V-number and the number of modes in an optical fiber of core diameter 50 μm, core and cladding having refractive indices 1.41 and 1.4 respectively at wavelength of 820 nm.

Given

$n_1 = 1.41,\ n_2 = 1.40,\ \lambda = 820nm = 820 \times 10^{-9}m,\ d = 50\,\mu m = 50 \times 10^{-6}m.$

$N.A = ?\quad V = ?\quad$ Number of Modes $N = ?$

Solution

The Numerical aperture is

$$N.A = \sqrt{n_1^2 - n_2^2}$$

$$N.A = \sqrt{1.41^2 - 1.4^2}$$

$$N.A = 0.167$$

The expression for V-number is,

$$V = \frac{\pi d}{\lambda}\frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$V = \frac{3.14 \times 50 \times 10^{-6}}{820 \times 10^{-9}}\frac{\sqrt{1.41^2 - 1.40^2}}{1}$$

$$V = 32$$

The number of Modes is

$$N = \frac{V^2}{2} = \frac{32^2}{2}$$

$$N = 512$$

10. Find the attenuation in an optical fiber of length 500 m, when a light signal of power 100 mW emerges out of the fiber with a power of 90 mW.

Given

$L = 500\, m = \mathbf{0.5\, km}$, $P_{in} = 100mW = 100 \times 10^{-3}W$, $P_{out} = 90mW = 90 \times 10^{-3}W$

$\alpha = ?$

Solution

The Attenuation in an Optical fiber is

$$\alpha = \frac{-10}{L}\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\alpha = \frac{-10}{0.5}\log_{10}\left(\frac{90 \times 10^{-3}}{100 \times 10^{-3}}\right)$$

$$\alpha = 0.915\ \mathrm{dB / km}.$$

11. The attenuation in an Optical fiber is 3.6 dB/km. What fraction of its initial intensity remains after 1 km and after 3 km respectively.

Given

$\alpha = 3.6$ dB / km , $\frac{P_{out}}{P_{in}} = ?$ after Stage $L = 1\ km$ and Stage $L = 3\ km$

Solution

WKT., $\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$

$$\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = \frac{-\alpha L}{10}$$

$$\left(\frac{P_{out}}{P_{in}}\right) = 10^{\left(\frac{-\alpha L}{10}\right)}$$

At the end of 1km distance, $\boldsymbol{L = 1\ km}$.

$$\left(\frac{P_{out}}{P_{in}}\right) = 10^{\left(\frac{-3.6 \times 1}{10}\right)}$$

$$\left(\frac{P_{out}}{P_{in}}\right) = 0.436.$$

At the end of 3 km distance, $\boldsymbol{L = 3\ km}$.

$$\left(\frac{P_{out}}{P_{in}}\right) = 10^{\left(\frac{-3.6 \times 3}{10}\right)}$$

$$\left(\frac{P_{out}}{P_{in}}\right) = 0.0832.$$

12. The attenuation in an Optical fiber is 2.2 dB/km. What fraction of its initial intensity remains after 2 km and after 6 km respectively.

Given

$\alpha = 2.2 \text{ dB / km}$, $\frac{P_{out}}{P_{in}} = ?$ after Stage $L = 2\ km$ and Stage $L = 6\ km$

Solution

WKT., $\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$

$$\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = \frac{-\alpha L}{10}$$

$$\left(\frac{P_{out}}{P_{in}}\right) = 10^{\left(\frac{-\alpha L}{10}\right)}$$

At the end of 2 km distance, $L = 2\ km$

$$\left(\frac{P_{out}}{P_{in}}\right) = 10^{\left(\frac{-2.2 \times 2}{10}\right)}$$

$$\left(\frac{P_{out}}{P_{in}}\right) = 0.363$$

At the end of 6 km distance, $L = 6\ km$

$$\left(\frac{P_{out}}{P_{in}}\right) = 10^{\left(\frac{-2.2 \times 6}{10}\right)}$$

$$\left(\frac{P_{out}}{P_{in}}\right) = 0.0478$$

Numerical problems on LASERs

Formulae

Energy of Photon $E = h\nu$

The ratio of population $\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}$

Condition for Resonance $L = \frac{n\lambda}{2}$

The frequency difference between two adjacent modes is, $\Delta f = \frac{v}{2L}$

Energy of Photons, $E = Nh\nu = \frac{Nhc}{\lambda}$ here 'N' is number of photons

$Power = \frac{Energy}{Time} = \frac{E}{t}$

Out Power of Laser, Wavelength of Laser and Number of photons emitted

$\therefore p = \frac{Nhc}{\lambda t}$ OR $\lambda = \frac{Nhc}{pt}$ OR $N = \frac{\lambda p t}{hc}$

1. Find the ratio of population of two energy levels in a medium at thermal equilibrium, if the wavelength of light emitted at 291 K is 6928 Å.

Given

$$\frac{N_2}{N_1} = ?,\ \lambda = 6928\ \text{Å},\ T = 291\text{K}$$

Solution

The ratio of population is

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}$$

$$\frac{N_2}{N_1} = e^{-\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{6928\times10^{-10}\times1.38\times10^{-23}\times291}\right)}$$

$$\frac{N_2}{N_1} = 9.34\times10^{-32}$$

2. Find the ratio of population of two energy levels in a LASER if the transition between them produces light of wavelength 6493 Å, assuming the ambient temperature at 27°C.

Given

$$\frac{N_2}{N_1} = ?,\ \lambda = 6493\ \text{Å},\ t = 27°C, T = 300K$$

Solution

The ratio of population is

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}$$

$$\frac{N_2}{N_1} = e^{-\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{6493\times10^{-10}\times1.38\times10^{-23}\times300}\right)}$$

$$\frac{N_2}{N_1} = 7.67\times10^{-33}$$

3. The ratio of population of two energy levels out of which one corresponds to metastable state is 1.059×10^{-30} . Find the wavelength of light emitted at 330 K.

Given

$$\frac{N_2}{N_1} = 1.059 \times 10^{-30},\ \lambda = ?,\ T = 330K$$

Solution

The ratio of population is

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}$$

Take log_e on both side, we get

$$\log_e\left(\frac{N_2}{N_1}\right) = \frac{-hc}{\lambda kT}$$

$$\lambda = \frac{-hc}{kT \log_e\left(\frac{N_2}{N_1}\right)}$$

$$\lambda = \frac{-6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 330 \times \log_e(1.059 \times 10^{-30})}$$

$$\lambda = 632.4\ nm$$

4. Find the ratio of population of two energy levels in a medium at thermal equilibrium, if the wavelength of light emitted at 300 K is 10 μm. Also find the effective temperature when energy levels are equally populated.

Given

$$\frac{N_2}{N_1} = ?,\ \lambda = 10\mu m,\ T = 300K\ also\ T_{eff} = ?\ when\ N_1 = N_2$$

Solution

The ratio of population is

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}$$

$$\frac{N_2}{N_1} = e^{-\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{10\times10^{-6}\times1.38\times10^{-23}\times300}\right)}$$

$$\frac{N_2}{N_1} = 0.00821$$

Also $T_{eff} = ?$ when $N_1 = N_2$

Consider $e^{\frac{-hc}{\lambda kT}} = \frac{N_2}{N_1}$

Take $N_1 = N_2$ we get,

$$e^{\frac{-hc}{\lambda kT_{eff}}} = 1$$

w.k.t $e^0 = 1$

$$e^{\frac{-hc}{\lambda kT_{eff}}} = e^0$$

Compare the powers

$$\frac{-hc}{\lambda kT_{eff}} = 0$$

$$T_{eff} = \infty$$

5. The average power output of a LASER beam of wavelength 6500 Å is 10 mW. Find the number of photons emitted per second by the LASER source.

Given

$\lambda = 6500$ Å, $p = 10$ mW, $N = ?$, $t = 1$s

Solution

Number of Photons emitted per sec is given by,

$$N = \frac{\lambda p t}{hc}$$

$$N = \frac{6500 \times 10^{-10} \times 10 \times 10^{-3} \times 1}{6.626 \times 10^{-34} \times 3 \times 10^{8}}$$

$N = 3.27 \times 10^{16}$ Photons emitted / sec

6. The average power of a LASER beam of wavelength 6328 Å is 5 mW. Find the number of photons emitted per second by the LASER source.

Given

$\lambda = 6328$ Å, $p = 5$ mW, $N = ?$, $t = 1$s

Solution

Number of Photons emitted per sec is given by,

$$N = \frac{\lambda p t}{hc}$$

$$N = \frac{6328 \times 10^{-10} \times 5 \times 10^{-3} \times 1}{6.626 \times 10^{-34} \times 3 \times 10^{8}}$$

$$N = 1.591 \times 10^{16} \text{ Photons emitted / sec}$$

7. A pulsed LASER has an average power output 1.5 mW per pulse and pulse duration is 20 ns. The number of photons emitted per pulse is estimated to be $1.047 x 10^8$. Find the wavelength of the emitted LASER.

Given

p = 1.5mW, t = 20 ns, λ = ? N = 1.047×10^8

Solution

Wavelength of Laser is given by,

$$\lambda = \frac{Nhc}{pt}$$

$$\lambda = \frac{1.047\times 10^8\times 6.626\times 10^{-34}\times 3\times 10^8}{1.5\times 10^{-3}\times 20\times 10^{-9}}$$

$$\lambda = 693.7\ nm$$

8. In a LASER system when the energy difference between two energy levels is 2 x 10^{-19} J, the average power output of LASER beam is found to be 4 mW. Calculate number of photons emitted per second.

Given

$E_2 - E_1 = 2 \times 10^{-19}\, J$, p = 4 mW, N = ? t = 1s

Solution

The energy of Photon is

$$\frac{hc}{\lambda} = E_2 - E_1$$

The reciprocal of above eqn is

$$\frac{\lambda}{hc} = \frac{1}{E_2 - E_1} \quad \text{........(1)}$$

Number of Photons emitted per sec is given by,

$$N = \frac{\lambda p t}{hc}$$

$$N = \frac{pt}{E_2 - E_1} \quad \text{using eqn (1)}$$

$$N = \frac{4 \times 10^{-3} \times 1}{2 \times 10^{-19}}$$

$$N = 2 \times 10^{16}$$

9. Find number of modes of standing waves and their frequency separation in resonant cavity of length 1 m, in He-Ne LASER operating at wavelength of 6328 Å.

Given

$n = ?\quad \Delta f = ?\quad L = 1m,\ \lambda = 6328\mathring{A}$

Solution

Laser cavity Resonate when,

$$L = \frac{n\lambda}{2}$$

$$n = \frac{2L}{\lambda}$$

$$n = \frac{2\times1}{6328\times10^{-10}}$$

$$n = 3.16\times10^{6}$$

for frequency separation we have,

$$\Delta f = \frac{v}{2L}$$

$v = c$ for photon

$$\Delta f = \frac{c}{2L} = \frac{3\times10^{8}}{2\times1}$$

$$\Delta f = 150\ \text{MHz}$$

Module 3

Maxwell’s Equations

Syllabus

- Maxwell's equations: Fundamentals of vector calculus. Divergence and curl of electric field and magnetic field (static), Gauss' divergence theorem and Stokes' theorem. Description of laws of electrostatics, magnetism and Faraday's laws of EMI. Current density & equation of Continuity; displacement current (with derivation) Maxwell's equations in vacuum

- EM Waves: The wave equation in differential form in free space (Derivation of the equation using Maxwell's equations), Plane electromagnetic waves in vacuum, their transverse nature, polarization of EM waves(Qualitative)

Course outcome:

CO1: Describe the fundamental principles of Quantum Mechanics and the essentials of Photonics

CO2: Elucidate the concepts of dielectrics and superconductivity

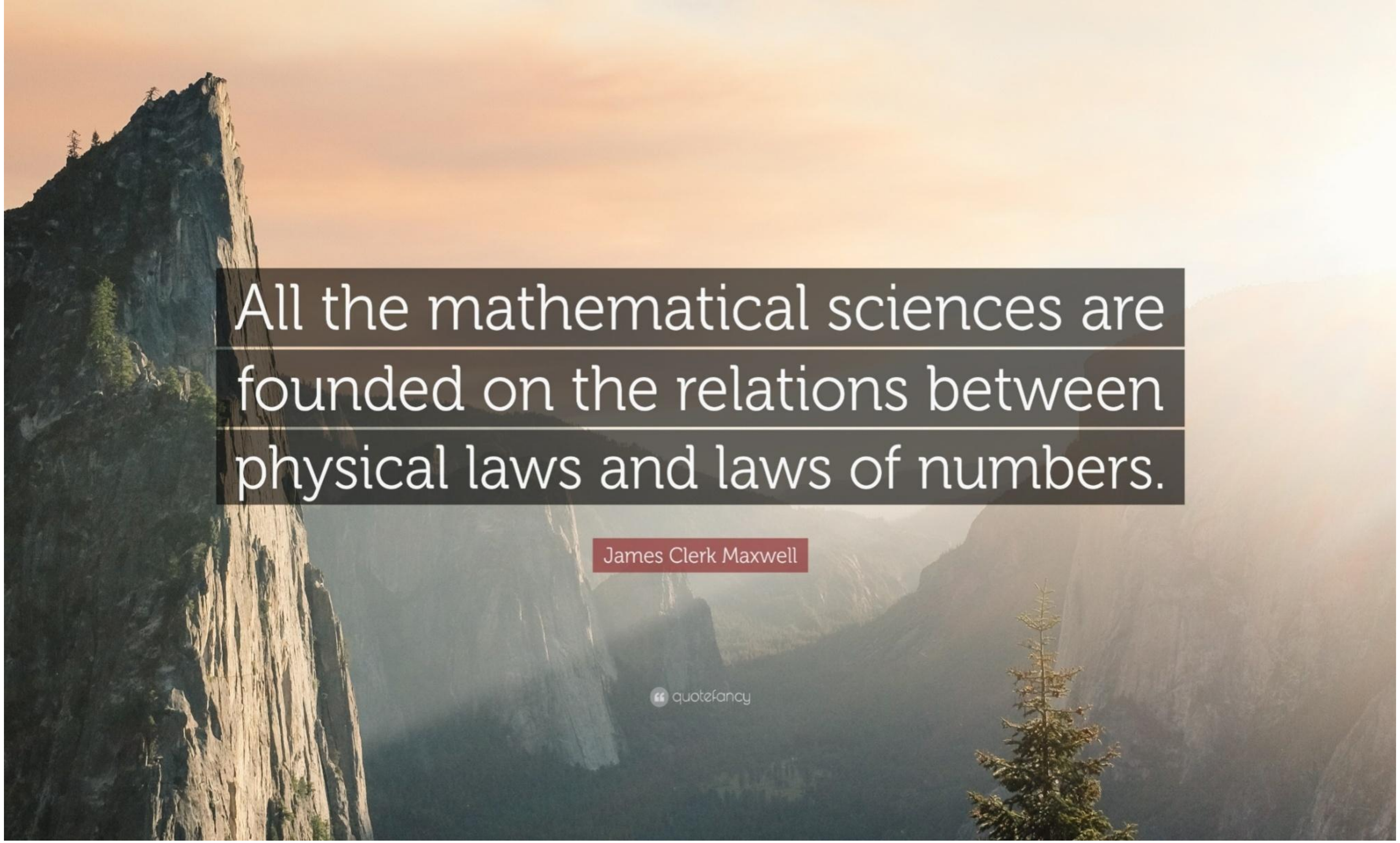
CO3: Discuss the fundamentals of vector calculus and their applications in Maxwell's Equations and EM Waves

CO4: Summarize the properties of semiconductors and the working principles of semiconductor devices

CO5: Practice working in groups to conduct experiments in physics and perform precise and honest measurements

Introduction

Discussion on The Set of Mathematical Equations that Revolutionized the world



Introduction

James Clerk Maxwell
(1831–1879)
Pioneer who laid the foundation for unification of Electricity and Magnetism

Introduction

Statue of James Clerk Maxwell

George Street, Edinburgh, the birthplace of Maxwell

Introduction

A new Science of electromagnetism was developed.

In 1864, Maxwell proposed Electromagnetic Theory of Light.

Maxwell put the great work of Oerstead, Gauss, Ampere, Henry, Faraday and others in mathematical form called Maxwell equations.

Maxwell showed that Light is an electromagnetic wave

Pioneers in the field of electricity and magnetism


Hans Christian Oersted


Ampere


James Clerk Maxwell


Carl Friedrich Gauss


Michael Faraday

Using the laws and theorems discussed in this chapter Four Maxwell's equations could be written as

Time varying fields

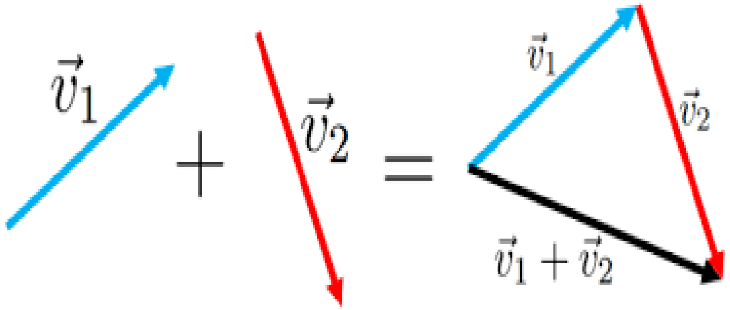
1. Gauss' Law of Electrostatics $\nabla.\vec{D} = \rho_v$
2. Faraday's Law $(\nabla \times \vec{E}) = -\frac{\partial \vec{B}}{\partial t}$
3. Gauss' Law of Magnetic fields $\nabla.\vec{B} = 0$
4. Maxwell - Ampere Law $\nabla \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

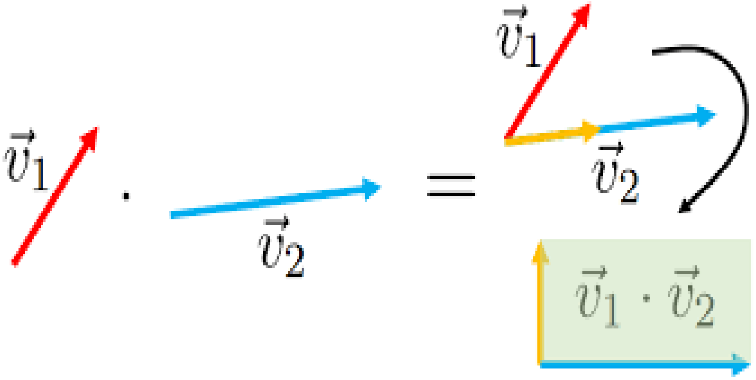
Static fields

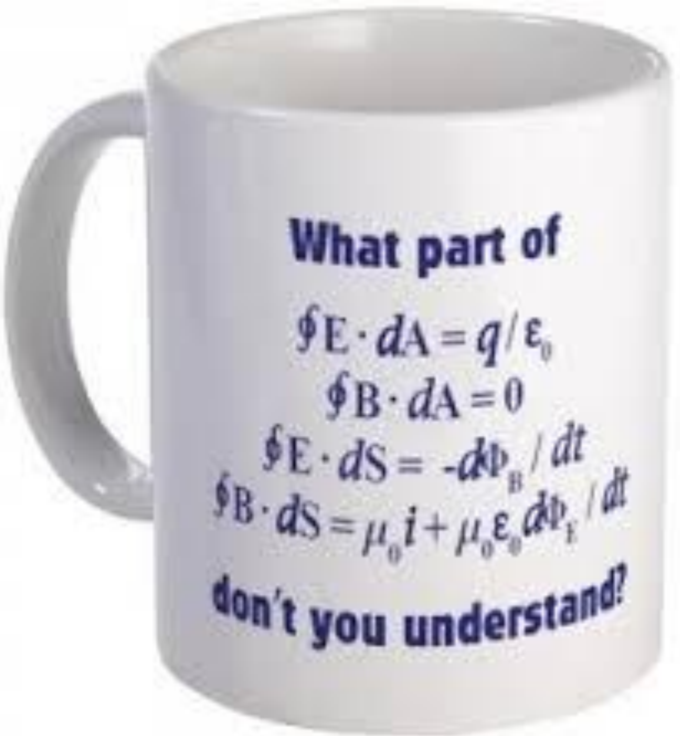
1. Gauss' Law of Electrostatics $\nabla.\vec{D} = \rho_v$
2. Faraday's Law $(\nabla \times \vec{E}) = 0$
3. Gauss' Law of Magnetic fields $\nabla.\vec{B} = 0$
4. Ampere's Law $\nabla \times \vec{H} = \vec{J}$

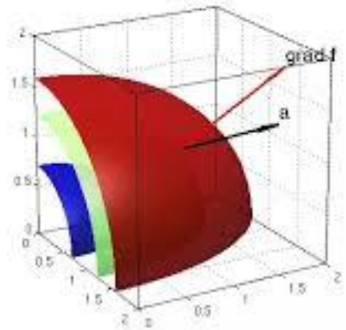
Need to know..

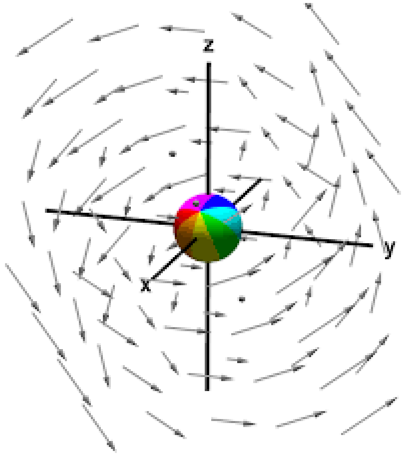
VECTOR CALCULUS

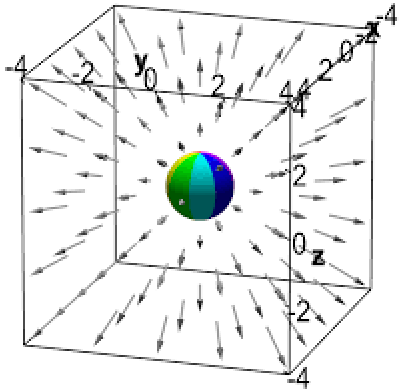






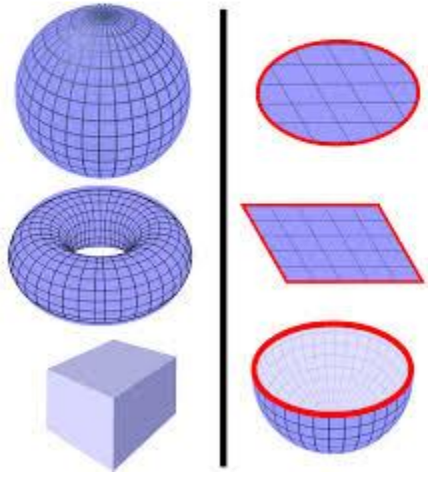


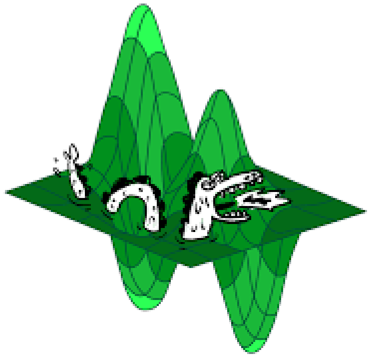




Vector Calculus

- Vector calculus deals with the study of scalar and vector field with respect to time and position.
- It is nothing but directional differentiation and directional integration.
- Some of the vector fields (not all) can be obtained from scalar fields by performing specific operations in vector calculus.





Types of Quantities..

Physical quantities	Psychological quantities
Completely specified, can be defined, measured and expressed using proper units	Not a specific quantity, can not be measured and expressed in terms of units; but only experienced
Mass, length, temperature, speed, time taken, force,….	Odor, taste, Loudness of sound,…

Physical quantities

- The physical quantity having only magnitude is known as Scalar.
  Mass, length, time, speed, work done,…
- The physical quantity having both magnitude and direction is known as Vector.
  Velocity, acceleration, momentum, force, torque,..

Dot product

The dot product of two vectors is defined as follows

$$\vec{a}.\vec{b} = ab\cos\theta$$

here θ is the angle between two vectors.

a & b are the magnitudes of $\vec{a}$ & $\vec{b}$

If $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

And $\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$

then the dot product or scalar product is given by

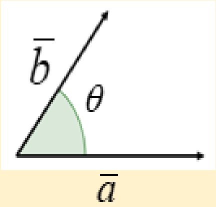
$$\vec{a}.\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

Dot Product

If $\bar{a} =< a_1, a_2, a_3 >$ and $\bar{b} =< b_1, b_2, b_3 >$

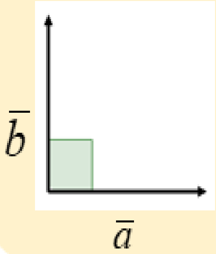
then the dot product is

$$\bar{a} \bullet \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$



If θ is the angle between $\bar{a}$ and $\bar{b}$ then

$$\bar{a} \bullet \bar{b} = |\bar{a}||\bar{b}|\cos\theta$$

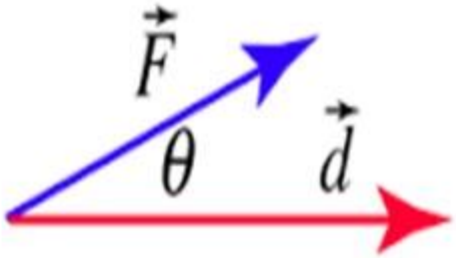


$\bar{a} \bullet \bar{b}$ are orthogonal (perpendicular) if and only if $\bar{a} \bullet \bar{b} = 0$

Dot product

For example the work done is maximum when the displacement is along the force. Thus work done is defined as the dot product of force $\vec{F}$ and displacement $\vec{d}$ and is a scalar quantity.

Hence $\boxed{W = \vec{F}.\vec{d}}$



$$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

Work done by a constant force, straight line motion.

Cross product

The cross product of two vectors is defined as follows

$$\vec{a} \times \vec{b} = ab \sin\theta \, \hat{n}$$

here θ is the angle between two vectors.

a & b are the magnitudes of $\vec{a}$ & $\vec{b}$ and $\hat{n}$ is

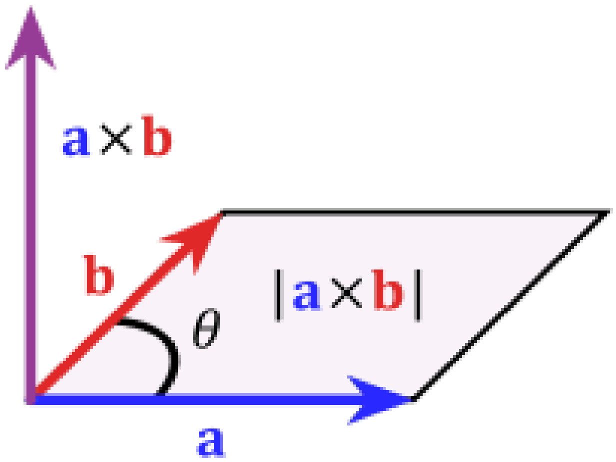
The unit vector normal to both $\vec{a}$ & $\vec{b}$

If $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

And $\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$

then the cross product or vector product is given by

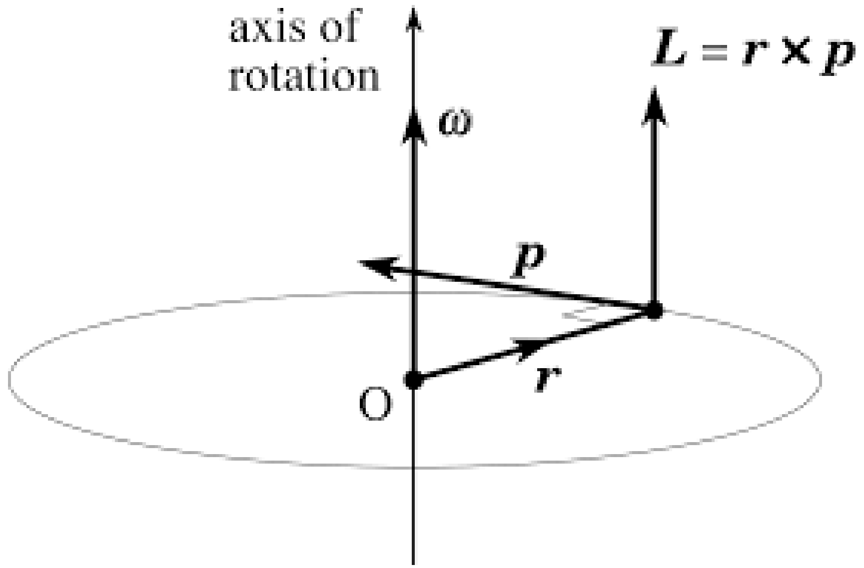
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



Cross product

Physical Significance The cross product is put forward in mathematics an could be applied in physics under suitable circumstances. For a rotating body the moment of linear momentum is the angular momentum.

The angular momentum acts in a direction perpendicular to momentum and the radius vector. Thus angular momentum ($\vec{L}$) is given by the cross product of radius vector ($\vec{r}$) and linear momentum ($\vec{p}$) and hence $\vec{L} = \vec{r} \times \vec{p}$



Scalar and Vector fields

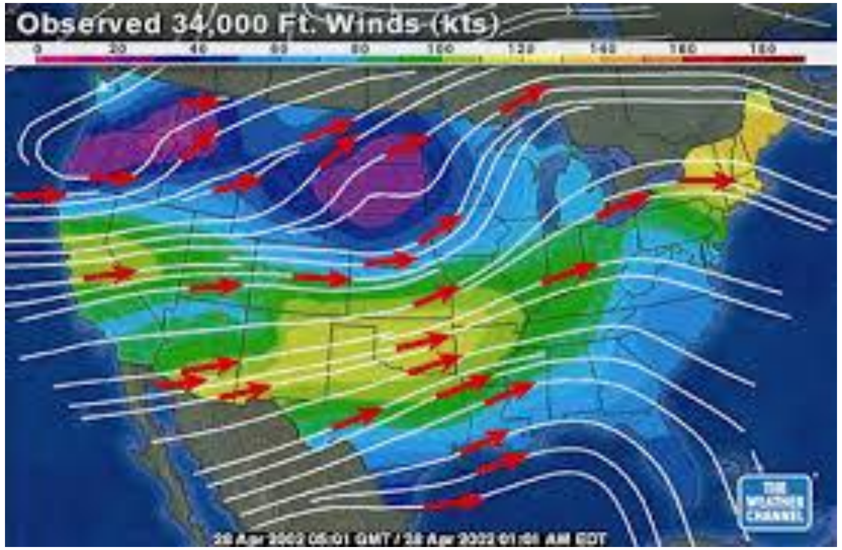
A point in space is described by three independent parameters.

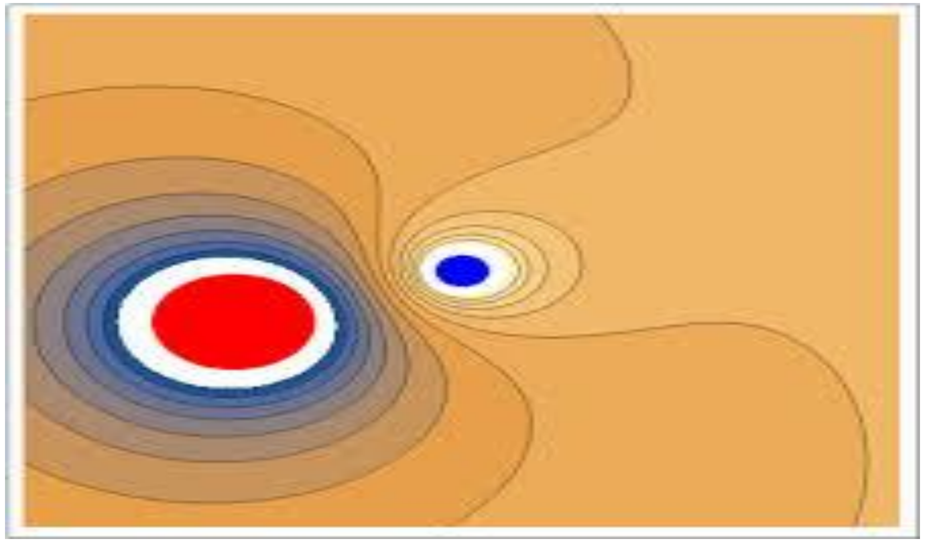
A continuous function of the position of a point in space is known as Point function.

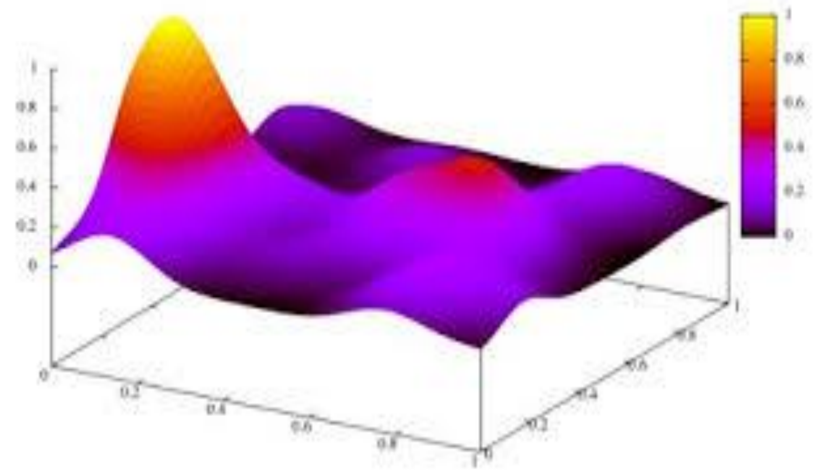
The region of space in which the point function represents a physical quantity is known as field.

Scalar fields

- It is a function of a space whose value at each point is a scalar quantity.
- Scalar fields are represented by Surfaces.
- Ex: temperature distribution throughout the space, pressure of air, density of gas, electrostatic potential,..

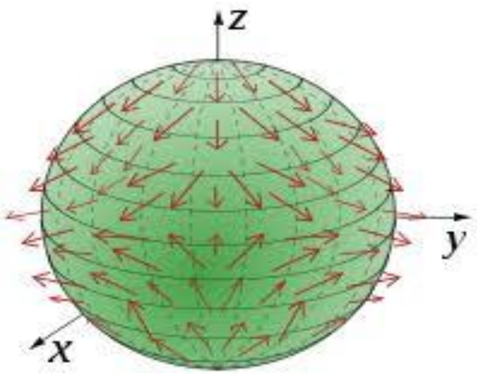


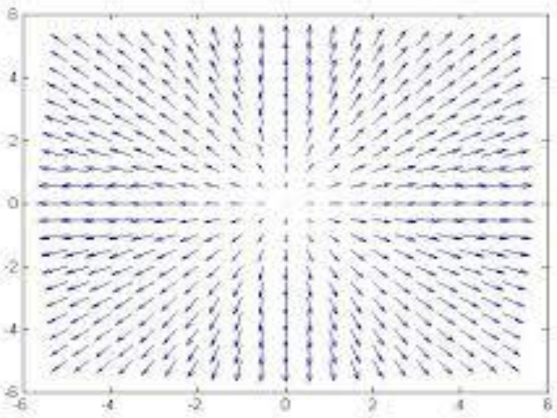


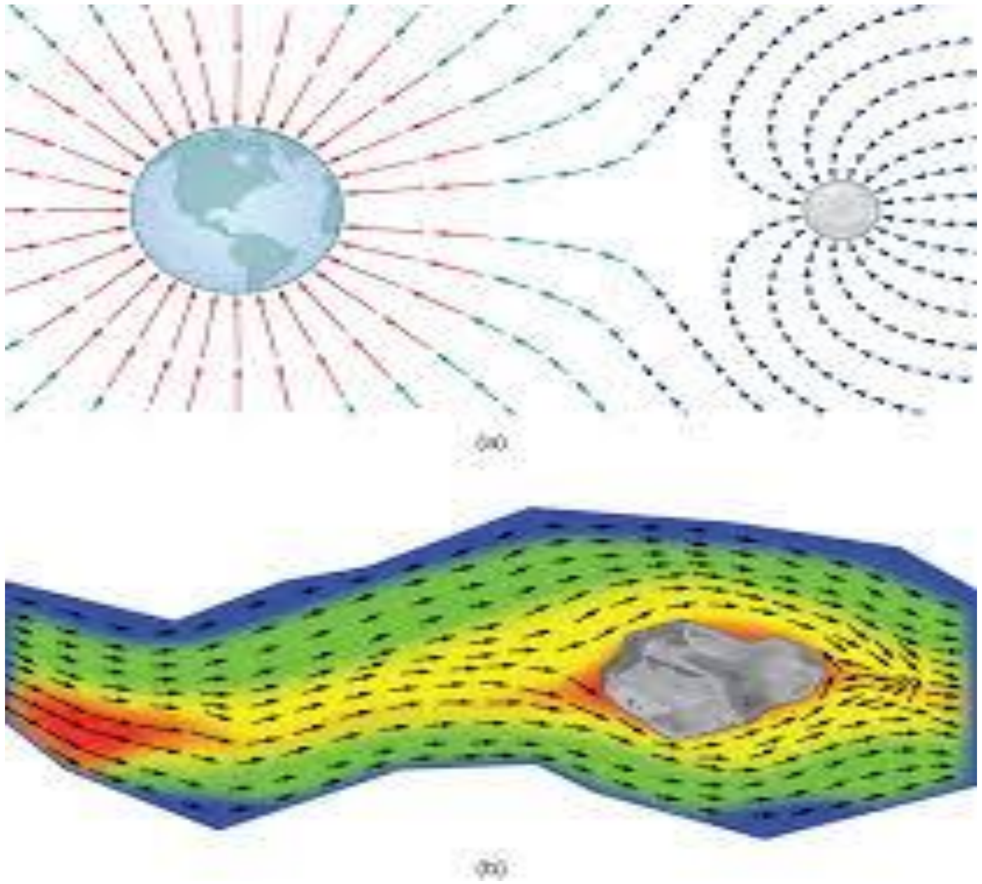


Vector fields

- It is a function of a space whose value at each point is a vector quantity.
- Vector field is represented by Vector lines.
- Ex: velocity of water flow, velocity vector of rotating body, force acting on test charge in an electric field, intensity of magnetic field around bar magnet, gravitational force,…



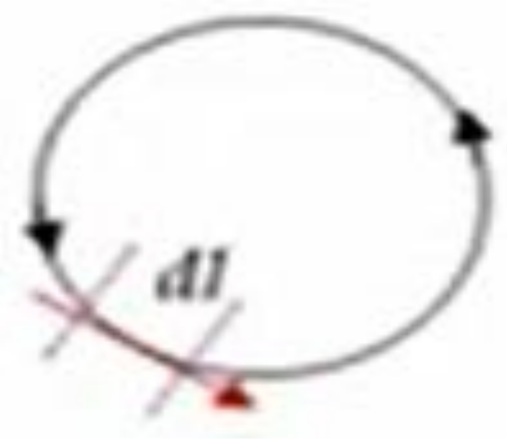


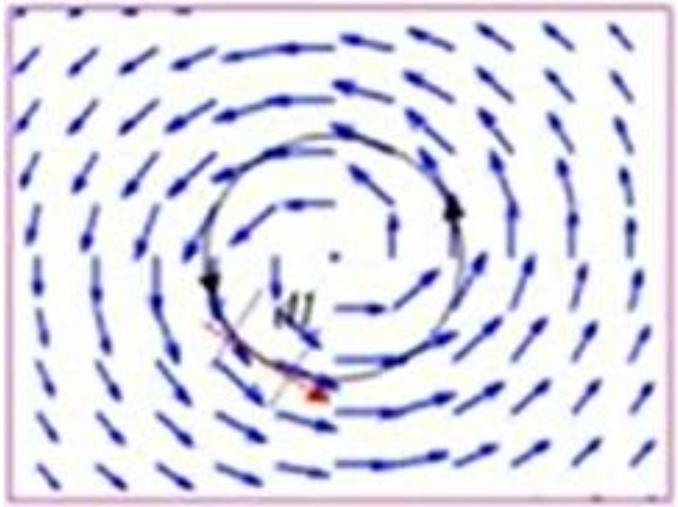


Circulation

- For any vector field the circulation around any imaginary closed curve is defined as the product of average tangential component of vector and elementary length on the circumference of the loop.

$$circulation = \vec{v}_1.\vec{dl}_1 + \vec{v}_2.\vec{dl}_2 + \vec{v}_3.\vec{dl}_3 + ----= \oint \vec{v}_i.\vec{dl}_i$$

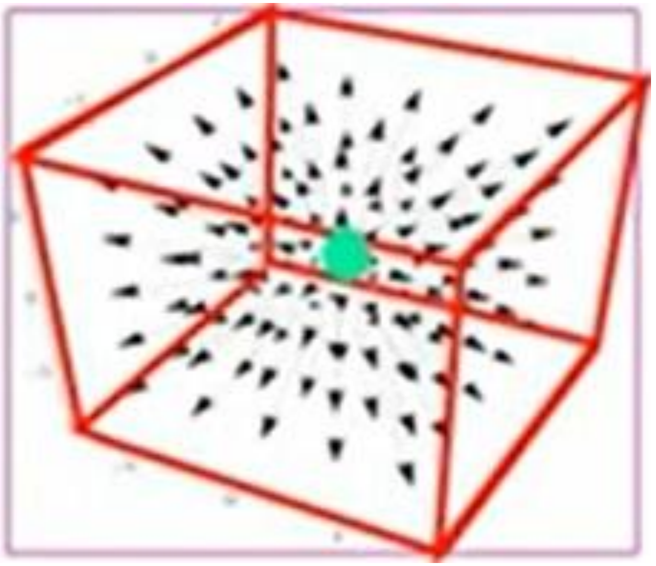


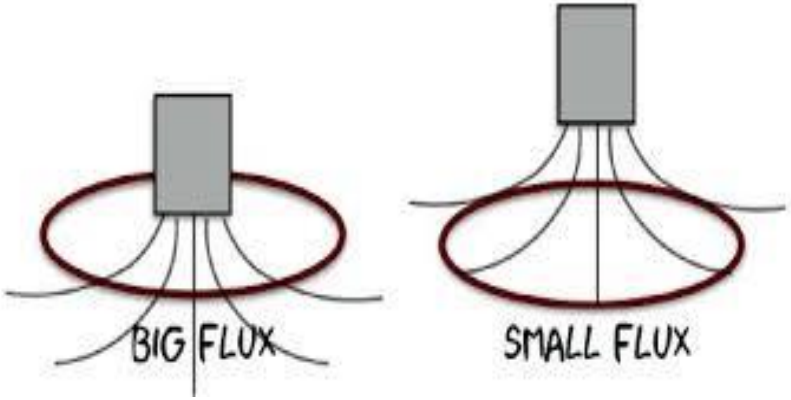


Flux

- **Flux** describes any effect that appears to pass or travel through a surface or substance.
- For an arbitrary closed surface, the flux (either outward or inward) is the product of average normal component of the vector and surface area.
- The outward flux is positive
- The inward flux is negative

$$flux = \vec{v}_1.\overrightarrow{dS}_1 + \vec{v}_2.\overrightarrow{dS}_2 + \vec{v}_3.\overrightarrow{dS}_3 + ---= \oint \vec{v}_i.\overrightarrow{dS}_i$$





∇ Operator

- **Del**, or **nabla (∇)** is an operator in mathematics like addition, subtraction, summation, differentiation, integration,…
- It is a vector operator which operates on both scalar and vectors. It is given by,

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

- ∇ alone has no meaning. $T\nabla \neq \nabla T$ $\quad m\vec{a} = \vec{a}m$

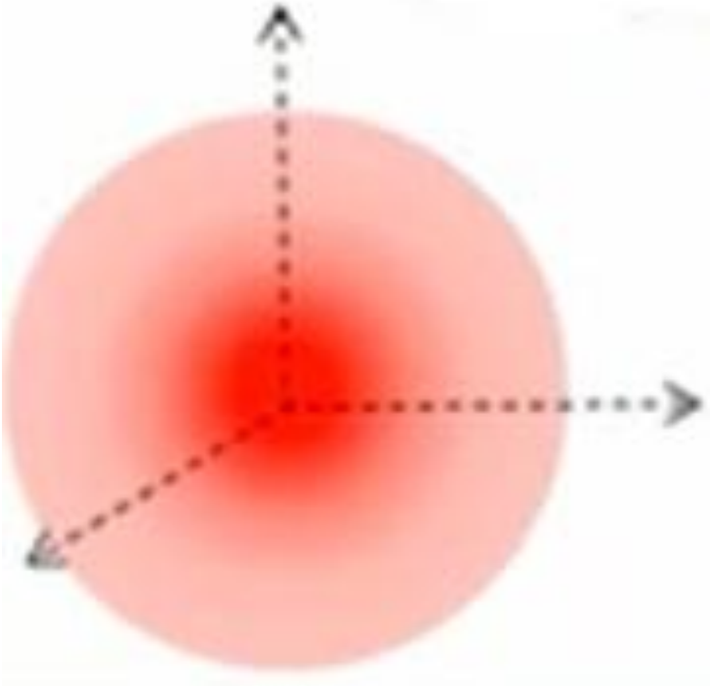
- It is most commonly used to simplify expressions for the gradient, divergence, curl, directional derivative, and Laplacian.

∇ Operator

- Consider the variation of temperature at different positions around a red hot iron sphere. It is decreasing along x, y and z axes.

- The rate at which it decreases along x-axis does not depends on y and z axes. Similarly for other axes.

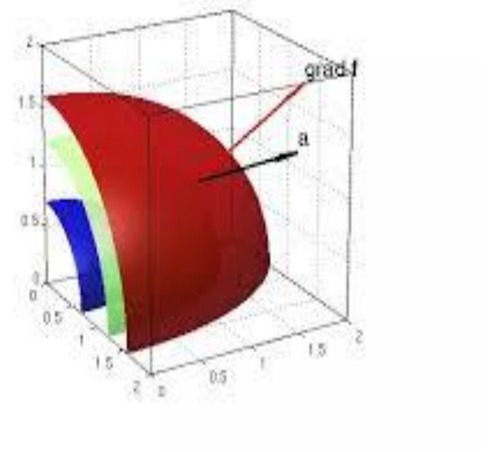
- The variation of temperature between any two points is given by

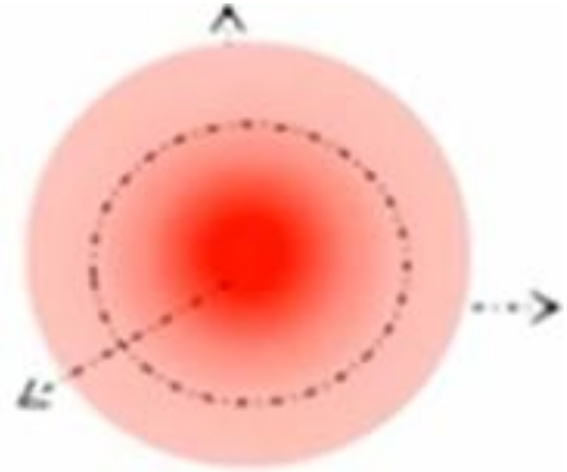


$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz = \nabla T$$

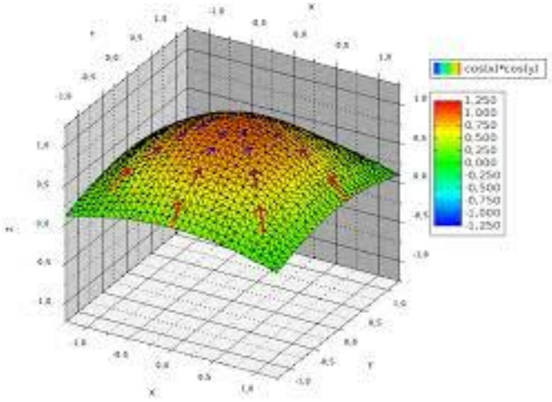
Gradient

- Gradient at any point in the scalar field is equal to the rate of change of scalar along the normal to the surface at that point.





$$\nabla V = \hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}$$



- Consider the case of Temperature variation around a red hot iron block. The decrease in the temperature along any direction represents the gradient.

Significance of Gradient

- Consider a positive point charge in space. Let the potential set up by the charge in the surrounding be V and is a scalar quantity. The potential decreases as the distance from the charge increases. Thus the gradient of potential results in the electric field strength which is a vector quantity. This could be written as

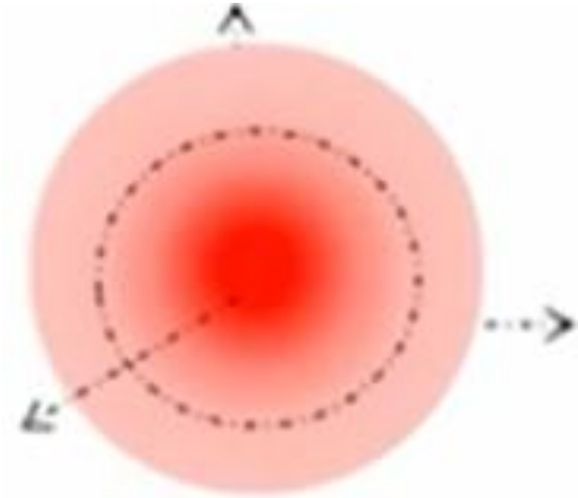
$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$$

- Here r is the position vector and r cap is the unit vector along position vector. The negative sign indicates the decrease in potential. Thus the above equation could be written as

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Significance of Gradient

- Gradient of a Scalar field is a vector. It is directed along the increasing scalar field.
- In case of red hot iron sphere ∇ T increases as we move towards the sphere. Hence T (x, y, z) also increases.
- It T (x, y, z) is a continuously differentiable real value function like temperature around the red hot sphere, then $\nabla T \neq 0$.
- Electric field is actually negative gradient of potential.
- There is no meaning for gradient of vector.

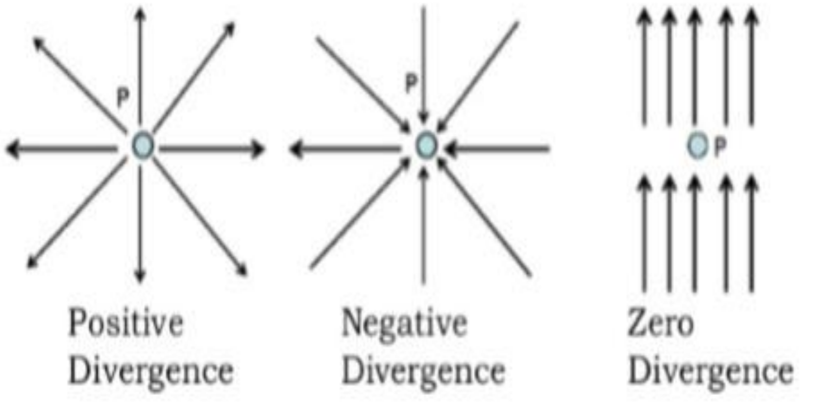


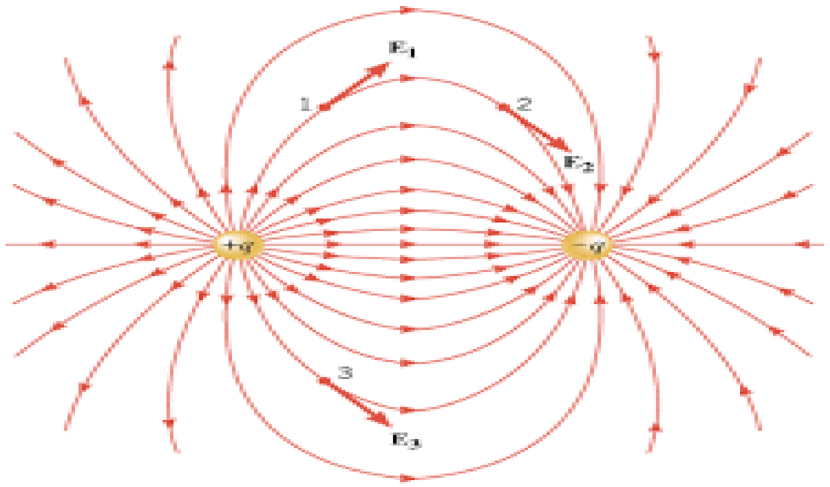
Divergence

- Divergence of a vector point function is dot product of ∇ and a vector. i.e $\nabla . \vec{A}$
- The divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.
- consider air as it is heated or cooled. The velocity of the air at each point defines a vector field. While air is heated in a region, it expands in all directions, and thus the velocity field points outward from that region. The divergence of the velocity field in that region would thus have a positive value. While the air is cooled and thus contracting, the divergence of the velocity has a negative value.

Divergence

- **Divergence** is a vector operator that operates on a vector field, producing a scalar field.
- A point at which the flux is outgoing has positive divergence, and is often called a "source" of the field. A point at which the flux is directed inward has negative divergence, and is often called a "sink" of the field.
- The greater the flux of field the greater the value of divergence at that point.
- A field which has zero divergence everywhere is called Solenoidal.



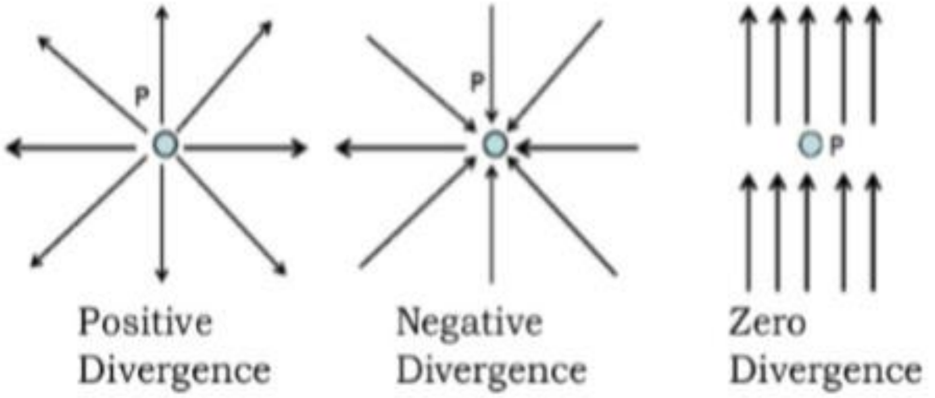


Divergence

- Divergence measures how much the vector E spreads out (diverges) from a point of consideration.
- For example if we consider a positive charge in space the field lines diverge and hence it is positive divergence. For a negative charge the field lines converge and hence it is negative divergence. If the field lines or parallel then it is zero divergence.
- Mathematically, the divergence of vector field is $\nabla . \vec{E}$
- The vector field $\vec{E}$ is represented as $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}$
- From the definition the divergence is

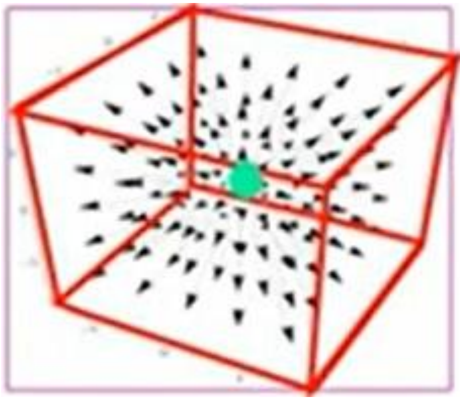
$$\nabla.\vec{E} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}\right)$$

$$\nabla.\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$



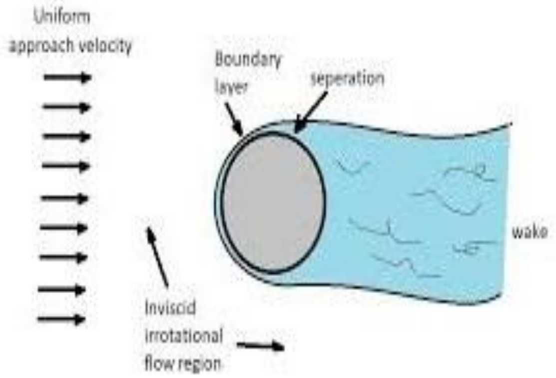
Significance of divergence

- Divergence of a vector represents how much of vector field is flowing out or flowing in over a surface area.
- It is always associated with flux.
- If the net flow is outward then divergence is positive. If the net flow is inward then the divergence is negative.
- The point of positive divergence is "Source" and the point of negative divergence is "Sink".
- If the flow is steady (net inflow is equal to outflow) then, divergence is zero. Such vector function is called "Solenoidal".
- There is no meaning for divergence of a scalar.



Curl

- Curl of a vector point function is the cross product of ∇ and a vector. i.e $\nabla \times \vec{H}$
- Curl is a vector operator that describes the infinitesimal rotation of a vector field in three dimension.
- At every point in the field, the curl of that point is represented by a vector.
- The direction of the curl is the axis of rotation. That can be determined by Right-hand rule.
- The magnitude of curl is the magnitude of rotation.
- A vector field whose curl is Zero is called irrotational.



Curl

- The curl of a vector field could be constructed as follows

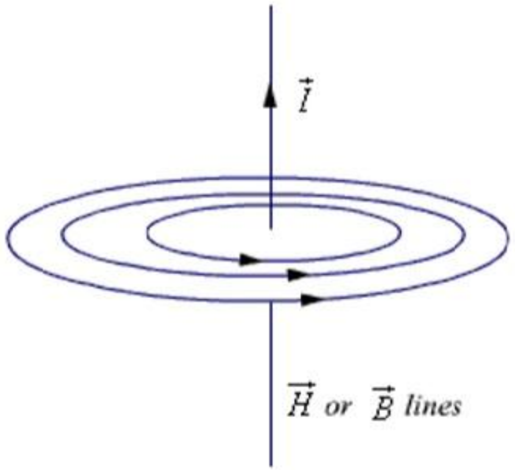
$$\nabla \times \vec{H} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(H_x\hat{\imath} + H_y\hat{\jmath} + H_z\hat{k}\right)$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

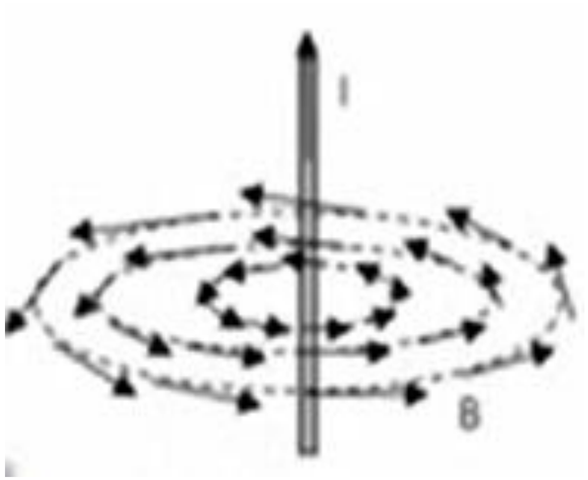
The curl of a vector function is a measure how much field swirls (curls) around the point of consideration.

Curl

- Consider a wire carrying electric current. This sets magnetic field surrounding the wire.



- Consider a point on the wire. The magnetic field lines curl or swirl around the point. Higher the value of vector H around the point stronger will be the curl.



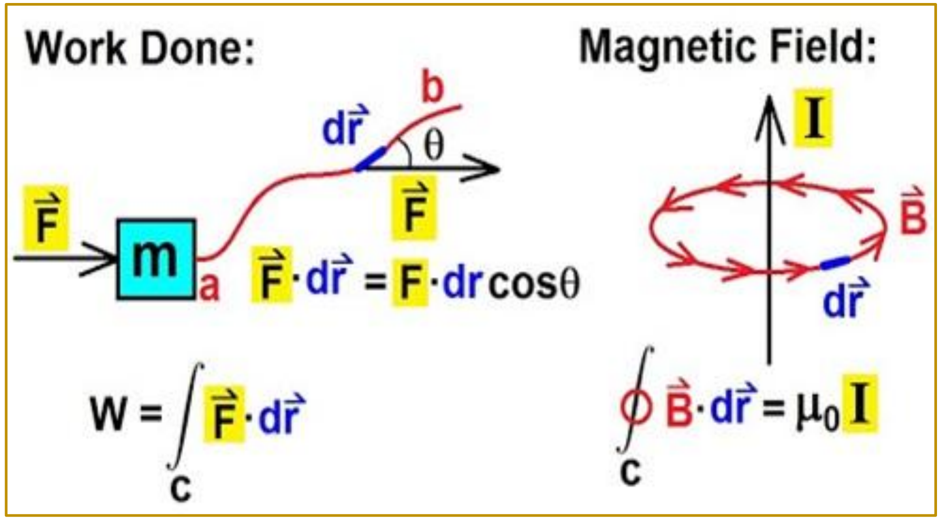
- If the field lines purely parallel then it represents zero curl around the point.

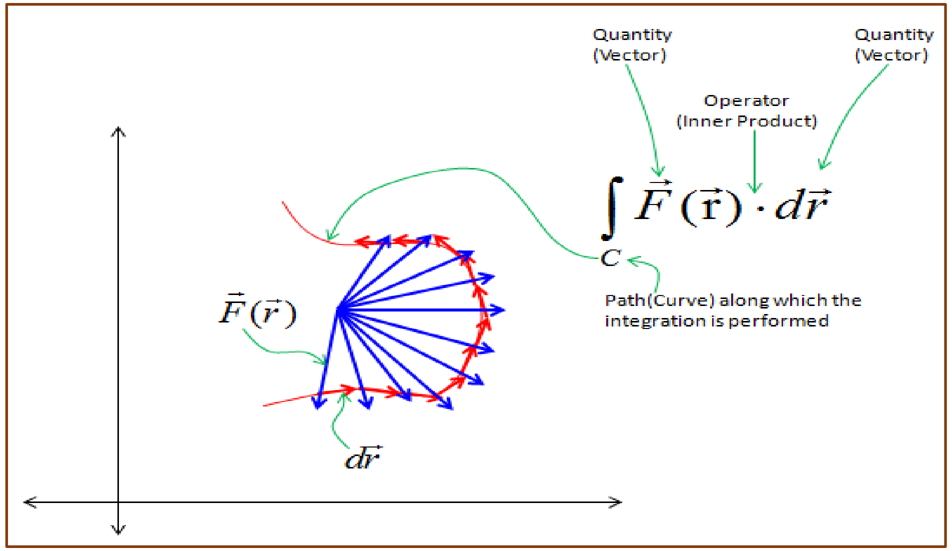
Curl

- Curl of a vector represents the rotational effect of a vector field.
- If the field is uniform (like electric field due to infinite charge) the curl is Zero.
- A vector field is said to be irrotational if the curl is zero. i.e., $\nabla \times \vec{H} = 0$.
- For certain vector field, (non uniform fields) the curl of a vector is non zero.
- Curl is a measure of the velocity of the field at the point (eg: a measure of the rate of rotational spin in a fluid.
- There is no meaning for curl of scalar.

Line integral

- An integral when, you integrate something over a line or a 1 D object like the path or circumference of a circle (called a closed line integral), Mathematically, it means you integrate a function w.r.t just one variable.
- A line integral is the generalization of simple integral. It is also called path integral.
- The case when the line integral deals with vectors. The integrand has the form F*dr where * indicates inner product. This is really just a projection of one vector onto another. So you're summing up all the projections.
- Even the work done on the charged particle in an electric field is determined using Line integral.





Line integral

A line integral is an integration of a function along a curve. One can integrate scalar valued functions (like mass of a wire) or vector valued functions (like work done by a force) along a curve.

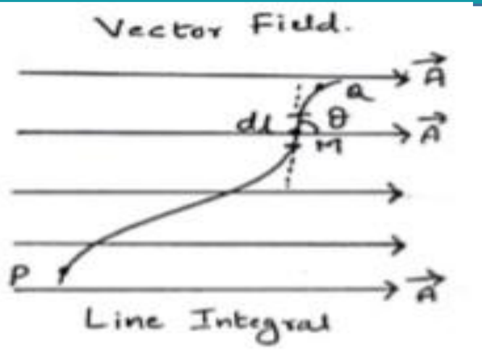
Line integral is an expression of the form

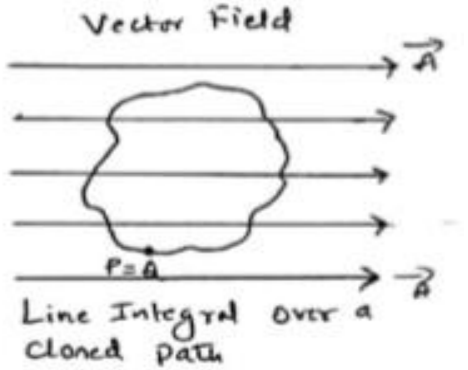
$$\int_P^A \vec{A}.\vec{dl}$$

here $\vec{A}$ represents the vector field and $\vec{dl}$ represents a infinitesimally small length at a point M along the path PQ in the field. The dot product of $\vec{A}$ and $\vec{dl}$ is given by $\vec{A}.\vec{dl} = A\ dl\ Cos\theta$, Here θ is the angle made $\vec{dl}$ with $\vec{A}$. For a closed path the integral is written as

$$\oint \vec{A}.\vec{dl}$$

$\oint$ is the symbol used for closed contour integral. This is also called as circulation of $\vec{A}$ around the closed path.



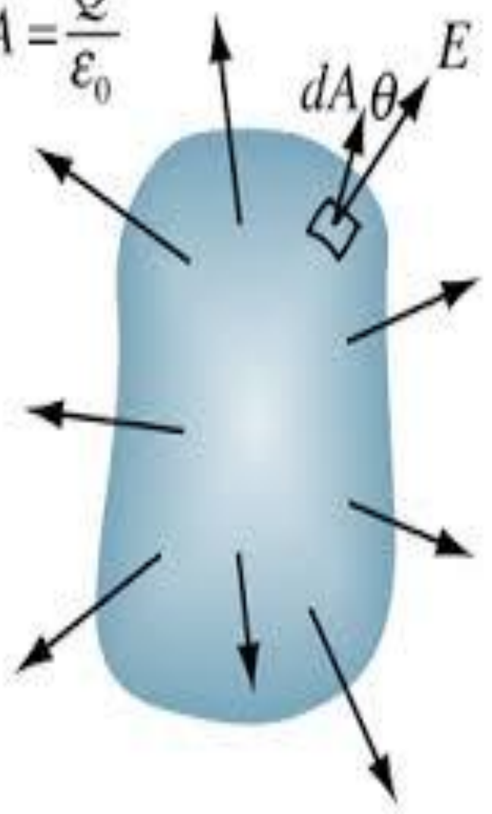


- The line integral concept can be applied to calculate the potential difference between to points in an electric field.

Surface integral

- A surface integral is generalization of double integral.
- The surface integral is taken over a surface. Often in physics, the surface integral is of a vector field, so the surface integral is the limit of the sum of scalar products, *i.e.*, *projections*, of the field vector onto the (normal) surface area vector for the patch of surface, again, as the patches shrink to zero.
- The surface integral of a vector field is, intuitively, an evaluation of "how many" field lines are passing through the surface. This is often called the flux of the vector field through the surface.

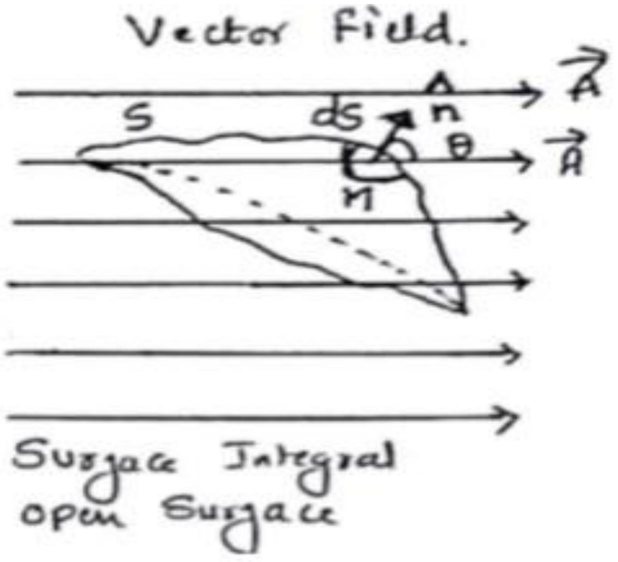
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$



Surface integral

In Surface integral, we add up the vector components that are flowing through the surface area like line integral stands for work done, surface integral represents flux. Hence it is know as flux integral.

- Consider a surface of area S in a vector field A. consider a small infinitesimal area dS on the surface around point M as in the figure.



Consider $\hat{n}$ a unit vector normal to ds and $dS\ \hat{n}$ represents area vector of dS. The surface integral over the entire surface S is given by

$$\int_s \vec{A}.\vec{dS}$$

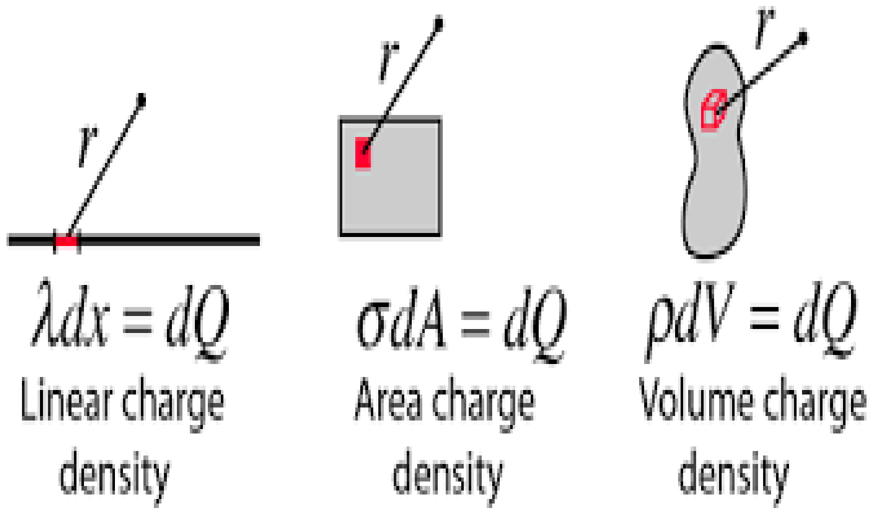
Here $\int_s$ is the symbol used for surface integral. The surface integral gives the net outward flux of the vector field through the surface. For a closed surface the surface integral is given by

$$\oint_s \vec{A}.\vec{dS}$$

In case of surface integral for a closed surface the $\hat{n}$ chosen outwards. The surface integral could be applied to calculate the net flux of the electric field through a surface in the electric field.

Volume integral

- A volume integral is generalization of triple integral.
- Volume integral is an integral over a 3 D domain. It can be calculated by finding a triple integral within a region D of a function f(x, y, z) dx dy dz.
- If there is distribution of charges in arbitrary shaped volume , then one can find the total charge enclosed by it by using volume integral.



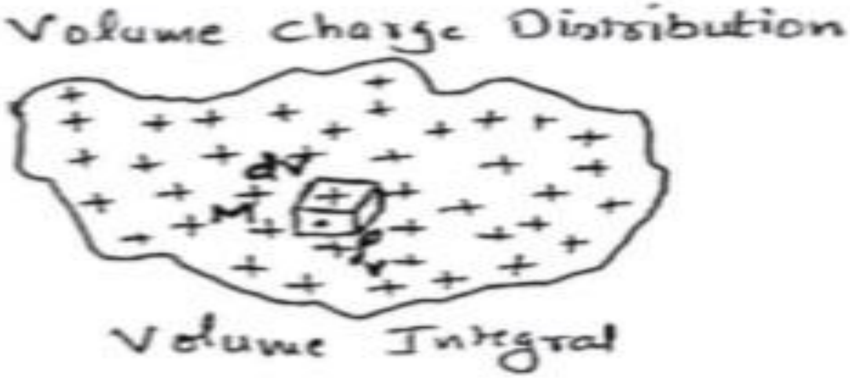
Volume integral

A Volume integral refers to an integral over 3 dimensional domain. It is a special case of multiple integrals. It is used to find volume of solids, volume of revolution,…

Consider a volume charge distribution in which charges are continuously distributed. Let v be the volume through which the charges are distributed. Consider a point M inside the charge distribution. Let dv be a small volume around a point M. let ρ_v be the density of charges at M and is a scalar quantity. The net charge in the volume is given by volume integral of the form

$$\oint_v \rho_v dv$$

here $\oint_v$ is the symbol for volume integral.



Important concepts

- **Electric field**

It is a region surrounding the charge, where if one keep a test charge will experience a force.

- **Electric flux**

Electric flux is the rate of flow of the electric field through a given area. In general it represents the flow of field lines from the charges.

- **Electric flux density** (Electric Displacement)

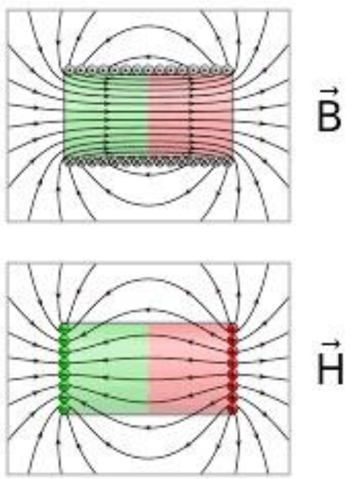
Electric flux density is the amount of flux passing through a defined area that is perpendicular to the direction of the flux.

B and H

Magnetic field is a place in space near a **magnet** or an electric current where a physical **field** is created from a moving electric charge that creates force on another moving electric charge. An example of a **magnetic field** is the Earth's **magnetic field**.

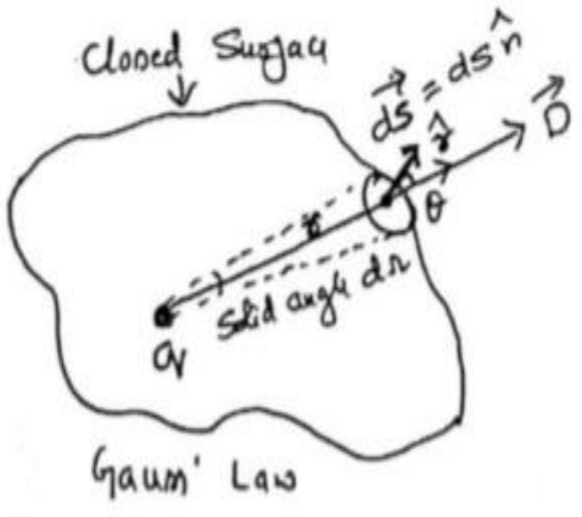
B is **magnetic** flux density, whereas **H** is **magnetic field** intensity. **H** has units of amp-turn/meter, whereas **B** has units of weber/turn-meter^2. In non-ferrous materials they have a simple inter-relation given by **B** = μ**H**

a measure of the actual **magnetic field** within a material considered as a concentration of **magnetic field** lines, or flux, per unit cross-sectional area



Gauss law in Electrostatics

- Consider a region in space consisting of charges. Let a surface of any shape enclose these charges and is called a Gaussian surface. Let Q be the net charge enclosed by the Gaussian surface S. The closed surface could be considered to be made up of number of elementary surfaces dS. If vector D is the electric flux density at dS then the surface integral gives the total electric flux over the surface S could be obtained as

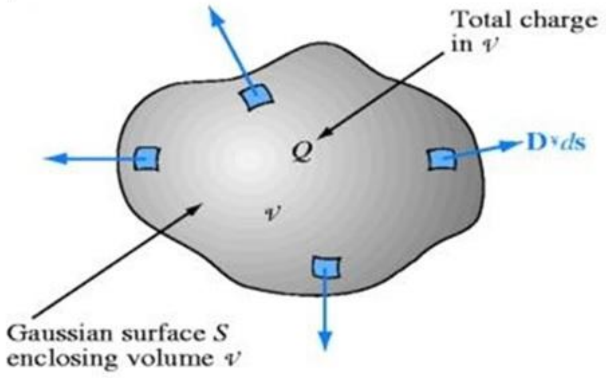


$$\phi = \oint_S \vec{D}.\vec{dS} = \sum q = Q$$

here ϕ is the total flux and

$\sum q = (q_1 + q_2 + \ldots)$ is the total

charge enclosed by the surface.



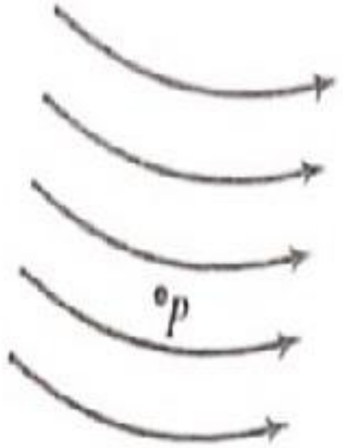
Gauss divergence theorem

Consider a vector field $\vec{D}$. Consider a point P in the vector field. Let ρ_v be the density of charges at the point P. It can be shown that the divergence of the $\vec{D}$ is given by

$$\nabla.\vec{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \rho_v$$

$$\nabla.\vec{D} = \rho_v$$

This is the Maxwell's first equation.



POINT IN A REGION OF CHARGES

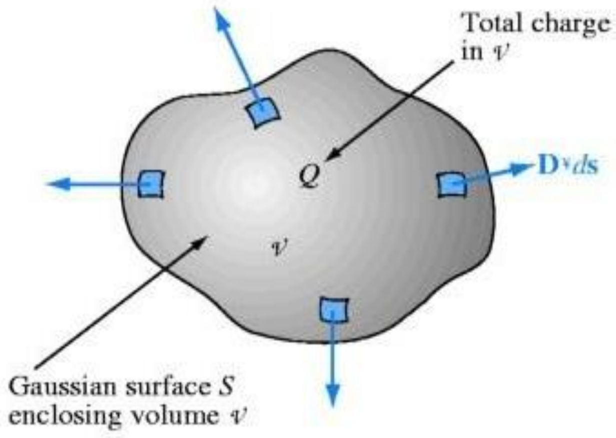
Gauss divergence theorem

Statement: The Gauss divergence theorem states that the integral of the normal component of the flux density over a closed surface of any shape in an electric field is equal to the volume integral of the divergence of the flux throughout the space enclosed by the Gaussian surface. Mathematically

$$\oint_s \vec{D}.\vec{dS} = \oint_v (\nabla.\vec{D})\, dv$$

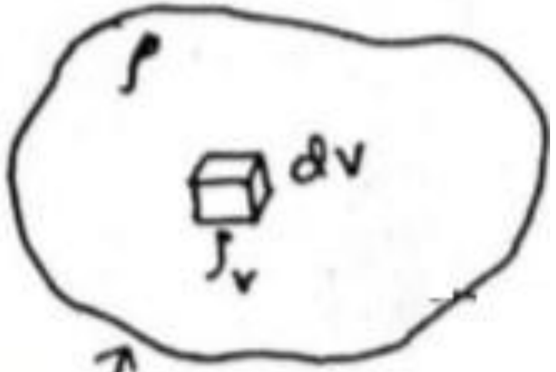
Gauss divergence theorem

Consider a volume v enclosed by a Gaussian surface S. Let a charge dQ be enclosed by a small volume dv inside the Gaussian surface. If ρ is the density of charges and may vary inside the volume v then the charge density associated with volume dv is given by



$$\rho_v = \frac{dQ}{dv}$$

$$dQ = \rho_v \, dv$$



Gauss divergence theorem

Thus the total charge enclosed by the Gaussian surface is give by

$$Q = \oint_v dQ = \oint_v \rho_v \, dv$$

Substituting for ρ_v from Maxwell's First equation we get

$$Q = \oint_v (\nabla.\vec{D}) \, dv$$

According to Gauss' law of electrostatics we have

$$Q = \oint_s \vec{D}.\vec{dS}$$

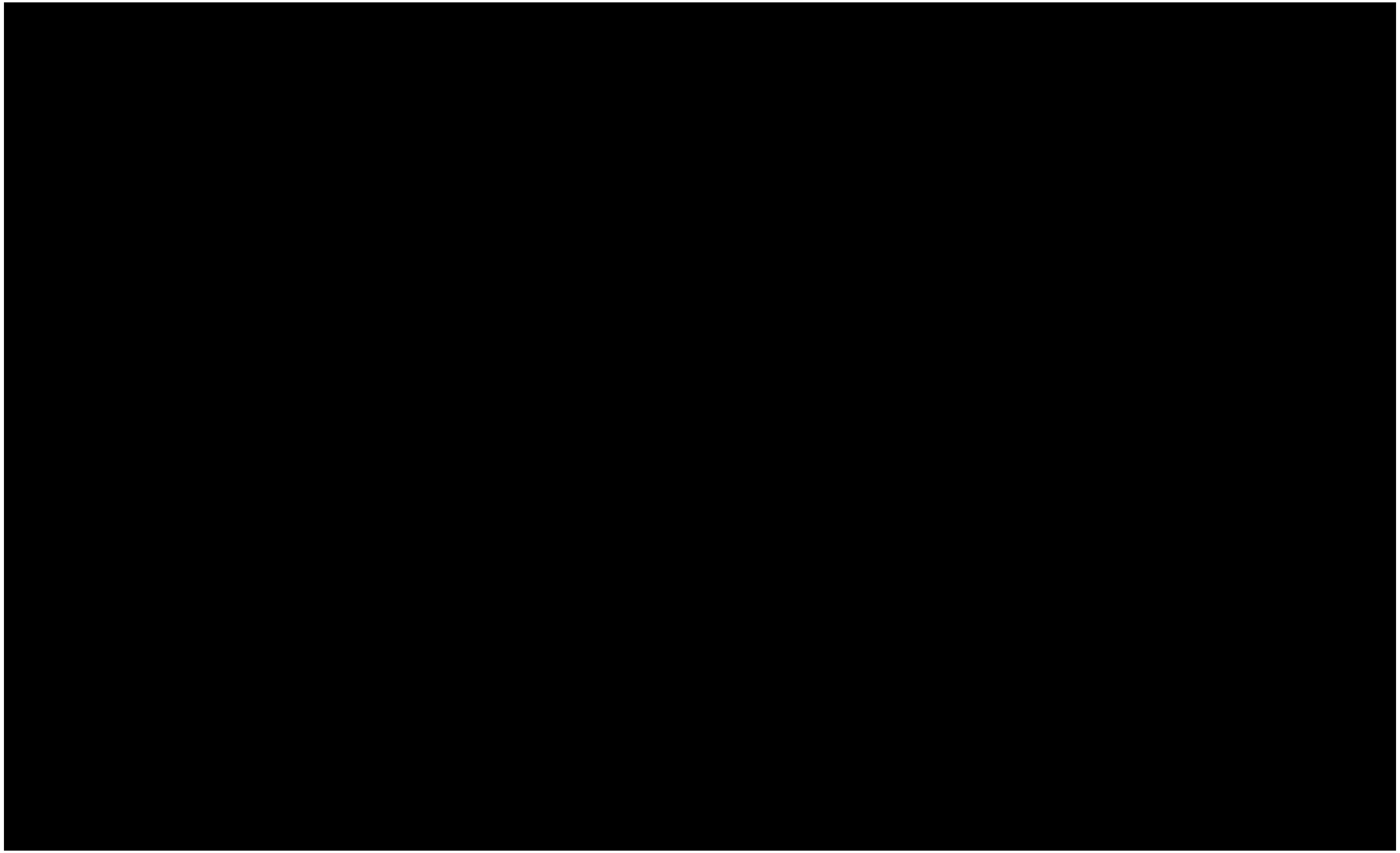
Thus equating the equations for Q we get

$$\oint_s \vec{D}.\vec{dS} = \oint_v (\nabla.\vec{D}) \, dv$$

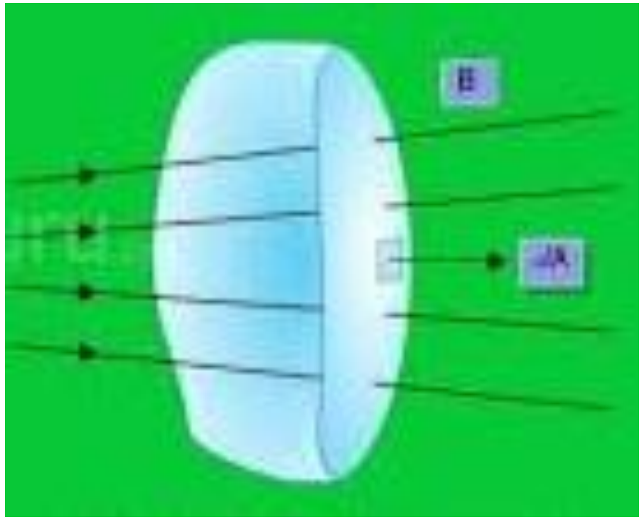
Thus Gauss divergence theorem. Divergence theorem relates the surface integral with volume integral.

Oersted experiment

In 1820, a Danish physicist, Hans Christian **Oersted**, discovered that there was a relationship between electricity and magnetism. By setting up a compass through a wire carrying an electric current, **Oersted** showed that moving electrons can create a magnetic field.



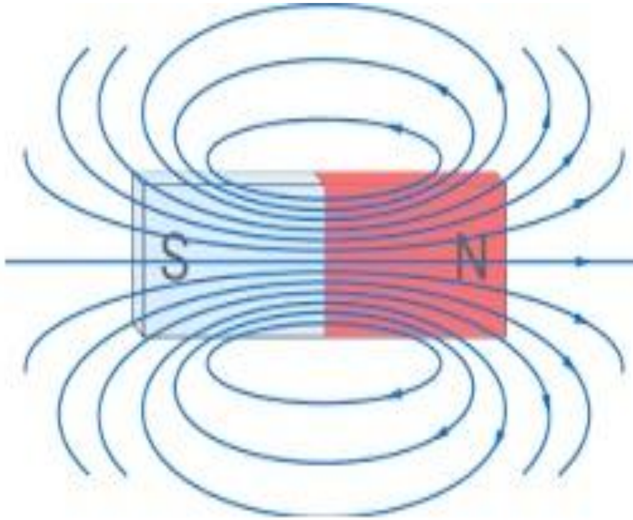
Gauss Law of magnetostatics



Consider a closed Gaussian surface of any shape in a magnetic field.

The magnetic fields lines exist in closed loops. Hence for every flux line that enters the closed surface a flux line emerges out else where.

The net flux out of any closed surface is zero.



Magnetic sources are dipole sources and magnetic field lines loops;

we can not isolate ‘N’ and ‘S’ monopoles (unlike electric sources, point charges-protons and electrons).

Thus for a closed surface in a magnetic field the total inward flux (Positive) is equal to total outward flux (Negative).

Gauss Law of magnetostatics

Thus the net flux through the Gaussian surface is zero. Thus it could be written

$$\oint_s \vec{B}.\vec{dS} = 0 \qquad \text{........(1)}$$

$\vec{B}$ is magnetic flux density.

W.k.t Gauss divergence theorem is

$$\oint_s \vec{D}.\vec{dS} = \oint_v (\nabla.\vec{D})\, dv$$

Thus equation (1) could be written as

$$\oint_s \vec{B}.\vec{dS} = \oint_v (\nabla.\vec{B})\, dv = 0$$

Hence

$$\nabla.\vec{B} = 0$$

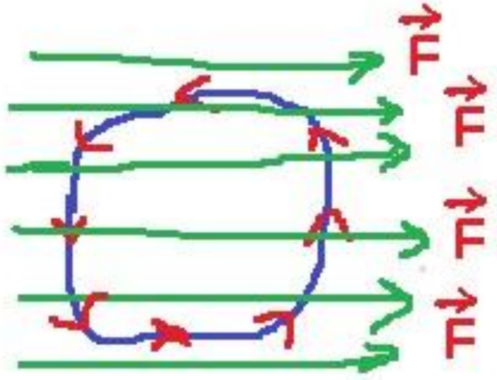
This is one of the Maxwell's equation

Stokes Theorem

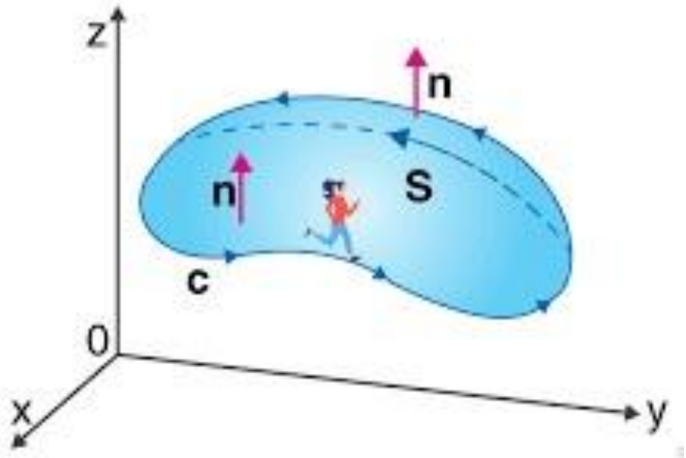
Stokes theorem relates surface integral with line integral (Circulation of a vector field around a closed path).

Statement: The surface integral of curl of $\vec{F}$ throughout a chosen surface is equal to the circulation of the $\vec{F}$ around the boundary of the chosen surface.
Mathematically

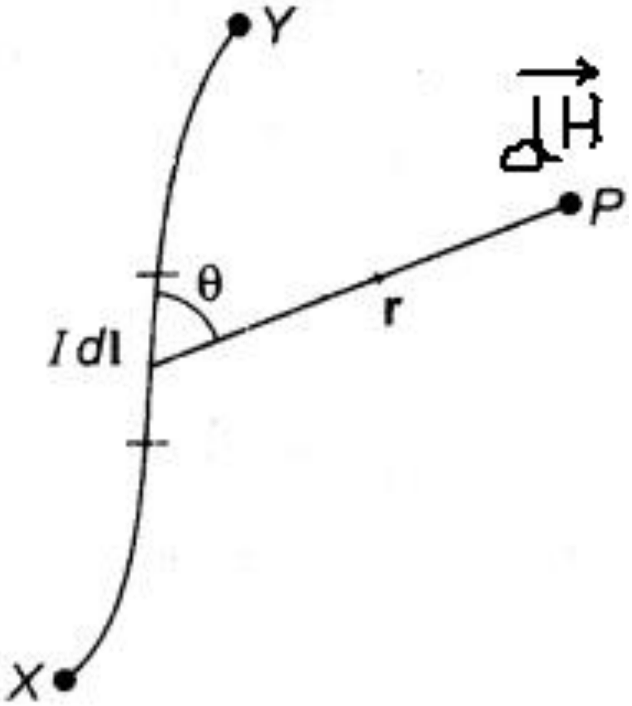
$$\int_s (\nabla \times \vec{F}).\vec{dS} = \oint \vec{F}.\vec{dl}$$

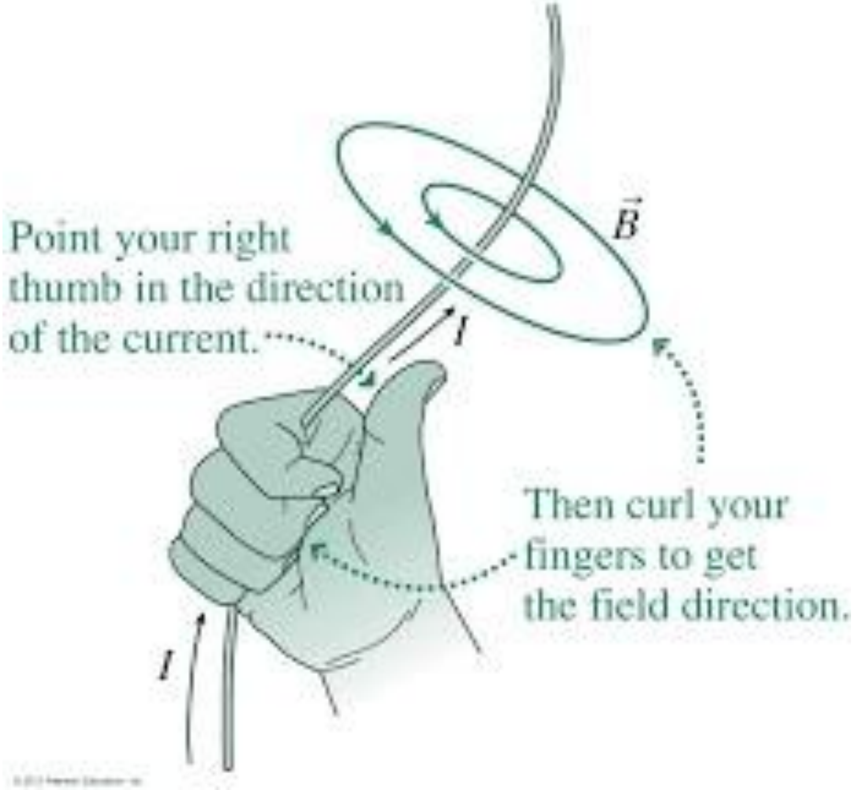


closed loop - circulation



Biot-Savart Law





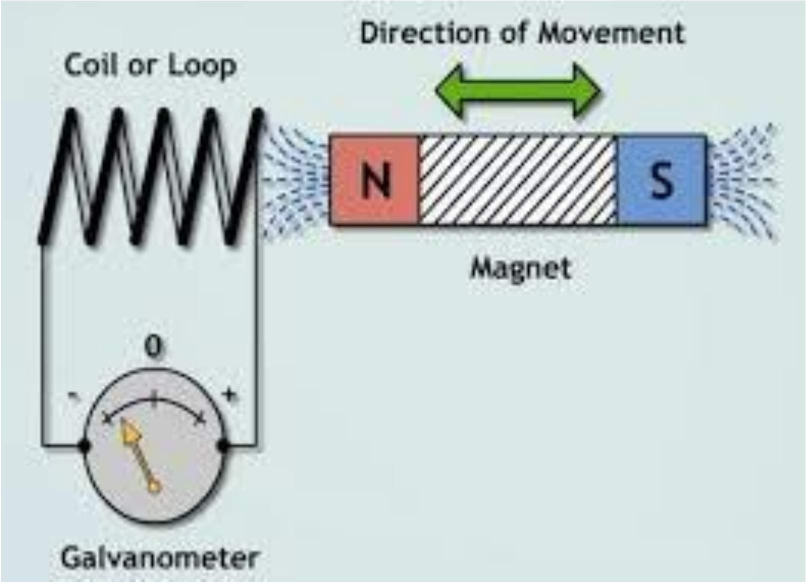
$$dH \propto \frac{I\ dl\ Sin(\theta)}{r^2}$$

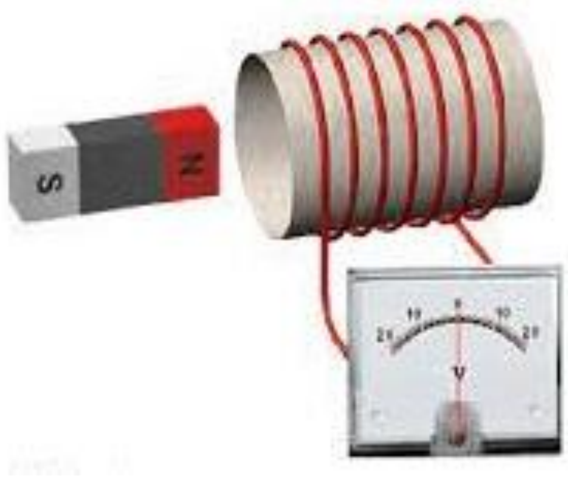
$$dH = \frac{\mu_0}{4\pi}\frac{I\ dl\ Sin(\theta)}{r^2}$$

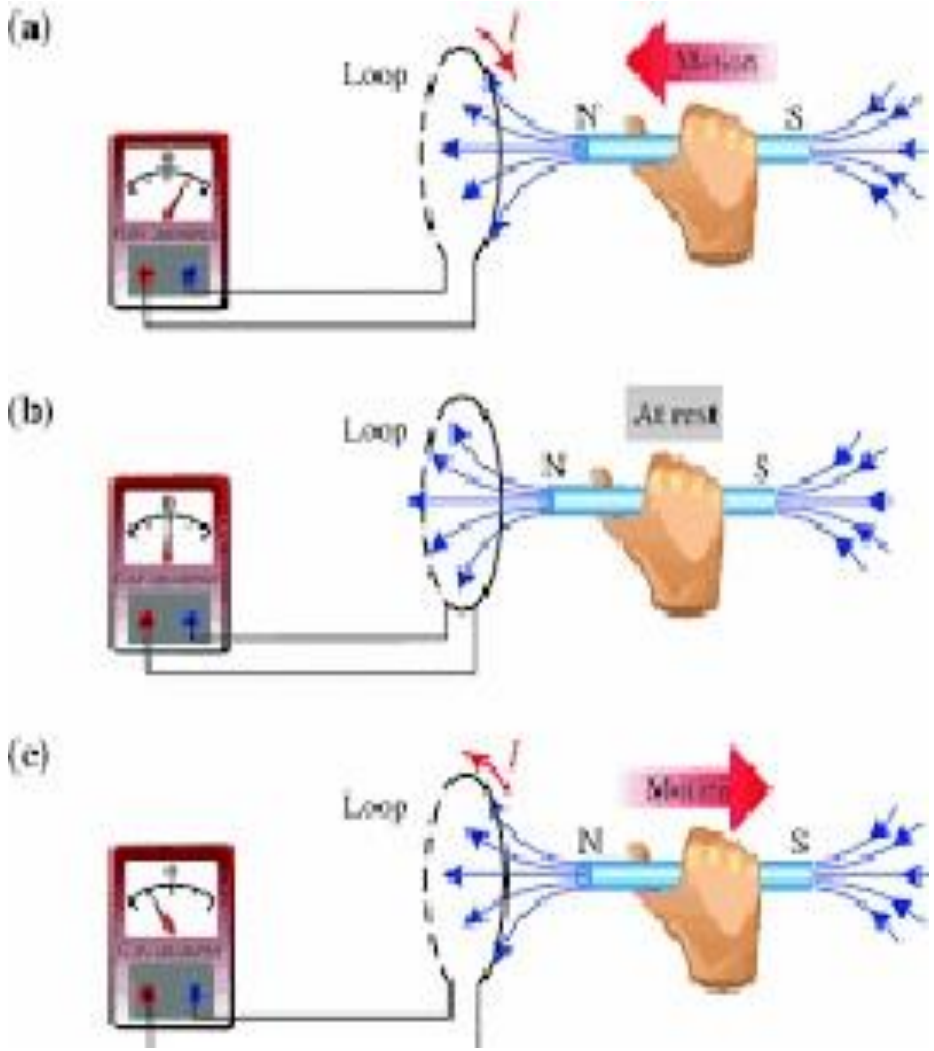
$$d\vec{H} = \frac{\mu_0}{4\pi}\frac{I\ \vec{dl} \times \hat{r}}{r^2}$$

$$d\vec{H} = \frac{\mu_0}{4\pi}\frac{I\ \vec{dl} \times \vec{r}}{r^3}$$

Faraday's Law of electro-magnetic induction







Faraday's Law of electro-magnetic induction

Statement :

1. When ever there is a change in magnetic flux linked with the circuit an emf (e) is induced and is equal to rate of change of magnetic flux.
2. The emf induced is in such a direction that it opposes the cause.

Mathematically the induced emf is given by

$$e = -\frac{d\phi}{dt}$$

φ is magnetic flux linked with the circuit.

For a coil of N turns the induced emf due to rate of change of flux is given by

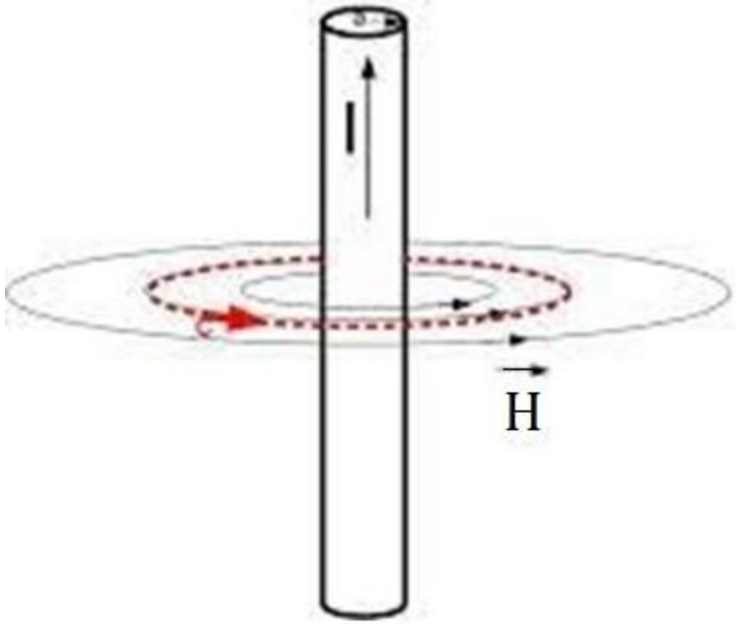
$$e = -N\frac{d\phi}{dt}$$

Faraday's law in differential (Point form).

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This is one of the Maxwell's equations.

Ampere's Law



$$H_{total} \propto I$$

$$H_{total} = \mu_0 I$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

The strength of magnetic field is

$$H = \frac{H_{total}}{total\ length}$$

$$H \times total\ length = H_{total}$$

$$\oint \vec{H}.\vec{dl} = H_{total}$$

$$\oint \vec{H}.\vec{dl} = \mu_0 I$$

$$\oint \vec{H}.\vec{dl} = I_{enc}$$

Also I_{enc} could be written as

$$\oint_s \rho_v\, \vec{ds} = I_{enc}$$

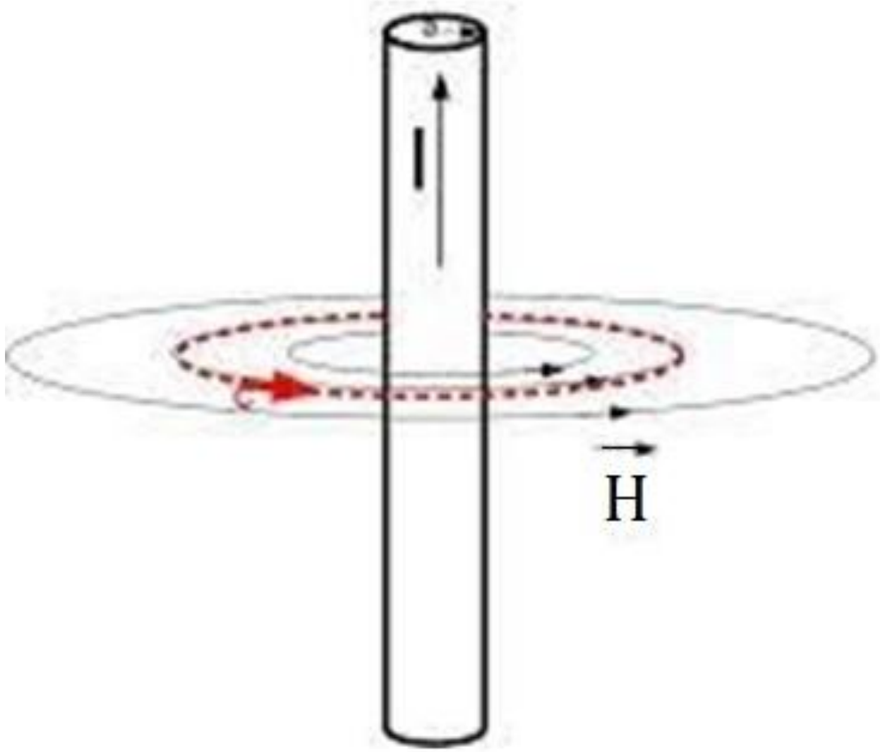
$J = \rho_v$ is the current density

$$\oint_s \vec{J}.\vec{ds} = I_{enc}$$

Ampere's Law

Statement: The circulation of magnetic field strength H along a closed path is equal to the net current enclosed (Ienc) by the loop. Mathematically

$$\oint \vec{H}.\vec{dl} = I_{enc}$$



Ampere's Law

$$\oint \vec{H}.\vec{dl} = I_{enc}$$

By applying stokes' theorem we get

$$\int_s (\nabla \times \vec{H}).\vec{dS} = I_{enc}$$

The equation for Ienc could be obtained as

$$I_{enc} = \oint_s \vec{J}.\vec{dS}$$

$$\int_s (\nabla \times \vec{H}).\vec{dS} = \oint_s \vec{J}.\vec{dS}$$

$$\int_s (\nabla \times \vec{H}).\vec{dS} - \oint_s \vec{J}.\vec{dS} = 0$$

$$\int_s (\nabla \times \vec{H} - \vec{J}).\vec{dS} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

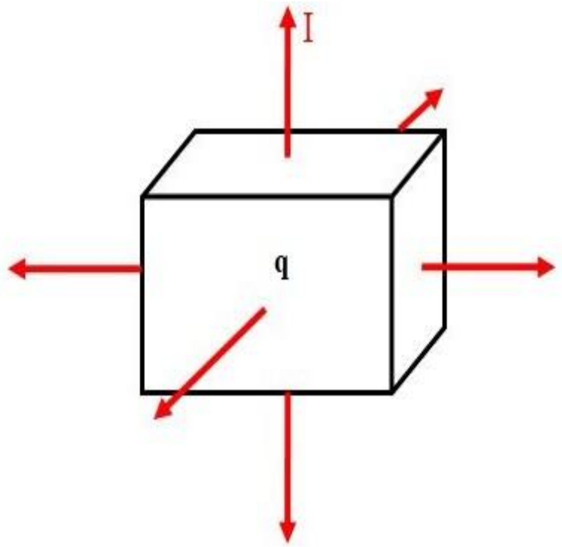
is another Maxwell's equation.

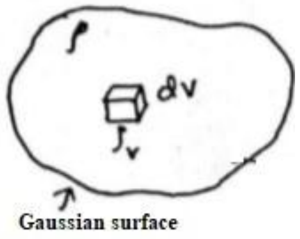
Equation of Continuity

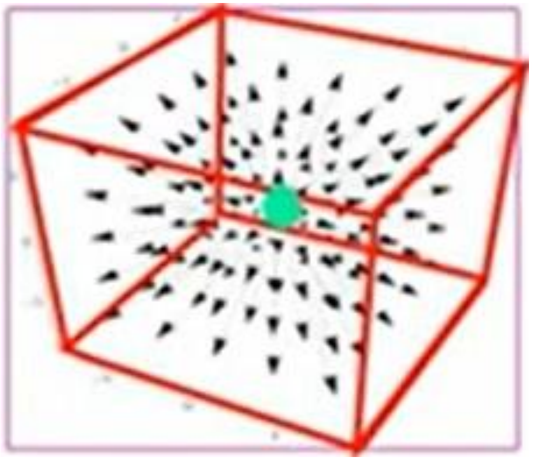
In all processes involving motion of charge carriers the net charge is always conserved and is called the law of conservation of charges.

Let us consider a volume V. Let the charges flow into and out of the volume V. Then the equation for the law of conservation could be written in the integral form as

$$\oint_s \vec{J}.\vec{dS} = -\frac{\partial}{\partial t}\int_v \rho_v \, dV \qquad \text{-------(1)}$$







The negative sign indicates that the current density is due to the decrease in positive charge density inside the volume.

Equation of Continuity

$$I = -\frac{\partial q}{\partial t}$$

$$q = \oint_v \rho \, dv$$

is the volume charge density

$$I = \oint_s \vec{J} \cdot \vec{ds}$$

From law of conservation of charges

$$\oint_s \vec{J} \cdot \vec{ds} = -\frac{\partial q}{\partial t}$$

$$\oint_s \vec{J} . d\vec{S} = -\frac{\partial}{\partial t}\int_v \rho_v \, dV \quad \text{-------(1)}$$

Using the Gauss divergence theorem we can write

$$\oint_s \vec{J} . d\vec{S} = \oint_v (\nabla . \vec{J}) . dV$$

Now equation (1) could be written as

$$\oint_v (\nabla . \vec{J}) . dV = -\frac{\partial}{\partial t}\int_v \rho_v \, dV$$

$$\oint_v (\nabla . \vec{J}) . dV = -\int_v \frac{\partial \rho_v}{\partial t} \, dV$$

$$\oint_v (\nabla . \vec{J}) . dV + \int_v \frac{\partial \rho_v}{\partial t} \, dV = 0$$

$$\oint_v \left[(\nabla . \vec{J}) + \frac{\partial \rho_v}{\partial t} \right] . dV = 0$$

$$\nabla . \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

law of conservation of charges.

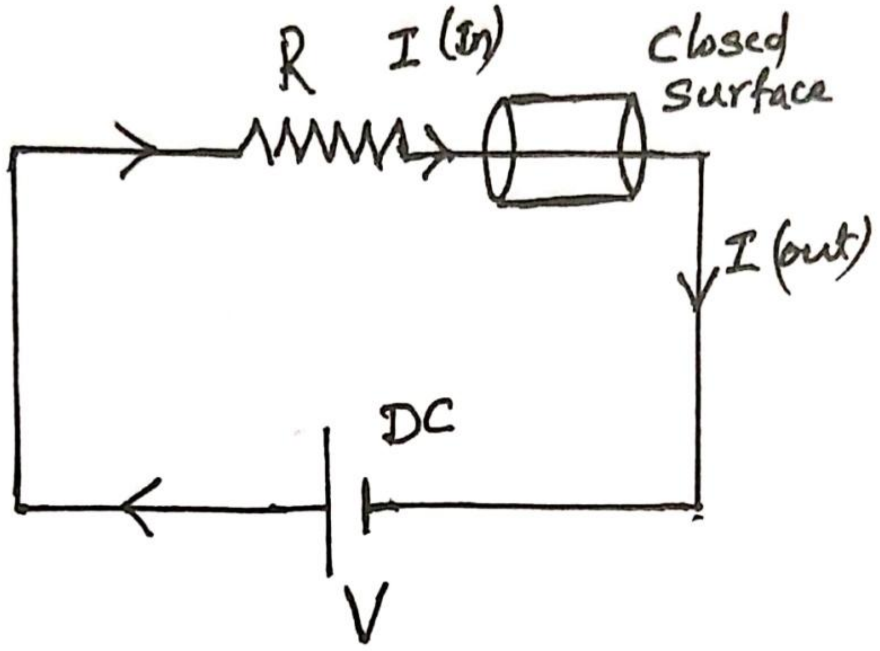
Discussion on continuity equation

- In case of DC circuits for steady current the inward flow of charges is equal to the outward flow through a closed surface and hence

$$\frac{\partial q}{\partial t} = 0$$

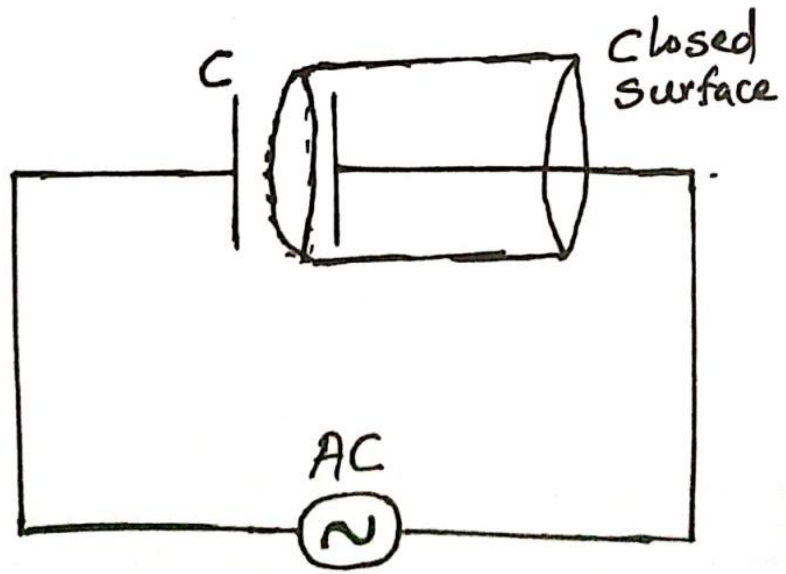
- Thus the equation of continuity becomes

$$\nabla . \vec{J} = 0$$



Discussion on continuity equation

- In case of AC circuits containing capacitors the equation fails as follows.
- During the *positive half cycle*, say, the *capacitor charges.* If we imagine a closed surface enclosing the capacitor plate and the attached conductor there will be inward flow of charges into the closed surface but no outward flow.
- Thus in order to rescue the equation of continuity Maxwell introduced the concept of *displacement current density*.

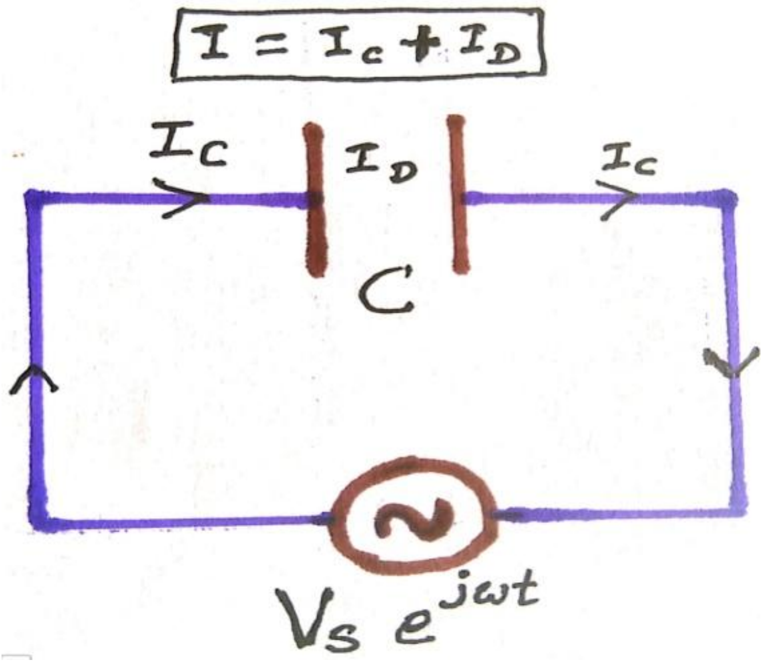


Displacement Current

- In electromagnetism, displacement current is a quantity appearing in Maxwell's equations that is defined in terms of the rate of change of electric displacement field.

- Displacement current has the units of electric current density, and it has an associated magnetic field just as actual currents do.

- Displacement current is associated with magnetic field but it does not describe the flow of charge.



Maxwell-Ampere Law

Introducing the concept of displacement current for time varying circuits, Maxwell suggested corrections to the Amperes law. According to Gauss' Law

$$\nabla . \vec{D} = \rho_v$$

Differentiate w.r.t time t we get,

$$\frac{\partial(\nabla . \vec{D})}{\partial t} = \frac{\partial \rho_v}{\partial t}$$

$$\nabla . \frac{\partial \vec{D}}{\partial t} = \frac{\partial \rho_v}{\partial t} \text{ ------------(1)}$$

The equation of continuity is given by

$$\nabla . \vec{J} = -\frac{\partial \rho_v}{\partial t} \text{ ----------(2)}$$

using equation (1) & (2) we get,

$$\nabla . \vec{J} = -\nabla . \frac{\partial \vec{D}}{\partial t}$$

$$\nabla . \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = 0$$

For time varying circuits $\nabla . \vec{J} = 0$ does not hold good

$\therefore \nabla . \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = 0$ has to be used

Ampere's circuital law is

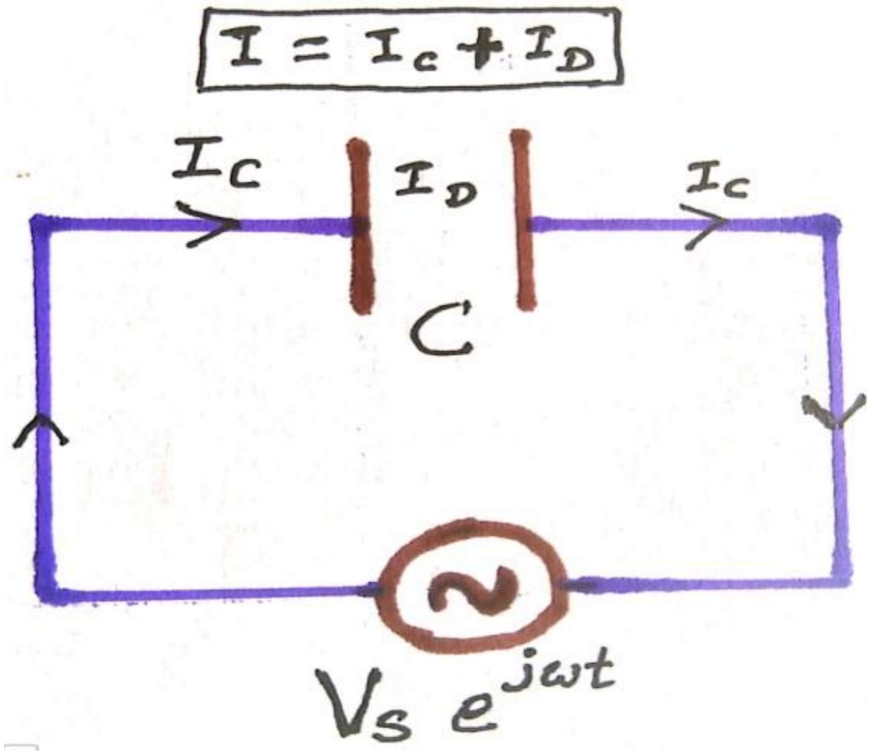
$$\nabla \times \vec{H} = \vec{J}$$

Now, Maxwell-Ampere law could be written as

$$\boxed{\nabla \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)}$$

Is the Maxwell's fourth equation

Expression for displacement current



Consider an AC circuit containing a capacitor.

Let 'A' be the area of each plate, the potential be

$V_s e^{j\omega t}$

The electric flux density D is give by

$D = \epsilon E - - - - -(1)$

Here E is the electric field strength

For parallel plate capacitor electric field strength

E is given by

$E = \frac{V}{d} - - - - -(2)$

here *d* is separation between the plates

Expression for displacement current

The applied potential V is

$$V = V_s e^{j\omega t} - - - - (3)$$

Using equation (2) in equation (1) we get,

$$D = \frac{\epsilon V}{d}$$

$$\therefore D = \frac{\epsilon V_s e^{j\omega t}}{d} - - (4)$$

If I_D is the displacement current then,

Displacement current density $= \frac{I_D}{A}$

But displacement current $= \frac{\partial D}{\partial t}$

$$\therefore \frac{I_D}{A} = \frac{\partial D}{\partial t}$$

$$I_D = A\left(\frac{\partial D}{\partial t}\right)$$

Using equation (4) we have,

$$I_D = A\frac{\partial}{\partial t}\left(\frac{\epsilon V_s e^{j\omega t}}{d}\right)$$

Executing differentiation

$$\therefore \boxed{I_D = \frac{j\omega\epsilon A}{d} V_s e^{j\omega t}}$$

This is the expression for Displacement current

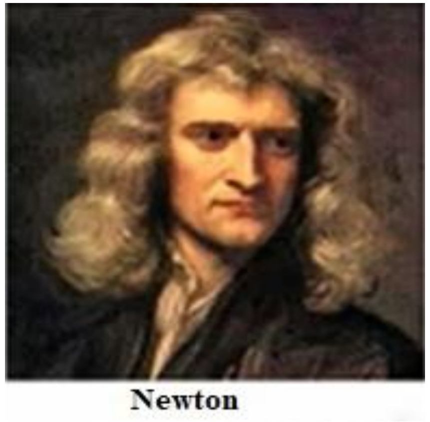
Using the laws and theorems discussed in this chapter Four Maxwell's equations could be written as

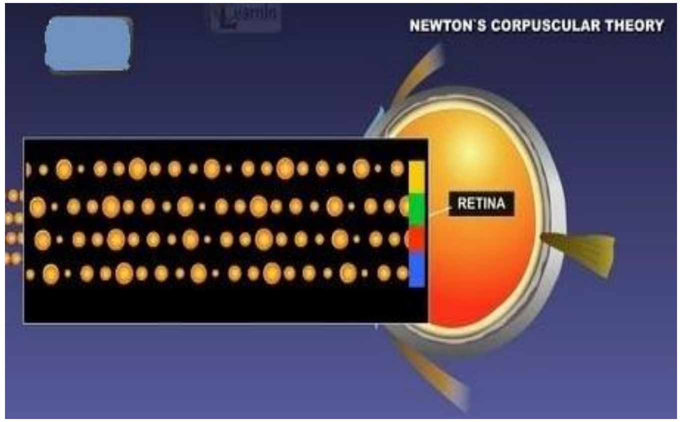
Time varying fields		Static fields	
1. Gauss' Law of Electrostatics	$\nabla.\vec{D} = \rho_v$	1. Gauss' Law of Electrostatics	$\nabla.\vec{D} = \rho_v$
2. Faraday's Law	$(\nabla \times \vec{E}) = -\frac{\partial \vec{B}}{\partial t}$	2. Faraday's Law	$(\nabla \times \vec{E}) = 0$
3. Gauss' Law of Magnetic fields	$\nabla.\vec{B} = 0$	3. Gauss' Law of Magnetic fields	$\nabla.\vec{B} = 0$
4. Maxwell - Ampere Law	$\nabla \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$	4. Ampere's Law	$\nabla \times \vec{H} = \vec{J}$

JAMES CLERK MAXWELL'S ELECTROMAGNETIC THEORY OF LIGHT

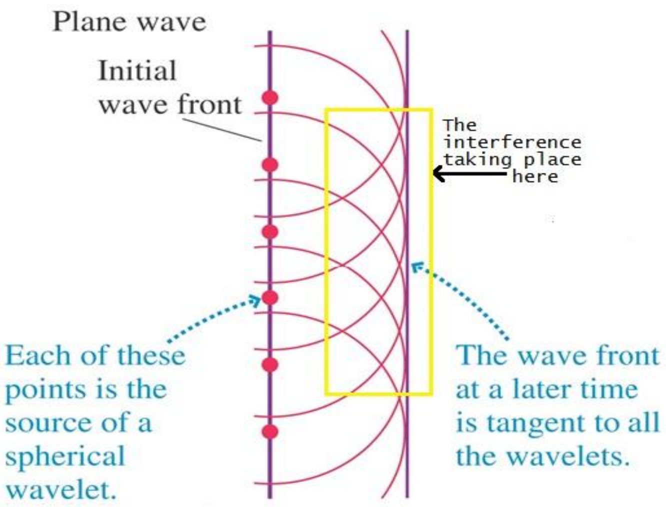
Two theories In 17th Century

In 1675, Corpuscular theory by Newton



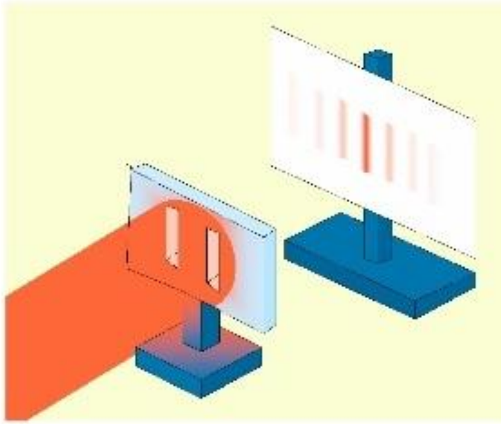


In 1690, Wave theory by Huygen



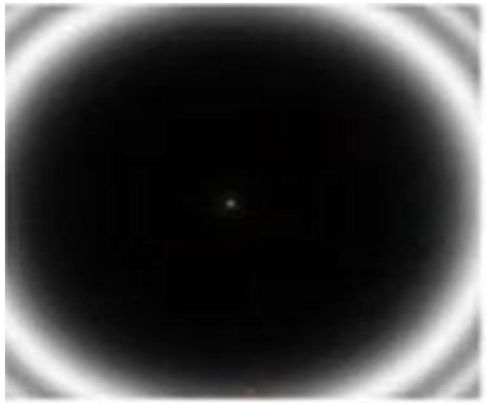
Wave theory gained momentum as time progress

In 1801 Young's double slit experiment



In 1819 Fresnel submitted his work on diffraction using wave theory to the French Academy of Science.

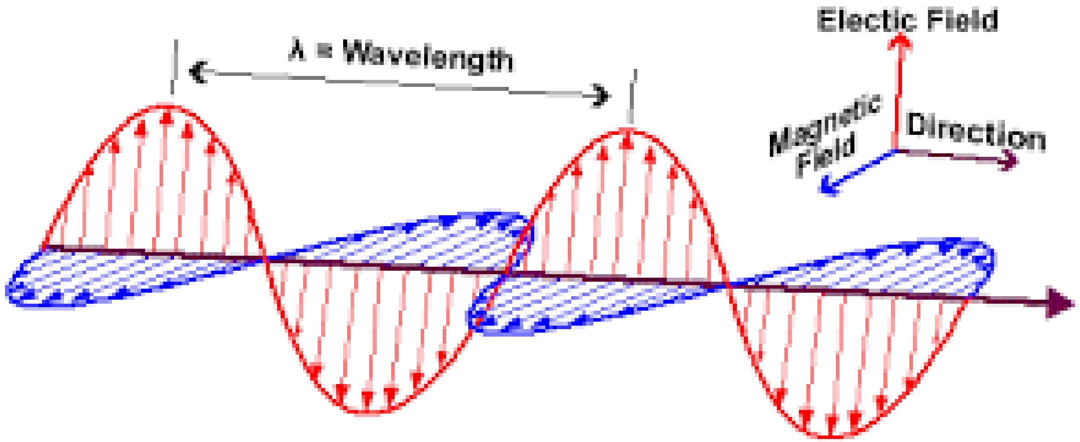
Fresnel

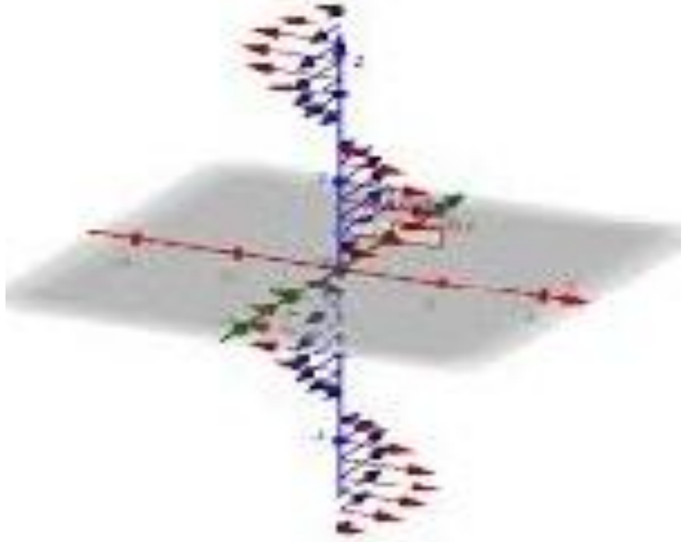


Bright spot at the center of shadow

- Maxwell's four equations describe the electric and magnetic fields arising from distributions of electric charges and currents, and how those fields change in time.
- Maxwell's own contribution to these equations is just the last term of the last equation but the addition of that term had dramatic consequences.
- It made evident for the first time that, the varying electric and magnetic fields could feed off each other these fields could propagate indefinitely through space, far from the varying charges and currents where they originated.
- Maxwell's new term (called displacement current) feed them to move through space in a self-sustaining fashion, and even predicted their velocity it was the velocity of light!

EM waves





EM Waves

Consider differential form of Maxwell's equation.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \dots\dots\dots(1)$$

$$\nabla \times \vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \dots\dots\dots(2)$$

The relation between electric flux density and electric field is

$$D = \epsilon E ----(3)$$

The relation between Magnetic flux density and magnetic field is

$$B = \mu H ----(4)$$

Using equation (4) and (3) in equation (1) and (2) we get,

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \dots\dots\dots(5)$$

$$\nabla \times \vec{H} = \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) \dots\dots\dots(6)$$

Equation (5) and (6) are known as coupled equation

To derive wave equation in terms of electric field, the term $\vec{H}$ has to be eliminated. Taking curl on both sides the equation (5) we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H}) \dots\dots\dots(7)$$

Using equation (6) we have,

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) \dots\dots\dots(8)$$

To solve LHS of above equation

We have $A \times B \times C = B(A.C) - C(A.B)$ thus

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla.\vec{E}) - \nabla^2\vec{E} \dots\dots\dots(9)$$

EM Waves

The LHS in equation (11) represents a propagating wave and the RHS the source of origin of the wave. Here μ and ϵ are respectively Absolute permeability and Absolute permittivity of isotropic homogeneous medium.

In case of propagation of EM wave in free space $(\vec{J} = 0\ and\ \rho_v = 0)$

$\therefore$ Equation (11) reduces to

$$\nabla^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0 \text{.........} (12)$$

The general wave equation is given by

$$\nabla^2\vec{F} - \frac{1}{v^2}\frac{\partial^2\vec{F}}{\partial t^2} = 0 \text{.........} (13)$$

On comparing equation (12) and (13)

We get

$$\mu\epsilon = \frac{1}{v^2}$$

$$\therefore v = \frac{1}{\sqrt{\mu\epsilon}}$$

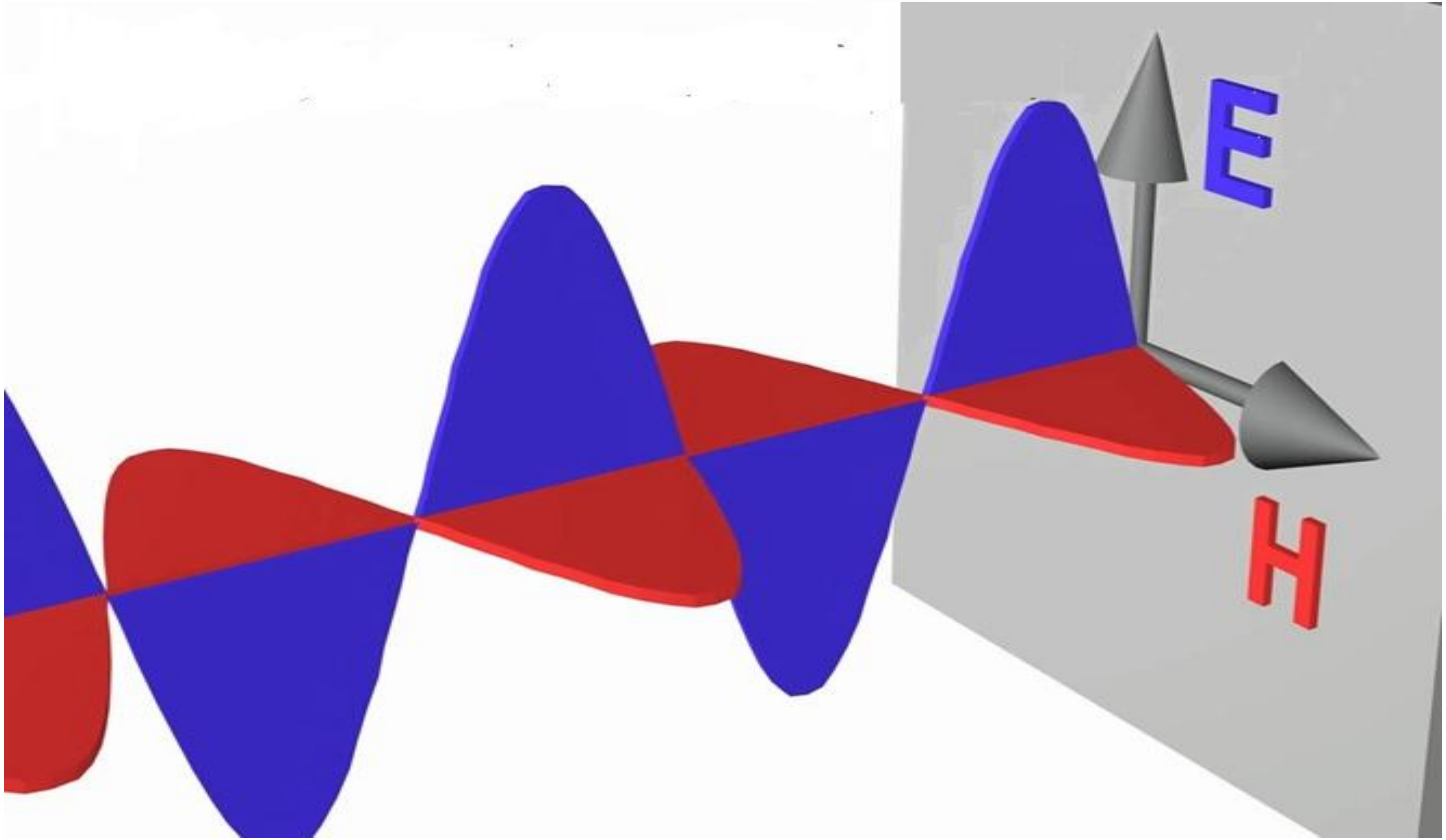
In vacuum, $\mu = \mu_0 = 4\pi \times 10^{-7} Hm^{-1}$,

$$\epsilon = \epsilon_0 = 8.854 \times 10^{-12}\ Fm^{-1}\ and\ v = c$$

$$\therefore c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{4 \times 3.14 \times 10^{-7} \times 8.854 \times 10^{-12}}}$$

$$c = 2.998 \times 10^8\ ms^{-1}$$

EM Waves



Polarization of EM Waves

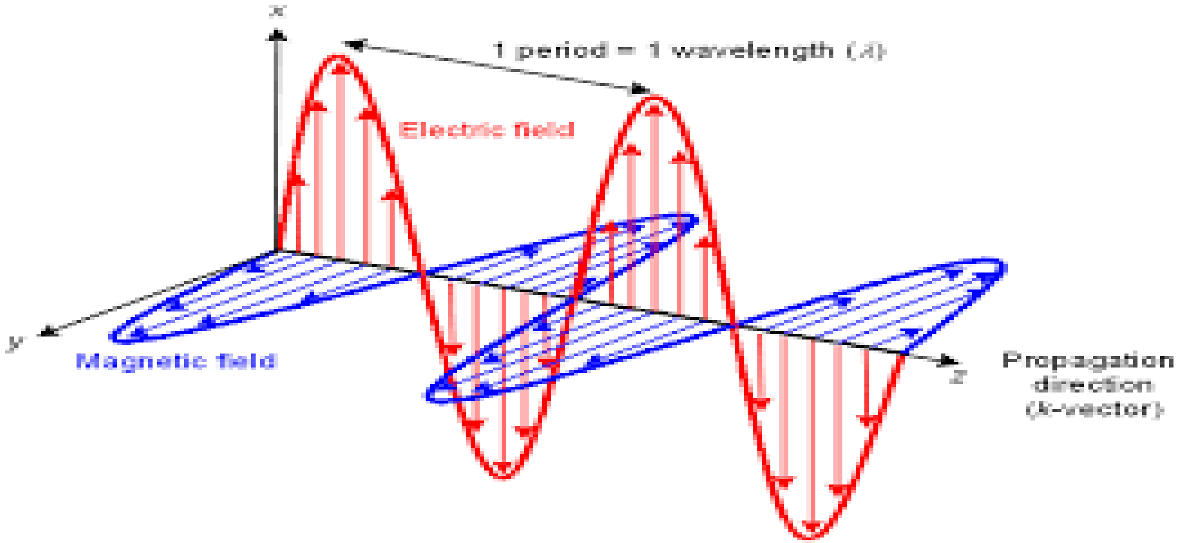
- Electromagnetic waves also exhibit polarization. Consider an electromagnetic wave propagating along z-axis. The electric field vector of this electromagnetic wave makes an angle theta with respect to x-axis, say. This electric vector could be resolved into two perpendicular components Ex and Ey along x and y axes respectively.
- Based on the magnitudes of the components and the phase difference between the components there are three kinds of Polarization of electromagnetic waves.

- They includes

- 1. Linearly Polarized EM waves
- 2. Circularly Polarized EM waves
- 3. Electrically Polarized EM waves

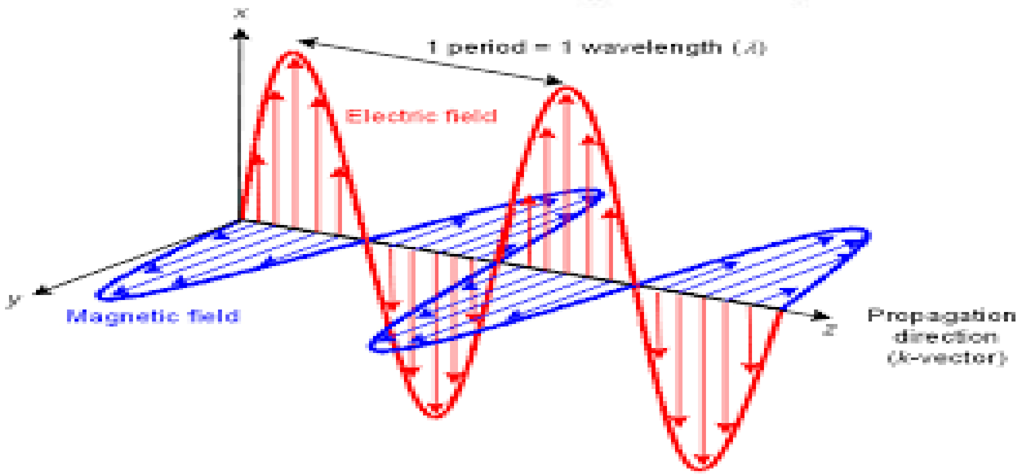
Plane EM Waves

- Electromagnetic waves that travels in one direction and uniform in the other two orthogonal directions is called plane electromagnetic waves.
- For example, consider a plane electromagnetic wave traveling along z axis the electric and magnetic vibrations are uniform and confined to xz and yz planes respectively.



Plane EM Waves

Consider a plane electromagnetic wave propagating along +ve z-axis. If the time varying electric and magnetic fields are along x and y axes respectively then we can write



$$\boldsymbol{E} = A\cos\left(\frac{2\pi}{\lambda}(x - ct)\right)\boldsymbol{i}$$

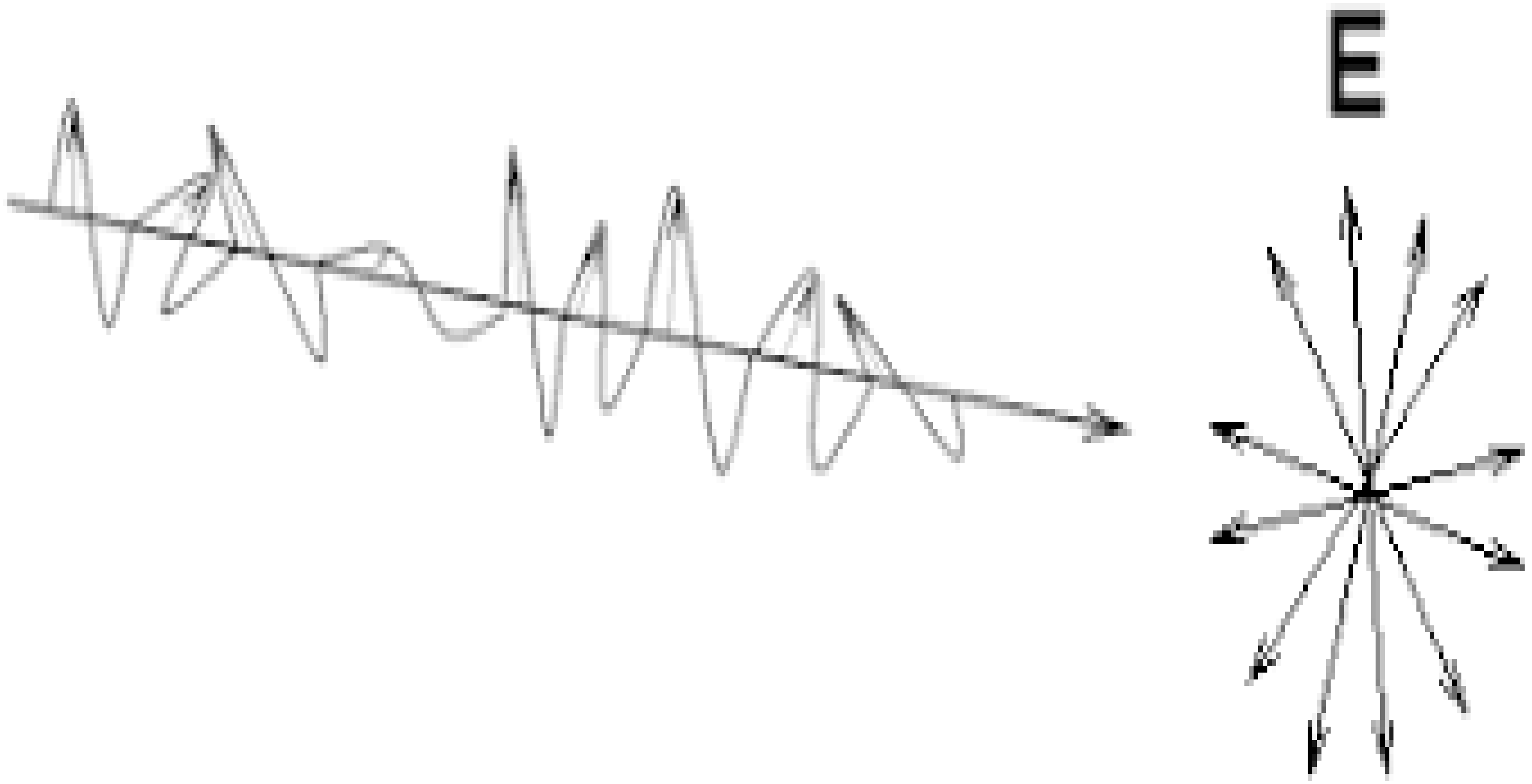
$$\boldsymbol{B} = \frac{1}{c}A\cos\left(\frac{2\pi}{\lambda}(x - ct)\right)\boldsymbol{j}$$

The ratio of magnitudes from the above equations gives,

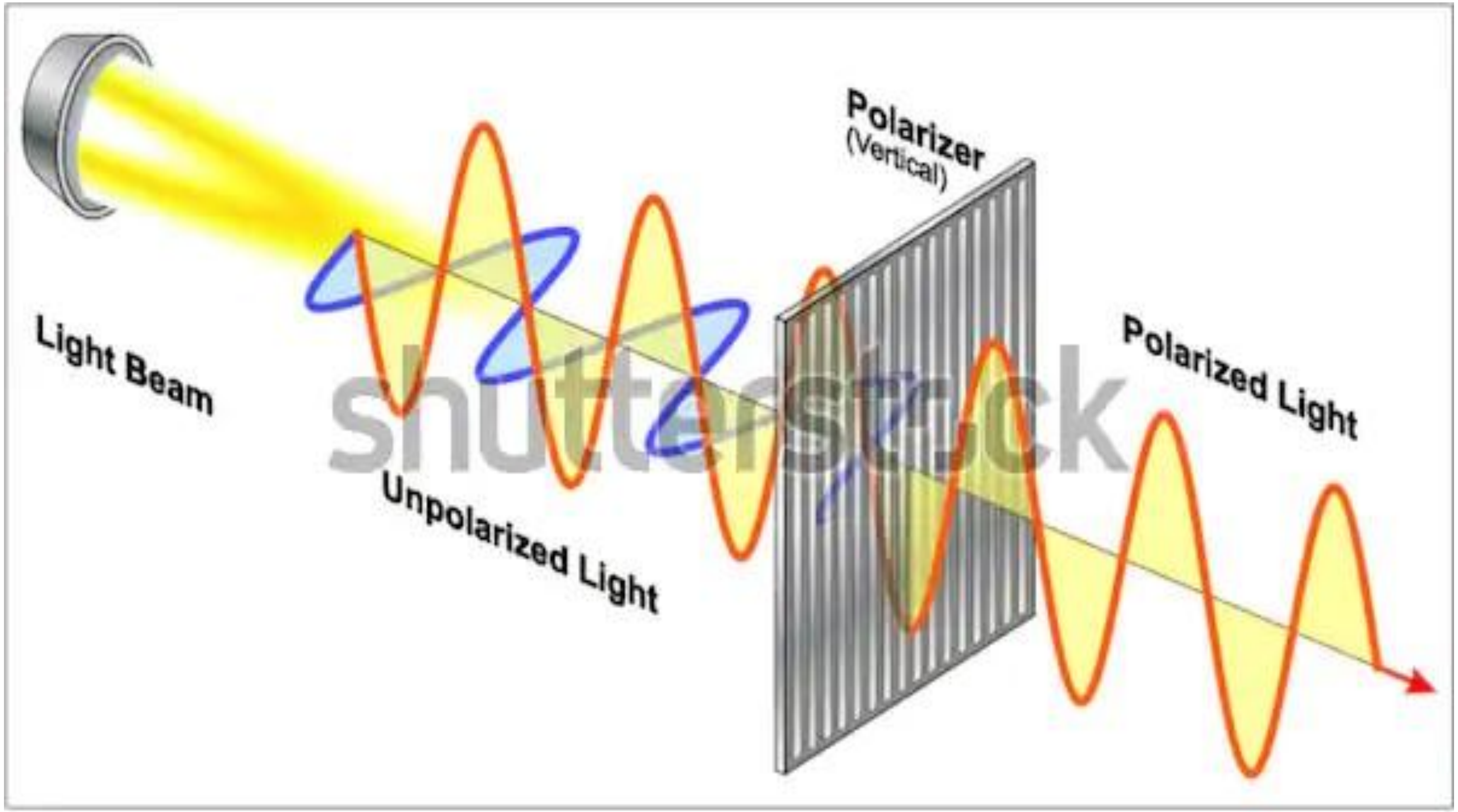
$$\frac{\boldsymbol{E_x}}{\boldsymbol{B_y}} = \boldsymbol{c}$$

Here c is the speed of light.

Unpolarized light



Polarized light



Outline

1. Review of the Previous Discussion
2. Semiconductors
    - Introduction
    - Intrinsic Semiconductors
    - Extrinsic Semiconductors
        - n-Type Semiconductors
        - p-Type Semiconductor
    - Significance of Femi level in Semiconductors
    - Carrier Concentration in Intrinsic Semiconductors
    - Expression for Electron concentration (N_e)
    - Expression for Hole concentration (N_e)
    - Expression relating Fermi energy and Energy gap for an intrinsic semiconductor

Introduction to Semiconductors

- Semiconductors are the materials which posses negative temperature co-efficient of resistance.
- Their electrical properties are different from that conductors and insulators.
- Their electrical properties lie between that of conductors and insulators.
- Semiconductors are insulators at temperatures close to absolute zero.
- At room temperature, a semiconductor has enough free electrons to allow it to conduct current.
- The resistivity of semiconductors decreases with increase in temperature.
- The electrical conductivity of semiconductors increases with increase in temperature.
- Most of the applications of Semiconductors are in electronic devices.

Intrinsic Semiconductors

- The pure form of semiconductors are known as intrinsic semiconductors.
- Example Germanium and Silicon are most frequently used semiconductors.
- Semiconductors are tetra-valent.
- The conductivity in intrinsic semiconductors is due to both the motion of electrons in the conduction band and the motion of holes in the valence band.
- The electrons in the valence band of intrinsic semiconductors need more energy to move to conduction band.

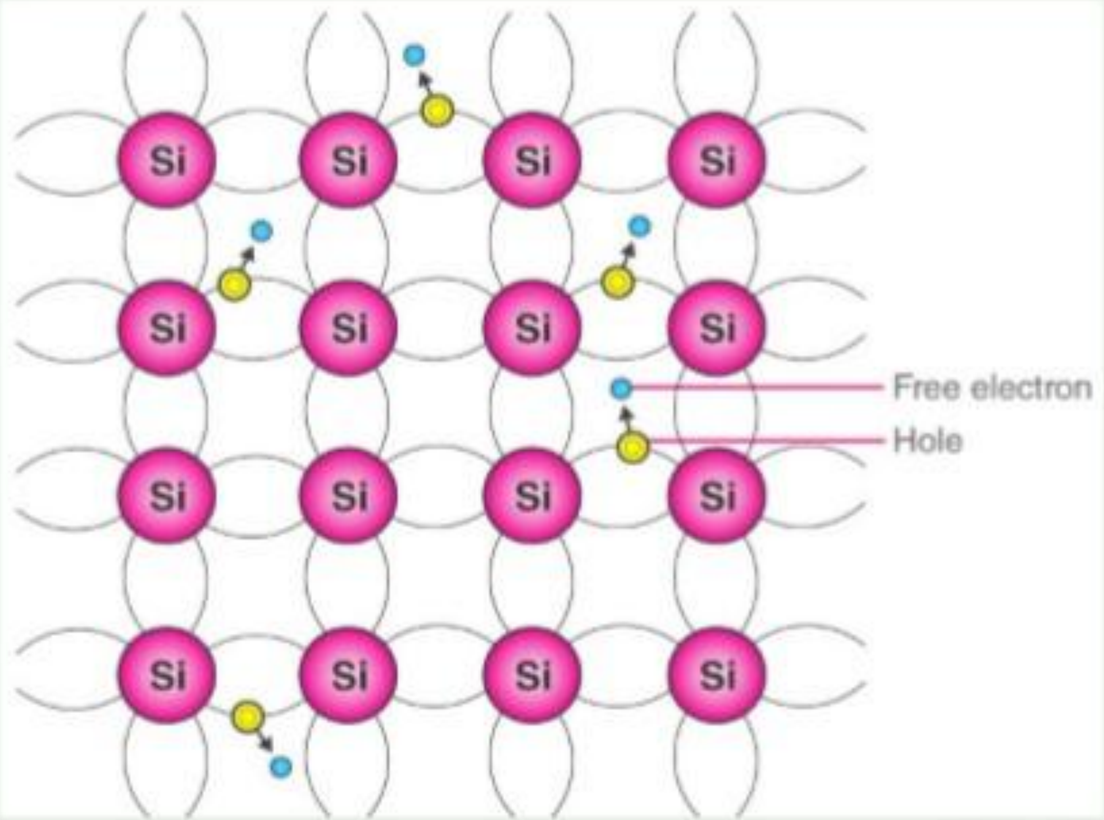


FIGURE: 2-D Intrinsic Semiconductor

Extrinsic Semiconductors : n-Type

- Doping of pentavalent impurity atoms to pure semiconductor results in the fifth electron loosely bound to impurity atoms.
- The energy of these electrons lie in the energy gap of semiconductor close to the conduction band hence donor level is created.
- Given a small amount of energy electrons move from donor level to conduction band.
- Large number of free electrons are found in the conduction band.
- Thus electrons are the majority charge carriers in n-type semiconductors.

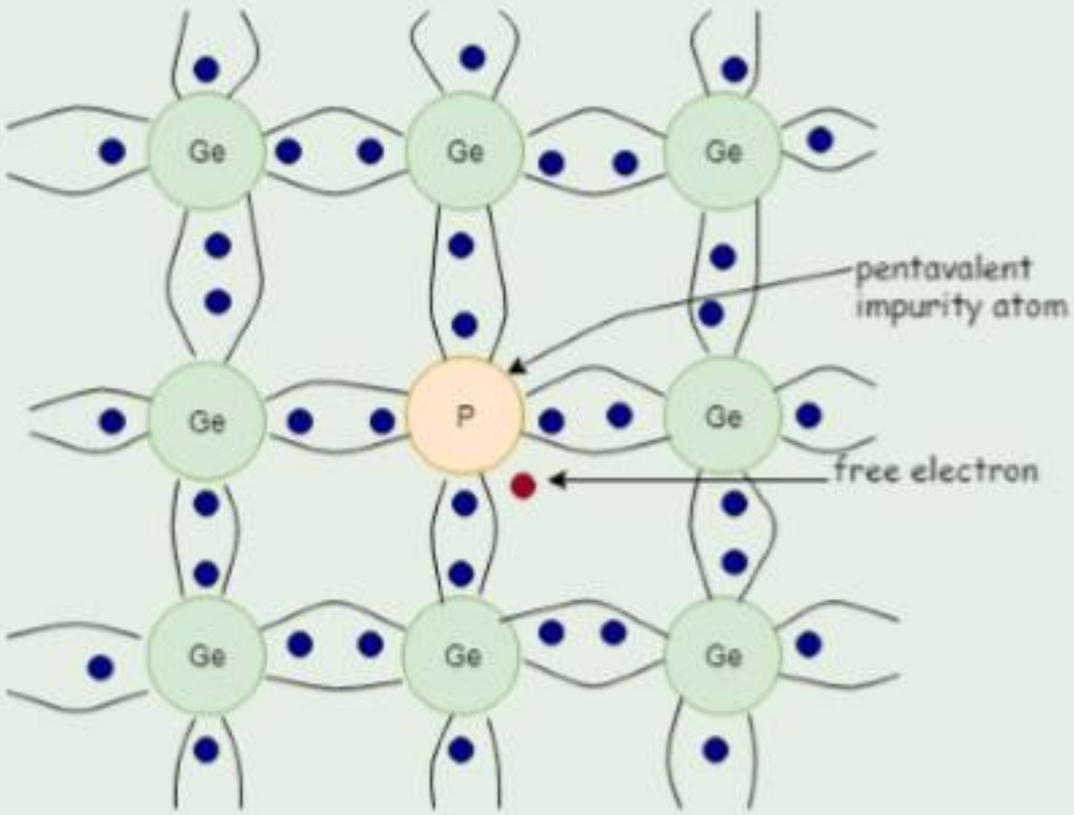


Figure: 2-D n-Type Semiconductor

Activate

Extrinsic Semiconductors : p-Type

- The doping of trivalent impurity atoms to pure semiconductor results in the formation of the acceptor level near the upper edge of the valence band.
- Holes are created at the incomplete covalent bond locations.
- Holes are the majority carriers in p-type semiconductors.
- The electric current is due to the motion of holes in the valence band.

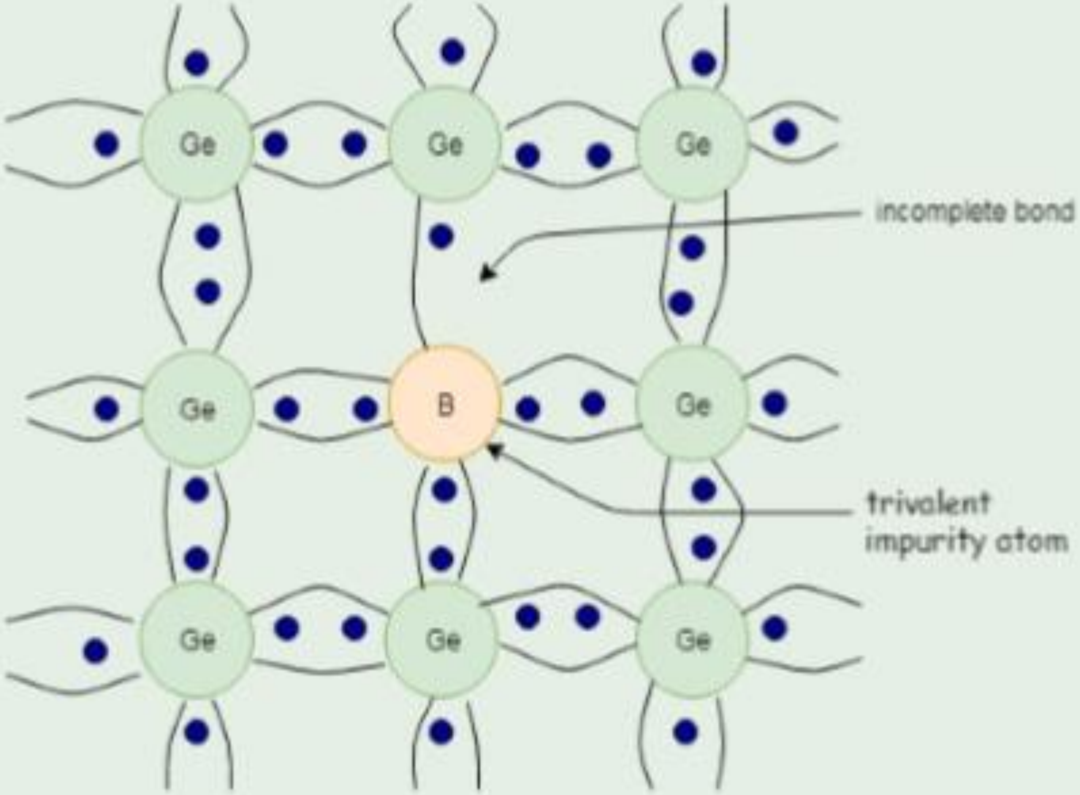


FIGURE: 2-D p-Type Semiconductor

Significance of Fermi Level In Semiconductors

- Fermi level acts as a distinguishing energy position between filled and unfilled energy states in metals and semiconductors.
- In case of pure semiconductors, at normal temperatures, electrons are most probably found either in conduction band or in valence band.
- This is because of electrons in the top most levels of valence band absorb energy and move to conduction band.
- The electrons cannot spend more time there and hence they return to a lower energy level in valence band.
- The electrons will be under constant excitation and de-excitation process.

Fermi level in Intrinsic Semiconductor

Thus the average energy of the electron lies at the center of the energy gap and called Fermi energy.

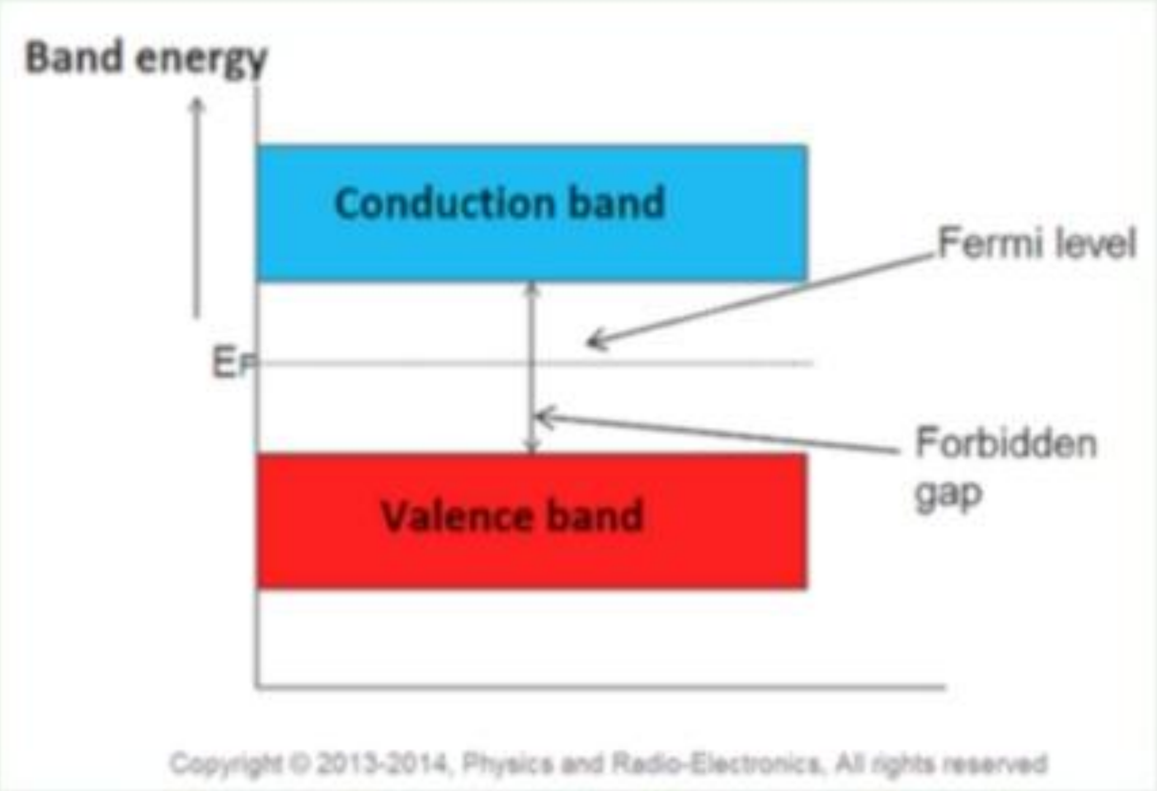


FIGURE: Fermi Level in Intrinsic Semiconductors

Fermi level in Extrinsic Semiconductor

In case of extrinsic semiconductors the Fermi energy lies towards conduction band in n-type and towards valence band in p-type semiconductors. Fermi level depends on temperature and changes with temperature.

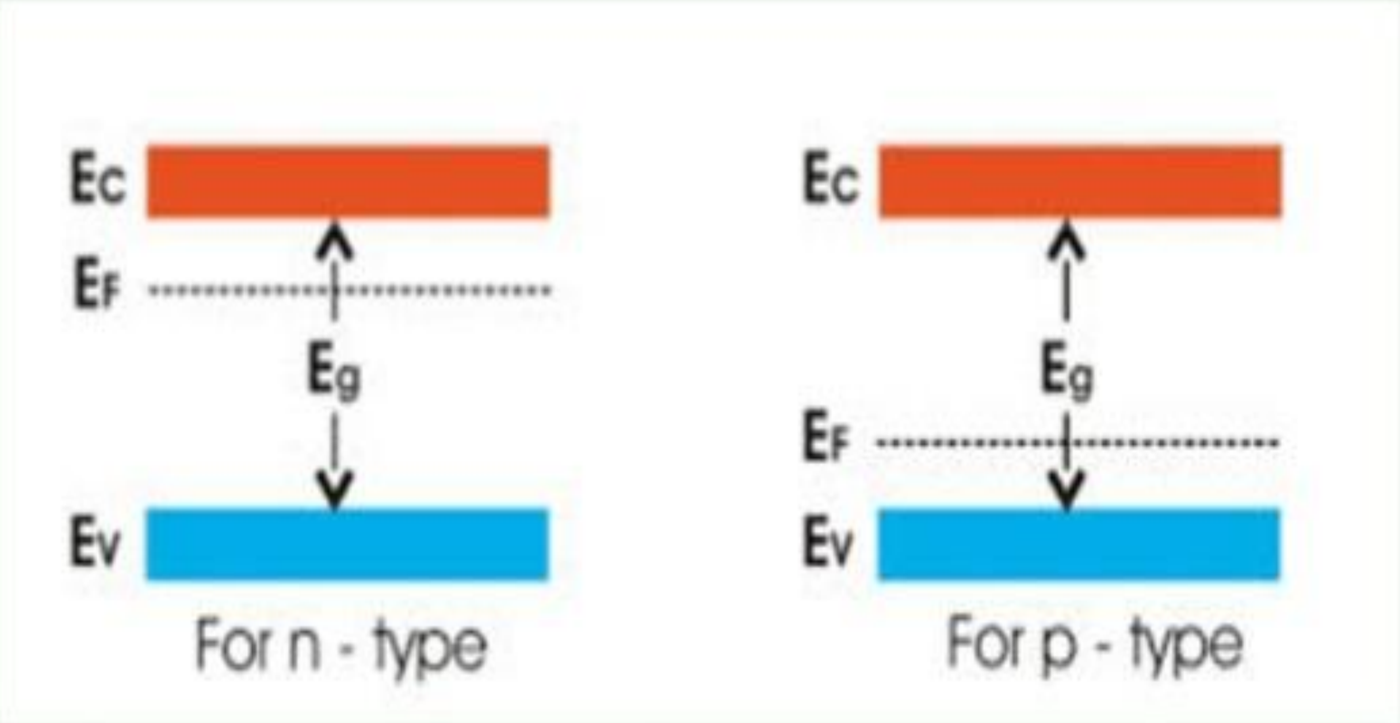


Figure: Fermi Level in Extrinsic semiconductors

Carrier Concentration in Intrinsic Semiconductors

- When sufficient energy is supplied to the intrinsic semiconductor the electrons in the valence band they absorb energy and move to conduction band leaving behind a hole.
- The number of electrons in the conduction band is equal to number of holes in the valence band.
- The total number of electrons in the conduction band per unit volume of the semiconductor is known as electron concentration.
- Similarly number of holes in the valance band per unit volume of the semiconductor is known as hole concentration.
- The total number of charge carriers per unit volume in the semiconductor is called carrier concentration.
- In case of pure semiconductors electron concentration and hole concentration are equal.

Expression for Electron concentration (N_e)

The following equation determines the electron concentration in intrinsic semiconductors.

$$N_e = \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{\frac{3}{2}} e^{\left(\frac{E_F - E_g}{kT}\right)} \quad (1)$$

Here
h is Planck's constant
m_e^* is the effective mass of the electron
k is Boltzmann constant
T is absolute temperature
E_F is Fermi energy
E_g is the energy gap of the semiconductor.

Expression for Hole concentration (N_e)

The following equation 2 determines the electron concentration in intrinsic semiconductors.

$$N_h = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{\frac{3}{2}} e^{\left(\frac{-E_F}{kT}\right)} \tag{2}$$

Here

h is Planck's constant

m_h^* is the effective mass of the electron

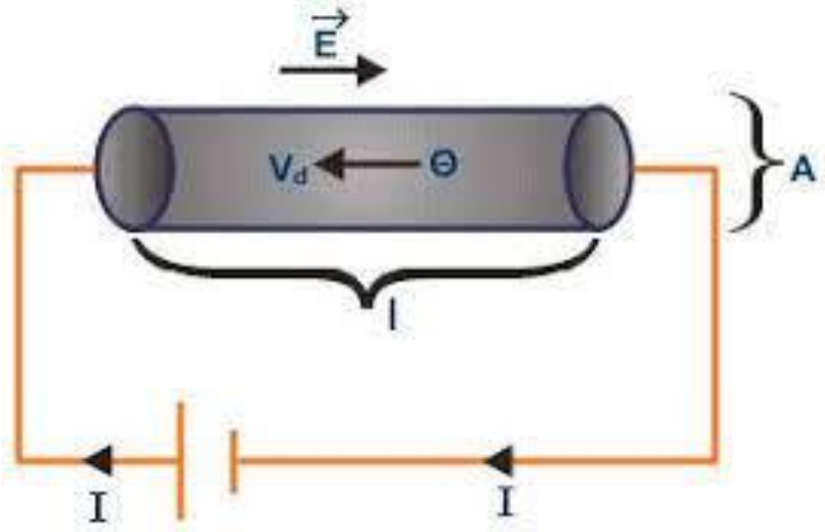
k is Boltzmann constant

T is absolute temperature

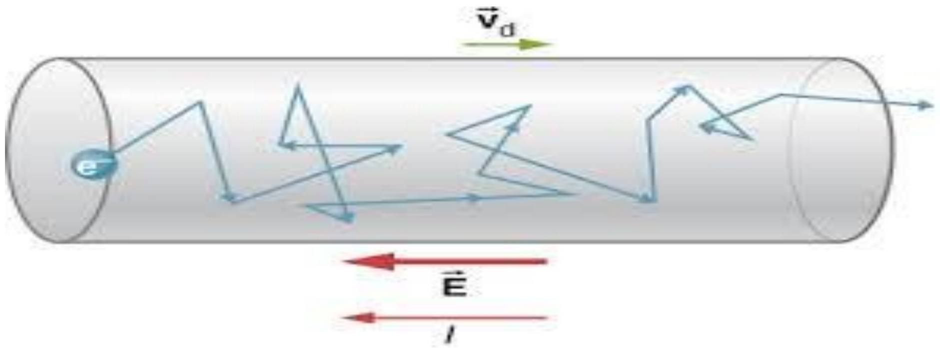
E_F is Fermi energy

E_g is the energy gap of the semiconductor.

Current density: defined as **the amount of electric current flowing through a unit cross-sectional area.** It is a vector quantity. The SI unit of electric current density is ampere per square meter.

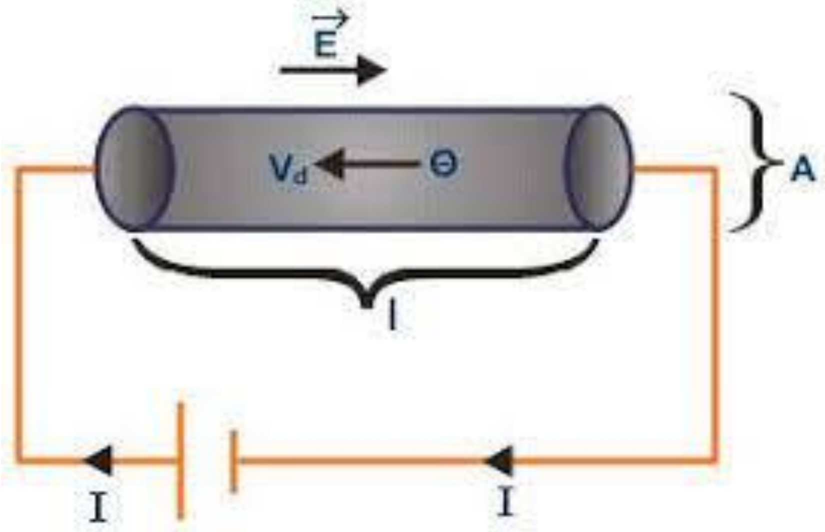


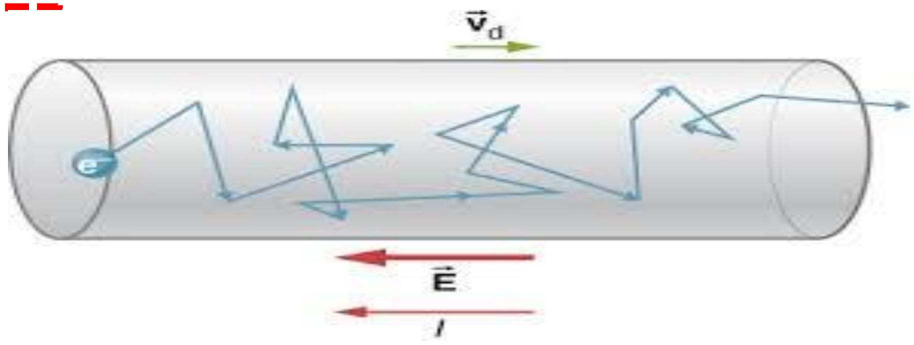
Drift velocity is **the average velocity with which electrons 'drift' in the presence of an electric field.** It's the drift velocity (or drift speed) that contributes to the electric current.



The measurement of how fast an electron can move through a semiconductor or a metal which is under the influence of an external electric field is known as electron mobility. We can show electron mobility mathematically by the equation,

$\mu = v/E$.





Expression for Electrical conductivity

The conductivity in semiconductors is due two kinds of charge carries electrons and holes. Let us calculate the contribution of electrons for conduction. Consider a semiconductor of area of cross section A and carrying current I. Let us assume that the current is only due to electrons. let v be the drift velocity of the electrons. Let N_e is the electron concentration. The current through the semi conductor is given by

$$I = \frac{q}{T}$$

Here q is the amount of charge crossing the given cross section in T second. In unit time the distance l traveled by the electron will be numerically equal to drift velocity (v). Thus the number of electrons in the volume swept by the electrons in one second is the rate of flow of charge. The volume swept by the electrons in one second is given by Av. Since e is the charge on electron then, rate of flow of charge is given by

$$I = N_e eAv \quad (1)$$

Activate W

Intrinsic Semiconductors

We know that the expression for the current density is given by $j = \frac{I}{A}$. Thus the expression for current density is given by

$$j = N_e e v \tag{2}$$

The drift velocity v is related to the mobility of the electrons by the equation $v = \mu_e E$. Here μ_e is the mobility of electrons in semiconductor. E is the applied electric field strength. Thus substituting for v in equation 2 we get

$$j = N_e e \mu_e E \tag{3}$$

It is also known that

$$j = \sigma_e E \tag{4}$$

Here σ_e conductivity due to electrons.

Expression for Electrical Conductivity

Comparing equations 3 and 4 we get

$$\sigma_e = N_e e \mu_e \tag{5}$$

Extending the same treatment for the conduction of holes we get an expression for electrical conductivity due to holes as

$$\sigma_h = N_h e \mu_h \tag{6}$$

Here μ_h is the mobility of holes. Thus the electrical conductivity of the semiconductor σ is given

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = N_e e \mu_e + N_h e \mu_h$$

$$\sigma = e(N_e \mu_e + N_h \mu_h) \tag{7}$$

Expression for Electrical Conductivity

The equation 7 determines the electrical conductivity of semiconductors in general.For an intrinsic semiconductors, we know that $N_e = N_h = n_i$. Thus the expressions for electrical conductivity of intrinsic semiconductor is given by

$$\sigma = n_i e(\mu_e + \mu_h) \tag{8}$$

Thus the expression for electrical conductivity in extrinsic and intrinsic semiconductors.

Hall Effect - Introduction

1. This was observed first by E. H. Hall in the year 1879.
2. When electric current is passed through a conductor or semiconductor placed in a magnetic field, a potential difference proportional to the current and to the magnetic field is developed across the material in a direction perpendicular to both the current and to the magnetic field. This effect is known as the Hall effect.
3. The potential difference developed in a direction perpendicular to both current and magnetic field is called Hall voltage and the corresponding electric field is called Hall field.

Hall Effect - Introduction

1. Consider a conductor carrying current along +ve X axis. Magnetic field is applied along +ve Z axis. Thus electrons experience Lorentz force along the -ve Y axis.
2. the electrons accumulate on the lower surface which results in the accumulation of +ve charges on the upper surface and hence electric field is developed between the surfaces.

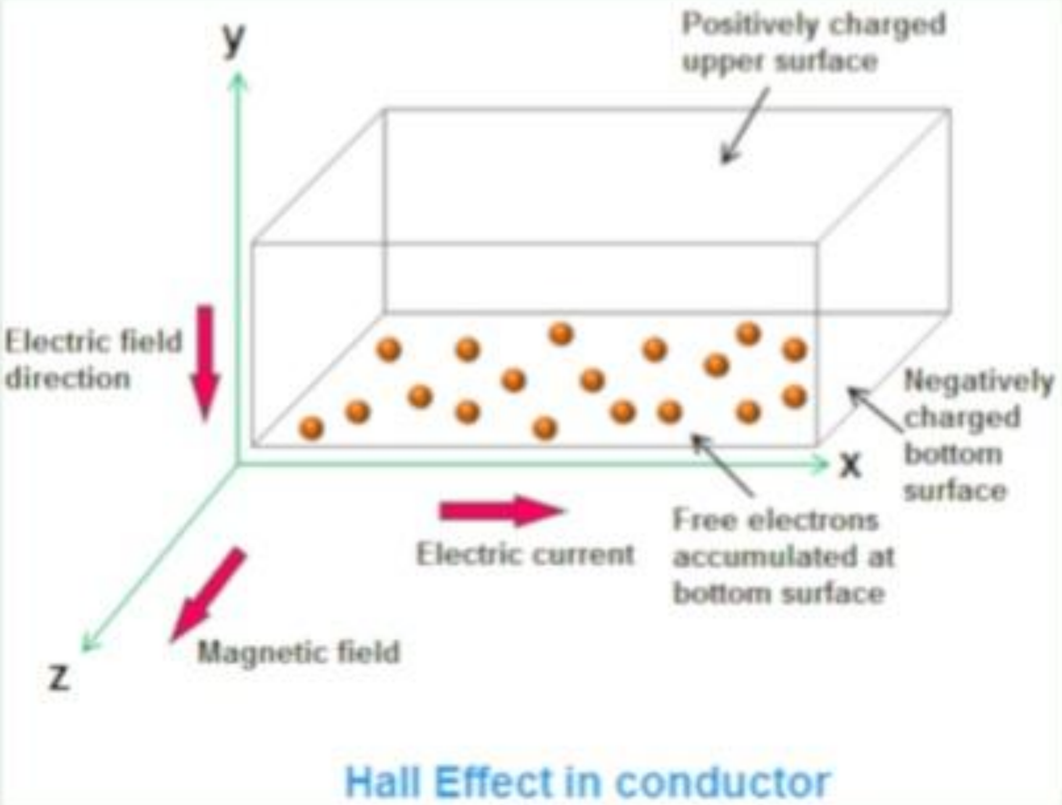


Figure: Hall effect in conductors

Hall Effect - Explanation

1. The force on the electrons due to electric field further apposes the force due to magnetic field.
2. A stage is reached where in both the forces are equal and an equilibrium state is reached. The electric field developed across the material at equilibrium is called Hall Field.

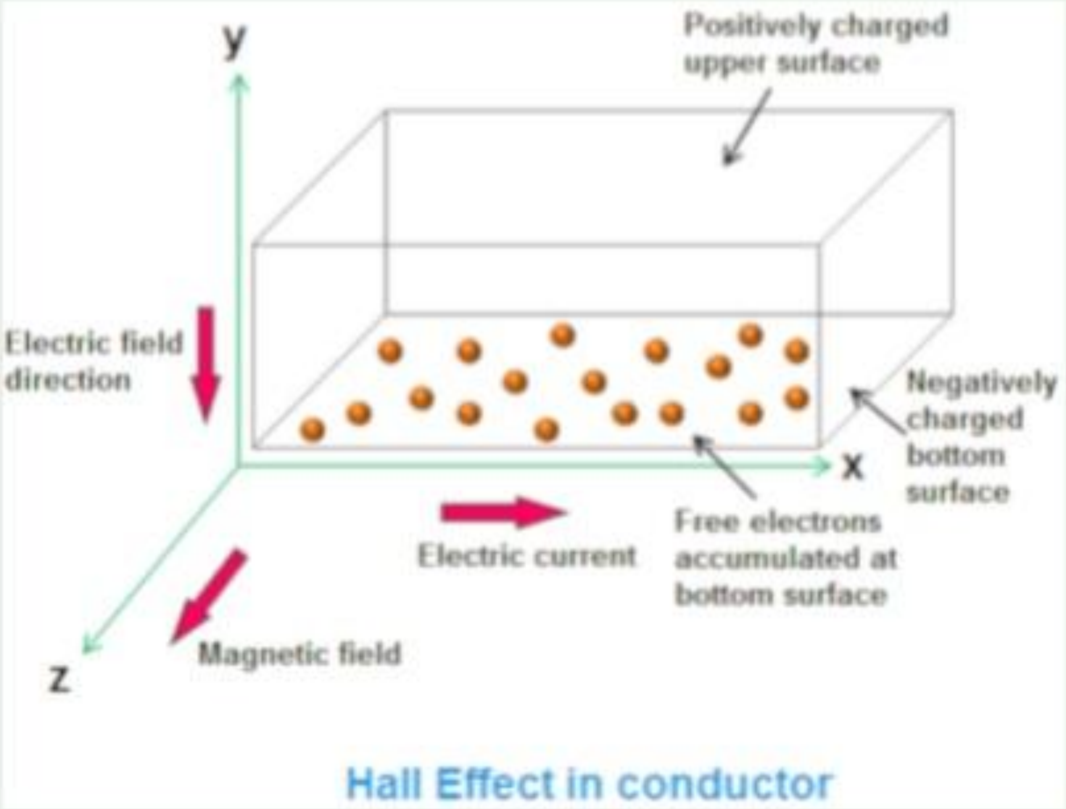


FIGURE: Hall effect in conductors

Hall Effect - Expression for Hall Co-efficient

Under equilibrium the Lorentz force on electrons is equal to force due to hall field

$$eE_H = Bev \tag{9}$$

Here e is electronic charge, E_H is Hall Field, B is Magnetic field, v is the velocity of electrons. The current density J is given by

$$J = nev \tag{10}$$

Here n is the numbmer density of charges. Dividing equation 9 by 10 we get

$$\frac{eE_H}{J} = \frac{Bev}{nev} \tag{11}$$

$$\frac{eE_H}{J} = \frac{B}{n} \tag{12}$$

Hall Effect - Expression for Hall Co-efficient

$$E_H = \frac{BJ}{ne} \tag{13}$$

$$E_H = R_H BJ \tag{14}$$

Here R_H is called Hall coefficient and is given by

$$R_H = \frac{1}{ne} \tag{15}$$

The Hall voltage is given by the equation

$$V_H = E_H d = R_H BJd \tag{16}$$

Here d is the thickness of the material along y-axis. **Note:** In case of n-type semiconductors R_H is negative and for p-type semiconductors R_H is positive.

Thank You

Dept. of Physics,
ATMECE, Mysuru